# Model Question Paper (3rd Set Solution)
## Mathematics HSSC-I

## Section – A

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | B | 2 | B | 3 | C | 4 | B | 5 | B | 6 | A | 7 | A | 8 | C | 9 | C | 10 | A |
| 11 | B | 12 | B | 13 | D | 14 | B | 15 | B | 16 | C | 17 | A | 18 | B | 19 | B | 20 | D |

## SECTION-B

**Q2.**

(i) $z = \frac{5-3i}{1+I} \times \frac{1-i}{1+i} = \frac{5-8i+3}{1-(-1)} = 1 - 4i$

a) Modulus of $z = |z| = \sqrt{(1)^2 + (-4)^2} = \sqrt{17}$

b) Argument of $z = tan\theta = -\frac{4}{1} \quad \Longrightarrow \theta = 275.60^\circ$

(ii) **Closure Property**:

Let $x, y$ be any two elements of G such that $x = a + \sqrt{5}b, y = c + \sqrt{5}d \ \ \forall a, b, c, d \in Q$

$x + y = (a + \sqrt{5}b) + (c + \sqrt{5}d) = (a + c) + \sqrt{5}(b + d)$

Since $a + c \ and \ b + d$ are element of $Q$

$\Longrightarrow \quad [(a + c) + \sqrt{5}(b + d)] \in$ G

$\therefore$ G is closed with respect to $' + '$.

**Associative Property:**

Let $(a + \sqrt{5}b), (c + \sqrt{5}d), (e + \sqrt{5}f) \in$ G, $\quad \forall a, b, c, d, e, f \in Q$

L.H.S $= [(a + \sqrt{5}b) + (c + \sqrt{5}d)] + (e + \sqrt{5}f)$

$= [(a + c) + \sqrt{5}(b + d)] + (\text{e} + \sqrt{5}f)$

$= (a + c + e) + \sqrt{5}(b + d + f) \qquad \rightarrow eqn(1)$

R.H.S $= (a + \sqrt{5}b) + [( c + \sqrt{5}d) + (e + \sqrt{5}f)]$

$= (a + \sqrt{5}b) + [( c + e) + \sqrt{5(}d + f)]$

$= (a + c + e) + \sqrt{5}(b + d + f) \qquad \rightarrow eqn(2)$

From Equations (1) & (2), G is Associative with respect to $' + '$.

**Existence of Identity:**

$\forall\ (a+\sqrt{5}b) \in$ G, there exist $(0+\sqrt{5}0) \in$ G such that

$[(a+\sqrt{5}b)+(0+\sqrt{5}0)] = [(0+\sqrt{5}0)+(a+\sqrt{5}b)] = (a+\sqrt{5}b)$

$\therefore\ (0+\sqrt{5}0)$ is an identity element in G with respect to $'+'$

**Existence of Inverse:**

For each $(a+\sqrt{5}b) \in$ G there exist $(-a-\sqrt{5}\,b) \in$ G

$(a+\sqrt{5}b)+[(-a)+\sqrt{5}(-b)] = [a+(-a)]+\sqrt{5}[(-b)+b] = 0+\sqrt{5}0 =$ Identity

$\therefore (-a)+\sqrt{5}(-b)$ is the inverse of $a+\sqrt{5}b$

Hence G is a group with respect to $'+'$.

(iii) $\begin{vmatrix} 4 & 4 & 4 & m \\ 4 & 4 & m & 4 \\ 4 & m & 4 & 4 \\ m & 4 & 4 & 4 \end{vmatrix} = 0$

$\begin{vmatrix} 4 & 4 & 4 & m+12 \\ 4 & 4 & m & m+12 \\ 4 & m & 4 & m+12 \\ m & 4 & 4 & m+12 \end{vmatrix} = 0$ Add $C_1, C_2, C_3$ in $C_4$

$(m+12)\begin{vmatrix} 4 & 4 & 4 & 1 \\ 4 & 4 & m & 1 \\ 4 & m & 4 & 1 \\ m & 4 & 4 & 1 \end{vmatrix} = 0$ Take $(m+12)$ common from $C_4$

$R_2 - R_1,\ R_3 - R_1, R_4 - R_1$

$(m+12)\begin{vmatrix} 4 & 4 & 4 & 1 \\ 0 & 0 & m-4 & 0 \\ 0 & m-4 & 0 & 0 \\ m-4 & 0 & 0 & 0 \end{vmatrix} = 0$

