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AN INTRODUCTION TO ASTRONOMY

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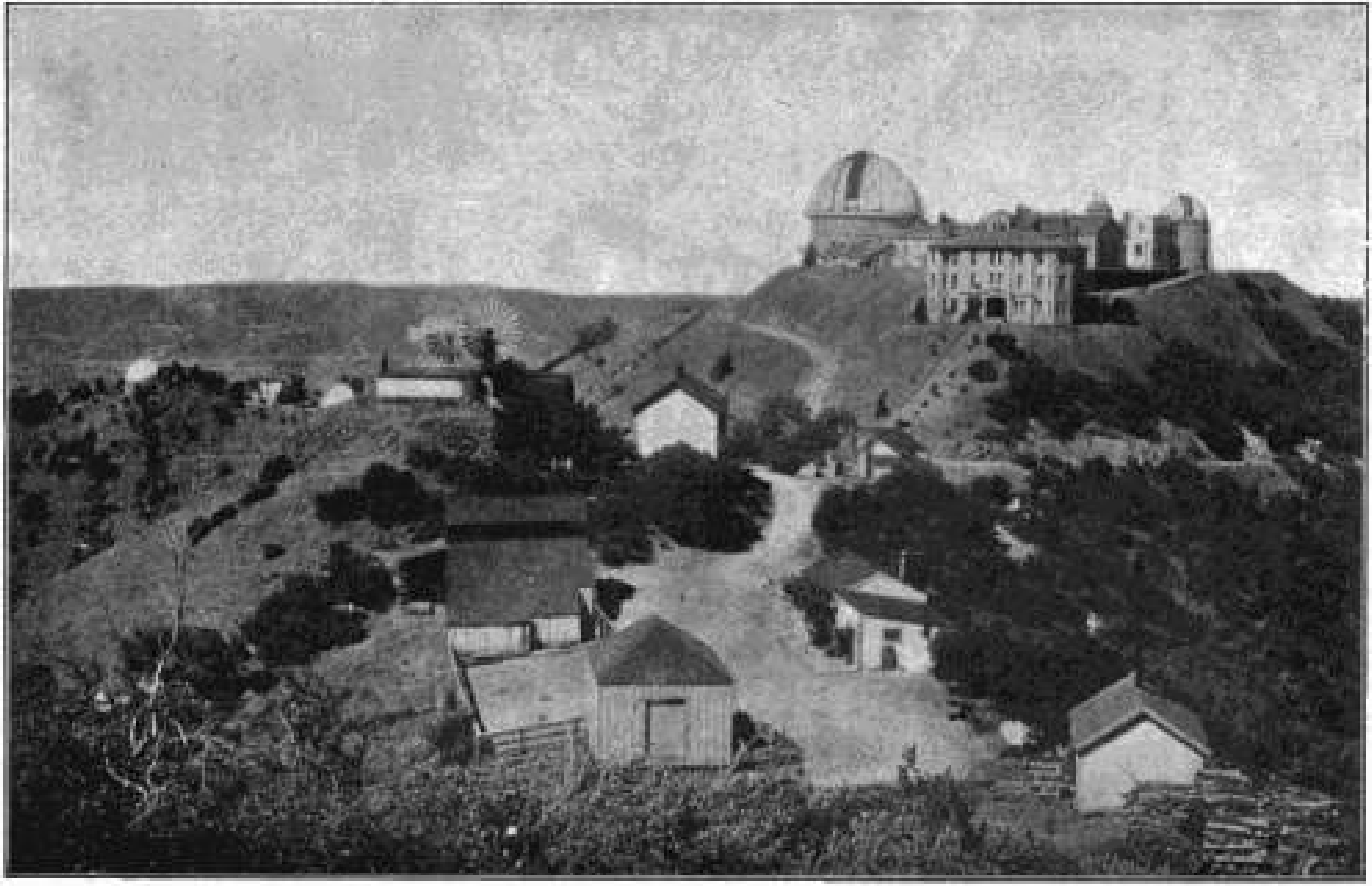

FIG. 1.—The Lick Observatory, Mount Hamilton, California.

AN INTRODUCTION

TO

ASTRONOMY

BY

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PREFACE

The necessity for a new edition of "An Introduction to Astronomy" has furnished an opportunity for entirely rewriting it. As in the first edition, the aim has been to present the great subject of astronomy so that it can be easily comprehended even by a person who has not had extensive scientific training. It has been assumed that the reader has no intention of becoming an astronomer, but that he has an interest in the wonderful universe which surrounds him, and that he has arrived at such a stage of intellectual development that he demands the reasons for whatever conclusions he is asked to accept. The first two of these assumptions have largely determined the subject matter which is presented; the third has strongly influenced the method of presenting it.

While the aims have not changed materially since the first edition was written, the details of the attempt to accomplish them have undergone many, and in some cases important, modifications. For example, the work on reference points and lines has been deferred to Chapter IV. If one is to know the sky, and not simply know about it, a knowledge of the coördinate systems is indispensable, but they always present some difficulties when they are encountered at the beginning of the subject. It is believed that the present treatment prepares so thoroughly for their study and leads so naturally to them that their mastery will not be found difficult. The chapter on telescopes has been regretfully omitted because it was not necessary for understanding the remainder of the work, and because the space it occupied was needed for treating more vital parts of the subject. The numerous discoveries in the sidereal universe during the last ten years have made it necessary greatly to enlarge the last chapter.

As now arranged, the first chapters are devoted to a discussion of the earth and its motions. They present splendid examples of the characteristics and methods of science, and amply illustrate the care with which scientific theories are established. The conclusions which are set forth are bound up with the development of science from the dawn of recorded history to the recent experiments on the rigidity and the elasticity of the earth. They show how closely various sciences are interlocked, and how much an understanding of the earth depends upon its relations to the sky. They lead naturally to a more formal treatment of the celestial sphere and a study of the constellations. A familiarity with the brighter stars and the more conspicuous constellations is regarded as important. One who has become thoroughly acquainted with them will always experience a thrill when he looks up at night into a cloudless sky.

The chapter on the sun has been postponed until after the treatment of the moon, planets, and comets. The reason is that the discussion of the sun necessitates the introduction of many new and difficult topics, such as the conservation of energy, the disintegration of radioactive elements, and the principles of spectrum analysis. Then follows the evolution of the solar system. In this chapter new and more serious demands are made on the reasoning powers and the imagination. Its study in a measure develops a point of view and prepares the way for the consideration, in the last chapter, of the transcendental and absorbingly interesting problems respecting the organization and evolution of the sidereal universe.

Lists of problems have been given at the ends of the principal divisions of the chapters. They cannot be correctly answered without a real comprehension of the principles which they involve, and in very many cases, especially in the later chapters, they lead to important supplementary results. It is strongly recommended that they be given careful consideration.

The author is indebted to Mr. Albert Barnett for the new star maps and the many drawings with which the book is illustrated, with the exception of Figs. 23 and 30, which were kindly furnished by Mr. George Otis. He is indebted to Professor David Eugene Smith for photographs of Newton, Kepler, Herschel, Adams, and Leverrier. He is indebted to the Lick, Lowell, Solar, and Yerkes observatories for a large amount of illustrative material which was very generously furnished. He is under deeper obligations to his colleague, Professor W. D. MacMillan, than this brief acknowledgment can express for assistance on the manuscript, on the proofs, and in preparing the many problems which appear in the book.

F. R. MOULTON.

THE UNIVERSITY OF CHICAGO,
September 25, 1916.

CONTENTS

CHAPTER I

Preliminary Considerations

CHAPTER II

THE EARTH

I. The Shape of the Earth

II. The Mass of the Earth and the Condition of its Interior

III. The Earth's Atmosphere

CHAPTER III

THE MOTIONS OF THE EARTH

I. The Rotation of the Earth

II. The Revolution of the Earth

CHAPTER IV

Reference Points and Lines

CHAPTER V

The Constellations

CHAPTER VI

Time

CHAPTER VII

The Moon

CHAPTER VIII

THE SOLAR SYSTEM

I. The Law of Gravitation

II. Orbits, Dimensions, and Masses of the Planets

CHAPTER IX

THE PLANETS

I. Mercury and Venus

II. Mars

III. Jupiter

IV. Saturn

V. Uranus and Neptune

CHAPTER X

COMETS AND METEORS

I. Comets

II. Meteors

CHAPTER XI

THE SUN

I. The Sun's Heat

II. Spectrum Analysis

III. The Constitution of the Sun

CHAPTER XII

EVOLUTION OF THE SOLAR SYSTEM

I. General Considerations on Evolution

II. Data of Problem of Evolution of Solar System

III. The Planetesimal Theory

IV. Historical Cosmogonies

CHAPTER XIII

THE SIDEREAL UNIVERSE

I. The Apparent Distribution of the Stars

II. Distances and Motions of the Stars

III. The Stars

IV. The Nebulæ

LIST OF TABLES

LIST OF PHOTOGRAPHIC ILLUSTRATIONS

AN INTRODUCTION TO ASTRONOMY

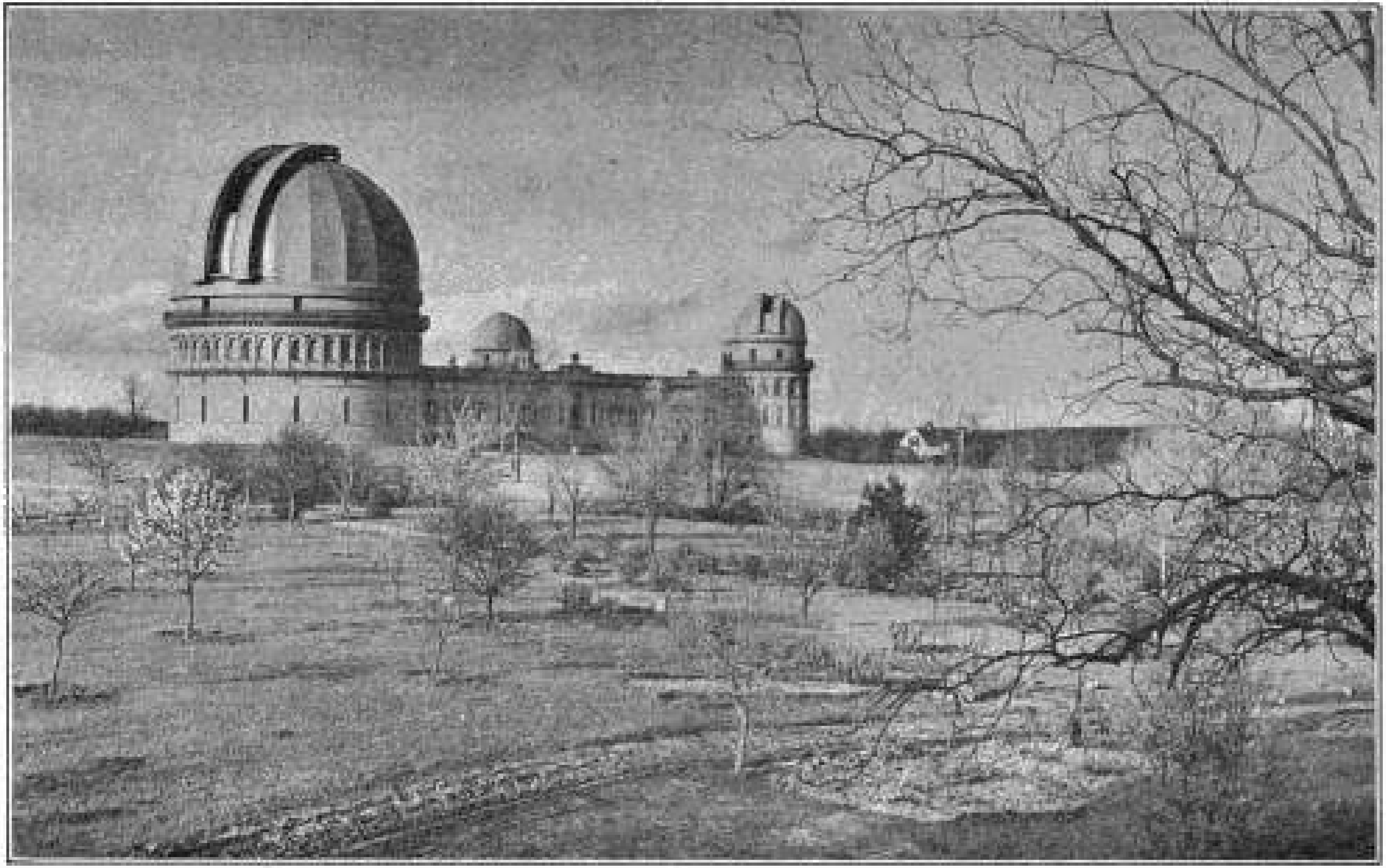


Fig. 2.—The Yerkes Observatory of the University of Chicago, Williams Bay, Wisconsin.

AN INTRODUCTION TO ASTRONOMY

CHAPTER I

PRELIMINARY CONSIDERATIONS

1. Science.—The progress of mankind has been marked by a number of great intellectual movements. At one time the ideas of men were expanding with the knowledge which they were obtaining from the voyages of Columbus, Magellan, and the long list of hardy explorers who first visited the remote parts of the earth. At another, millions of men laid down their lives in order that they might obtain toleration in religious beliefs. At another, the struggle was for political freedom. It is to be noted with satisfaction that those movements which have involved the great mass of people, from the highest to the lowest, have led to results which have not been lost.

The present age is known as the age of science. Never before have so many men been actively engaged in the pursuit of science, and never before have its results contributed so enormously to the ordinary affairs of life. If all its present-day applications were suddenly and for a considerable time removed, the results would be disastrous. With the stopping of trains and steamboats the food supply in cities would soon fail, and there would be no fuel with which to heat the buildings. Water could no longer be pumped, and devastating fires might follow. If people escaped to the country, they would perish in large numbers because without modern machinery not enough food could be raised to supply the population. In fact, the more the subject is considered, the more clearly it is seen that at the present time the lives of civilized men are in a thousand ways directly dependent on the things produced by science.

Astronomy is a science. That is, it is one of those subjects, such as physics, chemistry, geology, and biology, which have made the present age in very many respects altogether different from any earlier one. Indeed, it is the oldest science and the parent of a number of the others, and, in many respects, it is the most perfect one. For these reasons it illustrates most simply and clearly the characteristics of science. Consequently, when one enters on the study of astronomy he not only begins an acquaintance with a subject which has always been noted for its lofty and unselfish ideals, but, at the same time, he becomes familiar with the characteristics of the scientific movement.

2. The Value of Science.—The importance of science in changing the relations of men to the physical universe about them is easy to discern and is generally more or less recognized. That the present conditions of life are better than those which prevailed in earlier times proves the value of science, and the more it is considered from this point of view, the more valuable it is found to be.

The changes in the mode of living of man which science has brought about, will probably in the course of time give rise to marked alterations in his physique; for, the better food supply, shelter, clothing, and sanitation which have recently been introduced as a consequence of scientific discoveries, correspond in a measure to the means by which the best breeds of domestic animals have been developed, and without which they degenerate toward the wild stock from which they have been derived. And probably, also, as the factors which cause changes in living organisms become better known through scientific investigations, man will consciously direct his own evolution.

But there is another less speculative respect in which science is important and in which its importance will enormously increase. It has a profound influence on the minds of those who devote themselves to it, and the number of those who are interested in it is rapidly increasing. In the first place, it exalts truth and honestly seeks it, wherever the search may lead. In the second place, its subject matter often gives a breadth of vision which is not otherwise obtained. For example, the complexity and adaptability of living beings, the irresistible forces which elevate the mountains, or the majestic motions of the stars open an intellectual horizon far beyond that which belongs to the ordinary affairs of life. The conscious and deliberate search for truth and the contemplation of the wonders of nature change the mental habits of a man. They tend to make him honest with himself, just in his judgment, and serene in the midst of petty annoyances. In short, the study of science makes character, as is splendidly illustrated in the lives of many celebrated scientific men. It would undoubtedly be of very great benefit to the world if every one could have the discipline of the sincere and honest search for the truth which is given by scientific study, and the broadening influence of an acquaintance with scientific theories.

There is an important possible indirect effect of science on the intellectual development of mankind which should not be overlooked. One of the results of scientific discoveries has been the greatly increased productivity of the human race. All of the necessities of life and many of its luxuries can now be supplied by the expenditure of much less time than was formerly required to produce the bare means of existence. This leaves more leisure for intellectual pursuits. Aside from its direct effects, this is, when considered in its broad aspects, the most important benefit conferred by science, because, in the final analysis, intellectual experiences are the only things in which men have an interest. As an illustration, any one would prefer a normal conscious life for one year rather than an existence of five hundred years with the certainty that he would be completely unconscious during the whole time.

It is often supposed that science and the fine arts, whose importance every one recognizes, are the antitheses of each other. The arts are believed to be warm and human,—science, cold and austere. This is very far from being the case. While science is exacting in its demands for precision, it is not insensible to the beauties of its subject. In all branches of science there are wonderful harmonies which appeal strongly to those who fully comprehend them. Many of the great scientists have expressed themselves in their writings as being deeply moved by the æsthetic side of their subject. Many of them have had more than ordinary taste for art. Mathematicians are noted for being gifted in music, and there are numerous examples of scientific men who were fond of painting, sculpture, or poetry. But even if the common opinion that science and art are opposites were correct, yet science would contribute indirectly to art through the leisure it furnishes men.

3. The Origin of Science.—It is doubtful if any important scientific idea ever sprang suddenly into the mind of a single man. The great intellectual movements in the world have had long periods of preparation, and often many men were groping for the same truth, without exactly seizing it, before it was fully comprehended.

The foundation on which all science rests is the principle that the universe is orderly, and that all phenomena succeed one another in harmony with invariable laws. Consequently, science was impossible until the truth of this principle was perceived, at least as applied to a limited part of nature.

The phenomena of ordinary observation, as, for example, the weather, depend on such a multitude of factors that it was not easy for men in their primitive state to discover that they occur in harmony with fixed laws. This was the age of superstition, when nature was supposed to be controlled by a great number of capricious gods whose favor could be won by childish ceremonies. Enormous experience was required to dispel such errors and to convince men that the universe is one vast organization whose changes take place in conformity with laws which they can in no way alter.

The actual dawn of science was in prehistoric times, probably in the civilizations that flourished in the valleys of the Nile and the Euphrates. In the very earliest records of these people that have come down to modern times it is found that they were acquainted with many astronomical phenomena and had coherent ideas with respect to the motions of the sun, moon, planets, and stars. It is perfectly clear from their writings that it was from their observations of the heavenly bodies that they first obtained the idea that the universe is not a chaos. Day and night were seen to succeed each other regularly, the moon was found to pass through its phases systematically, the seasons followed one another in order, and in fact the more conspicuous celestial phenomena were observed to occur in an orderly sequence. It is to the glory of astronomy that it first led men to the conclusion that law reigns in the universe.

The ancient Greeks, at a period four or five hundred years preceding the Christian era, definitely undertook to find from systematic observation how celestial phenomena follow one another. They determined very accurately the number of days in the year, the period of the moon's revolution, and the paths of the sun and the moon among the stars; they correctly explained the cause of eclipses and learned how to predict them with a considerable degree of accuracy; they undertook to measure the distances to the heavenly bodies, and to work out a complete system that would represent their motions. The idea was current among the Greek philosophers that the earth was spherical, that it turned on its axis, and, among some of them, that it revolved around the sun. They had true science in the modern acceptance of the term, but it was largely confined to the relations among celestial phenomena. The conception that the heavens are orderly, which they definitely formulated and acted on with remarkable success, has been extended, especially in the last two centuries, so as to include the whole universe. The extension was first made to the inanimate world and then to the more complicated phenomena associated with living beings. Every increase in carefully recorded experience has confirmed and strengthened the belief that nature is perfectly orderly, until now every one who has had an opportunity of becoming familiar with any science is firmly convinced of the truth of this principle, which is the basis of all science.

4. The Methods of Science.—Science is concerned with the relations among phenomena, and it must therefore rest ultimately upon observations and experiments. Since its ideal is exactness, the observations and experiments must be made with all possible precision and the results must be carefully recorded. These principles seem perfectly obvious, yet the world has often ignored them. One of the chief faults of the scientists of ancient times was that they indulged in too many arguments, more or less metaphysical in character, and made too few appeals to what would now seem obvious observation or experiment. A great English philosopher, Roger Bacon (1214–1294), made a powerful argument in favor of founding science and philosophy on experience.

It must not be supposed that the failure to rely on observations and experiment, and especially to record the results of experience, are faults that the world has outgrown. On the contrary, they are still almost universally prevalent among men. For example, there are many persons who believe in dreams or premonitions because once in a thousand cases a dream or a premonition comes true. If they had written down in every case what was expected and what actually happened, the absurdity of their theory would have been evident. The whole mass of superstitions with which mankind has burdened itself survives only because the results of actual experience are ignored.

In scientific work great precision in making observations and experiments is generally of the highest importance. Every science furnishes examples of cases where the data seemed to have been obtained with greater exactness than was really necessary, and where later the extra accuracy led to important discoveries. In this way the foundation of the theory of the motion of the planets was laid. Tycho Brahe was an observer not only of extraordinary industry, but one who did all his work with the most painstaking care. Kepler, who had been his pupil and knew of the excellence of his measurements, was a computer who sought to bring theory and observation into exact harmony. He found it impossible by means of the epicycles and eccentrics, which his predecessors had used, to represent exactly the observation of Tycho Brahe. In spite of the fact that the discrepancies were small and might easily have been ascribed to errors of observation, Kepler had absolute confidence in his master, and by repeated trials and an enormous amount of labor he finally arrived at the true laws of planetary motion (Art. 145). These laws, in the hands of the genius Newton, led directly to the law of gravitation and to the explanation of all the many peculiarities of the motions of the moon and planets (Art. 146).

Observations alone, however carefully they may have been made and recorded, do not constitute science. First, the phenomena must be related, and then, what they have in common must be perceived. It might seem that it would be a simple matter to note in what respects phenomena are similar, but experience has shown that only a very few have the ability to discover relations that are not already known. If this were not true, there would not be so many examples of new inventions and discoveries depending on very simple things that have long been within the range of experience of every one. After the common element in the observed phenomena has been discovered the next step is to infer, by the process known as induction, that the same thing is true in all similar cases. Then comes the most difficult thing of all. The vital relationships of the one class of phenomena with other classes of phenomena must be discovered, and the several classes must be organized into a coherent whole.

An illustration will make the process clearer than an extended argument. Obviously, all men have observed moving bodies all their lives, yet the fact that a moving body, subject to no exterior force, proceeds in a straight line with uniform speed was not known until about the time of Galileo (1564–1642) and Newton (1643–1727). When the result is once enunciated it is easy to recall many confirmatory experiences, and it now seems remarkable that so simple a fact should have remained so long undiscovered. It was also noted by Newton that when a body is acted on by a force it has an acceleration (acceleration is the rate of change of velocity) in the direction in which the force acts, and that the acceleration is proportional to the magnitude of the force. Dense bodies left free in the air fall toward the earth with accelerated velocity, and they are therefore subject to a force toward the earth. Newton observed these things in a large number of cases, and he inferred by induction that they are universally true. He focused particularly on the fact that every body is subject to a force directed toward the earth.

If taken alone, the fact that bodies are subject to forces toward the earth is not so very important; but Newton used it in connection with many other phenomena. For example, he knew that the moon is revolving around the earth in an approximately circular orbit. At P, in Fig. 3, it is moving in the direction PQ around the earth, E. But it actually moves from P to R. That is, it has fallen toward the earth through the distance QR. Newton perceived that this motion is analogous to that of a body falling near the surface of the earth, or rather to the motion of a body which has been started in a horizontal direction from p near the surface of the earth. For, if the body were started horizontally, it would continue in the straight line pq, instead of curving downward to r, if it were not acted upon by a force directed toward the earth. Newton also knew Kepler's laws of planetary motion. By combining with wonderful insight a number of classes of phenomena which before his time had been supposed to be unrelated, he finally arrived at the law of gravitation—"Every particle of matter in the universe attracts every other particle with a force which is directly proportional to the product of their masses and inversely proportional to the square of their distance apart." Thus, by perceiving the essentials in many kinds of phenomena and by an almost unparalleled stroke of genius in combining them, he discovered one of the relations which every particle of matter in the universe has to all the others. By means of the laws of motion (Art. 40) and the law of gravitation, the whole problem of the motions of bodies was systematized.

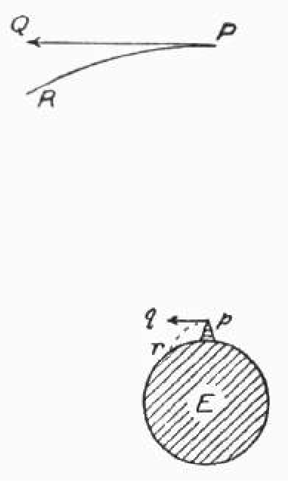


FIG. 3.—The motion of the moon from P to R around E is similar to that of a body projected horizontally from p.

There is still another method employed in science which is often very important. After general principles have been discovered they can be used as the basis for deducing particular conclusions. The value of the particular conclusions may consist in leading to the accomplishment of some desired end. For example, since a moving body tends to continue in a straight line, those who build railways place the outside rails on curves higher than those on the inside so that trains will not leave the track. Or, the knowledge of the laws of projectiles enables gunners to hit invisible objects whose positions are known.

The value of particular conclusions may consist in enabling men to adjust themselves to phenomena over which they have no control. For example, in many harbors large boats can enter or depart only when the tide is high, and the knowledge of the times when the tides will be high is valuable to navigators. After the laws of meteorology have become more perfectly known, so that approaching storms, or frosts, or drouths, or hot waves can be accurately foretold a considerable time in advance, the present enormous losses due to these causes will be avoided.

The knowledge of general laws may lead to information regarding things which are altogether inaccessible to observation or experiment. For example, it is very important for the geologist to know whether the interior of the earth is solid or liquid; and, if it is solid, whether it is elastic or viscous. Although at first thought it seems impossible to obtain reliable information on this subject, yet by a number of indirect processes (Arts. 25, 26) based on laws established from observation, it has been possible to prove with certainty that the earth, through and through, is about as rigid as steel, and that it is highly elastic.

Another important use of the deductive process in science is in drawing consequences of a theory which must be fulfilled in experience if the theory is correct, and which may fail if it is false. It is, indeed, the most efficient means of testing a theory. Some of the most noteworthy examples of its application have been in connection with the law of gravitation. Time after time mathematicians, using this law as a basis for their deductions, have predicted phenomena that had not been observed, and time after time their predictions have been fulfilled. This is one of the reasons why the truth of the law of gravitation is regarded as having been firmly established.

5. The Imperfections of Science.—One of the characteristics of science is its perfect candor and fairness. It would not be in harmony with its spirit to attempt to lead one to suppose that it does not have sources of weakness. Besides, if its possible imperfections are analyzed, they can be more easily avoided, and the real nature of the final conclusions will be better understood.

It must be observed, in the first place, that science consists of men's theories regarding what is true in the universe about them. These theories are based on observation and experiment and are subject to the errors and incompleteness of the data on which they are founded. The fact that it is not easy to record exactly what one may have attempted to observe is illustrated by the divergence in the accounts of different witnesses of anything except the most trivial occurrence. Since men are far from being perfect, errors in the observations cannot be entirely avoided, but in good science every possible means is taken for eliminating them.

In addition to this source of error, there is another more insidious one that depends upon the fact that observational data are often collected for the purpose of testing a specific theory. If the theory in question is due to the one who is making the observations or experiments, it is especially difficult for him to secure data uninfluenced by his bias in its favor. And even if the observer is not the author of the theory to which the observations relate, he is very apt to be prejudiced either in its favor or against it.

Even if the data on which science is based were always correct, they would not be absolutely exhaustive, and the inductions to general principles from them would be subject to corresponding uncertainties. Similarly, the general principles, derived from various classes of phenomena, which are used in formulating a complete scientific theory, do not include all the principles which are involved in the particular domain of the theory. Consequently it may be imperfect for this reason also.

The sources of error in scientific theories which have been enumerated are fundamental and will always exist. The best that can be done is to recognize their existence and to minimize their effects by all possible means. The fact that science is subject to imperfections does not mean that it is of little value or that less effort should be put forth in its cultivation. Wood and stone and brick and glass have never been made into a perfect house; yet houses have been very useful and men will continue to build them.

There are many examples of scientific theories which it has been found necessary to modify or even to abandon. These changes have not been more numerous than they have been in other domains of human activities, but they have been, perhaps, more frankly confessed. Indeed, there are plenty of examples where scientists have taken evident satisfaction in the alterations they have introduced. The fact that scientific theories have often been found to be imperfect and occasionally positively wrong, have led some persons who have not given the question serious consideration to suppose that the conclusions of science are worthy of no particular respect, and that, in spite of the pretensions of scientists, they are actually not far removed from the level of superstitions. The respect which scientific theories deserve and the gulf that separates them from superstitions will be evident from a statement of their real nature.

Suppose a person were so situated that he could look out from an upper window over a garden. He could make a drawing of what he saw that would show exactly the relative positions of the walks, shrubs, and flowers. If he were color blind, the drawing could be made in pencil so as to satisfy perfectly all his observations. But suppose some one else who was not color blind should examine the drawing. He would legitimately complain that it was not correct because the colors were not shown. If the colors were correctly given, both observers would be completely satisfied. Now suppose a third person should look at the drawing and should then go down and examine the garden in detail. He would find that the various objects in it not only have positions but also various heights. He would at once note that the heights were not represented in the drawing, and a little reflection would convince him that the three-dimensional garden could not be completely represented in a two-dimensional drawing. He would claim that that method of trying to give a correct idea of what was in the garden was fundamentally wrong, and he might suggest a model of suitable material in three dimensions. Suppose the three-dimensional model were made satisfying the third observer. It is important to note that it would correctly represent all the relative positions observed by the first one and all the colors observed by the second one, as well as the additional information obtained by the third one.

A scientific theory is founded on the work of one or more persons having only limited opportunities for observation and experiment. It is a picture in the imagination, not on paper, of the portion of the universe under consideration. It represents all the observed relations, and it is assumed that it will represent the relations that might be observed in all similar circumstances. Suppose some new facts are discovered which are not covered by the theory, just as the second observer in the garden saw colors not seen by the first. It will be necessary to change the scientific theory so as to include them. Perhaps it can be done simply by adding to the theory. But if the new facts correspond to the things discovered by the third observer in the garden, it will be necessary to abandon the old theory and to construct an entirely new one. The new one must preserve all the relations represented by the old one, and it must represent the new ones as well.

In the light of this discussion it may be asked in what sense scientific theories are true. The answer is that they are all true to the extent that they picture nature. The relations are the important things. When firmly established they are a permanent acquisition; however the mode of representing them may change, they remain. A scientific theory is a convenient and very useful way of describing the relations on which it is based. It correctly represents them, and in this respect differs from a superstition which is not completely in harmony with its own data. It implies many additional things and leads to their investigation. If the implications are found to hold true in experience, the theory is strengthened; if not, it must be modified. Hence, there should be no reproach in the fact that a scientific theory must be altered or abandoned. The necessity for such a procedure means that new information has been obtained, not that the old was false.[1]

6. Great Contributions of Astronomy to Science.—As was explained in Art. 3, science started in astronomy. Many astronomical phenomena are so simple that it was possible for primitive people to get the idea from observing them that the universe is orderly and that they could discover its laws. In other sciences there are so many varying factors that the uniformity in a succession of events would not be discovered by those who were not deliberately looking for it. It is sufficient to consider the excessive complexities of the weather or of the developments of plants or animals, to see how hopeless would be the problem which a people without a start on science would face if they were cut off from celestial phenomena. It is certain that if the sky had always been covered by clouds so that men could not have observed the regular motions of the sun, moon, and stars, the dawn of science would have been very much delayed. It is entirely possible, if not probable, that without the help of astronomy the science of the human race would yet be in a very primitive state.

