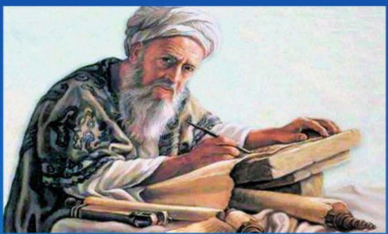
" Your Best Study Experience begins here."

SEVEN MUSLIMS NOTES

# PHYSICS 11



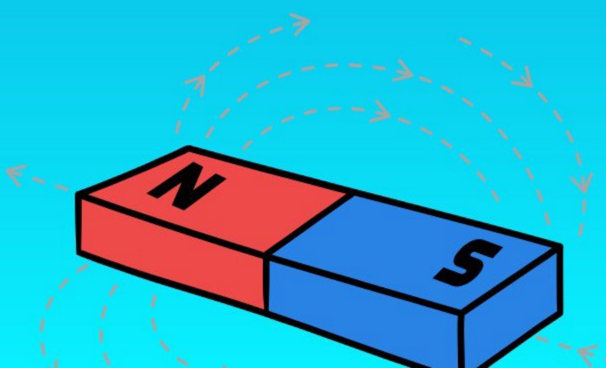
**Al-Biruni (973–1048)** calculated the Earth's radius and worked on the physics of planetary motion.

## Best Regards to

# Sir Muhammad Ali

( Physics Lecturer KIPS College )

$E=mc^2$



# CHAPTER # 3

# MOTION AND FORCE

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## INTRODUCTION

**Motion:**

"If a body changes its position with respect to its surroundings, the body is said to be in motion."

**Rest:**

"If a body does not change its position with respect to its surroundings, the body is said to be at rest."

**Q.1: Define the terms**

**(i) Distance** **(ii) Displacement**

**(iii) Speed** **(iv) Velocity**

**(v) Average velocity** **(vi) Instantaneous velocity**

**(vii) Uniform velocity** **(viii) Variable velocity**



**i. DISTANCE**

"Total length between two points is called distance."

**Representation:**

It is denoted by "S".

**SI Unit:**

Its SI unit is meter (m).

**Quantity:**

It is a scalar quantity.

**ii. DISPLACEMENT**

"The change in the position of a body from initial to final point is called displacement."

Or

"The shortest distance between two points is called displacement."

**Representation:**

It is denoted by "d" ($d = r_2 - r_1$).

**SI unit:**

Its SI unit is meter (m).

**Quantity:**

It is a vector quantity.

### iii. SPEED

"Distance covered by a body in unit time is called speed."

**Formula:**

$$v = \frac{\Delta S}{\Delta t}$$

**SI unit:**

Its SI unit is $ms^{-1}$.

**Quantity:**

It is a scalar quantity.

### iv. VELOCITY

"The rate of change of displacement is called velocity."

**Formula:**

$$v = \frac{\Delta d}{\Delta t}$$

**SI unit:**

Its SI unit is $ms^{-1}$.

**Quantity:**

It is a vector quantity.

### v. AVERAGE VELOCITY

"The ratio of total displacement to the total time taken to cover displacement is called average velocity."

$$\overline{v}_{avg} = \frac{\overline{d}}{t}$$

**SI unit:**

Its SI unit is $ms^{-1}$.

### vi. INSTANTANEOUS VELOCITY

"Velocity of an object at any instant of time is called instantaneous velocity."

### vii. UNIFORM/CONSTANT VELOCITY

"An object is said to be moving with uniform velocity if it is moving in a straight line at a constant speed, with no change in direction or acceleration."

### viii. VARIABLE VELOCITY

"An object is said to be moving with variable velocity if it is moving in a straight line with a changing speed or moving in a curve."

**Q.2: Define the following**

(i) Acceleration
(ii) Average acceleration
(iii) Instantaneous acceleration
(iv) Uniform acceleration
(v) Variable acceleration
(vi) Positive acceleration
(vii) Negative acceleration

## i. ACCELERATION

"The time rate of change of velocity is called acceleration."

$$\bar{a} = \frac{\Delta\bar{v}}{\Delta t}$$

It is a vector quantity and its direction is same as that of change in velocity.

