CARS MISSING DATA WORKSHOP ANALYSES

LOAD LIBRARIES

```r
# install.packages('summarytools')
# install.packages('ggplot2')
# install.packages('psych')
# install.packages('lavaan')
# install.packages('semTools')
# install.packages('mice')
# install.packages('remotes')
# remotes::install_github('bkeller2/fdir')
# requires blimp installation from www.appliedmissingdata.com/blimp
# remotes::install_github('blimp-stats/rblimp')

library(fdir)
library(ggplot2)
library(summarytools)
library(psych)
library(lavaan)
library(semTools)
library(rblimp)
library(mice)
```

READ DATA AND RECODE VARIABLES

```r
set()
carsdat <- read.csv('CARS Data.csv')
```

```r
# Recode admit type
carsdat$admit_type <- factor(carsdat$admit_type)
carsdat$admit_type[carsdat$admit_type == 'RTN'] <- NA
carsdat$admit_type <- droplevels(carsdat$admit_type)
carsdat$admit_type_num <- as.numeric(carsdat$admit_type)

# Recode gender
carsdat$gender <- factor(carsdat$gender)
carsdat$gender[carsdat$gender == 'P' | carsdat$gender == 'U'] <- NA
carsdat$gender <- droplevels(carsdat$gender)

# Create binary gender dummy code (male) and separate complete code for 'other' (nonbinary)
carsdat$nonbinary <- with(carsdat,ifelse(is.na(gender) | gender %in% c('F','M'), 0, 1))
carsdat$male <- ifelse(carsdat$gender == 'M', 1, ifelse(carsdat$gender == 'F', 0, NA_real_))
```

INSPECT DATA

Missing data handling procedures require distributional assumptions for incomplete variables. We typically try to preserve the associations among variables whenever possible. Estimating associations between pairs of categorical variables requires sufficient data in every cell combination. Screening variables by examining crosstab tables and group-level descriptive statistics can identify associations that lack support from the data.

```r
# Define variable sets
vars2analyze <- c('info_t1','info_t2','info_t3')
auxvars <- c('extra_t3','cont_t3','om_t3','ag_t3','ne_t3')

# Frequency distributions (summarytools package)
freq(carsdat$admit_type)
```

```
## Frequencies
## carsdat$admit_type
## Type: Factor
##
##               Freq   % Valid   % Valid Cum.   % Total   % Total Cum.
## ----------- ------ --------- -------------- --------- --------------
##         FYR   5717     86.00          86.00     85.00          85.00
##         TRN    931     14.00         100.00     13.84          98.84
##        <NA>     78                               1.16         100.00
##       Total   6726    100.00         100.00    100.00         100.00
```

```r
freq(carsdat$gender)
```

```
## Frequencies
## carsdat$gender
## Type: Factor
##
##               Freq   % Valid   % Valid Cum.   % Total   % Total Cum.
## ----------- ------ --------- -------------- --------- --------------
##           F   3744     56.30          56.30     55.66          55.66
##           M   2894     43.52          99.82     43.03          98.69
##           O     12      0.18         100.00      0.18          98.87
##        <NA>     76                               1.13         100.00
##       Total   6726    100.00         100.00    100.00         100.00
```

```r
freq(carsdat$nonbinary)

## Frequencies
## carsdat$nonbinary
## Type: Numeric
##
##               Freq   % Valid   % Valid Cum.   % Total   % Total Cum.
## ----------- ------ --------- -------------- --------- --------------
##           0   6714     99.82          99.82     99.82          99.82
##           1     12      0.18         100.00      0.18         100.00
##        <NA>      0                              0.00         100.00
##       Total   6726    100.00         100.00    100.00         100.00

freq(carsdat$male)

## Frequencies
## carsdat$male
## Type: Numeric
##
##               Freq   % Valid   % Valid Cum.   % Total   % Total Cum.
## ----------- ------ --------- -------------- --------- --------------
##           0   3744     56.40          56.40     55.66          55.66
##           1   2894     43.60         100.00     43.03          98.69
##        <NA>     88                              1.31         100.00
##       Total   6726    100.00         100.00    100.00         100.00
```

The three-categorical `gender` variable has low cell counts with `admit_type`. These variables could not be used in the same analysis because their correlation is likely inestimable.

```
ctable(carsdat$admit_type, carsdat$gender)

## Cross-Tabulation, Row Proportions
## admit_type * gender
## Data Frame: carsdat
##
## ------------ -------- -------------- -------------- ----------- ------------ ---------------
##                gender              F              M           O         <NA>           Total
##   admit_type
##          FYR            3240 (56.7%)   2443 (42.7%)    8 (0.1%)   26 ( 0.5%)   5717 (100.0%)
##          TRN             486 (52.2%)    436 (46.8%)    4 (0.4%)    5 ( 0.5%)    931 (100.0%)
##         <NA>              18 (23.1%)     15 (19.2%)    0 (0.0%)   45 (57.7%)     78 (100.0%)
##        Total            3744 (55.7%)   2894 (43.0%)   12 (0.2%)   76 ( 1.1%)   6726 (100.0%)
## ------------ -------- -------------- -------------- ----------- ------------ ---------------
```

In contrast, the `admit_type` by `male` dummy code crosstab table has sufficient coverage in each cell.

```
ctable(carsdat$admit_type, as.factor(carsdat$male))

## Cross-Tabulation, Row Proportions
## admit_type * as.factor(x = carsdat$male)
##
## ------------ ----------------------------- -------------- -------------- ------------ ---------------
##                as.factor(x = carsdat$male)              0              1         <NA>           Total
##   admit_type
##          FYR                                 3240 (56.7%)   2443 (42.7%)   34 ( 0.6%)   5717 (100.0%)
##          TRN                                  486 (52.2%)    436 (46.8%)    9 ( 1.0%)    931 (100.0%)
##         <NA>                                   18 (23.1%)     15 (19.2%)   45 (57.7%)     78 (100.0%)
##        Total                                 3744 (55.7%)   2894 (43.0%)   88 ( 1.3%)   6726 (100.0%)
## ------------ ----------------------------- -------------- -------------- ------------ ---------------
```

The group-specific descriptive statistics show that `info_t1` is essentially completely missing for the TRN group. As such, the `admit_type` and `info_t1` variables cannot be used in the same model because their bivariate association is inestimable. Including them in the same analysis would require a specification that specifically omits their bivariate association.

```
# Descriptive statistics by group
describeBy(carsdat[,c(vars2analyze,auxvars)], group = carsdat$admit_type)
```

```
##  Descriptive statistics by group
## group: FYR
##          vars    n  mean    sd median trimmed   mad   min max range  skew kurtosis   se

## info_t1     1 4425 67.04 15.10  70.00   68.90  9.88  6.67 100 93.33 -1.14     1.24 0.23
## info_t2     2 3003 64.63 19.11  70.00   66.61 14.83 10.00 100 90.00 -0.86    -0.18 0.35
## info_t3     3 3520 69.26 17.26  73.33   71.58 14.83  6.67 100 93.33 -1.16     0.86 0.29
## extra_t3    4 3520  3.35  0.65   3.33    3.36  0.74  1.17   5  3.83 -0.07    -0.09 0.01
## cont_t3     5 3520  3.53  0.66   3.50    3.52  0.74  1.50   5  3.50 -0.01    -0.47 0.01
## om_t3       6 3520  3.63  0.62   3.67    3.62  0.74  1.50   5  3.50  0.06    -0.45 0.01
## ag_t3       7 3520  3.72  0.59   3.67    3.71  0.74  1.17   5  3.83 -0.11    -0.49 0.01
## ne_t3       8 3520  2.77  0.67   2.83    2.77  0.74  1.00   5  4.00  0.06     0.08 0.01
## ------------------------------------------------------------
## group: TRN
##          vars   n  mean    sd median trimmed   mad   min    max range  skew kurtosis   se
## info_t1     1   2 71.67  2.36  71.67   71.67  2.47 70.00  73.33  3.33  0.00    -2.75 1.67
## info_t2     2 437 62.93 18.60  66.67   64.71 19.77 10.00  93.33 83.33 -0.78    -0.31 0.89
## info_t3     3 704 67.76 16.93  70.00   69.79 14.83 13.33 100.00 86.67 -1.04     0.62 0.64
## extra_t3    4 704  3.27  0.64   3.17    3.27  0.49  1.17   5.00  3.83  0.02     0.12 0.02
## cont_t3     5 704  3.46  0.65   3.50    3.46  0.74  1.67   5.00  3.33  0.02    -0.52 0.02
## om_t3       6 704  3.64  0.61   3.58    3.62  0.62  1.83   5.00  3.17  0.16    -0.46 0.02
## ag_t3       7 704  3.71  0.60   3.67    3.70  0.74  1.83   5.00  3.17 -0.03    -0.80 0.02
## ne_t3       8 704  2.79  0.70   2.83    2.79  0.74  1.00   5.00  4.00  0.05    -0.07 0.03
```

```r
describeBy(carsdat[,c(vars2analyze,auxvars)], group = carsdat$gender)
```

```
##  Descriptive statistics by group
## group: F
##          vars    n  mean    sd median trimmed   mad   min max range  ske kurtosis   se
## info_t1     1 2554 67.95 13.83  70.00   69.59  9.88 10.00 100 90.00 -1.2     1.80 0.27
## info_t2     2 1994 66.56 17.42  70.00   68.71 14.83 10.00 100 90.00 -1.0     0.48 0.39
## info_t3     3 2475 70.89 15.27  73.33   73.04  9.88  6.67 100 93.33 -1.3     1.91 0.31
## extra_t3    4 2475  3.36  0.66   3.33    3.37  0.74  1.17   5  3.83 -0.1    -0.14 0.01
## cont_t3     5 2475  3.65  0.65   3.67    3.65  0.74  1.67   5  3.33 -0.1    -0.59 0.01
## om_t3       6 2475  3.68  0.61   3.67    3.68  0.74  1.67   5  3.33 -0.0    -0.34 0.01
## ag_t3       7 2475  3.85  0.56   3.83    3.86  0.49  2.00   5  3.00 -0.2    -0.54 0.01
## ne_t3       8 2475  2.88  0.67   3.00    2.88  0.74  1.00   5  4.00  0.0    -0.02 0.01
## ------------------------------------------------------------
## group: M
##          vars    n  mean    sd median trimmed   mad   min    max range  skew kurtosis   se
## info_t1     1 1841 65.65 16.58  70.00   67.58 14.83  6.67  93.33 86.67 -1.01     0.55 0.39
## info_t2     2 1422 61.26 20.73  66.67   62.70 19.77 10.00  96.67 86.67 -0.57    -0.82 0.55
## info_t3     3 1755 66.34 19.32  70.00   68.34 19.77  6.67 100.00 93.33 -0.84    -0.17 0.46
## extra_t3    4 1755  3.31  0.62   3.17    3.31  0.49  1.33   5.00  3.67  0.06     0.06 0.01
## cont_t3     5 1755  3.34  0.63   3.33    3.33  0.49  1.50   5.00  3.50  0.13    -0.04 0.02
## om_t3       6 1755  3.56  0.63   3.50    3.53  0.74  1.50   5.00  3.50  0.25    -0.46 0.02
## ag_t3       7 1755  3.53  0.58   3.50    3.51  0.74  1.17   5.00  3.83  0.15    -0.30 0.01
## ne_t3       8 1755  2.62  0.65   2.67    2.63  0.49  1.00   5.00  4.00 -0.01     0.07 0.02
## ------------------------------------------------------------
## group: O
##          vars n  mean   sd median trimmed  mad   min   max range  skew kurtosis   se
## info_t1     1 8 80.83 9.88  80.00   80.83 9.88 66.67 96.67 30.00  0.16    -1.46 3.49
## info_t2     2 6 82.78 6.80  81.67   82.78 4.94 73.33 93.33 20.00  0.19    -1.39 2.78
## info_t3     3 7 79.52 4.48  80.00   79.52 4.94 73.33 86.67 13.33  0.22    -1.44 1.69
## extra_t3    4 7  2.55 0.97   2.83    2.55 1.24  1.33  4.00  2.67  0.05    -1.69 0.37
## cont_t3     5 7  3.00 0.62   2.67    3.00 0.49  2.33  3.83  1.50  0.39    -1.82 0.24
## om_t3       6 7  4.29 0.33   4.33    4.29 0.25  3.83  4.83  1.00  0.23    -1.29 0.12
## ag_t3       7 7  3.64 0.91   3.83    3.64 0.74  1.83  4.33  2.50 -0.96    -0.61 0.34
## ne_t3       8 7  3.69 0.92   3.50    3.69 0.74  2.67  5.00  2.33  0.37    -1.76 0.35
```

```r
describeBy(carsdat[,c(vars2analyze,auxvars)], group = carsdat$male)
```

```
##  Descriptive statistics by group
## group: 0
##          vars    n  mean    sd median trimmed   mad   min max range  skew kurtosis   se
## info_t1     1 2554 67.95 13.83  70.00   69.59  9.88 10.00 100 90.00 -1.21     1.80 0.27
## info_t2     2 1994 66.56 17.42  70.00   68.71 14.83 10.00 100 90.00 -1.05     0.48 0.39
## info_t3     3 2475 70.89 15.27  73.33   73.04  9.88  6.67 100 93.33 -1.35     1.91 0.31
## extra_t3    4 2475  3.36  0.66   3.33    3.37  0.74  1.17   5  3.83 -0.12    -0.14 0.01
## cont_t3     5 2475  3.65  0.65   3.67    3.65  0.74  1.67   5  3.33 -0.14    -0.59 0.01
## om_t3       6 2475  3.68  0.61   3.67    3.68  0.74  1.67   5  3.33 -0.03    -0.34 0.01
## ag_t3       7 2475  3.85  0.56   3.83    3.86  0.49  2.00   5  3.00 -0.24    -0.54 0.01
## ne_t3       8 2475  2.88  0.67   3.00    2.88  0.74  1.00   5  4.00  0.07    -0.02 0.01
## ------------------------------------------------------------
## group: 1
##          vars    n  mean    sd median trimmed   mad   min    max range  skew kurtosis   se
## info_t1     1 1841 65.65 16.58  70.00   67.58 14.83  6.67  93.33 86.67 -1.01     0.55 0.39
## info_t2     2 1422 61.26 20.73  66.67   62.70 19.77 10.00  96.67 86.67 -0.57    -0.82 0.55
## info_t3     3 1755 66.34 19.32  70.00   68.34 19.77  6.67 100.00 93.33 -0.84    -0.17 0.46
## extra_t3    4 1755  3.31  0.62   3.17    3.31  0.49  1.33   5.00  3.67  0.06     0.06 0.01
## cont_t3     5 1755  3.34  0.63   3.33    3.33  0.49  1.50   5.00  3.50  0.13    -0.04 0.02
## om_t3       6 1755  3.56  0.63   3.50    3.53  0.74  1.50   5.00  3.50  0.25    -0.46 0.02
## ag_t3       7 1755  3.53  0.58   3.50    3.51  0.74  1.17   5.00  3.83  0.15    -0.30 0.01
## ne_t3       8 1755  2.62  0.65   2.67    2.63  0.49  1.00   5.00  4.00 -0.01     0.07 0.02
```

ML DESCRIPTIVE STATISTICS

This section illustrates descriptive statistics with and without auxiliary variables. The conditionally MAR assumption is defined with respect to the observed variables in each model. `lavaan` has no missing data handling for categorical variables (WLS is not a desirable option because it assumes an unsystematic MCAR mechanism). This analysis uses the `male` dummy code as an auxiliary variable (0 = female, 1 = male, NA = else). Binary variables must be treated as normally distributed for missing data handling. Conceptually, this creates imputes that are decimals rather than 0s and 1s.

```
# Define primary and auxiliary variable sets
vars2analyze <- c('info_t1','info_t2','info_t3')
auxvars <- c('extra_t3','cont_t3','om_t3','ag_t3','ne_t3','male')
```

This function uses the mice package to get a table of missing data patterns. The table denotes 1 = observed, 0 = missing.

```
# Missing data patterns
md.pattern(carsdat[, c(vars2analyze)], plot = FALSE)

##      info_t1 info_t3 info_t2
## 1909       1       1       1    0
## 895        1       1       0    1
## 397        1       0       1    1
## 1226       1       0       0    2
## 430        0       1       1    1
## 1068       0       1       0    2
## 704        0       0       1    2
## 97         0       0       0    3
##         2299    2424    3286 8009

md.pattern(carsdat[, c(vars2analyze,auxvars)], plot = FALSE)

##      male info_t1 info_t3 extra_t3 cont_t3 om_t3 ag_t3 ne_t3 info_t2
## 1898    1       1       1        1       1     1     1     1       1    0
## 887     1       1       1        1       1     1     1     1       0    1
## 389     1       1       0        0       0     0     0     0       1    6
## 1221    1       1       0        0       0     0     0     0       0    7
## 428     1       0       1        1       1     1     1     1       1    1
## 1017    1       0       1        1       1     1     1     1       0    2
## 701     1       0       0        0       0     0     0     0       1    7
## 97      1       0       0        0       0     0     0     0       0    8
```

```
## 11      0       1       1        1       1     1     1     1       1     1
## 8       0       1       1        1       1     1     1     1       0     2
## 8       0       1       0        0       0     0     0     0       1     7
## 5       0       1       0        0       0     0     0     0       0     8
## 2       0       0       1        1       1     1     1     1       1     2
## 50      0       0       1        1       1     1     1     1       0     3
## 1       0       0       1        0       0     0     0     0       0     8
## 3       0       0       0        0       0     0     0     0       1     8
##         88    2299    2424     2425    2425  2425  2425  2425    3286 20222
```

This code constructs syntax for a saturated model from the `vars2analyze` vector. No modification is needed to the code.

```
# Model includes all possible means, variances, and covariances among the primary variable set
mean_lines <- paste0(vars2analyze, ' ~ 1')
var_lines <- paste0(vars2analyze, ' ~~ ', vars2analyze)
covar_lines <- combn(vars2analyze, 2, FUN = function(x) paste(x[1], '~~', x[2]))
syntax_saturated <- paste(c(mean_lines, var_lines, covar_lines), collapse = '\n')
```

The `missing = ml` option invokes full information maximum likelihood estimation. `lavaan` issues a warning that several cases have no data on the analysis variables.

```
fiml_saturated <- sem(
  model = syntax_saturated,
  data = carsdat,
  missing = 'ml')

## Warning: lavaan->lav_data_full():
##    some cases are empty and will be ignored: 114 227 331 390 416 427 454 463
##    474 497 548 560 606 625 665 761 865 891 901 1107 1118 1137 1180 1411 1437
##    1564 1643 1738 1950 2002 2205 2239 2253 2299 2301 2514 2868 2870 3005 3009
##    3079 3128 3129 3130 3133 3135 3206 3253 3504 3587 3588 3592 3594 3595 3714
##    3722 3930 3931 3933 4094 4137 4263 4379 4498 4591 4971 4972 4981 5058 5203
##    5205 5206 5209 5360 5446 5455 5456 5457 5458 5462 5672 5725 5726 5728 5758
##    5821 5822 5830 5908 5967 6028 6073 6116 6260 6546 6547 6549.
```

```r
summary(fiml_saturated, standardized = TRUE)

## lavaan 0.6-19 ended normally after 74 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                         9
##
##                                                   Used       Total
##   Number of observations                          6629        6726
##   Number of missing patterns                         7
##
## Model Test User Model:
##
##   Test statistic                                 0.000
##   Degrees of freedom                                 0
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Observed
##   Observed information based on                Hessian
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   info_t1 ~~
##     info_t2         123.599    5.849   21.131    0.000  123.599    0.428
##     info_t3         104.252    4.801   21.716    0.000  104.252    0.401
##   info_t2 ~~
##     info_t3         160.102    6.457   24.795    0.000  160.102    0.483
##
```

```
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     info_t1         67.005    0.219  305.349    0.000   67.005    4.455
##     info_t2         63.804    0.309  206.159    0.000   63.804    3.324
##     info_t3         68.814    0.254  270.967    0.000   68.814    3.986
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     info_t1        226.222    4.757   47.558    0.000  226.222    1.000
##     info_t2        368.476    8.875   41.518    0.000  368.476    1.000
##     info_t3        298.051    6.422   46.412    0.000  298.051    1.000
```

The `sem.auxiliary` function uses Graham's (2001) saturated correlates model. 1) All auxiliary variables correlate with each other. 2) Auxiliary variables correlate with exogenous predictors. 3) Auxiliary variables correlate with residuals of all outcomes. The analysis summary table will include these additional parameters, which can be ignored. Binary dummy codes like `male` must be treated as normally distributed auxiliary variables.

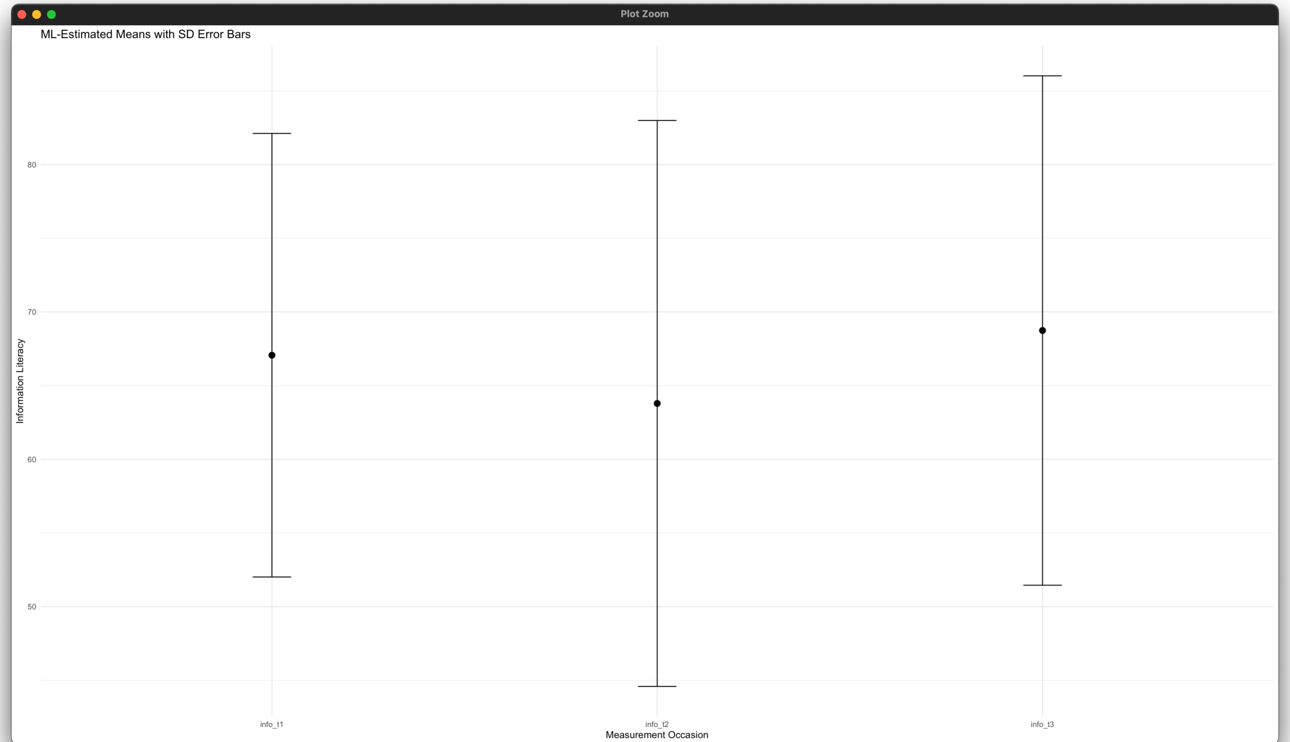
```r
# Maximum likelihood estimation with auxillary variables
fiml_saturated_aux <- sem.auxiliary(
  model = syntax_saturated,
  data = carsdat,
  fixed.x = FALSE,
  aux = auxvars)
summary(fiml_saturated_aux, standardized = TRUE)
```

The following code extracts `lavaan` means and standard deviations for graphing. The code uses the table of `lavaan` estimates to construct a data frame containing the means and standard deviations. Estimates are stored in `parameterEstimates()`.

```
est <- parameterEstimates(fiml_saturated_aux)
means <- est[ est$op == '~1', c('lhs','est') ]
colnames(means) <- c('variable','mean')
vars  <- est[ est$op == '~~' & est$lhs == est$rhs, c('lhs','est') ]
colnames(vars)  <- c('variable','variance')
summary_stats <- merge(means, vars, by = 'variable')
summary_stats$sd <- sqrt(summary_stats$variance)
summary_stats <- summary_stats[ summary_stats$variable %in% vars2analyze, ]
```

The `ggplot2` function produces a graph of the means with standard deviation error bars.

```
# Plot means and standard deviation error bars.
ggplot(summary_stats, aes(x = variable, y = mean)) +
  geom_point(size = 3) +
  geom_errorbar(aes(ymin = mean - sd, ymax = mean + sd), width = 0.1) +
  labs(
    x = 'Measurement Occasion',
    y = 'Information Literacy',
    title = 'ML-Estimated Means with SD Error Bars'
  ) + theme_minimal()
```



ML REPEATED MEASURES ANALYSIS

This section illustrates a repeated measures analysis with and without auxiliary variables. The conditionally MAR assumption is defined with respect to the observed variables in each model. The `lavaan` function has no missing data handling for categorical variables (WLS is not a desirable option because it assumes an unsystematic MCAR mechanism). This analysis uses the `male` dummy code as an auxiliary variable (0 = female, 1 = male, NA = else). Binary variables must be treated as normally distributed for missing data handling. Conceptually, this creates imputes that are decimals rather than 0s and 1s.

```r
# Define auxiliary variables for use with sem.auxiliary function
auxvars <- c('extra_t3','cont_t3','om_t3','ag_t3','ne_t3','male')
```

A latent variable named `subjects` represents the random subjects factor in a within-subjects design. Constraints on residual variance create the compound symmetry assumption, which can be relaxed. Means are labeled and used to obtain a custom ANOVA-style significance test.

```r
# Repeated measures syntax
syntax_repeated <- '
  # Random subjects factor with mean = 0 and loadings = 1
  subjects =~ 1*info_t1 + 1*info_t2 + 1*info_t3
  subjects ~ 0*1
  # Label the means
  info_t1 ~ mu1*1
  info_t2 ~ mu2*1
  info_t3 ~ mu3*1
  # Equal residual variances gives compound symmetry assumption, relax if needed
  info_t1 ~~ res*info_t1
  info_t2 ~~ res*info_t2
  info_t3 ~~ res*info_t3
'
```

The `missing = ml` input invokes full information maximum likelihood estimation. `lavaan` issues a warning that several cases have no data on the analysis variables.

```r
# Maximum likelihood estimation
fiml_repeated <- sem(
  model = syntax_repeated,
  data = carsdat,
  missing = 'ml')
```

```
## Warning: lavaan->lav_data_full():
##    some cases are empty and will be ignored: 114 227 331 390 416 427 454 463
##    474 497 548 560 606 625 665 761 865 891 901 1107 1118 1137 1180 1411 1437
##    1564 1643 1738 1950 2002 2205 2239 2253 2299 2301 2514 2868 2870 3005 3009
##    3079 3128 3129 3130 3133 3135 3206 3253 3504 3587 3588 3592 3594 3595 3714
##    3722 3930 3931 3933 4094 4137 4263 4379 4498 4591 4971 4972 4981 5058 5203
##    5205 5206 5209 5360 5446 5455 5456 5457 5458 5462 5672 5725 5726 5728 5758
##    5821 5822 5830 5908 5967 6028 6073 6116 6260 6546 6547 6549.
```

```
summary(fiml_repeated, standardized = TRUE)
```

```
## lavaan 0.6-19 ended normally after 35 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                         7
##   Number of equality constraints                     2
##
##                                                   Used       Total
##   Number of observations                          6629        6726
##   Number of missing patterns                         7
##
## Model Test User Model:
##
##   Test statistic                               269.386
##   Degrees of freedom                                 4
##   P-value (Chi-square)                           0.000
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Observed
##   Observed information based on                Hessian
##
```

```
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   subjects =~
##     info_t1           1.000                               11.070    0.650
##     info_t2           1.000                               11.070    0.650
##     info_t3           1.000                               11.070    0.650
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     subjects          0.000                                0.000    0.000
##    .info_t1  (mu1)   66.979    0.248  270.013    0.000   66.979    3.931
##    .info_t2  (mu2)   63.946    0.277  231.155    0.000   63.946    3.753
##    .info_t3  (mu3)   68.826    0.251  273.990    0.000   68.826    4.040
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info_t1  (res)  167.707    3.106   53.986    0.000  167.707    0.578
##    .info_t2  (res)  167.707    3.106   53.986    0.000  167.707    0.578
##    .info_t3  (res)  167.707    3.106   53.986    0.000  167.707    0.578
##     subjects        122.536    4.206   29.136    0.000    1.000    1.000
```