Astronomy has made positive and important contributions to science within historical times. Spherical trigonometry was invented and developed because of its uses in determining the relations among the stars on the vault of the heavens. Very many things in calculus and still higher branches of mathematics were suggested by astronomical problems. The mathematical processes developed for astronomical applications are, of course, available for use in other fields. But the great science of mathematics does not exist alone for its applications, and to have stimulated its growth is an important contribution. While many parts of mathematics did not have their origin in astronomical problems, it is certain that had it not been for these problems mathematical science would be very different from what it now is.

The science of dynamics is based on the laws of motion. These laws were first completely formulated by Newton, who discovered them and proved their correctness by considering the revolutions of the moon

[1]The comparison of scientific theories with the picture of the objects seen in the garden is for the purpose of making clear one of their particular features. It must be remembered that in most respects the comparison with so trivial a thing is very imperfect and unfair to science.

and planets, which describe their orbits under the ideal condition of motion in a vacuum without any friction. The immense importance of mechanics in modern life is a measure of the value of this contribution of astronomy to science.

The science of geography owes much to astronomy, both directly and indirectly. A great period of exploration followed the voyages of Columbus. It took courage of the highest order to sail for many weeks over an unknown ocean in the frail boats of his time. He had good reasons for thinking he could reach India, to the eastward, by sailing westward from Spain. His reasons were of an astronomical nature. He had seen the sun rise from the ocean in the east, travel across the sky and set in the west; he had observed that the moon and stars have similar motions; and he inferred from these things that the earth was of finite extent and that the heavenly bodies moved around it. This led him to believe it could be circumnavigated. Relying upon the conclusions that he drew from his observations of the motions of the heavenly bodies, he maintained control of his mutinous sailors during their perilous voyage across the Atlantic, and made a discovery that has been of immense consequence to the human race.

One of the most important influences in modern scientific thought is the doctrine of evolution. It has not only largely given direction to investigations and speculations in biology and geology, but it has also been an important factor in the interpretation of history, social changes, and even religion. The first clear ideas of the orderly development of the universe were obtained by contemplating the relatively simple celestial phenomena, and the doctrine of evolution was current in astronomical literature more than half a century before it appeared in the writings of Darwin, Spencer, and their contemporaries. In fact, it was carried directly from astronomy over into geology, and from geology into the biological sciences (Art. 242).

7. The Present Value of Astronomy.—From what has been said it will be admitted that astronomy has been of great importance in the development of science, but it is commonly believed that at the present time it is of little practical value to mankind. While its uses are by no means so numerous as those of physics and chemistry, it is nevertheless quite indispensable in a number of human activities.

Safe navigation of the seas is absolutely dependent upon astronomy. In all long voyages the captains of vessels frequently determine their positions by observations of the celestial bodies. Sailors use the nautical mile, or knot, which approximately equals one and one sixth ordinary miles. The reason they employ the nautical mile is that this is the distance which corresponds to a change of one minute of arc in the apparent positions of the heavenly bodies. That is, if, for simplicity, the sun were over a meridian, its altitude as observed from two vessels a nautical mile apart on that meridian would differ by one minute of arc.

Navigation is not only dependent on simple observations of the sun, moon, and stars, but the mathematical theory of the motions of these bodies is involved. The subject is so difficult and intricate that for a long time England and France offered substantial cash prizes for accurate tables of the positions of the moon for the use of their sailors.

Just as a sea captain determines his position by astronomical observations, so also are geographical positions located. For example, explorers of the polar regions find how near they have approached to the pole by observations of the altitude of the sun. International boundary lines in many cases are defined by latitudes and longitudes, instead of being determined by natural barriers, as rivers, and in all such cases they are located by astronomical observations.

It might be supposed that even though astronomy is essential to navigation and geography, it has no value in the ordinary activities of life. Here, again, first impressions are erroneous. It is obvious that railway trains must be run according to accurate time schedules in order to avoid confusion and wrecks. There are also many other things in which accurate time is important. Now, time is determined by observations of the stars. The millions of clocks and watches in use in the world are all ultimately corrected and controlled by comparing them with the apparent diurnal motions of the stars. For example, in the United States, observations are made by the astronomers of the Naval Observatory, at Washington, on every clear night, and from these observations their clocks are corrected. These clocks are in electrical connection with more than 30,000 other clocks in various parts of the country. Every day time signals are sent out from Washington and these 30,000 clocks are automatically corrected, and all other timepieces are directly or indirectly compared with them.

It might be inquired whether some other means might not be devised of measuring time accurately. It might be supposed that a clock could be made that would run so accurately as to serve all practical purposes. The fact is, however, no clock ever was made which ran accurately for any considerable length of time. No two clocks have been made which ran exactly alike. In order to obtain a satisfactory measure of time it is necessary to secure the ideal conditions under which the earth rotates and the heavenly bodies move, and there is no prospect that it ever will be possible to use anything else, as the fundamental basis, than the apparent motions of the stars.

Astronomy is, and will continue to be, of great importance in connection with other sciences. It supplies most of the fundamental facts on which meteorology depends. It is of great value to geology because it furnishes the geologist information respecting the origin and pre-geologic history of the earth, it determines for him the size and shape of the earth, it measures the mass of the earth, and it proves important facts respecting the condition of the earth's interior. It is valuable in physics and chemistry because the universe is a great laboratory which, with modern instruments, can be brought to a considerable extent within reach of the investigator. For example, the sun is at a higher temperature than can be produced by any known means on the earth. The material of which it is composed is in an incandescent state, and the study of the light received from it has proved the existence, in a number of instances, of chemical elements which had not been known on the earth. In fact, their discovery in the sun led to their detection on the earth. It seems probable that similar discoveries will be made many times in the future. The sun's corona and the nebulæ contain material which seems to be in a more primitive state than any known on the earth, and the revelations afforded by these objects are having a great influence on physical theories respecting the ultimate structure of matter.

Astronomy is of greatest value to mankind, however, in an intellectual way. It furnishes men with an idea of the wonderful universe in which they live and of their position in it. Its effects on them are analogous to those which are produced by travel on the earth. If a man visits various countries, he learns many things which he does not and cannot apply on his return home, but which, nevertheless, make him a broader and better man. Similarly, though what one may learn about the millions of worlds which occupy the almost infinite space within reach of the great telescopes of modern times cannot be directly applied in the ordinary affairs of life, yet the contemplation of such things, in which there is never anything that is low or mean or sordid, makes on him a profound impression. It strongly modifies the particular philosophy which he has more or less definitely formulated in his consciousness, and in harmony with which he orders his life.

8. The Scope of Astronomy.—The popular conception of astronomy is that it deals in some vague and speculative way with the stars. Since it is obviously impossible to visit them, it is supposed that all conclusions respecting them, except the few facts revealed directly by telescopes, are pure guesses. Many people suppose that astronomers ordinarily engage in the harmless and useless pastime of gazing at the stars with the hope of discovering a new one. Many of those who do not have this view suppose that astronomers control the weather, can tell fortunes, and are very shrewd to have discovered the names of so many stars. As is true of most conclusions that are not based on evidence, these conceptions of astronomy and astronomers are absurd.

Astronomy contains a great mass of firmly established facts. Astronomers demand as much evidence in support of their theories as is required by other scientists. They have actually measured the distances to the moon, sun, and many of the stars. They have discovered the laws of their motions and have determined the masses of the principal members of the solar system. The precision attained in much of their work is beyond that realized in most other sciences, and their greatest interest is in measurable things and not in vague speculations.

A more extended preliminary statement of the scope of astronomy is necessary in order that its study may be entered on without misunderstandings. Besides, the relations among the facts with which a science deals are very important, and a preliminary outline of the subject will make it easier to place in their proper position in an organized whole all the various things which may be set forth in the detailed discussions.

The most accessible and best-known astronomical object is the earth. Those facts respecting it that are determined entirely or in large part by astronomical means are properly regarded as belonging to astronomy. Among them are the shape and size of the earth, its average density, the condition of its interior, the height of its atmosphere, its rotation on its axis and revolution around the sun, and the climatic conditions of its surface so far as they are determined by its relation to the sun.

The nearest celestial body is the moon. Astronomers have found by fundamentally the same methods as those which surveyors employ that its distance from the earth averages about 240,000 miles, that its diameter is about 2160 miles, and that its mass is about one eightieth that of the earth. The earth holds the moon in its orbit by its gravitational control, and the moon in turn causes the tides on the earth. It is found that there is neither atmosphere nor water on the moon, and the telescope shows that its surface is covered with mountains and circular depressions, many of great size, which are called craters.

The earth is one of the eight planets which revolve around the sun in nearly circular orbits. Three of them are smaller than the earth and four are larger. The smallest, Mercury, has a volume about one twentieth that of the earth, and the largest, Jupiter, has a volume about one thousand times that of the earth. The great sun, whose mass is seven hundred times that of all of the planets combined, holds them in their orbits and lights and warms them with its abundant rays. Those nearest the sun are heated much more than the earth, but remote Neptune gets only one nine-hundredth as much light and heat per unit area as is received by the earth. Some of the planets have no moons and others have several. The conditions on one or two of them seem to be perhaps favorable for the development of life, while the others certainly cannot be the abode of such life as flourishes on the earth.

FIG. 4.—The moon 1.5 days after the first quarter. *Photographed with the 40-inch telescope of the Yerkes Observatory.*

In addition to the planets, over eight hundred small planets, or planetoids, and a great number of comets circulate around the sun in obedience to the same law of gravitation. The orbits of nearly all the small planets lie between the orbits of Mars and Jupiter; the orbits of the comets are generally very elongated and are unrelated to the other members of the system. The phenomena presented by the comets, for example the behavior of their tails, raise many interesting and puzzling questions.

The dominant member of the solar system is the sun. Its volume is more than a million times that of the earth, its temperature is far higher than any that can be produced on the earth, even in the most efficient electrical furnaces, and its surface is disturbed by the most violent storms. Often masses of this highly heated material, in volumes greater than the whole earth, move along or spout up from its surface at the rate of several hundreds miles a minute. The spectroscope shows that the sun contains many of the elements, particularly the metals, of which the earth is composed. The consideration of the possible sources of the sun's heat leads to the conclusion that it has supplied the earth with radiant energy for many millions of years, and that the supply will not fail for at least a number of million years in the future.

The stars that seem to fill the sky on a clear night are suns, many of which are much larger and more brilliant than our own sun. They appear to be relatively faint points of light because of their enormous distances from us. The nearest of them is so remote that more than four years are required for its light to come to the solar system, though light travels at the rate of 186,330 miles per second; and others, still within the range of large telescopes, are certainly a thousand times more distant. At these vast distances such a tiny object as the earth would be entirely invisible even though astronomers possessed telescopes ten thousand times as powerful as those now in use. Sometimes stars appear to be close together, as in the case of the Pleiades, but their apparent proximity is due to their immense distances from the observer. There are doubtless regions of space from which the sun would seem to be a small star forming a close group with a number of others. There are visible to the unaided eye in all the sky only about 5000 stars, but the great photographic telescopes with which modern observatories are equipped show several hundreds of millions of them. It might be supposed that telescopes with twice the light-gathering power would show proportionately more stars, and so on indefinitely, but this is certainly not true, for there is evidence that points to the conclusion that they do not extend indefinitely, at least with the frequency with which they occur in the region around the sun. The visible stars are not uniformly scattered throughout the space which they occupy, but form a great disk-like aggregation lying in the plane of the Milky Way.

Many stars, instead of being single isolated masses, like the sun, are found on examination with highly magnifying telescopes to consist of two suns revolving around their common center of gravity. In most cases the distances between the two members of a double star is several times as great as the distance from the earth to the sun. The existence of double stars which may be much closer together than those which are visible through telescopes has also been shown by means of instruments called spectroscopes. It has been found that a considerable fraction, probably one fourth, of all the nearer stars are double stars. There are also triple and quadruple stars; and in some cases thousands of suns, all invisible to the unaided eye, occupy a part of the sky apparently smaller than the moon. Even in such cases the distances between the stars are enormous, and such clusters, as they are called, constitute larger and more wonderful aggregations of matter than any one ever dreamed existed before they were revealed by modern instruments.

While the sun is the center around which the planets and comets revolve, it is not fixed with respect to the other stars. Observations with both the telescope and the spectroscope prove that it is moving, with respect to the brighter stars, approximately in the direction of the brilliant Vega in the constellation Lyra. It is found by use of the spectroscope that the rate of motion is about 400,000,000 miles per year. The other stars are also in motion with an average velocity of about 600,000,000 miles per year, though some of them move much more slowly than this and some of them many times faster. One might think that the great speed of the sun would in a century or two so change its relations to the stars that the appearance of the sky would be entirely altered. But the stars are so remote that in comparison the distance traveled by the sun in a year is negligible. When those who built the pyramids turned their eyes to the sky at night they saw the stars grouped in constellations almost exactly as they are seen at present. During the time covered by observations accurate enough to show the motion of the sun it has moved sensibly in a straight line, though in the course of time the direction of its path will doubtless be changed by the attractions of the other stars. Similarly, the other stars are moving in sensibly straight lines in every direction, but not altogether at random, for it has been found that there is a general tendency for them to move in two or more roughly parallel streams.

In addition to learning what the universe is at present, one of the most important and interesting objects of astronomy is to find out through what great series of changes it has gone in its past evolution, and what will take place in it in the future. As a special problem, the astronomer tries to discover how the earth originated, how long it has been in existence, particularly in a state adapted to the abode of life, and what reasonably may be expected for the future. These great problems of cosmogony have been of deep interest to mankind from the dawn of civilization; with increasing knowledge of the wonders of the universe and of the laws by which alone such questions can be answered, they have become more and more absorbingly attractive.

I. QUESTIONS

1. Enumerate as many ways as possible in which science is beneficial to men.

2. What is the fundamental basis on which science rests, and what are its chief characteristics?

3. What is induction? Give examples. Can a science be developed without inductions? Are inductions always true?

4. What is deduction? Give examples. Can a science be developed without deductions? Are deductions always true?

5. In what respects may science be imperfect? How may its imperfections be most largely eliminated? Are any human activities perfect?

6. Name some superstition and show in what respects it differs from scientific conclusions.

7. Why did science originate in astronomy?

8. Are conclusions in astronomy firmly established, as they are in other sciences?

9. In what fundamental respects do scientific laws differ from civil laws?

10. What advantages may be derived from a preliminary outline of the scope of astronomy? Would they hold in the case of a subject not a science?

11. What questions respecting the earth are properly regarded as belonging to astronomy? To what other sciences do they respectively belong? Is there any science which has no common ground with some other science?

12. What arts are used in astronomy? Does astronomy contribute to any art?

13. What references to astronomy in the sacred or classical literatures do you know?

14. Has astronomy exerted any influence on philosophy and religion? Have they modified astronomy?

CHAPTER II
THE EARTH

I. The Shape of the Earth

9. Astronomical Problems respecting the Earth.—The earth is one of the objects belonging to the field of astronomical investigations. In the consideration of it astronomy has its closest contact with some of the other sciences, particularly with geology and meteorology. Those problems respecting the earth that can be solved for other planets also, or that are essential for the investigation of other astronomical questions, are properly considered as belonging to the field of astronomy.

The astronomical problems respecting the earth can be divided into two general classes. The first class consists of those which can be treated, at least to a large extent, without regarding the earth as a member of a family of planets or considering its relations to them and the sun. Such problems are its shape and size, its mass, its density, its interior temperature and rigidity, and the constitution, mass, height, and effects of its atmosphere. These problems will be treated in this chapter. The second class consists of the problems involved in the relations of the earth to other bodies, particularly its rotation, revolution around the sun, and the consequences of these motions. The treatment of these problems will be reserved for the next chapter.

It would be an easy matter simply to state the astronomical facts respecting the earth, but in science it is necessary not only to say what things are true but also to give the reasons for believing that they are true. Therefore one or more proofs will be given for the conclusions astronomers have reached respecting the earth. As a matter of logic one complete proof is sufficient, but it must be remembered that a scientific doctrine consists of, and rests on, a great number of theories whose truth may be more or less in question, and consequently a number of proofs is always desirable. If they agree, their agreement confirms belief in the accuracy of all of them. It will not be regarded as a burden to follow carefully these proofs; in fact, one who has arrived at a mature stage of intellectual development instinctively demands the reasons we have for believing that our conclusions are sound.

10. The Simplest and most Conclusive Proof of the Earth's

Sphericity.[1]—Among the proofs that the earth is found, the simplest and most conclusive is that *the plane of the horizon, or the direction of the plumb line, changes by an angle which is directly proportional to the distance the observer travels along the surface of the earth, whatever the direction and distance of travel.*

It will be shown first that if the earth were a true sphere the statement would be true. For simplicity, suppose the observer travels along a meridian. If the statement is true for this case, it will be true for all others, because a sphere has the same curvature in every direction. Suppose the observer starts from O_1, Fig. 5, and travels northward to O_2. The length of the arc O_1O_2 is proportional to the angle a which it subtends at the center of the sphere. The planes of the horizon of O_1 and O_2 are respectively O_1H_1 and O_2H_2. These lines are respectively perpendicular to CO_1 and CO_2. Therefore the angle between them equals the angle a. That is, the distance traveled is proportional to the change of direction of the plane of the horizon.

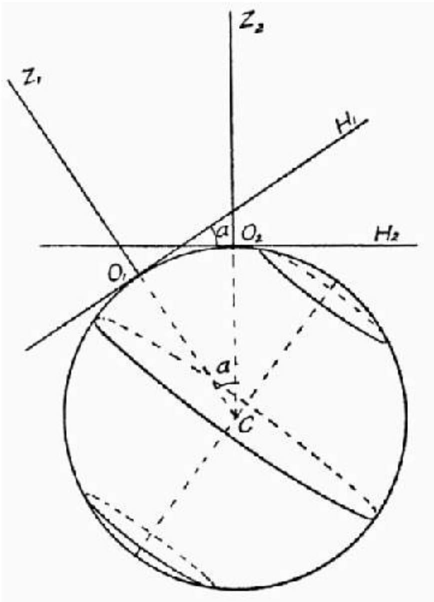


FIG. 5.—The change in the direction of the plumb line is proportional to the distance traveled along the surface of the earth.

The plumb lines at O_1 and O_2 are respectively O_1Z_1 and O_2Z_2, and the angle between these lines is a. Hence the distance traveled is proportional to the change in the direction of the plumb line.

It will be shown now that if the surface of the earth were not a true sphere the change in the direction of the plane of the horizon would not be proportional to the distance traveled on the surface. Suppose Fig. 6 represents a plane section through the non-spherical earth along whose surface the observer travels. Since the earth is not a sphere, the curvature of its surface will be different at different places. Suppose that O_1O_2 is one of the flatter regions and O_3O_4 is one of the more

[1]The earth is not exactly round, but the departure from sphericity will be neglected for the moment.

convex ones. In the neighborhood of O_1O_2 the direction of the plumb line changes slowly, while in the neighborhood of O_3O_4 its direction changes more rapidly. The large arc O_1O_2 subtends an angle at C_1 made by the respective perpendiculars to the surface which exactly equals the angle at C_3 subtended by the smaller arc O_3O_4. Therefore in this case the change in direction of the plumb line is *not* proportional to the distance traveled, for the same angular change corresponds to two different distances. The same result is true for the plane of the horizon because it is always perpendicular to the plumb line.

Since the conditions of the statement would be satisfied in case the earth were spherical, and only in case it were spherical, the next question is what the observations show. Except for irregularities of the surface, which are not under consideration here, and the oblateness, which will be discussed in Art. 12, the observations prove absolutely that the change in direction of the plumb line is proportional to the arc traversed.

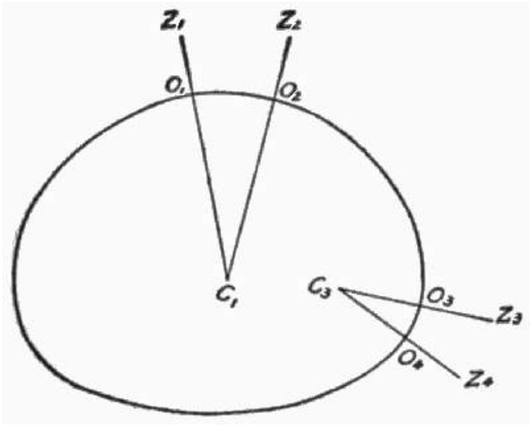


FIG. 6.—If the earth were not spherical, equal angles would be subtended by arcs of different lengths.

Two practical problems are involved in carrying out the proof which has just been described. The first is that of measuring the distance between two points along the surface of the earth, and the second is that of determining the change in the direction of the plumb line. The first is a refined problem of surveying; the second is solved by observations of the stars.

All long distances on the surface of the earth are determined by a process known as triangulation. It is much more convenient than direct measurement and also much more accurate. A fairly level stretch of country, A_1 and A_2 in Fig. 7, a few miles long is selected, and the distance between the two points, which must be visible from each other, is measured with the greatest possible accuracy. This line is called the *base line*. Then a point A_3 is taken which can be seen from both A_1 and A_2. A telescope is set up at A_1 and pointed at A_2. It has a circle

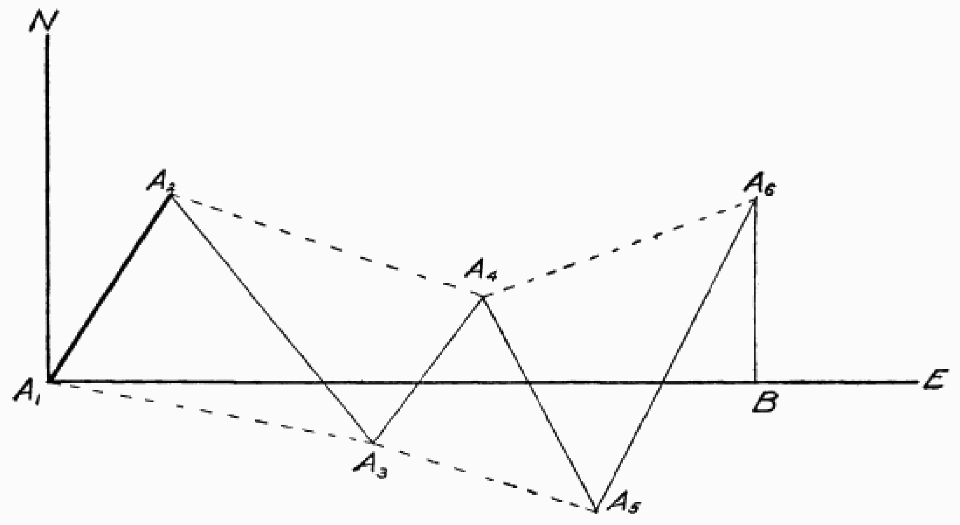


FIG. 7.—The base line A_1A_2 is measured directly and the other distances are obtained by triangulation.

parallel to the surface of the earth on which the degrees are marked. The position of the telescope with respect to this circle is recorded. Then the telescope is turned until it points toward A_3. The difference of its position with respect to the circle when pointed at A_2 and at A_3 is the angle $A_2A_1A_3$. Similarly, the telescope is set up at A_2 and the angle $A_1A_2A_3$ is measured. Then in the triangle $A_1A_2A_3$ two angles and the included side are known. By plane geometry, two triangles that have two angles and the included side of one respectively equal to two angles and the included side of the other are exactly alike in size and shape. This simply means that when two angles and the included side of the triangle are given, the triangle is uniquely defined. The remaining parts can be computed by trigonometry. In the present case suppose the distance A_2A_3 is computed.

Now suppose a fourth point A_4 is taken so that it is visible from both A_2 and A_3. Then, after the angles at A_2 and A_3 in the triangle $A_2A_3A_4$ have been measured, the line A_3A_4 can be computed. This process evidently can be continued, step by step, to any desired distance.

Suppose A_1 is regarded as the original point from which measurements are to be made. Not only have various distances been determined, but also their directions with respect to the north-south line are known. Consequently, it is known how far north and how far east

A_2 is from A_1. The next step gives how far south and how far east A_3 is from A_2. By combining the two results it is known how far south and how far east A_3 is from A_1, and so on for succeeding points.

The convenience in triangulation results partly from the long distances that can be measured, especially in rough country. It is sometimes advisable to go to the trouble of erecting towers in order to make it possible to use stations separated by long distances. The accuracy arises, at least in part, from the fact that the angles are measured by instruments which magnify them. The fact that the stations are not all on the same level, and the curvature of the earth, introduce little difficulties in the computations that must be carefully overcome.

The direction of the plumb line at the station A_1, for example, is determined by noting the point among the stars at which it points. The plumb line at A_2 will point to a different place among the stars. The difference in the two places among the stars gives the difference in the directions of the plumb lines at the two stations. The stars apparently move across the sky from east to west during the night and are not in the same positions at the same time of the day on different nights. Hence, there are here also certain circumstances to which careful attention must be given in order to get accurate results.

11. Other Proofs of the Earth's Sphericity.—There are many reasons given for believing that the earth is not a plane, and that it is, indeed, some sort of a convex figure; but most of them do not prove that it is actually spherical. It will be sufficient to mention them.

(*a*) The earth has been circumnavigated, but so far as this fact alone is concerned it might be the shape of a cucumber. (*b*) Vessels disappear below the horizon hulls first and masts last, but this only proves the convexity of the surface. (*c*) The horizon appears to be a circle when viewed from an elevation above the surface of the water. This is theoretically good but observationally it is not very exact. (*d*) The shadow of the earth on the moon at the time of a lunar eclipse is always an arc of a circle, but this proof is very inconclusive, in spite of the fact that it is often mentioned, because the shadow has no very definite edge and its radius is large compared to that of the moon.

12. Proof of the Oblateness of the Earth by Arcs of Latitude.—The latitude of a place on the earth is determined by observations of the direction of the plumb line with respect to the stars. This is the reason that a sea captain refers to the heavenly bodies in order to find his location on the ocean. It is found by actual observations of the stars and measurements of arcs that the length of a degree of arc is longer the farther it is from the earth's equator. This proves that the earth is less curved at the poles than it is at the equator. A body which is thus flattened at the poles and bulged at the equator is called *oblate*.

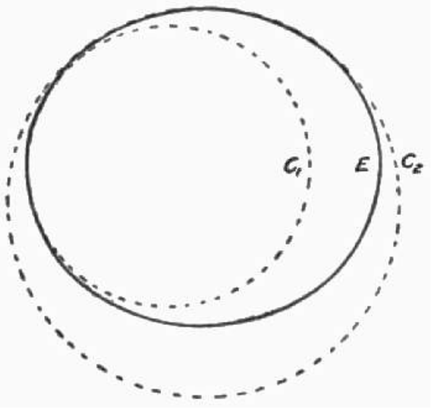


FIG. 8.—The length of a degree of latitude is least at the equator and greatest at the poles.

In order to see that in the case of an oblate body a degree of latitude is longer near the poles than it is at the equator, consider Fig. 8. In this figure E represents a plane section of the body through its poles. The curvature at the equator is the same as the curvature of the circle C_1, and a degree of latitude on E at its equator equals a degree of latitude on C_1. The curvature of E at its pole is the same as the curvature of the circle C_2, and a degree of latitude on E at its pole equals a degree of latitude on C_2. Since C_2 is greater than C_1, a degree of latitude near the pole of the oblate body is greater than a degree of latitude near its equator.

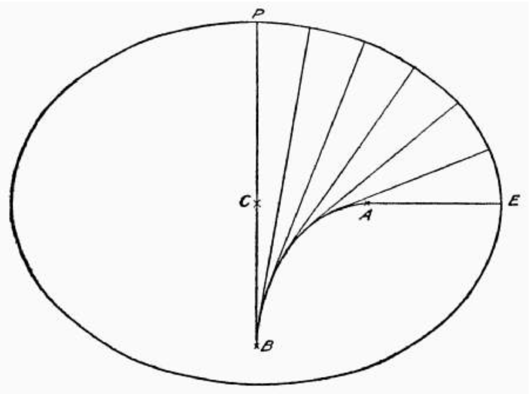


FIG. 9.—Perpendiculars to the surface of an oblate body, showing that equal arcs subtend largest angles at its equator and smallest at its poles.

A false argument is sometimes made which leads to the opposite conclusion. Lines are drawn from the center of the oblate body dividing the quadrant into a number of equal angles. Then it is observed that the arc intercepted between the two lines nearest the equator is longer than that intercepted between the two lines nearest the pole. The error of this argument lies in the fact that, with the exception of those drawn to the equator and poles, these lines are not perpendicular to the surface. Figure 9 shows an oblate body with a number of lines drawn perpendicular to its surface. Instead of their all passing through the center of the body, they are tangent to the curve AB. The line AE equals the radius of a circle having the same curvature as the oblate body at E, and BP is the radius of the circle having the curvature at P.

13. Size and Shape of the Earth.—The size and shape of the earth can both be determined from measurements of arcs. If the earth were spherical, a degree of arc would have the same length everywhere on its surface, and its circumference would be 360 times the length of one degree. Since the earth is oblate, the matter is not quite so simple. But from the lengths of arcs in different latitudes both the size and the shape of the earth can be computed.

It is sufficiently accurate for ordinary purposes to state that the diameter of the earth is about 8000 miles, and that the difference between the equatorial and polar diameters is 27 miles.

The dimensions of the earth have been computed with great accuracy by Hayford, who found for the equatorial diameter 7926.57 miles, and for the polar diameter 7899.98 miles. The error in these results cannot exceed a thousand feet. The equatorial circumference is 24,901.7 miles, and the length of one degree of longitude at the equator is 69.17 miles. The lengths of degrees of latitude at the equator and at the poles are respectively 69.40 and 68.71 miles. The total area of the earth is about 196,400,000 square miles. The volume of the earth is equal to the volume of a sphere whose radius is 3958.9 miles.

14. Newton's Proof of the Oblateness of the Earth.—The first proof that the earth is oblate was due to Newton. He based his demonstration on the laws of motion, the law of gravitation, and the rotation of the earth. It is therefore much more complicated than that depending on the lengths of degrees of latitude, which is purely geometrical. It has the advantage, however, of not requiring any measurements of arcs.

Suppose the earth, Fig. 10, rotates around the axis PP'. Imagine that a tube filled with water exists reaching from the pole P to the center C, and then to the surface on the equator at Q. The water in this tube exerts a pressure toward the center because of the attraction of the earth for it. Consider a unit volume in the part CP at any distance D from the center; the pressure it exerts toward the center equals the earth's attraction for it because it is subject to no other forces. Suppose for the moment that the earth is a sphere, as it would be if it were not rotating on its axis, and consider a unit volume in the part CQ at the distance D from the center. Because of the symmetry of the sphere it will be subject to an attraction equal to that on the corresponding unit in CP. But, in addition to the earth's attraction, this mass of water is subject to the centrifugal force due to the earth's rotation, which to some extent counter-balances the attraction. Therefore, the pressure it exerts toward the center is less than that exerted by the corresponding unit in CP. If the earth were spherical, all units in the two columns could be paired in this way. The result would be that the pressure exerted by PC would be greater than that exerted by QC; but such a condition would not be one of equilibrium, and water would flow out of the mouth of the tube from the center to the equator. In order that the two columns of water shall be in equilibrium the equatorial column must be longer than the polar.

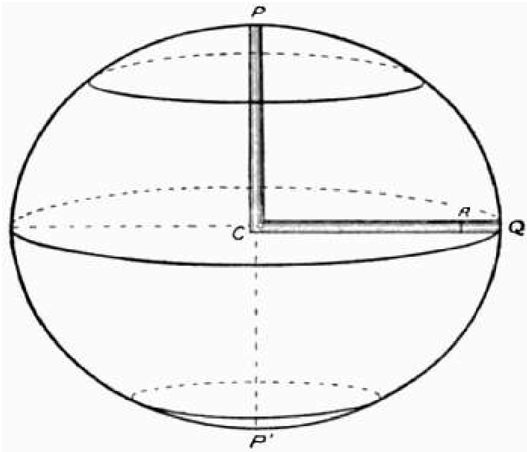


FIG. 10.—Because of the earth's rotation around PP' the column CQ must be longer than PC.