## ii. AVERAGE ACCELERATION

"The ratio of the total change in velocity to the total time taken is called average acceleration."

$$\bar{a}_{avg} = \frac{\bar{v}_2 - \bar{v}_1}{\Delta t} = \frac{\Delta\bar{v}}{\Delta t}$$

## iii. INSTANTANEOUS ACCELERATION

"Acceleration of a body at any instant of time is called instantaneous acceleration."

**For example:**

If Δv is the change in velocity in very short interval of time Δt such that Δt approaches to zero but not equal to zero, then mathematically the Instantaneous acceleration is expressed as

$$\bar{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\bar{v}}{\Delta t}$$

## iv. UNIFORM ACCELERATION

"It is defined as the constant change in velocity over a specified period of time."

Or

"Uniform acceleration occurs when the velocity of an object changes at a constant rate over a specified period of time."

## v. VARIABLE ACCELERATION

"Variable acceleration occurs when the velocity of an object changes at a non-constant rate over a specified period of time."

### vi. POSITIVE ACCELERATION

"If velocity of a body is increasing then its acceleration is positive."

### vii. NEGATIVE ACCELERATION

"If velocity of a body is decreasing then its acceleration is negative."

Negative acceleration is also called deceleration and retardation .

## Q.3: What is velocity time graph

### VELOCITY-TIME GRAPH

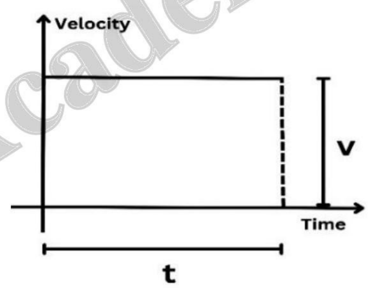
" Graph which shows the variation of velocity of an object with time is called velocity-time graph."

In such graphs, the time is taken along positive x-axis because it is the independent quantity.

1. **Body moving with Constant velocity**

   v-t graph for a body moving with constant velocity is a horizontal line parallel to time axis.

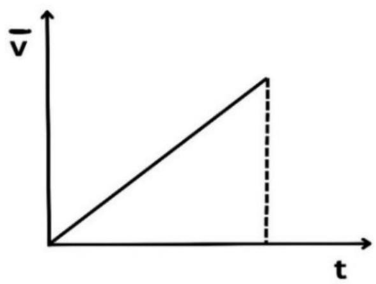
$$A = v \times t = S$$




2. **Body moving with uniform acceleration**

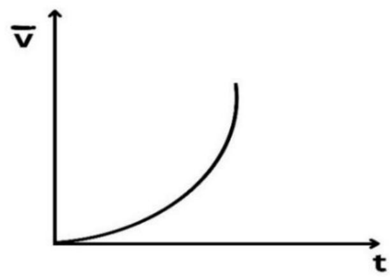
   v-t graph for a body moving with constant acceleration is a straight line.

$$\text{Slope} = \frac{\Delta v}{\Delta t} = a$$




3. **Body moving with increasing acceleration**

   v-t graph for a body moving with increasing acceleration is a curved line.




**Significance of velocity time graph:**

Velocity time graph is used

- To find the average acceleration of an object from the slope of v-t graph.
- To find the distance by calculating the area under the v-t graph.

## Q.4: Write down the equations of motion for uniformly accelerated bodies.

## EQUATIONS OF MOTION

Consider an object moving with a constant acceleration " a " along a straight path. Its initial velocity is " $v_i$ " and its final velocity after a time interval " t " is " $v_f$ ". The distance traveled during this interval is " S ", then we have

1. $v_f = v_i + at$
2. $S = v_i t + \frac{1}{2}at^2$
3. $2aS = v_f^2 - v_i^2$

**Limitations:**

These equations hold for linear motion and uniform acceleration.

## Q.5: State newton's law of motion.

## NEWTON'S LAWS OF MOTION

**Newton's first law of motion**

"A body continues its state of rest or of uniform motion until or unless an unbalanced force acts on it." It is also known as law of inertia.

$$a = 0$$

**Inertia:**

"Ability of a body to maintain its state of rest or motion is called inertia."

**Inertial frame of reference:**

"Set of coordinate axis in which law of inertia holds is called inertial frame of reference."

**Example:**

A frame of reference fixed on the earth is approximately an inertial frame of reference because the rotational acceleration of earth is very small.