The `sem.auxiliary` function uses Graham's (2001) saturated correlates model: 1) All auxiliary variables correlate with each other 2) Auxiliary variables correlate with exogenous predictors 3) Auxiliary variables correlate with residuals of all outcomes Analysis summary table will include these additional parameters, which can be ignored. Binary dummy codes like `male` must be treated as normally distributed auxiliary variables. `lavaan` issues a warning that several cases have no data on the analysis variables. As well, `lavaan` issues a warning that variables have very different variances (the variance of the `male` dummy code is much smaller).

```r
# Maximum likelihood estimation with auxiliary variables
fiml_repeated_aux <- sem.auxiliary(
  model = syntax_repeated,
  data = carsdat,
  aux = auxvars)
```

```
## Warning: lavaan->lav_data_full():
##    some cases are empty and will be ignored: 114 227 331 390 416 427 454 463
##    474 497 548 560 606 625 665 761 865 891 901 1107 1118 1137 1180 1411 1437
##    1564 1643 1738 1950 2002 2205 2239 2253 2299 2301 2514 2868 2870 3005 3009
##    3079 3128 3129 3130 3133 3135 3206 3253 3504 3587 3588 3592 3594 3595 3714
##    3722 3930 3931 3933 4094 4137 4263 4379 4498 4591 4971 4972 4981 5058 5203
##    5205 5206 5209 5360 5446 5455 5456 5457 5458 5462 5672 5725 5726 5728 5758
##    5821 5822 5830 5908 5967 6028 6073 6116 6260 6546 6547 6549.

## Warning: lavaan->lav_data_full():
##    some observed variances are (at least) a factor 1000 times larger than
##    others; use varTable(fit) to investigate
## Warning: lavaan->lav_data_full():
##    some observed variances are (at least) a factor 1000 times larger than
##    others; use varTable(fit) to investigate

summary(fiml_repeated_aux, standardized = TRUE)

## lavaan 0.6-19 ended normally after 197 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        52
##   Number of equality constraints                     2
##
##   Number of observations                          6726
##   Number of missing patterns                        16
##
## Model Test User Model:
##
##   Test statistic                               268.441
##   Degrees of freedom                                 4
##   P-value (Chi-square)                           0.000
##
## Parameter Estimates:
```

```
##
##   Standard errors                             Standard
##   Information                                 Observed
##   Observed information based on                Hessian
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   subjects =~
##     info_t1           1.000                               11.115    0.652
##     info_t2           1.000                               11.115    0.652
##     info_t3           1.000                               11.115    0.652
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   extra_t3 ~~
##     cont_t3           0.139    0.007   20.226    0.000    0.139    0.324
##     om_t3             0.101    0.006   15.961    0.000    0.101    0.251
##     ag_t3             0.077    0.006   12.865    0.000    0.077    0.200
##     ne_t3            -0.164    0.007  -22.917    0.000   -0.164   -0.373
##     male             -0.012    0.005   -2.451    0.014   -0.012   -0.038
##   cont_t3 ~~
##     om_t3             0.051    0.006    8.173    0.000    0.051    0.125
##     ag_t3             0.160    0.006   24.807    0.000    0.160    0.407
##     ne_t3            -0.159    0.007  -21.898    0.000   -0.159   -0.354
##     male             -0.074    0.005  -14.620    0.000   -0.074   -0.227
##   om_t3 ~~
##     ag_t3             0.112    0.006   19.278    0.000    0.112    0.306
##     ne_t3            -0.018    0.006   -2.823    0.005   -0.018   -0.043
##     male             -0.032    0.005   -6.819    0.000   -0.032   -0.105
##   ag_t3 ~~
##     ne_t3            -0.092    0.006  -14.633    0.000   -0.092   -0.229
##     male             -0.080    0.005  -17.492    0.000   -0.080   -0.272
##   ne_t3 ~~
##     male             -0.064    0.005  -12.318    0.000   -0.064   -0.192
##   extra_t3 ~~
```

```
##    .info_t1          -0.196    0.236   -0.831    0.406   -0.196   -0.023
##   cont_t3 ~~
##    .info_t1          -0.040    0.238   -0.168    0.867   -0.040   -0.005
##   om_t3 ~~
##    .info_t1           1.999    0.217    9.222    0.000    1.999    0.250
##   ag_t3 ~~
##    .info_t1           0.884    0.209    4.225    0.000    0.884    0.115
##   ne_t3 ~~
##    .info_t1           0.407    0.248    1.642    0.101    0.407    0.046
##   male ~~
##    .info_t1          -0.581    0.140   -4.151    0.000   -0.581   -0.091
##   extra_t3 ~~
##    .info_t2          -0.277    0.195   -1.420    0.156   -0.277   -0.033
##   cont_t3 ~~
##    .info_t2           0.808    0.197    4.096    0.000    0.808    0.094
##   om_t3 ~~
##    .info_t2           2.044    0.183   11.191    0.000    2.044    0.255
##   ag_t3 ~~
##    .info_t2           1.320    0.173    7.632    0.000    1.320    0.172
##   ne_t3 ~~
##    .info_t2           0.499    0.202    2.466    0.014    0.499    0.057
##   male ~~
##    .info_t2          -1.036    0.124   -8.376    0.000   -1.036   -0.162
##   extra_t3 ~~
##    .info_t3           0.505    0.169    2.995    0.003    0.505    0.060
##   cont_t3 ~~
##    .info_t3           1.701    0.171    9.939    0.000    1.701    0.199
##   om_t3 ~~
##    .info_t3           3.442    0.161   21.412    0.000    3.442    0.430
##   ag_t3 ~~
##    .info_t3           2.879    0.153   18.851    0.000    2.879    0.376
##   ne_t3 ~~
##    .info_t3          -0.201    0.176   -1.143    0.253   -0.201   -0.023
##   male ~~
##    .info_t3          -1.133    0.125   -9.098    0.000   -1.133   -0.177
```

```
##
## Intercepts:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     subjects         0.000                               0.000    0.000
##    .info_t1  (mu1)   67.046    0.249  269.753    0.000   67.046    3.932
##    .info_t2  (mu2)   63.933    0.276  231.307    0.000   63.933    3.750
##    .info_t3  (mu3)   68.749    0.251  273.848    0.000   68.749    4.032
##     extra_t3         3.337    0.010  337.243    0.000    3.337    5.148
##     cont_t3          3.507    0.010  350.526    0.000    3.507    5.301
##     om_t3            3.628    0.009  386.696    0.000    3.628    5.857
##     ag_t3            3.707    0.009  415.870    0.000    3.707    6.257
##     ne_t3            2.772    0.010  269.717    0.000    2.772    4.091
##     male             0.436    0.006   71.793    0.000    0.436    0.880
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info_t1  (res)  167.178    3.088   54.142    0.000  167.178    0.575
##    .info_t2  (res)  167.178    3.088   54.142    0.000  167.178    0.575
##    .info_t3  (res)  167.178    3.088   54.142    0.000  167.178    0.575
##     subjects       123.538    4.210   29.345    0.000    1.000    1.000
##     extra_t3         0.420    0.009   46.372    0.000    0.420    1.000
##     cont_t3          0.438    0.009   46.450    0.000    0.438    1.000
##     om_t3            0.384    0.008   46.385    0.000    0.384    1.000
##     ag_t3            0.351    0.008   46.535    0.000    0.351    1.000
##     ne_t3            0.459    0.010   46.341    0.000    0.459    1.000
##     male             0.245    0.004   57.719    0.000    0.245    1.000
```

The remaining code in this section performs Wald chi-square tests, which are analogous to F statistics. A Wald test is a chi-square statistic equal to the sum of squared standardized differences between the model parameters and the null. A single-df Wald test is a squared z-statistic. The Wald tests use parameter labels to specify ANOVA-style significance tests. A significant test statistic refutes the null, indicating a difference or effect.

```r
# Chi-square test of omnibus null hypothesis that all means are equal
lavTestWald(fiml_repeated_aux, 'mu1 == mu2; mu2 == mu3')

## $stat
## [1] 229.2187
##
## $df
## [1] 2
##
## $p.value
## [1] 0
##
## $se
## [1] "standard"

# Pairwise comparisons
lavTestWald(fiml_repeated_aux, 'mu1 == mu2')

## $stat
## [1] 94.87408
##
## $df
## [1] 1
##
## $p.value
## [1] 0
##
## $se
## [1] "standard"
```

```r
lavTestWald(fiml_repeated_aux, 'mu1 == mu3')

## $stat
## [1] 32.48125
##
## $df
## [1] 1
##
## $p.value
## [1] 1.20348e-08
##
## $se
## [1] "standard"

lavTestWald(fiml_repeated_aux, 'mu2 == mu3')

## $stat
## [1] 226.9077
##
## $df
## [1] 1
##
## $p.value
## [1] 0
##
## $se
## [1] "standard"
```

ML REPEATED MEASURES ANALYSIS WITH BETWEEN-SUBJECTS PREDICTOR

This section illustrates a repeated measures analysis with a between-group predictor, with and without auxiliary variables. The conditionally MAR assumption is defined with respect to the observed variables in each model. This analysis uses the `male` dummy code as a between-group predictor (0 = female, 1 = male, NA = else). Missing predictor variables require their own model for missing data handling. `lavaan` has no missing data handling for categorical variables (WLS is not a desirable option because it assumes an unsystematic MCAR process). Binary variables must be treated as normally distributed for missing data handling. Conceptually, this creates imputes that are decimals rather than 0s and 1s.

```
# Define auxiliary variables for use with sem.auxiliary function
auxvars <- c('extra_t3','cont_t3','om_t3','ag_t3','ne_t3')
```

A latent variable named `subjects` represents the random subjects factor in a within-subjects design. Constraints on residual variance create the compound symmetry assumption, which can be relaxed. Means and mean differences are labeled and used to obtain ANOVA-style significance tests. Group means are defined as additional parameters.

```
# Repeated measures syntax
syntax_repeated_gender <- '
  # Random subjects factor with mean = 0 and loadings = 1
  subjects =~ 1*info_t1 + 1*info_t2 + 1*info_t3
  subjects ~ 0*1
  # Label means and mean differences
  info_t1 ~ mu1*1 + dif1*male
  info_t2 ~ mu2*1 + dif2*male
  info_t3 ~ mu3*1 + dif3*male
  # Equal residual variances gives compound symmetry assumption, relax if needed
  info_t1 ~~ res*info_t1
  info_t2 ~~ res*info_t2
  info_t3 ~~ res*info_t3
  # Define group means
  fem_mu1 := mu1
  fem_mu2 := mu2
  fem_mu3 := mu3
  male_mu1 := mu1 + dif1
  male_mu2 := mu2 + dif2
  male_mu3 := mu3 + dif3
'
```

The `fixed.x = FALSE` input tells the program to estimate all possible means, variances, and covariances among exogenous predictors. This specification invokes multivariate normality for all analysis variables, including the binary dummy code. To apply an appropriate categorical variable model to the gender dummy code, you would need to use MCMC. `lavaan` issues a warning that variables have very different variances (the variance of `male` is much smaller).

```
# Maximum likelihood estimation
fiml_repeated_gender <- sem(
  model = syntax_repeated_gender,
  data = carsdat,
  fixed.x = FALSE,
  missing = 'ml')

## Warning: lavaan->lav_data_full():
##    some observed variances are (at least) a factor 1000 times larger than
##    others; use varTable(fit) to investigate

summary(fiml_repeated_gender, standardized = TRUE)

## lavaan 0.6-19 ended normally after 59 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        12
##   Number of equality constraints                     2
##
##   Number of observations                          6726
##   Number of missing patterns                        15
##
## Model Test User Model:
##
##   Test statistic                               251.540
##   Degrees of freedom                                 4
##   P-value (Chi-square)                           0.000
##
```

```
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Observed
##   Observed information based on                Hessian
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   subjects =~
##     info_t1           1.000                               10.936    0.645
##     info_t2           1.000                               10.936    0.639
##     info_t3           1.000                               10.936    0.640
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   info_t1 ~
##     male    (dif1)   -2.034    0.501   -4.058    0.000   -2.034   -0.060
##   info_t2 ~
##     male    (dif2)   -5.270    0.560   -9.405    0.000   -5.270   -0.153
##   info_t3 ~
##     male    (dif3)   -4.922    0.510   -9.651    0.000   -4.922   -0.143
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     subjects          0.000                                0.000    0.000
##    .info_t1  (mu1)   67.887    0.324  209.373    0.000   67.887    4.006
##    .info_t2  (mu2)   66.171    0.361  183.398    0.000   66.171    3.866
##    .info_t3  (mu3)   70.901    0.328  216.260    0.000   70.901    4.148
##     male              0.436    0.006   71.607    0.000    0.436    0.879
##
```

```
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info_t1  (res)  166.580    3.087   53.962    0.000  166.580    0.580
##    .info_t2  (res)  166.580    3.087   53.962    0.000  166.580    0.569
##    .info_t3  (res)  166.580    3.087   53.962    0.000  166.580    0.570
##     subjects        119.592    4.146   28.843    0.000    1.000    1.000
##     male              0.246    0.004   57.610    0.000    0.246    1.000
##
## Defined Parameters:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     fem_mu1          67.887    0.324  209.373    0.000   67.887    4.006
##     fem_mu2          66.171    0.361  183.398    0.000   66.171    3.866
##     fem_mu3          70.901    0.328  216.260    0.000   70.901    4.148
##     male_mu1         65.853    0.381  172.815    0.000   65.853    3.946
##     male_mu2         60.901    0.427  142.585    0.000   60.901    3.713
##     male_mu3         65.979    0.388  169.880    0.000   65.979    4.005
```

The `fixed.x = FALSE` option tells the program to estimate all possible means, variances, and covariances among exogenous predictors. This specification invokes multivariate normality for all analysis variables, including the binary dummy code. The `sem.auxiliary` function uses Graham's (2001) saturated correlates model: 1) All auxiliary variables correlate with each other 2) Auxiliary variables correlate with exogenous predictors 3) Auxiliary variables correlate with residuals of all outcomes Analysis summary table will include these additional parameters, which can be ignored. `lavaan` issues a warning that variables have very different variances (the variance of `male` is much smaller).

```
# Maximum likelihood estimation with auxiliary variables
fiml_repeated_gender_aux <- sem.auxiliary(syntax_repeated_gender,
                                  data = carsdat,
                                  fixed.x = FALSE,
                                  aux = auxvars)
```

```
## Warning: lavaan->lav_data_full():
##    some observed variances are (at least) a factor 1000 times larger than
##    others; use varTable(fit) to investigate
## Warning: lavaan->lav_data_full():
##    some observed variances are (at least) a factor 1000 times larger than
##    others; use varTable(fit) to investigate
## Warning: lavaan->lav_data_full():
##    some observed variances are (at least) a factor 1000 times larger than
##    others; use varTable(fit) to investigate
```

```
summary(fiml_repeated_gender_aux, standardized = TRUE)
```

```
## lavaan 0.6-19 ended normally after 202 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        52
##   Number of equality constraints                     2
##
##   Number of observations                          6726
##   Number of missing patterns                        16
##
## Model Test User Model:
##
##   Test statistic                               250.060
##   Degrees of freedom                                 4
##   P-value (Chi-square)                           0.000
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Observed
##   Observed information based on                Hessian
##
```

```
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   subjects =~
##     info_t1           1.000                               10.956    0.646
##     info_t2           1.000                               10.956    0.640
##     info_t3           1.000                               10.956    0.641
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   info_t1 ~
##     male    (dif1)   -2.030    0.501   -4.054    0.000   -2.030   -0.059
##   info_t2 ~
##     male    (dif2)   -5.301    0.560   -9.474    0.000   -5.301   -0.153
##   info_t3 ~
##     male    (dif3)   -4.896    0.510   -9.606    0.000   -4.896   -0.142
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   extra_t3 ~~
##     cont_t3           0.139    0.007   20.212    0.000    0.139    0.324
##     om_t3             0.101    0.006   15.962    0.000    0.101    0.251
##     ag_t3             0.077    0.006   12.853    0.000    0.077    0.200
##     ne_t3            -0.164    0.007  -22.918    0.000   -0.164   -0.373
##   cont_t3 ~~
##     om_t3             0.052    0.006    8.229    0.000    0.052    0.126
##     ag_t3             0.160    0.006   24.808    0.000    0.160    0.408
##     ne_t3            -0.158    0.007  -21.860    0.000   -0.158   -0.353
##   om_t3 ~~
##     ag_t3             0.113    0.006   19.301    0.000    0.113    0.307
##     ne_t3            -0.018    0.006   -2.790    0.005   -0.018   -0.043
##   ag_t3 ~~
##     ne_t3            -0.092    0.006  -14.586    0.000   -0.092   -0.228
##   extra_t3 ~~
##    .info_t1          -0.217    0.233   -0.933    0.351   -0.217   -0.026
##   cont_t3 ~~
```

```
##    .info_t1          -0.210    0.232   -0.904    0.366   -0.210   -0.025
##   om_t3 ~~
##    .info_t1           1.910    0.214    8.937    0.000    1.910    0.239
##   ag_t3 ~~
##    .info_t1           0.690    0.202    3.421    0.001    0.690    0.090
##   ne_t3 ~~
##    .info_t1           0.248    0.242    1.025    0.305    0.248    0.028
##   extra_t3 ~~
##    .info_t2          -0.328    0.194   -1.691    0.091   -0.328   -0.039
##   cont_t3 ~~
##    .info_t2           0.506    0.192    2.628    0.009    0.506    0.059
##   om_t3 ~~
##    .info_t2           1.919    0.181   10.611    0.000    1.919    0.240
##   ag_t3 ~~
##    .info_t2           0.992    0.168    5.912    0.000    0.992    0.130
##   ne_t3 ~~
##    .info_t2           0.223    0.198    1.124    0.261    0.223    0.025
##   extra_t3 ~~
##    .info_t3           0.451    0.167    2.697    0.007    0.451    0.054
##   cont_t3 ~~
##    .info_t3           1.365    0.166    8.232    0.000    1.365    0.160
##   om_t3 ~~
##    .info_t3           3.304    0.159   20.825    0.000    3.304    0.413
##   ag_t3 ~~
##    .info_t3           2.519    0.146   17.227    0.000    2.519    0.329
##   ne_t3 ~~
##    .info_t3          -0.501    0.172   -2.917    0.004   -0.501   -0.057
##   extra_t3 ~~
##     male             -0.012    0.005   -2.443    0.015   -0.012   -0.038
##   cont_t3 ~~
##     male             -0.075    0.005  -14.732    0.000   -0.075   -0.229
##   om_t3 ~~
##     male             -0.033    0.005   -7.000    0.000   -0.033   -0.108
##   ag_t3 ~~
##     male             -0.081    0.005  -17.611    0.000   -0.081   -0.275
```

```
##   ne_t3 ~~
##     male             -0.064    0.005  -12.316    0.000   -0.064   -0.192
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     subjects          0.000                               0.000    0.000
##    .info_t1  (mu1)   67.946    0.324  209.452    0.000   67.946    4.008
##    .info_t2  (mu2)   66.218    0.361  183.582    0.000   66.218    3.867
##    .info_t3  (mu3)   70.879    0.328  216.146    0.000   70.879    4.146
##     male              0.436    0.006   71.720    0.000    0.436    0.879
##     extra_t3          3.337    0.010  337.243    0.000    3.337    5.148
##     cont_t3           3.507    0.010  350.394    0.000    3.507    5.298
##     om_t3             3.628    0.009  386.559    0.000    3.628    5.854
##     ag_t3             3.707    0.009  415.607    0.000    3.707    6.251
##     ne_t3             2.772    0.010  269.718    0.000    2.772    4.091
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info_t1  (res)  166.359    3.079   54.037    0.000  166.359    0.579
##    .info_t2  (res)  166.359    3.079   54.037    0.000  166.359    0.567
##    .info_t3  (res)  166.359    3.079   54.037    0.000  166.359    0.569
##     subjects        120.032    4.147   28.941    0.000    1.000    1.000
##     male              0.246    0.004   57.642    0.000    0.246    1.000
##     extra_t3          0.420    0.009   46.373    0.000    0.420    1.000
##     cont_t3           0.438    0.009   46.405    0.000    0.438    1.000
##     om_t3             0.384    0.008   46.356    0.000    0.384    1.000
##     ag_t3             0.352    0.008   46.449    0.000    0.352    1.000
##     ne_t3             0.459    0.010   46.345    0.000    0.459    1.000
##
## Defined Parameters:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     fem_mu1          67.946    0.324  209.452    0.000   67.946    4.008
##     fem_mu2          66.218    0.361  183.582    0.000   66.218    3.867
##     fem_mu3          70.879    0.328  216.146    0.000   70.879    4.146
##     male_mu1         65.916    0.381  173.052    0.000   65.916    3.949
```

```
##     male_mu2          60.917    0.427  142.766    0.000   60.917    3.713
##     male_mu3          65.983    0.388  169.984    0.000   65.983    4.004
```

The remaining code in this section performs Wald chi-square tests, which are analogous to F statistics. A Wald test is a chi-square statistic equal to the sum of squared standardized differences between the model parameters and the null. A single-df Wald test is a squared z-statistic. The Wald tests use parameter labels to specify ANOVA-style significance tests. A significant test statistic refutes the null, indicating a difference or effect.

```
# Chi-square test of the group-by-time interaction (no occasion-specific gender differences)
lavTestWald(fiml_repeated_gender_aux, 'dif1 == dif2; dif2 == dif3')

## $stat
## [1] 32.65378
##
## $df
## [1] 2
##
## $p.value
## [1] 8.115618e-08
##
## $se
## [1] "standard"
```

ML MULTIPLE REGRESSION MODEL

This section illustrates a multiple regression analysis with a continuous and binary predictor, with and without auxiliary variables. The conditionally MAR assumption is defined with respect to the observed variables in each model. This analysis uses the `male` dummy code as a between-group predictor (0 = female, 1 = male, NA = else). `lavaan` has no missing data handling for categorical variables (WLS is not a desirable option because it assumes an unsystematic MCAR process). Binary variables must be treated as normally distributed for missing data handling. Conceptually, this creates imputes that are decimals rather than 0s and 1s. `lavaan` cannot accommodate interactions with incomplete variables, but MCMC can.

```r
# Define auxiliary variables for use with sem.auxiliary function
auxvars <- c('extra_t3','cont_t3','om_t3','ag_t3','ne_t3','admit_type_num')

# Regression model syntax
syntax_multreg <- 'info_t3 ~ b1*effort_t3 + b2*male'
```

The `fixed.x = FALSE` input tells the program to estimate all possible means, variances, and covariances among exogenous predictors. This specification invokes multivariate normality for all analysis variables, including the binary dummy code. To apply an appropriate categorical variable model to the gender dummy code, you would need to use MCMC. `lavaan` issues a warning that several cases have no data on the analysis variables. `lavaan` issues a warning that variables have very different variances (the variance of `male` is much smaller).

```r
# Maximum likelihood estimation
fiml_multreg <- sem(
  model = syntax_multreg,
  data = carsdat,
  fixed.x = FALSE, # ml missing data requires predictor vars/covs/means
  missing = 'ml'
)

## Warning: lavaan->lav_data_full():
##    some observed variances are (at least) a factor 1000 times larger than
##    others; use varTable(fit) to investigate

## Warning: lavaan->lav_data_full():
##    some cases are empty and will be ignored: 947 1111 1706 1906 1907 2637
##    2989 3475 3507 4645 4666 4817 5001 5255 6425 6429.

summary(fiml_multreg, standardized = TRUE)

## lavaan 0.6-19 ended normally after 21 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
```

```
##   Number of model parameters                         9
##
##                                                   Used       Total
##   Number of observations                          6710        6726
##   Number of missing patterns                         4
##
## Model Test User Model:
##
##   Test statistic                                 0.000
##   Degrees of freedom                                 0
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Observed
##   Observed information based on                Hessian
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   info_t3 ~
##     effort_t3 (b1)   10.007    0.337   29.688    0.000   10.007    0.413
##     male      (b2)   -2.548    0.488   -5.217    0.000   -2.548   -0.073
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   effort_t3 ~~
##     male             -0.049    0.005   -9.005    0.000   -0.049   -0.140
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info_t3          33.081    1.312   25.213    0.000   33.081    1.924
##     effort_t3         3.694    0.011  342.292    0.000    3.694    5.206
##     male              0.436    0.006   71.632    0.000    0.436    0.879
##
```

```
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info_t3         241.163    5.200   46.373    0.000  241.163    0.816
##     effort_t3         0.504    0.011   46.366    0.000    0.504    1.000
##     male              0.246    0.004   57.609    0.000    0.246    1.000
```

The `fixed.x = FALSE` input tells the program to estimate all possible means, variances, and covariances among exogenous predictors. This specification invokes multivariate normality for all analysis variables, including the binary dummy code. The sem.auxiliary function uses Graham's (2001) saturated correlates model: 1) All auxiliary variables correlate with each other 2) Auxiliary variables correlate with exogenous predictors 3) Auxiliary variables correlate with residuals of all outcomes Analysis summary table will include these additional parameters, which can be ignored. The `admit_type` variable can be used as an auxiliary variable because `info_t1` is not in the model. Note that `admit_type` must be treated as a normally distributed auxiliary variable. `lavaan` issues a warning that variables have very different variances (the variance of `male` is much smaller).

```
# Maximum likelihood estimation with auxiliary variables
fiml_reg_aux <- sem.auxiliary(
  model = syntax_multreg,
  data = carsdat,
  fixed.x = FALSE, # ml missing data requires predictor vars/covs/means
  aux = auxvars)

## Warning: lavaan->lav_data_full():
##    some observed variances are (at least) a factor 1000 times larger than
##    others; use varTable(fit) to investigate

## Warning: lavaan->lav_data_full():
##    some cases are empty and will be ignored: 947 1111 1706 1906 1907 2637
##    2989 3475 3507 4645 4666 4817 5001 5255 6425 6429.
```

```
## Warning: lavaan->lav_data_full():
##    some observed variances are (at least) a factor 1000 times larger than
##    others; use varTable(fit) to investigate
## Warning: lavaan->lav_data_full():
##    some observed variances are (at least) a factor 1000 times larger than
##    others; use varTable(fit) to investigate
```

```
summary(fiml_reg_aux, standardized = TRUE)
```

```
## lavaan 0.6-19 ended normally after 122 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        54
##
##   Number of observations                          6726
##   Number of missing patterns                         7
##
## Model Test User Model:
##
##   Test statistic                                 0.000
##   Degrees of freedom                                 0
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Observed
##   Observed information based on                Hessian
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   info_t3 ~
##     effort_t3 (b1)   10.010    0.337   29.699    0.000   10.010    0.413
##     male      (b2)   -2.497    0.488   -5.114    0.000   -2.497   -0.072
##
```

```
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   effort_t3 ~~
##     male              -0.049    0.005   -8.996    0.000   -0.049   -0.139
##   extra_t3 ~~
##     cont_t3            0.139    0.007   20.182    0.000    0.139    0.323
##     om_t3              0.101    0.006   15.980    0.000    0.101    0.251
##     ag_t3              0.077    0.006   12.835    0.000    0.077    0.200
##     ne_t3             -0.164    0.007  -22.924    0.000   -0.164   -0.373
##     admit_type_num    -0.009    0.003   -2.948    0.003   -0.009   -0.042
##   cont_t3 ~~
##     om_t3              0.053    0.006    8.355    0.000    0.053    0.128
##     ag_t3              0.161    0.006   24.829    0.000    0.161    0.409
##     ne_t3             -0.158    0.007  -21.848    0.000   -0.158   -0.353
##     admit_type_num    -0.007    0.003   -2.268    0.023   -0.007   -0.032
##   om_t3 ~~
##     ag_t3              0.113    0.006   19.292    0.000    0.113    0.308
##     ne_t3             -0.019    0.006   -2.888    0.004   -0.019   -0.044
##     admit_type_num     0.001    0.003    0.282    0.778    0.001    0.004
##   ag_t3 ~~
##     ne_t3             -0.092    0.006  -14.609    0.000   -0.092   -0.228
##     admit_type_num    -0.000    0.003   -0.088    0.930   -0.000   -0.001
##   ne_t3 ~~
##     admit_type_num     0.003    0.003    0.867    0.386    0.003    0.012
##   extra_t3 ~~
##    .info_t3           -0.071    0.152   -0.465    0.642   -0.071   -0.007
##   cont_t3 ~~
##    .info_t3            0.252    0.147    1.711    0.087    0.252    0.024
##   om_t3 ~~
##    .info_t3            2.131    0.144   14.819    0.000    2.131    0.221
##   ag_t3 ~~
##    .info_t3            1.164    0.128    9.114    0.000    1.164    0.126
##   ne_t3 ~~
##    .info_t3            0.035    0.156    0.223    0.824    0.035    0.003
##   admit_type_num ~~
```

```
##    .info_t3          -0.129    0.077   -1.677    0.093   -0.129   -0.024
##   extra_t3 ~~
##     effort_t3         0.054    0.007    7.701    0.000    0.054    0.118
##   cont_t3 ~~
##     effort_t3         0.135    0.007   18.091    0.000    0.135    0.287
##   om_t3 ~~
##     effort_t3         0.128    0.007   18.350    0.000    0.128    0.291
##   ag_t3 ~~
##     effort_t3         0.158    0.007   23.050    0.000    0.158    0.376
##   ne_t3 ~~
##     effort_t3        -0.042    0.007   -5.685    0.000   -0.042   -0.087
##   admit_type_num ~~
##     effort_t3        -0.003    0.004   -0.855    0.392   -0.003   -0.012
##   extra_t3 ~~
##     male             -0.012    0.005   -2.443    0.015   -0.012   -0.038
##   cont_t3 ~~
##     male             -0.075    0.005  -14.723    0.000   -0.075   -0.229
##   om_t3 ~~
##     male             -0.032    0.005   -6.653    0.000   -0.032   -0.103
##   ag_t3 ~~
##     male             -0.080    0.005  -17.469    0.000   -0.080   -0.273
##   ne_t3 ~~
##     male             -0.064    0.005  -12.298    0.000   -0.064   -0.191
##   admit_type_num ~~
##     male              0.005    0.002    2.472    0.013    0.005    0.030
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info_t3          33.077    1.312   25.210    0.000   33.077    1.924
##     effort_t3         3.694    0.011  341.451    0.000    3.694    5.206
##     male              0.436    0.006   71.741    0.000    0.436    0.880
##     extra_t3          3.339    0.010  336.815    0.000    3.339    5.151
##     cont_t3           3.508    0.010  349.632    0.000    3.508    5.298
##     om_t3             3.630    0.009  383.582    0.000    3.630    5.858
##     ag_t3             3.708    0.009  414.001    0.000    3.708    6.249
```

```
##     ne_t3             2.772    0.010  269.122    0.000    2.772    4.091
##     admit_type_num    1.140    0.004  267.868    0.000    1.140    3.285
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .info_t3         241.188    5.201   46.378    0.000  241.188    0.816
##     effort_t3         0.504    0.011   46.367    0.000    0.504    1.000
##     male              0.246    0.004   57.643    0.000    0.246    1.000
##     extra_t3          0.420    0.009   46.385    0.000    0.420    1.000
##     cont_t3           0.438    0.009   46.378    0.000    0.438    1.000
##     om_t3             0.384    0.008   46.367    0.000    0.384    1.000
##     ag_t3             0.352    0.008   46.375    0.000    0.352    1.000
##     ne_t3             0.459    0.010   46.360    0.000    0.459    1.000
##     admit_type_num    0.120    0.002   57.654    0.000    0.120    1.000
```

The remaining code in this section performs a Wald chi-square test, which is analogous to an F statistic. A Wald test is a chi-square statistic (analogous to an F test) equal to the sum of squared standardized differences between the model parameters and the null. A single-df Wald test is a squared z-statistic. The Wald tests use parameter labels to specify ANOVA-style significance tests. A significant test statistic refutes the null, indicating a difference or effect.

```
# Chi-square test of null hypothesis that both slopes equal zero
lavTestWald(fiml_reg_aux, 'b1 == 0; b2 == 0')
```

```
## $stat
## [1] 969.4948
##
## $df
## [1] 2
##
## $p.value
## [1] 0
##
## $se
## [1] "standard"
```

MCMC DESCRIPTIVE STATISTICS

This section illustrates descriptive statistics with and without auxiliary variables. The conditionally mar assumption is defined with respect to the observed variables in each model. MCMC can accommodate continuous and discrete variables.

```
# MCMC estimation
mcmc_saturated <- rblimp(
  data = carsdat,
  model = 'info_t1 info_t2 info_t3 ~~ info_t1 info_t2 info_t3;', # All possible covariances (and means/vars)
  seed = 90291, # Integer random number seed
  burn = 5000, # Warm up iterations
  iter = 10000) # Iterations for analysis
```