Newton computed the amount RQ by which the one tube must be longer than the other in order that for a body having the mass, dimensions, and rate of rotation of the earth, there should be equilibrium. This gave him the oblateness of the earth. In spite of the fact that his data were not very exact, he obtained results which agree very well with those furnished by modern measurements of arcs.

The objection at once arises that the tubes did not actually exist and that they could not possibly be constructed, and therefore that the conclusion was as insecure as those usually are which rest on imaginary conditions. But the fears aroused by these objections are dissipated by a little more consideration of the subject. It is not necessary that the tubes should run in straight lines from the surface to the center in order that the principle should apply. They might bend in any manner and the results would be the same, just as the level to which the water rises in the spout of a teakettle does not depend on its shape. Suppose the tubes are deformed into a single one connecting P and Q along the surface of the earth. The principles still hold; but the ocean connection of pole and equator may be considered as being a tube. Hence the earth must be oblate or the ocean would flow from the poles toward the equator.

15. Pendulum Proof of the Oblateness of the Earth.—It seems strange at first that the shape of the earth can be determined by means of the pendulum. Evidently the method cannot rest on such simple geometrical principles as were sufficient in using the lengths of arcs. It will be found that it involves the laws of motion and the law of gravitation.

The time of oscillation of a pendulum depends on the intensity of the force acting on the bob and on the distance from the point of support to the bob. It is shown in analytic mechanics that the formula for a complete oscillation is

$$t = 2\pi\sqrt{l/g},$$

where t is the time, $\pi = 3.1416$, l is the length of the pendulum, and g is the resultant acceleration[1] produced by all the forces to which the pendulum is subject. If l is determined by measurement and t is found by observations, the resultant acceleration is given by

$$g = \frac{4\pi^2 l}{t^2}.$$

Consequently, the pendulum furnishes a means of finding the gravity g at any place.

In order to treat the problem of determining the shape of the earth from a knowledge of g at various places on its surface, suppose first that it is a homogeneous sphere. If this were its shape, its attraction would be equal for all points on its surface. But the gravity g would not be the same at all places, because it is the resultant of the earth's attraction and the centrifugal acceleration due to the earth's rotation. The gravity g would be the greatest at the poles, where there is no centrifugal acceleration, and least at the equator, where the attraction is exactly opposed by the centrifugal acceleration. Moreover, the value of g would vary from the poles to the equator in a perfectly definite manner which could easily be determined from theoretical considerations.

Now suppose the earth is oblate. It can be shown mathematically that the attraction of an oblate body for a particle at its pole is greater than that of a sphere of equal volume and density for a particle on

[1] Force equals mass times acceleration. On a large pendulum the force of gravity is greater but the acceleration is the same.

its surface, and that at its equator the attraction is less. Therefore at the pole, where there is no centrifugal acceleration, g is greater on an oblate body than it is on an equal sphere. On the other hand, at the equator g is less on the oblate body than on the sphere both because the attraction of the former is less, and also because its equator is farther from its axis so that the centrifugal acceleration is greater. That is, the manner in which g varies from pole to equator depends upon the oblateness of the earth, and it can be computed when the oblateness is given. Conversely, when g has been found by experiment, the shape of the earth can be computed.

Very extensive determinations of g by means of the pendulum, taken in connection with the mathematical theory, not only prove that the earth is oblate, but give a degree of flattening agreeing closely with that obtained from the measurement of arcs.

The question arises why g is determined by means of the pendulum. Its variations cannot be found by using balance scales, because the forces on both the body to be weighed and the counter weights vary in the same proportion. However, the variations in g can be determined with some approximation by employing the spring balance. The choice between the spring balance and the pendulum is to be settled on the basis of convenience and accuracy. It is obvious that spring balances are very convenient, but they are not very accurate. On the other hand, the pendulum is capable of furnishing the variation of g with almost indefinite precision by the period in which it vibrates. Suppose the pendulum is moved from one place to another where g differs by one hundred-thousandth of its value. This small difference could not be detected by the use of spring balances, however many times the attempt might be made. It follows from the formula that the time of a swing of the pendulum would be changed by about one two-hundred-thousandth of its value. If the time of a complete oscillation were a second, for example, the difference could not be detected in a second; but the deviation for the following second would be equal to that in the first, and the difference would be doubled. The effect would accumulate, second after second, and in a day of 86,400 seconds it would amount to nearly one half of a second, a quantity which is easily measured. In ten days the difference would amount to about 4.3 seconds. The important point in the pendulum method is that the effects of the quantities to be measured accumulate until they become observable.

16. The Theoretical Shape of the Earth.—The oblateness of the earth is not an accident; its shape depends on its size, mass, distribution of density, and rate of rotation. If it were homogeneous,

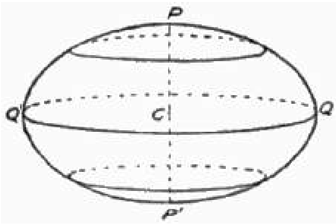


FIG. 11.—Oblate spheroid.

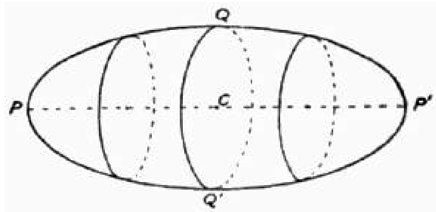


FIG. 12.—Prolate spheroid.

its shape could be theoretically determined without great difficulty. It has been found from mathematical discussions that if a homogeneous fluid body is slowly rotating it may have either of two forms of equilibrium, one of which is nearly spherical while the other is very much flattened like a discus. These figures are not simply oblate, but they are figures known as spheroids. A spheroid is a solid generated by the rotation of an ellipse (Art. 53) about one of its diameters. Figure 11 is an *oblate* spheroid generated by the rotation of the ellipse $PQP'Q'$ about its shortest diameter PP'. Its equator is its largest circumference. Figure 12 is a *prolate* spheroid generated by the rotation of the ellipse $PQP'Q'$ about its longest diameter PP'. The equator of this figure is its smallest circumference. The oblate and prolate spheroids are fundamentally different in shape.

Of the two oblate spheroids which theory shows are figures of equilibrium for slow rotation, that which is the more nearly spherical is stable, while the other is unstable. That is, if the former were disturbed a little, it would retake its spheroidal form, while if the latter were deformed a little, it would take an entirely different shape, or might even break all to pieces. In spite of the fact that the earth is neither a fluid nor homogeneous, its shape is almost exactly that of the more nearly spherical oblate spheroid corresponding to its density and rate of rotation. This fact might tempt one to the conclusion that it was formerly in a fluid state. But this conclusion is not necessarily sound, because, in such an enormous body, the strains which would result from appreciable departure from the figure of equilibrium would be so great that they could not be withstood by the strongest material known. Besides this, if the conditions for equilibrium were not exactly satisfied by the solid parts of the earth, the water and atmosphere would move and make compensation.

The sun, moon, and planets are bodies whose forms can likewise be compared with the results furnished by theory. Their figures agree closely with the theoretical forms. The only appreciable disagreements are in the case of Jupiter and Saturn, both of which are more nearly spherical than the corresponding homogeneous bodies would be. The reason for this is that these planets are very rare in their outer parts and relatively dense at their centers. It is probable that they are even more stable than the corresponding homogeneous figures.

17. Different Kinds of Latitude.—It was seen in Art. 12 that perpendiculars to the water-level surface of the earth, except on the equator and at the poles, do not pass through the center of the earth. This leads to the definition of different kinds of latitude.

The geometrically simplest latitude is that defined by a line from the center of the earth to the point on its surface occupied by the observer. Thus, in Fig. 13, PC is the earth's axis of rotation, QC is in the plane of its equator, and O is the position of the observer. The angle l is called the *geocentric latitude.*

The observer at O cannot see the center of the earth and cannot locate it by any kind of observation made at his station alone. Consequently, he cannot directly determine l. All he has is the perpendicular to the surface defined by his plumb line which strikes the line CQ at A. The angle l_1 between this line and CQ is his *astronomical latitude.* The difference between the geocentric and astronomical latitudes varies from zero at the poles and equator to about 11′ in latitude 45°.

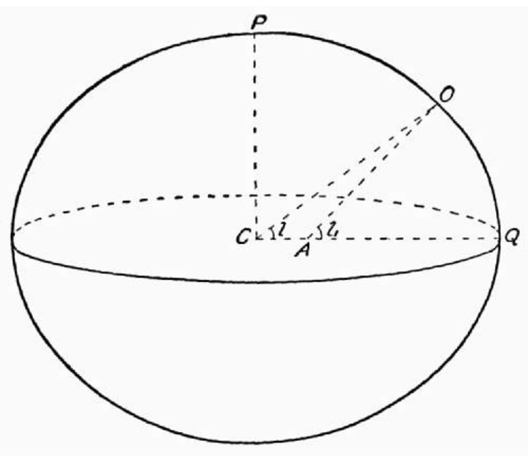


FIG. 13.—Geocentric and astronomical latitudes.

Sometimes the plumb line has an abnormal direction because of the attractions of neighboring mountains, or because of local excesses or deficiencies of matter under the surface. The astronomical latitude, when corrected for these anomalies, is called the *geographical latitude.* The astronomical and geographical latitudes rarely differ by more than a few seconds of arc.

18. Historical Sketch of Measurements of the Earth.—While the earth was generally supposed to be flat down to the time of Columbus, yet there were several Greek philosophers who believed that it was a sphere. The earliest philosopher who is known certainly to have maintained that the earth is spherical was Pythagoras, author of the famous Pythagorean proposition of geometry, who lived from about 569 to 490 B.C. He was followed in this conclusion, among others, by Eudoxus (407–356 B.C.), by Aristotle (384–322 B.C.), the most famous philosopher of antiquity if not of all time, and by Aristarchus of Samos (310–250 B.C.). But none of these men seems to have had so clear convictions as Eratosthenes (275–194 B.C.), who not only believed in the earth's sphericity but undertook to determine its dimensions. He had noticed that the altitude of the pole star was less when he was in Egypt than when he was farther north in Greece, and he correctly interpreted this as meaning that in traveling northward he journeyed around the curved surface of the earth. By very crude means he undertook to measure the length of a degree in Egypt, and in spite of the fact that he had neither accurate instruments for obtaining the distances on the surface of the earth, nor telescopes with which to determine the changes of the direction of the plumb line with respect to the stars, he secured results that were not surpassed in accuracy until less than 300 years ago.

After the decline of the Greek civilization and science, no progress was made in proving the earth is spherical until the voyage of Columbus in 1492. His ideas regarding the size of the earth were very erroneous, as is shown by the fact that he supposed he had reached India by crossing the Atlantic Ocean. The great explorations and geographical discoveries that quickly followed the voyages of Columbus convinced men that the earth is at least globular and gave them some idea of its dimensions.

There were no serious attempts made to obtain accurate knowledge of the shape and size of the earth until about the middle of the seventeenth century. The first results of any considerable degree of accuracy were obtained in 1671 by Picard from a measurement of an arc in France.

In spite of the fact that Newton proved in 1686 that the earth is oblate, the conclusion was by no means universally accepted. Imperfections in the measures of the French led Cassini to maintain until about 1745 that the earth is prolate. But the French were taking hold of the question in earnest and they finally agreed with the conclusion of Newton. They extended the arc that Picard had started from the Pyrenees to Dunkirk, an angular distance of 9°. The results were published in

1720. They sent an expedition to Peru, on the equator, in 1735, under Bouguer, Condamine, and Godin. By 1745 these men had measured an arc of 3°. In the meantime an expedition to Lapland, near the Arctic circle, had measured an arc of 1°. On comparing these measurements it was found that a degree of latitude is greater the farther it is from the equator.

In the last century all the principal governments of the world have carried out very extensive and accurate surveys of their possessions. The English have not only triangulated the British Isles but they have done an enormous amount of work in India and Africa. The Coast and Geodetic Survey in the United States has triangulated with unsurpassed precision a great part of the country. They have run a level from the Atlantic Ocean to the Pacific. The names most often encountered in this connection are Clarke of England, Helmert of Germany, and Hayford of the United States. Hayford has taken up an idea first thrown out by the English in connection with their work in India along the borders of the Himalaya Mountains, and by using an enormous amount of observational data and making appalling computations he has placed it on a firm basis. The observations in India showed that under the Himalaya Mountains the earth is not so dense as it is under the plains to the south. Hayford has proved that the corresponding thing is true in the United States, even in the case of very moderate elevations and depressions. Moreover, deficiency in density under the elevated places is just enough to offset the elevations, so that the total weight of the material along every radius from the surface of the earth to its center is almost exactly the same. This theory is known as the theory of isostasy, and the earth is said to be in almost perfect isostatic adjustment.

II. QUESTIONS

1. In order to prove the sphericity of the earth by measurement of arcs, would it be sufficient to measure only along meridians? (Consider the anchor ring.)

2. Do the errors in triangulation accumulate with the length of the distance measured? Do the errors in the astronomical determination of the angular length of the arc increase with its length?

3. How accurately must a base line of five miles be measured in order that it may not introduce an error in the determination of the earth's circumference of more than 1000 feet?

4. Which of the reasons given in Art. 11 actually prove, so far as they go, that the earth is spherical? What other reasons are there for believing

it is spherical?

5. The acceleration g in mid-latitudes is about 32.2 feet per second; how long would a pendulum have to be to swing in 1, 2, 3, 4 seconds?

6. Draw to scale a meridian section of a figure having the earth's oblateness.

7. Newton's proof of the earth's oblateness depends on the knowledge that the earth rotates; what proofs of it do not depend upon this knowledge?

8. Suppose time can be measured with an error not exceeding one tenth of a second; how accurately can g be determined by the pendulum in 10 days?

9. Suppose the solid part of the earth were spherical and perfectly rigid; what would be the distribution of land and water over the surface?

10. Is the astronomical latitude greater than, or equal to, the geocentric latitude for all points on the earth's surface?

11. What distance on the earth's surface corresponds to a degree of arc, a minute of arc, a second of arc?

12. Which of the proofs of the earth's sphericity depend upon modern discoveries and measurements?

II. The Mass of the Earth and the Condition of its Interior

19. The Principle by which Mass is Determined.—It is important to understand clearly the principles which are at the foundation of any subject in which one may be interested, and this applies in the present problem. The ordinary method of determining the mass of a body is to weigh it. That is the way in which the quantity of most commodities, such as coal or ice or sugar, is found. The reason a body has weight at the surface of the earth is that the earth attracts it. It will be seen later (Art. 40) that the body attracts the earth equally in the opposite direction. Consequently, the real property of a body by which its mass is determined is its attraction for some other body. The underlying principle is that *the mass of a body is proportional to the attraction which it has for another body.*

Now consider the problem of finding the mass of the earth, which must be solved by considering its attraction for some other body. Its attraction for any given mass, for example, a cubic inch of iron, can easily be measured. But this does not give the mass of the earth compared to the cubic inch of iron. It is necessary to compare the attraction of the earth for the iron with the attraction of some other fully known body, as a lead ball of given size, for the same unit of iron. Since the amount of attraction of one body for another depends upon their distance apart, it is necessary to measure the distance from the lead ball to the attracted body, and also to know the distance of the attracted body from the center of the earth. For this reason the mass of the earth could not be found until after its dimensions had been ascertained. By comparing the attractions of the earth and the lead ball for the attracted body, and making proper adjustments for the distances of their respective centers from it, the number of times the earth exceeds the lead ball in mass can be determined.

Not only is the mass of the earth computed from its attraction, but the same principle is the basis for determining the mass of every other celestial body. The masses of those planets that have satellites are easily found from their attractions for their respective satellites, and when two stars revolve around each other in known orbits their masses are defined by their mutual attractions. There is no means of determining the mass of a single star.

20. The Mass and Density of the Earth.—By applications (Arts. 21, 22) of the principle in Art. 19 the mass of the earth has been found. If it were weighed a small quantity at a time at the surface, its total weight in tons would be 6×10^{21}, or 6 followed by 21 ciphers. This makes no appeal to the imagination because the numbers are so extremely far beyond all experience. A much better method is to give its density, which is obtained by dividing its mass by its volume. With water at its greatest density as a standard, the average density of the earth is 5.53.

The average density of the earth to the depth of a mile or two is in the neighborhood of 2.75. Therefore there are much denser materials in the earth's interior; their greater density may be due either to their composition or to the great pressure to which they are subject. The density of quartz (sand) is 2.75, limestone 3.2, cast iron 7.1, steel 7.8, lead 11.3, mercury 13.6, gold 19.3, and platinum 21.5. It follows that no considerable part of the earth can be composed of such heavy substances as mercury, gold, and platinum, but, so far as these considerations bear on the question, it might be largely iron.

The distribution of density in the earth was worked out over 100 years ago by Laplace on the basis of a certain assumption regarding the compressibility of the matter of which it is composed. The results of this computation have been compared with all the phenomena on which the disposition of the mass of the earth has an influence, and the results have been very satisfactory. Hence, it is supposed that this law represents approximately the way the density of the earth increases from its surface to its center. According to this law, taking the density of the surface as 2.72, the densities at depths of 1000, 2000, 3000 miles, and the center of the earth are respectively 5.62, 8.30, 10.19, 10.87. At no depth is the average density so great as that of the heavier metals.

21. Determination of the Density of the Earth by Means of the Torsion Balance.—The whole difficulty in determining the density of the earth is due to the fact that the attractions of masses of moderate dimensions are so feeble that they almost escape detection with the most sensitive apparatus. The problem from an experimental point of view reduces to that of devising a means of measuring extremely minute forces. It has been solved most successfully by the torsion balance.

The torsion balance consists essentially of two small balls, *bb* in Fig. 14, connected by a rod which is suspended from the point *O* by

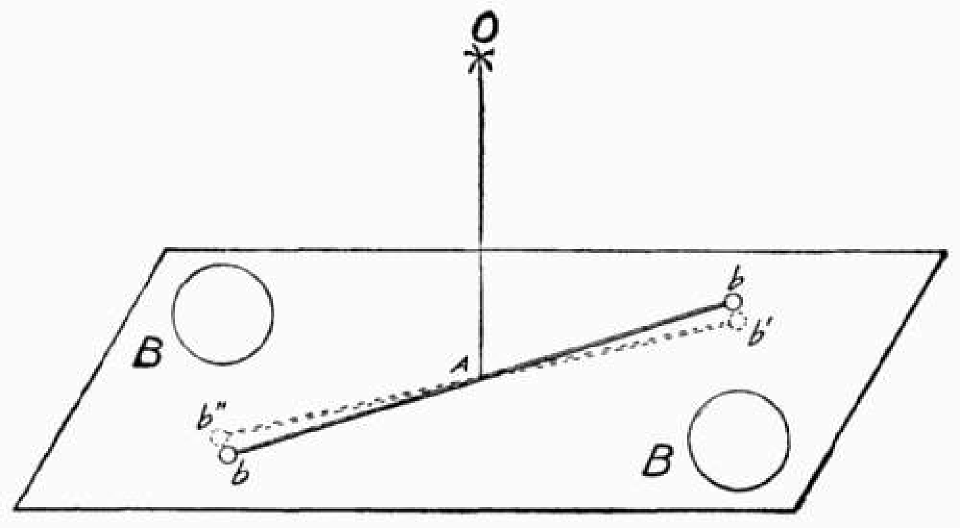


FIG. 14.—The torsion balance.

a quartz fiber *OA*. If the apparatus is left for a considerable time in a sealed case so that it is not disturbed by air currents, it comes to rest. Suppose the balls *bb* are at rest and that the large balls *BB* are carefully brought near them on opposite sides of the connecting rod, as shown in the figure. They exert slight attractions for the small balls and gradually move them against the feeble resistance of the quartz fiber to torsion (twisting) to the position $b'b''$. The resistance of the quartz fiber becomes greater the more it is twisted, and finally exactly balances the attraction of the large balls. The forces involved are so small that several hours may be required for the balls to reach their final positions of rest. But they will finally be reached and the angle through which the rod has been turned can be recorded.

The next problem is to determine from the deflection which the large balls have produced how great the force is which they have exerted. This would be a simple matter if it were known how much resistance the quartz fiber offers to twisting, but the resistance is so exceedingly small that it cannot be directly determined. However, it can be found by a very interesting indirect method.

Suppose the large balls are removed and that the rod connecting the small balls is twisted a little out of its position of equilibrium. It will then turn back because of the resistance offered to twisting by the quartz fiber, and will rotate past the position of equilibrium almost as far as it was originally displaced in the opposite direction. Then it will return and vibrate back and forth until friction destroys its motion. It is evident that the characteristics of the oscillations are much like those of a vibrating pendulum. The formula connecting the various quantities involved is

$$t = 2\pi\sqrt{l/f},$$

where t is the time of a complete oscillation of the rod joining b and b, l is the distance from A to b, and f is the resistance of torsion. This equation differs from that for the pendulum, Art. 15, only in that g has been replaced by f. Now l is measured, t is observed, and f is computed from the equation with great exactness however small it may be.

Now that f and g are known it is easy to compute the mass of the earth by means of the law of gravitation (Art. 146). Let E represent the mass of the earth, R its radius, $2B$ the mass of the two large balls, and r the distances from BB to bb respectively. Then, since gravitation is proportional to the attracting mass and inversely as the square of its distance from the attracted body, it follows that

$$\frac{E}{R^2} : \frac{2B}{r^2} = g : f.$$

In this proportion the only unknown is E, which can therefore be computed.

22. Determination of the Density of the Earth by the Mountain Method.—The characteristic of the torsion balance is that it is very delicate and adapted to measuring very small forces; the characteristic of the mountain method is that a very large mass is employed, and the forces are larger. In the torsion balance the balls BB are brought near those suspended by the quartz fiber and are removed at will. A mountain cannot be moved, and the advantage of using a large mass is at least partly counterbalanced by this disadvantage. The necessity for moving the attracting body (in this case the mountain) is obviated in a very ingenious manner.

For simplicity let the oblateness of the earth be neglected in explaining the mountain method. In Fig. 15, C is the center of the earth, M is the mountain, and O_1 and O_2 are two stations on opposite sides of the mountain at which plumb lines are suspended. If it were not for the attraction of the mountain they would hang in the directions O_1C and O_2C. The angle between these lines at C depends upon the distance between the stations O_1 and O_2. The distance between these stations, even though they are on opposite sides of the mountain, can be obtained by triangulation. Then, since the size of the earth is known, the angle at C can be computed.

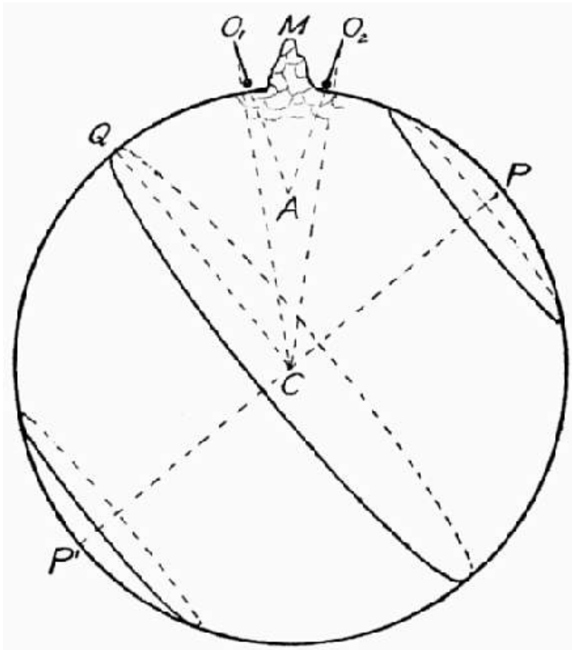


FIG. 15.—The mountain method of determining the mass of the earth.

But the attraction of the mountain for the plumb bobs causes the plumb lines to hang in the directions O_1A and O_2A. The directions of these lines with respect to the stars can easily be determined by observations, and the difference in their directions as thus determined is the angle at A.

What is desired is the deflections of the plumb line produced by the attractions of the mountain. It follows from elementary geometry that the sum of the two small deflections CO_1A and CO_2A equals the angle A minus the angle C. Suppose, for simplicity, that the mountain is symmetrical and that the deflections are equal. Then each one equals one half the difference of the angles A and C. Therefore the desired quantities have been found.

When the deflection has been found it is easy to obtain the relation of the force exerted by the mountain to that due to the earth. Let Fig. 16 represent the plumb line on a large scale. If it were not for the mountain it would hang in the direction O_1B_1; it actually hangs in the direction O_1B_1'. The earth's attraction is in the direction O_1B_1, and that of the mountain is in the direction B_1B_1'. The two forces are in the same ratio as O_1B_1 is to B_1B_1', for, by the law of the composition of forces, only then would the plumb line hang in the direction O_1B_1'.

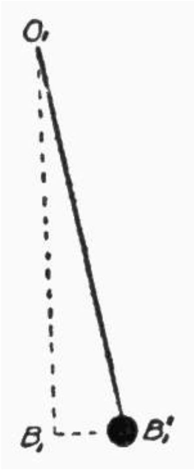


FIG. 16.—The deflection of a plumb line.

The problem of finding the mass of the earth compared to that of the mountain now proceeds just like that of finding the mass of the earth compared to the balls BB in the torsion-balance method. The mountain plays the rôle of the large balls. A mountain 5000 feet high and broad would cause nearly 800 times as much deflection as that produced by an iron ball a foot in diameter. The advantage of the large deflection is offset by not having very accurate means of measuring it, and also by the fact that it is necessary to determine the mass of a more or less irregular shaped mountain made up of materials which may lack much of being uniform in density. In spite of these drawbacks this method was the first one to give fairly accurate results.

23. Determination of the Density of the Earth by the Pendulum Method.—It was explained in Art. 15 that the pendulum furnishes a very accurate means of determining the force of gravity. Its delicacy arises from the fact that in using it the effects of the changes in the forces accumulate indefinitely; no such favorable circumstances were present in the methods of the torsion balance and the mountain.

Suppose a pendulum has been swung at the surface of the earth so long that the period of its oscillation has been accurately determined. Then suppose it is taken at the same place down into a deep pit or mine. The force to which it is subject will be changed for three different reasons. (a) The pendulum will be nearer the axis of rotation of the earth and the centrifugal acceleration to which it is subject will be diminished. The relative change in gravity due to this cause can be accurately computed from the latitude of the position and the depth of the pit or mine. (*b*) The pendulum will be nearer the center of the earth, and, so far as this factor alone is concerned, the force to which it is subject will be increased. Moreover, the relative change due to this cause also can be computed. (*c*) The pendulum will be below a certain amount of material whose attraction will now be in the opposite direction. This cannot be computed directly because the amount of attraction due to a ton of matter, for example, is unknown. This is what is to be found out. But from the time of the oscillation of the pendulum at the bottom of the pit or mine the whole force to which it is subject can be computed. Then, on making correction for the known changes (*a*) and (*b*), the unknown change (*c*) can be obtained simply by subtraction. From the amount of force exerted by the known mass above the pendulum, the density of the earth can be computed by essentially the same process as that employed in the case of the torsion-balance method and the mountain method.

24. Temperature and Pressure in the Earth's Interior.—There are many reasons for believing that the interior of the earth is very hot. For example, volcanic phenomena prove that at least in many localities the temperature is above the melting point of rock at a comparatively short distance below the earth's surface. Geysers and hot springs show that the interior of the earth is hot at many other places. Besides this, the temperature has been found to rise in deep mines at the rate of about one degree Fahrenheit for a descent of 100 feet, the amount depending somewhat on the locality.

Suppose the temperature should go on increasing at the rate of one degree for every hundred feet from the surface to the center of the earth. At a depth of ten miles it would be over 500 degrees, at 100 miles over 5000 degrees, at 1000 miles over 50,000 degrees, and at the center of the earth over 200,000 degrees. While there is no probability that the rate of increase of temperature which prevails near the surface keeps up to great depths, yet it is reasonably certain that at a depth of a few hundred miles it is several thousand degrees. Since almost every substance melts at a temperature below 5000 degrees, it has been supposed until recent times that the interior of the earth, below the depth of 100 miles, is liquid.

But the great pressure to which matter in the interior of the earth is subject is a factor that cannot safely be neglected. A cylinder one inch in cross section and 1728 inches, or 144 feet, in height has a volume of one cubic foot. If it is filled with water, the pressure on the bottom equals the weight of a cubic foot of water, or 62.5 pounds. The pressure per square inch on the bottom of the column 144 feet high having the density 2.75, or that of the earth's crust, is 172 pounds. The pressure per square inch at the depth of a mile is 6300 pounds, or 3 tons in round numbers. The pressure is approximately proportional to the depth for a considerable distance. Therefore, the pressure per square inch at the depth of 100 miles is approximately 300 tons, and at 1000 miles it is 3000 tons. However, the pressure is not strictly proportional to the depth, and more refined means must be employed to find how great it is at the earth's center. Moreover, the pressure at great depths depends upon the distribution of mass in the earth. On the basis of the Laplacian law of density, which probably is a good approximation to the truth, the pressure per square inch at the center of the earth is $3,000,000$ times the atmospheric pressure at the earth's surface, or $22,500$ tons.

It is a familiar fact that pressure increases the boiling points of liquids. It has been found recently by experiment that pressure increases the melting points of solids. Therefore, in view of the enormous pressures at moderate depths in the earth, it is not safe to conclude that its interior is molten without further evidence. The question cannot be answered directly because, in the first place, there is no very exact means of determining the temperature, and, in the second place, it is not possible to make experiments at such high pressures. There are, however, several methods of proving that the earth is solid through and through, and they will now be considered.

25. Proof of the Rigidity and Elasticity of the Earth by the Tide Experiment.—Among the several lines of attack that have been made on the question of the rigidity of the earth, the one depending on the tides generated in the earth by the moon and sun has been most satisfactory; and of the methods of this class, that devised by Michelson and carried out in collaboration with Gale, in 1913, has given by far the most exact results. Besides, it has settled one very important question, which no other method has been able to answer, namely, that the earth is highly elastic instead of being viscous. For these reasons the work of Michelson and Gale will be treated first.

The important difference between a solid and a liquid is that the former offers resistance to deforming forces while the latter does not. If a perfect solid existed, no force whatever could deform it; if a perfect liquid existed, the only resistance it would offer to deformation would be the inertia of the parts moved. Neither perfect solids nor absolutely perfect liquids are known. If a solid body has the property of being deformed more and more by a continually applied force, and if, on the application of the force being discontinued, the body not only does not retake its original form but does not even tend toward it, then it is said to be *viscous*. Putty is a good example of a material that is viscous. On the other hand, if on the application of a continuous force the body is deformed to a certain extent beyond which it does not go, and if, on the removal of the force, it returns absolutely to its original condition, it is said to be *elastic*. While there are no solid bodies which are either perfectly viscous or perfectly elastic, the distinction is a clear and important one, and the characteristics of a solid may be described by stating how far it approaches one or the other of these ideal states.

In order to find how the earth is deformed by forces it is necessary to consider what forces there are acting on it. The most obvious ones are the attractions of the sun and moon. But it is not clear in the first place that these attractions tend to deform the earth, and in the second place that, even if they have such a tendency, the result is at all appreciable. A ball of iron attracted by a magnet is not sensibly deformed, and it seems that the earth should behave similarly. But the earth is so large that one's intuitions utterly fail in such considerations. The sun and moon actually do tend to alter the shape of the earth, and the amount of its deformation due to their attractions is measurable. The forces are precisely those that produce the tides in the ocean.