**Non-inertial frame of reference:**

"Set of coordinate axis in which law of inertia does not hold is known as non-inertial frame of reference."

**For example:**

A car moving with increasing velocity is an example of non-inertial frame of reference.

**Newton's second law of motion:**

" When an unbalanced force acts on a body it produces acceleration which is directly proportional to the applied force and inversely proportional to the mass of the object."

$$F = ma$$

**Newton's third law of motion:**

"To every action, there is an equal but opposite reaction."

**For example:**

When you let the air out of a balloon, the air rushes out in one direction (action), and the balloon moves in the opposite direction (reaction).

## Q.6: What is Momentum? Write its formula and unit.

### MOMENTUM

"Quantity of motion possessed by a body is called its momentum."

**Formula:**

$P = mv$

Momentum is a vector quantity. Its units are $Kgms^{-1} = Ns$

**Dimensions:**

The dimensions of momentum are $[MLT^{-1}]$

## Q.7: State Newton's second law of motion in terms of momentum.

[ GRW. 2017 ]



**Statement:**

**"**Time rate of change of momentum is equal to applied force."

**Proof:**

Let us consider a force " F " acting on a body of mass " m " changes its velocity from " $v_i$ " to " $v_f$ " in time " t " then

$$a = \frac{v_f - v_i}{t} \quad \ldots\ldots\ldots..(1)$$

By newton's second law of motion

$$F = ma$$

Using equation 1

$$F = m\left(\frac{v_f - v_i}{t}\right)$$

$$F = \frac{mv_f - mv_i}{t}$$

## Q.8: Define impulse and show that it is change in momentum.

## IMPULSE

"When a large force acts on a body for a very short interval of time then the product of force and time is called impulse or impulse of force."

$$\text{Impulse} = I = F \times t$$

$$\bar{F} = \frac{mv_f - mv_i}{t}$$

$$F \times t = \frac{mv_f - mv_i}{t} \times t$$

$$F \times t = mv_f - mv_i$$

$$\text{Impulse} = \text{change in momentum}$$

**Unit:**

Its unit is $Kgms^{-1}$ or Na. "It is denoted by I and it is a vector quantity."

## Q.9: State and explain law of conservation of linear momentum.

[ LHR. GI, 2017 ] [ FSD. GII, SGD. GI, BWP. GI, 2019 ] [ FSD. GII, 2021 ] [ MLN. GI, DGK, GI, GRW. GII. 2022 ]

## LAW OF CONSERVATION OF MOMENTUM

**Isolated system:**

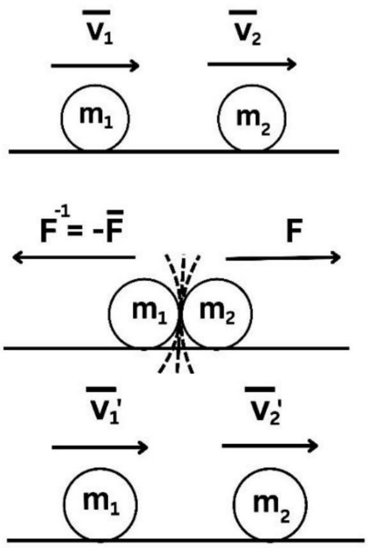
"A system in which no external force is involved is called an isolated system."

**Statement:**

Total linear momentum of an isolated system remains constant.

**Explanation:**

Consider an isolated system of two balls of masses $m_1$ and $m_2$ moving in a straight line in the same direction with velocities $\bar{v}_1$ and $\bar{v}_2 < \bar{v}_1$ respectively. They collide and after collision $m_1$ moves with velocity $\bar{v}_1{}'$ and $m_2$ moves with velocity $\bar{v}_2$.