Check the last row of the `BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT` table to verify that the value in the `Highest PSR` column is less than 1.05 (indicating convergence). The `DATA INFORMATION` section displays a table with missing data patterns. The `MODEL FIT` section contains custom Wald significance tests, if specified. Each outcome variable has its own summary table. Check the `N_Eff` diagnostic column to verify that all values are greater than 100 (indicating adequate support from the data). The `Estimate` column contains the Bayesian point estimates (posterior medians), the `StdDev` column contains "Bayesian standard errors", and the 2.5% and 97.5% columns are 95% credible intervals. The `ChiSq` and `PValue` columns are frequentist test statistics (squared z-statistics).

```
# Print output
output(mcmc_saturated)

##
## ---------------------------------------------------------------------------
##
##                                    Blimp
##                                   3.2.20
##
##              Blimp was developed with funding from Institute of
```

```
##            Education Sciences awards R305D150056 and R305D190002.
##
##             Craig K. Enders, P.I. Email: cenders@psych.ucla.edu
##            Brian T. Keller, Co-P.I. Email: btkeller@missouri.edu
##                  Han Du, Co-P.I. Email: hdu@psych.ucla.edu
##                  Roy Levy, Co-P.I. Email: roy.levy@asu.edu
##
##               Programming and Blimp Studio by Brian T. Keller
##
##                    There is no expressed license given.
##
## --------------------------------------------------------------------------
##
##
## ALGORITHMIC OPTIONS SPECIFIED:
##
##   Imputation method:                 Fully Bayesian model-based
##   MCMC algorithm:                    Full conditional Metropolis sampler with
##                                        Auto-Derived Conditional Distributions
##   Between-cluster imputation model:  Not applicable, single-level imputation
##   Prior for random effect variances: Not applicable, single-level imputation
##   Prior for residual variances:      Zero sum of squares, df = -2 (PRIOR2)
##   Prior for predictor variances:     Unit sum of squares, df = 2 (XPRIOR1)
##   Chain Starting Values:             Random starting values
##
##
##
## BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:
##
##   NOTE: Split chain PSR is being used. This splits each chain's
##         iterations to create twice as many chains.
##
##   Comparing iterations across 2 chains     Highest PSR   Parameter #
##                           126 to 250             1.100            16
##                           251 to 500             1.016             6
```

```
##                          376 to 750            1.022            16
##                          501 to 1000           1.016            18
##                          626 to 1250           1.026            18
##                          751 to 1500           1.013            16
##                          876 to 1750           1.009             9
##                         1001 to 2000           1.006            18
##                         1126 to 2250           1.005             5
##                         1251 to 2500           1.005            16
##                         1376 to 2750           1.003            16
##                         1501 to 3000           1.004            18
##                         1626 to 3250           1.004            10
##                         1751 to 3500           1.005             6
##                         1876 to 3750           1.004            10
##                         2001 to 4000           1.003            10
##                         2126 to 4250           1.003            16
##                         2251 to 4500           1.003            18
##                         2376 to 4750           1.004            16
##                         2501 to 5000           1.003            18
##
##
## DATA INFORMATION:
##
##   Sample Size:              6629
##   Missing Data Info:
##                          miss %      1     2     3     4     5     6     7
##                                 -------------------------------------------
##                   info_t1 = 33.2      -     -     M     -     M     M     -
##                   info_t2 = 48.1      -     M     M     M     -     -     -
##                   info_t3 = 35.1      -     M     -     -     M     -     M
##                                 -------------------------------------------
##                                 %  28.8  18.5  16.1  13.5  10.6   6.5   6.0
##
##
```

```
## MODEL INFORMATION:
##
##   NUMBER OF PARAMETERS
##     Outcome Models:         9
##     Predictor Models:       0
##
##   MODELS
##     [1]  info_t1 ~ Intercept
##     [2]  info_t2 ~ Intercept
##     [3]  info_t3 ~ Intercept
##     [4]  info_t1 info_t2 info_t3 <-> info_t1 info_t2 info_t3
##
##
##  WARNING MESSAGES:
##
##   WARNING: 97 observations have all variables in the imputation
##            model missing. They have been dropped from data set.
##
##
## MODEL FIT:
##
##
##   INFORMATION CRITERIA
##
##     Conditional Likelihood
##       DIC2                  168196.974
##       WAIC                  171180.026
##
##
## CORRELATIONS AMONG RESIDUALS:
##
##   No residual correlations.
##
##
##
```

```
## OUTCOME MODEL ESTIMATES:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##
## Outcome Variable:  info_t1
##
## Parameters                         Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                   -----------------------------------------------------------------------------
## Variances:
##   Residual Var.                     226.444      4.734    217.416    236.159        ---        ---   4882.396
##
## Coefficients:
##   Intercept                          67.003      0.218     66.577     67.431  94451.428      0.000   3819.654
##
## Proportion Variance Explained
##   by Coefficients                     0.000      0.000      0.000      0.000        ---        ---        nan
##   by Residual Variation               1.000      0.000      1.000      1.000        ---        ---        nan
##
##                                   -----------------------------------------------------------------------------
##
##
##
## Outcome Variable:  info_t2
##
## Parameters                         Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                   -----------------------------------------------------------------------------
## Variances:
##   Residual Var.                     369.316      8.838    352.310    387.156        ---        ---   3335.531
##
## Coefficients:
##   Intercept                          63.804      0.303     63.209     64.393  44434.876      0.000   2635.635
##
```

```
## Proportion Variance Explained
##   by Coefficients                   0.000      0.000      0.000      0.000        ---        ---        nan
##   by Residual Variation             1.000      0.000      1.000      1.000        ---        ---        nan
##
##                            -----------------------------------------------------------------------------
##
##
##
## Outcome Variable:  info_t3
##
## Parameters                       Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                            -----------------------------------------------------------------------------
## Variances:
##   Residual Var.                   298.383      6.500    286.028    311.585        ---        ---   4233.495
##
## Coefficients:
##   Intercept                        68.819      0.253     68.314     69.312  73824.711      0.000   3346.248
##
## Proportion Variance Explained
##   by Coefficients                   0.000      0.000      0.000      0.000        ---        ---        nan
##   by Residual Variation             1.000      0.000      1.000      1.000        ---        ---        nan
##
##                            -----------------------------------------------------------------------------
##
##
##
## Covariance Matrix: info_t1 info_t2 info_t3
##
## Parameters                       Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                            -----------------------------------------------------------------------------
## Covariances:
##   Cov(info_t1,info_t2)            123.800      5.899    112.335    135.481    440.775      0.000   1974.175
##   Cov(info_t1,info_t3)            104.326      4.790     95.051    113.892    474.707      0.000   3050.306
##   Cov(info_t2,info_t3)            160.243      6.427    148.064    173.196    622.505      0.000   2996.151
##
```

```
## Correlations:
##   Cor(info_t1,info_t2)           0.428     0.017     0.394     0.462    630.990     0.000   1698.481
##   Cor(info_t1,info_t3)           0.401     0.015     0.370     0.431    673.046     0.000   2663.417
##   Cor(info_t2,info_t3)           0.483     0.015     0.453     0.511   1066.399     0.000   2461.104
##
##                                  ------------------------------------------------------------------------------
```

The `ordinal` command invokes a categorical variable model for binary or ordered categorical variables. Alternatively, binary and multicategorical variables can be listed on the `nominal` line, which automatically creates dummy codes with lowest codes the reference. Auxiliary variables use a sequential specification (extra dependent variable) approach. Auxiliary variables are listed to the left of the tilde and primary variables to the right. For example, `A3 A2 A1 ~ Y X` produces three regressions: `A1 ~ Y X`, `A2 ~ A1 Y X`, and `A3 ~ A2 A1 Y X`. The output includes summary tables for these additional models, which can be ignored. Attempts to use the three-group gender variable an auxiliary variable (1 = female, 2 = male, 3 = non-binary) failed. Convergence required a long burn-in period, and MCMC diagnostics indicated a lack of support from the data (e.g., `N_eff` values less than 100). This analysis instead uses the `male` dummy code as an auxiliary variable (0 = female, 1 = male, NA = else).

```
# MCMC estimation with auxiliary variables
mcmc_saturated_aux <- rblimp(
  data = carsdat,
  ordinal = 'male', # Define binary or ordinal variable
  model = '
    primary: # This is an arbitrary and optional label that groups summary tables
    info_t1 info_t2 info_t3 ~~ info_t1 info_t2 info_t3; # All possible covariances (and means/vars)
    auxiliary: # This is an arbitrary and optional label that groups summary tables
    extra_t3 cont_t3 om_t3 ag_t3 ne_t3 male ~ info_t1 info_t2 info_t3', # Sequential auxiliary variable models
  seed = 90291, # Integer random number seed
  burn = 5000, # Warm up iterations
  iter = 10000) # Iterations for analysis
```

Check the last row of the `BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT` table to verify that the value in the `Highest PSR` column is less than 1.05 (indicating convergence). The `DATA INFORMATION` section displays a table with missing data patterns. The `MODEL FIT` section contains custom Wald significance tests, if specified. Each outcome variable has its own summary table. Check the `N_Eff` diagnostic column to verify that all values are greater than 100 (indicating adequate support from the data). The `Estimate` column contains the Bayesian point estimates (posterior medians), the `StdDev` column contains "Bayesian standard errors", and the 2.5% and 97.5% columns are 95% credible intervals. The `ChiSq` and `PValue` columns are frequentist test statistics (squared z-statistics).

```
# Print output
output(mcmc_saturated_aux)

##
## ---------------------------------------------------------------------------
##
##                                     Blimp
##                                    3.2.20
##
##                Blimp was developed with funding from Institute of
##              Education Sciences awards R305D150056 and R305D190002.
##
##               Craig K. Enders, P.I. Email: cenders@psych.ucla.edu
##              Brian T. Keller, Co-P.I. Email: btkeller@missouri.edu
##                    Han Du, Co-P.I. Email: hdu@psych.ucla.edu
##                    Roy Levy, Co-P.I. Email: roy.levy@asu.edu
##
##                 Programming and Blimp Studio by Brian T. Keller
##
##                      There is no expressed license given.
##
## ---------------------------------------------------------------------------
##
##
```

```
## ALGORITHMIC OPTIONS SPECIFIED:
##
##   Imputation method:                 Fully Bayesian model-based
##   MCMC algorithm:                    Full conditional Metropolis sampler with
##                                        Auto-Derived Conditional Distributions
##   Between-cluster imputation model:  Not applicable, single-level imputation
##   Prior for random effect variances: Not applicable, single-level imputation
##   Prior for residual variances:      Zero sum of squares, df = -2 (PRIOR2)
##   Prior for predictor variances:     Unit sum of squares, df = 2 (XPRIOR1)
##   Chain Starting Values:             Random starting values
##
##
##   NOTE: The default prior for regression coefficients
##         in categorical models is 'normal( 0.0, 5.0)'
##
##
## BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:
##
##   NOTE: Split chain PSR is being used. This splits each chain's
##         iterations to create twice as many chains.
##
##   Comparing iterations across 2 chains     Highest PSR   Parameter #
##                            126 to 250            1.107            23
##                            251 to 500            1.069            16
##                            376 to 750            1.036            80
##                            501 to 1000           1.021            33
##                            626 to 1250           1.017            81
##                            751 to 1500           1.021            75
##                            876 to 1750           1.018            78
##                           1001 to 2000           1.020            78
##                           1126 to 2250           1.014            46
##                           1251 to 2500           1.015            75
##                           1376 to 2750           1.015            77
##                           1501 to 3000           1.013            77
##                           1626 to 3250           1.011            46
```

```
##                          1751 to 3500            1.008            46
##                          1876 to 3750            1.007             5
##                          2001 to 4000            1.006            24
##                          2126 to 4250            1.006            24
##                          2251 to 4500            1.004            67
##                          2376 to 4750            1.004            18
##                          2501 to 5000            1.004            79
##
##
## METROPOLIS-HASTINGS ACCEPTANCE RATES:
##
##   Chain 1:
##
##     Variable                            Type    Probability   Target Value
##     male                          imputation          0.497          0.500
##
##   NOTE: Suppressing printing of 1 chains.
##         Use keyword 'tuneinfo' in options to override.
##
##
## DATA INFORMATION:
##
##   Sample Size:              6726
##   Missing Data Info:
##                           miss %      1     2     3     4     5     6     7     8     9    10
##                                   --------------------------------------------------------------
##                    info_t1 = 34.2      -     -     M     -     M     M     -     M     M     -
##                    info_t2 = 48.9      -     M     M     M     -     -     -     M     M     -
##                    info_t3 = 36.0      -     M     -     -     M     -     M     M     -     -
##                   extra_t3 = 36.1      -     M     -     -     M     -     M     M     -     -
##                    cont_t3 = 36.1      -     M     -     -     M     -     M     M     -     -
##                      om_t3 = 36.1      -     M     -     -     M     -     M     M     -     -
##                      ag_t3 = 36.1      -     M     -     -     M     -     M     M     -     -
##                      ne_t3 = 36.1      -     M     -     -     M     -     M     M     -     -
##                       male =  1.3      -     -     -     -     -     -     -     -     M     M
```

```
##                                  --------------------------------------------------------------
##                                  %  28.2  18.2  15.1  13.2  10.4   6.4   5.8   1.4   0.7   0.2
##
##   NOTE: Suppressing printing of 6 missing data patterns.
##         These patterns consist of 0.4% of the data.
##         Use keyword 'allpatterns' in options to print all patterns.
##
##
## MODEL INFORMATION:
##
##   NUMBER OF PARAMETERS
##     Outcome Models:         53
##     Predictor Models:       0
##
##   MODELS
##
##    primary:
##     [1]  info_t1 ~ Intercept
##     [2]  info_t2 ~ Intercept
##     [3]  info_t3 ~ Intercept
##     [4]  info_t1 info_t2 info_t3 <-> info_t1 info_t2 info_t3
##
##    auxiliary:
##     [5]  ag_t3 ~ Intercept ne_t3 male info_t1 info_t2 info_t3
##     [6]  cont_t3 ~ Intercept om_t3 ag_t3 ne_t3 male info_t1 info_t2 info_t3
##     [7]  extra_t3 ~ Intercept cont_t3 om_t3 ag_t3 ne_t3 male info_t1 info_t2
##                  info_t3
##     [8]  male ~ Intercept info_t1 info_t2 info_t3
##     [9]  ne_t3 ~ Intercept male info_t1 info_t2 info_t3
##    [10]  om_t3 ~ Intercept ag_t3 ne_t3 male info_t1 info_t2 info_t3
##
##
##  WARNING MESSAGES:
##
##   No warning messages.
```

```
##
##
## MODEL FIT:
##
##
##   INFORMATION CRITERIA
##
##     Conditional Likelihood
##       DIC2                  253874.345
##       WAIC                  264222.995
##
##
## CORRELATIONS AMONG RESIDUALS:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##
## Correlations                       Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                   ------------------------------------------------------------------------
##
##   info_t1, ag_t3                      -0.000      0.017     -0.034      0.034      0.000      0.992   8994.804
##   info_t1, cont_t3                     0.000      0.017     -0.034      0.033      0.000      0.997   9682.980
##   info_t1, extra_t3                   -0.000      0.017     -0.034      0.034      0.000      0.994  10440.415
##   info_t1, male                        0.000      0.017     -0.034      0.034      0.000      0.993   9810.597
##   info_t1, ne_t3                      -0.000      0.017     -0.033      0.033      0.000      0.988   8252.204
##   info_t1, om_t3                      -0.001      0.017     -0.034      0.033      0.000      0.983   8716.537
##   info_t2, ag_t3                       0.000      0.017     -0.034      0.034      0.000      0.997   8563.915
##   info_t2, cont_t3                    -0.000      0.018     -0.034      0.035      0.000      0.993   9660.441
##   info_t2, extra_t3                    0.000      0.017     -0.034      0.033      0.000      0.995   8336.188
##   info_t2, male                       -0.000      0.017     -0.034      0.033      0.000      0.998  10062.140
##   info_t2, ne_t3                      -0.000      0.017     -0.034      0.034      0.000      0.997   8552.724
##   info_t2, om_t3                      -0.000      0.017     -0.034      0.034      0.000      0.988   8905.562
##   info_t3, ag_t3                       0.000      0.017     -0.034      0.034      0.000      0.999   7268.493
##   info_t3, cont_t3                    -0.000      0.017     -0.033      0.034      0.000      0.999   8598.475
```

```
##   info_t3, extra_t3                  0.000     0.017    -0.034     0.033     0.000     0.991   8176.291
##   info_t3, male                     -0.000     0.017    -0.033     0.034     0.000     0.994   9794.118
##   info_t3, ne_t3                    -0.000     0.017    -0.034     0.034     0.000     0.999   7951.293
##   info_t3, om_t3                    -0.001     0.017    -0.034     0.034     0.001     0.981   8506.816
##   ag_t3, cont_t3                     0.000     0.017    -0.034     0.034     0.000     0.996   7372.545
##   ag_t3, extra_t3                    0.000     0.017    -0.033     0.034     0.000     1.000   7363.820
##   ag_t3, male                        0.000     0.017    -0.034     0.034     0.000     0.994   9749.972
##   ag_t3, ne_t3                      -0.000     0.017    -0.033     0.034     0.000     0.998   9603.053
##   ag_t3, om_t3                      -0.000     0.017    -0.034     0.034     0.000     0.998   7696.407
##   cont_t3, extra_t3                  0.000     0.017    -0.034     0.034     0.000     0.995   7178.581
##   cont_t3, male                      0.000     0.017    -0.034     0.034     0.000     0.999   9932.884
##   cont_t3, ne_t3                     0.000     0.017    -0.034     0.033     0.000     0.997   9754.299
##   cont_t3, om_t3                     0.000     0.017    -0.034     0.034     0.000     0.988   9159.514
##   extra_t3, male                     0.000     0.017    -0.034     0.033     0.000     0.996   9336.293
##   extra_t3, ne_t3                    0.000     0.017    -0.033     0.034     0.000     0.996   9942.143
##   extra_t3, om_t3                    0.000     0.017    -0.033     0.035     0.000     0.992   9768.075
##   male, ne_t3                        0.000     0.017    -0.034     0.035     0.000     0.998   9667.368
##   male, om_t3                       -0.000     0.017    -0.034     0.033     0.000     0.984   9731.068
##   ne_t3, om_t3                      -0.000     0.017    -0.035     0.034     0.000     0.989   7648.882
##
##                              -----------------------------------------------------------------------------
##
##
##
```

```
## OUTCOME MODEL ESTIMATES:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##   primary block:
##
## Outcome Variable:  info_t1
##
## Parameters                         Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                   -----------------------------------------------------------------------------
## Variances:
##   Residual Var.                     226.960      4.864    217.814    236.646        ---        ---   4878.289
##
## Coefficients:
##   Intercept                          67.059      0.219     66.627     67.494  93730.685      0.000   3734.402
##
## Proportion Variance Explained
##   by Coefficients                     0.000      0.000      0.000      0.000        ---        ---        nan
##   by Residual Variation               1.000      0.000      1.000      1.000        ---        ---        nan
##
##                                   -----------------------------------------------------------------------------
##
##
##
## Outcome Variable:  info_t2
##
## Parameters                         Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                   -----------------------------------------------------------------------------
## Variances:
##   Residual Var.                     369.317      8.807    352.737    387.329        ---        ---   3445.387
##
## Coefficients:
##   Intercept                          63.794      0.309     63.179     64.394  42589.707      0.000   2671.121
##
```

```
## Proportion Variance Explained
##   by Coefficients                   0.000      0.000      0.000      0.000        ---        ---        nan
##   by Residual Variation             1.000      0.000      1.000      1.000        ---        ---        nan
##
##                            -------------------------------------------------------------------------------
##
##
##
## Outcome Variable:  info_t3
##
## Parameters                       Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                            -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                   299.273      6.430    287.060    312.294        ---        ---   4579.102
##
## Coefficients:
##   Intercept                        68.752      0.253     68.249     69.238  73828.355      0.000   3195.129
##
## Proportion Variance Explained
##   by Coefficients                   0.000      0.000      0.000      0.000        ---        ---        nan
##   by Residual Variation             1.000      0.000      1.000      1.000        ---        ---        nan
##
##                            -------------------------------------------------------------------------------
##
##
##
## Covariance Matrix: info_t1 info_t2 info_t3
##
## Parameters                       Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                            -------------------------------------------------------------------------------
## Covariances:
##   Cov(info_t1,info_t2)            124.845      5.940    113.184    136.793    441.996      0.000   2115.487
##   Cov(info_t1,info_t3)            105.674      4.775     96.490    115.207    490.340      0.000   2691.648
##   Cov(info_t2,info_t3)            160.611      6.542    147.933    173.700    602.934      0.000   2375.172
##
```

```
## Correlations:
##   Cor(info_t1,info_t2)              0.431     0.017     0.397     0.464   650.657     0.000  1750.850
##   Cor(info_t1,info_t3)              0.406     0.015     0.375     0.436   708.698     0.000  2417.405
##   Cor(info_t2,info_t3)              0.483     0.015     0.453     0.513  1016.530     0.000  1929.971
##
##                                -------------------------------------------------------------------------------
##
##
##   auxiliary block:
##
## Outcome Variable:  ag_t3
##
## Parameters                       Estimate    StdDev      2.5%     97.5%     ChiSq    PValue     N_Eff
##                                -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                     0.277     0.006     0.265     0.289       ---       ---  4686.411
##
## Coefficients:
##   Intercept                         3.995     0.059     3.880     4.111  4584.153     0.000  3847.140
##   ne_t3                            -0.245     0.012    -0.268    -0.221   401.388     0.000  4379.282
##   male                             -0.351     0.017    -0.383    -0.317   434.256     0.000  4028.256
##   info_t1                          -0.001     0.001    -0.003     0.000     2.309     0.129  2340.952
##   info_t2                           0.001     0.001    -0.001     0.002     0.631     0.427  2163.674
##   info_t3                           0.009     0.001     0.007     0.010   199.939     0.000  2768.199
##
## Standardized Coefficients:
##   ne_t3                            -0.279     0.013    -0.305    -0.253   437.927     0.000  4134.726
##   male                             -0.292     0.013    -0.317    -0.266   490.238     0.000  3988.955
##   info_t1                          -0.031     0.020    -0.070     0.008     2.309     0.129  2334.426
##   info_t2                           0.017     0.022    -0.026     0.059     0.631     0.427  2159.905
##   info_t3                           0.250     0.017     0.216     0.283   209.439     0.000  2747.761
##
```

```
## Proportion Variance Explained
##   by Coefficients                  0.217      0.011      0.196      0.239        ---        ---   4112.239
##   by Residual Variation            0.783      0.011      0.761      0.804        ---        ---   4112.239
##
##                                  -----------------------------------------------------------------------------
##
##
##
## Outcome Variable:  cont_t3
##
## Parameters                      Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                  -----------------------------------------------------------------------------
## Variances:
##   Residual Var.                    0.315      0.007      0.302      0.328        ---        ---   4875.090
##
## Coefficients:
##   Intercept                        3.426      0.096      3.241      3.615   1268.186      0.000   4351.553
##   om_t3                           -0.004      0.015     -0.034      0.026      0.071      0.791   4606.166
##   ag_t3                            0.289      0.017      0.256      0.322    294.328      0.000   4398.454
##   ne_t3                           -0.326      0.014     -0.353     -0.300    571.405      0.000   4802.325
##   male                            -0.285      0.019     -0.323     -0.247    220.368      0.000   4345.013
##   info_t1                         -0.003      0.001     -0.004     -0.001      8.701      0.003   2166.579
##   info_t2                          0.001      0.001     -0.000      0.003      2.539      0.111   2043.322
##   info_t3                          0.002      0.001      0.001      0.003     10.279      0.001   3089.238
##
## Standardized Coefficients:
##   om_t3                           -0.004      0.014     -0.032      0.024      0.071      0.790   4607.546
##   ag_t3                            0.259      0.015      0.230      0.287    312.848      0.000   4466.327
##   ne_t3                           -0.333      0.013     -0.359     -0.308    643.179      0.000   4728.294
##   male                            -0.213      0.014     -0.241     -0.186    232.085      0.000   4385.359
##   info_t1                         -0.058      0.020     -0.097     -0.019      8.711      0.003   2154.774
##   info_t2                          0.034      0.021     -0.009      0.076      2.543      0.111   2043.182
##   info_t3                          0.055      0.017      0.021      0.089     10.283      0.001   3079.380
##
```

```
## Proportion Variance Explained
##   by Coefficients                      0.284      0.011      0.262      0.307        ---        ---   4319.779
##   by Residual Variation                0.716      0.011      0.693      0.738        ---        ---   4319.779
##
##                               -----------------------------------------------------------------------------
##
##
##
## Outcome Variable:  extra_t3
##
## Parameters                          Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                               -----------------------------------------------------------------------------
## Variances:
##   Residual Var.                        0.322      0.007      0.309      0.336        ---        ---   4248.947
##
## Coefficients:
##   Intercept                            2.912      0.112      2.696      3.130    680.738      0.000   4017.184
##   cont_t3                              0.188      0.015      0.158      0.218    150.720      0.000   4653.388
##   om_t3                                0.251      0.016      0.221      0.281    261.755      0.000   4736.024
##   ag_t3                               -0.015      0.017     -0.049      0.020      0.714      0.398   4734.938
##   ne_t3                               -0.291      0.015     -0.320     -0.262    400.125      0.000   4869.236
##   male                                -0.058      0.019     -0.095     -0.020      8.930      0.003   4576.534
##   info_t1                             -0.000      0.001     -0.002      0.001      0.177      0.674   2447.510
##   info_t2                             -0.002      0.001     -0.004     -0.001      7.823      0.005   2307.684
##   info_t3                             -0.001      0.001     -0.003     -0.000      4.789      0.029   3793.943
##
## Standardized Coefficients:
##   cont_t3                              0.192      0.015      0.161      0.222    156.100      0.000   4739.053
##   om_t3                                0.240      0.014      0.212      0.268    276.723      0.000   4684.458
##   ag_t3                               -0.013      0.016     -0.045      0.018      0.714      0.398   4734.214
##   ne_t3                               -0.304      0.015     -0.332     -0.275    437.250      0.000   4797.243
##   male                                -0.044      0.015     -0.073     -0.015      8.945      0.003   4559.829
##   info_t1                             -0.009      0.020     -0.048      0.031      0.177      0.674   2446.687
##   info_t2                             -0.061      0.022     -0.104     -0.017      7.838      0.005   2308.988
##   info_t3                             -0.039      0.018     -0.074     -0.004      4.794      0.029   3786.924
```

```
## Proportion Variance Explained
##   by Coefficients                   0.235      0.011      0.214      0.256        ---        ---   4718.739
##   by Residual Variation             0.765      0.011      0.744      0.786        ---        ---   4718.739
##
##                          -------------------------------------------------------------------------------
##
##
##
## Outcome Variable:  male
##
## Parameters                       Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                          -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                     1.000      0.000      1.000      1.000        ---        ---        nan
##
## Coefficients:
##   Intercept                         0.665      0.092      0.483      0.842     51.939      0.000   2849.583
##   info_t1                           0.001      0.002     -0.002      0.004      0.439      0.508   1938.660
##   info_t2                          -0.006      0.001     -0.009     -0.003     17.552      0.000   1337.417
##   info_t3                          -0.007      0.001     -0.010     -0.005     26.729      0.000   1835.901
##
## Thresholds:
##   Tau 1                             0.000      0.000      0.000      0.000        ---        ---        nan
##
## Standardized Coefficients:
##   info_t1                           0.015      0.023     -0.030      0.061      0.438      0.508   1941.678
##   info_t2                          -0.115      0.027     -0.167     -0.061     17.801      0.000   1334.140
##   info_t3                          -0.125      0.024     -0.173     -0.079     27.076      0.000   1827.829
##
## Proportion Variance Explained
##   by Coefficients                   0.041      0.007      0.028      0.056        ---        ---   1922.583
##   by Residual Variation             0.959      0.007      0.944      0.972        ---        ---   1922.583
##
##                          -------------------------------------------------------------------------------
##
```

```
## Outcome Variable:  ne_t3
##
## Parameters                          Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                        0.441      0.010      0.423      0.461        ---        ---   4259.791
##
## Coefficients:
##   Intercept                            2.909      0.061      2.790      3.030   2253.723      0.000   2624.305
##   male                                -0.266      0.021     -0.309     -0.226    156.653      0.000   3518.474
##   info_t1                              0.001      0.001     -0.001      0.003      2.184      0.139   2010.600
##   info_t2                              0.001      0.001     -0.000      0.003      2.422      0.120   1930.382
##   info_t3                             -0.003      0.001     -0.004     -0.001     15.861      0.000   3379.479
##
## Standardized Coefficients:
##   male                                -0.195      0.015     -0.225     -0.166    165.520      0.000   3528.427
##   info_t1                              0.033      0.022     -0.011      0.075      2.186      0.139   2009.073
##   info_t2                              0.037      0.024     -0.010      0.085      2.425      0.119   1938.926
##   info_t3                             -0.075      0.019     -0.112     -0.038     15.963      0.000   3379.129
##
## Proportion Variance Explained
##   by Coefficients                      0.042      0.006      0.031      0.055        ---        ---   3182.514
##   by Residual Variation                0.958      0.006      0.945      0.969        ---        ---   3182.514
##
##                                    -------------------------------------------------------------------------------
##
##
##
## Outcome Variable:  om_t3
##
## Parameters                          Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                        0.322      0.007      0.309      0.337        ---        ---   4967.293
##
```

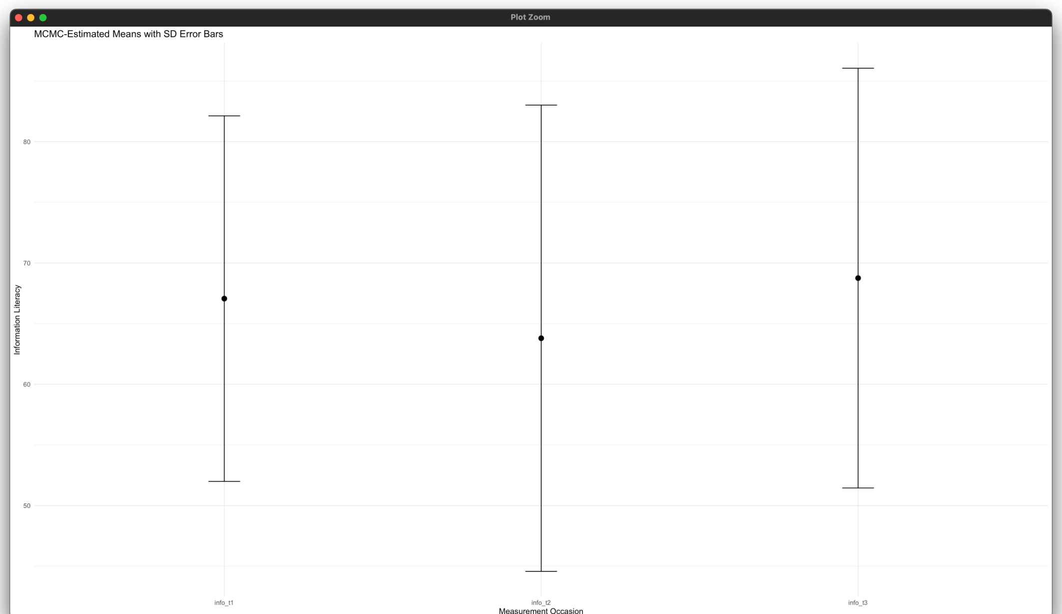
```
## Coefficients:
##   Intercept                        1.905     0.094     1.723     2.091   414.780     0.000   3815.255
##   ag_t3                            0.247     0.017     0.214     0.279   221.810     0.000   4759.461
##   ne_t3                            0.010     0.014    -0.017     0.037     0.555     0.456   4211.143
##   male                             0.001     0.019    -0.036     0.038     0.003     0.960   4248.600
##   info_t1                          0.002     0.001     0.000     0.004     5.461     0.019   2148.241
##   info_t2                          0.002     0.001     0.000     0.003     5.877     0.015   1973.498
##   info_t3                          0.008     0.001     0.006     0.009   139.904     0.000   3200.318
##
## Standardized Coefficients:
##   ag_t3                            0.236     0.015     0.205     0.266   233.709     0.000   4754.031
##   ne_t3                            0.011     0.015    -0.018     0.041     0.555     0.456   4215.856
##   male                             0.001     0.015    -0.029     0.030     0.003     0.960   4249.386
##   info_t1                          0.047     0.020     0.008     0.086     5.475     0.019   2153.830
##   info_t2                          0.054     0.022     0.011     0.098     5.894     0.015   1967.861
##   info_t3                          0.217     0.018     0.181     0.253   143.673     0.000   3148.740
##
## Proportion Variance Explained
##   by Coefficients                  0.165     0.010     0.146     0.186       ---       ---   3998.278
##   by Residual Variation            0.835     0.010     0.814     0.854       ---       ---   3998.278
##
##                                -----------------------------------------------------------------------------
```