It will be sufficient at present to give a rough idea, correct so far as it goes, of the reason that the moon and sun raise tides in the earth, reserving for Arts. 263, 264 a more complete treatment of the question. In Fig. 17 let E represent the center of the earth, the arrow the direction toward the moon, and A and B the points where the line from E to the moon pierces the earth's surface. The moon is 4000 miles nearer to A than it is to E, and 4000 miles nearer to E than it is to B. Therefore the attraction of the moon for a unit mass at A is greater than it is for a unit mass at E, and greater for a unit mass at E than it is for one at B. Since the distance from the earth to the moon is $240,000$ miles, the distance of the moon from A is fifty-nine sixtieths of its distance from E. Since the attraction varies inversely as the square of the distance, the force on A is about one thirtieth greater than that on E, and the difference between the forces on E and B is only slightly less.

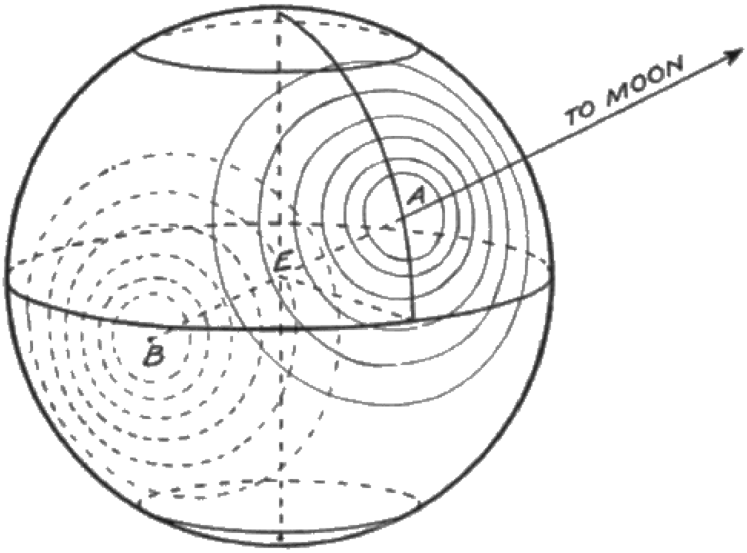


FIG. 17.—The tidal bulges at A and B on the earth produced by the moon.

It follows from the relation of the attraction of the moon for masses at A, E, and B that it tends to pull the nearer material at A away from the center of the earth E, and the center of the earth away from the more remote material at B. Since the forces are known, it is possible to compute the elongation the earth would suffer if it were a perfect fluid. The result is two elevations, or tidal bulges, at A and B. The concentric lines shown in Fig. 17 are the lines of equal elevation. A rather difficult mathematical discussion shows that the radii EA and EB would each be lengthened by about four feet. Since the earth possesses at least some degree of rigidity its actual tidal elongation is somewhat less than four feet. When it is remembered that the uncertainty in the diameter of the earth, in spite of the many years that have been devoted to determining it, is still several hundred feet, the problem of finding how much the earth's elongation, as a consequence of the rapidly changing tidal forces, falls short of four feet seems altogether hopeless of solution. Nevertheless the problem has been solved.

Suppose a pipe half filled with water is fastened in a horizontal position to the surface of the earth. The water in the pipe is subject to the attraction of the moon. To fix the ideas, suppose the pipe lies in the east-and-west direction in the same latitude as the point A, Fig. 17. Suppose, first, that the earth is absolutely rigid so that it is not deformed by the moon, and consider what happens to the water in the pipe as the rotation of the earth carries it past the point A. When the pipe is to the west of A the water rises in its eastern end, and settles correspondingly in its western end, because the moon tends to make an elevation on the earth at A. When the pipe is carried past A to the east the water rises in its western end and settles in its eastern end. Since the earth is not absolutely rigid the magnitudes of the tides under the hypothesis that it is rigid cannot be experimentally determined; but, since all the forces that are involved are known, the heights the tides would be on a rigid earth can be computed.

Suppose now that the earth yields perfectly to the disturbing forces of the moon. Its surface is in this case always the exact figure of equilibrium. Consider the pipe, which is attached to this surface, when it is to the west of A. The water would be high in its eastern end if the shape of the surface of the earth were unchanged. But the surface to the east of it is elevated and the pipe is raised with it. Moreover, the elevation of the surface is, under the present hypothesis, just that necessary for equilibrium. Therefore, in this case there is no tide at all with respect to the pipe.

The actual earth is neither absolutely rigid nor perfectly fluid. Consequently the tides in the pipe will actually be neither their theoretical maximum nor zero. The amount by which they fall short of the value they would have if the earth were perfectly rigid depends upon the extent to which it yields to the moon's forces, and is a measure of this yielding. Therefore the problem of finding how much the earth is deformed by the moon is reduced to computing how great the tides in the pipe would be if the earth were absolutely rigid, and then comparing these results with the actual tides in the pipe as determined by direct experiment. After the amount the earth yields has been determined in this way, its rigidity can be found by the theory of the deformation of solid bodies.

In the experiment of Michelson and Gale two pipes were used, one lying in the plane of the meridian and the other in the east-and-west direction. In order to secure freedom from vibrations due to trains and heavy wagons they were placed on the grounds of the Yerkes Observatory, and to avoid variations in temperature they were buried a number of feet in the ground. Since the tidal forces are very small, pipes 500 feet long were used, and even then the maximum tides were only about two thousandths of an inch.

An ingenious method of measuring these small changes in level was devised. The ends of the pipes were sealed with plane glass windows through which their interiors could be viewed. Sharp pointers, fastened to the pipe, were placed just under the surface of the water near the windows. When viewed from below the level of the water the pointer and its reflected image could be seen. Figure 18 shows an end of one of the pipes, S is the surface of the water, P is the pointer, and P' is its reflected image. The distances of P and P' from the surface S are equal. Now suppose the water rises; since P and P' are equidistant from S, the change in their apparent distance is twice the change in the water level. The distances between P and P' were accurately measured with the help of permanently fixed microscopes, and the variations in the water level were determined within one per cent of their whole amount.

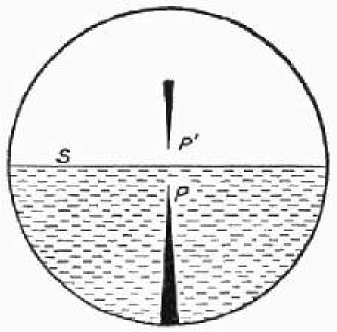


FIG. 18.—End of pipe in the Michelson-Gale tide experiment.

In order to make clear the accuracy of the results, the complicated nature of the tides must be pointed out. Consider the tidal bulges A and B, Fig. 17, which give an idea of what happened to the water in the pipes. For simplicity, fix the attention on the east-and-west pipe, which in the experiment was about 13° north of the highest latitude A ever attains. The rotating earth carried it daily across the meridian of A to the north of A, and similarly across the meridian of B. When the relations were as represented in the diagram there were considerable tides in the pipe before and after it crossed the meridian at A because it was, so to speak, well on the tidal bulge. On the other hand, when it crossed the meridian of B about 12 hours later, the tides were very small because the bulge B was far south of the equator. But the moon was not all the time north of the plane of the earth's equator. Once each month it was 28° north and once each month 28° south, and it varied from hour to hour in a rather irregular manner. Moreover, its distance, on which the magnitudes of the tidal forces depend, also changed continuously. Then add to all these complexities the corresponding ones due to the sun, which are unrelated to those of the moon, and which mix up with them and make the phenomena still more involved. Finally, consider the north-and-south pipe and notice, by the help of Fig. 17, that its tides are altogether distinct in character from those in the east-and-west pipe. With all this in mind, remember that the observations made every two hours of the day for a period of several months agreed perfectly in all their characteristics with the results given by theory. The only difference was that the observed tides were reduced in a constant ratio by the yielding of the earth.

The perfection of this domain of science is proved by the satisfactory coördination in this experiment of a great many distinct theories. The perfect agreement in their characteristics of more than a thousand observed tides with their computed values depended on the correctness of the laws of motion, the truth of the law of gravitation, the size of the earth, the distance of the moon and the theory of its motion, the mass of the moon, the distance to the sun and the theory of the earth's motion around it, the mass of the sun, the theory of tides, the numerous observations, and the lengthy calculations. How improbable that there would be perfect harmony between observation and theory in so many cases unless scientific conclusions respecting all these things are correct!

The extent to which the earth yields to the forces of the moon was obtained from the amount by which the observed tides were less than their theoretical values for an unyielding earth. It was found that in the east-and-west pipe the observed tides were about 70 per cent of the computed, while in the north-and-south pipe the observed tides were only about 50 per cent of the computed. This led to the astonishing conclusion, which, however, had been reached earlier by Schweydar on the basis of much less certain observational data, that the earth's resistance to deformation in the east-and-west direction is greater than it is in the north-and-south direction. Love has suggested that the difference may be due indirectly to the effects of the oceanic tides on the general body of the earth.

On using the amount of the yielding of the earth established by observations and the magnitude of the forces exerted by the moon and sun, it was found by the mathematical processes which are necessary in treating such problems, that the earth, taken as a whole, is as rigid as steel. That is, it resists deformation as much as it would if it were made of solid steel having throughout the properties of ordinary good steel.

The work of Michelson and Gale for the first time gave a reliable answer to the question whether the earth is viscous or elastic. It had almost invariably been supposed that the earth is viscous, because it was thought that even if the enormous pressure keeps the highly heated material of its interior in a solid state, yet it would be only stiff like a solid is when its temperature approaches the melting point. In fact, Sir George Darwin had built up an elaborate theory of tidal evolution (Arts. 265, 266), at the cost of a number of years of work, on the hypothesis that the earth is viscous. But the experiments of Michelson and Gale prove that it is very elastic.

If the earth were viscous, it would yield somewhat slowly to the forces of the moon and sun. Consequently, the tilting of the surface, which carries the pipes, would lag behind the forces which caused both the tilting and the tides in the pipes. There is no appreciable lag of a water tide in the pipe only 500 feet long, and consequently the observed and computed tides would not agree in phase. On the other hand, if the earth were elastic, there would be agreement in phase between the observed and computed tides. It is more difficult practically to determine accurately the phase of the tides than it is to measure their magnitudes, but the observations showed that there is no appreciable difference in the phases of the observed and computed tides. These results force the conclusion that the elasticity of the earth, taken as a whole, cannot be less than that of steel,—a result obviously of great interest to geologists.

26. Other Proofs of the Earth's Rigidity.—(*a*) There is a method of finding how much the earth yields to the forces of the moon and sun which is fundamentally equivalent to that of measuring tides in a pipe. It depends on the fact that the position of a pendulum depends upon all the forces acting on it, and, if the earth were in equilibrium, the line of its direction would always be perpendicular to the water-level surface. Consequently, if the earth yielded perfectly to the forces of the moon and sun, a pendulum would constantly remain perpendicular to its water-level surface. But if the earth did not yield perfectly, the pendulum would undergo very minute oscillations with respect to the solid part analogous to those of the water in the pipes. A modification of the ordinary pendulum, known as the horizontal pendulum, was found to be sensitive enough to show the oscillations, giving the rigidity of the earth but no satisfactory evidence regarding its elasticity.

(*b*) The principles at the basis of the method of employing tides in pipes apply equally well to tides in the ocean. Longer columns of water are available in this case, but there is difficulty in obtaining the exact heights of the actual tides, and very much greater difficulty in determining their theoretical heights on a shelving and irregular coast where they would necessarily be observed. In fact, it has not yet been found possible to predict in advance with any considerable degree of accuracy the height of tides where they have not been observed. Yet, Lord Kelvin with rare judgment inferred on this basis that the earth is very rigid.

(c) Earthquakes are waves in the earth which start from some restricted region and spread all over the earth, diminishing in intensity as they proceed. Modern instruments, depending primarily on some adaptation of the horizontal pendulum, can detect important earthquakes to a distance of thousands of miles from their origin. Earthquake waves are of different types; some proceed through the surface rocks around the earth in undulations like the waves in the ocean, while others, compressional in character like waves of sound in the air, radiate in straight lines from their sources.

The speed of a wave depends upon the density and the rigidity of the medium through which it travels. This principle applies to earthquake waves, and when tested on those which travel in undulations through the surface rocks there is good agreement between observation and theory. Consider its application to the compressional waves that go through the earth. The time required for them to go from the place of their origin to the place where they are observed is given by the observations. The density of the earth is known. If its rigidity were known, the time could be computed; but the time being known, the rigidity can be computed. While the results are subject to some uncertainties, they agree with those found by other methods.

(d) The attraction of the moon for the equatorial bulge slowly changes the plane of the earth's equator (Art. 47). The magnitude of the force that causes this change is known. If the earth consisted of a crust not more than a few hundred miles deep floating on a liquid interior, the forces would cause the crust to slip on the liquid core, just as a vessel containing water can be rotated without rotating the water. If the crust of the earth alone were moved, it would be shifted rapidly because the mass moved would not be great. But the rate at which the plane of the earth's equator is moved, as given by the observations, taken together with the forces involved, proves that the whole earth moves. When the effects of forces acting on such an enormous body are considered, it is found that this fact means that the earth has a considerable degree of rigidity.

(e) Every one knows that a top may be spun so that its axis remains stationary in a vertical direction, or so that it wabbles. Similarly, a body rotating freely in space may rotate steadily around a fixed axis, or its axis of rotation may wabble. The period of the wabbling depends upon the size, shape, mass, rate of rotation, and rigidity of the body. In the case of the earth all these factors except the last may be regarded as known. If it were known, the rate of wabbling could be computed; or, if the rate of wabbling were found from observation, the rigidity could be computed. It has recently been found that the earth's axis of rotation wabbles slightly, and the rate of this motion proves that the rigidity of the earth is about that of steel.

27. Historical Sketch on the Mass and Rigidity of the Earth.—The history of correct methods of attempting to find the mass of the earth necessarily starts with Newton, because the ideas respecting mass were not clearly formulated before his time, and because the determination of mass depends on the law of gravitation which he discovered. By some general but inconclusive reasoning he arrived at the conjecture that the earth is five or six times as dense as water.

The first scientific attempt to determine the density of the earth was made by Maskelyne, who used the mountain method, in 1774, in Scotland. He found 4.5 for the density of the earth. The torsion balance, devised by Michell, was first employed by Cavendish, in England, in 1798. His result agreed closely with those obtained by later experimenters, among whom may be mentioned Baily (1840) in England, and Reich (1842) in Germany, Cornu (1872) in France, Wilsing (1887) in Germany, Boys (1893) in England, and Braun (1897) in Austria. The pendulum method, using either a mountain or a mine to secure difference in elevation, has been employed a number of times.

Lord Kelvin (then Sir William Thomson) first gave in 1863 good reasons for believing the earth is rigid. His conclusion was based on the height of the oceanic tides, as outlined in Art. 26 (*b*). The proof by means of the rate of transmission of earthquake waves owes its possibility largely to John Milne, an Englishman who long lived in Japan, which is frequently disturbed by earthquakes. His interest in the character of earthquakes stimulated him to the invention of instruments, known as seismographs, for detecting and recording faint earth tremors. The change of the position of the plane of the earth's equator, known as the precession of the equinoxes, has been known observationally ever since the days of the ancient Greeks, and its cause was understood by Newton, but it has not been used to prove the rigidity of the earth because it takes place very slowly. The wabbling of the axis of the earth was first established observationally, in 1888, by Chandler of Cambridge, Mass., and Küstner of Berlin. The theoretical applications of the rigidity of the earth were made first by Newcomb of Washington, and then more completely by S. S. Hough of England. The first attempt at the determination of the rigidity of the earth by the amount it yields to the tidal forces of the moon and sun was made unsuccessfully in 1879 by George and Horace Darwin, in England. Notable success has been achieved only in the last 15 years, and that by improvements in the horizontal pendulum and by taking great care in keeping the instruments from being disturbed. The names that stand out are von Rebeur-Paschwitz, Ehlert, Kortozzi, Schweydar, Hecker, and Orloff. The observations of Hecker at Potsdam, Germany, were especially good, and Schweydar made two exhaustive mathematical discussions of the subject.

III. QUESTIONS

1. What is the difference between mass and weight? Does the weight of a body depend on its position? Does the inertia of a body depend on its position?

2. Can the mass of a small body be determined from its inertia? Can the mass of the earth be determined in the same way?

3. What is the average weight of a cubic mile of the earth?

4. Discuss the relative advantages of the torsion-balance method and mountain method in determining the density of the earth. Which one has the greater advantages?

5. What is the pressure at the bottom of an ocean six miles deep?

6. Discuss the character of the tides in east-and-west and north-and-south pipes during a whole day when the moon is in the position indicated in Fig. 17, and when it is over the earth's equator.

7. What are the advantages and disadvantages of a long pipe in the tide experiment?

8. If a body is at A, Fig. 17, is its weight greater or less than normal as determined by spring balances? By balance scales? What are the facts, if it is at B?

9. Enumerate the scientific theories and facts involved in the tide experiment.

10. List the principles on which the several proofs of the earth's rigidity depend. How many fundamentally different methods are there of determining its rigidity?

III. The Earth's Atmosphere

28. Composition and Mass of the Earth's Atmosphere.—The atmosphere is the gaseous envelope which surrounds the earth. Its chief constituents are the elements nitrogen and oxygen, but there are also minute quantities of argon, helium, neon, krypton, xenon, and some other very rare constituents. When measured by volume at the earth's surface, 78 per cent of the atmosphere is nitrogen, 21 per cent is oxygen, 0.94 per cent is argon, and the remaining elements occur in much smaller quantities.

Nitrogen, oxygen, etc., are elements; that is, they are substances which are not broken up into more fundamental units by any physical or any chemical changes. The thousands of different materials that are found on the earth are all made up of about 90 elements, only about half of which are of very frequent occurrence. The union of elements into a chemical compound is a very fundamental matter, for the compound may have properties very unlike those of any of the elements of which it is composed. For example, hydrogen, carbon, and nitrogen are in almost all food, but hydrocyanic acid, which is composed of these elements alone, is a deadly poison.

Besides the elements which have been enumerated, the atmosphere contains some carbon dioxide, which is a compound of carbon and oxygen, and water vapor, which is a compound of oxygen and hydrogen. In volume three hundredths of one per cent of the earth's atmosphere is carbon dioxide; but this compound is heavier than nitrogen and oxygen, and by weight, 0.05 per cent of the atmosphere is carbon dioxide. The amount of water vapor in the air varies greatly with the position on the earth's surface and with the time. There are also small quantities of dust, soot, ammonia, and many other things which occur in variable quantities and which are considered as impurities.

The pressure of the atmosphere at sea level is about 15 pounds per square inch and its density is about one eight-hundredth that of water. This means that the weight of a column of air reaching from the earth's surface to the limits of the atmosphere and having a cross section of one square inch weighs 15 pounds. The total mass of the atmosphere can be obtained by multiplying the weight of one column by the total area of the earth. In this way it is found that the mass of the earth's atmosphere is nearly $6,000,000,000,000,000$ tons, or approximately one millionth the mass of the solid earth. The total mass of even the carbon dioxide of the earth's atmosphere is approximately $3,000,000,000,000$ tons.

29. Determination of Height of Earth's Atmosphere from Observations of Meteors.—Meteors, or shooting stars as they are commonly called, are minute bodies, circulating in interplanetary space, which become visible only when they penetrate the earth's atmosphere and are made incandescent by the resistance which they encounter. The great heat developed is a consequence of their high velocities, which ordinarily are in the neighborhood of 25 miles per second.

Let m, Fig. 19, represent the path of a meteor before it encounters the atmosphere at A. Until it reaches A it is invisible, but at A it begins to glow and continues luminous until it is entirely burned up at B. Suppose it is observed from the two stations O_1 and O_2 which are at a known distance apart. The observations at O_1 give the angle AO_1O_2, and those at O_2 give the angle AO_2O_1. From these data the other parts of the triangle can be computed (compare Art. 10). After the distance O_1A has been computed the perpendicular height of A from the surface of the earth can be computed by using the angle AO_1O_2. Similarly, the height of B above the surface of the earth can be determined.

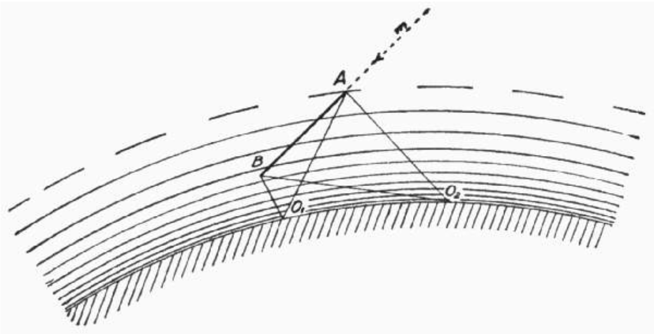


FIG. 19.—Determination of the height of meteors.

Observations of meteors from two stations show that they ordinarily become visible at a height of from 60 to 100 miles. Therefore, the atmosphere is sufficiently dense to a height of about 100 miles to offer sensible resistance to meteors. Meteors usually disappear by the time they have descended to within thirty or forty miles of the earth's surface.

30. Determination of Height of Earth's Atmosphere from Observations of Auroræ.—Auroræ are almost certainly electrical phenomena of the very rare upper atmosphere, though their nature is not yet very well understood. Their altitude can be computed from simultaneous observations made at different stations. The method is the same as that in obtaining the height of a meteor.

The southern ends of auroral streamers are usually more than 100 miles in height, and they are sometimes found at an altitude of 500 or 600 miles. Their northern ends are much lower. This means that the density required to make meteors incandescent is considerably greater than that which is sufficient for auroral phenomena.

31. Determination of Height of Earth's Atmosphere from

the Duration of Twilight.—Often after sunset, even to the east of the observer, high clouds are brilliantly illuminated by the rays of the sun which still fall on them. The higher the clouds are, the longer they are illuminated. Similarly, the sun shines on the upper atmosphere for a considerable time after it has set or before it rises, and gives the twilight. The duration of twilight depends upon the height of the atmosphere. While it is difficult to determine the instant at which the twilight ceases to be visible, observations show that under favorable weather conditions it does not disappear until the sun is 18 degrees below the horizon.

In order to see how the height of the atmosphere can be determined from the duration of the twilight, consider Fig. 20. The sun's rays come in from the left in lines that are sensibly parallel. The observer at O can see the illuminated atmosphere at P; but if the atmosphere were much shallower, it would not be visible to him. The region P is midway between O and the sunset point. Since O is 18 degrees from the sunset point, it is possible to compute the height of the plane of the horizon at P above the surface of the earth. It is found that 18 degrees corresponds to an altitude of 50 miles. That is, the atmosphere extends to a height of 50 miles above the earth's surface in quantities sufficient to produce twilight.

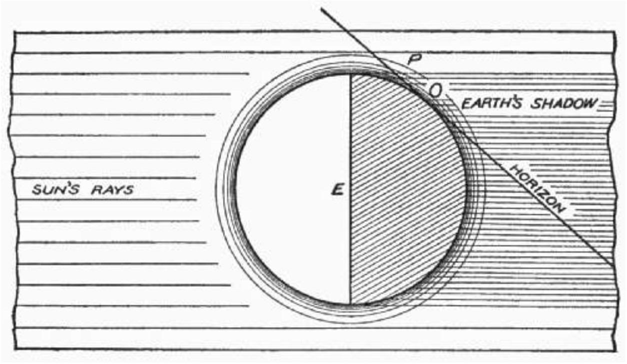


FIG. 20.—Determination of the height of the atmosphere from the duration of twilight.

The results obtained by the various methods for determining the height of the atmosphere disagree because its density decreases with altitude, as is found by ascending in balloons, and different densities are required to produce the different phenomena. It will convey the correct idea for most applications to state that the atmosphere does not extend in appreciable quantities beyond 100 miles above the earth's surface. At this altitude its density is of the order of one four-millionth of that at the surface. When the whole earth is considered it is found that the atmosphere forms a relatively thin layer. If the earth is represented by a globe 8 inches in diameter, the thickness of the atmosphere on the same scale is only about one tenth of an inch.

32. The Kinetic Theory of Gases.—It has been stated that every known substance on the earth is composed of about 90 fundamental elements. A chemical combination of atoms is called a molecule. A molecule of oxygen consists of two atoms of oxygen, a molecule of water of two atoms of hydrogen and one of oxygen, and similarly for all substances. Some molecules contain only a few atoms and others a great many; for example, a molecule of cane sugar is composed of 12 atoms of carbon, 22 of hydrogen, and 11 of oxygen. As a rule the compounds developed in connection with the life processes contain many atoms.

The molecules are all very minute, though their dimensions doubtless vary with the number and kind of atoms they contain. Lord Kelvin devised a number of methods of determining their size, or at least the distances between their centers. In water, for example, there are in round numbers $500,000,000$ in a line of them one inch long, or the cube of this number in a cubic inch.

In solids the molecules are constrained to keep essentially the same relations to one another, though they are capable of making complicated small vibrations. In liquids the molecules continually suffer restraints from neighboring molecules, but their relative positions are not fixed and they move around among one another, though not with perfect freedom. In gases the molecules are perfectly free from one another except when they collide. They move with great speed and collide with extraordinary frequency; but, in spite of the frequency of the collisions, the time during which they are uninfluenced by their neighbors is very much greater than that in which they are in effective contact.

The pressure exerted by a gas is due to the impact of its molecules on the walls of the retaining vessel. To make the ideas definite, consider a cubic foot of atmosphere at sea-level pressure. Its weight is about one and one fourth ounces, but it exerts a pressure of 15 pounds on each square inch of each of its six surfaces, or a total pressure on the surface of the cube of more than six tons. This implies that the molecules move with enormous speed. They do not all move with the same speed, but some travel slowly while others go much faster than the average. Theoretically, at least, in every gas there are molecules moving with every velocity, however great, but the number of those having any given velocity diminishes rapidly as its difference from the average velocity increases. The average velocity of molecules in common air at ordinary temperature and pressure is more than 1600 feet per second, and on the average each molecule has 5,000,000,000 collisions per second. Therefore the average distance traveled between collisions is only about $\frac{1}{250000}$ of an inch.

From the kinetic theory of gases it is possible to determine how fast the density of the air diminishes with increase of altitude. It is found that about one half of the earth's atmosphere is within the first 3.5 miles of its surface, that one half of the remainder is contained in the next 3.5 miles, and so on until it is so rare that the kinetic theory no longer applies without sensible modifications.

33. The Escape of Atmospheres.—Suppose a body is projected upward from the surface of the earth. The height to which it rises depends upon the speed with which it is started. The greater the initial speed, the higher it will rise, and there is a certain definite initial velocity for which, neglecting the resistance of the air, it will leave the earth and never return. This is the velocity of escape, and for the earth it is a little less than 7 miles per second.

The molecules in the earth's atmosphere may be considered as projectiles which dart in every direction. It has been seen that there is a small fraction of them which move with a velocity as great as 7 miles per second. Half of these will move toward points in the sky and consequently would escape from the earth if they did not encounter other molecules. But in view of the great frequency of collisions of molecules, it is evident that only a very small fraction of those which move with high velocities can escape from the earth. However, it seems certain that some molecules will be lost in this way, and, so far as this factor is concerned, the earth's atmosphere is being continually depleted. The process is much more rapid in the case of bodies, such as the moon, for example, whose masses and attractions are much smaller, and for which, therefore, the velocity of escape is lower.

It should not be inferred from this that the earth's atmosphere is diminishing in amount even if possible replenishment from the rocks and its interior is neglected. When a molecule escapes from the earth it is still subject to the attraction of the sun and goes around it in an orbit which crosses that of the earth. Therefore the earth has a chance of acquiring the molecule again by collision. The only exception to this statement is when the molecule escapes with a velocity so high that the sun's attraction cannot control it. The velocity necessary in order that the molecule shall escape both the earth and the sun depends upon its direction of motion, but averages about 25 miles per second and cannot be less than 19 miles per second. But besides the molecules that have escaped from the earth there are doubtless many others revolving around the sun near the orbit of the earth. These also can be acquired by collision. The earth is so old and there has been so much time for losing and acquiring an atmosphere, molecule by molecule, that probably an equilibrium has been reached in which the number of molecules lost equals the number gained. The situation is analogous to a large vessel of water placed in a sealed room. The water evaporates until the air above it becomes so nearly saturated that the vessel acquires as many molecules of water vapor by collisions as it loses by evaporation.

The doctrine of the escape of atmospheres implies that bodies of small mass will have limited and perhaps inappreciable atmospheres, and that those of large mass will have extensive atmospheres. The implications of the theory are exactly verified in experience. For example, the moon, with a mass $\frac{1}{80}$ that of the earth and a velocity of escape of about 1.5 miles per second, has no sensible atmosphere. On the other hand, Jupiter, with a mass 318 times that of the earth and a velocity of escape of 37 miles per second, has an enormous atmosphere. These examples are typical of the facts furnished by all known celestial bodies.

34. Effects of the Atmosphere on Climate.—Aside from the heat received from the sun, the most important factor affecting the earth's climate is its atmosphere. It tends to equalize the temperature in three important ways. (*a*) It makes the temperature at any one place more uniform than it would otherwise be, and (*b*) it reduces to a large extent the variations in temperature in different latitudes that would otherwise exist. And (*c*) it distributes water over the surface of the earth.

(*a*) Consider the day side of the earth. The rays of the sun are partly absorbed by the atmosphere and the heating of the earth's surface is thereby reduced. The amount absorbed at sea level is possibly as much as 40 per cent. Every one is familiar with the fact that on a mountain, above a part of the atmosphere, sunlight is more intense than it is at lower levels. But at night the effects are reversed. The heat that the atmosphere has absorbed in the daytime is radiated in every direction, and hence some of it strikes the earth and warms it. Besides this, at night the earth radiates the heat it has received in the daytime. The atmosphere above reflects some of the radiated heat directly back to the earth. Another portion of it is absorbed and radiated in every direction, and consequently in part back to the earth. In short, the atmosphere acts as a sort of blanket, keeping out part of the heat in the daytime, and helping to retain at night that which has been received. Its action is analogous to that of a glass with which the gardener covers his hotbed. The results are that the variations in temperature between night and day are reduced, and the average temperature is raised.

(*b*) The unequal heating of the earth's atmosphere in various latitudes is the primary cause of the winds. The warmer air moves toward the cooler regions, and the cold air of the higher latitudes returns toward the equator. The trade winds are examples of these movements. Their importance will be understood when it is remembered that wind velocities of 15 or 20 miles an hour are not uncommon, and that there is about 15 pounds of air above every square inch of the earth's surface.

One of the effects of the winds is the production of the ocean currents which are often said to be dominant factors in modifying climate, but which are, as a matter of fact, relatively unimportant consequences of the air currents. A south wind will often in the course of a few hours raise the temperature of the air over thousands of square miles of territory by 20 degrees, or even more. In order to raise the temperature of the atmosphere at constant pressure, over one square mile through 20 degrees by the combustion of coal it would be necessary to burn ten thousand tons. This illustration serves to give some sort of mental image of the great influence of air currents on climatic conditions, and if it were not for them, it is probable that both the equatorial and polar regions would be uninhabitable by man.

35. Importance of the Constitution of the Atmosphere.—The blanketing effect of the atmosphere depends to a considerable extent on its constitution. Every one is familiar with the fact that the early autumn frosts occur only when the air is clear and has low humidity. The reason is that water vapor is less transparent to the earth's radiations than are nitrogen and oxygen gas. On the other hand, there is not so much difference in their absorption of the rays that come from the sun. The reason is that the very hot sun's rays are largely of short wave length (Art. 211); that is, they are to a considerable extent in the blue end of the spectrum, while the radiation from the cooler earth is almost entirely composed of the much longer heat rays. Ordinary glass has the same property, for it transmits the sun's rays almost perfectly, while it is a pretty good screen for the rays emitted by a stove or radiator.