**Mathematical expression:**

**For mass $m_1$:**

$$\bar{F}' \times t = m_1\bar{v}_1' - m_1\bar{v}_1 \quad \ldots\ldots\ldots\ldots (1)$$

**For mass $m_2$:**

$$\bar{F} \times t = m_2\bar{v}_2' - m_2\bar{v}_2 \quad \ldots\ldots\ldots (2)$$

Adding equation no. 1 and 2

$$\bar{F}' \times t + \bar{F} \times t = m_1\bar{v}_1{}' - m_1\bar{v}_1 + m_2\bar{v}_2{}' - m_2\bar{v}_2$$

$$(\bar{F}' + \bar{F})t = m_1\bar{v}_1{}' - m_1\bar{v}_1 + m_2\bar{v}_2{}' - m_2\bar{v}_2$$

$$(-\bar{F} + F)t = m_1\bar{v}_1{}' - m_1\bar{v}_1 + m_2\bar{v}_2{}' - m_2\bar{v}_2$$

$$0 = m_1\bar{v}_1{}' - m_1\bar{v}_1 + m_2\bar{v}_2{}' - m_2\bar{v}_2$$

$$m_1\bar{v}_1 + m_2\bar{v}_2 = m_1\bar{v}_1' + m_2\bar{v}_2{}'$$

**Conclusion:**

The above equations shows that total linear momentum of the system before collision is equal to the total linear momentum of the system after collision i.e. the linear momentum remains constant.

**Q.10: Define Elastic and inelastic collision. Prove that for elastic collision in one dimension, relative velocity of approach is equal to the relative velocity of separation.** [ SGD. GII, 2017 ] [ LHR. GI, RWP. GI, 2018 ]



## ELASTIC AND INELASTIC COLLISION

**Elastic collision:**

"Type of collision in which the kinetic energy of the system remains constant is called elastic collision."

**Example:**

Ball dropped from a certain height bounces back to the same height.

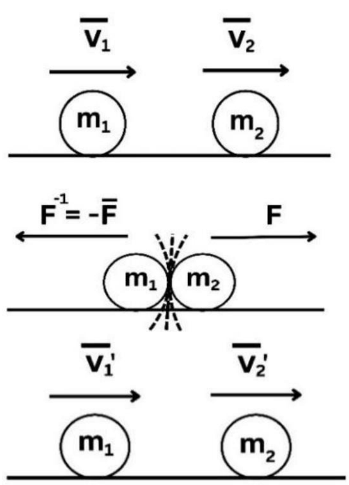
**Inelastic collision:**

"Type of collision in which kinetic energy of the system does not remains constant is called inelastic collision."

**Example:**

Ball dropped from a certain height does not bounces back to the same height.

## ELASTIC COLLISION IN ONE DIMENSION




**Explanation:**

Consider two balls of masses $m_1$ and $m_2$ moving in a straight line in same direction with initial velocities $\bar{v}_1$ and $\bar{v}_2$. They collide and after collision $m_1$ moves with velocity $\bar{v}_1{}'$ and $m_2$ moves with velocity $\bar{v}_2{}'$.

**Mathematical deviation:**

Using law of conservation of momentum:

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2{}'$$

$$m_1\overline{v}_1 - m_1\overline{v}_1' = m_2\overline{v}_2' - m_2\overline{v}_2$$

$$m_1(\overline{v}_1 - \overline{v}_1') = m_2(\overline{v}_2' - \overline{v}_2) \quad \ldots\ldots\ldots\ldots (1)$$

Using conservation of K.E

$$\frac{1}{2}m_1\overline{v}_1^2 + \frac{1}{2}m_2\overline{v}_2^2 = \frac{1}{2}m_1\overline{v}_1'^2 + \frac{1}{2}m_2\overline{v}_2'^2$$

$$m_1\overline{v}_1^2 - m_1\overline{v}_1'^2 = m_2\overline{v}_2'^2 - m_2\overline{v}_2^2$$

$$m_1(\overline{v}_1^2 - \overline{v}_1'^2) = m_2(\overline{v}_2'^2 - \overline{v}_2^2)$$

$$m_1(\overline{v}_1 + \overline{v}_1')(\overline{v}_1 - \overline{v}_1') = m_2(\overline{v}_2' + \overline{v}_2)(\overline{v}_2' - \overline{v}_2) \quad \ldots\ldots\ldots\ldots (2)$$

Dividing equation no 2 by equation no 1

$$\frac{m_1(\overline{v}_1 + \overline{v}_1')(\overline{v}_1 - \overline{v}_1')}{m_1(\overline{v}_1 - \overline{v}_1')} = \frac{m_2(\overline{v}_2' + \overline{v}_2)(\overline{v}_2' - \overline{v}_2)}{m_2(\overline{v}_2' - \overline{v}_2)}$$

$$\overline{v}_1 + \overline{v}_1' = \overline{v}_2' + \overline{v}_2$$

$$\overline{v}_1 - \overline{v}_2 = \overline{v}_2' - \overline{v}_1' \quad \ldots\ldots\ldots\ldots (3)$$

$$\overline{v}_1 - \overline{v}_2 = -(\overline{v}_1' - \overline{v}_2')$$

Negative sign shows that velocities of both balls are interchanged.

