

Basic

Z.R. Bhatti

GROUP THEORY

For

B.A./B.Sc.

BS 4-Years

M.Sc. Mathematics

Third Edition



ILMI KITAB KHANA

Kabir Street, Urdu Bazar, Lahore.

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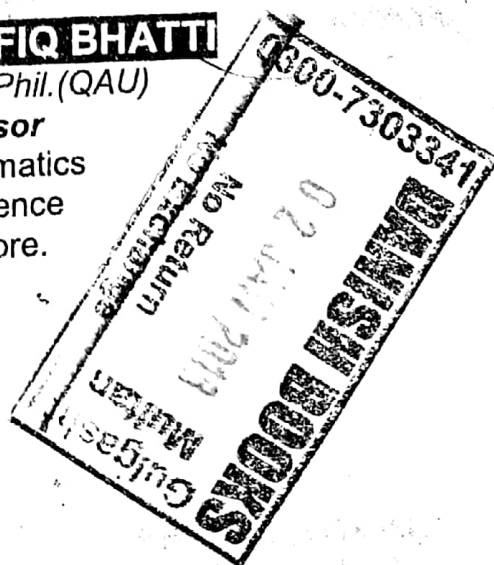
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MUHAMMAD ZAHID RAFIQ BHATTI

M.Sc.(PU), B.Ed.(PU), M.Phil.(QAU)

Associate Professor

Department of Mathematics
Govt. College of Science
Wahdat Road, Lahore.



ILMI KITAB KHANA

Kabir Street, Urdu Bazar, Lahore, Ph: 042-37353510, 37248129

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ILMI KITAB KHANA

Kabir Street, Urdu Bazar, Lahore.

Ph. 042-37353510, 042-37248129

Website: ilmikitabkhana.com



BASIC GROUP THEORY



DEDICATION

*I dedicate this humble work to my
esteemed teacher*

Mr. Amjad Ali

*who guided me in my school education
devotedly and it is due to his endeavour
that I could do some work*

Preface

(Third Edition)

The book is revised thoroughly keeping in mind the paper patterns of B.A./B.Sc., BS 4-Years, and M.Sc. Mathematics of University of the Punjab, and different universities of Pakistan. Each exercise is divided into three parts. First part of each exercise consists of MCQs, second part consists of Short Questions and third part consists of Long Questions.

Keeping in mind the difficulties of students of BS Mathematics, as they have no proper material covering their syllabus, I tried my best to include their topics in detail.

All the topics covering the syllabi of B.A./B.Sc., BS Mathematics (Semesters IV and V) and M.Sc. Mathematics are given in 8 chapters.

The details of chapters covering the syllabi of above three different disciplines are given below:

S. No.	Discipline	Chapters
1	B.A./B.Sc.	1-3
2	BS 4-Years (Semester IV)	1-3
3	BS 4-Years (Semester V)	3-8
4	M.Sc. Mathematics	1-8

I wish to thank to all my friends and colleagues for their appreciation and valuable suggestions.

Lahore
December, 2013

Z. R. BHATTI
Associate Professor
Department of Mathematics
Govt. College of Science
Wahdat Road, Lahore.

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SETS, FUNCTIONS AND BINARY OPERATIONS

Chapter

1

Set theory has been usefully employed in various branches of natural sciences. It forms a basis of all the fundamental concepts of mathematics specially, topology, real analysis, functional analysis, algebra and group theory. Set theory also plays a key role in solving the problems of applied mathematics, physics, chemistry and many other scientific disciplines. It is obviously impossible to give a complete discussion on various aspects of set theory and its uses in the different scientific disciplines in this elementary book. However, some basic concepts concerning to group theory are presented in this introductory chapter.

1-1 Definitions and Set Operations

1-1.1 Definition: The collection of well defined objects is called a *set*. The well defined objects of this collection are said to be the *elements* or *points* of the set. Sets are denoted by capital letters while their elements are denoted by small letters. If x is an element of the set A , we write $x \in A$ and read ' x belongs to A '. If x is not an element of a set A , we write $x \notin A$ and read ' x does not belong to A '.

If A is a set and P is a statement which applies to some of the elements of A , then the set of elements x of A for which $P(x)$ is true is denoted by $\{x \in A : P(x)\}$.

Thus if N is the set of positive integers, the positive divisors of 16 form the set $\{x \in N : xy = 16, \text{ for some } y \in N\}$. In the case of small sets it is easy to describe the set by listing its elements in brackets. Thus the set just given above is the set $\{1, 2, 4, 8, 16\}$. The three ways of writing sets are given in the following definitions.