The water-vapor content of the atmosphere varies and cannot surpass a certain amount. But carbon dioxide has the same absorbing properties as water vapor, and in spite of the fact that it makes up only a very small part of the earth's atmosphere, Arrhenius believes that it has important climatic effects. He concluded that if the quantity of it in the air were doubled the climate would be appreciably warmer, and that if half of it were removed the average temperature of the earth would fall. Chamberlin has shown that there are reasons for believing that the amount of carbon dioxide has varied in long oscillations, and he suggested that this may be the explanation of the ice ages, with intervening warm epochs, which the middle latitudes have experienced.

If the effect of carbon dioxide on the climate has been correctly estimated, its production by the recent enormous consumption of coal raises the interesting question whether man at last is not in this way seriously interfering with the cosmic processes. At the present time about $1,000,000,000$ tons of coal are mined and burned annually. In order to burn 12 pounds of coal 32 pounds of oxygen are required, and the result of the combustion is $12 + 32 = 44$ pounds of carbon dioxide. Consequently, by the combustion of coal there is now annually produced by man about $3,670,000,000$ tons of carbon dioxide. On referring to the total amount of carbon dioxide now in the air (Art. 28), it is seen that at the present rate of combustion of coal it will be doubled in 800 years. Consequently, there are grounds for believing that modern industry may have sensible climatic effects in a few centuries.

36. Rôle of the Atmosphere in Life Processes.—Oxygen is an indispensable element in the atmosphere for all higher forms of animal life. It is taken into the blood stream through the lungs and is used in the tissues. Its proportion in the atmosphere is probably not very important, for it seems probable that if it had always been much more or much less, animals would have become adapted to the different condition. But if the earth's crust had contained enough material which readily unites with oxygen, such as hydrogen, silicon, or iron, to have exhausted the supply, it seems certain that animals with warm, red blood could not have developed. Such considerations are of high importance in speculating on the question of the habit-

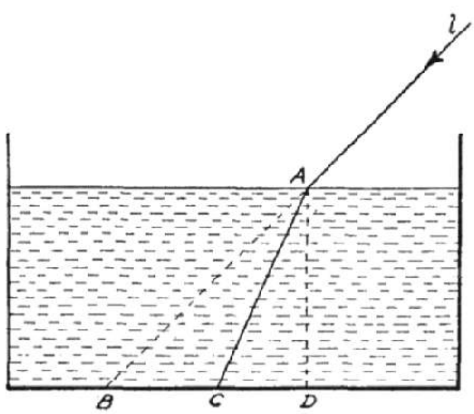


FIG. 21.—The refraction of light.

ability of other planets.

The higher forms of vegetable matter are largely composed of carbon and water. The carbon is obtained from the carbon dioxide in the atmosphere. The carbon and oxygen are separated in the cells of the plants, the carbon is retained, and the oxygen is given back to the air.

37. Refraction of Light by the Atmosphere.—When light passes from a rarer to a denser medium it is bent toward the perpendicular to the surface between the two media, and in general the greater the difference in the densities of the two media, the greater is the bending, which is called refraction. Thus, in Fig. 21, the ray l which strikes the surface of the denser medium at A is bent from the direction AB toward the perpendicular to the surface AD and takes the direction AC.

Now consider a ray of light striking the earth's atmosphere obliquely. The density of the air increases from its outer borders to the surface of the earth. Consequently, a ray of light is bent more and more as it proceeds down through the air. Let l, Fig. 22, represent a ray of light coming from a star S to an observer at O. The star is really in the direction OS'', but it appears to be in the direction OS' from which the light comes when it strikes the observer's eye. The angle between OS'' and OS' is the angle of refraction. It is zero for a star at the zenith and increases to a little over one-half of a degree for one at the horizon. For this reason a celestial body apparently rises before it is actually above the horizon, and is visible until after it has really set. If the sun or moon is on the horizon, its bottom part is apparently raised more than its top part by refraction, so that it seems to be flattened in the vertical direction, as is shown in Fig. 23.

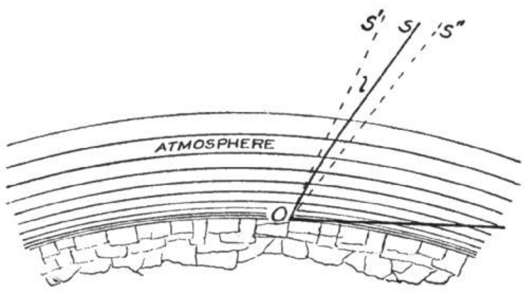


FIG. 22.—Refraction of light by the earth's atmosphere.

38. The Twinkling of the Stars.—The atmosphere is not only of variable density from its highest regions to the surface of the earth, but it is always disturbed by waves which cause the density at a given point to vary continually. These variations in density cause constant small changes in the refraction of light, and consequently alterations in the direction from which the light appears to come. When the source is a point of light, as a star, it twinkles or scintillates. The twinkling of the

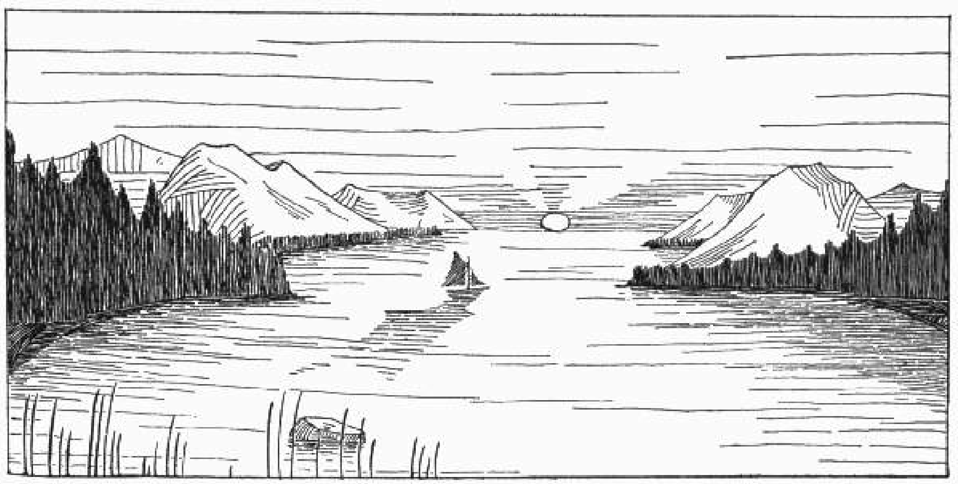


FIG. 23.—The sun is apparently flattened by refraction when it is on the horizon.

stars is particularly noticeable in winter time on nights when the air is cold and unsteady. The variation in refraction is different for different colors, and consequently when a star twinkles it flashes sometimes blue or green and at other times red or yellow. Objects that have disks, even though they are too small to be discerned with the unaided eye, appear much steadier than stars because the irregular refractions from various parts seldom agree in direction, and consequently do not displace the whole object.

IV. QUESTIONS

1. What is the weight of the air in a room 16 feet square and 10 feet high?

2. How many pounds of air pass per minute through a windmill 12 feet in diameter in a breeze of 20 miles per hour?

3. Compute the approximate total atmospheric pressure to which a person is subject.

4. What is the density of the air, compared to its density at the surface, at heights of 50, 100, and 500 miles, the density being determined by the law given at the end of Art. 32? This gives an idea of the density required for the phenomena of twilight, of meteors, and of auroræ.

5. Draw a diagram showing the earth and its atmosphere to scale.

6. The earth's mass is slowly growing by the acquisition of meteors; if there is nothing to offset this growth, will its atmosphere have a tendency to increase or to decrease in amount?

7. If the earth's atmosphere increases or decreases, as the case may be, what will be the effect on the mean temperature, the daily range at any place, and the range over the earth's whole surface?

8. If the earth's surface were devoid of water, what would be the effect on the mean temperature, the daily range at any place, and the range over its whole surface?

CHAPTER III
THE MOTIONS OF THE EARTH

I. The Rotation of the Earth

39. The Relative Rotation of the Earth.—The most casual observer of the heavens has noticed that not only the sun and moon, but also the stars, rise in the east, pass across the sky, and set in the west. At least this is true of those stars which cross the meridian south of the zenith. Figure 24 is a photograph of Orion in which the telescope was kept fixed while the stars passed in front of it, and the horizontal streaks are the images traced out by the stars on the photographic plate.

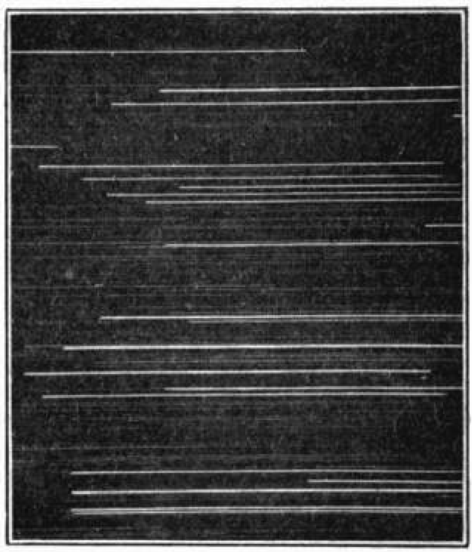


Fig. 24.—Star trails of brighter stars in Orion (Barnard).

The stars in the northern heavens describe circles around the north pole of the sky as a center. Two hours of observation of the position of the Big Dipper will show the character of the motion very clearly. Figure 25 shows circumpolar star trails secured by pointing a fixed telescope toward the pole star and giving an exposure of a little over an hour. The conspicuous streak a little below and to the left of the center is the trail of the pole star, which therefore is not exactly at the pole of the heavens. A comparison of this picture with the northern sky will show that most of the stars whose trails are seen are quite invisible to the unaided eye.

Since all the heavenly bodies rise in the east (except those so near the pole that they simply go around it), travel across the sky, and set in the west, to reappear again in the east, it follows that either they go around the earth from east to west, or the earth turns from west to east. So far as the simple motions of the sun, moon, and stars are concerned both hypotheses are in perfect harmony with the observations, and it is not possible to decide which of them is correct without additional data. All the apparent motions prove is that there is a relative motion of the earth with respect to the heavenly bodies.

It is often supposed that the ancients were unscientific, if not stupid, because they believed that the earth was fixed and that the sky went around it, but it has been seen that so far as their data bore on the question one theory was as good as the other. In fact, not all of them thought that the earth was fixed. The earliest philosopher who is known to have believed in the rotation of the earth was Philolaus, a Pythagorean, who lived in the fifth century B.C. His ideas were more or less mystical, but they seem to have had some influence, for they were quoted by Copernicus (1473–1543) in his great work on the theory of the motions in the solar system. Aristotle (384–322 B.C.) recognized the fact that the apparent motions of the stars can be explained either by their revolution around the earth, or by the rotation of the earth on its axis. Aristarchus of Samos (310–250 B.C.) made the clearest statements regarding both the rotation and the revolution of the earth of any philosopher of antiquity. But Hipparchus (180–110 B.C.), who was the greatest astronomer of antiquity, and whose discoveries were very numerous and valuable, believed in the fixity of the earth. He was followed in this opinion by Ptolemy (100–170 A.D.) and every other astronomer of note down to Copernicus, who believed the earth rotated and revolved around the sun.

FIG. 25. — Circumpolar star trails (Ritchey).

40. The Laws of Motion.—One method of attacking the question of whether or not any particular body, such as the earth, moves is to consider the laws of motion of bodies in general, and then to answer it on the basis of, and in harmony with, these laws. The laws of nature are in a fundamental respect different from civil laws, and it is unfortunate that the same term is used for both of them. A civil law prescribes or forbids a mode of conduct, with penalties if it is violated. It can be violated at pleasure if one is willing to run the chance of suffering the penalty. On the other hand, a law of nature does not prescribe or compel anything, but is a description of the way all phenomena of a certain class succeed one another.

The laws of motion are statements of the way bodies actually move. They were first given by Newton in 1686, although they were to some extent understood by his predecessor Galileo. Newton called them *axioms* although they are by no means self-evident, as is proved by the fact that for thousands of years they were quite unknown. The laws, essentially as Newton gave them, are:

LAW I. *Every body continues in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by an exterior force acting upon it.*

LAW II. *The rate of change of motion of a body is directly proportional to the force applied to it and inversely proportional to its mass, and the change of motion takes place in the direction of the line in which the force acts.*

LAW III. *To every action there is an equal and oppositely directed reaction; or, the mutual actions of two bodies are always equal and oppositely directed.*

The importance of the laws of motion can be seen from the fact that every astronomical and terrestrial phenomenon involving the motion of matter is interpreted by using them as a basis. They are, for example, the foundation of all mechanics. A little reflection will lead to the conclusion that there are few, if indeed any, phenomena that do not in some way, directly or indirectly, depend upon the motion of matter.

The first law states the important fact that if a body is at rest it will never begin to move unless some force acts upon it, and that if it is in motion it will forever move with uniform speed in a straight line unless some exterior force acts upon it. In two respects this law is contradictory to the ideas generally maintained before the time of Newton. In the first place, it had been supposed that bodies near the earth's surface would descend, because it was natural for them to do so, even though no forces were acting upon them. In the second place, it had been supposed that a moving body would stop unless some force were continually applied to keep it going. These errors kept the predecessors of Newton from getting any satisfactory theories regarding the motions of the heavenly bodies.

The second law defines how the change of motion of a body, in both direction and amount, depends upon the applied force. It asserts what happens when any force is acting, and this means that the statement is true whether or not there are other forces. In other words, the momentary effects of forces can be considered independently of one another. For example, if two forces, PA and PB in Fig. 26, are acting on a body at P, it will move in the direction PA just as though PB were not acting on it, and it will move in the direction PB just as though PA were not acting on it. The result is that when they are both acting it will go from P to C along PC. Since $PACB$ is a parallelogram, this is called the parallelogram law of the composition of forces.

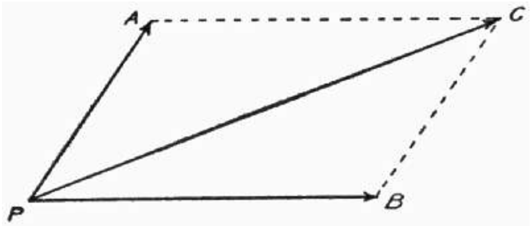


FIG. 26.—The parallelogram of forces.

The first two laws refer to the motion of a single body; the third expresses the way in which two bodies act on each other. It means essentially that if one body changes the state of motion of another body, its own state of motion is also changed reciprocally in a definite way. The term "action" in the law means the mass times the rate of change of motion (acceleration) of the body. Hence the third law might read that if two bodies act on each other, then the product of the mass and acceleration in one is equal and opposite to the product of the mass and acceleration in the other. This is a complete statement of the way two bodies act upon each other. But the second law states that the product of the mass and acceleration of a body is proportional to the force acting on it. Hence it follows that the third law might read that if two bodies act on each other, then the force exerted by the first on the second is equal and opposite to the force exerted by the second on the first. This statement is not obviously true because it seems to contradict ordinary experience. For example, the law states that if a strong man and a weak man are pulling on a rope (weight of the rope being neglected) against each other, the strong man cannot pull any more than the weak man. The reason is, of course, that the weak man does not give the strong one an opportunity to use his full strength. If the strong man is heavier than the weak one and pulls enough, he will move the latter while he remains in his tracks. This seems to contradict the statement of the law in terms of the acceleration; but the contradiction disappears when it is remembered that the men are subject not only to the forces they exert on each other, but also to their friction with the earth. If they were in canoes in open water, they would both move, and, if the weights of the canoes were included, their motions would be in harmony with the third law.

Since the laws of motion are to be used fundamentally in considering the motion of the earth, the question of their truth at once arises. When they are applied to the motions of the heavenly bodies, everything becomes orderly. Besides this, they have been illustrated millions of times in ordinary experience on the earth and they have been tested in laboratories, but nothing has been found to indicate they are not in harmony with the actual motions of material bodies. In fact, they are now supported by such an enormous mass of experience that they are among the most trustworthy conclusions men have reached.

41. Rotation of the Earth Proved by Its Shape.—The shape of the earth can be determined without knowing whether or not it rotates. The simple measurements of arcs (Art. 12) prove that the earth is oblate.

It can be shown that it follows from the laws of motion and the law of gravitation that the earth would be spherical if it were not rotating. Since it is not spherical, it must be rotating. Moreover, it follows from the laws of motion that if it is rotating it will be bulged at the equator. Hence the oblateness of the earth proves that it rotates and determines the position of its axis, but does not determine in which direction it turns.

42. Rotation of the Earth Proved by the Eastward Deviation of Falling Bodies.—Let OP, Fig. 27, represent a tower from whose top a ball is dropped. Suppose that while the ball is falling to the foot of the tower the earth rotates through the angle QEQ'. The top of the tower is carried from P to P', and its foot from O to O'. The distance PP' is somewhat greater than the distance OO'. Now consider the falling body. It tends to move in the direction PP' in accordance with the first law of motion because, at the time it is dropped, it is carried in this direction by the rotation of the earth. Moreover, PP' is the distance through which it would be carried if it were not dropped.

But the earth's attraction causes it to descend, and the force acts at *right angles* to the line PP'. Therefore, by the second law of motion, the attraction of the earth does not have any influence on the motion in the direction PP'. Consequently, while it is descending it moves in a horizontal direction a distance equal to PP' and strikes the surface at O'' to the east of the foot of the tower O'. The eastward deviation is the distance $O'O''$. The small diagram at the right shows the tower and the path of the falling body on a larger scale.

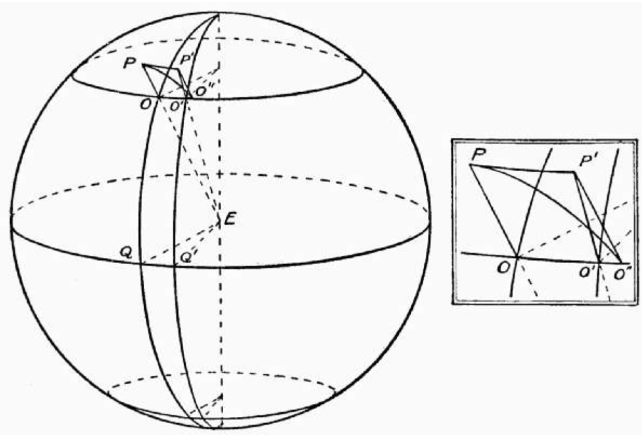


FIG. 27. — The eastward deviation of falling bodies proves the eastward rotation of the earth.

The foregoing reasoning has been made on the assumption that the earth rotates to the eastward. The question arises whether the conclusions are in harmony with experience. The experiment for determining the deviation of falling bodies is complicated by air currents and the resistance of the air. Furthermore, the eastward deviation is very small, being only 1.2 inches for a drop of 500 feet in latitude 40°. In spite of these difficulties, the experiment for moderate heights proves that the earth rotates to the eastward. Father Hagen, of Rome, has devised an apparatus, having analogies with Atwood's machine in physics, which avoids most of the disturbances to which a freely falling body is subject. The largest free fall so far tried was in a vertical mine shaft, near Houghton, Mich., more than 4000 feet deep. In spite of the fact that the diameter of the mine shaft was many times the deviation for that distance, the experiment utterly failed because the balls which were dropped never reached the bottom. It is probable that when they had fallen far enough to acquire high speed the air packed up in front of them until they were suddenly deflected far enough from their course to hit the walls and become imbedded.

43. Rotation of the Earth Proved by Foucault's Pendulum.— One of the most ingenious and convincing experiments for proving the rotation of the earth was devised in 1851 by the French physicist Foucault. It depends upon the fact that according to the laws of motion a freely swinging pendulum tends constantly to move in the same plane.

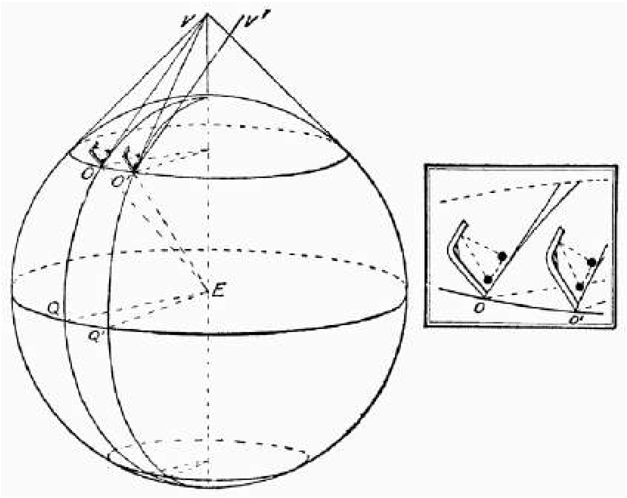


FIG. 28.—The Foucault pendulum.

Suppose a pendulum suspended at O, Fig. 28, is started swinging in the meridian OQ. Let OV be the tangent at O drawn in the plane of the meridian. After a certain interval the meridian OQ will have rotated to the position $O'Q'$. The line $O'V'$ is drawn parallel to the line OV. Consequently the pendulum will be swinging in the plane $EO'V'$. The tangent to the meridian at O' is $O'V$. Consequently, the angle between this line and the plane in which the pendulum will be swinging is $V'O'V$, which equals OVO'. That is, the angle at V between the meridian tangents equals the apparent deviation of the plane of the pendulum from the meridian. For points in the northern hemisphere the deviation is from a north-and-south direction toward a northeast-and-southwest direction. The angle around the cone at V equals the total deviation in one rotation of the earth. If O is at the earth's pole, the daily deviation is 360 degrees. If O is on the earth's equator, the point V is infinitely far away and the deviation is zero.

Foucault suspended a heavy iron ball by a steel wire about 200 feet long, and the deviation became evident in a few minutes. The experiment is very simple and has been repeated in many places. It proves that the earth rotates eastward, and the rate of deviation of the pendulum proves that the relative motion of the earth with respect to the stars is due entirely to its rotation and not at all to the motions of the stars around it.

44. Consequences of the Earth's Rotation.—An important consequence of the earth's rotation is the direction of air currents at considerable distances from the equator in both northern and southern latitudes. Suppose the unequal heating of the atmosphere causes a certain portion of it to move northward from O, Fig. 29, with such a velocity that if the earth were not rotating, it would arrive at A in a certain interval of time. Suppose that in this interval of time the meridian OQ rotates to the position $O'Q'$. Hence the mass of air under consideration actually had the velocities OA and OO' when it started from O, the former with respect to the surface of the earth and the latter because of the rotation of the earth. By the laws of motion these motions, being at right angles to each other, are mutually independent, and the air will move over both distances during the interval of time and arrive at the point A'', which is east of A'. Consequently, the mass of air that started straight northward with respect to the surface of the earth along the meridian OA will have deviated eastward by the amount $A'A''$.

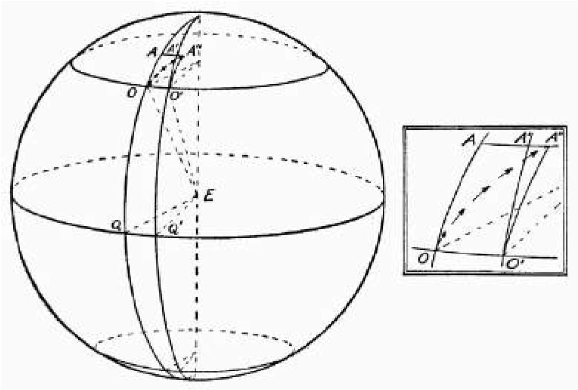


FIG. 29.—The deviation of air currents.

The deviation for northward motion in the northern hemisphere is toward the east; for southward motion, it is toward the west. In both cases it is toward the right. For similar reasons, in the southern hemisphere the deviation is toward the left.

The deviations in the directions of air currents are evidently greater the higher the latitude, because near the poles a given distance along the earth's surface corresponds to an almost equal change in the distance from the axis of rotation, while at the equator there is no change in the distance from the earth's axis. It might be supposed that in middle latitudes a moderate northward or southward displacement of the air would cause no appreciable change in its direction of motion. But a point on the equator moves eastward at the rate of over 1000 miles an hour, at latitude 60 degrees the eastward velocity is half as great, and at the pole it is zero. If it were not for friction with the earth's surface, a mass of air moving from latitude 40 degrees to latitude 45 degrees, a distance less than 350 miles, would acquire an eastward velocity with respect to the surface of the earth of over 40 miles an hour. The prevailing winds of the northern hemisphere in middle latitudes are to the northeast, and the eastward component has been found to be strong for the very high currents.

Obviously the same principles apply to water currents and to air currents. Consequently water currents, such as rivers, tend to deviate toward the right in the northern hemisphere. It has been found by examining the Mississippi and Yukon rivers that the former to some extent, and the latter to a much greater extent, on the whole scour their right-hand banks.

All the proofs of the earth's rotation so far given depend upon the laws of motion. There is one independent reason for believing the earth rotates, though it falls a little short of proof. It has been found by observations involving only geometrical principles that the sun, moon, and planets are comparable to the earth in size, some being larger and others smaller. Direct observations with the telescope show that a number of these bodies rotate on their axes, the remainder being either very remote or otherwise unfavorably situated for observation. The conclusion by analogy is that the earth also rotates.

45. The Uniformity of the Earth's Rotation.—It follows from the laws of motion, and in particular from the first law, that if the earth were subject to no external forces and were invariable in size, shape, and distribution of mass, it would rotate on its axis with absolute uniformity. Since the earth is a fundamental means of measuring time its rotation cannot be tested by clocks. Its rotation might be compared with other celestial phenomena, but then the question of their uniformity would arise. The only recourse is to make an examination of the possible forces and changes in the earth which are capable of altering the rate of its rotation.

The earth is subject to the attractions of the sun, moon, and planets. But these attractions do not change its rate of rotation because the forces pulling on opposite sides balance, just as the earth's attraction for a rotating wheel whose plane is vertical neither retards nor accelerates its motion.

The earth is struck by millions of small meteors daily coming in from all sides. They virtually act as a resisting medium and slightly retard its rotation, just as a top spinning in the air is retarded by the molecules impinging on it. But the mass of the earth is so large and the meteors are so small that, at their present rate of infall, the length of the day cannot be changed by this cause so much as a second in 100,000,000 years.

The moon and the sun generate tides in the water around the earth and the waves beat in upon the shores and are gradually destroyed by friction. The energy of the waves is transformed into heat. This means that something else has lost energy, and a mathematical treatment of the subject shows that the earth has suffered the loss. Consequently its rotation is diminished. But as great and irresistible as the tides may be, their energies are insignificant compared to that of the rotating earth, and according to the work of MacMillan the day is not increasing in length from this cause more than one second in 500,000 years.

Before discussing the effects of a change in the size of the earth or in the distribution of its mass, it is necessary to explain a very important property of the motion of rotating bodies. It can be shown from the laws of motion that if a body is not subject to any exterior forces, its total quantity of rotation always remains the same no matter what changes may take place in the body itself. The quantity of rotation of a body, or *moment of momentum*, as it is technically called in mechanics, is the sum of the rotations of all its parts. The rotation of a single part, or particle, is the product of its mass, its distance from the axis of rotation passing through the center of gravity of the body, and the speed with which it is moving at right angles to the line joining it to the axis of rotation. It can be shown that in the case of a body rotating as a solid, the quantity of rotation is proportional to the product of the square of the radius and the angular velocity of rotation, the angular velocity of rotation being the angle through which the body turns in a unit of time.

Now apply this principle of the conservation of the moment of momentum to the earth. If it should lose heat and shrink so that its radius were diminished in length, then the angular velocity of rotation would increase, for the product of the square of the radius and the rate of rotation must be constant. On the other hand, if the radioactive substances in the earth should cause its temperature to rise and its radius to expand, then the rate of rotation would decrease. Neither of these causes can make a sensible change in the rotation in 1,000,000 years. Similarly, if a river rising in low latitudes should carry sediment to higher latitudes and deposit it nearer the earth's axis, then the rate of rotation of the earth would be increased. While such factors are theoretically effective in producing changes in the rotation of the earth, from a practical point of view they are altogether negligible.

It follows from this discussion that there are some influences tending to decrease the rate of the earth's rotation, and others tending to increase it, but that they are all so small as to have altogether inappreciable effects even in a period as long as 100,000 years.

46. The Variation of Latitude.—It was mentioned in connection with the discussion of the rigidity of the earth (Arts. 25, 26), that its axis of rotation is not exactly fixed. This does not mean that the direction of the axis changes, but that the position of the earth itself changes so that its axis of rotation continually pierces different parts of its surface. That is, the poles of the earth are not fixed points on its surface. Since the earth's equator is 90 degrees from its poles, the position of the equator also continually changes. Therefore the latitude of any fixed point on the surface of the earth undergoes continual variation. The fact was discovered by very accurate determinations of latitude, and for this reason is known as the variation of latitude.

The pole wanders from its mean position not more than 30 feet, corresponding to a change of latitude of 0.3 of a second of arc. This is such a small quantity that it can be measured only by the most refined means, and accounts for the failure to discover it until the work of Chandler and Küstner about 1885.

In 1891 Chandler took up the problem of finding from the observations how the pole actually moves. The variation in its position is very complicated, Fig. 30 showing it from 1906 to 1913. Chandler found that this complicated motion is the result of two simpler ones. The first is a yearly motion in an ellipse (Art. 53) whose longest radius is 14 feet and shortest radius 4 feet; and the second is a motion in a circle of radius 15 feet, which is described in about 428 days. More recent discussions, based on observations secured by the coöperation of the astronomers of several countries, have modified these results to some extent and have added other minor terms.

The problem is to account for the variation of latitude and for the different periods. Unless a freely rotating oblate rigid body is started turning exactly around its shortest axis, it will undergo an oscillation with respect to its axis of rotation in a period which depends upon its figure, mass, and speed of rotation. Hence it might be supposed that the earth in some way originally started rotating in this manner. But since the earth is not perfectly rigid and unyielding, friction would in the course of time destroy the wabbling. In view of the fact that the earth is certainly many millions of years old, it seems that friction should long ago have reduced its rotation to sensible uniformity around a fixed

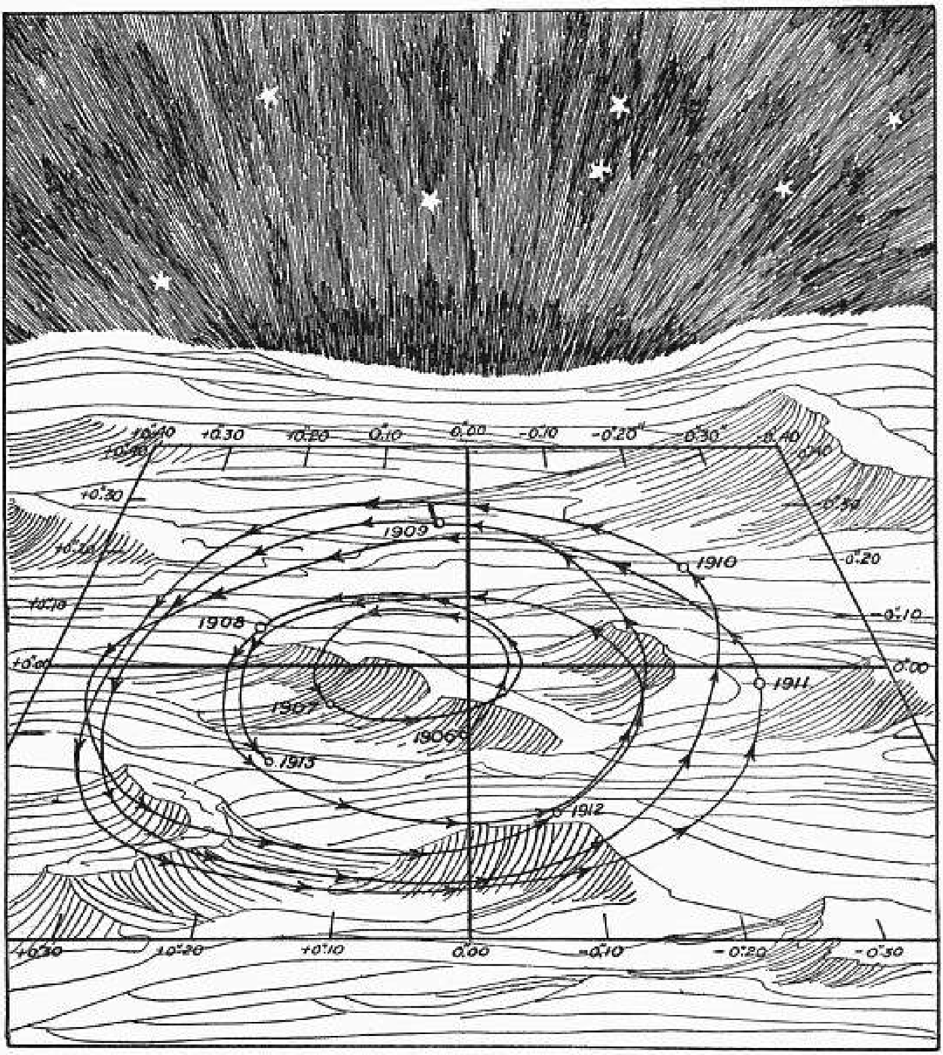


FIG. 30.—The position of the pole from 1906 to 1913.

axis, and this is true unless it is very elastic instead of being somewhat viscous. The tide experiment (Art. 25) proves that the earth is very elastic and suggests that perhaps the earth's present irregularities of rotation have been inherited from greater ones produced at the time of its origin, possibly by the falling together of scattered meteoric masses. But the fact that the earth has two different variations of latitude of almost equal magnitude is opposed to this conclusion. The one which has the period of a year is probably produced by meteorological causes, as Jeffreys infers from a quantitative examination of the question. The one whose period is 428 days, the natural period of variation of latitude for a body having the dynamical properties of the earth, is probably the consequence of the other. In order to understand their relations consider a pendulum which naturally oscillates in seconds. Suppose it starts from rest and is disturbed by a small periodic force whose period is two thirds of a second. Presently it will be moving, not like an undisturbed pendulum, but with one oscillation in two thirds of a second, and with another oscillation having an approximately equal magnitude, in its natural period, or one second.