The remaining code in this section uses the table of `rblimp` estimates to construct a data frame containing the means and standard deviations. Estimates are stored in `model_object@estimates`. The `ggplot2` function produces a graph of the means with standard deviation error bars.

```r
# Extract rblimp means and standard deviations for graphing
est <- mcmc_saturated_aux@estimates[,1]
int_names <- grep('^info.*~ Intercept$', names(est), value = TRUE)
res_names <- grep('^info.*residual variance$', names(est), value = TRUE)
means <- est[int_names]
sds   <- sqrt(est[res_names])
vars <- sub(' ~ Intercept$', '', int_names)
```

```r
summary_stats <- data.frame(
  variable = vars,
  mean     = as.numeric(means),
  sd       = as.numeric(sds)
)

# Graph mcmc means and sds
ggplot(summary_stats, aes(x = variable, y = mean)) +
  geom_point(size = 3) +
  geom_errorbar(aes(ymin = mean - sd, ymax = mean + sd), width = 0.1) +
  labs(
    x = 'Information Literacy',
    y = 'Measurement Occasion',
    title = 'MCMC-Estimated Means with SD Error Bars'
  ) + theme_minimal()
```



MCMC REPEATED MEASURES ANALYSIS

This section illustrates a repeated measures analysis with and without auxiliary variables. The conditionally MAR assumption is defined with respect to the observed variables in each model. A latent variable named `subjects` represents the random subjects factor in a within-subjects design. The @ symbol is used to label parameters and fix them at specific values. Constraints on residual variance creates the compound symmetry assumption, which can be relaxed. Means are labeled and used to obtain a custom ANOVA-style significance test. The `waldtest` command specifies Wald chi-square tests, which are analogous to F statistics. A Wald test is a chi-square statistic equal to the sum of squared standardized differences between the model parameters and the null. A single-df Wald test is a squared z-statistic. The Wald tests use parameter labels to specify ANOVA-style significance tests. A significant test statistic refutes the null, indicating a difference or effect.

```
# MCMC estimation
mcmc_repeated <- rblimp(
  data = carsdat,
  latent = 'subjects', # Define latent variable for subjects factor
  model = '
    # Random subjects factor with mean = 0 and loadings = 1
    subjects ~ intercept@0;
    info_t1 ~ intercept@mu1 subjects@1; # @ labels the means and sets subject factor loadings to 1
    info_t2 ~ intercept@mu2 subjects@1;
    info_t3 ~ intercept@mu3 subjects@1;
    # Equal residual variances gives compound symmetry assumption, relax if needed
    info_t1 ~~ info_t1@res;  # @ labels the variances, setting them equal
    info_t2 ~~ info_t2@res;
    info_t3 ~~ info_t3@res;',
  waldtest = c(
    # Test of null that all means are equal
    'mu1 = mu2; mu2 = mu3',
    # Tests of pairwise comparisons
    'mu1 = mu2','mu1 = mu3','mu2 = mu3'),
```

```r
  seed = 90291, # Integer random number seed
  burn = 5000, # Warm up iterations
  iter = 10000) # Iterations for analysis
```

Check the last row of the `BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT` table to verify that the value in the `Highest PSR` column is less than 1.05 (indicating convergence). The `DATA INFORMATION` section displays a table with missing data patterns. The `MODEL FIT` section contains custom Wald significance tests, if specified. Each outcome variable has its own summary table. Check the `N_Eff` diagnostic column to verify that all values are greater than 100 (indicating adequate support from the data). The `Estimate` column contains the Bayesian point estimates (posterior medians), the `StdDev` column contains "Bayesian standard errors", and the 2.5% and 97.5% columns are 95% credible intervals. The `ChiSq` and `PValue` columns are frequentist test statistics (squared z-statistics).

```r
# Print output
output(mcmc_repeated)
```

```
##
## ---------------------------------------------------------------------------
##
##                                    Blimp
##                                   3.2.20
##
##              Blimp was developed with funding from Institute of
##            Education Sciences awards R305D150056 and R305D190002.
##
##             Craig K. Enders, P.I. Email: cenders@psych.ucla.edu
##            Brian T. Keller, Co-P.I. Email: btkeller@missouri.edu
##                  Han Du, Co-P.I. Email: hdu@psych.ucla.edu
##                  Roy Levy, Co-P.I. Email: roy.levy@asu.edu
##
##               Programming and Blimp Studio by Brian T. Keller
##
##                    There is no expressed license given.
```

```
##
## --------------------------------------------------------------------------
##
##
## ALGORITHMIC OPTIONS SPECIFIED:
##
##   Imputation method:                 Fully Bayesian model-based
##   MCMC algorithm:                    Full conditional Metropolis sampler with
##                                        Auto-Derived Conditional Distributions
##   Between-cluster imputation model:  Not applicable, single-level imputation
##   Prior for random effect variances: Not applicable, single-level imputation
##   Prior for residual variances:      Zero sum of squares, df = -2 (PRIOR2)
##   Prior for predictor variances:     Unit sum of squares, df = 2 (XPRIOR1)
##   Chain Starting Values:             Random starting values
##
##
##
## BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:
##
##   NOTE: Split chain PSR is being used. This splits each chain's
##         iterations to create twice as many chains.
##
##   Comparing iterations across 2 chains     Highest PSR   Parameter #
##                             126 to 250           1.967             7
##                             251 to 500           2.170             4
##                             376 to 750           1.013             4
##                             501 to 1000          1.008            11
##                             626 to 1250          1.013             4
##                             751 to 1500          1.011             7
##                             876 to 1750          1.009             8
##                            1001 to 2000          1.019             1
##                            1126 to 2250          1.006            17
##                            1251 to 2500          1.006             4
##                            1376 to 2750          1.019             8
##                            1501 to 3000          1.005            11
```

```
##                          1626 to 3250            1.005              5
##                          1751 to 3500            1.008              5
##                          1876 to 3750            1.008              5
##                          2001 to 4000            1.010              7
##                          2126 to 4250            1.007              7
##                          2251 to 4500            1.007             17
##                          2376 to 4750            1.008             17
##                          2501 to 5000            1.006             17
##
##
## DATA INFORMATION:
##
##   Sample Size:              6629
##   Missing Data Info:
##                           miss %       1     2     3     4     5     6     7
##                                   -------------------------------------------
##                   info_t1 = 33.2       -     -     M     -     M     M     -
##                   info_t2 = 48.1       -     M     M     M     -     -     -
##                   info_t3 = 35.1       -     M     -     -     M     -     M
##                                   -------------------------------------------
##                                   %  28.8  18.5  16.1  13.5  10.6   6.5   6.0
##
##
## MODEL INFORMATION:
##
##   NUMBER OF PARAMETERS
##     Outcome Models:         5
##     Predictor Models:       0
##
##   FACTORS
##     Level-1:                subjects
##
##   MODELS
##     [1]  subjects ~ Intercept@0
##     [2]  info_t1 ~ Intercept@mu1 subjects@1
```

```
##     [3]  info_t2 ~ Intercept@mu2 subjects@1
##     [4]  info_t3 ~ Intercept@mu3 subjects@1
##
##
##  WARNING MESSAGES:
##
##   WARNING: 97 observations have all variables in the imputation
##            model missing. They have been dropped from data set.
##
##
## MODEL FIT:
##
##
##   INFORMATION CRITERIA
##
##     Conditional Likelihood
##       DIC2                  213415.453
##       WAIC                  218961.432
##
##   WALD TESTS (Asparouhov & Muthén, 2021)
##
##   Test #1
##
##     Full:
##       [1]  info_t1 ~ Intercept@mu1 subjects@1
##       [2]  info_t2 ~ Intercept@mu2 subjects@1
##       [3]  info_t3 ~ Intercept@mu3 subjects@1
##
##     Restricted:
##       [1]  info_t1 ~ Intercept@mu1 subjects@1
##       [2]  info_t2 ~ Intercept@mu2 subjects@1
##       [3]  info_t3 ~ Intercept@mu3 subjects@1
##
```

```
##     Constraints in Restricted:
##       [1]  mu1 = mu2
##       [2]  mu2 = mu3
##
##
##     Wald Statistic (Chi-Square)                238.691
##     Number of Parameters Tested (df)                 2
##     Probability                                  0.000
##
##   Test #2
##
##     Full:
##       [1]  info_t1 ~ Intercept@mu1 subjects@1
##       [2]  info_t2 ~ Intercept@mu2 subjects@1
##
##     Restricted:
##       [1]  info_t1 ~ Intercept@mu1 subjects@1
##       [2]  info_t2 ~ Intercept@mu2 subjects@1
##
##     Constraints in Restricted:
##       [1]  mu1 = mu2
##
##
##     Wald Statistic (Chi-Square)                 94.112
##     Number of Parameters Tested (df)                 1
##     Probability                                  0.000
##
##   Test #3
##
##     Full:
##       [1]  info_t1 ~ Intercept@mu1 subjects@1
##       [2]  info_t3 ~ Intercept@mu3 subjects@1
##
##     Restricted:
##       [1]  info_t1 ~ Intercept@mu1 subjects@1
```

```
##       [2]  info_t3 ~ Intercept@mu3 subjects@1
##
##     Constraints in Restricted:
##       [1]  mu1 = mu3
##
##
##     Wald Statistic (Chi-Square)                38.101
##     Number of Parameters Tested (df)                1
##     Probability                                 0.000
##
##   Test #4
##
##     Full:
##       [1]  info_t2 ~ Intercept@mu2 subjects@1
##       [2]  info_t3 ~ Intercept@mu3 subjects@1
##
##     Restricted:
##       [1]  info_t2 ~ Intercept@mu2 subjects@1
##       [2]  info_t3 ~ Intercept@mu3 subjects@1
##
##     Constraints in Restricted:
##       [1]  mu2 = mu3
##
##
##     Wald Statistic (Chi-Square)               236.745
##     Number of Parameters Tested (df)                1
##     Probability                                 0.000
##
##
## CORRELATIONS AMONG RESIDUALS:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##
```

```
## Correlations                                 Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                  ------------------------------------------------------------------------------
##
##   subjects, info_t1                              -0.094      0.016     -0.126     -0.062     32.528      0.000   6439.139
##   subjects, info_t2                               0.074      0.016      0.042      0.105     20.734      0.000   8249.452
##   subjects, info_t3                               0.012      0.016     -0.019      0.044      0.563      0.453   7677.388
##   info_t1, info_t2                               -0.012      0.017     -0.044      0.021      0.480      0.488   9599.896
##   info_t1, info_t3                               -0.004      0.017     -0.036      0.029      0.060      0.806   7772.047
##   info_t2, info_t3                                0.015      0.017     -0.018      0.047      0.771      0.380   7048.147
##
##                                  ------------------------------------------------------------------------------
##
##
##
## OUTCOME MODEL ESTIMATES:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##
## Latent Variable:   subjects
##
## Parameters                                   Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                  ------------------------------------------------------------------------------
## Variances:
##   Residual Var.                                 122.690      4.282    114.538    131.485        ---        ---   1020.252
##
## Proportion Variance Explained
##   by Coefficients                                 0.000      0.000      0.000      0.000        ---        ---        nan
##   by Residual Variation                           1.000      0.000      1.000      1.000        ---        ---        nan
##
##                                  ------------------------------------------------------------------------------
##
##
##
```

```
## Outcome Variable:  info_t1
##
## Parameters                          Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                 -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                      167.706      3.145    161.659    174.032        ---        ---   1084.421
##
## Coefficients:
##   Intercept                           66.992      0.248     66.514     67.489  73161.396      0.000   2204.650
##   subjects                           @ 1.000        ---        ---        ---        ---        ---        ---
##
## Standardized Coefficients:
##   subjects                             0.650      0.008      0.634      0.665   6674.929      0.000    805.313
##
## Proportion Variance Explained
##   by Coefficients                      0.422      0.010      0.402      0.443        ---        ---    804.452
##   by Residual Variation                0.578      0.012      0.554      0.602        ---        ---   1099.231
##
##                                 -------------------------------------------------------------------------------
##
##
##
## Outcome Variable:  info_t2
##
## Parameters                          Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                 -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                      167.706      3.145    161.659    174.032        ---        ---   1084.421
##
## Coefficients:
##   Intercept                           63.949      0.274     63.409     64.485  54513.844      0.000   1798.334
##   subjects                           @ 1.000        ---        ---        ---        ---        ---        ---
##
```

```
## Standardized Coefficients:
##   subjects                            0.650      0.008      0.634      0.665   6674.929      0.000    805.313
##
## Proportion Variance Explained
##   by Coefficients                     0.422      0.010      0.402      0.443        ---        ---    804.452
##   by Residual Variation               0.578      0.012      0.554      0.602        ---        ---   1099.231
##
##                                 -----------------------------------------------------------------------------
##
##
##
## Outcome Variable:  info_t3
##
## Parameters                         Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                 -----------------------------------------------------------------------------
## Variances:
##   Residual Var.                     167.706      3.145    161.659    174.032        ---        ---   1084.421
##
## Coefficients:
##   Intercept                          68.832      0.251     68.346     69.336  75144.230      0.000   2121.179
##   subjects                          @ 1.000        ---        ---        ---        ---        ---        ---
##
## Standardized Coefficients:
##   subjects                            0.650      0.008      0.634      0.665   6674.929      0.000    805.313
##
## Proportion Variance Explained
##   by Coefficients                     0.422      0.010      0.402      0.443        ---        ---    804.452
##   by Residual Variation               0.578      0.012      0.554      0.602        ---        ---   1099.231
##
##                                 -----------------------------------------------------------------------------
```

See the previous analysis for a description of the repeated measures analysis components. The `ordinal` command invokes a categorical variable model for binary or ordered categorical variables. Alternatively, binary and multicategorical variables can be listed on the `nominal` line, which automatically creates dummy codes with lowest codes the reference. Auxiliary variables use a sequential specification (extra dependent variable) approach. Auxiliary variables are listed to the left of the tilde and primary variables to the right. For example, `A3 A2 A1 ~ Y X` produces three regressions: `A1 ~ Y X`, `A2 ~ A1 Y X`, and `A3 ~ A2 A1 Y X`. The output includes summary tables for these additional models, which can be ignored. Attempts to use the three-group gender variable an auxiliary variable (1 = female, 2 = male, 3 = non-binary) failed. Convergence required a long burn-in period, and MCMC diagnostics indicated a lack of support from the data (e.g., `N_eff` values less than 100). This analysis instead uses the `male` dummy code as an auxiliary variable (0 = female, 1 = male, NA = else).

```
# MCMC estimation with auxiliary variables
mcmc_repeated_aux <- rblimp(
  data = carsdat,
  ordinal = 'male', # Define binary or ordinal variable
  latent = 'subjects', # Define latent variable for subjects factor
  model = '
    primary: # This is an arbitrary and optional label that groups summary tables
    # Random subjects factor with mean = 0 and loadings = 1
    subjects ~ intercept@0;
    info_t1 ~ intercept@mu1 subjects@1;  # @ labels the means and sets subject factor loadings to 1
    info_t2 ~ intercept@mu2 subjects@1;
    info_t3 ~ intercept@mu3 subjects@1;
    # Equal residual variances gives compound symmetry assumption, relax if needed
    info_t1 ~~ info_t1@res;  # @ labels the variances, setting them equal
    info_t2 ~~ info_t2@res;
    info_t3 ~~ info_t3@res;
    auxiliary: # This is an arbitrary and optional label that groups summary tables
    extra_t3 cont_t3 om_t3 ag_t3 ne_t3 male ~ info_t1 info_t2 info_t3;', # Sequential auxiliary variable models
  waldtest = c(
    # Test of null that all means are equal
    'mu1 = mu2; mu2 = mu3',
    # Tests of pairwise comparisons
    'mu1 = mu2','mu1 = mu3','mu2 = mu3'),
  seed = 90291, # Integer random number seed
  burn = 5000, # Warm up iterations
  iter = 10000) # Iterations for analysis
```

Check the last row of the `BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT` table to verify that the value in the `Highest PSR` column is less than 1.05 (indicating convergence). The `DATA INFORMATION` section displays a table with missing data patterns. The `MODEL FIT` section contains custom Wald significance tests, if specified. Each outcome variable has its own summary table. Check the `N_Eff` diagnostic column to verify that all values are greater than 100 (indicating adequate support from the data). The `Estimate` column contains the Bayesian point estimates (posterior medians), the `StdDev` column contains "Bayesian standard errors", and the 2.5% and 97.5% columns are 95% credible intervals. The `ChiSq` and `PValue` columns are frequentist test statistics (squared z-statistics).

```
# Print output
output(mcmc_repeated_aux)

##
## ---------------------------------------------------------------------------
##
##                                    Blimp
##                                   3.2.20
##
##               Blimp was developed with funding from Institute of
##             Education Sciences awards R305D150056 and R305D190002.
##
##              Craig K. Enders, P.I. Email: cenders@psych.ucla.edu
##             Brian T. Keller, Co-P.I. Email: btkeller@missouri.edu
##                   Han Du, Co-P.I. Email: hdu@psych.ucla.edu
##                   Roy Levy, Co-P.I. Email: roy.levy@asu.edu
##
##                Programming and Blimp Studio by Brian T. Keller
##
##                      There is no expressed license given.
##
## ---------------------------------------------------------------------------
##
##
```

```
## ALGORITHMIC OPTIONS SPECIFIED:
##
##   Imputation method:                 Fully Bayesian model-based
##   MCMC algorithm:                    Full conditional Metropolis sampler with
##                                        Auto-Derived Conditional Distributions
##   Between-cluster imputation model:  Not applicable, single-level imputation
##   Prior for random effect variances: Not applicable, single-level imputation
##   Prior for residual variances:      Zero sum of squares, df = -2 (PRIOR2)
##   Prior for predictor variances:     Unit sum of squares, df = 2 (XPRIOR1)
##   Chain Starting Values:             Random starting values
##
##
##   NOTE: The default prior for regression coefficients
##         in categorical models is 'normal( 0.0, 5.0)'
##
##
## BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:
##
##   NOTE: Split chain PSR is being used. This splits each chain's
##         iterations to create twice as many chains.
##
##   Comparing iterations across 2 chains     Highest PSR   Parameter #
##                          126 to 250            2.160             7
##                          251 to 500            1.390             1
##                          376 to 750            1.044             7
##                          501 to 1000           1.029           103
##                          626 to 1250           1.024            78
##                          751 to 1500           1.016            11
##                          876 to 1750           1.029           102
##                         1001 to 2000           1.015             1
##                         1126 to 2250           1.019             7
##                         1251 to 2500           1.013           108
##                         1376 to 2750           1.009           108
##                         1501 to 3000           1.013            42
##                         1626 to 3250           1.012             1
```

```
##                          1751 to 3500           1.011             78
##                          1876 to 3750           1.011              1
##                          2001 to 4000           1.006             82
##                          2126 to 4250           1.005              7
##                          2251 to 4500           1.004             42
##                          2376 to 4750           1.006              8
##                          2501 to 5000           1.014              1
##
##
## METROPOLIS-HASTINGS ACCEPTANCE RATES:
##
##   Chain 1:
##
##     Variable                            Type    Probability   Target Value
##     male                          imputation          0.499          0.500
##
##   NOTE: Suppressing printing of 1 chains.
##         Use keyword 'tuneinfo' in options to override.
##
##
```

```
## DATA INFORMATION:
##
##   Sample Size:              6726
##   Missing Data Info:
##                          miss %      1     2     3     4     5     6     7     8     9    10
##                                 -------------------------------------------------------------
##                    info_t1 = 34.2      -     -     M     -     M     M     -     M     M     -
##                    info_t2 = 48.9      -     M     M     M     -     -     -     M     M     -
##                    info_t3 = 36.0      -     M     -     -     M     -     M     M     -     -
##                   extra_t3 = 36.1      -     M     -     -     M     -     M     M     -     -
##                    cont_t3 = 36.1      -     M     -     -     M     -     M     M     -     -
##                      om_t3 = 36.1      -     M     -     -     M     -     M     M     -     -
##                      ag_t3 = 36.1      -     M     -     -     M     -     M     M     -     -
##                      ne_t3 = 36.1      -     M     -     -     M     -     M     M     -     -
##                       male =  1.3      -     -     -     -     -     -     -     -     M     M
##                                 -------------------------------------------------------------
##                                 %  28.2  18.2  15.1  13.2  10.4   6.4   5.8   1.4   0.7   0.2
##
##   NOTE: Suppressing printing of 6 missing data patterns.
##         These patterns consist of 0.4% of the data.
##         Use keyword 'allpatterns' in options to print all patterns.
##
##
## MODEL INFORMATION:
##
##   NUMBER OF PARAMETERS
##     Outcome Models:         49
##     Predictor Models:       0
##
##   FACTORS
##     Level-1:                subjects
##
```

```
##   MODELS
##
##    primary:
##     [1]  subjects ~ Intercept@0
##     [2]  info_t1 ~ Intercept@mu1 subjects@1
##     [3]  info_t2 ~ Intercept@mu2 subjects@1
##     [4]  info_t3 ~ Intercept@mu3 subjects@1
##
##    auxiliary:
##     [5]  ag_t3 ~ Intercept ne_t3 male info_t1 info_t2 info_t3
##     [6]  cont_t3 ~ Intercept om_t3 ag_t3 ne_t3 male info_t1 info_t2 info_t3
##     [7]  extra_t3 ~ Intercept cont_t3 om_t3 ag_t3 ne_t3 male info_t1 info_t2
##                  info_t3
##     [8]  male ~ Intercept info_t1 info_t2 info_t3
##     [9]  ne_t3 ~ Intercept male info_t1 info_t2 info_t3
##    [10]  om_t3 ~ Intercept ag_t3 ne_t3 male info_t1 info_t2 info_t3
##
##
##  WARNING MESSAGES:
##
##   No warning messages.
##
##
## MODEL FIT:
##
##
##   INFORMATION CRITERIA
##
##     Conditional Likelihood
##       DIC2                  299743.084
##       WAIC                  312692.003
##
```

```
##   WALD TESTS (Asparouhov & Muthén, 2021)
##
##   Test #1
##
##     Full:
##       [1]  info_t1 ~ Intercept@mu1 subjects@1
##       [2]  info_t2 ~ Intercept@mu2 subjects@1
##       [3]  info_t3 ~ Intercept@mu3 subjects@1
##
##     Restricted:
##       [1]  info_t1 ~ Intercept@mu1 subjects@1
##       [2]  info_t2 ~ Intercept@mu2 subjects@1
##       [3]  info_t3 ~ Intercept@mu3 subjects@1
##
##     Constraints in Restricted:
##       [1]  mu1 = mu2
##       [2]  mu2 = mu3
##
##
##     Wald Statistic (Chi-Square)               228.009
##     Number of Parameters Tested (df)                2
##     Probability                                 0.000
##
##   Test #2
##
##     Full:
##       [1]  info_t1 ~ Intercept@mu1 subjects@1
##       [2]  info_t2 ~ Intercept@mu2 subjects@1
##
##     Restricted:
##       [1]  info_t1 ~ Intercept@mu1 subjects@1
##       [2]  info_t2 ~ Intercept@mu2 subjects@1
##
##     Constraints in Restricted:
##       [1]  mu1 = mu2
```

```
##
##
##     Wald Statistic (Chi-Square)                94.617
##     Number of Parameters Tested (df)                1
##     Probability                                 0.000
##
##   Test #3
##
##     Full:
##       [1]  info_t1 ~ Intercept@mu1 subjects@1
##       [2]  info_t3 ~ Intercept@mu3 subjects@1
##
##     Restricted:
##       [1]  info_t1 ~ Intercept@mu1 subjects@1
##       [2]  info_t3 ~ Intercept@mu3 subjects@1
##
##     Constraints in Restricted:
##       [1]  mu1 = mu3
##
##
##     Wald Statistic (Chi-Square)                33.071
##     Number of Parameters Tested (df)                1
##     Probability                                 0.000
##
##   Test #4
##
##     Full:
##       [1]  info_t2 ~ Intercept@mu2 subjects@1
##       [2]  info_t3 ~ Intercept@mu3 subjects@1
##
##     Restricted:
##       [1]  info_t2 ~ Intercept@mu2 subjects@1
##       [2]  info_t3 ~ Intercept@mu3 subjects@1
##
```

```
##     Constraints in Restricted:
##       [1]  mu2 = mu3
##
##
##     Wald Statistic (Chi-Square)              225.931
##     Number of Parameters Tested (df)               1
##     Probability                                0.000
##
##
## CORRELATIONS AMONG RESIDUALS:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##
## Correlations                          Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    -------------------------------------------------------------------------------
##
##   subjects, info_t1                     -0.092      0.016     -0.125     -0.060     31.676      0.000   6200.436
##   subjects, info_t2                      0.072      0.016      0.040      0.103     20.290      0.000   7911.091
##   subjects, info_t3                      0.013      0.016     -0.019      0.044      0.586      0.444   7068.506
##   subjects, ag_t3                       -0.000      0.017     -0.033      0.035      0.000      0.997   9861.353
##   subjects, cont_t3                      0.000      0.017     -0.034      0.034      0.000      0.999   9214.660
##   subjects, extra_t3                    -0.000      0.017     -0.034      0.033      0.000      0.992   9321.815
##   subjects, male                        -0.000      0.017     -0.034      0.033      0.000      0.993   9504.400
##   subjects, ne_t3                        0.000      0.017     -0.034      0.033      0.000      0.993   9204.255
##   subjects, om_t3                       -0.000      0.017     -0.033      0.033      0.000      0.991   8555.733
##   info_t1, info_t2                      -0.011      0.016     -0.043      0.022      0.426      0.514   8491.344
##   info_t1, info_t3                      -0.003      0.016     -0.035      0.029      0.033      0.855   7600.125
##   info_t1, ag_t3                         0.001      0.017     -0.034      0.034      0.000      0.985   8728.411
##   info_t1, cont_t3                      -0.000      0.017     -0.034      0.033      0.000      0.991   9644.866
##   info_t1, extra_t3                      0.000      0.017     -0.034      0.034      0.000      0.997   7643.664
##   info_t1, male                          0.000      0.017     -0.034      0.033      0.000      0.994  10205.876
##   info_t1, ne_t3                         0.000      0.017     -0.033      0.033      0.000      0.995   8796.184
##   info_t1, om_t3                         0.000      0.017     -0.034      0.034      0.000      0.992   9331.463
```

```
##   info_t2, info_t3                   0.014      0.017     -0.019      0.046      0.682      0.409   7964.467
##   info_t2, ag_t3                     0.000      0.017     -0.034      0.034      0.000      1.000   9041.914
##   info_t2, cont_t3                   0.000      0.017     -0.033      0.034      0.000      0.997   9011.665
##   info_t2, extra_t3                 -0.001      0.017     -0.034      0.033      0.001      0.978   9124.027
##   info_t2, male                      0.000      0.017     -0.034      0.034      0.000      0.996  10243.016
##   info_t2, ne_t3                     0.000      0.017     -0.034      0.034      0.000      1.000   9612.020
##   info_t2, om_t3                     0.000      0.017     -0.033      0.034      0.000      0.996   9343.772
##   info_t3, ag_t3                    -0.000      0.017     -0.034      0.034      0.000      0.997   8616.091
##   info_t3, cont_t3                   0.000      0.017     -0.034      0.034      0.000      0.998   8848.751
##   info_t3, extra_t3                  0.000      0.017     -0.034      0.034      0.000      0.999   8331.796
##   info_t3, male                     -0.000      0.017     -0.035      0.034      0.000      0.997   9792.332
##   info_t3, ne_t3                     0.000      0.017     -0.033      0.034      0.000      0.995   8520.537
##   info_t3, om_t3                    -0.000      0.017     -0.034      0.033      0.000      0.994   7526.161
##   ag_t3, cont_t3                     0.000      0.017     -0.034      0.033      0.000      0.996   7172.254
##   ag_t3, extra_t3                    0.000      0.017     -0.034      0.033      0.000      0.999   7048.595
##   ag_t3, male                        0.000      0.017     -0.035      0.033      0.000      0.991   9281.945
##   ag_t3, ne_t3                      -0.000      0.017     -0.034      0.033      0.000      0.990   9099.477
##   ag_t3, om_t3                       0.000      0.017     -0.034      0.034      0.000      0.996   7744.517
##   cont_t3, extra_t3                  0.000      0.017     -0.034      0.034      0.000      0.999   7756.853
##   cont_t3, male                      0.000      0.017     -0.033      0.035      0.000      0.987   8264.280
##   cont_t3, ne_t3                    -0.000      0.017     -0.034      0.034      0.000      0.990  10071.303
##   cont_t3, om_t3                     0.000      0.017     -0.034      0.034      0.000      0.999   9326.306
##   extra_t3, male                     0.000      0.017     -0.034      0.034      0.000      0.998   9774.632
##   extra_t3, ne_t3                    0.000      0.017     -0.033      0.035      0.000      0.991   9888.159
##   extra_t3, om_t3                    0.000      0.017     -0.034      0.033      0.000      0.999   9090.814
##   male, ne_t3                        0.000      0.017     -0.033      0.034      0.000      0.993   9567.853
##   male, om_t3                       -0.000      0.017     -0.034      0.033      0.000      0.985   9875.394
##   ne_t3, om_t3                      -0.000      0.017     -0.035      0.033      0.000      0.983   7479.601
##
##                               ----------------------------------------------------------------------------
##
##
##
```

```
## OUTCOME MODEL ESTIMATES:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##   primary block:
##
## Latent Variable:   subjects
##
## Parameters                          Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    -----------------------------------------------------------------------------
## Variances:
##   Residual Var.                      123.561      4.112    115.359    131.540        ---        ---   1047.857
##
## Proportion Variance Explained
##   by Coefficients                      0.000      0.000      0.000      0.000        ---        ---        nan
##   by Residual Variation                1.000      0.000      1.000      1.000        ---        ---        nan
##
##                                    -----------------------------------------------------------------------------
##
##
##
## Outcome Variable:  info_t1
##
## Parameters                          Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    -----------------------------------------------------------------------------
## Variances:
##   Residual Var.                      167.366      3.045    161.584    173.545        ---        ---   1281.032
##
## Coefficients:
##   Intercept                           67.046      0.249     66.550     67.528  72212.585      0.000   2373.977
##   subjects                           @ 1.000        ---        ---        ---        ---        ---        ---
##
## Standardized Coefficients:
##   subjects                             0.652      0.008      0.636      0.666   7364.361      0.000    887.714
```

```
## Proportion Variance Explained
##   by Coefficients                    0.425      0.010      0.405      0.443        ---        ---    887.884
##   by Residual Variation              0.575      0.012      0.554      0.599        ---        ---   1231.701
##
##                                    -------------------------------------------------------------------------------
##
##
##
## Outcome Variable:  info_t2
##
## Parameters                        Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                    167.366      3.045    161.584    173.545        ---        ---   1281.032
##
## Coefficients:
##   Intercept                         63.945      0.271     63.400     64.457  55476.123      0.000   1914.174
##   subjects                         @ 1.000        ---        ---        ---        ---        ---        ---
##
## Standardized Coefficients:
##   subjects                           0.652      0.008      0.636      0.666   7364.361      0.000    887.714
##
## Proportion Variance Explained
##   by Coefficients                    0.425      0.010      0.405      0.443        ---        ---    887.884
##   by Residual Variation              0.575      0.012      0.554      0.599        ---        ---   1231.701
##
##                                    -------------------------------------------------------------------------------
##
##
##
```

```
## Outcome Variable:  info_t3
##
## Parameters                          Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    ------------------------------------------------------------------------------
## Variances:
##   Residual Var.                      167.366      3.045    161.584    173.545        ---        ---   1281.032
##
## Coefficients:
##   Intercept                           68.752      0.250     68.264     69.243  75631.476      0.000   2257.328
##   subjects                           @ 1.000        ---        ---        ---        ---        ---        ---
##
## Standardized Coefficients:
##   subjects                             0.652      0.008      0.636      0.666   7364.361      0.000    887.714
##
## Proportion Variance Explained
##   by Coefficients                      0.425      0.010      0.405      0.443        ---        ---    887.884
##   by Residual Variation                0.575      0.012      0.554      0.599        ---        ---   1231.701
##
##                                    ------------------------------------------------------------------------------
##
##
##   auxiliary block:
##
## Outcome Variable:  ag_t3
##
## Parameters                          Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    ------------------------------------------------------------------------------
## Variances:
##   Residual Var.                        0.277      0.006      0.265      0.288        ---        ---   4449.682
##
## Coefficients:
##   Intercept                            3.992      0.059      3.875      4.108   4561.074      0.000   2881.916
##   ne_t3                               -0.244      0.012     -0.269     -0.221    394.309      0.000   3979.386
##   male                                -0.350      0.017     -0.383     -0.318    433.923      0.000   4125.184
##   info_t1                             -0.001      0.001     -0.003      0.000      2.556      0.110   1557.848
```

```
##   info_t2                          0.001      0.001     -0.001      0.002      0.657      0.418   2094.361
##   info_t3                          0.009      0.001      0.008      0.010    213.909      0.000   3262.595
##
## Standardized Coefficients:
##   ne_t3                           -0.279      0.013     -0.305     -0.253    432.379      0.000   4106.745
##   male                            -0.292      0.013     -0.318     -0.266    487.477      0.000   4137.343
##   info_t1                         -0.034      0.021     -0.076      0.007      2.558      0.110   1560.812
##   info_t2                          0.017      0.020     -0.024      0.056      0.657      0.418   2092.062
##   info_t3                          0.251      0.017      0.218      0.284    224.935      0.000   3269.395
##
## Proportion Variance Explained
##   by Coefficients                  0.217      0.011      0.196      0.238        ---        ---   3896.614
##   by Residual Variation            0.783      0.011      0.762      0.804        ---        ---   3896.614
##
##                                -------------------------------------------------------------------------------
##
##
##
## Outcome Variable:  cont_t3
##
## Parameters                      Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                    0.315      0.007      0.302      0.328        ---        ---   4538.900
##
## Coefficients:
##   Intercept                        3.419      0.096      3.235      3.612   1268.591      0.000   3794.677
##   om_t3                           -0.003      0.015     -0.033      0.027      0.043      0.837   4517.832
##   ag_t3                            0.288      0.017      0.256      0.322    292.138      0.000   4412.510
##   ne_t3                           -0.326      0.013     -0.352     -0.300    593.729      0.000   4208.406
##   male                            -0.285      0.019     -0.323     -0.248    221.995      0.000   3979.468
##   info_t1                         -0.003      0.001     -0.004     -0.001      9.033      0.003   1866.573
##   info_t2                          0.001      0.001     -0.000      0.003      2.334      0.127   1953.631
##   info_t3                          0.002      0.001      0.001      0.004     12.359      0.000   3291.572
##
```

```
## Standardized Coefficients:
##   om_t3                         -0.003     0.014    -0.031     0.025     0.043     0.837   4512.667
##   ag_t3                          0.258     0.015     0.230     0.287   310.404     0.000   4406.675
##   ne_t3                         -0.333     0.013    -0.358    -0.308   672.617     0.000   4082.180
##   male                          -0.213     0.014    -0.240    -0.186   235.082     0.000   4099.362
##   info_t1                       -0.061     0.020    -0.102    -0.022     9.034     0.003   1855.839
##   info_t2                        0.030     0.020    -0.008     0.070     2.335     0.126   1950.069
##   info_t3                        0.060     0.017     0.026     0.093    12.396     0.000   3287.340
##
## Proportion Variance Explained
##   by Coefficients                0.284     0.011     0.263     0.306       ---       ---   3723.590
##   by Residual Variation          0.716     0.011     0.694     0.737       ---       ---   3723.590
##
##                            ------------------------------------------------------------------------------
##
##
##
## Outcome Variable:  extra_t3
##
## Parameters                    Estimate    StdDev      2.5%     97.5%     ChiSq    PValue      N_Eff
##                            ------------------------------------------------------------------------------
## Variances:
##   Residual Var.                  0.323     0.007     0.309     0.337       ---       ---   4303.299
##
## Coefficients:
##   Intercept                      2.927     0.111     2.707     3.144   692.242     0.000   3841.033
##   cont_t3                        0.188     0.016     0.157     0.218   146.027     0.000   4796.542
##   om_t3                          0.251     0.015     0.221     0.280   269.225     0.000   4481.307
##   ag_t3                         -0.014     0.018    -0.049     0.020     0.664     0.415   4537.628
##   ne_t3                         -0.291     0.015    -0.320    -0.262   394.992     0.000   4682.757
##   male                          -0.058     0.020    -0.097    -0.019     8.700     0.003   4078.061
##   info_t1                       -0.001     0.001    -0.002     0.001     0.428     0.513   1753.034
##   info_t2                       -0.002     0.001    -0.003    -0.000     6.847     0.009   1857.099
##   info_t3                       -0.002     0.001    -0.003    -0.000     5.308     0.021   3317.679
##
```

```
## Standardized Coefficients:
##   cont_t3                          0.191     0.016     0.160     0.222   149.769     0.000   4745.899
##   om_t3                            0.239     0.014     0.211     0.267   285.080     0.000   4460.173
##   ag_t3                           -0.013     0.016    -0.045     0.019     0.664     0.415   4532.165
##   ne_t3                           -0.304     0.015    -0.333    -0.275   430.756     0.000   4566.455
##   male                            -0.044     0.015    -0.074    -0.015     8.722     0.003   4078.385
##   info_t1                         -0.014     0.021    -0.054     0.028     0.428     0.513   1751.569
##   info_t2                         -0.055     0.021    -0.096    -0.014     6.859     0.009   1848.300
##   info_t3                         -0.041     0.018    -0.074    -0.006     5.313     0.021   3316.816
##
## Proportion Variance Explained
##   by Coefficients                  0.236     0.011     0.214     0.257       ---       ---   4635.554
##   by Residual Variation            0.764     0.011     0.743     0.786       ---       ---   4635.554
##
##                                -------------------------------------------------------------------------------
##
##
##
## Outcome Variable:  male
##
## Parameters                      Estimate    StdDev      2.5%     97.5%     ChiSq    PValue      N_Eff
##                                -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                    1.000     0.000     1.000     1.000       ---       ---        nan
##
## Coefficients:
##   Intercept                        0.708     0.090     0.530     0.882    61.534     0.000   2763.140
##   info_t1                          0.001     0.002    -0.002     0.004     0.250     0.617   1790.489
##   info_t2                         -0.006     0.001    -0.009    -0.003    19.569     0.000   1630.530
##   info_t3                         -0.008     0.001    -0.010    -0.005    31.020     0.000   1866.857
##
## Thresholds:
##   Tau 1                            0.000     0.000     0.000     0.000       ---       ---        nan
```

```
## Standardized Coefficients:
##   info_t1                             0.012      0.024     -0.035      0.058      0.249      0.617   1791.765
##   info_t2                            -0.109      0.024     -0.158     -0.062     19.856      0.000   1631.822
##   info_t3                            -0.129      0.023     -0.175     -0.085     31.574      0.000   1854.470
##
## Proportion Variance Explained
##   by Coefficients                     0.040      0.007      0.027      0.054        ---        ---   2316.473
##   by Residual Variation               0.960      0.007      0.946      0.973        ---        ---   2316.473
##
##                                ------------------------------------------------------------------------------
##
##
##
## Outcome Variable:  ne_t3
##
## Parameters                         Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                ------------------------------------------------------------------------------
## Variances:
##   Residual Var.                       0.441      0.010      0.423      0.460        ---        ---   4548.032
##
## Coefficients:
##   Intercept                           2.905      0.061      2.784      3.023   2248.396      0.000   2491.780
##   male                               -0.267      0.021     -0.306     -0.226    168.503      0.000   3514.261
##   info_t1                             0.002      0.001     -0.000      0.004      2.161      0.142   1603.186
##   info_t2                             0.001      0.001     -0.000      0.003      2.531      0.112   2213.204
##   info_t3                            -0.003      0.001     -0.004     -0.002     16.294      0.000   3265.648
##
## Standardized Coefficients:
##   male                               -0.195      0.015     -0.223     -0.166    177.070      0.000   3460.093
##   info_t1                             0.035      0.024     -0.011      0.082      2.163      0.141   1596.276
##   info_t2                             0.036      0.023     -0.009      0.081      2.533      0.111   2215.816
##   info_t3                            -0.075      0.019     -0.111     -0.038     16.366      0.000   3261.046
##
## Proportion Variance Explained
##   by Coefficients                     0.042      0.006      0.031      0.055        ---        ---   3357.638
```

```
##   by Residual Variation             0.958     0.006     0.945     0.969       ---       ---  3357.638
##
##                                    -------------------------------------------------------------------------------
##
##
##
## Outcome Variable:  om_t3
##
## Parameters                       Estimate    StdDev      2.5%     97.5%     ChiSq    PValue     N_Eff
##                                    -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                     0.322     0.007     0.309     0.336       ---       ---  4855.543
##
## Coefficients:
##   Intercept                         1.900     0.092     1.718     2.078   424.624     0.000  3592.720
##   ag_t3                             0.248     0.017     0.215     0.280   218.257     0.000  4455.998
##   ne_t3                             0.010     0.014    -0.017     0.037     0.546     0.460  4304.425
##   male                              0.001     0.019    -0.037     0.038     0.001     0.977  4052.634
##   info_t1                           0.002     0.001     0.000     0.004     6.037     0.014  1961.858
##   info_t2                           0.002     0.001     0.000     0.003     5.620     0.018  2410.222
##   info_t3                           0.008     0.001     0.007     0.009   141.384     0.000  3349.828
##
## Standardized Coefficients:
##   ag_t3                             0.237     0.016     0.206     0.267   232.150     0.000  4470.708
##   ne_t3                             0.011     0.015    -0.018     0.041     0.546     0.460  4298.176
##   male                              0.000     0.015    -0.029     0.030     0.001     0.977  4050.883
##   info_t1                           0.053     0.021     0.011     0.094     6.053     0.014  1963.183
##   info_t2                           0.049     0.021     0.009     0.090     5.632     0.018  2407.597
##   info_t3                           0.216     0.018     0.180     0.250   146.377     0.000  3277.671
##
## Proportion Variance Explained
##   by Coefficients                   0.164     0.010     0.144     0.184       ---       ---  3631.591
##   by Residual Variation             0.836     0.010     0.816     0.856       ---       ---  3631.591
##
##                                    -------------------------------------------------------------------------------
```