**Final velocities:**

Solving equation 1 and 3 simultaneously

$$\overline{v}_1' = \frac{m_1 - m_2}{m_1 + m_2}\overline{v}_1 + \frac{2m_2}{m_1 + m_2}\overline{v}_2$$

$$\overline{v}_2' = \frac{m_2 - m_1}{m_1 + m_2}\overline{v}_2 + \frac{2m_1}{m_1 + m_2}\overline{v}_1$$

## Q.11: Discuss the various cases of elastic collision in dimensions.

# SPECIAL CASES

1. **When $m_1 = m_2$**
   If $m_1 = m_2$
   Then
   $v_1' = v_2$
   $v_2' = v_1$
   Thus velocities of both the balls are interchanged.
2. **When $m_1 = m_2$ and $v_2 = 0$**
   If $m_1 = m_2$ ; $v_2 = 0$
   Then
   $v_1' = 0$
   $v_2' = v_1$

After collision, mass $m_1$ comes to rest and $m_2$ starts moving with $v_1$.

3. **When a light body collides with a massive body at rest**

   If light mass collides with heavy mass at rest

$$m_1 << m_2$$

$m_1 \approx 0$ ; $v_2 = 0$

Then

$$v_1' = - v_1$$
$$v_2' = 0$$

Thus after collision, mass $m_1$ moves back with $v_1$.

4. **When a massive body collides with light stationary body**

   If a heavy mass collides with light mass at rest.

$$m_1 >> m_2$$

$m_2 \approx 0$ ; $v_2 = 0$

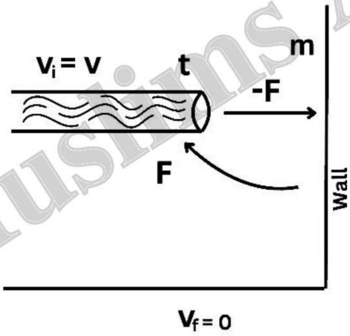
Then

$$v_1' = v_1$$
$$v_2' = 2v_1$$

After collision lighter mass will move with velocity double than the initial velocity of mass $m_1$.

**Q.12: Drive a relation for the force due to water flow on the wall.**

## FORCE DUE TO WATER FLOW




Let mass " m " of water flowing through a horizontal pipe with velocity " v " strikes perpendicularly to a wall in time " t " then force applied by the wall is

$$F_{wall} = \frac{mv_f - mv_i}{t} = -\frac{mv}{t}$$

$$F_{water} = -F_{wall} = -\left(-\frac{mv}{t}\right)$$

$$F_{water} = -F_{wall} = \frac{mv}{t}$$

$$F_{water} = \frac{m}{t} \times v$$

( Force = Mass per second × Change in velocity )

**Q.13: Show that the momentum is conserved when explosive forces changes the momentum within an isolated system.**

## MOMENTUM AND EXPLOSIVE FORCES

**Principle:**

In an isolated system, the total momentum remains constant due to explosive forces.

**Examples:**

There are many examples of momentum and explosive forces which are as follows

1. **Explosion of a bomb:**

   If a bomb explodes in mid-air, the fragments fly in different directions, but their total momentum is still the same as the initial momentum.

2. **Firing of rifle:**

   Let a bullet of mass " m " fired from a rifle of mass " M " with velocity " v ". Initially, the total momentum of the bullet and rifle is zero. From the principle of conservation of linear momentum, when the bullet is fired, the total momentum of bullet and rifle still remains zero, since no external force has acted on them. Thus " V' " is the velocity of the rifle then,

$$0 + 0 = mv + MV'$$
$$0 = mv + MV'$$
$$MV' = -mv$$
$$V' = -\frac{mv}{M}$$

   The momentum of the rifle is thus equal and opposite to that of the bullet. Since mass of rifle is much greater than bullet, it follows that the rifle moves back with very small velocity than that of the bullet.