Euler showed about 1770 that if the earth were absolutely rigid the natural period of oscillation of its pole would be 305 days. The increase of period to 428 days is due to the fact that the earth yields partially to disturbing forces (Art. 25).

Many parts of the earth have experienced wide variations in climate during geological ages, and it has often been suggested that these great changes in temperature were produced by the wandering of its poles. There are no known forces which could produce any greater variations in latitude than those which have been considered, and there is not the slightest probability that the earth's poles ever have been far from their present position on the surface of the earth.

47. Precession of the Equinoxes and Nutation.—There is one more phenomenon to be considered in connection with the rotation of the earth. In the variation of latitude the poles of the earth are slightly displaced on its surface; now the changes in the direction of its axis with respect to the stars are under consideration.

The axis of the earth can be changed in direction only by forces exterior to itself. The only important exterior forces to which the earth is subject are the attractions of the moon and sun. If the earth were a sphere, these bodies would have no effect upon its axis of rotation, but its oblateness gives rise to very important consequences.

Let O, Fig. 31, represent a point on the equator of the oblate earth, and suppose the moon M is in the plane of the meridian which passes through O. The point O is moving in the direction OA as a consequence of the earth's rotation. The attraction of the moon for a particle at O is in the direction OM. By the resolution of forces (the inverse of the parallelogram of forces law) the force along OM can be resolved in two others, one along OE and the other along the line OB perpendicular to OE. The former of these two forces has no effect on the rotation; the latter tends to move the particle in the direction OB, and this tendency,

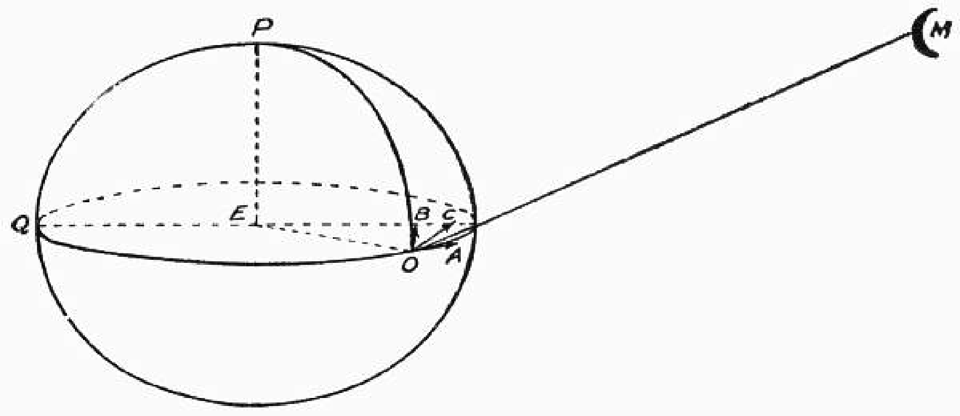


FIG. 31. — The attraction of the moon for the earth's equatorial bulge causes the precession of the equinoxes.

combined with the velocity OA, causes it to move in the direction OC (the change is greatly exaggerated). Therefore the direction of motion of O is changed; that is, the plane of the equator is changed.

The moon, however, attracts every particle in the equatorial bulge of the earth, and its effects vary with the position of the particles. It can be shown by a mathematical discussion that cannot be taken up here that the combined effect on the entire bulge is to change the plane of the equator. It is evident from Fig. 31 that the effect vanishes when the moon is in the plane of the earth's equator. Therefore it is natural to take the plane of the moon's orbit as a plane of reference. These two planes intersect in a certain line whose position changes as the plane of the earth's equator is shifted. The plane of the earth's equator shifts in such a way that the angle between it and the plane of the moon's orbit is constant, while the line of intersection of the two planes rotates in the direction opposite to that in which the earth turns on its axis.

The plane in which the sun moves is called the *plane of the ecliptic*, and the moon is always near this plane. For the moment neglect its departure from the plane of the ecliptic. Then the moon, and the sun similarly, cause the line of the intersection of the plane of the earth's equator and the plane of the ecliptic, called the *line of the equinoxes*, to rotate in the direction opposite to that of the rotation of the earth. This is the precession of the equinoxes, four fifths of which is due to the moon and one fifth of which is due to the sun. Since the axis of the earth is perpendicular to the plane of its equator, the point in the sky toward which the axis is directed describes a circle among the stars.

The mass of the earth is so great, the equatorial bulge is relatively so small, and the forces due to the moon and sun are so feeble that the precession is very slow, amounting only to 50.2 seconds of arc per year, from which it follows that the line of the equinoxes will make a complete rotation only after more than 25,800 years have passed.

The precession of the equinoxes was discovered by Hipparchus about 120 B.C. from a comparison of his observations with those made by earlier astronomers, but the cause of it was not known until it was explained by Newton, in 1686, in his Principia. The theoretical results obtained for the precession are in perfect harmony with the observations, and the weight of this statement will be appreciated when it is remembered that the calculations depend upon the size of the earth, its density, the distribution of mass in it, the laws of motion, the rate of rotation of the earth and its oblateness, the distances to the moon and sun, their apparent motions with respect to the earth, and the law of gravitation.

The moon does not move exactly in the plane of the ecliptic, but deviates from it as much as 5 degrees, and consequently the precession which it produces is not exactly with respect to the ecliptic. This circumstance would not be particularly important if it were not for the further fact that the plane of the moon's orbit has a sort of precession with respect to the ecliptic, completing a cycle in 18.6 years. This introduces a variation in the character of the precession which is periodic with the same period of 18.6 years. This variation in the precession, which at its maximum amounts to 9.2 seconds of arc, is called the *nutation.* It was discovered by the great English astronomer Bradley from observations made during the period from 1727 to 1747. The cause of it was first explained by D'Alembert, a famous French mathematician.

V. QUESTIONS

1. Which of the proofs of the rotation of the earth depend upon the laws of motion?

2. Give three practical illustrations (one a train moving around a curve) of the first law of motion.

3. Give three illustrations of the second law of motion.

4. Why is the kick in a heavy gun, for a given charge, less than in a light gun?

5. If a man fixed on the shore pulls a boat by a rope, do the interactions not violate the third law of motion?

6. For a body falling from a given height, in what latitude will the eastward deviation be the greatest?

7. For what latitude will the rotation of the Foucault pendulum be most rapid, and where would the experiment fail entirely?

8. In what latitude will the easterly (or westerly) deviation of wind or water currents be most pronounced?

9. Is it easier to stop a large or small wheel of the same mass rotating at the same rate?

10. If a wheel rotating without friction should diminish in size, would its rate of rotation be affected?

11. Are boundaries that are defined by latitudes affected by the wabbling of the earth's axis? By the precession of the equinoxes?

12. Would the precession be faster or slower if the earth were more oblate? If the moon were nearer? If the earth were denser?

II. The Revolution of the Earth

48. Relative Motion of the Earth with Respect to the Sun.— The diurnal motion of the sun is so obvious that the most careless observer fully understands it. But it is not so well known that the sun has an apparent eastward motion among the stars analogous to that of the moon, which every one has noticed. The reason that people are not so familiar with the apparent motion of the sun is that stars cannot be observed in its neighborhood without telescopic aid, and, besides, it moves slowly. However, the fact that it apparently moves can be established without the use of optical instruments; indeed, it was known in very ancient times. Suppose on a given date certain stars are seen directly south on the meridian at 8 o'clock at night. The sun is therefore 120° west of the star; or, what is equivalent, the stars in question are 120° east of the sun. A month later at 8 o'clock at night the observed stars will be found to be 30° west of the meridian. Since at that time in the evening the sun is 120° west of the meridian, the stars are $120° - 30° = 90°$ east of the sun. That is, during a month the sun apparently has moved 30° eastward with respect to the stars.

The question arises whether or not the sun's apparent motion eastward is produced by its actual motion around the earth. It will be shown that the hypothesis that it actually moves around the earth satisfies all the data so far mentioned. Suppose E, Fig. 32, represents the earth, assumed fixed, and S_1 the position of the sun at a certain time.

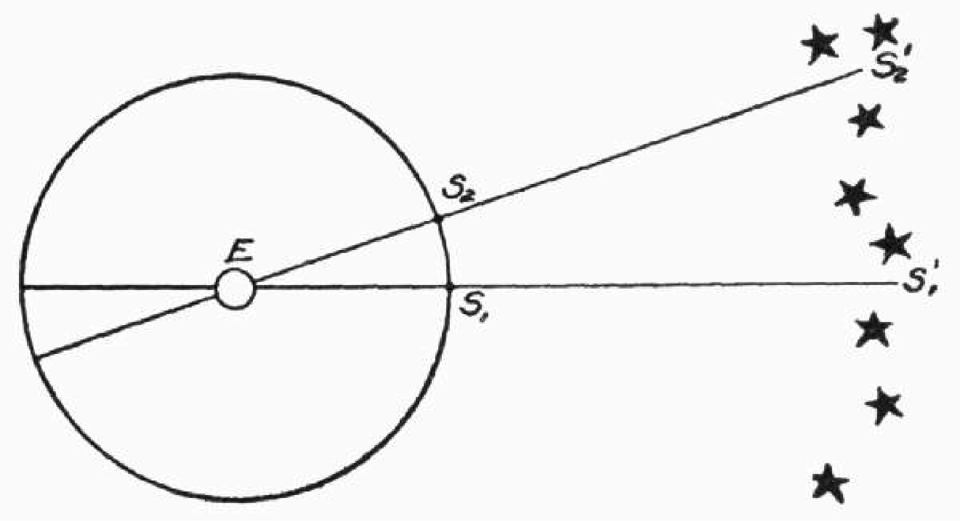


FIG. 32.—The hypothesis that the sun revolves around the earth explains the apparent eastward motion of the sun with respect to the stars.

As seen from the earth it will appear to be on the sky among the stars at S_1'. Suppose that at the end of 25 days the sun has moved forward in a path around the earth to the position S_2; it will then appear to be among the stars at S_2'. That is, it will appear to have moved eastward among the stars in perfect accordance with the observations of its apparent motion.

It will now be shown that the same observations can be satisfied completely by the hypothesis that the earth revolves around the sun. Let S, Fig. 33, represent the sun, assumed fixed, and suppose E_1 is the position of the earth at a certain time. The sun will appear to be among the stars at S_1'. Suppose that at the end of 25 days the earth has moved forward in a path around the sun to E_2; the sun will then appear to be among the stars at S_2'. That is, it will appear to have moved eastward among the stars in perfect accordance with the observations of its apparent motion. It is noted that the assumed actual motion of the earth is in the same direction as the sun's apparent motion; or, to explain the apparent motion of the sun by the motion of the earth, the earth must be supposed to move eastward in its orbit.

Since all the data satisfy two distinct and mutually contradictory hypotheses, new data must be employed in order to determine which of them is correct. The ancients had no facts by which they could disprove

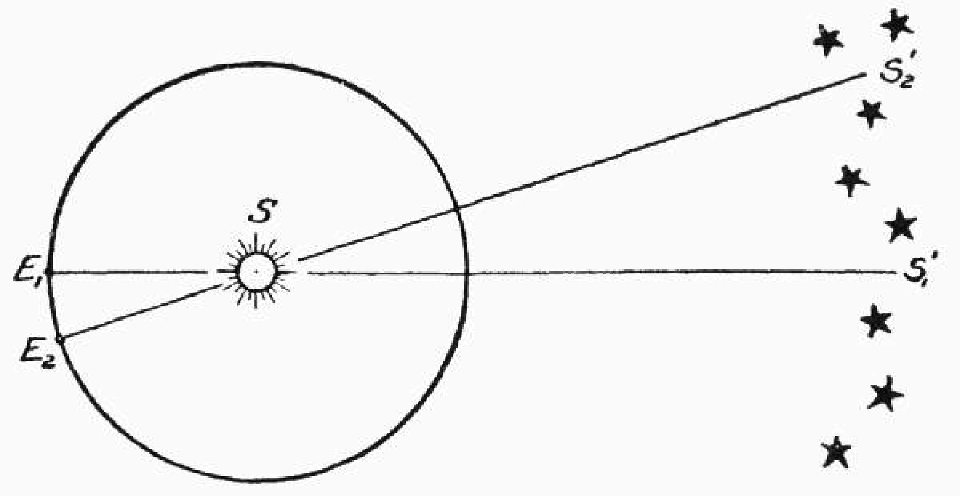


FIG. 33.—The hypothesis that the earth revolves around the sun explains the apparent eastward motion of the sun with respect to the stars.

one of these hypotheses and establish the truth of the other.

49. Revolution of the Earth Proved from the Laws of Motion.—The first actual proof that the earth revolves around the sun was based on the laws of motion in 1686, though the fact was generally believed by astronomers somewhat earlier (Art. 62). It must be confessed at once, however, that the statement requires a slight correction because the sun and earth actually revolve around the center of gravity of the two bodies, which is very near the center of the sun because of the sun's relatively enormous mass.

It can be shown by measurements that have no connection with the motion of the sun or earth that the volume of the sun is more than a million times that of the earth. Hence, unless it is extraordinarily rare, its mass is much greater than that of the earth. In view of the fact that it is opaque, the only sensible conclusion is that it has an appreciable density. Hence, in the motion of the earth and sun around their common center of gravity, the sun is nearly fixed while the earth moves in an enormous orbit.

50. Revolution of the Earth Proved by the Aberration of Light.—The second proof that the earth revolves was made in 1728 when Bradley discovered what is known as the *aberration of light.* This proof has the advantage of depending neither on an assumption regarding the density of the sun nor on the laws of motion.

Suppose rain falls vertically and that one stands still in it; then it appears to him that it comes straight down. Suppose he walks rapidly through it; then it appears to fall somewhat obliquely, striking him in the face. Suppose he rides through it rapidly; then it appears to descend more obliquely.

In order to get at the matter qualitatively suppose T_1, Fig. 34, is a tube at rest which is to be placed in such a position that drops of rain shall descend through it without striking the sides. Clearly it must be vertical. Suppose T_2 is a tube which is being carried to the right with moderate speed. It is evident that the tube must be tilted slightly in the direction of motion. Suppose the tube T_3 is being transported still more rapidly; it must be given a greater deviation from the vertical. The distance A_3C_3 is the distance the tube moves while the drop descends its length. Hence A_3C_3 is to B_3C_3 as the velocity of the tube is to the velocity of the drops. From the given velocity of the rain and the velocity of the tube at right angles to the direction of the rain, the angle of the deviation from the vertical, namely $A_3B_3C_3$, can be computed.

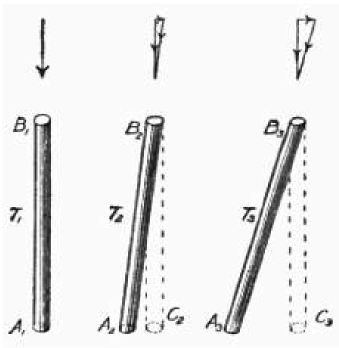


FIG. 34.—Explanation of the aberration of light.

Now suppose light from a distant star is considered instead of falling rain, and let the tube represent a telescope. All the relations will be qualitatively as in the preceding case because the velocity of light is not infinite. In fact, it has been found by experiments on the earth, which in no way depend upon astronomical observations or theory, that light travels in a vacuum at the rate of 186,330 miles per second. Hence, if the earth moves, stars should appear displaced in the direction of its motion, the amount of the displacement depending upon the velocity of the earth and the velocity of light. Bradley observed such displacements, at one time of the year in one direction and six months later, when the earth was on the other side of its orbit, in the opposite direction. The maximum displacement of a star for this reason is 20.47 seconds of arc which, at the present time, is very easy to observe because measurements of position are now accurate to one hundredth of this amount. Moreover, it is a quantity which does not depend on the brightness or the distance of the star, and it can be checked by observing as many stars as may be desired.

The aberration of light not only proves the revolution of the earth, but its amount enables the astronomer to compute the speed with which the earth moves. The result is accurate to within about one tenth of one per cent. Since the earth's period around the sun is known, this result gives the circumference of the earth's orbit, from which the distance from the earth to the sun can be computed. The distance of the sun as found in this way agrees very closely with that found by other methods.

There is, similarly, a small aberration due to the earth's rotation, which, for a point on the earth's equator, amounts at its maximum to 0.31 second of arc.

51. Revolution of the Earth Proved by the Parallax of the Stars.—The most direct method of testing whether or not the earth moves is to find whether the direction of a star is the same when observed at different times of the year. This was the first method tried, but for a long time it failed because the stars are exceedingly remote. Even with all the resources of modern instrumental equipment fewer than 100 stars are known which are so near that their differences

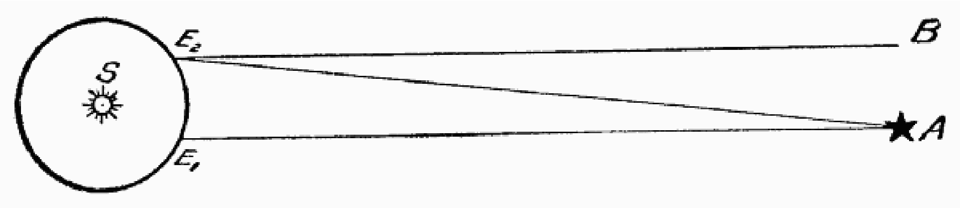


FIG. 35.—The parallax of A is the angle E_1AE_2.

in direction at different times of the year can be measured with any considerable accuracy. Yet the observations succeed in a considerable number of cases and really prove the motion of the earth by purely geometrical means.

The angular difference in direction of a star as seen from two points on the earth's orbit, which, in the direction perpendicular to the line to the star, are separated from each other by the distance from the earth to the sun, is the *parallax* of the star. In Fig. 35 let S represent the sun, A a star, and E_1 and E_2 two positions of the earth such that the line E_1E_2 is perpendicular to SA and such that E_1E_2 equals E_1S. Let E_2B be parallel to E_1A. Then, by definition, the angle AE_2B is the parallax of A. This angle equals E_1AE_2. Therefore an alternative definition of the parallax of a star is that it is the angle subtended by the radius of the earth's orbit as seen from the star.

It is obvious that the parallax is smaller the more remote the star. The nearest known star, Alpha Centauri, in the southern heavens, has a parallax of only 0.75 second of arc, from which it can be shown that its distance is 275,000 times as great as that from the earth to the sun, or about 25,600,000,000,000 miles. Suppose a point of light is seen first with one eye and then with the other. If its distance from the observer is about 11 miles, then its difference in direction as seen with the two eyes is 0.75 second of arc, the parallax of Alpha Centauri. This gives an idea of the difficulties that must be overcome in order to measure the distance of even the nearest star, especially when it is recalled that the observations must be extended over several months. The first success with this method was obtained by Henderson about 1840.

52. Revolution of the Earth Proved by the Spectroscope.—The spectroscope is an instrument of modern invention which, among other things, enables the astronomer to determine whether he and the source of light he may be examining are relatively approaching toward, or receding from, each other. Moreover, it enables him to measure the speed of relative approach or recession irrespective of their distance apart. (Art. 226.)

Consider the observation of a star A, Fig. 36, in the plane of the earth's orbit when the earth is at E_1, and again when it is at E_2. In the first position the earth is moving toward the star at the rate of 18.5 miles per second, and in the second position it is moving away from the star at the same rate. Since in the case of many stars the motion can be determined to within one tenth of a mile per second, the observational difficulties are not serious. If the star is not in the plane of the earth's orbit, a correction must be made in order to find what fraction of the earth's motion is toward or from the star. The method is independent of the distance of the star and can be applied to all stars which are bright enough except those whose directions from the sun are nearly perpendicular to the plane of the earth's orbit.

FIG. 36.—Motion of the earth toward and from a star.

Since 1890 the spectroscope has been so highly perfected that the spectroscopic proof of the earth's revolution has been made with thousands of stars. This method gives the earth's speed, and therefore the circumference of its orbit and its distance from the sun. It should be stated, however, that the motion of the earth was long ago so firmly established that it has not been considered necessary to use the spectroscope to give additional proof of it. Rather, it has been used to determine how the stars move individually (Art. 273) and how the sun moves with respect to them as a whole (Art. 274). In order to obtain the motion of a star with respect to the sun it is sufficient to observe it when the earth is at E, Fig. 36. Then correction for the earth's motion can be applied to the observations made when the earth is at E_1 or E_2.

53. Shape of the Earth's Orbit.—It has been tacitly assumed so far that the earth's orbit is a circle with the sun at the center. If this assumption were true, the apparent diameter of the sun would be the same all the year because the earth's distance from it would be constant. On the other hand, if the sun were not at the center of the circle, or if the orbit were not a circle, the apparent size of the sun would vary with changes in the earth's distance from it. It is clear that the shape of the earth's orbit can easily be established by observation of the apparent diameter and position of the sun.

It is found from the changes in the apparent diameter of the sun that the earth's orbit is not exactly a circle. These changes and the apparent motion of the sun together prove that the earth moves around it in an elliptical orbit which differs only a little from a circle. An ellipse is a plane curve such that the sum of the distances from two fixed points in its interior, known as *foci*, to any point on its circumference is always the same.

In Fig. 37, E represents an ellipse and F and F' its two foci. The definition of an ellipse suggests a convenient way of drawing one. Two pins are put in drawing paper at a convenient distance apart and a loop of thread somewhat longer than twice this distance is placed over them. Then a pencil P is placed inside the thread and the curve is drawn, keeping the thread taut. The curve obtained in this way is obviously an ellipse because the length of the thread is constant, and this means that the sum of the distances from

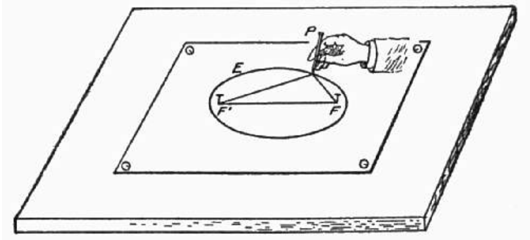


FIG. 37.—An ellipse.

F and F' to the pencil P is the same for all points of the curve.

54. Motion of the Earth in Its Orbit.—The earth moves in its orbit around the sun in such a way that the line drawn from the sun to the earth sweeps over, or describes, equal areas in equal intervals of time. Thus, in Fig. 38, if the three shaded areas are equal, the intervals of time required for the earth to move over the corresponding arcs of its orbit are also equal. This implies that the earth moves fastest when it is at P, the point nearest the sun, and slowest when it is at A, the point farthest from the sun. The former is called the *perihelion point*, and the latter the *aphelion point*.

It is obvious that an ellipse may be very nearly round or much elongated. The extent of the elongation is defined by what is known as the eccentricity, which is the ratio CS divided by CP. If the line CS is very short for a given line CP, the eccentricity is small and the ellipse is nearly circular. In fact, a circle may be considered as being an ellipse whose eccentricity is zero.

The eccentricity of the earth's orbit is very slight, being only 0.01677. That is, the distance CS, Fig. 38, in the case of the earth's orbit is about $\frac{1}{60}$ of CP. Hence, if the earth's orbit were drawn to scale, its elongation would be so slight that it would not be obvious by simple inspection.

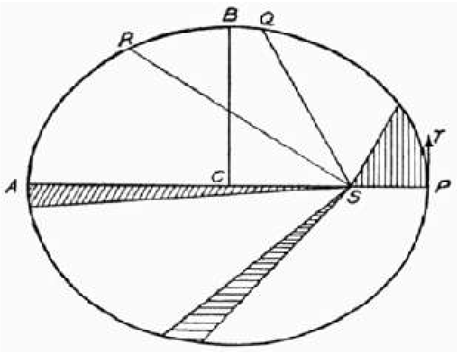


FIG. 38.—The earth moves so that the line from the sun to the earth sweeps over equal areas in equal intervals of time.

The question arises as to what occupies the second focus of the elliptical orbit of the earth. The answer is that there is no body there; nor is it absolutely fixed in position because the earth's orbit is continually modified to a very slight extent by the attractions of the other planets.

It is easy to see how the earth might revolve around the sun in a circle if it were started with the right velocity. But it is not so easy to understand how it can revolve in an elliptical orbit with the sun at one of the foci. While the matter cannot be fully explained without some rather formidable mathematical considerations, it can, at least, be made plausible by a little reflection. Suppose a body is at P, Fig. 38, and moving in the direction PT. If its speed is exactly such that its centrifugal acceleration balances the attraction of the sun, it will revolve around the sun in a circle.

But suppose the initial velocity is a little greater than that required for motion in a circular orbit. In this case the sun's attraction does not fully counterbalance the centrifugal acceleration, and the distance of the body from the sun increases. Consider the situation when the body has moved around in its orbit to the point Q. At this point the centrifugal acceleration is still greater than the attraction of the sun, and the distance of the body from the sun is increasing. It will be observed that the sun's attraction no longer acts at right angles to the direction of motion of the body, but that it tends to diminish its speed. It can be shown by a suitable mathematical discussion, which must be omitted here, that the diminution of the speed of the body more than offsets the decreasing attraction of the sun due to the increasing distance of the body, and that in elliptical orbits a time comes in which the attraction and the centrifugal acceleration balance. Suppose this takes place when the body is at R. Since its speed is still being diminished by the attraction of the sun from that point on, the attraction will more than counterbalance the centrifugal acceleration. Eventually at A the distance of the body from the sun will cease to increase. That is, it will again be moving at right angles to a line joining it to the sun; but its velocity will be so low that the sun will pull it inside of a circular orbit tangent at that point. It will then proceed back to the point P, its velocity increasing as it decreases in distance while going from P to A. The motion out from the sun and back again is analogous to that of a ball projected obliquely upward from the surface of the earth; its speed decreases to its highest point, and then increases again as it it descends.

55. Inclination of the Earth's Orbit.—The plane of the earth's orbit is called the *plane of the ecliptic*, and the line in which this plane intersects the sky is called the *ecliptic*. In Fig. 39 it is the circle $RAR'V$. The plane of the earth's equator cuts the sky in a circle which is called the *celestial equator*. In the figure it is $QAQ'V$. The angle between the plane of the equator and the plane of the ecliptic is 23.5 degrees. This angle is called the *inclination or obliquity of the ecliptic*.

The point on the sky pierced by a line drawn perpendicular to the plane of the ecliptic is called the *pole of the ecliptic*, and the point where the earth's axis, extended, pierces the sky is called the pole of the equator or, simply, *the celestial pole*. The orbit of the earth is so very small in comparison with the distance to the sky that the motion of the earth in its orbit has no sensible effects on the position of the

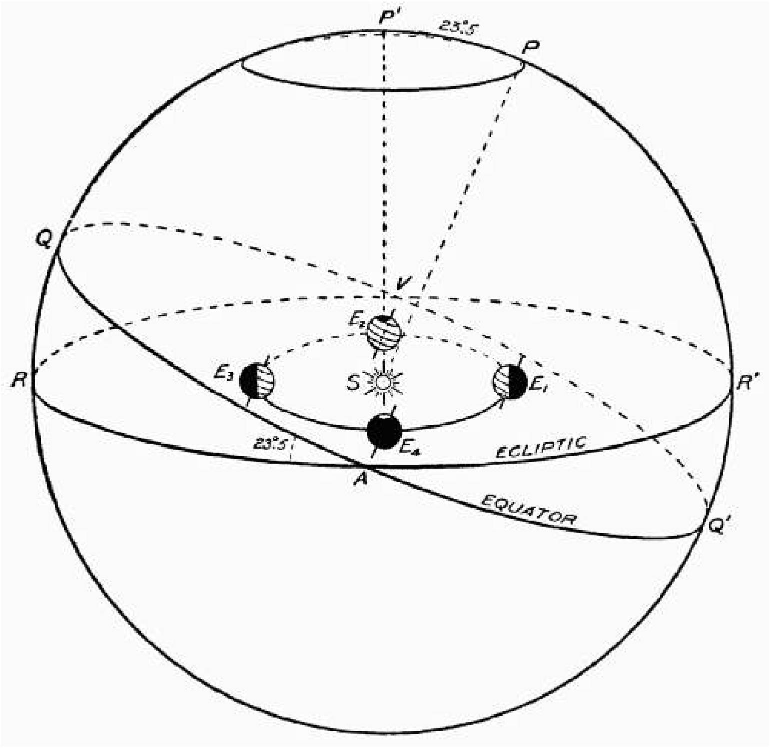


FIG. 39.—The ecliptic, celestial equator, and celestial pole.

celestial pole and it may be regarded as a fixed point. In Fig. 39, P' is the pole of the ecliptic and P is the pole of the equator. The angle between these lines is the same as the angle between the planes, or 23.5 degrees.

Now consider the precession of the equinoxes (Art. 47). The pole of the ecliptic remains fixed. As a consequence of the precession of the equinoxes the pole P describes a circle around it with a radius of 23.5 degrees, and the direction of the motion is opposite to that of the direction of the motion of the earth around the sun. Or, the points A and V, which are the equinoxes, continually move backward along the ecliptic in the direction opposite to that of the revolution of the earth.

56. Cause of the Seasons.—Let the upper part of the earth E, Fig. 39, represent its north pole. When the earth is at E_1 its north pole is turned away from the sun so that it is in continual darkness; but, on the other hand, the south pole is continually illuminated. At this time of the year the northern hemisphere has its winter and the southern hemisphere its summer. The conditions are reversed when the earth is at E_3. When the earth is at E_2 the plane of its equator passes through the sun, and it is the spring season in the northern hemisphere. Similarly, when the earth is at E_4 the equator also passes through the sun and it is autumn in the northern hemisphere.