MCMC REPEATED MEASURES ANALYSIS WITH BETWEEN-SUBJECTS PREDICTOR

This section illustrates a repeated measures analysis with a between-group predictor, with and without auxiliary variables. The conditionally MAR assumption is defined with respect to the observed variables in each model. Missing predictor variables require their own model for missing data handling. MCMC can accommodate continuous and discrete predictors. Attempts to use the three-group gender variable an auxiliary variable (1 = female, 2 = male, 3 = non-binary) failed. Convergence required a long burn-in period, and MCMC diagnostics indicated a lack of support from the data (e.g., `N_eff` values less than 100). This analysis instead uses the `male` dummy code as an auxiliary variable (0 = female, 1 = male, NA = else).

The `ordinal` command invokes a categorical variable model for binary or ordered categorical variables. Alternatively, binary and multicategorical variables can be listed on the `nominal` line, which automatically creates dummy codes with lowest codes the reference. A latent variable named `subjects` represents the random subjects factor in a within-subjects design. The @ symbol is used to label parameters and fix them at specific values. Constraints on residual variance creates the compound symmetry assumption, which can be relaxed. Means are labeled and used to obtain a custom ANOVA-style significance test. Group means are defined as additional parameters. The `waldtest` command specifies Wald chi-square tests, which are analogous to F statistics. A Wald test is a chi-square statistic equal to the sum of squared standardized differences between the model parameters and the null. A single-df Wald test is a squared z-statistic. The Wald tests use parameter labels to specify ANOVA-style significance tests. A significant test statistic refutes the null, indicating a difference or effect.

```
# MCMC estimation
mcmc_repeated_gender <- rblimp(
  data = carsdat,
  ordinal = 'male', # Define binary or ordinal variable
  latent = 'subjects', # Define latent variable for subjects factor
  model = '
    # Random subjects factor with mean = 0 and loadings = 1
```

```
    subjects ~ intercept@0;
    info_t1 ~ intercept@mu1 subjects@1 male@dif1; # @ labels means and differences and fixes subject loadings to 1
    info_t2 ~ intercept@mu2 subjects@1 male@dif2;
    info_t3 ~ intercept@mu3 subjects@1 male@dif3;
    # Equal residual variances gives compound symmetry assumption, relax if needed
    info_t1 ~~ info_t1@res;  # @ labels the variances, setting them equal
    info_t2 ~~ info_t2@res;
    info_t3 ~~ info_t3@res;',
  waldtest = 'dif1 = dif2; dif2 = dif3;', # Test of the group-by-time interaction
  parameters = ' # Define group means
    fem_mu1 = mu1;
    fem_mu2 = mu2;
    fem_mu3 = mu3;
    male_mu1 = mu1 + dif1;
    male_mu2 = mu2 + dif2;
    male_mu3 = mu3 + dif3;',
  seed = 90291, # Integer random number seed
  burn = 5000, # Warm up iterations
  iter = 10000) # Iterations for analysis
```

Check the last row of the `BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT` table to verify that the value in the `Highest PSR` column is less than 1.05 (indicating convergence). The `DATA INFORMATION` section displays a table with missing data patterns. The `MODEL FIT` section contains custom Wald significance tests, if specified. Each outcome variable has its own summary table. Check the `N_Eff` diagnostic column to verify that all values are greater than 100 (indicating adequate support from the data). The `Estimate` column contains the Bayesian point estimates (posterior medians), the `StdDev` column contains "Bayesian standard errors", and the 2.5% and 97.5% columns are 95% credible intervals. The `ChiSq` and `PValue` columns are frequentist test statistics (squared z-statistics). Group means appear in a table near the bottom of the output labeled `GENERATED PARAMETERS`.

```
# Print output
output(mcmc_repeated_gender)
```

```
##
## ---------------------------------------------------------------------------
##
##                                    Blimp
##                                   3.2.20
##
##              Blimp was developed with funding from Institute of
##            Education Sciences awards R305D150056 and R305D190002.
##
##             Craig K. Enders, P.I. Email: cenders@psych.ucla.edu
##            Brian T. Keller, Co-P.I. Email: btkeller@missouri.edu
##                  Han Du, Co-P.I. Email: hdu@psych.ucla.edu
##                  Roy Levy, Co-P.I. Email: roy.levy@asu.edu
##
##               Programming and Blimp Studio by Brian T. Keller
##
##                    There is no expressed license given.
##
## ---------------------------------------------------------------------------
##
##
## ALGORITHMIC OPTIONS SPECIFIED:
##
##   Imputation method:                 Fully Bayesian model-based
##   MCMC algorithm:                    Full conditional Metropolis sampler with
##                                        Auto-Derived Conditional Distributions
##   Between-cluster imputation model:  Not applicable, single-level imputation
##   Prior for random effect variances: Not applicable, single-level imputation
##   Prior for residual variances:      Zero sum of squares, df = -2 (PRIOR2)
##   Prior for predictor variances:     Unit sum of squares, df = 2 (XPRIOR1)
##   Chain Starting Values:             Random starting values
##
##
##   NOTE: The default prior for regression coefficients
##         in categorical models is 'normal( 0.0, 5.0)'
```

```
##
##
## BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:
##
##   NOTE: Split chain PSR is being used. This splits each chain's
##         iterations to create twice as many chains.
##
##   Comparing iterations across 2 chains     Highest PSR   Parameter #
##                            126 to 250            1.781            24
##                            251 to 500            1.211             1
##                            376 to 750            1.030             4
##                            501 to 1000           1.041             4
##                            626 to 1250           1.046             8
##                            751 to 1500           1.021            13
##                            876 to 1750           1.008            13
##                           1001 to 2000           1.025            16
##                           1126 to 2250           1.011            31
##                           1251 to 2500           1.006            24
##                           1376 to 2750           1.005             5
##                           1501 to 3000           1.008            23
##                           1626 to 3250           1.016            10
##                           1751 to 3500           1.006            33
##                           1876 to 3750           1.012            21
##                           2001 to 4000           1.005            21
##                           2126 to 4250           1.005            15
##                           2251 to 4500           1.005            26
##                           2376 to 4750           1.006             1
##                           2501 to 5000           1.006            18
##
##
```

```
## METROPOLIS-HASTINGS ACCEPTANCE RATES:
##
##   Chain 1:
##
##     Variable                            Type    Probability   Target Value
##     male                          imputation          0.502          0.500
##
##   NOTE: Suppressing printing of 1 chains.
##         Use keyword 'tuneinfo' in options to override.
##
##
## DATA INFORMATION:
##
##   Sample Size:              6726
##   Missing Data Info:
##                          miss %      1     2     3     4     5     6     7     8     9    10
##                                 --------------------------------------------------------------
##                  info_t1 = 34.2      -     -     M     -     M     M     -     M     M     -
##                  info_t2 = 48.9      -     M     M     M     -     -     -     M     M     -
##                  info_t3 = 36.0      -     M     -     -     M     -     M     M     -     -
##                     male =  1.3      -     -     -     -     -     -     -     -     M     M
##                                 --------------------------------------------------------------
##                                 %  28.2  18.2  15.1  13.2  10.4   6.4   5.8   1.4   0.8   0.2
##
##   NOTE: Suppressing printing of 5 missing data patterns.
##         These patterns consist of 0.4% of the data.
##         Use keyword 'allpatterns' in options to print all patterns.
##
##
## MODEL INFORMATION:
##
##   NUMBER OF PARAMETERS
##     Outcome Models:         8
##     Generated Parameters:   6
##     Predictor Models:       1
```

```
##
##   PREDICTORS
##     Incomplete ordinal:     male
##
##   FACTORS
##     Level-1:                subjects
##
##   MODELS
##     [1]  subjects ~ Intercept@0
##     [2]  info_t1 ~ Intercept@mu1 subjects@1 male@dif1
##     [3]  info_t2 ~ Intercept@mu2 subjects@1 male@dif2
##     [4]  info_t3 ~ Intercept@mu3 subjects@1 male@dif3
##
##   GENERATED PARAMETERS
##     [1]  fem_mu1 = mu1
##     [2]  fem_mu2 = mu2
##     [3]  fem_mu3 = mu3
##     [4]  male_mu1 = mu1+dif1
##     [5]  male_mu2 = mu2+dif2
##     [6]  male_mu3 = mu3+dif3
##
##
##  WARNING MESSAGES:
##
##   No warning messages.
##
##
## MODEL FIT:
##
##
##   INFORMATION CRITERIA
##
##     Conditional Likelihood
##       DIC2                  216299.477
##       WAIC                  221994.197
```

```
##
##   WALD TESTS (Asparouhov & Muthén, 2021)
##
##   Test #1
##
##     Full:
##       [1]  info_t1 ~ Intercept@mu1 subjects@1 male@dif1
##       [2]  info_t2 ~ Intercept@mu2 subjects@1 male@dif2
##       [3]  info_t3 ~ Intercept@mu3 subjects@1 male@dif3
##
##     Restricted:
##       [1]  info_t1 ~ Intercept@mu1 subjects@1 male@dif1
##       [2]  info_t2 ~ Intercept@mu2 subjects@1 male@dif2
##       [3]  info_t3 ~ Intercept@mu3 subjects@1 male@dif3
##
##     Constraints in Restricted:
##       [1]  dif1 = dif2
##       [2]  dif2 = dif3
##
##
##     Wald Statistic (Chi-Square)                32.048
##     Number of Parameters Tested (df)                2
##     Probability                                 0.000
##
##
## CORRELATIONS AMONG RESIDUALS:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##
```

```
## Correlations                              Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                  -------------------------------------------------------------------------------
##
##   subjects, info_t1                        -0.088      0.016     -0.119     -0.056     28.836      0.000   7152.203
##   subjects, info_t2                         0.070      0.016      0.039      0.101     19.412      0.000   8760.187
##   subjects, info_t3                         0.011      0.016     -0.021      0.042      0.442      0.506   6636.115
##   info_t1, info_t2                         -0.009      0.017     -0.042      0.022      0.336      0.562   8324.493
##   info_t1, info_t3                         -0.002      0.016     -0.034      0.030      0.018      0.893   8303.831
##   info_t2, info_t3                          0.011      0.017     -0.021      0.043      0.473      0.492   7294.851
##
##                                  -------------------------------------------------------------------------------
##
##
##
## OUTCOME MODEL ESTIMATES:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##
## Latent Variable:   subjects
##
## Parameters                                Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                  -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                           119.741      4.228    111.573    128.202        ---        ---    870.186
##
## Proportion Variance Explained
##   by Coefficients                           0.000      0.000      0.000      0.000        ---        ---        nan
##   by Residual Variation                     1.000      0.000      1.000      1.000        ---        ---        nan
##
##                                  -------------------------------------------------------------------------------
##
##
##
```

```
## Outcome Variable:  info_t1
##
## Parameters                        Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                   ------------------------------------------------------------------------------
## Variances:
##   Residual Var.                    166.720      3.113    160.678    173.021        ---        ---    971.520
##
## Coefficients:
##   Intercept                         67.881      0.322     67.249     68.524  44446.849      0.000   2073.418
##   subjects                        @ 1.000        ---        ---        ---        ---        ---        ---
##   male                              -2.019      0.506     -3.000     -1.041     15.910      0.000   1942.129
##
## Standardized Coefficients:
##   subjects                           0.645      0.008      0.629      0.661   6288.266      0.000    678.774
##   male                              -0.059      0.015     -0.088     -0.031     15.971      0.000   1935.208
##
## Proportion Variance Explained
##   by Coefficients                    0.420      0.010      0.399      0.440        ---        ---    684.285
##   by Residual Variation              0.580      0.012      0.556      0.604        ---        ---    920.134
##
##                                   ------------------------------------------------------------------------------
##
##
##
## Outcome Variable:  info_t2
##
## Parameters                        Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                   ------------------------------------------------------------------------------
## Variances:
##   Residual Var.                    166.720      3.113    160.678    173.021        ---        ---    971.520
##
## Coefficients:
##   Intercept                         66.175      0.358     65.475     66.898  34164.674      0.000   2029.921
##   subjects                        @ 1.000        ---        ---        ---        ---        ---        ---
##   male                              -5.269      0.578     -6.394     -4.145     83.030      0.000   1775.493
```

```
## Standardized Coefficients:
##   subjects                          0.639      0.008      0.622      0.654   6157.732      0.000    684.998
##   male                             -0.153      0.016     -0.184     -0.120     85.739      0.000   1765.218
##
## Proportion Variance Explained
##   by Coefficients                   0.432      0.011      0.411      0.452        ---        ---    719.210
##   by Residual Variation             0.568      0.012      0.545      0.593        ---        ---    949.507
##
##                                   -------------------------------------------------------------------------------
##
##
##
## Outcome Variable:  info_t3
##
## Parameters                       Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                   -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                   166.720      3.113    160.678    173.021        ---        ---    971.520
##
## Coefficients:
##   Intercept                        70.890      0.327     70.263     71.541  46954.771      0.000   2009.250
##   subjects                        @ 1.000        ---        ---        ---        ---        ---        ---
##   male                             -4.919      0.509     -5.928     -3.923     93.297      0.000   1953.644
##
## Standardized Coefficients:
##   subjects                          0.640      0.008      0.623      0.656   6217.873      0.000    679.294
##   male                             -0.143      0.015     -0.171     -0.114     95.711      0.000   1924.900
##
## Proportion Variance Explained
##   by Coefficients                   0.430      0.010      0.409      0.450        ---        ---    713.657
##   by Residual Variation             0.570      0.012      0.547      0.594        ---        ---    944.851
##
##                                   -------------------------------------------------------------------------------
##
##
```

```
## GENERATED PARAMETERS:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##
## Parameters                          Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    -------------------------------------------------------------------------------
##
##   fem_mu1                             67.881      0.322     67.249     68.524  44446.849      0.000   2073.418
##   fem_mu2                             66.175      0.358     65.475     66.898  34164.674      0.000   2029.921
##   fem_mu3                             70.890      0.327     70.263     71.541  46954.771      0.000   2009.250
##   male_mu1                            65.861      0.389     65.110     66.629  28645.249      0.000   1827.840
##   male_mu2                            60.905      0.446     60.045     61.777  18669.053      0.000   1594.066
##   male_mu3                            65.983      0.392     65.204     66.757  28380.563      0.000   1991.902
##
##                                    -------------------------------------------------------------------------------
##
##
## PREDICTOR MODEL ESTIMATES:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##
##
## Missing predictor:  male
##
## Parameters                          Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    -------------------------------------------------------------------------------
##
## Grand Mean                            -0.162      0.015     -0.193     -0.132    110.488      0.000   4302.707
##
## Level 1:
##   Residual Var.                        1.000      0.000      1.000      1.000        ---        ---        nan
```

```
## Thresholds:
##   Tau 1                          0.000     0.000     0.000     0.000        ---        ---        nan
##
##                             ------------------------------------------------------------------------------
```

See the previous analysis for a description of the repeated measures analysis components. The `ordinal` command invokes a categorical variable model for binary or ordered categorical variables. Alternatively, binary and multicategorical variables can be listed on the `nominal` line, which automatically creates dummy codes with lowest codes the reference. Auxiliary variables use a sequential specification (extra dependent variable) approach. Auxiliary variables are listed to the left of the tilde and primary variables to the right. For example, `A3 A2 A1 ~ Y X` produces three regressions: `A1 ~ Y X`, `A2 ~ A1 Y X`, and `A3 ~ A2 A1 Y X`. The output includes summary tables for these additional models, which can be ignored.

```
# MCMC estimation with auxiliary variables
mcmc_repeated_gender_aux <- rblimp(
  data = carsdat,
  ordinal = 'male', # Define binary or ordinal variable
  latent = 'subjects', # Define latent variable for subjects factor
  model = '
    primary: # This is an arbitrary and optional label that groups summary tables
    # Random subjects factor with mean = 0 and loadings = 1
    subjects ~ intercept@0;
    info_t1 ~ intercept@mu1 subjects@1 male@dif1; # @ labels means and differences and fixes subject loadings to 1
    info_t2 ~ intercept@mu2 subjects@1 male@dif2;
    info_t3 ~ intercept@mu3 subjects@1 male@dif3;
    # Equal residual variances gives compound symmetry assumption, relax if needed
    info_t1 ~~ info_t1@res;  # @ labels the variances, setting them equal
    info_t2 ~~ info_t2@res;
    info_t3 ~~ info_t3@res;
    auxiliary: # This is an arbitrary and optional label that groups summary tables
    # Auxiliary variables ~ primary variables
    extra_t3 cont_t3 om_t3 ag_t3 ne_t3 ~ info_t1 info_t2 info_t3 male;',
  waldtest = 'dif1 = dif2; dif2 = dif3;', # Test of the group-by-time interaction
  parameters = ' # Define group means
```

```
    fem_mu1 = mu1;
    fem_mu2 = mu2;
    fem_mu3 = mu3;
    male_mu1 = mu1 + dif1;
    male_mu2 = mu2 + dif2;
    male_mu3 = mu3 + dif3;',
  seed = 90291, # Integer random number seed
  burn = 5000, # Warm up iterations
  iter = 10000) # Iterations for analysis
```