$(m+12)[(m-4)^3] = 0$

(Product of the principle diagonal elements in a triangular matrix)

Either $(m+12) = 0$ or $(m-4)^3 = 0$

$m = -12, 4$

(iv) Let $P(x) = x^3 - 8sx^2 - 4tx + 9$

As $-2$ and 2 are the roots of $P(x)$

So $P(-2) = 0$ and $P(2) = 0$

$\therefore\ P(-2) = (-2)^3 - 8s(-2)^2 - 4t(-2) + 9 = 0$

$-32s + 8t + 1 = 0$

$32s - 8t = 1 \qquad \rightarrow eqn(1)$

Now $P(2) = (2)^3 - 8s(2)^2 - 4t(2) + 9 = 0$

$-32s - 8t + 17 = 0$

$32s + 8t = 17 \qquad \rightarrow eqn(2)$

On adding Equations (1) & (2), we get $s = \frac{9}{32}$

On subtracting Equations (1) from (2), we get $t = 1$

(v) If $\alpha, \beta$ are the roots of $x^2 - 3x + 2 = 0$ so $\alpha + \beta = 3$ & $\alpha\beta = 2$

If roots are $\left(1 + \frac{3}{\alpha}\right)$ and $\left(1 + \frac{3}{\beta}\right)$, then

Sum of roots: $S = \left(1 + \frac{3}{\alpha}\right) + \left(1 + \frac{3}{\beta}\right) = 2 + 3\left(\frac{\alpha+\beta}{\alpha\beta}\right) = 2 + 3\left(\frac{3}{2}\right) = \frac{13}{2}$

Product of roots: $P = \left(1 + \frac{3}{\alpha}\right)\left(1 + \frac{3}{\beta}\right) = 1 + 3\left(\frac{\alpha+\beta}{\alpha\beta}\right) + \frac{9}{\alpha\beta} = 1 + 3\left(\frac{3}{2}\right) + \frac{9}{2} = 10$

Required Equation: $x^2 - Sx + P = 0$

$$x^2 - \frac{13}{2}x + 10 = 0$$

$$2x^2 - 13x + 20 = 0$$

(vi) $\frac{7-3x+x^2}{(3+x)(1-x)^2} = \frac{A}{(3+x)} + \frac{B}{(1-x)} + \frac{C}{(1-x)^2} \qquad \rightarrow eqn(1)$

$7 - 3x + x^2 = A(1-x)^2 + B(3+x)(1-x) + C(3+x) \qquad \rightarrow eqn(2)$

For A put $3 + x = 0$ or $x = -3$ in $eqn(2)$

$7 - 3(-3) + (-3)^2 = A(1+3)^2 + 0 + 0 \qquad \Longrightarrow A = \frac{25}{16}$

For C put $1 - x = 0$ or $x = 1$ in $eqn(2)$

$7 - 3(1) + (1)^2 = 0 + 0 + C(3+1) \qquad \Longrightarrow C = \frac{5}{4}$

Simplifying $eqn(2)$ as

$7 - 3x - x^2 = A(1 + x^2 - 2x) + B(-x^2 - 2x + 3) + C(3+x)$

Equating the coefficients of $x^2$

$-1 = A - B \quad \Longrightarrow -1 = \frac{25}{16} - B \quad \Longrightarrow B = \frac{9}{16}$

Substituting the values of A, B and C in $eqn$ (1)

$\frac{7-3x+x^2}{(3+x)(1-x)^2} = \frac{25}{16(3+x)} + \frac{9}{16(1-x)} + \frac{5}{4(1-x)^2}$

(vii) In A.P $a_1 = 74$ $and$ $d = -6$

(a) $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$$S_n = \frac{n}{2}[2(74) + (n-1)(-6)]$$

$$S_n = n(77 - 3n)$$

(b) $S_n = n(77 - 3n)$

$380 = n(77 - 3n) \qquad \because S_n = 380$

$3n^2 - 77n + 380 = 0$

$n = 19$ or $\frac{20}{3}$

Since $n = \frac{20}{3} \notin \mathbb{N} \quad \Longrightarrow \quad n = 19$

(viii) G.P: $a_1, a_1r, a_1r^2, a_1r^3,,,,,,,,,,, a_1r^{n-1},,,,$

Given that $a_2 = a_1r = \frac{1}{4} \quad \Longrightarrow a_1 = \frac{1}{4r} \quad \rightarrow eqn(1)$