Consider a point in a medium northern latitude when the earth is at E_1, and the same position again when the earth is at E_3. At E_1 the sun's rays, when it is on the meridian, strike the surface of the earth at the point in question more obliquely than when the earth is at E_3. Their intensity is, therefore, less in the former case than it is in the latter; for, in the former, the rays whose cross section is PQ, Fig. 40, are spread out over the distance AB, while in the latter they extend over the smaller distance $A'B$. This fact, and the variations in the number of hours of sunshine per day (Art. 58), cause the changes in the seasons.

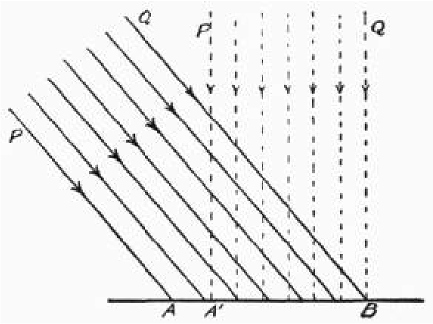


FIG. 40.—Effects of obliquity of sun's rays.

57. Relation of the Position of the Celestial Pole to the Latitude of the Observer.—In order to make clear the climatic effects of certain additional factors, consider the apparent position of the celestial pole as seen by an observer in any latitude. Since the pole is the place where the axis of the earth, extended, pierces the sky, it is obvious that, if an observer were at a pole of the earth, the celestial equator would be on his horizon and the celestial pole would be at his zenith; while, if he were on the equator of the earth, the celestial equator would pass through his zenith, and the celestial poles would be on his horizon, north and south.

Consider an observer at O, Fig. 41, in latitude l degrees north of the equator. The line $P'P$ points toward the north pole of the sky. Since the sky is extremely far away compared to the dimensions of the earth, the line from O to the celestial pole is essentially parallel to $P'P$. The angle between the plane of the horizon and the line to the pole is called the altitude of the pole. Since ON is perpendicular to EO, and $P'P$ is perpendicular to EQ, it follows that a equals l, or *the altitude of the pole equals the latitude of the observer.*

Consider also the altitude of the equator where it crosses the meridian directly south of the observer. It is represented by b in the diagram.

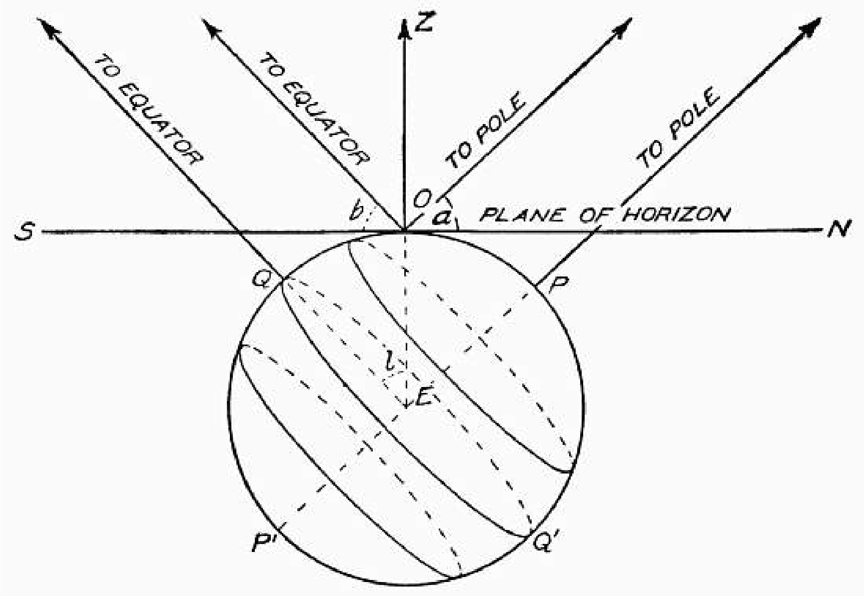


FIG. 41.—The altitude of the celestial pole equals the latitude of the observer.

It easily follows that $b = 90° - l$, or the altitude of the equator where it crosses the meridian equals 90° minus the latitude of the observer.

58. The Diurnal Circles of the Sun.—It is evident from Fig. 39 that when the earth is in the position E_1, the sun is seen south of the celestial equator; when the earth is at E_2 or E_4, the sun appears to be on the celestial equator; and when the earth is at E_3, the sun is seen north of the celestial equator. If the equator is taken as the line of reference and the apparent motion of the sun is considered, its position

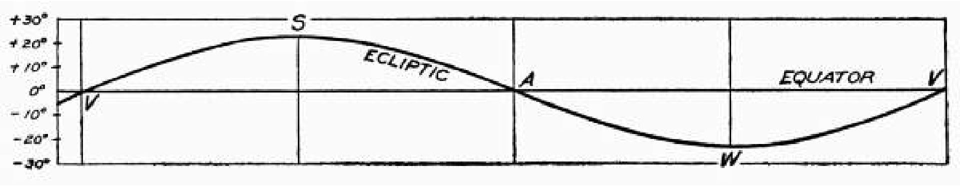


FIG. 42.—Relation of ecliptic and celestial equator.

with respect to the equator is represented in Fig. 42. The sun appears to be at V when the earth is at E_2, Fig. 39. The point V is called the *vernal equinox*, and the sun has this position on or within one day of March 21. The sun is at S, called the *summer solstice*, when the earth is at E_3, Fig. 39, and it is in this position about June 21. The sun is at A, called the *autumnal equinox*, when the earth is at E_4, and it has this position about September 23. Finally, the sun is at W, which is called the *winter solstice*, when the earth is at E_1. The angle between the ecliptic and the equator at V and A is 23°.5; and the perpendicular distance between the equator and the ecliptic at S and W is 23°.5. From these relations and those given in Art. 57 the diurnal paths of the sun can readily be constructed.

Suppose the observer is in north latitude 40°. Let O, Fig. 43, represent his position, and suppose his horizon is $SWNE$, where the letters stand for the four cardinal points. Then it follows from the relation of the altitude of the pole to the latitude of the observer that NP, where P represents the pole, is 40°. Likewise SQ, where Q represents the place at which the equator crosses the meridian, is 50°. The equator is everywhere 90 degrees from the pole and in the figure is represented by the circle $QWQ'E$.

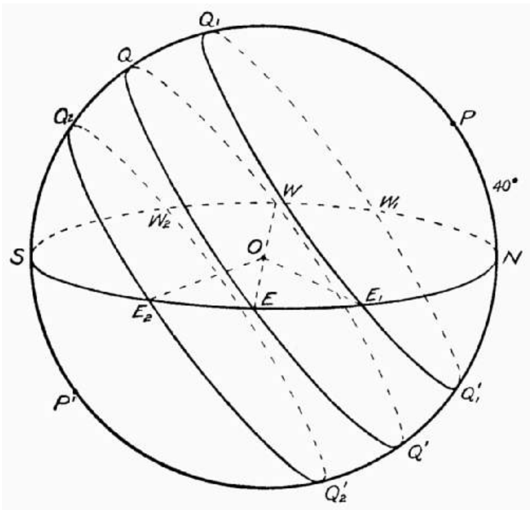


FIG. 43.—Diurnal circles of the sun.

Suppose the sun is on the equator at V or A, Fig. 42. Since it takes six months for it to move from V to A, its motion in one day is very small and may be neglected in the present discussion. Hence, without serious error, it may be supposed that the sun is on the equator all day. When this is the case, its apparent diurnal path, due to the rotation of the earth, is $EQWQ'$, Fig. 43. It will be noticed that it rises directly in the east and sets directly in the west, being exactly half the time above the horizon and half the time below it. This is true whatever the latitude of the observer. But the height at which it crosses the meridian depends, of course, upon the latitude of the observer, and is greater the nearer he is to the earth's equator.

Suppose now that it is June 21 and that the sun is at the summer solstice S, Fig. 42. It is then 23°.5 north of the equator and will have essentially this distance from the equator all day. The diurnal path of the sun in this case is $E_1Q_1W_1Q'_1$, Fig. 43, which is a circle parallel to, and 23°.5 north of, the equator. In this case the sun rises north of the east point by the angle EE_1, and sets an equal distance north of the west point. Moreover, it is more than half the twenty-four hours above the horizon. The fact that its altitude at noon is 23°.5 greater than it is when the sun is on the equator, and the longer time from sunrise to sunset, are the reasons that the temperature is higher in the summer than in the spring or autumn. It is obvious from Fig. 43 that the length of the day from sunrise to sunset depends upon the latitude of the observer, being greater the farther he is from the earth's equator.

When the sun is at the winter solstice W, Fig. 42, its diurnal path is $E_2Q_2W_2Q'_2$. At this time of the year it rises in the southeast, crosses the meridian at a low altitude, and sets in the southwest. The time during which it is above the horizon is less than that during which it is below the horizon, and the difference in the two intervals depends upon the latitude of the observer.

59. Hours of Sunlight in Different Latitudes.—It follows from Fig. 43 that when the sun is north of the celestial equator, an observer north of the earth's equator receives more than 12 hours of sunlight per day; and when the sun is south of the celestial equator, he receives less than 12 hours of sunlight per day. It might be suspected that the excess at one time exactly balances the deficiency at the other. This suspicion is strengthened by the obvious fact that, a point at the equator receives 12 hours of sunlight per day every day in the year, and at the pole the sun shines continuously for six months and is below the horizon for six months, giving the same total number of hours of sunshine in these two extreme positions on the earth. The conclusion is correct, for it can be shown that the total number of hours of sunshine in a year is the same at all places on the earth's surface. This does not, of course, mean that the same amount of sunshine is received at all places, because at positions near the poles the sun's rays always strike the surface very obliquely, while at positions near the equator, for at least part of the time they strike the surface perpendicularly. The intensity of sunlight at the earth's equator when the sun is at the zenith is 2.5 times its maximum intensity at the earth's poles; and the amount received per unit area on the equator in a whole year is about 2.5 times that received at the poles.

If the obliquity of the ecliptic were zero, the sun would pass every day through the zenith of an observer at the earth's equator; but actually, it passes through the zenith only twice a year. Consequently, the effect of the obliquity of the ecliptic is to diminish the amount of heat received on the earth's equator. Therefore some other places on the earth, which are obviously the poles, must receive a larger amount than they would if the equator and the ecliptic were coincident. That is, the obliquity of the ecliptic causes the climate to vary less in different latitudes than it would if the obliquity were zero.

60. Lag of the Seasons.—From the astronomical point of view March 21 and September 23, the times at which the sun passes the two equinoxes are corresponding seasons. The middle of the summer is when the sun is at the summer solstice, June 21, and the middle of the winter when it is at the winter solstice, December 21. But from the climatic standpoint March 21 and September 23 are not corresponding seasons, and June 21 and December 21 are not the middle of summer and winter respectively. The climatic seasons lag behind the astronomical.

The cause of the lag of the seasons is very simple. On June 21 any place on the earth's surface north of the Tropic of Cancer is receiving the largest amount of heat it gets at any time in the year. On account of the blanketing effect of the atmosphere, less heat is radiated than is received; hence the temperature continues to rise. But after that date less and less heat is received as day succeeds day; on the other hand, more is radiated daily, for the hotter a body gets, the faster it radiates. In a few weeks the loss equals, and then exceeds, that which is received, after which the temperature begins to fall. The same reasoning applies for all the other seasons. This phenomenon is quite analogous to the familiar fact that the maximum daily temperature normally occurs somewhat after noon.

If there were no atmosphere and if the earth radiated heat as fast as it was acquired, there would be no lag in the seasons. In high altitudes, where the air is thin and dry, this condition is nearly realized and the lag of the seasons is small, though the phenomenon is very much disturbed by the great air currents which do much to equalize temperatures.

61. The Effect of the Eccentricity of the Earth's Orbit on the Seasons.—It is found from observations of the apparent diameter of the sun that the earth is at its perihelion on or about January 3, and at its aphelion on or about July 4. It follows from the way the earth describes its orbit, as explained in Art. 54, that the time required for it to move from P to Q, Fig. 44, is exactly equal to that required for it to move from Q to P. But the line joining the vernal and autumnal equinoxes, which passes through the sun, is nearly at right angles to the line joining the perihelion and aphelion points, and is represented

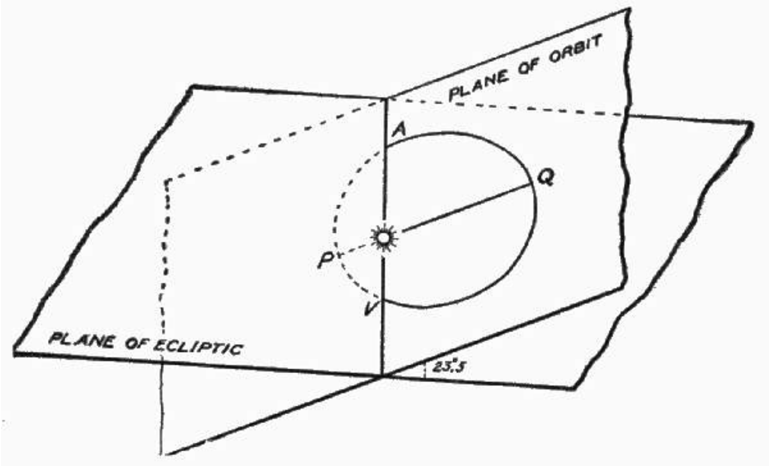


FIG. 44.—Because of the eccentricity of the earth's orbit, summers in the northern hemisphere are longer than the winters.

by VA, Fig. 44. Since the area swept over by the radius from the sun to the earth, while the earth is moving over the arc VQA, is greater than the area described while it goes over the arc APV, it follows that the interval of time in the former case is greater than that in the latter. That is, since V is the vernal equinox, the summer in the northern hemisphere is longer than the winter. The difference in length is greatly exaggerated in the figure, but it is found that the interval from vernal equinox to autumnal equinox is actually about 186.25 days, while that from autumnal equinox to vernal equinox is only 179 days. The difference is, therefore, about 7.25 days.

Since the summers are longer than the winters in the northern hemisphere while the reverse is true in the southern hemisphere, it might be supposed that points in corresponding latitudes receive more heat in the northern hemisphere than in the southern hemisphere. But it will be noticed from Fig. 44 that, although the summer is longer in the northern hemisphere than it is in the southern, the earth is then farther from the sun. It can be shown from a discussion of the way in which the earth's distance from the sun varies and from the rate at which it moves at different points in its orbit, that the longer summer season in the northern hemisphere is exactly counterbalanced by the greater distance the earth is then from the sun. The result is that points in corresponding latitudes north and south of the equator receive in the whole year exactly the same amount of light and heat from the sun.

There is, however, a difference in the seasons in the northern and southern hemispheres which depends upon the eccentricity of the earth's orbit. When the sun is north of the celestial equator so that its rays strike the surface in northern latitudes most nearly perpendicularly, a condition that tends to produce high temperatures, the greater distance of the sun reduces them somewhat. Therefore, the temperature does not rise in the summer so high as it would if the earth's orbit were circular. In the winter time, at the same place, when the sun's rays strike the surface slantingly, the earth is nearer to the sun than the average, and consequently the temperature does not fall so low as it would if the eccentricity of the earth's orbit were zero. The result is that the seasonal variations in the northern hemisphere are less extreme than they would be if the earth's orbit were circular; and, for the opposite reason, in the southern hemisphere they are more extreme. This does not mean that actually there are greater extremes in the temperature south of the equator than there are north of the equator. The larger proportion of water in the southern hemisphere, which tends to make temperature conditions uniform, may more than offset the effects of the eccentricity of the earth's orbit.

The attractions of the other planets for the earth change very slowly both the eccentricity and the direction of the perihelion of the earth's orbit. It has been shown by mathematical discussions of these influences that the relation of the perihelion to the line of the equinoxes will be reversed in about 50,000 years. In fact, there is a cyclical change in these relations with a period of somewhat more than 100,000 years. It was suggested by James Croll that the condition of long winter and short summer, such as now prevails in the southern hemisphere, especially when the eccentricity of the earth's orbit was greatest, produced the glaciation which large portions of the earth's surface are known to have experienced repeatedly in the past. This theory has now been abandoned because, on other grounds, it is extremely improbable.

62. Historical Sketch of the Motions of the Earth.—The history of the theory of the motion of the earth is intimately associated with that of the motions of the planets, and the whole problem of the relations of the members of the solar system to one another may well be considered together.

The planets are readily found by observations, even without telescopes, to be moving among the stars. Theories respecting the meanings of these motions date back to the very dawn of history. Many of the simpler phenomena of the sun, moon, and planets had been carefully observed by the Chaldeans and Egyptians, but it remained for the brilliant and imaginative Greeks to organize and generalize experience and to develop theories. Thales is credited with having introduced Egyptian astronomy into Greece more than 600 years before the Christian era. The Pythagoreans followed a century later and made important contributions to the philosophy of the science, but very few to its data. Their success was due to the weakness of their method; for, not being too much hampered by the facts of observation, they gave free rein to their imaginations and introduced numerous ideas into a budding science which, though often erroneous, later led to the truth. They believed that the earth was round, immovable, at the center of the universe, and that the heavenly bodies moved around it on crystalline spheres.

Following the Pythagoreans came Eudoxus (409–356 B.C.), Aristotle (384–322 B.C.), and Aristarchus (310–250 B.C.), who were much more scientific, in the modern sense of the term, and who made serious attempts to secure perfect agreement between the observations and theory. Aristarchus was the first to show that the apparent motions of the sun, moon, and stars could be explained by the theory that the earth rotates on its axis and revolves around the sun. Aristotle's objection was that if this theory were true the stars would appear to be in different directions at different times of the year; the reply of Aristarchus was that the stars were infinitely remote, a valid answer to a sensible criticism. Aristarchus was a member of the Alexandrian school, founded by Alexander the Great, and to which the geometer Euclid belonged. His astronomy had the formal perfection which would be natural in a school where geometry was so splendidly systematized that it has required almost no modification for 2000 years.

The rather formal astronomy which resulted from the influence of the mathematics of Alexandria was succeeded by an epoch in which the greatest care was taken to secure observations of the highest possible precision. Hipparchus (180–110 B.C.), who belonged to this period, is universally conceded to have been the greatest astronomer of antiquity. His observations in both extent and accuracy had never been approached before his time, nor were they again equaled until the time of the Arab, Albategnius (850–929 A.D.). He systematically and critically compared his observations with those of his predecessors. He developed trigonometry without which precise astronomical calculations cannot be made. He developed an ingenious scheme of eccentrics and epicycles (which will be explained presently) to represent the motions of the heavenly bodies.

Ptolemy (100–170 A.D.) was the first astronomer of note after Hipparchus, and the last important astronomer of the Alexandrian period. From his time until that of Copernicus (1473–1543) not a single important advance was made in the science of astronomy. From Pythagoras to Ptolemy was 700 years, from Ptolemy to Copernicus was 1400 years, and from Copernicus to the present time is 400 years. The work of Ptolemy, which is preserved in the *Almagest* (*i.e.* The Greatest Composition), was the crowning achievement of the second period, and that of Copernicus was the first of the modern period; or, perhaps it would be more accurate to say that the work of Copernicus constituted the transition from ancient to modern astronomy, which was really begun by Kepler (1571–1630) and Galileo (1564–1642).

The most elaborate theory of ancient times for explaining the motions of the heavenly bodies was due to Ptolemy. He supposed that the earth was a fixed sphere situated at the center of the universe. He supposed that the sun and moon moved around the earth in circles. It does not seem to have occurred to the ancients that the orbits of the heavenly bodies could be anything but circles, which were supposed to be perfect curves. In order to explain the varying distances of the sun and moon, which were proved by the variations in their apparent diameters, he supposed that the earth was somewhat out of the centers of the circles in which the various bodies were supposed to move around it. It is clear that such motion, called eccentric motion, would have considerable similarity to motion in an ellipse around a body at one of its foci.

Another device used by Ptolemy for the purpose of explaining the motions of the planets was the epicycle. In this system the body was supposed to travel with uniform speed along a small circle, the epicycle, whose center moved with uniform speed along a large circle, the deferent, around the earth. By carefully adjusting the dimensions and inclinations of the epicycle and the deferent, together with the rates of motion along them, Ptolemy succeeded in getting a very satisfactory theory for the motions of the sun, moon, and planets so far as they were then known.

Copernicus was not a great, or even a skillful, observer, but he devoted many years of his life to the study of the apparent motions of the heavenly bodies with a view to discovering their real motions. The invention of printing about 1450 had made accessible the writings of the Greek philosophers, and Copernicus gradually became convinced that the suggestion that the sun is the center, and that the earth both rotates on its axis and revolves around the sun, explains in the simplest possible way all the observed phenomena. It must be insisted that Copernicus had no rigorous proof that the earth revolved, but the great merit of his work consisted in the faithfulness and minute care with which he showed that the heliocentric theory would satisfy the observation as well as the geocentric theory, and that from the standpoint of common sense it was much more plausible.

The immediate successor of Copernicus was Tycho Brahe (1546–1610), who rejected the heliocentric theory both for theological reasons and because he could not observe any displacements of the stars due to the annual motion of the earth. He contributed nothing of value to the theory of astronomy, but he was an observer of tireless industry whose work had never been equaled in quality or quantity. For example, he determined the length of the year correctly to within one second of time.

Between the time of Tycho Brahe and that of Newton (1643–1727), who finally laid the whole foundation for mechanics and particularly the theory of motions of the planets, there lived two great astronomers, Galileo (1564–1642) and Kepler (1571–1630), who by work in quite different directions led to the complete overthrow of the Ptolemaic theory of eccentrics and epicycles. These two men had almost no characteristics in common. Galileo was clear, penetrating, brilliant; Kepler was mystical, slow, but endowed with unwearying industry. Galileo, whose active mind turned in many directions, invented the telescope and the pendulum clock, to some extent anticipated Newton in laying the foundation of dynamics, proved that light and heavy bodies fall at the same rate, covered the field of mathematical and physical science, and defended the heliocentric theory in a matchless manner in his *Dialogue on the Two Chief Systems of the World.* Kepler confined his attention to devising a theory to account for the apparent motions of sun and planets, especially as measured by his preceptor, Tycho Brahe. With an honesty and thoroughness that could not be surpassed, he tested one theory after another and found them unsatisfactory. Once he had reduced everything to harmony except some of the observations of Mars by Tycho Brahe (of course without a telescope), and there the discrepancy was below the limits of error of all observers except Tycho Brahe. Instead of ascribing the discrepancies to minute errors by Tycho Brahe, he had implicit faith in the absolute reliability of his master and passed on to the consideration of new theories. In his books he set forth the complete record of his successes and his failures with a childlike candor not found in any other writer. After nearly twenty years of computation he found the three laws of planetary motion (Art. 145) which paved the way for Newton. Astronomy owes much to the thoroughness of Kepler.

VI. QUESTIONS

1. Note carefully the position of any conspicuous star at 8 P.M. and verify the fact that in a month it will be 30° farther west at the same time in the evening.

2. From which of the laws of motion does it follow that two attracting bodies revolve around their common center of gravity?

3. What are the fundamental principles on which each of the four proofs of the revolution of the earth depend? How many really independent proofs of the revolution of the earth are there?

4. Which of the proofs of the revolution of the earth give also the size of its orbit?

5. The aberration of light causes a star apparently to describe a small curve near its true place; what is the character of the curve if the star is at the pole of the ecliptic? If it is in the plane of the earth's orbit?

6. Discuss the questions corresponding to question 5 for the small curve described as a consequence of the parallax of a star. Do aberration and parallax have their maxima and minima at the same times, or are their phases such that they can be separated?

7. Discuss the climatic conditions if the day were twice as long as it is at present.

8. If the eccentricity of the earth's orbit were zero, in what respects would the seasons differ from those which we have now?

9. If the inclination of the equator to the ecliptic were zero, in what respects would the seasons differ from those which we have now?

10. Suppose the inclination of the equator to the ecliptic were 90°; describe the phenomena which would correspond to our day and to our seasons.

11. Draw diagrams giving the diurnal circles of the sun when the sun is at an equinox and both solstices, for an observer at the earth's equator, in latitude 75° north, and at the north pole.

12. At what times of the year is the sun's motion northward or southward slowest (see Fig. 42)? For what latitude will it then pass through or near the zenith? This place will then have its highest temperature. Compare the amount of heat it receives with that received by the equator during an equal interval when the sun is near the equinox. Which will have the higher temperature?

CHAPTER IV
REFERENCE POINTS AND LINES

63. Object and Character of Reference Points and Lines.—One of the objects at which astronomers aim is a knowledge of the motions of the heavenly bodies. In order fully to determine their motions it is necessary to learn how their apparent positions change with the time. Another important problem of the astronomer is the measurement of the distances of the celestial objects, for without a knowledge of their distances, their dimensions and many other of their properties cannot be determined. In order to measure the distance of a celestial body it is necessary to find how its apparent direction differs as seen from different points on the earth's surface (Art. 123), or from different points in the the earth's orbit (Art. 51). For both of these problems it is obviously important to have a precise and convenient means of describing the apparent positions of the heavenly bodies.

Not only are systems of reference points and lines important for certain kinds of serious astronomical work, but they are also indispensable to those who wish to get a reasonable familiarity with the wonders of the sky. Any one who has traveled and noticed the stars has found that their apparent positions are different when viewed from different latitudes on the earth. It can be verified by any one on a single clear evening that the stars apparently move during the night. And if the sky is examined at the same time of night on different dates the stars will be found to occupy different places. That is, there is considerable complexity in the apparent motions of the stars, and any such vague directions as are ordinarily made to suffice for describing positions on the earth would be absolutely useless when applied to the heavens.

Although the celestial bodies differ greatly in distance from the earth, some being millions of times as far away as others, they all seem to be at about the same distance on a spherical surface, which is called the *celestial sphere*. In fact, the ancients actually assumed that the stars are attached to a crystalline sphere. The celestial sphere is not a sphere at any particular large distance; it is an imaginary surface beyond all the stars and on which they are all projected, at such an enormous distance from the earth that two lines drawn toward a point on it from any two points on the earth, or from any two points on the earth's orbit, are so nearly parallel that their convergence can never be detected with any instrument. For short, it is said to be an infinite sphere.

While the real problem giving rise to reference points and lines is that of describing accurately and concisely the directions of celestial objects from the observer, its solution is equivalent to describing their apparent positions on the celestial sphere. Since it is much easier to imagine a position on a sphere than it is to think of the direction of lines radiating from its center, the heavenly bodies are located in direction by describing their projected positions on the celestial sphere. Fortunately, a similar problem has been solved in locating positions on the surface of the earth, and the astronomical problem is treated similarly.

64. The Geographical System.—Every one is familiar with the method of locating a position on the surface of the earth by giving its latitude and longitude. Therefore it will be sufficient to point out here the essential elements of this process.

The geographical lines that cover the earth are composed of two distinct sets which have quite different properties. The first set consists of the equator, which is a great circle, and the parallels of latitude, which are small circles parallel to the equator. If the equator is defined in any way, the two associated poles, which are 90° from it, are also uniquely located. Or, if there is any natural way in which the poles are defined, the equator is itself given. In the case of the earth the poles are the points on its surface at the ends of its axis of rotation, and these points consequently have properties not possessed by any others. If they are regarded as being defined in this way, the equator is defined as the great circle 90° from them.

The second set of circles on the surface of the earth consists of great circles, called meridians, passing through the poles and cutting the equator at right angles. All the meridians are similar to one another, and a convenient one is chosen as a line from which to measure longitudes. The distances from the fundamental meridian to the other meridians are given in degrees and are most conveniently measured in arcs along the equator.

The fundamental meridian generally used as a standard is that one which passes through the observatory at Greenwich, England. However, in many cases, other countries use the meridians of their own national observatories. For example, in the United States, the meridian of the Naval Observatory at Washington is frequently employed.

In order to locate uniquely a point on the surface of the earth, it is sufficient to give its *latitude*, which is the angular distance from the equator, and its *longitude*, which is the angular distance east or west of the standard meridian. These distances are called the *coördinates* of the point. It is customary to measure the longitude either east or west, as may be necessary in order that it shall not be greater than 180°. In many respects it would be simpler if longitude were counted from the fundamental meridian in a single direction.

65. The Horizon System.—The horizon, which separates the visible portion of the sky from that which is invisible, is a curve that cannot escape attention. If it were a great circle, it might be taken as the principal circle for a system of coördinates on the sky. But on the land the contour of the horizon is subject to the numerous irregularities of surface, and on the sea it is always viewed from at least some small altitude above the surface of the water. For this reason it is called the sensible horizon to distinguish it from the astronomical horizon, which will be defined in the next paragraph.

The direction defined by the plumb line at any place is perfectly definite. The point where the plumb line, if extended upward, pierces the celestial sphere is called the *zenith*, and the opposite point is called the *nadir*. These two points will be taken as poles of the first set of coördinates in the horizon system, and the horizon is defined as the great circle on the celestial sphere 90° from the zenith. The small circles parallel to the horizon are called *altitude circles* or, sometimes, almucantars.

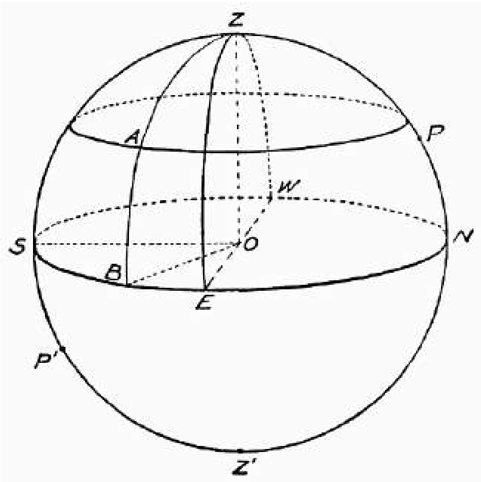


FIG. 45.—The horizon system.

The second set of circles in the horizon system consists of the great circles which pass through the zenith and the nadir and cut the horizon at right angles. They are called *vertical circles.* The fundamental vertical circle from which distances along the horizon are measured is that one which passes through the pole of the sky; that is, the point where the axis of the earth, prolonged, cuts the celestial sphere, and it is called the *meridian.*

The coördinates of a point in the horizon system are (a) the angular distance above or below the horizon, which is called *altitude*, and (b) the angular distance west from the south point along the horizon to the place where the vertical circle through the object crosses the horizon. This is called the *azimuth* of the object.

In Fig. 45, O represents the position of the observer, $SWNE$ his horizon, and Z his zenith. The point where the earth's axis pierces the sky is perfectly definite and is represented by P in the diagram. The vertical circle which passes through Z and P is the meridian. The points at which the meridian cuts the horizon are the north and south points. The north point, for positions in the northern hemisphere of the earth, is the one nearest the pole P. In this way the cardinal points are uniquely defined.

Consider a star at A. Its altitude is BA, which, in this case, is about 40°, and its azimuth is $SWNEB$, which, in this case, is about 300°. It is, of course, understood that the object might be below the horizon and the azimuth might be anything from zero to 360°. When the object is above the horizon, its altitude is considered as being positive, and when below, as being negative.

66. The Equator System.—The poles of the sky have been defined as the points where the earth's axis prolonged intersects the celestial sphere. It might be supposed at first that these would not be conspicuous points because the earth's axis is a line which of course cannot be seen. But the rotation of the earth causes an apparent motion of the stars around the pole of the sky. Consequently, an equally good definition of the poles is that they are the common centers of the diurnal circles of the stars. That pole which is visible from the position of an observer is a point no less conspicuous than the zenith.

The celestial equator is a great circle 90° from the poles of the sky. An alternative definition is that the celestial equator is the great circle in which the plane of the earth's equator intersects the celestial sphere. The small circles parallel to the celestial equator are called *declination circles.*

The second set of circles in the equatorial system consists of those which pass through the poles and are perpendicular to the celestial equator. They are called *hour circles.* The fundamental hour circle, called the *equinoctial colure*, from which all others are measured, is that one which passes through the vernal equinox, that is, the place at which the sun in its apparent annual motion around the sky crosses the celestial equator from south to north.