Check the last row of the `BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT` table to verify that the value in the `Highest PSR` column is less than 1.05 (indicating convergence). The `DATA INFORMATION` section displays a table with missing data patterns. The `MODEL FIT` section contains custom Wald significance tests, if specified. Each outcome variable has its own summary table. Check the `N_Eff` diagnostic column to verify that all values are greater than 100 (indicating adequate support from the data). The `Estimate` column contains the Bayesian point estimates (posterior medians), the `StdDev` column contains "Bayesian standard errors", and the 2.5% and 97.5% columns are 95% credible intervals. The `ChiSq` and `PValue` columns are frequentist test statistics (squared z-statistics). Group means appear in a table near the bottom of the output labeled `GENERATED PARAMETERS`.

```
# Print output
output(mcmc_repeated_gender_aux)

##
## ---------------------------------------------------------------------------
##
##                                    Blimp
##                                   3.2.20
##
##               Blimp was developed with funding from Institute of
##             Education Sciences awards R305D150056 and R305D190002.
##
##              Craig K. Enders, P.I. Email: cenders@psych.ucla.edu
```

```
##            Brian T. Keller, Co-P.I. Email: btkeller@missouri.edu
##                  Han Du, Co-P.I. Email: hdu@psych.ucla.edu
##                  Roy Levy, Co-P.I. Email: roy.levy@asu.edu
##
##               Programming and Blimp Studio by Brian T. Keller
##
##                    There is no expressed license given.
##
## ---------------------------------------------------------------------------
##
##
## ALGORITHMIC OPTIONS SPECIFIED:
##
##   Imputation method:                 Fully Bayesian model-based
##   MCMC algorithm:                    Full conditional Metropolis sampler with
##                                        Auto-Derived Conditional Distributions
##   Between-cluster imputation model:  Not applicable, single-level imputation
##   Prior for random effect variances: Not applicable, single-level imputation
##   Prior for residual variances:      Zero sum of squares, df = -2 (PRIOR2)
##   Prior for predictor variances:     Unit sum of squares, df = 2 (XPRIOR1)
##   Chain Starting Values:             Random starting values
##
##
##   NOTE: The default prior for regression coefficients
##         in categorical models is 'normal( 0.0, 5.0)'
##
##
## BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:
##
##   NOTE: Split chain PSR is being used. This splits each chain's
##         iterations to create twice as many chains.
##
##   Comparing iterations across 2 chains     Highest PSR   Parameter #
##                           126 to 250            2.253            16
##                           251 to 500            4.191            11
```

```
##                            376 to 750            1.062             1
##                            501 to 1000           1.040            26
##                            626 to 1250           1.031            18
##                            751 to 1500           1.018            13
##                            876 to 1750           1.021            18
##                           1001 to 2000           1.014            23
##                           1126 to 2250           1.021            16
##                           1251 to 2500           1.007            13
##                           1376 to 2750           1.023            13
##                           1501 to 3000           1.033            16
##                           1626 to 3250           1.014            16
##                           1751 to 3500           1.008            48
##                           1876 to 3750           1.013             1
##                           2001 to 4000           1.015            24
##                           2126 to 4250           1.028            16
##                           2251 to 4500           1.019            16
##                           2376 to 4750           1.008            16
##                           2501 to 5000           1.007            16
##
##
## METROPOLIS-HASTINGS ACCEPTANCE RATES:
##
##   Chain 1:
##
##     Variable                            Type    Probability   Target Value
##     male                          imputation          0.496          0.500
##
##   NOTE: Suppressing printing of 1 chains.
##         Use keyword 'tuneinfo' in options to override.
##
##
## DATA INFORMATION:
##
##   Sample Size:              6726
##   Missing Data Info:
```

```
##                          miss %       1     2     3     4     5     6     7     8     9    10
##                                  -------------------------------------------------------------
##                  info_t1 = 34.2       -     -     M     -     M     M     -     M     M     -
##                  info_t2 = 48.9       -     M     M     M     -     -     -     M     M     -
##                  info_t3 = 36.0       -     M     -     -     M     -     M     M     -     -
##                 extra_t3 = 36.1       -     M     -     -     M     -     M     M     -     -
##                  cont_t3 = 36.1       -     M     -     -     M     -     M     M     -     -
##                    om_t3 = 36.1       -     M     -     -     M     -     M     M     -     -
##                    ag_t3 = 36.1       -     M     -     -     M     -     M     M     -     -
##                    ne_t3 = 36.1       -     M     -     -     M     -     M     M     -     -
##                     male =  1.3       -     -     -     -     -     -     -     -     M     M
##                                  -------------------------------------------------------------
##                                  %  28.2  18.2  15.1  13.2  10.4   6.4   5.8   1.4   0.7   0.2
##
##   NOTE: Suppressing printing of 6 missing data patterns.
##         These patterns consist of 0.4% of the data.
##         Use keyword 'allpatterns' in options to print all patterns.
##
##
## MODEL INFORMATION:
##
##   NUMBER OF PARAMETERS
##     Outcome Models:         48
##     Generated Parameters:   6
##     Predictor Models:       1
##
##   PREDICTORS
##     Incomplete ordinal:     male
##
##   FACTORS
##     Level-1:                subjects
##
##   MODELS
##
##    primary:
```

```
##     [1]  subjects ~ Intercept@0
##     [2]  info_t1 ~ Intercept@mu1 subjects@1 male@dif1
##     [3]  info_t2 ~ Intercept@mu2 subjects@1 male@dif2
##     [4]  info_t3 ~ Intercept@mu3 subjects@1 male@dif3
##
##    auxiliary:
##     [5]  ag_t3 ~ Intercept ne_t3 info_t1 info_t2 info_t3 male
##     [6]  cont_t3 ~ Intercept om_t3 ag_t3 ne_t3 info_t1 info_t2 info_t3 male
##     [7]  extra_t3 ~ Intercept cont_t3 om_t3 ag_t3 ne_t3 info_t1 info_t2 info_t3
##                 male
##     [8]  ne_t3 ~ Intercept info_t1 info_t2 info_t3 male
##     [9]  om_t3 ~ Intercept ag_t3 ne_t3 info_t1 info_t2 info_t3 male
##
##   GENERATED PARAMETERS
##     [1]  fem_mu1 = mu1
##     [2]  fem_mu2 = mu2
##     [3]  fem_mu3 = mu3
##     [4]  male_mu1 = mu1+dif1
##     [5]  male_mu2 = mu2+dif2
##     [6]  male_mu3 = mu3+dif3
##
##
##  WARNING MESSAGES:
##
##   No warning messages.
##
##
## MODEL FIT:
##
##
##   INFORMATION CRITERIA
##
##     Conditional Likelihood
##       DIC2                  278349.136
##       WAIC                  288741.492
```

```
##   WALD TESTS (Asparouhov & Muthén, 2021)
##
##   Test #1
##
##     Full:
##       [1]  info_t1 ~ Intercept@mu1 subjects@1 male@dif1
##       [2]  info_t2 ~ Intercept@mu2 subjects@1 male@dif2
##       [3]  info_t3 ~ Intercept@mu3 subjects@1 male@dif3
##
##     Restricted:
##       [1]  info_t1 ~ Intercept@mu1 subjects@1 male@dif1
##       [2]  info_t2 ~ Intercept@mu2 subjects@1 male@dif2
##       [3]  info_t3 ~ Intercept@mu3 subjects@1 male@dif3
##
##     Constraints in Restricted:
##       [1]  dif1 = dif2
##       [2]  dif2 = dif3
##
##
##     Wald Statistic (Chi-Square)                33.123
##     Number of Parameters Tested (df)                2
##     Probability                                 0.000
##
##
## CORRELATIONS AMONG RESIDUALS:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##
## Correlations                           Estimate     StdDev       2.5%      97.5%       ChiSq     PValue       N_Eff
##                                    -------------------------------------------------------------------------------
##
##   subjects, info_t1                      -0.088      0.016     -0.120     -0.056      28.714      0.000    6146.172
##   subjects, info_t2                       0.070      0.016      0.039      0.102      19.085      0.000    8377.671
```

```
##   subjects, info_t3          0.010     0.016    -0.021     0.041     0.376     0.540   7220.182
##   subjects, ag_t3           -0.000     0.017    -0.034     0.034     0.000     0.995   9329.574
##   subjects, cont_t3         -0.000     0.017    -0.033     0.034     0.000     1.000   9423.070
##   subjects, extra_t3         0.000     0.017    -0.033     0.034     0.000     0.999   9711.871
##   subjects, ne_t3           -0.000     0.017    -0.034     0.033     0.000     0.986   8964.222
##   subjects, om_t3           -0.000     0.017    -0.034     0.034     0.000     0.990   9290.870
##   info_t1, info_t2          -0.010     0.017    -0.042     0.024     0.319     0.572   8072.052
##   info_t1, info_t3          -0.002     0.017    -0.034     0.031     0.013     0.908   8207.700
##   info_t1, ag_t3             0.000     0.017    -0.034     0.034     0.000     0.995   9072.630
##   info_t1, cont_t3           0.000     0.017    -0.033     0.034     0.000     0.992   8305.718
##   info_t1, extra_t3          0.000     0.017    -0.034     0.033     0.000     0.999   9263.560
##   info_t1, ne_t3            -0.000     0.017    -0.034     0.034     0.000     0.990   9259.490
##   info_t1, om_t3            -0.000     0.017    -0.033     0.034     0.000     0.993   9956.157
##   info_t2, info_t3           0.011     0.016    -0.021     0.044     0.471     0.492   8386.591
##   info_t2, ag_t3            -0.000     0.017    -0.034     0.034     0.000     1.000   8937.092
##   info_t2, cont_t3           0.000     0.017    -0.034     0.034     0.000     0.997   9117.482
##   info_t2, extra_t3          0.000     0.017    -0.034     0.034     0.000     0.999   9074.064
##   info_t2, ne_t3            -0.000     0.017    -0.033     0.034     0.000     0.998   9262.153
##   info_t2, om_t3             0.000     0.017    -0.034     0.033     0.000     0.999   9266.978
##   info_t3, ag_t3            -0.000     0.017    -0.033     0.034     0.000     0.996   8741.642
##   info_t3, cont_t3           0.000     0.017    -0.034     0.033     0.000     0.997   8181.713
##   info_t3, extra_t3          0.000     0.017    -0.034     0.034     0.000     0.995   7760.476
##   info_t3, ne_t3             0.000     0.017    -0.034     0.033     0.000     0.996   7873.397
##   info_t3, om_t3             0.000     0.017    -0.034     0.034     0.000     1.000   8328.146
##   ag_t3, cont_t3            -0.000     0.017    -0.034     0.034     0.000     0.994   7741.582
##   ag_t3, extra_t3           -0.000     0.017    -0.034     0.034     0.000     0.996   7256.506
##   ag_t3, ne_t3               0.000     0.017    -0.034     0.034     0.000     1.000   9989.274
##   ag_t3, om_t3              -0.000     0.017    -0.034     0.035     0.000     0.992   7403.291
##   cont_t3, extra_t3          0.000     0.017    -0.034     0.034     0.000     0.999   7076.566
##   cont_t3, ne_t3             0.001     0.017    -0.033     0.034     0.000     0.983   9714.471
##   cont_t3, om_t3             0.000     0.017    -0.034     0.034     0.000     0.999   9528.172
##   extra_t3, ne_t3           -0.000     0.017    -0.034     0.034     0.000     0.991  10000.869
##   extra_t3, om_t3           -0.000     0.017    -0.034     0.033     0.000     0.986   9540.128
##   ne_t3, om_t3               0.000     0.017    -0.033     0.034     0.000     0.990   8191.014
##
```

```
##                                 --------------------------------------------------------------------------------
##
##
##
## OUTCOME MODEL ESTIMATES:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##   primary block:
##
## Latent Variable:   subjects
##
## Parameters                         Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                 --------------------------------------------------------------------------------
## Variances:
##   Residual Var.                     120.175      4.288    111.844    128.672        ---        ---    944.671
##
## Proportion Variance Explained
##   by Coefficients                     0.000      0.000      0.000      0.000        ---        ---        nan
##   by Residual Variation               1.000      0.000      1.000      1.000        ---        ---        nan
##
##                                 --------------------------------------------------------------------------------
##
##
##
## Outcome Variable:  info_t1
##
## Parameters                         Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                 --------------------------------------------------------------------------------
## Variances:
##   Residual Var.                     166.571      3.161    160.439    172.945        ---        ---   1112.374
##
```

```
## Coefficients:
##   Intercept                         67.954      0.322     67.322     68.598  44445.588      0.000   2157.623
##   subjects                         @ 1.000        ---        ---        ---        ---        ---        ---
##   male                              -2.038      0.499     -3.024     -1.062     16.695      0.000   2062.233
##
## Standardized Coefficients:
##   subjects                           0.646      0.008      0.630      0.662   6202.366      0.000    778.239
##   male                              -0.060      0.015     -0.088     -0.031     16.762      0.000   2054.457
##
## Proportion Variance Explained
##   by Coefficients                    0.421      0.011      0.400      0.441        ---        ---    784.686
##   by Residual Variation              0.579      0.012      0.555      0.603        ---        ---   1052.147
##
##                                -----------------------------------------------------------------------------
##
##
##
## Outcome Variable:  info_t2
##
## Parameters                        Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                -----------------------------------------------------------------------------
## Variances:
##   Residual Var.                    166.571      3.161    160.439    172.945        ---        ---   1112.374
##
## Coefficients:
##   Intercept                         66.233      0.362     65.513     66.938  33523.741      0.000   1569.466
##   subjects                         @ 1.000        ---        ---        ---        ---        ---        ---
##   male                              -5.303      0.562     -6.415     -4.220     89.203      0.000   1587.404
##
## Standardized Coefficients:
##   subjects                           0.640      0.008      0.623      0.655   6040.640      0.000    815.201
##   male                              -0.154      0.016     -0.185     -0.123     92.286      0.000   1581.583
##
```

```
## Proportion Variance Explained
##   by Coefficients                  0.433      0.011      0.412      0.453        ---        ---    777.480
##   by Residual Variation            0.567      0.012      0.543      0.592        ---        ---   1030.909
##
##                             -------------------------------------------------------------------------------
##
##
##
## Outcome Variable:  info_t3
##
## Parameters                      Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                             -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                  166.571      3.161    160.439    172.945        ---        ---   1112.374
##
## Coefficients:
##   Intercept                       70.894      0.326     70.255     71.530  47302.378      0.000   2123.967
##   subjects                       @ 1.000        ---        ---        ---        ---        ---        ---
##   male                            -4.914      0.502     -5.892     -3.921     95.720      0.000   1979.720
##
## Standardized Coefficients:
##   subjects                         0.641      0.008      0.624      0.656   6143.042      0.000    795.942
##   male                            -0.142      0.014     -0.170     -0.114     98.396      0.000   1971.855
##
## Proportion Variance Explained
##   by Coefficients                  0.431      0.011      0.410      0.451        ---        ---    791.496
##   by Residual Variation            0.569      0.012      0.546      0.594        ---        ---   1048.678
##
##                             -------------------------------------------------------------------------------
##
##
```

```
##   auxiliary block:
##
## Outcome Variable:  ag_t3
##
## Parameters                          Estimate     StdDev       2.5%      97.5%       ChiSq     PValue      N_Eff
##                                  -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                        0.277      0.006      0.265      0.289         ---        ---   4563.922
##
## Coefficients:
##   Intercept                            3.992      0.060      3.874      4.108    4400.251      0.000   3101.126
##   ne_t3                               -0.245      0.012     -0.269     -0.221     407.911      0.000   4414.893
##   info_t1                             -0.001      0.001     -0.003      0.000       2.287      0.130   1744.944
##   info_t2                              0.001      0.001     -0.001      0.002       0.619      0.432   2028.196
##   info_t3                              0.009      0.001      0.007      0.010     217.664      0.000   3287.460
##   male                                -0.350      0.017     -0.383     -0.317     430.108      0.000   4055.075
##
## Standardized Coefficients:
##   ne_t3                               -0.279      0.013     -0.305     -0.253     446.173      0.000   4337.087
##   info_t1                             -0.033      0.022     -0.075      0.010       2.287      0.130   1740.606
##   info_t2                              0.016      0.021     -0.025      0.057       0.619      0.431   2030.543
##   info_t3                              0.251      0.017      0.217      0.282     228.882      0.000   3179.356
##   male                                -0.292      0.013     -0.318     -0.266     481.615      0.000   4145.406
##
## Proportion Variance Explained
##   by Coefficients                      0.217      0.011      0.196      0.238         ---        ---   4378.866
##   by Residual Variation                0.783      0.011      0.762      0.804         ---        ---   4378.866
##
##                                  -------------------------------------------------------------------------------
##
##
##
```

```
## Outcome Variable:  cont_t3
##
## Parameters                        Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                   -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                      0.315      0.007      0.301      0.328        ---        ---   4355.963
##
## Coefficients:
##   Intercept                          3.418      0.095      3.233      3.604   1300.552      0.000   4078.657
##   om_t3                             -0.003      0.015     -0.033      0.027      0.037      0.847   4581.108
##   ag_t3                              0.289      0.017      0.256      0.321    302.602      0.000   4565.874
##   ne_t3                             -0.326      0.014     -0.352     -0.299    576.353      0.000   4522.222
##   info_t1                           -0.003      0.001     -0.004     -0.001      9.569      0.002   1836.239
##   info_t2                            0.001      0.001     -0.000      0.003      2.471      0.116   2201.516
##   info_t3                            0.002      0.001      0.001      0.004     12.064      0.001   3153.485
##   male                              -0.285      0.019     -0.321     -0.247    223.795      0.000   4146.280
##
## Standardized Coefficients:
##   om_t3                             -0.003      0.014     -0.031      0.025      0.037      0.847   4583.278
##   ag_t3                              0.259      0.014      0.230      0.287    318.858      0.000   4550.383
##   ne_t3                             -0.333      0.013     -0.358     -0.307    655.687      0.000   4459.299
##   info_t1                           -0.062      0.020     -0.100     -0.022      9.565      0.002   1825.874
##   info_t2                            0.032      0.020     -0.008      0.071      2.471      0.116   2198.186
##   info_t3                            0.059      0.017      0.026      0.093     12.094      0.001   3143.066
##   male                              -0.213      0.014     -0.239     -0.185    236.438      0.000   4086.544
##
## Proportion Variance Explained
##   by Coefficients                    0.285      0.011      0.263      0.306        ---        ---   4533.153
##   by Residual Variation              0.715      0.011      0.694      0.737        ---        ---   4533.153
##
##                                   -------------------------------------------------------------------------------
##
##
##
```

```
## Outcome Variable:  extra_t3
##
## Parameters                             Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    -----------------------------------------------------------------------------
## Variances:
##   Residual Var.                           0.322      0.007      0.309      0.337        ---        ---   4889.301
##
## Coefficients:
##   Intercept                               2.924      0.109      2.711      3.139    719.882      0.000   3795.709
##   cont_t3                                 0.188      0.016      0.157      0.218    146.033      0.000   4661.104
##   om_t3                                   0.251      0.015      0.220      0.281    266.571      0.000   4454.749
##   ag_t3                                  -0.015      0.018     -0.049      0.020      0.698      0.403   4150.418
##   ne_t3                                  -0.291      0.015     -0.319     -0.263    401.842      0.000   4784.044
##   info_t1                                -0.000      0.001     -0.002      0.001      0.340      0.560   1819.468
##   info_t2                                -0.002      0.001     -0.003     -0.001      7.850      0.005   1807.785
##   info_t3                                -0.002      0.001     -0.003     -0.000      5.267      0.022   3034.452
##   male                                   -0.059      0.020     -0.097     -0.020      8.741      0.003   4215.538
##
## Standardized Coefficients:
##   cont_t3                                 0.192      0.016      0.160      0.222    150.419      0.000   4694.159
##   om_t3                                   0.240      0.014      0.211      0.267    281.052      0.000   4453.002
##   ag_t3                                  -0.014      0.016     -0.045      0.018      0.699      0.403   4153.995
##   ne_t3                                  -0.303      0.014     -0.332     -0.276    443.856      0.000   4613.094
##   info_t1                                -0.012      0.021     -0.053      0.028      0.340      0.560   1818.144
##   info_t2                                -0.057      0.020     -0.096     -0.017      7.859      0.005   1808.393
##   info_t3                                -0.041      0.018     -0.075     -0.006      5.275      0.022   3026.431
##   male                                   -0.045      0.015     -0.074     -0.015      8.772      0.003   4223.519
##
## Proportion Variance Explained
##   by Coefficients                         0.235      0.011      0.214      0.257        ---        ---   4257.798
##   by Residual Variation                   0.765      0.011      0.743      0.786        ---        ---   4257.798
##
##                                    -----------------------------------------------------------------------------
##
##
```

```
## Outcome Variable:  ne_t3
##
## Parameters                        Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                 ------------------------------------------------------------------------------
## Variances:
##   Residual Var.                      0.441      0.010      0.423      0.460        ---        ---   4449.380
##
## Coefficients:
##   Intercept                          2.902      0.060      2.784      3.022   2306.390      0.000   2647.479
##   info_t1                            0.002      0.001     -0.001      0.003      2.213      0.137   1776.785
##   info_t2                            0.001      0.001     -0.000      0.003      2.746      0.097   2010.736
##   info_t3                           -0.003      0.001     -0.004     -0.002     15.967      0.000   3141.569
##   male                              -0.266      0.021     -0.308     -0.225    159.723      0.000   3364.553
##
## Standardized Coefficients:
##   info_t1                            0.035      0.024     -0.012      0.081      2.216      0.137   1777.572
##   info_t2                            0.037      0.022     -0.007      0.081      2.746      0.098   2008.970
##   info_t3                           -0.075      0.019     -0.112     -0.038     16.044      0.000   3134.353
##   male                              -0.195      0.015     -0.224     -0.165    168.807      0.000   3330.010
##
## Proportion Variance Explained
##   by Coefficients                    0.042      0.006      0.031      0.055        ---        ---   3110.066
##   by Residual Variation              0.958      0.006      0.945      0.969        ---        ---   3110.066
##
##                                 ------------------------------------------------------------------------------
##
##
##
## Outcome Variable:  om_t3
##
## Parameters                        Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                 ------------------------------------------------------------------------------
## Variances:
##   Residual Var.                      0.322      0.007      0.309      0.336        ---        ---   4847.409
##
```

```
## Coefficients:
##   Intercept                         1.902      0.092      1.717      2.081    425.764      0.000   4038.836
##   ag_t3                             0.247      0.017      0.214      0.279    220.843      0.000   4568.167
##   ne_t3                             0.010      0.014     -0.017      0.037      0.490      0.484   4748.791
##   info_t1                           0.002      0.001      0.000      0.004      6.208      0.013   2174.188
##   info_t2                           0.002      0.001      0.000      0.003      5.230      0.022   2188.901
##   info_t3                           0.008      0.001      0.007      0.009    142.063      0.000   3437.170
##   male                              0.001      0.020     -0.037      0.039      0.004      0.952   3953.836
##
## Standardized Coefficients:
##   ag_t3                             0.236      0.015      0.206      0.266    232.713      0.000   4577.672
##   ne_t3                             0.011      0.015     -0.019      0.040      0.490      0.484   4747.198
##   info_t1                           0.053      0.021      0.012      0.095      6.221      0.013   2169.633
##   info_t2                           0.049      0.021      0.007      0.090      5.238      0.022   2186.269
##   info_t3                           0.216      0.018      0.181      0.251    147.985      0.000   3456.252
##   male                              0.001      0.016     -0.030      0.031      0.004      0.952   3954.803
##
## Proportion Variance Explained
##   by Coefficients                   0.164      0.010      0.144      0.184        ---        ---   4020.283
##   by Residual Variation             0.836      0.010      0.816      0.856        ---        ---   4020.283
##
##                              ------------------------------------------------------------------------------
##
##
##
```

```
## GENERATED PARAMETERS:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##
## Parameters                            Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    -------------------------------------------------------------------------------
##
##   fem_mu1                               67.954      0.322     67.322     68.598  44445.588      0.000   2157.623
##   fem_mu2                               66.233      0.362     65.513     66.938  33523.741      0.000   1569.466
##   fem_mu3                               70.894      0.326     70.255     71.530  47302.378      0.000   2123.967
##   male_mu1                              65.918      0.385     65.162     66.672  29283.751      0.000   1986.912
##   male_mu2                              60.924      0.424     60.094     61.763  20657.140      0.000   1547.318
##   male_mu3                              65.984      0.386     65.225     66.747  29225.214      0.000   1752.829
##
##                                    -------------------------------------------------------------------------------
##
##
## PREDICTOR MODEL ESTIMATES:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##
##
## Missing predictor:  male
##
## Parameters                            Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    -------------------------------------------------------------------------------
##
## Grand Mean                              -0.161      0.016     -0.191     -0.130    106.567      0.000   4529.865
##
## Level 1:
##   Residual Var.                          1.000      0.000      1.000      1.000        ---        ---        nan
```

```
## Thresholds:
##   Tau 1                              0.000      0.000      0.000      0.000        ---        ---        nan
##
##                                -------------------------------------------------------------------------------
```

MCMC MULTIPLE REGRESSION MODEL

This section illustrates a multiple regression analysis with a continuous and binary predictor, with and without auxiliary variables. The conditionally MAR assumption is defined with respect to the observed variables in each model. Missing predictor variables require their own model for missing data handling. MCMC can accommodate continuous and discrete predictors. Attempts to use the three-group gender variable an auxiliary variable (1 = female, 2 = male, 3 = non-binary) failed. Convergence required a long burn-in period, and MCMC diagnostics indicated a lack of support from the data (e.g., `N_eff` values less than 100). This analysis instead uses the `male` dummy code as an auxiliary variable (0 = female, 1 = male, NA = else). See the MCMC repeated measures analysis for illustration that incorporates nonbinary group as a fixed predictor.

The `ordinal` command invokes a categorical variable model for binary or ordered categorical variables. Alternatively, binary and multicategorical variables can be listed on the `nominal` line, which automatically creates dummy codes with lowest codes the reference. The `center` command performs iterative grand mean centering. The `waldtest` command specifies Wald chi-square tests, which are analogous to F statistics. A Wald test is a chi-square statistic equal to the sum of squared standardized differences between the model parameters and the null. A single-df Wald test is a squared z-statistic. The Wald tests use parameter labels to specify ANOVA-style significance tests. A significant test statistic refutes the null, indicating a difference or effect.

```r
# MCMC estimation
mcmc_multreg <- rblimp(
  data = carsdat,
  ordinal = 'male', # Define binary or ordinal variable
  center = 'effort_t3', # Iterative grand mean centering
  model = 'info_t3 ~ effort_t3@b1 male@b2', # Focal model with labels
  waldtest = 'b1:b2 = 0', # Test that both slopes = 0
  seed = 90291, # Integer random number seed
  burn = 5000, # Warm-up iterations
  iter = 10000) # Iterations for analysis
```

Check the last row of the `BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT` table to verify that the value in the `Highest PSR` column is less than 1.05 (indicating convergence). The `DATA INFORMATION` section displays a table with missing data patterns. The `MODEL FIT` section contains custom Wald significance tests, if specified. Each outcome variable has its own summary table. Check the `N_Eff` diagnostic column to verify that all values are greater than 100 (indicating adequate support from the data). The `Estimate` column contains the Bayesian point estimates (posterior medians), the `StdDev` column contains "Bayesian standard errors", and the 2.5% and 97.5% columns are 95% credible intervals. The `ChiSq` and `PValue` columns are frequentist test statistics (squared z-statistics).

```r
# Print output
output(mcmc_multreg)
```

```
##
## ---------------------------------------------------------------------------
##
##                                   Blimp
##                                  3.2.20
##
##              Blimp was developed with funding from Institute of
##            Education Sciences awards R305D150056 and R305D190002.
##
##             Craig K. Enders, P.I. Email: cenders@psych.ucla.edu
```

```
##            Brian T. Keller, Co-P.I. Email: btkeller@missouri.edu
##                  Han Du, Co-P.I. Email: hdu@psych.ucla.edu
##                  Roy Levy, Co-P.I. Email: roy.levy@asu.edu
##
##               Programming and Blimp Studio by Brian T. Keller
##
##                    There is no expressed license given.
##
## ---------------------------------------------------------------------------
##
##
## ALGORITHMIC OPTIONS SPECIFIED:
##
##   Imputation method:                 Fully Bayesian model-based
##   MCMC algorithm:                    Full conditional Metropolis sampler with
##                                        Auto-Derived Conditional Distributions
##   Between-cluster imputation model:  Not applicable, single-level imputation
##   Prior for random effect variances: Not applicable, single-level imputation
##   Prior for residual variances:      Zero sum of squares, df = -2 (PRIOR2)
##   Prior for predictor variances:     Unit sum of squares, df = 2 (XPRIOR1)
##   Chain Starting Values:             Random starting values
##
##
##   NOTE: The default prior for regression coefficients
##         in categorical models is 'normal( 0.0, 5.0)'
##
##
## BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:
##
##   NOTE: Split chain PSR is being used. This splits each chain's
##         iterations to create twice as many chains.
##
##   Comparing iterations across 2 chains     Highest PSR   Parameter #
##                           126 to 250             1.078             9
##                           251 to 500             1.089             9
```

```
##                            376 to 750             1.061            10
##                            501 to 1000            1.056            10
##                            626 to 1250            1.019            13
##                            751 to 1500            1.020            13
##                            876 to 1750            1.010            11
##                           1001 to 2000            1.004             2
##                           1126 to 2250            1.012             9
##                           1251 to 2500            1.014             9
##                           1376 to 2750            1.026             9
##                           1501 to 3000            1.004             7
##                           1626 to 3250            1.012             9
##                           1751 to 3500            1.012             9
##                           1876 to 3750            1.010             9
##                           2001 to 4000            1.005             9
##                           2126 to 4250            1.003             9
##                           2251 to 4500            1.004             9
##                           2376 to 4750            1.007             9
##                           2501 to 5000            1.010             9
##
##
## METROPOLIS-HASTINGS ACCEPTANCE RATES:
##
##   Chain 1:
##
##     Variable                            Type    Probability   Target Value
##     effort_t3                     imputation          0.502          0.500
##     effort_t3                      parameter          0.495          0.500
##     male                          imputation          0.501          0.500
##
##   NOTE: Suppressing printing of 1 chains.
##         Use keyword 'tuneinfo' in options to override.
##
##
```

```
## DATA INFORMATION:
##
##   Sample Size:              6710
##   Missing Data Info:
##                          miss %       1     2     3     4
##                                  --------------------------
##                  info_t3 = 35.9       -     M     -     -
##                effort_t3 = 35.9       -     M     -     M
##                     male =  1.1       -     -     M     M
##                                  --------------------------
##                                  %  63.0  35.9   1.1   0.0
##
##
## MODEL INFORMATION:
##
##   NUMBER OF PARAMETERS
##     Outcome Models:         4
##     Predictor Models:       5
##
##   PREDICTORS
##     Incomplete continuous:  effort_t3
##     Incomplete ordinal:     male
##
##   CENTERED PREDICTORS
##     Grand Mean Centered:    effort_t3
##
##   MODELS
##     [1]  info_t3 ~ Intercept effort_t3@b1 male@b2
##
##
##  WARNING MESSAGES:
##
##   WARNING: 16 observations have all variables in the imputation
##            model missing. They have been dropped from data set.
##
```

```
##
## MODEL FIT:
##
##
##   INFORMATION CRITERIA
##
##     Conditional Likelihood
##       DIC2                  56601.426
##       WAIC                  57532.113
##
##   WALD TESTS (Asparouhov & Muthén, 2021)
##
##   Test #1
##
##     Full:
##       [1]  info_t3 ~ Intercept effort_t3@b1 male@b2
##
##     Restricted:
##       [1]  info_t3 ~ Intercept effort_t3@b1 male@b2
##
##     Constraints in Restricted:
##       [1]  b1 = 0
##       [2]  b2 = 0
##
##
##     Wald Statistic (Chi-Square)                983.818
##     Number of Parameters Tested (df)                 2
##     Probability                                  0.000
##
##
## CORRELATIONS AMONG RESIDUALS:
##
##   No residual correlations.
##
##
```

```
## OUTCOME MODEL ESTIMATES:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##
## Outcome Variable:  info_t3
##
## Grand Mean Centered: effort_t3
##
##
## Parameters                           Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                       241.557      5.227    231.751    252.295        ---        ---   4048.785
##
## Coefficients:
##   Intercept                            70.043      0.326     69.405     70.690  46251.967      0.000   2377.124
##   effort_t3                            10.020      0.335      9.346     10.677    891.598      0.000   4933.718
##   male                                 -2.544      0.485     -3.485     -1.583     27.369      0.000   4529.024
##
## Standardized Coefficients:
##   effort_t3                             0.413      0.012      0.388      0.437   1126.314      0.000   4607.383
##   male                                 -0.073      0.014     -0.100     -0.045     27.562      0.000   4512.849
##
## Proportion Variance Explained
##   by Coefficients                       0.185      0.010      0.164      0.205        ---        ---   4484.980
##   by Residual Variation                 0.815      0.010      0.795      0.836        ---        ---   4484.980
##
##                                    -------------------------------------------------------------------------------
##
##
##
##
```

```
## PREDICTOR MODEL ESTIMATES:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##
##
## Missing predictor:  effort_t3
##
## Parameters                        Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                  -----------------------------------------------------------------------------
##
## Grand Mean                           3.694      0.011      3.673      3.717  1.110e+05      0.000    632.861
##
## Level 1:
##   male                              -0.122      0.013     -0.148     -0.096     88.432      0.000   1512.859
##   Residual Var.                      0.488      0.011      0.468      0.510        ---        ---   2320.011
##
##                                  -----------------------------------------------------------------------------
##
##
##
```

```
## Missing predictor:  male
##
## Parameters                         Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                  ------------------------------------------------------------------------------
##
## Grand Mean                           -0.163      0.016     -0.194     -0.132    105.621      0.000   3781.825
##
## Level 1:
##   effort_t3                          -0.249      0.027     -0.301     -0.197     86.470      0.000   1360.647
##   Residual Var.                       1.000      0.000      1.000      1.000        ---        ---        nan
## Thresholds:
##   Tau 1                               0.000      0.000      0.000      0.000        ---        ---        nan
##
##                                  ------------------------------------------------------------------------------
```

The `ordinal` command invokes a categorical variable model for binary or ordered categorical variables. Alternatively, binary and multicategorical variables can be listed on the `nominal` line, which automatically creates dummy codes with lowest codes the reference. Auxiliary variables use a sequential specification (extra dependent variable) approach. Auxiliary variables are listed to the left of the tilde and primary variables to the right. For example, `A3 A2 A1 ~ Y X` produces three regressions: `A1 ~ Y X`, `A2 ~ A1 Y X`, and `A3 ~ A2 A1 Y X`. The output includes summary tables for these additional models, which can be ignored.

```
# MCMC estimation with auxiliary variables
mcmc_multreg <- rblimp(
  data = carsdat,
  ordinal = 'male admit_type', # Define binary or ordinal variables
  center = 'effort_t3', # Iterative grand mean centering
  model = '
    primary: # This is an arbitrary and optional label that groups summary tables
    info_t3 ~ effort_t3@b1 male@b2;  # Focal model with labels
    auxiliary: # This is an arbitrary and optional label that groups summary tables
    extra_t3 cont_t3 om_t3 ag_t3 ne_t3 admit_type ~ info_t3 effort_t3 male;', # Sequential auxiliary variable models
  waldtest = 'b1:b2 = 0', # Test that both slopes = 0
```

```
  seed = 90291, # Integer random number seed
  burn = 5000, # Warm up iterations
  iter = 10000) # Iterations for analysis
```