G.S: $a_1 + a_1r + a_1r^2 + a_1r^3 +,,,,,,, a_1r^{n-1} +,,,,$

Taking $S_\infty = \frac{a_1}{1-r}$

$2 = \frac{a_1}{1-r} \qquad \because S_\infty = 2$

$2(1-r) = a_1$

Using $eqn(1)$

$2(1-r) = \frac{1}{4r}$

$8r^2 - 8r + 1 = 0$

$r = \frac{2 \pm \sqrt{2}}{4}$

Substituting it in $eqn(1)$

$a_1 = \frac{1}{2 \pm \sqrt{2}}$

(ix) Total arrangements $= \frac{7!}{3!.2!.1!.1!} = 420$

Numbers less than $20000000$(when we put 0 at extreme left position) $= \frac{6!}{3!.2!.1!.} = 60$

Numbers greater than $20000000 = 420 - 60 = 360$

(x) Sample space $n(s) = 52$

Let $A$ be the event that the selected card is a heart card $\Longrightarrow n(A) = 13$

Let $B$ be the event that the selected card is a face card $\Longrightarrow n(B) = 12$

$P(A) = \frac{n(A)}{n(S)} = \frac{13}{52}$ and $P(B) = \frac{n(B)}{n(S)} = \frac{12}{52}$

$A \cap B = \{king\ of\ heart\ , jack\ of\ heart\ , queen\ of\ heart\ \} \Longrightarrow n(A \cap B) = 3$

$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{52}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{11}{26}$

(xi) $(1 - kx)^n = 1 - 10x + 60x^2 + \cdots.$

Comparing with

$(1 - kx)^n = 1 - nkx + \frac{n(n-1)}{2\,!}k^2x^2 + \cdots.$

We get $nk = 10 \Longrightarrow k = \frac{10}{n} \qquad \rightarrow eqn(1)$

and $\frac{n(n-1)}{2\,!}k^2 = 60 \Longrightarrow \frac{n(n-1)}{2\,!}\left(\frac{10}{n}\right)^2 = 60$ using $eqn(1)$

$\frac{5(n-1)}{n} = 6 \Longrightarrow n = -5$

$k = \frac{10}{-5} = -2$ Using $eqn(1)$

$(xii)$ $$\frac{24\cos\theta-5\sin^2\theta}{\cos^2\theta+5\cos\theta}=5-\sec\theta$$

$$L.H.S=\frac{24\cos\theta-5\sin^2\theta}{\cos^2\theta+5\cos\theta}$$

$$L.H.S\ =\frac{24\cos\theta-5(1-\cos^2\theta)}{\cos\theta\,(\cos\theta+5)}$$

$$L.H.S\ =\frac{5\cos^2\theta+24\cos\theta-5}{\cos\theta\,(\cos\theta+5)}$$

$$L.H.S\ =\frac{5\cos^2\theta+25\cos\theta-cos\theta-5}{\cos\theta\,(\cos\theta+5)}$$

$$L.H.S\ =\frac{(5\cos\theta-1)(\cos\theta+5)}{\cos\theta\,(\cos\theta+5)}$$

$$L.H.S=\frac{5\cos\theta}{cos\theta}-\frac{1}{\cos\theta}=5-\sec\theta$$

$(xiii)$ $$\frac{1-\cos 2x+sin2x}{1+\cos 2x+sin2x}=tanx$$

$$L.H.S=\frac{1-\cos 2x+sin2x}{1+\cos 2x+sin2x}$$

$$L.H.S=\frac{1-(1-2\sin^2 x)+2sinxcosx}{1+(1-2\sin^2 x)+2sinxcosx}$$

$$L.H.S=\frac{2\sin^2 x+2sinxcosx}{2(1-\sin^2 x)+2sinxcosx}$$

$$L.H.S=\frac{2sinx(sinx+cosx)}{2\cos^2 x+2sinxcosx}$$

$$L.H.S=\frac{2sinx(sinx+cosx)}{2cosx(cosx+sinx)}$$

$L.H.S=tanx$

$L.H.S=R.H.S$

(xiv) Let $a_1=a+d, b_1=a+2d, c_1=a+3d$

$$\text{Cos}\,\gamma=\frac{{a_1}^2+{b_1}^2-{c_1}^2}{2\,a_1b_1}$$

$$\cos\gamma=\frac{(a+d)^2+(a+2d)^2-(a+3d)^2}{2\,(a+d)(a+2d)}\qquad \textit{substituting the values}$$

$$\cos\gamma=\frac{a^2+d^2+2ad+a^2+4d^2+4ad-a^2-9d^2-6ad}{2\,(a+d)(a+2d)}$$

$$\cos\gamma=\frac{a^2-4d^2}{2\,(a+d)(a+2d}=\frac{(a-2d)(a+2d)}{2(a+d)(a+2d)}=\frac{a-2d}{2(a+d)}=\frac{a}{2(a+d)}-\frac{d}{(a+d)}$$