## Q.14: Explain the propulsion of rocket in space.

**Important points to learn:**

- The rocket gains momentum equal to the momentum of the gas expelled from the engine but in opposite direction
- Instead of travelling at steady speed the rocket gets faster and faster so long the engines are operating.
- A rocket carries its own Fuel in the form of a liquid or solid and oxygen.
- A typical rocket consumes about $10000 kgs^{-1}$ of fuel and ejects the burnt gases at speeds of over $4000\ ms^{-1}$.
- In effect more than 80% of the launch mass of a rocket consists of fuel only.

## Q.15: Define projectile motion. Drive the relation for the following terms:

**(i) Time of flight (ii) Height of flight (iii) Range of flight**

[ LHR. GII, 2017 ] [ DGK. GI, 2021 ] [ BWP. GII, 2022]

**(Or) What is projectile motion? Drive expressions for its height and range.** [ DGK. GI, 2021 ]

**PROJECTILE MOTION**

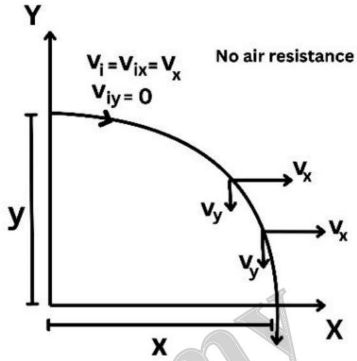
"Two dimensional motion of a body under constant acceleration due to gravity is called projectile motion."

**Example:**

- Football kicked by a player.
- Bullet fired from a gun.

**Explanation:**

1. **Body projected horizontally:**
Consider a body is projected horizontally from point " A " with initial velocity " $v_x$ " ($v_y = 0$) it moves forward as well as downward under the action of gravity.




**Along x-axis:**
Ignoring air friction

$$\because \quad F_x = 0$$

$$\because \quad a_x = 0$$

Thus from 2nd equation of motion distance covered by projectile along x-axis is:

$$x = v_{ix}t + \frac{1}{2}a_x t^2$$

$$\because \quad v_x = \text{Constant}$$

$$\because \quad a_x = 0$$

$$\therefore \quad x = v_x t$$

This is the horizontal displacement of projectile in time t.

**Along y- axis:**
As the ball moves downward under the action of gravity, so its acceleration in this direction is " g ".



$$S = v_i t + \frac{1}{2}at^2$$

$$\because \quad F_y \neq 0$$

$$\because \quad a_y = g$$

$$\because \quad v_{iy} = 0$$

Now the distance covered along y- axis is given as

$$y = v_{iy}t + \frac{1}{2}a_y t^2$$

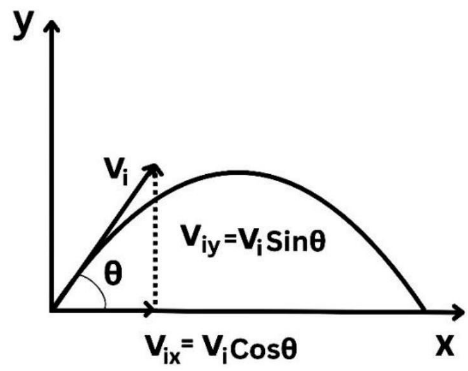
$$\therefore \quad y = (0)t + \frac{1}{2}gt^2$$

$$y = \frac{1}{2}gt^2$$

This is the vertical displacement of projectile in time t.

**2. Body projected at some angle:**

Now consider a body projected at some angle " θ " with horizontal. Let " $v_i$ " be the initial velocity of the body.




$$v_{ix} = v_i\cos\theta$$
$$v_{iy} = v_i\sin\theta$$

**Along x-axis:**

Ignoring air friction

$$\because \quad F_x = 0$$
$$\because \quad a_x = 0$$

Using first equation of motion

$$v_{fx} = v_{ix} + a_xt$$
$$\therefore \quad v_{fx} = v_{ix} + (0)t$$
$$v_{fx} = v_{ix} = v_i\cos\theta$$