Check the last row of the `BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT` table to verify that the value in the `Highest PSR` column is less than 1.05 (indicating convergence). The `DATA INFORMATION` section displays a table with missing data patterns. The `MODEL FIT` section contains custom Wald significance tests, if specified. Each outcome variable has its own summary table. Check the `N_Eff` diagnostic column to verify that all values are greater than 100 (indicating adequate support from the data). The `Estimate` column contains the Bayesian point estimates (posterior medians), the `StdDev` column contains "Bayesian standard errors", and the 2.5% and 97.5% columns are 95% credible intervals. The `ChiSq` and `PValue` columns are frequentist test statistics (squared z-statistics).

```
# Print output
output(mcmc_multreg)

##
## ---------------------------------------------------------------------------
##
##                                    Blimp
##                                   3.2.20
##
##              Blimp was developed with funding from Institute of
##            Education Sciences awards R305D150056 and R305D190002.
##
##             Craig K. Enders, P.I. Email: cenders@psych.ucla.edu
##            Brian T. Keller, Co-P.I. Email: btkeller@missouri.edu
##                  Han Du, Co-P.I. Email: hdu@psych.ucla.edu
##                  Roy Levy, Co-P.I. Email: roy.levy@asu.edu
##
##               Programming and Blimp Studio by Brian T. Keller
##
##                    There is no expressed license given.
```

```
##
## ---------------------------------------------------------------------------
##
##
## ALGORITHMIC OPTIONS SPECIFIED:
##
##   Imputation method:                  Fully Bayesian model-based
##   MCMC algorithm:                     Full conditional Metropolis sampler with
##                                         Auto-Derived Conditional Distributions
##   Between-cluster imputation model:  Not applicable, single-level imputation
##   Prior for random effect variances: Not applicable, single-level imputation
##   Prior for residual variances:      Zero sum of squares, df = -2 (PRIOR2)
##   Prior for predictor variances:     Unit sum of squares, df = 2 (XPRIOR1)
##   Chain Starting Values:             Random starting values
##
##
##   NOTE: The default prior for regression coefficients
##         in categorical models is 'normal( 0.0, 5.0)'
##
##
## BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:
##
##   NOTE: Split chain PSR is being used. This splits each chain's
##         iterations to create twice as many chains.
##
##   Comparing iterations across 2 chains     Highest PSR   Parameter #
##                          126 to 250             1.078            100
##                          251 to 500             1.078            100
##                          376 to 750             1.105            100
##                          501 to 1000            1.048            100
##                          626 to 1250            1.051            100
##                          751 to 1500            1.018            100
##                          876 to 1750            1.052            100
##                         1001 to 2000            1.013            101
##                         1126 to 2250            1.009             10
```

```
##                          1251 to 2500          1.019          104
##                          1376 to 2750          1.016          104
##                          1501 to 3000          1.013          102
##                          1626 to 3250          1.010          104
##                          1751 to 3500          1.011          102
##                          1876 to 3750          1.007           73
##                          2001 to 4000          1.008          102
##                          2126 to 4250          1.009          102
##                          2251 to 4500          1.007          100
##                          2376 to 4750          1.006           16
##                          2501 to 5000          1.010          100
##
##
## METROPOLIS-HASTINGS ACCEPTANCE RATES:
##
##   Chain 1:
##
##     Variable                            Type    Probability   Target Value
##     admit_type                    imputation          0.500          0.500
##     effort_t3                     imputation          0.499          0.500
##     effort_t3                      parameter          0.518          0.500
##     male                          imputation          0.500          0.500
##
##   NOTE: Suppressing printing of 1 chains.
##         Use keyword 'tuneinfo' in options to override.
##
##
## DATA INFORMATION:
##
##   Sample Size:              6726
##   Missing Data Info:
##                             miss %      1     2     3     4     5     6     7
##                                    --------------------------------------------
##                     info_t3 = 36.0      -     M     -     -     -     M     -
##                    extra_t3 = 36.1      -     M     -     -     -     M     M
```

```
##                   cont_t3 = 36.1       -     M     -     -     -     M     M
##                     om_t3 = 36.1       -     M     -     -     -     M     M
##                     ag_t3 = 36.1       -     M     -     -     -     M     M
##                     ne_t3 = 36.1       -     M     -     -     -     M     M
##                admit_type =  1.2       -     -     M     M     -     -     M
##                 effort_t3 = 36.1       -     M     -     -     -     M     M
##                      male =  1.3       -     -     M     -     M     M     M
##                                  --------------------------------------------
##                                  %  62.4  35.8   0.7   0.5   0.4   0.2   0.0
##
##
## MODEL INFORMATION:
##
##   NUMBER OF PARAMETERS
##     Outcome Models:         48
##     Predictor Models:       5
##
##   PREDICTORS
##     Incomplete continuous:  effort_t3
##     Incomplete ordinal:     male
##
##   CENTERED PREDICTORS
##     Grand Mean Centered:    effort_t3
##
##   MODELS
##
##    primary:
##     [1]  info_t3 ~ Intercept effort_t3@b1 male@b2
##
##    auxiliary:
##     [2]  admit_type ~ Intercept info_t3 effort_t3 male
##     [3]  ag_t3 ~ Intercept ne_t3 admit_type info_t3 effort_t3 male
##     [4]  cont_t3 ~ Intercept om_t3 ag_t3 ne_t3 admit_type info_t3 effort_t3
##                  male
##     [5]  extra_t3 ~ Intercept cont_t3 om_t3 ag_t3 ne_t3 admit_type info_t3
```

```
##                   effort_t3 male
##     [6]  ne_t3 ~ Intercept admit_type info_t3 effort_t3 male
##     [7]  om_t3 ~ Intercept ag_t3 ne_t3 admit_type info_t3 effort_t3 male
##
##
##  WARNING MESSAGES:
##
##   No warning messages.
##
##
## MODEL FIT:
##
##
##   INFORMATION CRITERIA
##
##     Conditional Likelihood
##       DIC2                  138748.507
##       WAIC                  146382.089
##
##   WALD TESTS (Asparouhov & Muthén, 2021)
##
##   Test #1
##
##     Full:
##       [1]  info_t3 ~ Intercept effort_t3@b1 male@b2
##
##     Restricted:
##       [1]  info_t3 ~ Intercept effort_t3@b1 male@b2
##
##     Constraints in Restricted:
##       [1]  b1 = 0
##       [2]  b2 = 0
##
##
```

```
##     Wald Statistic (Chi-Square)              963.017
##     Number of Parameters Tested (df)               2
##     Probability                                0.000
##
##
## CORRELATIONS AMONG RESIDUALS:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##
## Correlations                           Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                   ------------------------------------------------------------------------------
##
##   info_t3, admit_type                    -0.000      0.017     -0.034      0.034      0.000      0.994   9878.774
##   info_t3, ag_t3                         -0.000      0.017     -0.033      0.034      0.000      0.984   7537.990
##   info_t3, cont_t3                        0.000      0.017     -0.033      0.034      0.000      0.987   7481.318
##   info_t3, extra_t3                      -0.000      0.017     -0.034      0.034      0.000      0.992   7640.325
##   info_t3, ne_t3                          0.000      0.017     -0.033      0.034      0.000      0.985   7747.425
##   info_t3, om_t3                          0.000      0.017     -0.034      0.033      0.000      0.998   7694.649
##   admit_type, ag_t3                       0.000      0.017     -0.034      0.033      0.000      0.991   9947.155
##   admit_type, cont_t3                    -0.000      0.017     -0.034      0.034      0.000      0.996   9345.000
##   admit_type, extra_t3                   -0.000      0.017     -0.035      0.033      0.000      0.986   9866.564
##   admit_type, ne_t3                       0.000      0.017     -0.034      0.034      0.000      0.999   9681.666
##   admit_type, om_t3                       0.000      0.017     -0.033      0.033      0.000      0.995   9768.255
##   ag_t3, cont_t3                          0.000      0.017     -0.034      0.035      0.000      0.996   7421.283
##   ag_t3, extra_t3                         0.000      0.017     -0.034      0.033      0.000      0.994   6842.501
##   ag_t3, ne_t3                           -0.000      0.017     -0.034      0.034      0.000      0.999   9818.272
##   ag_t3, om_t3                            0.000      0.017     -0.034      0.033      0.000      0.998   7240.711
##   cont_t3, extra_t3                      -0.000      0.017     -0.034      0.034      0.000      0.994   7596.482
##   cont_t3, ne_t3                         -0.000      0.017     -0.034      0.035      0.000      0.998   9930.270
##   cont_t3, om_t3                         -0.000      0.017     -0.034      0.033      0.000      0.986   9038.842
##   extra_t3, ne_t3                        -0.000      0.017     -0.034      0.033      0.000      0.989  10119.141
##   extra_t3, om_t3                        -0.000      0.017     -0.034      0.034      0.000      0.990   9323.098
##   ne_t3, om_t3                           -0.000      0.017     -0.034      0.033      0.000      0.993   7829.847
```

```
##
##                                    ------------------------------------------------------------------------------
##
##
##
## OUTCOME MODEL ESTIMATES:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##   primary block:
##
## Outcome Variable:  info_t3
##
## Grand Mean Centered: effort_t3
##
##
## Parameters                          Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    ------------------------------------------------------------------------------
## Variances:
##   Residual Var.                      241.488      5.176    231.542    251.849        ---        ---   4643.665
##
## Coefficients:
##   Intercept                           70.061      0.330     69.413     70.705  45175.243      0.000   2561.768
##   effort_t3                           10.010      0.338      9.341     10.679    878.118      0.000   4431.186
##   male                                -2.511      0.488     -3.453     -1.541     26.349      0.000   4143.186
##
## Standardized Coefficients:
##   effort_t3                            0.413      0.012      0.388      0.436   1128.004      0.000   4108.547
##   male                                -0.072      0.014     -0.099     -0.044     26.567      0.000   4155.379
##
## Proportion Variance Explained
##   by Coefficients                      0.184      0.010      0.164      0.204        ---        ---   3978.725
##   by Residual Variation                0.816      0.010      0.796      0.836        ---        ---   3978.725
##
```

```
##                                   ------------------------------------------------------------------------------
##
##
##   auxiliary block:
##
## Outcome Variable:  admit_type
##
## Grand Mean Centered: effort_t3
##
##
## Parameters                           Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                   ------------------------------------------------------------------------------
## Variances:
##   Residual Var.                         1.000      0.000      1.000      1.000        ---        ---        nan
##
## Coefficients:
##   Intercept                            -0.957      0.100     -1.153     -0.760     92.141      0.000   1838.154
##   info_t3                              -0.002      0.001     -0.005      0.000      2.907      0.088   1754.795
##   effort_t3                             0.004      0.035     -0.066      0.072      0.013      0.911   1216.441
##   male                                  0.085      0.039      0.007      0.161      4.829      0.028   2597.804
##
## Thresholds:
##   Tau 1                                 0.000      0.000      0.000      0.000        ---        ---        nan
##
## Standardized Coefficients:
##   info_t3                              -0.040      0.024     -0.088      0.006      2.916      0.088   1756.554
##   effort_t3                             0.003      0.025     -0.047      0.051      0.013      0.911   1217.024
##   male                                  0.042      0.019      0.004      0.079      4.845      0.028   2597.079
##
## Proportion Variance Explained
##   by Coefficients                       0.005      0.003      0.001      0.011        ---        ---   2142.757
##   by Residual Variation                 0.995      0.003      0.989      0.999        ---        ---   2142.757
##
##                                   ------------------------------------------------------------------------------
##
```

```
## Outcome Variable:  ag_t3
##
## Grand Mean Centered: effort_t3
##
##
## Parameters                          Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                   -----------------------------------------------------------------------------
## Variances:
##   Residual Var.                        0.258      0.006      0.248      0.270        ---        ---   4486.521
##
## Coefficients:
##   Intercept                            4.096      0.055      3.988      4.205   5518.630      0.000   4085.082
##   ne_t3                               -0.223      0.012     -0.246     -0.200    356.660      0.000   4143.796
##   admit_type                           0.030      0.021     -0.011      0.072      2.001      0.157   4931.881
##   info_t3                              0.005      0.000      0.004      0.006     96.020      0.000   4777.584
##   effort_t3                            0.214      0.012      0.190      0.239    306.171      0.000   4465.785
##   male                                -0.320      0.016     -0.353     -0.288    380.753      0.000   3868.286
##
## Standardized Coefficients:
##   ne_t3                               -0.255      0.013     -0.280     -0.229    388.034      0.000   4212.305
##   admit_type                           0.017      0.012     -0.006      0.042      2.003      0.157   4925.107
##   info_t3                              0.141      0.014      0.113      0.169     98.016      0.000   4678.292
##   effort_t3                            0.256      0.014      0.228      0.283    327.826      0.000   4515.642
##   male                                -0.267      0.013     -0.293     -0.241    422.355      0.000   3874.123
##
## Proportion Variance Explained
##   by Coefficients                      0.268      0.011      0.246      0.290        ---        ---   3710.987
##   by Residual Variation                0.732      0.011      0.710      0.754        ---        ---   3710.987
##
##                                   -----------------------------------------------------------------------------
##
##
##
```

```
## Outcome Variable:  cont_t3
##
## Grand Mean Centered: effort_t3
##
##
## Parameters                          Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    -----------------------------------------------------------------------------
## Variances:
##   Residual Var.                        0.308      0.007      0.296      0.322        ---        ---   4848.153
##
## Coefficients:
##   Intercept                            3.710      0.097      3.520      3.901   1460.224      0.000   4700.204
##   om_t3                               -0.023      0.015     -0.052      0.007      2.289      0.130   4497.074
##   ag_t3                                0.252      0.017      0.219      0.286    218.428      0.000   4828.394
##   ne_t3                               -0.322      0.013     -0.348     -0.296    573.226      0.000   4830.050
##   admit_type                          -0.037      0.023     -0.081      0.009      2.555      0.110   5097.334
##   info_t3                              0.000      0.001     -0.001      0.001      0.006      0.940   4227.923
##   effort_t3                            0.139      0.014      0.112      0.167     98.842      0.000   4488.957
##   male                                -0.281      0.019     -0.318     -0.244    222.479      0.000   4447.258
##
## Standardized Coefficients:
##   om_t3                               -0.021      0.014     -0.049      0.006      2.291      0.130   4491.543
##   ag_t3                                0.226      0.015      0.197      0.255    228.313      0.000   4833.768
##   ne_t3                               -0.329      0.013     -0.355     -0.303    640.248      0.000   4879.920
##   admit_type                          -0.020      0.012     -0.043      0.005      2.556      0.110   5085.433
##   info_t3                              0.001      0.014     -0.027      0.030      0.006      0.940   4229.838
##   effort_t3                            0.149      0.015      0.120      0.178    100.503      0.000   4494.054
##   male                                -0.210      0.014     -0.237     -0.183    235.559      0.000   4461.089
##
## Proportion Variance Explained
##   by Coefficients                      0.298      0.011      0.276      0.320        ---        ---   4309.756
##   by Residual Variation                0.702      0.011      0.680      0.724        ---        ---   4309.756
##
##                                    -----------------------------------------------------------------------------
##
```

```
## Outcome Variable:  extra_t3
##
## Grand Mean Centered: effort_t3
##
## Parameters                          Estimate     StdDev       2.5%      97.5%       ChiSq     PValue       N_Eff
##                                    -----------------------------------------------------------------------------
## Variances:
##   Residual Var.                        0.323      0.007      0.310      0.337         ---        ---    4188.616
##
## Coefficients:
##   Intercept                            2.914      0.116      2.690      3.142     635.531      0.000    4816.680
##   cont_t3                              0.187      0.016      0.156      0.218     143.962      0.000    4669.672
##   om_t3                                0.248      0.016      0.218      0.279     253.550      0.000    4866.129
##   ag_t3                               -0.012      0.018     -0.046      0.023       0.446      0.504    4776.819
##   ne_t3                               -0.293      0.014     -0.322     -0.265     410.687      0.000    4961.287
##   admit_type                          -0.063      0.024     -0.109     -0.018       7.192      0.007    5288.360
##   info_t3                             -0.003      0.001     -0.004     -0.001      21.020      0.000    4876.437
##   effort_t3                           -0.005      0.014     -0.032      0.024       0.098      0.754    4674.389
##   male                                -0.052      0.020     -0.091     -0.012       6.818      0.009    4571.094
##
## Standardized Coefficients:
##   cont_t3                              0.191      0.016      0.160      0.221     149.001      0.000    4629.017
##   om_t3                                0.237      0.014      0.208      0.265     270.194      0.000    4815.097
##   ag_t3                               -0.011      0.016     -0.042      0.021       0.447      0.504    4772.570
##   ne_t3                               -0.306      0.014     -0.334     -0.278     445.925      0.000    4888.624
##   admit_type                          -0.034      0.013     -0.058     -0.009       7.204      0.007    5280.139
##   info_t3                             -0.069      0.015     -0.098     -0.039      21.081      0.000    4885.391
##   effort_t3                           -0.005      0.016     -0.035      0.026       0.098      0.754    4674.537
##   male                                -0.040      0.015     -0.070     -0.010       6.826      0.009    4568.408
##
## Proportion Variance Explained
##   by Coefficients                      0.233      0.011      0.212      0.254         ---        ---    4774.376
##   by Residual Variation                0.767      0.011      0.746      0.788         ---        ---    4774.376
##
##                                    -----------------------------------------------------------------------------
```

```
## Outcome Variable:  ne_t3
##
## Grand Mean Centered: effort_t3
##
##
## Parameters                          Estimate     StdDev       2.5%      97.5%       ChiSq     PValue      N_Eff
##                                   ------------------------------------------------------------------------------
## Variances:
##   Residual Var.                        0.437      0.009      0.419      0.456         ---        ---   4623.333
##
## Coefficients:
##   Intercept                            2.848      0.058      2.735      2.963    2405.154      0.000   4495.351
##   admit_type                           0.033      0.027     -0.019      0.087       1.433      0.231   4426.800
##   info_t3                              0.000      0.001     -0.001      0.001       0.054      0.816   4682.577
##   effort_t3                           -0.112      0.015     -0.143     -0.082      52.149      0.000   4305.882
##   male                                -0.284      0.021     -0.325     -0.244     184.618      0.000   3571.365
##
## Standardized Coefficients:
##   admit_type                           0.017      0.014     -0.010      0.044       1.434      0.231   4421.974
##   info_t3                              0.004      0.016     -0.029      0.036       0.054      0.816   4679.226
##   effort_t3                           -0.117      0.016     -0.149     -0.086      52.779      0.000   4264.070
##   male                                -0.207      0.015     -0.236     -0.179     198.291      0.000   3581.530
##
## Proportion Variance Explained
##   by Coefficients                      0.051      0.006      0.039      0.064         ---        ---   3584.960
##   by Residual Variation                0.949      0.006      0.936      0.961         ---        ---   3584.960
##
##                                   ------------------------------------------------------------------------------
##
##
##
```

```
## Outcome Variable:  om_t3
##
## Grand Mean Centered: effort_t3
##
##
## Parameters                           Estimate      StdDev        2.5%       97.5%       ChiSq      PValue       N_Eff
##                                   -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                         0.320       0.007       0.306       0.333         ---         ---    4804.436
##
## Coefficients:
##   Intercept                             2.241       0.092       2.057       2.418     589.388       0.000    4260.142
##   ag_t3                                 0.211       0.017       0.179       0.245     155.543       0.000    4437.534
##   ne_t3                                 0.016       0.014      -0.011       0.042       1.353       0.245    4174.279
##   admit_type                            0.021       0.023      -0.025       0.066       0.797       0.372    5333.203
##   info_t3                               0.008       0.001       0.007       0.009     196.466       0.000    4896.339
##   effort_t3                             0.110       0.014       0.083       0.137      62.098       0.000    4382.917
##   male                                  0.001       0.019      -0.037       0.038       0.002       0.966    4100.224
##
## Standardized Coefficients:
##   ag_t3                                 0.202       0.016       0.171       0.233     161.061       0.000    4341.769
##   ne_t3                                 0.017       0.015      -0.012       0.046       1.353       0.245    4171.330
##   admit_type                            0.012       0.013      -0.014       0.037       0.797       0.372    5328.359
##   info_t3                               0.217       0.015       0.187       0.246     205.820       0.000    4868.859
##   effort_t3                             0.125       0.016       0.095       0.157      63.345       0.000    4317.023
##   male                                  0.001       0.015      -0.030       0.031       0.002       0.966    4100.729
##
## Proportion Variance Explained
##   by Coefficients                       0.170       0.010       0.150       0.190         ---         ---    4673.726
##   by Residual Variation                 0.830       0.010       0.810       0.850         ---         ---    4673.726
##
##                                   -------------------------------------------------------------------------------
##
##
##
```

```
## PREDICTOR MODEL ESTIMATES:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##
##
## Missing predictor:  effort_t3
##
## Parameters                          Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    -----------------------------------------------------------------------------
##
## Grand Mean                             3.694      0.011      3.672      3.715  1.155e+05      0.000    609.535
##
## Level 1:
##   male                                -0.122      0.013     -0.148     -0.097     87.701      0.000   1422.541
##   Residual Var.                        0.488      0.011      0.468      0.509        ---        ---   2216.213
##
##                                    -----------------------------------------------------------------------------
##
##
## Missing predictor:  male
##
## Parameters                          Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    -----------------------------------------------------------------------------
##
## Grand Mean                            -0.163      0.016     -0.194     -0.133    108.015      0.000   3909.328
##
## Level 1:
##   effort_t3                           -0.250      0.027     -0.303     -0.197     85.270      0.000   1420.319
##   Residual Var.                        1.000      0.000      1.000      1.000        ---        ---        nan
## Thresholds:
##   Tau 1                                0.000      0.000      0.000      0.000        ---        ---        nan
##
##                                    -----------------------------------------------------------------------------
```

MCMC MULTIPLE REGRESSION MODEL WITH INTERACTION

This section illustrates a moderated regression analysis with a continuous and binary predictor and an incomplete interaction, with and without auxiliary variables. The conditionally mar assumption is defined with respect to the observed variables in each model. Missing predictor variables require their own model for missing data handling. MCMC can accommodate continuous and discrete predictors. Attempts to use the three-group gender variable an auxiliary variable (1 = female, 2 = male, 3 = non-binary) failed. Convergence required a long burn-in period, and MCMC diagnostics indicated a lack of support from the data (e.g., `N_eff` values less than 100). This analysis instead uses the `male` dummy code as an auxiliary variable (0 = female, 1 = male, NA = else).

The `ordinal` command invokes a categorical variable model for binary or ordered categorical variables. Alternatively, binary and multicategorical variables can be listed on the `nominal` line, which automatically creates dummy codes with lowest codes the reference. The `center` command performs iterative grand mean centering. Product terms are specified by joining two variables with an asterisk. The `simple` command produces simple intercepts and slopes for each group. The simple command structure specifies the focal predictor to the left of the pipe and the moderator to the right (i.e., `focal | moderator`).

```
# MCMC estimation
mcmc_modreg <- rblimp(
  data = carsdat,
  ordinal = 'male', # Define binary or ordinal variable
  center = 'effort_t3', # Iterative grand mean centering
  model = 'info_t3 ~ effort_t3 male effort_t3*male', # Focal model with product
  simple = 'effort_t3 | male', # Simple effects
  seed = 90291, # Integer random number seed
  burn = 5000, # Warm up iterations
  iter = 10000) # Iterations for analysis
```

Check the last row of the `BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT` table to verify that the value in the `Highest PSR` column is less than 1.05 (indicating convergence). The `DATA INFORMATION` section displays a table with missing data patterns. The `MODEL FIT` section contains custom Wald significance tests, if specified. Each outcome variable has its own summary table. Check the `N_Eff` diagnostic column to verify that all values are greater than 100 (indicating adequate support from the data). The `Estimate` column contains the Bayesian point estimates (posterior medians), the `StdDev` column contains "Bayesian standard errors", and the 2.5% and 97.5% columns are 95% credible intervals. The `ChiSq` and `PValue` columns are frequentist test statistics (squared z-statistics).

```
# Print output
output(mcmc_modreg)

##
## ---------------------------------------------------------------------------
##
##                                    Blimp
##                                   3.2.20
##
##              Blimp was developed with funding from Institute of
##            Education Sciences awards R305D150056 and R305D190002.
##
##             Craig K. Enders, P.I. Email: cenders@psych.ucla.edu
##            Brian T. Keller, Co-P.I. Email: btkeller@missouri.edu
##                  Han Du, Co-P.I. Email: hdu@psych.ucla.edu
##                  Roy Levy, Co-P.I. Email: roy.levy@asu.edu
##
##               Programming and Blimp Studio by Brian T. Keller
##
##                    There is no expressed license given.
##
## ---------------------------------------------------------------------------
##
##
```

```
## ALGORITHMIC OPTIONS SPECIFIED:
##
##   Imputation method:                 Fully Bayesian model-based
##   MCMC algorithm:                    Full conditional Metropolis sampler with
##                                        Auto-Derived Conditional Distributions
##   Between-cluster imputation model:  Not applicable, single-level imputation
##   Prior for random effect variances: Not applicable, single-level imputation
##   Prior for residual variances:      Zero sum of squares, df = -2 (PRIOR2)
##   Prior for predictor variances:     Unit sum of squares, df = 2 (XPRIOR1)
##   Chain Starting Values:             Random starting values
##
##
##   NOTE: The default prior for regression coefficients
##         in categorical models is 'normal( 0.0, 5.0)'
##
##
## BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:
##
##   NOTE: Split chain PSR is being used. This splits each chain's
##         iterations to create twice as many chains.
##
##   Comparing iterations across 2 chains     Highest PSR   Parameter #
##                             126 to 250           1.133            12
##                             251 to 500           1.067            11
##                             376 to 750           1.033            11
##                             501 to 1000          1.016            13
##                             626 to 1250          1.033            11
##                             751 to 1500          1.017            12
##                             876 to 1750          1.028            12
##                            1001 to 2000          1.009            12
##                            1126 to 2250          1.032            11
##                            1251 to 2500          1.014            11
##                            1376 to 2750          1.011            11
##                            1501 to 3000          1.006            11
##                            1626 to 3250          1.011            11
```

```
##                            1751 to 3500            1.003            11
##                            1876 to 3750            1.004            11
##                            2001 to 4000            1.003            11
##                            2126 to 4250            1.006            13
##                            2251 to 4500            1.008            13
##                            2376 to 4750            1.003            13
##                            2501 to 5000            1.002            12
##
##
## METROPOLIS-HASTINGS ACCEPTANCE RATES:
##
##   Chain 1:
##
##     Variable                            Type    Probability   Target Value
##     effort_t3                     imputation          0.499          0.500
##     effort_t3                      parameter          0.460          0.500
##     male                          imputation          0.503          0.500
##
##   NOTE: Suppressing printing of 1 chains.
##         Use keyword 'tuneinfo' in options to override.
##
##
## DATA INFORMATION:
##
##   Sample Size:              6710
##   Missing Data Info:
##                             miss %       1     2     3     4
##                                    --------------------------
##                    info_t3 = 35.9       -     M     -     -
##                  effort_t3 = 35.9       -     M     -     M
##                       male =  1.1       -     -     M     M
##                                    --------------------------
##                                    %  63.0  35.9   1.1   0.0
##
##
```

```
## MODEL INFORMATION:
##
##   NUMBER OF PARAMETERS
##     Outcome Models:         5
##     Predictor Models:       5
##
##   PREDICTORS
##     Incomplete continuous:  effort_t3
##     Incomplete ordinal:     male
##
##   CENTERED PREDICTORS
##     Grand Mean Centered:    effort_t3
##
##   MODELS
##     [1]  info_t3 ~ Intercept effort_t3 male effort_t3*male
##
##
##  WARNING MESSAGES:
##
##   WARNING: 16 observations have all variables in the imputation
##            model missing. They have been dropped from data set.
##
##
## MODEL FIT:
##
##
##   INFORMATION CRITERIA
##
##     Conditional Likelihood
##       DIC2                  56558.179
##       WAIC                  57487.798
##
##
```

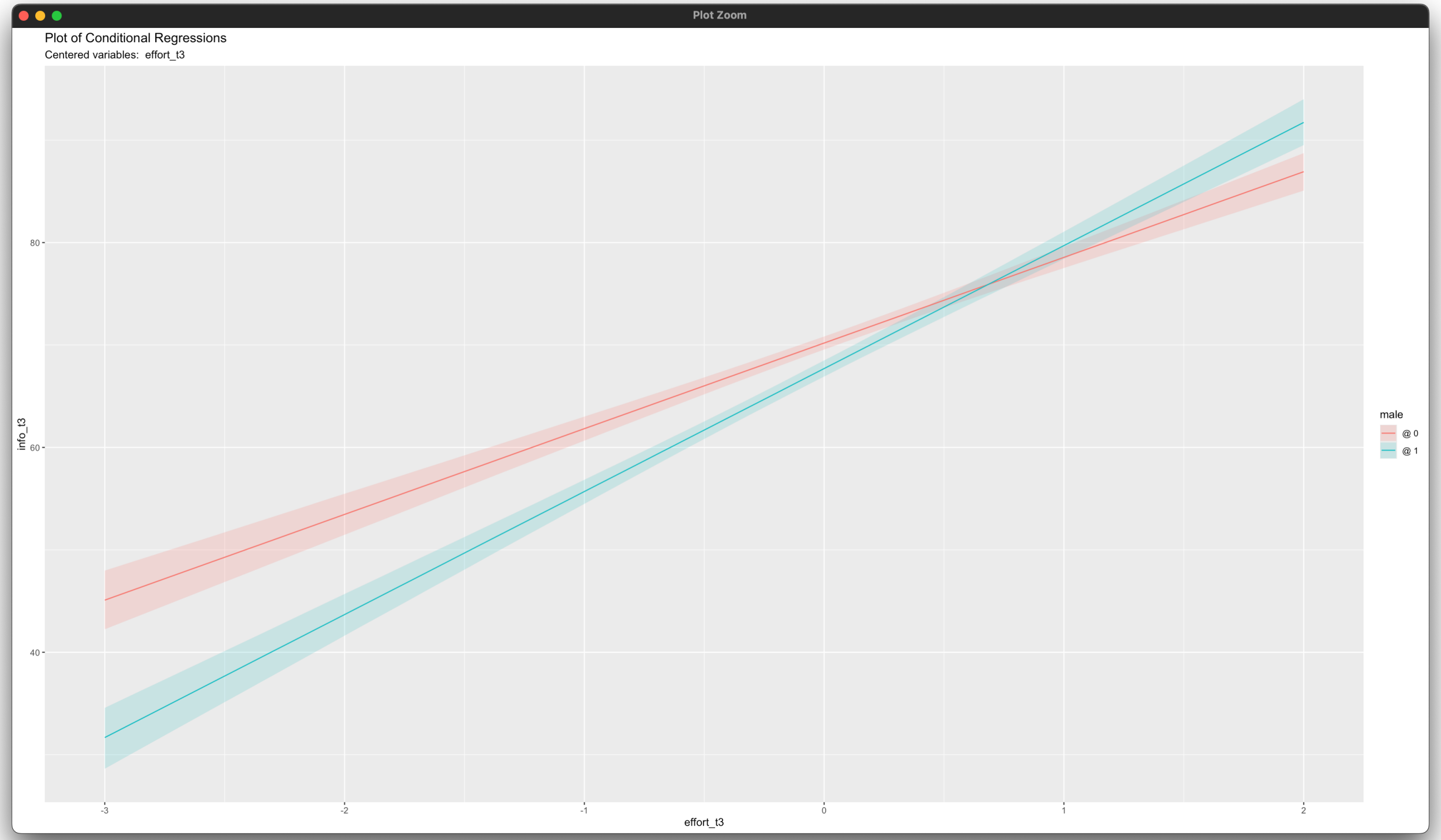
```
## CORRELATIONS AMONG RESIDUALS:
##
##   No residual correlations.
##
##
##
## OUTCOME MODEL ESTIMATES:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##
## Outcome Variable:  info_t3
##
## Grand Mean Centered: effort_t3
##
##
## Parameters                        Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                   -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                    239.896      5.175    229.733    250.055        ---        ---   4712.933
##
## Coefficients:
##   Intercept                         70.193      0.324     69.562     70.828  46994.319      0.000   3130.225
##   effort_t3                          8.366      0.456      7.464      9.261    336.080      0.000   4898.563
##   male                              -2.488      0.490     -3.443     -1.530     25.813      0.000   4144.788
##   effort_t3*male                     3.653      0.677      2.336      5.002     29.183      0.000   4720.269
##
## Standardized Coefficients:
##   effort_t3                          0.345      0.018      0.309      0.380    362.445      0.000   4777.509
##   male                              -0.072      0.014     -0.099     -0.044     26.007      0.000   4164.860
##   effort_t3*male                     0.102      0.019      0.065      0.139     29.471      0.000   4722.109
##
```

```
## Proportion Variance Explained
##   by Coefficients                     0.191      0.010      0.171      0.211        ---        ---   4227.327
##   by Residual Variation               0.809      0.010      0.789      0.829        ---        ---   4227.327
##
##                                    ------------------------------------------------------------------------------
##
##
##
## Conditional Effects                Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    ------------------------------------------------------------------------------
##   effort_t3 | male @ 0
##     Intercept                        70.193      0.324     69.562     70.828  46994.319      0.000   3130.225
##     Slope                             8.366      0.456      7.464      9.261    336.080      0.000   4898.563
##
##   effort_t3 | male @ 1
##     Intercept                        67.700      0.397     66.926     68.487  29095.096      0.000   2396.749
##     Slope                            12.013      0.499     11.037     13.019    579.862      0.000   4369.591
##
##                                    ------------------------------------------------------------------------------
##
##                                    NOTE: Intercepts are computed by setting all predictors
##                                          not involved in the conditional effect to zero.
##
##
##
##
```

```
## PREDICTOR MODEL ESTIMATES:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##
##
## Missing predictor:  effort_t3
##
## Parameters                           Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    -------------------------------------------------------------------------------
##
## Grand Mean                              3.693      0.011      3.672      3.714  1.167e+05      0.000    811.199
##
## Level 1:
##   male                                 -0.122      0.013     -0.147     -0.096     89.732      0.000   1625.865
##   Residual Var.                         0.488      0.011      0.468      0.510        ---        ---   2413.521
##
##                                    -------------------------------------------------------------------------------
##
##
## Missing predictor:  male
##
## Parameters                           Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    -------------------------------------------------------------------------------
##
## Grand Mean                             -0.163      0.016     -0.194     -0.132    107.983      0.000   4258.248
##
## Level 1:
##   effort_t3                            -0.249      0.027     -0.302     -0.196     86.378      0.000   1584.853
##   Residual Var.                         1.000      0.000      1.000      1.000        ---        ---        nan
## Thresholds:
##   Tau 1                                 0.000      0.000      0.000      0.000        ---        ---        nan
##
##                                    -------------------------------------------------------------------------------
```