(xv) $\tan^{-1}\left(\frac{1+\frac{x}{2}}{1-\frac{x}{2}}\right)+\tan^{-1}\left(\frac{x}{2}\right)$

$$=\tan^{-1}\left(\frac{2+x}{2-x}\right)+\tan^{-1}\left(\frac{x}{2}\right)$$

$$=\tan^{-1}\left(\frac{\frac{2+x}{2-x}+\frac{x}{2}}{1-\left(\frac{2+x}{2-x}\right)\left(\frac{x}{2}\right)}\right) \quad \because \tan^{-1}A+\tan^{-1}B=\tan^{-1}\left(\frac{A+B}{1-AB}\right)$$

$$=\tan^{-1}\left(\frac{2(2+x)+x(2-x)}{2(2-x)-x(2+x)}\right)$$

$$=tan^{-1}\left(\frac{4+4x-x^2}{4-4x-x^2}\right)$$

(xvi) $2\cos 2\theta=-3-4cos\theta$

$2(2\cos^2\theta-1)+3+4cos\theta=0$

$4\cos^2\theta+4cos\theta+1=0$

$(2cos\theta+1)^2=0$

$2cos\theta+1=0$

$\cos\theta=-\frac{1}{2}$

Here $\cos\theta<0 \Longrightarrow \theta\ lies\ in\ Quadrants\ II\ \&\ III$ with reference angle $\frac{\pi}{3}$

In Quad-II: $\theta=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$

In Quad-III: $\theta=\pi+\frac{\pi}{3}=\frac{4\pi}{3}$

Solution Set= $\left\{\frac{2\pi}{3},\frac{4\pi}{3}\right\}$

## SECTION-C

Q3. $A=\begin{bmatrix}1&2&3\\2&5&3\\1&0&8\end{bmatrix}$

Using Row Operations

$$[A\,\vdots\,I]=\begin{bmatrix}1&2&3&\vdots&1&0&0\\2&5&3&\vdots&0&1&1\\1&0&8&\vdots&0&0&1\end{bmatrix}$$

$$R\sim\begin{bmatrix}1&2&3&\vdots&1&0&0\\0&1&-3&\vdots&-2&1&0\\0&-2&5&\vdots&-1&0&1\end{bmatrix} \qquad R_2-2R_1\ ;\ R_3-R_1$$

$$R\sim\begin{bmatrix}1&0&9&\vdots&5&-2&0\\0&1&-3&\vdots&-2&1&0\\0&0&-1&\vdots&-5&2&1\end{bmatrix} \qquad R_1-2R_2;\ R_3+2R_2$$

$$R\sim\begin{bmatrix}1&0&9&\vdots&5&-2&0\\0&1&-3&\vdots&-2&1&0\\0&0&1&\vdots&5&-2&-1\end{bmatrix} \qquad (-1)R_3$$

$$R\sim\begin{bmatrix}1&0&0&\vdots&-40&16&9\\0&1&0&\vdots&13&-5&-3\\0&0&1&\vdots&5&-2&-1\end{bmatrix} \qquad R_1-9R_3;\ R_2+3R_3$$

$= [I \vdots A^{-1}]$

Hence $A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$

Q4. (a) Number of men = 6, Number of women = 4

(i) Number of ways that a 5 member Committee including exactly 3 men = $\binom{6}{3} \times \binom{4}{2} = 120$

(ii) Number of ways that a 5 member Committee including at least 2 women

$= \binom{6}{3} \times \binom{4}{2} + \binom{6}{2} \times \binom{4}{3} + \binom{6}{1} \times \binom{4}{4} = 120 + 60 + 6 = 166$

(b) $S = \{(H,1),(H,2),(H,3),(H,4),(H,5),(H,6),(T,1),(T,2),(T,3),(T,4),(T,5),(T,6)\}$

$\Longrightarrow n(S) = 12$

Since A be the event that a Tail appear in tossing a coin.