**Along y-axis:**

$$\because \quad F_y = F_g$$
$$\because \quad a_y = -g$$

Using first equation of motion

$$v_{fy} = v_{iy} + a_yt$$
$$\therefore \quad v_{fy} = v_i\sin\theta - gt$$

The final velocity is then given as

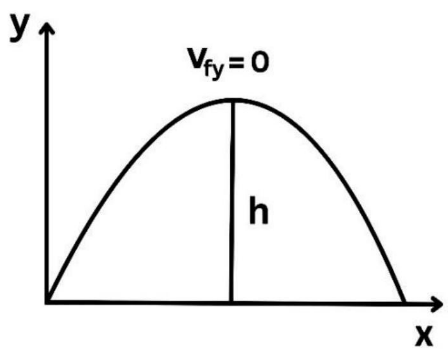
$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$$

Also

$$\theta = \tan^{-1}\frac{v_{fx}}{v_{fy}}$$

**3. Height of projectile:**

"Maximum vertical distance covered by a projectile is called height of the projectile."




**Derivation:**

Using 3rd equation of motion

$$2a_yS = v_{fy}^2 - v_{iy}^2$$

As

$$a_y = -g$$
$$v_{iy} = v_i\sin\theta$$
$$S = h$$
$$v_{fy} = 0$$

By putting these values in the above equation

$$-2gh = 0 - v_i^2\sin^2\theta$$
$$-2gh = -v_i^2\sin^2\theta$$
$$h = \frac{v_i^2\sin^2\theta}{2g}$$

**4. Time of flight:**

"Time taken by a projectile to cover distance between point of projection and the point where it bits the ground is called time of flight."

**Derivation:**

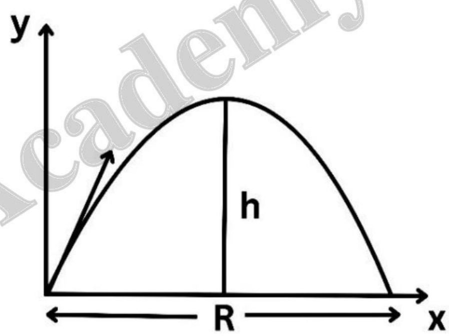
$$\because \quad S = 0$$
$$\because \quad v_{iy} = v_i \sin\theta$$
$$\because \quad a_y = -g$$

Using 2nd equation of motion

$$S = v_{iy}t + \frac{1}{2}a_y t^2$$
$$\therefore \quad 0 = v_i \sin\theta t - \frac{1}{2}gt^2$$
$$\frac{1}{2}gt^2 = v_i \sin\theta t$$
$$t = \frac{2v_i \sin\theta}{g}$$

5. **Range of projectile:**

"The maximum horizontal distance covered by a projectile is called range of projectile."




**Derivation:**

$$\because \quad a_x = 0$$
$$\because \quad v_{ix} = v_i \cos\theta$$
$$\because \quad S = R$$

Using second equation of motion

$$S = v_{ix}t + \frac{1}{2}a_x t^2$$
$$\therefore \quad R = v_i \cos\theta t + \frac{1}{2}(0)t^2$$
$$R = v_i \cos\theta t$$
$$= v_i \cos\theta \left(\frac{2v_i \sin\theta}{g}\right)$$
$$R = \frac{v_i^2(2\sin\theta\cos\theta)}{g}$$

As $\sin 2\theta = 2\sin\theta\cos\theta$

By putting in the above equation

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

## Q.16: Prove that range is maximum at 45°

At

$$\theta = 45°$$
$$R = \frac{v_i^2 \sin 2\theta}{g}$$

$$R = \frac{v_i^2 \sin 2(45)}{g}$$

$$= \frac{v_i^2 \sin 90}{g}$$

$$R = \frac{v_i^2}{g} = \text{maximum range}$$

## Q.17: Describe the application to ballistic missile?

**Ballistic missile:**

"The unpowered and unguided missile is called ballistic missile."

**Ballistic trajectory:**

"The path followed by the ballistic missile is called ballistic trajectory."

For flat earth / short range the trajectory of projectile is parabolic and for spherical it is elliptical.

**Ballistic flight:**

"A flight in which a projectile is given an initial push and is then allowed to move freely due to inertia under the action of gravity is called ballistic flight."

**Use of ballistic missile:**

The ballistic missiles are used only for short ranges. For long ranges and greater precision, powered and remote control guided missiles are used.