The `simple_plot` is an rblimp function for plotting conditional effects defined on the `simple` command. Because `male` is specified as binary on the `ordinal` command line, the plot will display group-specific regression lines. The function is specified as `simple_plot(simple slope equation | moderator, model_object)`.

```
# Plot group-specific regressions
simple_plot(info_t3 ~ effort_t3 | male, mcmc_modreg)
```



The `ordinal` command invokes a categorical variable model for binary or ordered categorical variables. Alternatively, binary and multicategorical variables can be listed on the `nominal` line, which automatically creates dummy codes with lowest codes the reference. The `center` command performs iterative grand mean centering. Product terms are specified by joining two variables with an asterisk. The `simple` command produces simple intercepts and slopes for each group. The simple command structure specifies the focal predictor to the left of the pipe and the moderator to the right (i.e., `focal | moderator`). Auxiliary variables use a sequential specification (extra dependent variable) approach. Auxiliary variables use a sequential specification (extra dependent variable) approach. Auxiliary variables are listed to the left of the tilde and primary variables to the right. For example, `A3 A2 A1 ~ Y X` produces three regressions: `A1 ~ Y X`, `A2 ~ A1 Y X`, and `A3 ~ A2 A1 Y X`. The output includes summary tables for these additional models, which can be ignored. The admit_type variable can be used as an auxiliary variable because info_t1 is not in the model. Output includes summary tables for these additional models, which can be ignored.

```
# MCMC estimation with auxiliary variables
mcmc_modreg_aux <- rblimp(
  data = carsdat,
  ordinal = 'male admit_type', # Define binary or ordinal variable
  center = 'effort_t3', # Iterative grand mean centering
  model = '
    primary: # This is an arbitrary and optional label that groups summary tables
    info_t3 ~ effort_t3 male effort_t3*male; # Focal model with product
    auxiliary: # This is an arbitrary and optional label that groups summary tables
    extra_t3 cont_t3 om_t3 ag_t3 ne_t3 admit_type ~ info_t3 effort_t3 male;', # Sequential auxiliary variable models
  simple = 'effort_t3 | male',
  seed = 90291, # Integer random number seed
  burn = 5000, # Warm up iterations
  iter = 10000) # Iterations for analysis
```

Check the last row of the `BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT` table to verify that the value in the `Highest PSR` column is less than 1.05 (indicating convergence). The `DATA INFORMATION` section displays a table with missing data patterns. The `MODEL FIT` section contains custom Wald significance tests, if specified. Each outcome variable has its own summary table. Check the `N_Eff` diagnostic column to verify that all values are greater than 100 (indicating adequate support from the data). The `Estimate` column contains the Bayesian point estimates (posterior medians), the `StdDev` column contains "Bayesian standard errors", and the 2.5% and 97.5% columns are 95% credible intervals. The `ChiSq` and `PValue` columns are frequentist test statistics (squared z-statistics).

```
# Print output
output(mcmc_modreg_aux)

##
## ---------------------------------------------------------------------------
##
##                                    Blimp
##                                   3.2.20
##
##              Blimp was developed with funding from Institute of
##            Education Sciences awards R305D150056 and R305D190002.
##
##             Craig K. Enders, P.I. Email: cenders@psych.ucla.edu
##            Brian T. Keller, Co-P.I. Email: btkeller@missouri.edu
##                  Han Du, Co-P.I. Email: hdu@psych.ucla.edu
##                  Roy Levy, Co-P.I. Email: roy.levy@asu.edu
##
##               Programming and Blimp Studio by Brian T. Keller
##
##                    There is no expressed license given.
##
## ---------------------------------------------------------------------------
##
##
## ALGORITHMIC OPTIONS SPECIFIED:
##
##   Imputation method:                 Fully Bayesian model-based
##   MCMC algorithm:                    Full conditional Metropolis sampler with
```

```
##                                        Auto-Derived Conditional Distributions
##   Between-cluster imputation model:  Not applicable, single-level imputation
##   Prior for random effect variances: Not applicable, single-level imputation
##   Prior for residual variances:      Zero sum of squares, df = -2 (PRIOR2)
##   Prior for predictor variances:     Unit sum of squares, df = 2 (XPRIOR1)
##   Chain Starting Values:             Random starting values
##
##
##   NOTE: The default prior for regression coefficients
##         in categorical models is 'normal( 0.0, 5.0)'
##
##
## BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:
##
##   NOTE: Split chain PSR is being used. This splits each chain's
##         iterations to create twice as many chains.
##
##   Comparing iterations across 2 chains     Highest PSR   Parameter #
##                         126 to 250               1.276           102
##                         251 to 500               1.058           102
##                         376 to 750               1.049            17
##                         501 to 1000              1.017           104
##                         626 to 1250              1.021            14
##                         751 to 1500              1.019            21
##                         876 to 1750              1.018            21
##                        1001 to 2000              1.014            21
##                        1126 to 2250              1.014            12
##                        1251 to 2500              1.009            20
##                        1376 to 2750              1.014            18
##                        1501 to 3000              1.007           102
##                        1626 to 3250              1.006            12
##                        1751 to 3500              1.005           102
##                        1876 to 3750              1.008           102
##                        2001 to 4000              1.005             4
##                        2126 to 4250              1.005             4
```

```
##                          2251 to 4500           1.004            4
##                          2376 to 4750           1.005          103
##                          2501 to 5000           1.007           18
##
##
## METROPOLIS-HASTINGS ACCEPTANCE RATES:
##
##   Chain 1:
##
##     Variable                            Type    Probability   Target Value
##     admit_type                    imputation          0.496          0.500
##     effort_t3                     imputation          0.500          0.500
##     effort_t3                      parameter          0.467          0.500
##     male                          imputation          0.494          0.500
##
##   NOTE: Suppressing printing of 1 chains.
##         Use keyword 'tuneinfo' in options to override.
##
## DATA INFORMATION:
##
##   Sample Size:              6726
##   Missing Data Info:
##                            miss %      1     2     3     4     5     6     7
##                                    --------------------------------------------
##                   info_t3 = 36.0      -     M     -     -     -     M     -
##                  extra_t3 = 36.1      -     M     -     -     -     M     M
##                   cont_t3 = 36.1      -     M     -     -     -     M     M
##                     om_t3 = 36.1      -     M     -     -     -     M     M
##                     ag_t3 = 36.1      -     M     -     -     -     M     M
##                     ne_t3 = 36.1      -     M     -     -     -     M     M
##                admit_type =  1.2      -     -     M     M     -     -     M
##                 effort_t3 = 36.1      -     M     -     -     -     M     M
##                      male =  1.3      -     -     M     -     M     M     M
##                                    --------------------------------------------
##                                    %  62.4  35.8   0.7   0.5   0.4   0.2   0.0
```

```
## MODEL INFORMATION:
##
##   NUMBER OF PARAMETERS
##     Outcome Models:         49
##     Predictor Models:       5
##
##   PREDICTORS
##     Incomplete continuous:  effort_t3
##     Incomplete ordinal:     male
##
##   CENTERED PREDICTORS
##     Grand Mean Centered:    effort_t3
##
##   MODELS
##
##    primary:
##     [1]  info_t3 ~ Intercept effort_t3 male effort_t3*male
##
##    auxiliary:
##     [2]  admit_type ~ Intercept info_t3 effort_t3 male
##     [3]  ag_t3 ~ Intercept ne_t3 admit_type info_t3 effort_t3 male
##     [4]  cont_t3 ~ Intercept om_t3 ag_t3 ne_t3 admit_type info_t3 effort_t3
##                 male
##     [5]  extra_t3 ~ Intercept cont_t3 om_t3 ag_t3 ne_t3 admit_type info_t3
##                 effort_t3 male
##     [6]  ne_t3 ~ Intercept admit_type info_t3 effort_t3 male
##     [7]  om_t3 ~ Intercept ag_t3 ne_t3 admit_type info_t3 effort_t3 male
##
##
##  WARNING MESSAGES:
##
##   No warning messages.
##
##
```

```
## MODEL FIT:
##
##
##   INFORMATION CRITERIA
##
##     Conditional Likelihood
##       DIC2                  138709.436
##       WAIC                  146343.508
##
##
## CORRELATIONS AMONG RESIDUALS:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##
## Correlations                           Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    -------------------------------------------------------------------------------
##
##   info_t3, admit_type                    -0.000      0.017     -0.034      0.034      0.000      0.998   9429.946
##   info_t3, ag_t3                          0.002      0.017     -0.031      0.036      0.015      0.904   6940.375
##   info_t3, cont_t3                        0.002      0.017     -0.032      0.035      0.009      0.926   7576.062
##   info_t3, extra_t3                       0.001      0.017     -0.033      0.035      0.002      0.968   7579.262
##   info_t3, ne_t3                          0.004      0.018     -0.030      0.038      0.044      0.834   8223.063
##   info_t3, om_t3                          0.000      0.017     -0.034      0.035      0.000      0.985   7899.509
##   admit_type, ag_t3                      -0.000      0.017     -0.034      0.034      0.000      1.000   9893.390
##   admit_type, cont_t3                     0.000      0.017     -0.034      0.034      0.000      0.999   9759.439
##   admit_type, extra_t3                   -0.000      0.017     -0.034      0.033      0.000      0.994   9778.605
##   admit_type, ne_t3                       0.000      0.017     -0.034      0.033      0.000      0.990   9728.824
##   admit_type, om_t3                      -0.000      0.017     -0.034      0.034      0.000      0.998   9432.381
##   ag_t3, cont_t3                         -0.000      0.018     -0.035      0.034      0.000      0.992   7688.865
##   ag_t3, extra_t3                        -0.000      0.017     -0.035      0.034      0.000      0.986   6877.912
##   ag_t3, ne_t3                            0.000      0.017     -0.033      0.035      0.000      0.994  10152.672
##   ag_t3, om_t3                           -0.000      0.017     -0.034      0.033      0.000      1.000   7864.160
##   cont_t3, extra_t3                       0.000      0.017     -0.034      0.034      0.000      0.998   7384.632
```

```
##   cont_t3, ne_t3                    -0.000      0.017     -0.035      0.034      0.000      0.994   9823.818
##   cont_t3, om_t3                    -0.000      0.017     -0.035      0.033      0.001      0.978   8562.240
##   extra_t3, ne_t3                   -0.000      0.017     -0.034      0.034      0.000      0.994   9854.675
##   extra_t3, om_t3                   -0.000      0.017     -0.034      0.034      0.000      1.000   9154.038
##   ne_t3, om_t3                      -0.000      0.017     -0.034      0.034      0.000      0.997   7605.964
##
##                                  ------------------------------------------------------------------------------
##
##
##
## OUTCOME MODEL ESTIMATES:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##   primary block:
##
## Outcome Variable:  info_t3
##
## Grand Mean Centered: effort_t3
##
##
## Parameters                        Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                  ------------------------------------------------------------------------------
## Variances:
##   Residual Var.                    240.003      5.223    229.951    250.561        ---        ---   4670.446
##
## Coefficients:
##   Intercept                         70.204      0.323     69.563     70.824  47153.662      0.000   2803.239
##   effort_t3                          8.353      0.456      7.450      9.241    335.042      0.000   4451.948
##   male                              -2.436      0.489     -3.393     -1.473     24.857      0.000   4488.756
##   effort_t3*male                     3.694      0.677      2.372      5.042     29.863      0.000   4261.002
##
```

```
## Standardized Coefficients:
##   effort_t3                          0.344      0.018      0.308      0.379    361.991      0.000   4356.903
##   male                              -0.070      0.014     -0.098     -0.042     25.049      0.000   4470.440
##   effort_t3*male                     0.103      0.019      0.066      0.140     30.097      0.000   4268.208
##
## Proportion Variance Explained
##   by Coefficients                    0.191      0.010      0.170      0.211        ---        ---   3691.631
##   by Residual Variation              0.809      0.010      0.789      0.830        ---        ---   3691.631
##
##                                -----------------------------------------------------------------------------
##
##
##
## Conditional Effects               Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                -----------------------------------------------------------------------------
##   effort_t3 | male @ 0
##     Intercept                       70.204      0.323     69.563     70.824  47153.662      0.000   2803.239
##     Slope                            8.353      0.456      7.450      9.241    335.042      0.000   4451.948
##
##   effort_t3 | male @ 1
##     Intercept                       67.765      0.392     66.989     68.526  29887.030      0.000   2383.933
##     Slope                           12.041      0.498     11.073     13.024    584.349      0.000   3993.951
##
##                                -----------------------------------------------------------------------------
##
##                                NOTE: Intercepts are computed by setting all predictors
##                                      not involved in the conditional effect to zero.
##
##
```

```
##   auxiliary block:
##
## Outcome Variable:  admit_type
##
## Grand Mean Centered: effort_t3
##
##
## Parameters                           Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                         1.000      0.000      1.000      1.000        ---        ---        nan
##
## Coefficients:
##   Intercept                            -0.954      0.102     -1.154     -0.757     87.531      0.000   2057.119
##   info_t3                              -0.002      0.001     -0.005      0.000      2.875      0.090   1983.293
##   effort_t3                             0.007      0.035     -0.062      0.077      0.037      0.847   1242.129
##   male                                  0.085      0.040      0.007      0.163      4.621      0.032   2396.559
##
## Thresholds:
##   Tau 1                                 0.000      0.000      0.000      0.000        ---        ---        nan
##
## Standardized Coefficients:
##   info_t3                              -0.041      0.024     -0.088      0.008      2.883      0.090   1984.152
##   effort_t3                             0.005      0.025     -0.044      0.055      0.037      0.847   1240.927
##   male                                  0.042      0.020      0.003      0.080      4.636      0.031   2395.950
##
## Proportion Variance Explained
##   by Coefficients                       0.005      0.003      0.001      0.012        ---        ---   2158.418
##   by Residual Variation                 0.995      0.003      0.988      0.999        ---        ---   2158.418
##
##                                    -------------------------------------------------------------------------------
##
##
##
```

```
## Conditional Effects                 Estimate    StdDev      2.5%     97.5%     ChiSq    PValue     N_Eff
##                                    -------------------------------------------------------------------------------
##   effort_t3 | male @ 0
##     Intercept                          -0.954     0.102    -1.154    -0.757    87.531     0.000  2057.119
##     Slope                               0.007     0.035    -0.062     0.077     0.037     0.847  1242.129
##
##   effort_t3 | male @ 1
##     Intercept                          -0.868     0.099    -1.065    -0.676    76.431     0.000  2026.045
##     Slope                               0.007     0.035    -0.062     0.077     0.037     0.847  1242.129
##
##                                    -------------------------------------------------------------------------------
##
##                                    NOTE: Intercepts are computed by setting all predictors
##                                          not involved in the conditional effect to zero.
##
##
##
## Outcome Variable:  ag_t3
##
## Grand Mean Centered: effort_t3
##
##
## Parameters                          Estimate    StdDev      2.5%     97.5%     ChiSq    PValue     N_Eff
##                                    -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                         0.258     0.006     0.248     0.269       ---       ---  4352.079
##
## Coefficients:
##   Intercept                             4.098     0.055     3.991     4.205  5495.552     0.000  4408.361
##   ne_t3                                -0.224     0.012    -0.247    -0.201   365.476     0.000  4465.082
##   admit_type                            0.030     0.021    -0.012     0.071     1.935     0.164  4979.583
##   info_t3                               0.005     0.001     0.004     0.006    93.471     0.000  4177.739
##   effort_t3                             0.214     0.012     0.190     0.239   306.639     0.000  4475.504
##   male                                 -0.321     0.016    -0.353    -0.289   390.012     0.000  4523.161
##
```

```
## Standardized Coefficients:
##   ne_t3                            -0.255      0.013     -0.280     -0.230    394.009      0.000   4466.741
##   admit_type                        0.017      0.012     -0.007      0.041      1.935      0.164   4967.338
##   info_t3                           0.141      0.014      0.113      0.169     95.036      0.000   4097.644
##   effort_t3                         0.256      0.014      0.228      0.284    327.099      0.000   4401.027
##   male                             -0.268      0.013     -0.293     -0.242    425.809      0.000   4533.370
##
## Proportion Variance Explained
##   by Coefficients                   0.269      0.011      0.247      0.290        ---        ---   3986.191
##   by Residual Variation             0.731      0.011      0.710      0.753        ---        ---   3986.191
##
##                                ----------------------------------------------------------------------------
##
##
##
## Conditional Effects              Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                ----------------------------------------------------------------------------
##   effort_t3 | male @ 0
##     Intercept                       4.098      0.055      3.991      4.205   5495.552      0.000   4408.361
##     Slope                           0.214      0.012      0.190      0.239    306.639      0.000   4475.504
##
##   effort_t3 | male @ 1
##     Intercept                       3.777      0.053      3.674      3.883   5040.421      0.000   4494.668
##     Slope                           0.214      0.012      0.190      0.239    306.639      0.000   4475.504
##
##                                ----------------------------------------------------------------------------
##
##                                NOTE: Intercepts are computed by setting all predictors
##                                      not involved in the conditional effect to zero.
##
##
##
```

```
## Outcome Variable:  cont_t3
##
## Grand Mean Centered: effort_t3
##
##
## Parameters                          Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                   -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                        0.308      0.007      0.296      0.322        ---        ---   4519.770
##
## Coefficients:
##   Intercept                            3.707      0.098      3.511      3.901   1418.367      0.000   4799.256
##   om_t3                               -0.023      0.015     -0.052      0.007      2.212      0.137   4240.854
##   ag_t3                                0.252      0.017      0.219      0.286    213.144      0.000   4658.305
##   ne_t3                               -0.322      0.013     -0.348     -0.295    572.992      0.000   4592.135
##   admit_type                          -0.037      0.023     -0.081      0.009      2.606      0.106   5744.235
##   info_t3                              0.000      0.001     -0.001      0.001      0.008      0.927   4830.931
##   effort_t3                            0.139      0.014      0.112      0.167    100.471      0.000   4841.849
##   male                                -0.281      0.019     -0.317     -0.244    227.898      0.000   4204.297
##
## Standardized Coefficients:
##   om_t3                               -0.021      0.014     -0.048      0.007      2.213      0.137   4223.985
##   ag_t3                                0.226      0.015      0.196      0.255    223.666      0.000   4676.986
##   ne_t3                               -0.329      0.013     -0.354     -0.303    647.092      0.000   4497.256
##   admit_type                          -0.019      0.012     -0.043      0.005      2.607      0.106   5736.907
##   info_t3                              0.001      0.015     -0.027      0.031      0.008      0.927   4837.217
##   effort_t3                            0.149      0.015      0.120      0.178    102.017      0.000   4785.546
##   male                                -0.210      0.014     -0.236     -0.183    239.704      0.000   4177.730
##
## Proportion Variance Explained
##   by Coefficients                      0.298      0.011      0.276      0.320        ---        ---   4387.038
##   by Residual Variation                0.702      0.011      0.680      0.724        ---        ---   4387.038
##
##                                   -------------------------------------------------------------------------------
##
```

```
## Conditional Effects                    Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                  ------------------------------------------------------------------------------
##   effort_t3 | male @ 0
##     Intercept                            3.707      0.098      3.511      3.901   1418.367      0.000   4799.256
##     Slope                                0.139      0.014      0.112      0.167    100.471      0.000   4841.849
##
##   effort_t3 | male @ 1
##     Intercept                            3.426      0.093      3.239      3.611   1346.035      0.000   4804.037
##     Slope                                0.139      0.014      0.112      0.167    100.471      0.000   4841.849
##
##                                  ------------------------------------------------------------------------------
##
##                                  NOTE: Intercepts are computed by setting all predictors
##                                        not involved in the conditional effect to zero.
##
##
##
## Outcome Variable:  extra_t3
##
## Grand Mean Centered: effort_t3
##
##
## Parameters                             Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                  ------------------------------------------------------------------------------
## Variances:
##   Residual Var.                          0.323      0.007      0.310      0.337        ---        ---   4646.876
##
## Coefficients:
##   Intercept                              2.914      0.115      2.688      3.140    639.545      0.000   4647.202
##   cont_t3                                0.187      0.016      0.157      0.218    143.096      0.000   4060.647
##   om_t3                                  0.247      0.016      0.217      0.278    254.122      0.000   4921.554
##   ag_t3                                 -0.011      0.018     -0.046      0.024      0.387      0.534   4625.785
##   ne_t3                                 -0.293      0.015     -0.322     -0.264    395.480      0.000   4753.721
##   admit_type                            -0.064      0.024     -0.110     -0.017      7.362      0.007   5365.915
##   info_t3                               -0.003      0.001     -0.004     -0.001     20.345      0.000   4502.077
```

```
##   effort_t3                           -0.004     0.014    -0.033     0.023     0.100     0.752   4664.448
##   male                                -0.051     0.020    -0.091    -0.014     6.885     0.009   4439.048
##
## Standardized Coefficients:
##   cont_t3                              0.191     0.016     0.160     0.222   147.486     0.000   4235.672
##   om_t3                                0.236     0.014     0.207     0.264   266.224     0.000   4885.994
##   ag_t3                               -0.010     0.016    -0.042     0.022     0.387     0.534   4626.815
##   ne_t3                               -0.306     0.015    -0.335    -0.277   434.591     0.000   4688.509
##   admit_type                          -0.034     0.013    -0.059    -0.009     7.375     0.007   5382.774
##   info_t3                             -0.069     0.015    -0.099    -0.039    20.450     0.000   4525.926
##   effort_t3                           -0.005     0.016    -0.036     0.026     0.100     0.752   4856.547
##   male                                -0.039     0.015    -0.069    -0.011     6.895     0.009   4417.270
##
## Proportion Variance Explained
##   by Coefficients                      0.234     0.011     0.212     0.255       ---       ---   4734.935
##   by Residual Variation                0.766     0.011     0.745     0.788       ---       ---   4734.935
##
##                                  ------------------------------------------------------------------------------
##
##
##
## Conditional Effects                 Estimate    StdDev      2.5%     97.5%     ChiSq    PValue      N_Eff
##                                  ------------------------------------------------------------------------------
##   effort_t3 | male @ 0
##     Intercept                          2.914     0.115     2.688     3.140   639.545     0.000   4647.202
##     Slope                             -0.004     0.014    -0.033     0.023     0.100     0.752   4664.448
##
##   effort_t3 | male @ 1
##     Intercept                          2.864     0.108     2.650     3.075   700.113     0.000   4773.679
##     Slope                             -0.004     0.014    -0.033     0.023     0.100     0.752   4664.448
##
##                                  ------------------------------------------------------------------------------
##
##                                  NOTE: Intercepts are computed by setting all predictors
##                                        not involved in the conditional effect to zero.
```

```
## Outcome Variable:  ne_t3
##
## Grand Mean Centered: effort_t3
##
##
## Parameters                          Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                    ------------------------------------------------------------------------------
## Variances:
##   Residual Var.                        0.437      0.009      0.419      0.456        ---        ---   4131.705
##
## Coefficients:
##   Intercept                            2.846      0.059      2.732      2.962   2353.536      0.000   4441.575
##   admit_type                           0.034      0.027     -0.019      0.088      1.538      0.215   4200.326
##   info_t3                              0.000      0.001     -0.001      0.001      0.062      0.804   4778.620
##   effort_t3                           -0.113      0.016     -0.143     -0.082     51.244      0.000   4412.623
##   male                                -0.284      0.021     -0.325     -0.243    186.386      0.000   3754.266
##
## Standardized Coefficients:
##   admit_type                           0.018      0.014     -0.010      0.045      1.539      0.215   4210.136
##   info_t3                              0.004      0.017     -0.029      0.037      0.062      0.804   4777.153
##   effort_t3                           -0.118      0.016     -0.149     -0.085     52.022      0.000   4386.780
##   male                                -0.208      0.015     -0.237     -0.178    198.682      0.000   3888.592
##
## Proportion Variance Explained
##   by Coefficients                      0.051      0.006      0.039      0.064        ---        ---   3792.139
##   by Residual Variation                0.949      0.006      0.936      0.961        ---        ---   3792.139
##
##                                    ------------------------------------------------------------------------------
##
##
##
```

```
## Conditional Effects                   Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                  -------------------------------------------------------------------------------
##   effort_t3 | male @ 0
##     Intercept                           2.846      0.059      2.732      2.962   2353.536      0.000   4441.575
##     Slope                              -0.113      0.016     -0.143     -0.082     51.244      0.000   4412.623
##
##   effort_t3 | male @ 1
##     Intercept                           2.562      0.058      2.450      2.679   1929.901      0.000   4295.048
##     Slope                              -0.113      0.016     -0.143     -0.082     51.244      0.000   4412.623
##
##                                  -------------------------------------------------------------------------------
##
##                                  NOTE: Intercepts are computed by setting all predictors
##                                        not involved in the conditional effect to zero.
##
##
##
## Outcome Variable:  om_t3
##
## Grand Mean Centered: effort_t3
##
##
## Parameters                            Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                  -------------------------------------------------------------------------------
## Variances:
##   Residual Var.                         0.320      0.007      0.307      0.333        ---        ---   4925.774
##
## Coefficients:
##   Intercept                             2.239      0.092      2.056      2.417    593.440      0.000   4694.937
##   ag_t3                                 0.211      0.017      0.179      0.245    153.381      0.000   4566.618
##   ne_t3                                 0.016      0.013     -0.011      0.042      1.394      0.238   4384.896
##   admit_type                            0.021      0.023     -0.025      0.066      0.812      0.368   5357.054
##   info_t3                               0.008      0.001      0.007      0.009    194.487      0.000   5057.970
##   effort_t3                             0.109      0.014      0.082      0.137     61.453      0.000   4702.564
##   male                                  0.001      0.019     -0.036      0.038      0.003      0.957   4337.473
```

```
## Standardized Coefficients:
##   ag_t3                                0.202      0.016      0.171      0.234    159.987      0.000   4553.218
##   ne_t3                                0.017      0.015     -0.012      0.046      1.396      0.237   4382.806
##   admit_type                           0.012      0.013     -0.014      0.037      0.812      0.367   5364.512
##   info_t3                              0.217      0.015      0.187      0.246    203.559      0.000   5095.025
##   effort_t3                            0.125      0.016      0.094      0.156     62.388      0.000   4688.947
##   male                                 0.001      0.015     -0.029      0.031      0.003      0.957   4335.661
##
## Proportion Variance Explained
##   by Coefficients                      0.170      0.010      0.151      0.190        ---        ---   4550.098
##   by Residual Variation                0.830      0.010      0.810      0.849        ---        ---   4550.098
##
##                                  ------------------------------------------------------------------------------
##
##
##
## Conditional Effects                 Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                  ------------------------------------------------------------------------------
##   effort_t3 | male @ 0
##     Intercept                          2.239      0.092      2.056      2.417    593.440      0.000   4694.937
##     Slope                              0.109      0.014      0.082      0.137     61.453      0.000   4702.564
##
##   effort_t3 | male @ 1
##     Intercept                          2.240      0.086      2.068      2.405    672.393      0.000   4748.711
##     Slope                              0.109      0.014      0.082      0.137     61.453      0.000   4702.564
##
##                                  ------------------------------------------------------------------------------
##
##                                  NOTE: Intercepts are computed by setting all predictors
##                                        not involved in the conditional effect to zero.
##
##
##
##
```

```
## PREDICTOR MODEL ESTIMATES:
##
##   Summaries based on 10000 iterations using 2 chains.
##   NOTE: Estimate column based on posterior median.
##
##
##
## Missing predictor:  effort_t3
##
## Parameters                         Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                   -----------------------------------------------------------------------------
##
## Grand Mean                            3.694      0.011      3.672      3.716  1.151e+05      0.000    564.139
##
## Level 1:
##   male                               -0.121      0.013     -0.147     -0.097     90.528      0.000   1695.053
##   Residual Var.                       0.488      0.011      0.468      0.510        ---        ---   2109.744
##
##                                   -----------------------------------------------------------------------------
##
##
## Missing predictor:  male
##
## Parameters                         Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
##                                   -----------------------------------------------------------------------------
##
## Grand Mean                           -0.163      0.015     -0.193     -0.133    111.960      0.000   4346.887
##
## Level 1:
##   effort_t3                          -0.249      0.027     -0.301     -0.195     86.272      0.000   1506.112
##   Residual Var.                       1.000      0.000      1.000      1.000        ---        ---        nan
## Thresholds:
##   Tau 1                               0.000      0.000      0.000      0.000        ---        ---        nan
##
##                                   -----------------------------------------------------------------------------
```

The `simple_plot` is an rblimp function for plotting conditional effects defined on the `simple` command. Because `male` is specified as binary on the `ordinal` command line, the plot will display group-specific regression lines. The function is specified as `simple_plot(simple slope equation | moderator, model_object)`.

```
# Plot group-specific regressions
simple_plot(info_t3 ~ effort_t3 | male, mcmc_modreg_aux)
```