$A = \{(T,1),(T,2),(T,3),(T,4),(T,5),(T,6)\} \Longrightarrow n(A) = 6$

Since B be the event that 4 dots appear in rolling a dice.

$B = \{(H,4),(T,4)\} \Longrightarrow n(B) = 2$

$A \cap B = \{(T,4)\} \Longrightarrow n(A \cap B) = 1$

$P(A) = \frac{6}{12} = \frac{1}{2}$, $P(B) = \frac{2}{12} = \frac{1}{6}$, $P(A \cap B) = \frac{1}{12}$

For Independent Events A and B

$P(A \cap B) = P(A) \times P(B)$

$$\frac{1}{12} = \left(\frac{1}{2}\right)\left(\frac{1}{6}\right)$$

$$\frac{1}{12} = \frac{1}{12}$$

Hence A and B are the independent events.

$Q5.$ $\left(\frac{3y^2}{2} - \frac{1}{3y}\right)^6$

$T_{r+1} = \binom{n}{r}(a)^{n-r}(b)^r$ where $a = \frac{3y^2}{2}, b = -\frac{1}{3y}, n = 6$

$$T_{r+1} = \binom{6}{r}\left(\frac{3y^2}{2}\right)^{6-r}\left(-\frac{1}{3y}\right)^r$$

$$T_{r+1} = \binom{6}{r}\frac{(3)^{6-r}y^{2(6-r)}(-1)^r}{(2)^{6-r}(3)^r(y)^r}$$

$$T_{r+1} = \binom{6}{r}\frac{(-1)^r(3)^{6-2r}(y)^{12-3r}}{(2)^{6-r}}$$

(i) For the term involving $y^3$ put $12 - 3r = 3 \Longrightarrow r = 3$

$$T_{3+1} = \binom{6}{3}\frac{(-1)^3(3)^{6-6}(y)^{12-9}}{(2)^{6-3}} = -\frac{5}{2}y^3$$

(ii) For the term independent of $y$ put $12 - 3r = 0 \Longrightarrow r = 4$

$$T_{4+1} = \binom{6}{4}\frac{(-1)^4(3)^{6-8}(y)^{12-12}}{(2)^{6-4}} = \frac{5}{12}$$

(iii) Middle term $= \left(\frac{n+2}{2}\right) th\ term$

Middle term$= \left(\frac{6+2}{2}\right) th = 4th\ term$

$$T_{r+1} = \binom{6}{r}\frac{(-1)^r(3)^{6-2r}(y)^{12-3r}}{(2)^{6-r}}$$

$$T_{3+1} = \binom{6}{3}\frac{(-1)^3(3)^{6-6}(y)^{12-9}}{(2)^{6-3}} = -\frac{5}{2}y^3$$

Q6. Statement: For any two angles $\alpha\ and\ \beta$ (real numbers) then,

$cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

Proof:

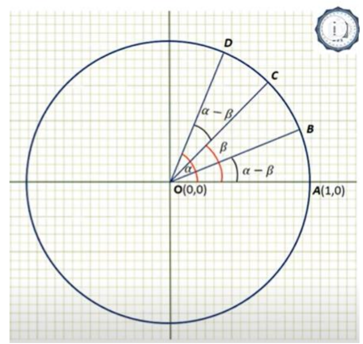
Consider four points A, B, C and D on a unit circle with center at O

Such that $Arc\ \widehat{AB} = Arc\ \widehat{CD}$ and $Chord\ \overline{AB} = Chord\ \overline{CD}$

Let $\angle AOD = \alpha\ and\ \angle AOC = \beta$

Then $\angle AOB = \alpha - \beta, \angle COD = \alpha - \beta$

Here $\alpha, \beta\ and\ \alpha - \beta$ are in standard position.




Co-ordinates of point A are $(1,0)$

Co-ordinates of point B are $(\cos(\alpha-\beta), \sin(\alpha-\beta))$

Co-ordinates of point C are $(\cos\beta, \sin\beta)$

Co-ordinate of point D are $(\cos\alpha, \sin\alpha)$

Since $|AB| = |CD|$ $\qquad \because Chord\ \overline{AB} = Chord\ \overline{CD}$

$|AB|^2 = |CD|^2$

$[\cos(\alpha-\beta) - 1]^2 + [\sin(\alpha-\beta) - 0]^2 = (\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2$

$\cos^2(\alpha-\beta) - 2cos(\alpha-\beta) + 1 + \sin^2(\alpha-\beta) = \cos^2\alpha + \cos^2\beta - 2\cos\alpha\cos\beta$
$+ \sin^2\alpha + \sin^2\beta - 2\sin\alpha\sin\beta$

$[\cos^2(\alpha-\beta) + \sin^2(\alpha-\beta)] - 2cos(\alpha-\beta) + 1 = (\cos^2\alpha + \sin^2\alpha) + (\cos^2\beta + \sin^2\beta)$
$-2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

$1 - 2\cos(\alpha-\beta) + 1 = 1 + 1 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

$cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

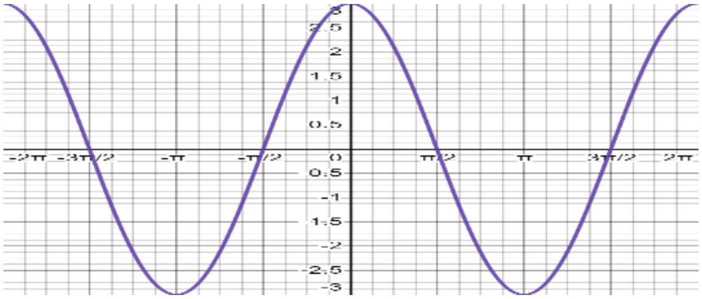
Q7. $y = 3\cos x\ \ for\ -2\pi \le x \le 2\pi$

(a) $3\cos x = 3\cos(x + 2\pi)$ $\qquad \because$ Period of cosine function is $2\pi$

Hence $2\pi$ is the period of $3\cos x$

(b) Table of values

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x$ | $0^\circ$ | $\pm30^\circ$ | $\pm60^\circ$ | $\pm90^\circ$ | $\pm120^\circ$ | $\pm150^\circ$ | $\pm180^\circ$ |
| $y$ | 3 | 2.6 | 1.5 | 0 | −1.5 | −2.6 | −3 |
| $x$ | $\pm210^\circ$ | $\pm240^\circ$ | $\pm270^\circ$ | $\pm300^\circ$ | $\pm330^\circ$ | $\pm360^\circ$ | $\pm210^\circ$ |
| $y$ | −2.6 | −1.5 | 0 | 1.5 | 2.6 | 3 | −2.6 |



Q8. $\cos^{-1}\left(\frac{3}{5}\right) - 2\tan^{-1}\left(\frac{3}{4}\right) = \cos^{-1}\left(\frac{117}{125}\right)$

Proof: L.H.S $= \cos^{-1}\left(\frac{3}{5}\right) - 2\tan^{-1}\left(\frac{3}{4}\right)$

L.H.S $= \cos^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}\right)$ $\qquad \because 2\tan^{-1}A = \tan^{-1}\left(\frac{2A}{1-A^2}\right)$

L.H.S$= \cos^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{24}{7}\right)$ $\qquad \to eqn(1)$

Let $\alpha = \cos^{-1}\left(\frac{3}{5}\right) iff cos\alpha = \frac{3}{5} > 0$ in $[0,\pi] \Longrightarrow \alpha \in Quad - I$

i.e. $sin\alpha = +\sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$

Let $\beta = \tan^{-1}\left(\frac{24}{7}\right)$ $iff\ \ tan\beta = \frac{24}{7} > 0$ in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right) \Longrightarrow \beta \in Quad - I$

$\sec\beta = +\sqrt{1+\tan^2\beta} = \sqrt{1+\frac{576}{49}} = \frac{25}{7}$

$sec\beta = \frac{25}{7} \quad \Longrightarrow \quad cos\beta = \frac{7}{25}$

$sin\beta = +\sqrt{1-cos^2\beta} = \sqrt{1-\frac{49}{625}} = \frac{24}{25}$

$\because \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos(\alpha-\beta) = \left(\frac{3}{5}\right)\left(\frac{7}{25}\right) + \left(\frac{4}{5}\right)\left(\frac{24}{25}\right)$

$\cos(\alpha-\beta) = \frac{117}{125}$

$\alpha - \beta = \cos^{-1}\frac{117}{25}$

$\cos^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{24}{7}\right) = \cos^{-1}\frac{117}{25}$

$\cos^{-1}\left(\frac{3}{5}\right) - 2\tan^{-1}\left(\frac{3}{4}\right) = \cos^{-1}\frac{117}{25}$