The coördinates in the equator system are (a) the angular distance north or south of the celestial equator, which is called declination, and (b) the angular distance eastward from the vernal equinox along the equator to the point where the hour circle through the object crosses the equator. This distance is called right ascension. The direction eastward is defined as that in which the sun moves in its apparent motion among the stars.

In Fig. 46, let O represent the position of the observer, $NESW$ his horizon, $PNQ'SQ$ his meridian. Suppose the star is at A and that the vernal equinox is at V. Then the declination of the star is the arc CA and its right ascension is VQC. In this case the declination is about 40° and the right ascension is about 75°. It is not customary to express the right ascension in degrees, but to give it in hours, where an hour equals 15°. In the present case the right ascension of A is, therefore, about 5 hours.

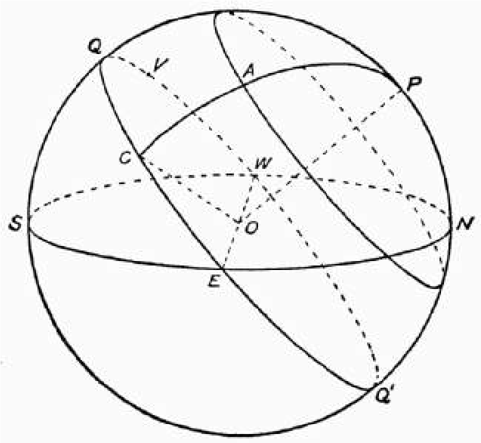


FIG. 46.—The equator system.

It is easy to understand why it is convenient to count right ascension in hours. The sky has an apparent motion westward because of the earth's actual rotation eastward, and it makes a complete circuit of 360° in 24 hours. Therefore it apparently moves westward 15° in one hour. It follows that a simple method of finding the right ascension of an object is to note when the vernal equinox crosses the meridian and to measure the time which elapses before the object is observed to cross the meridian. The interval of time is its right ascension expressed in hours.

67. The Ecliptic System.—The third system which is employed in astronomy, but much less frequently than the other two, is known as the ecliptic system because the fundamental circle in its first set is the ecliptic. The *ecliptic* is the great circle on the celestial sphere traced out by the sun in its apparent annual motion around the sky. The points on the celestial sphere 90° from the ecliptic are the poles of the ecliptic. The small circles parallel to the ecliptic are called *parallels of latitude*. The great circles which cross the ecliptic at right angles are called *longitude circles*.

The coördinates in the ecliptic system are the angular distance north or south of the ecliptic, which is called *latitude*, and the distance eastward from the vernal equinox along the ecliptic to the point where the longitude circle through the object intersects the ecliptic, which is called *longitude*.

In Fig. 47, O represents the position of the observer and $QEQ'W$ the celestial equator. Suppose that at the time in question the vernal equinox is at V and that the autumnal equinox is at A. Then, since the angle between the ecliptic and the equator is $23°.5$, the position of the ecliptic is $AX'VX$.

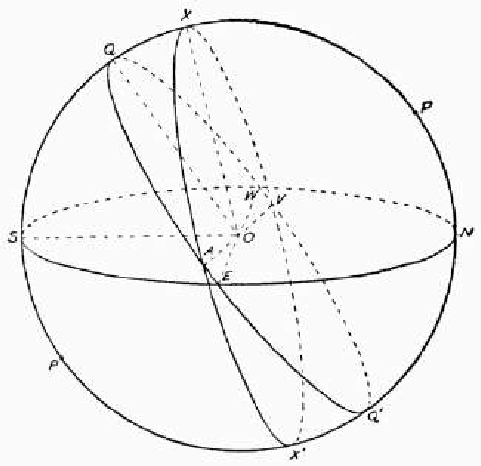


FIG. 47.—The ecliptic system.

68. Comparison of the Systems of Coördinates.—All three of the systems of coördinates are geometrically like the one used in geography; but there are important differences in the way in which they arise and in the purposes for which their use is convenient.

The horizon system depends upon the position of the observer and the direction of his plumb line. It always has the same relation to him, and if he travels he takes it with him. The equator system is defined by the apparent rotation of the sky, which is due, of course, to the actual rotation of the earth, and it is altogether independent of the position of the observer. The ecliptic system is defined by the apparent motion of the sun around the sky and also is independent of the position of the observer.

Since the horizon system depends upon the position of the observer, the altitude and azimuth of an object do not really locate it unless the place of the observer is given. Since the stars have diurnal motions across the sky, the time of day must also be given; and since different stars cross the meridian at different times on succeeding days, it follows that the day of the year must also be given. The inconvenience of the horizon system arises from the fact that its circles are not fixed on the sky. Yet it is important for the observer because the horizon is approximately the boundary which separates the visible from the invisible portion of the sky.

In the equator system the reference points and lines are fixed with respect to the stars. This statement, however, requires two slight corrections. In the first place, the earth's equator, and therefore the celestial equator, is subject to precession (Art. 47). In the second place, the stars have very small motions with reference to one another which become appreciable in work of extreme precision, generally in the course of a few years. But in the present connection these motions will be neglected and the equator coördinates will be considered as being absolutely fixed with respect to the stars. With this understanding the apparent position of an object is fully defined if its right ascension and declination are given. The reference points and lines of the ecliptic system also have the desirable quality of being fixed with respect to the stars.

From what has been said it might be inferred that the equator and ecliptic systems are equally convenient, but such is by no means the case. The equator always crosses the meridian at an altitude which is equal to 90° minus the latitude of the observer (Art. 57) and always passes through the east and west points of the horizon. Consequently, all objects having the same declination cross the meridian at the same altitude. Suppose, for example, that the observer is in latitude 40° north. Then the equator crosses his meridian at an altitude of 50°. If he observes that a star crosses the meridian at an altitude of 60°, he knows that it is 10° north of the celestial equator, or that its declination is 10°; and by noting the time that has elapsed from the time of the passage of the vernal equinox across the meridian to the passage of the star, he has its right ascension. Nothing could be simpler than getting the coördinates of an object in the equator system.

Now consider the ecliptic system. Suppose V, in Fig. 48, represents the position of the vernal equinox on a certain date and time of day. Then the pole of the ecliptic $XVX'A$ is at R and the ecliptic crosses the meridian below the equator. In this case the star might have north celestial latitude and be on the meridian south of the equator. Twelve hours later the vernal equinox has apparently rotated westward with the sky to the point V, Fig. 49. The pole of the ecliptic has gone around the pole P to the point R, and the ecliptic crosses the meridian north of the equator. It is clear from Figs. 48 and 49 that the position of the ecliptic with respect to the horizon system changes continually with the apparent rotation of the sky. It follows that for most purposes the ecliptic system is not convenient. Its use in astronomy is limited almost entirely to describing the position of the sun, which is always on the ecliptic, and the positions of the moon and planets, which are always near it.

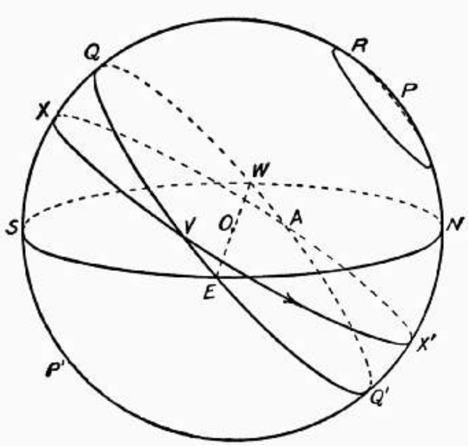


FIG. 48.

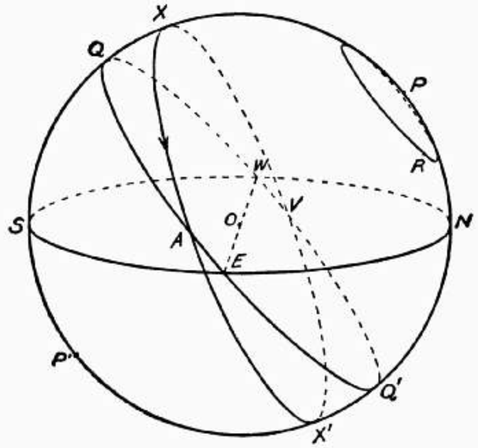


FIG. 49.

Equator and ecliptic.

69. Finding the Altitude and Azimuth when the Right Ascension, Declination, and Time are Given.—Suppose the right ascension and declination of a star are given and that its altitude and azimuth are desired. It is necessary also to have given the latitude of the observer, the time of day, and the time of year, because the altitude and azimuth depend on these quantities. Most of the difficulty of the problem arises from the fact that the vernal equinox has a diurnal motion around the sky and that it is a point which is not easily located. By computing the right ascension of the sun at the date in question, direct use of the vernal equinox may be avoided. It has been found convenient to solve the problem in four distinct steps.

Step 1. The right ascension of the sun on the date in question.—It has been found by observation that the sun passes the vernal equinox March 21. (The date may vary a day because of the leap year, but it will be sufficiently accurate for the present purposes to use March 21 for all cases.) In a year the sun moves around the sky 24 hours in right ascension, or at the rate of two hours a month. Although the rate of apparent motion of the sun is not perfectly uniform, the variations from it are small and will be neglected in the present connection. It follows from these facts that the right ascension of the sun on any date may be found by counting the number of months from March 21 to the date in question and multiplying the result by two. For example, October 6 is 6.5 months from March 21, and the right ascension of the sun on this date is, therefore, 13 hours.

Step 2. The right ascension of the meridian at the given time of day on the date in question.—Suppose the right ascension of the sun has been determined by Step 1. Since the sun moves 360° in 365 days, or only one degree per day, its motion during one day may be neglected. Suppose, for example, that it is 8 o'clock at night. Then the sun is 8 hours west of the meridian at the position indicated in Fig. 50. Since right ascension is counted eastward and the right ascension of the sun is known, the right ascension of Q may be found by adding the number of hours from the sun to Q to the right ascension of the sun. If the right ascension of the sun is 13 hours and the time of the day is 8 P.M., the right ascension of the meridian is $13 + 8 = 21$ hours. The general rule is, the right ascension of the meridian is obtained by adding to the right ascension of the sun the number of hours after noon.

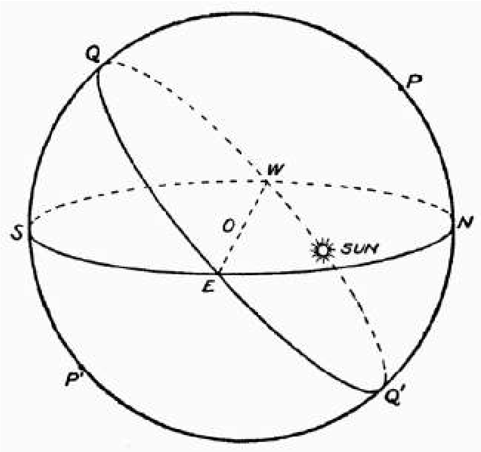


FIG. 50.—The right ascension of the meridian.

Step 3. The hour angle of the object.—Wherever the object may be, a certain hour circle passes through it and crosses the equator at some point. The distance from the meridian along the equator to this point is called the hour angle of the object. The hour angle is counted either east or west as may be necessary in order that the resulting number shall not exceed 12.

Suppose the right ascension of the meridian has been found by Step 2. The hour angle of the star is the difference between its right ascension, which is one of the quantities given in the problem, and the right ascension of the meridian. If the right ascension of the star is greater than that of the meridian, its hour angle is east, and if it is less than that of the meridian, its hour angle is west. There is one case which, in a way, is an exception to this statement. Suppose the right ascension of the meridian is 22 hours and the right ascension of the star is 2 hours. According to the rule the star is 20 hours west, which, of course, is the same as 4 hours east. But its right ascension of 2 hours may be considered as being a right ascension of 26 hours, just as 2 o'clock in the afternoon can be equally well called 14 o'clock.

When its right ascension is called 26 hours, the rule leads directly to the result that the hour angle is 4 hours east.

Step 4. Application of the declination and estimation of the altitude and azimuth.—In order to make the last step clear, consider a special example. Suppose the hour angle of the object has been found by Step 3 to be 7 hours east. This locates the point C, Fig. 51. Therefore the star is somewhere on the hour circle PCP'. The given declination determines where the star is on the circle. Suppose, for example, that the object is 35° north. In order to locate it, it is only necessary to measure off 35° from C along the circle CP. Hence the star is at A. Now draw a vertical circle from Z through A to the horizon at B. The altitude is BA and the azimuth is $SWNB$. These distances can be computed by spherical trigonometry, but they may be estimated closely enough for present purposes. In this problem the altitude is about 12° and the azimuth is about 230°. Whatever the data may be which are supplied by the problem, the method of procedure is always that which has been given in the present case.

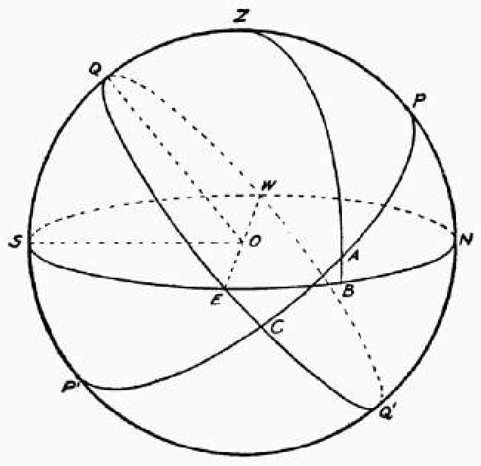


FIG. 51.—Application of the declination in finding the position of a star.

70. Illustrative Example for Finding Altitude and Azimuth.—In order to illustrate fully the processes that have been explained in Art. 69, an actual problem will be solved. Suppose the observer is in latitude 40° north. The altitude of the pole P, Fig. 52, as seen from his position, will be 40°, and the point Q, where the equator crosses the meridian, will have an altitude of 50°. Suppose the date on which the observation is made is June 21 and the time of day is 8 P.M. Suppose the right ascension of the star in question is approximately 16 hours and that its declination is −16°. The problem is to find its apparent altitude and azimuth.

The steps of the solution will be made in their natural order. (1) Since June 21 is three months after March 21, the right ascension of the sun on that date is 6 hours. (2) Since the time of day is 8 P.M., and the right ascension is counted eastward, the right ascension of the meridian is $6 + 8 = 14$ hours. (3) Since the right ascension of the star is 16 hours, its hour angle is 2 hours east, and it is on the hour circle PCP'. (4) Since its declination is $-16°$, it is 16° south from C toward P' and at the point A. Now draw a vertical circle from Z through A, cutting the horizon at B. The altitude is BA, which is about 22°. The azimuth is $SWNEB$, which is about 320°.

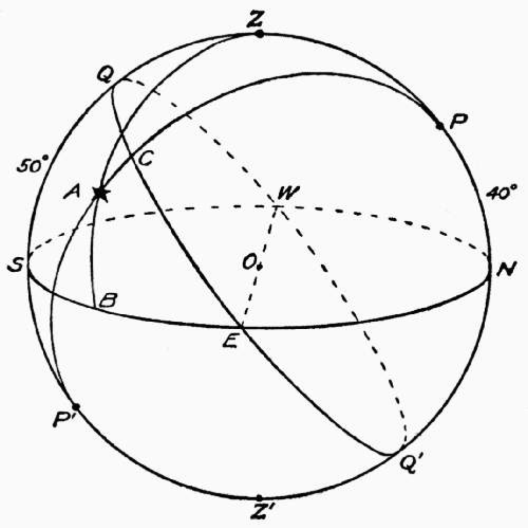


FIG. 52.—Finding the altitude and azimuth.

71. Finding the Right Ascension and Declination when the Altitude and Azimuth are Given.—The problem of finding the right ascension and declination when the altitude and azimuth are given is the converse of that treated in Art. 69. It can also be conveniently solved in four steps.

In the first step, the right ascension of the sun is obtained, and in the second, the right ascension of the meridian is found. These steps are, of course, the same as those given in Art. 69. The third step is to draw through the position of the given object an hour circle which, from its definition, reaches from one pole of the sky to the other and cuts the equator at right angles. The fourth step is to estimate the hour angle of the hour circle drawn in Step 3 and the distance of the star from the equator measured along the hour circle. Then the right ascension of the object is equal to the right ascension of the meridian plus the hour angle of the object if it is east, and minus the hour angle if it is west; and the declination of the object is simply its distance from the equator.

72. Illustrative Example for Finding Right Ascension and Declination.—Suppose the date of the observation is May 6 and that the time of day is 8 P.M. Suppose the observer's latitude is 40° north. Suppose he sees a bright star whose altitude is estimated to be 35° and whose azimuth is estimated to be 60°. Its right ascension and declination are required, and after they have been obtained it can be found from Table I, p. 118, what star is observed.

The right ascension of the sun on May 6 is 3 hours and the right ascension of the meridian at 8 P.M. is 11 hours. The star then is at the point A, Fig. 53, where $BA = 35°$ and $SB = 60°$. The part of the vertical circle BA is much less foreshortened than AZ by the projection of the celestial sphere on a plane, and this fact must be remembered in connection with the drawing. The hour circle PAP' cuts the equator at the point C. The arc QC is much more foreshortened by projection than CW. Consequently, it is seen that the hour angle of the star is 3.5 hours west. Therefore its right ascension is $11 - 3.5 = 7.5$ hours approximately. It is also seen that the star is approximately 5° north of the equator. On referring to Table I, it is found that this star must be Procyon.

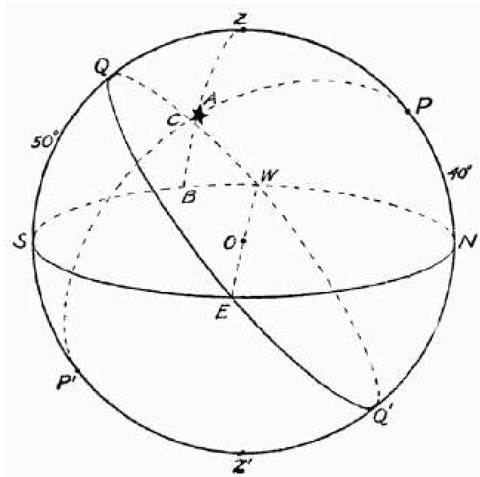


FIG. 53.—Finding the right ascension and declination.

All problems of the same class can be solved in a similar manner. But reliance should not be placed in the diagrams alone, especially because of the distortion to which certain of the lines are subject. The diagrams should be supplemented, if not replaced, by actually pointing out on the sky the various points and lines which are used. A little practice with this method will enable one to solve either the problem of finding the altitude and azimuth, or that of obtaining the right ascension and declination, with an error not exceeding 5° or 10°.

73. Other Problems Connected with Position.—There are two other problems of some importance which naturally arise. The first is that of finding the time of the year at which a star of given right ascension will be on the meridian at a time in the evening convenient for observation.

In order to make the problem concrete, suppose the time in question is 8 P.M. The right ascension of the sun is then 8 hours less than the right ascension of the meridian. Since the object is supposed to be on the meridian, the right ascension of the sun will be 8 hours less than that of the object. To find the time of the year at which the sun has a given right ascension, it is only necessary to count forward from March 21 two hours for each month. For example, if the object is Arcturus, whose right ascension is 14 hours, the right ascension of the sun is $14 - 8 = 6$ hours, and the date is June 21.

The second problem is that of finding the time of day at which an object whose right ascension is given will be on the meridian or horizon on a given date. A problem of this character will naturally arise in connection with the announcement of the discovery of a comet or some other object whose appearance in a given position would be conspicuous only for a short time. This problem is solved by first finding the right ascension of the sun on the date, and then taking the difference between this result and the right ascension of the object. This gives the hour angle of the sun at the required time. If the sun is west of the meridian, its hour angle is the time of day; if it is east of the meridian, its hour angle is the number of hours before noon.

VII. QUESTIONS

1. Make a table showing the correspondences of the points, circles, and coördinates of the horizon, equator, and ecliptic systems with those of the geographical system.

2. What are the altitude and azimuth of the zenith, the east point, the north pole? What are the angular distances from the zenith to the pole and to the point where the equator crosses the meridian in terms of the latitude l of the observer?

3. Estimate the horizon coördinates of the sun at 10 o'clock this morning; at 10 o'clock this evening.

4. Describe the complete diurnal motions of stars near the pole. What part of the sky for an observer in latitude 40° is always above the horizon? Always below the horizon?

5. How long is required for the sky apparently to turn 1°? Through what angle does it apparently turn in 1 minute?

6. Are there positions on the earth from which the diurnal motions of the stars are along parallels of altitude? Along vertical circles?

7. Develop a rule for finding the hour angle of the vernal equinox on any date at any time of day.

8. Find the altitude and azimuth of the vernal equinox at 9 A.M. to-day.

9. Given: Rt. asc. = 19 hrs., declination = +20°, date = July 21, time = 8 P.M.; find the altitude and azimuth.

10. Find the altitude and the azimuth (constructing a diagram) of each of the stars given in Table I, p. 118, at 8 P.M. to-day.

11. If a star whose right ascension is 18 hours is on the meridian at 8 P.M., what is the date?

12. At what time of the day is a star whose right ascension is 14 hours on the meridian on May 21?

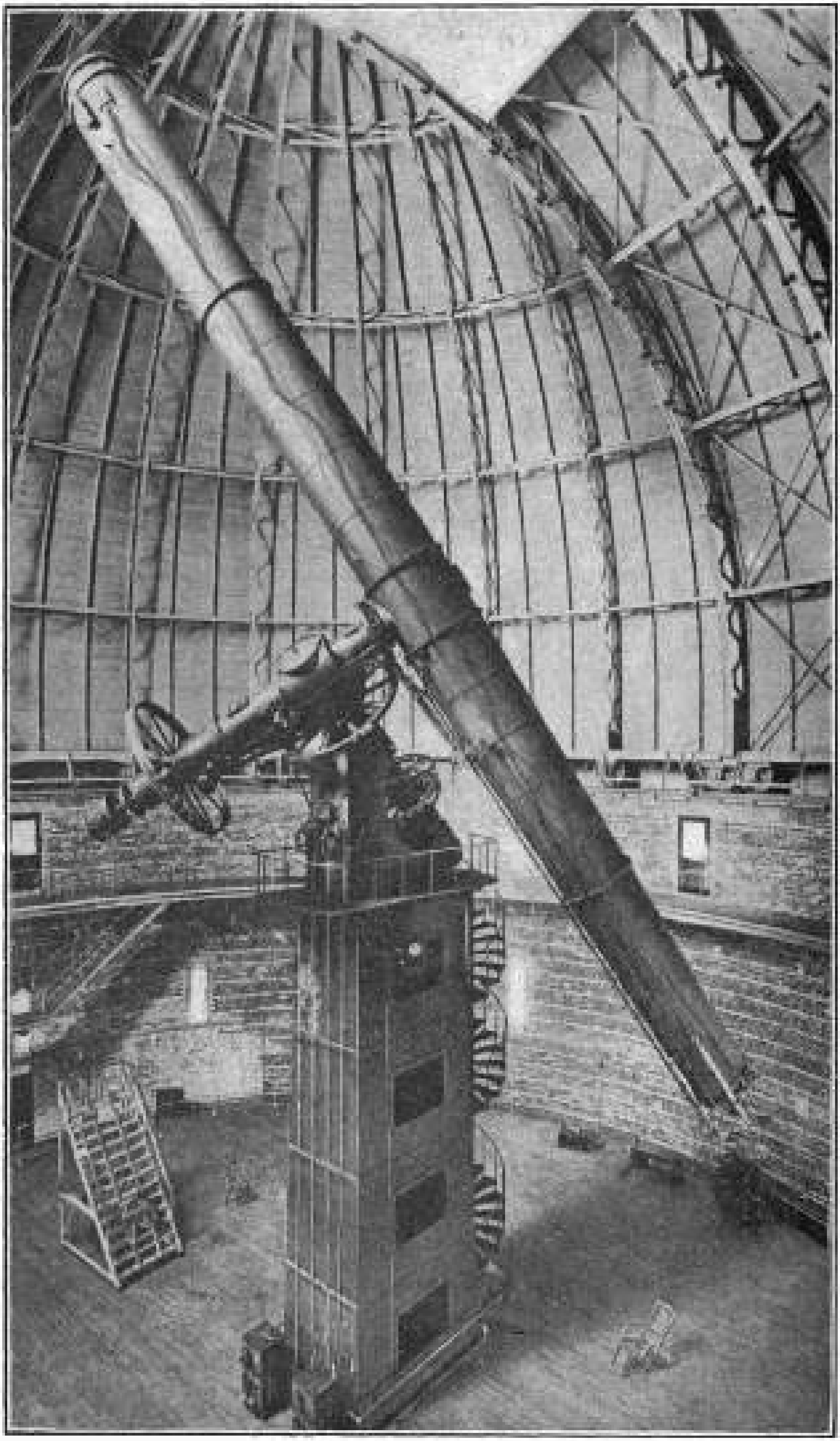
13. At what time of the day does a comet whose right ascension is 4 hours and declination is zero rise on Sept. 21?

14. The Leonid meteors have their radiant at right ascension 10 hours and they appear on Nov. 14. At what time of the night are they visible?

15. What is the right ascension of the point on the celestial sphere toward which the earth is moving on June 21?

16. What are the altitude and azimuth of the point toward which the earth is moving to-day at noon? At 6 P.M.? At midnight? At 6 A.M.?

17. Observe some conspicuous star (avoid the planets), estimate its altitude and azimuth, approximately determine its right ascension and declination (Art. 71), and with these data identify it in Table I, p. 118.



CHAPTER V

THE CONSTELLATIONS

74. Origin of the Constellations.—A moment's observation of the sky on a clear and moonless night shows that the stars are not scattered uniformly over its surface. Every one is acquainted with such groups as the Big Dipper and the Pleiades. This natural grouping of the stars was observed in prehistoric times by primitive and childlike peoples who imagined the stars formed outlines of various living creatures, and who often wove about them the most fantastic romances.

The earliest list of constellations, still in existence, is that of Ptolemy (about 140 A.D.), who enumerated, described, and located 48 of them. These constellations not only did not entirely cover the part of the sky visible from Alexandria, where Ptolemy lived, but they did not even occupy all of the northern sky. In order to fill the gaps and to cover the southern sky many other constellations were added from time to time, though some of them have now been abandoned. The lists of Argelander (1799–1875) in the northern heavens, and the more recent ones of Gould in the southern heavens, contain 80 constellations, and these are the ones now generally recognized.

75. Naming the Stars.—The ancients gave proper names to many of the stars, and identified the others by describing their relations to the anatomy of the fictitious creatures in which they were situated. For example, there were Sirius, Altair, Vega, etc., with proper names, and "The Star at the End of the Tail of the Little Bear" (Polaris), "The Star in the Eye of the Bull" (Aldebaran), etc., designated by their positions.

In modern times the names of 40 or 50 of the most conspicuous stars are frequently used by astronomers and writers on astronomy; the remainder are designated by letters and numbers. A system in very common use, that introduced by Bayer in 1603, is to give to the stars in each constellation, in the order of their brightness, the names of the letters of the Greek alphabet in their natural order. In connection with the Greek letters, the genitive of the name of the constellation is used. For example, the brightest star in the whole sky is Sirius, in Canis Major. Its name according to the system of Bayer is Alpha Canis Majoris. The second brightest star in Perseus, whose common name is Algol, in this system is called Beta Persei. After the Greek letters are exhausted the Roman letters are used, and then follow numbers for the stars in the order of their brightness. While this is the general rule,

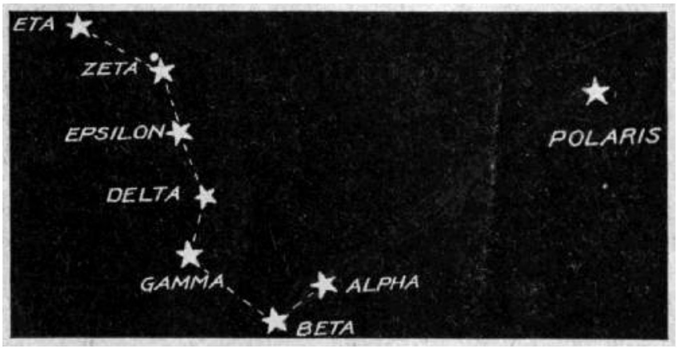


FIG. 55.—The Big Dipper and the Pole Star.

there are numerous exceptions in naming the stars, for example, in the case of the stars which constitute the Big Dipper (Fig. 55).

About 1700, Flamsteed published a catalogue of stars in which he numbered those in each constellation according to their right ascensions regardless of their brightness. In modern catalogues the stars are usually given in the order of their right ascension and no reference is made either to the constellation to which they belong or to their apparent brightness.

76. Star Catalogues.—Star catalogues are lists of stars, usually all above a given brightness, in certain parts of the sky, together with their right ascensions and declinations on a given date. It is necessary to give the date, for the stars slowly move with respect to one another, and the reference points and lines to which their positions are referred are not absolutely fixed. The most important variation in the position of the reference points and lines is due to the precession of the equinoxes (Art. 47).

The earliest known star catalogue is one of 1080 stars by Hipparchus for the epoch 125 B.C. Ptolemy revised it and reduced the star places to the epoch 150 A.D. Tycho Brahe made a catalogue of 1005 stars in 1580, about 30 years before the invention of the telescope. Since the invention of the telescope and the revival of science in Europe, numerous catalogues have been made, containing in some cases more than 100,000 stars. While the positions in all these catalogues are very accurately given, compared even to the work of Tycho Brahe, they are not accurate enough for certain of the most refined work in modern times. To meet these needs, a number of catalogues, containing a limited number of stars whose positions have been determined with the very greatest accuracy, have been made. The most accurate of these is the Preliminary General Catalogue of Boss, in which the positions of 6188 stars are given.

A project for photographing the whole heavens by international coöperation was formulated at Paris in 1887. The plan provided that each plate should cover 4 square degrees of the sky, and that they should overlap so that the whole sky would be photographed twice. The number of plates required, therefore, is nearly 22,000. On every plate a number of stars are photographed whose positions are already known from direct observations. The positions of the other stars on the plate can then be determined by measuring with a suitable machine their distances and directions from the known stars. This work can, of course, be carried out at leisure in an astronomical laboratory. On these plates, most of which have already been secured, there will be shown in all about 8,000,000 different stars. In the first catalogue based on them only about 1,300,000 of the brightest stars will be given.

The photographic catalogue was an indirect outgrowth of photographs of the great comet of 1882 taken by Gill at the Cape of Good Hope. The number of star images obtained on his plates at once showed the possibilities of making catalogues of stars by the photographic method. In 1889 he secured photographs of the whole southern sky from declination $-19°$ south, and the enormous labor of measuring the positions of the 350,000 star images on these plates was carried out by Kapteyn, of Groningen, Holland.

77. The Magnitudes of the Stars.—The magnitude of a star depends upon the amount of light received from it by the earth, and is not determined altogether by the amount of light it radiates, for a small star near the earth might give the observer more light than a much larger one farther away. It is clear from this fact that the magnitude of a star depends upon its actual brightness and also upon its distance from the observer.

The stars which are visible to the unaided eye are divided arbitrarily into 6 groups, or magnitudes, depending upon their apparent brightness. The 20 brightest stars constitute the first-magnitude group, and the faintest stars which can be seen by the ordinary eye on a clear night are of the sixth magnitude, the other four magnitudes being distributed between them so that the ratio of the brightness of one group to that of the next is the same for all consecutive magnitudes. The definition of what shall be exactly the first magnitude is somewhat arbitrary; but a first-magnitude star has been taken to be approximately equal to the average brightness of the first 20 stars. The sixth-magnitude stars are about $\frac{1}{100}$ as bright as the average of the first group, and, in order to make the ratio from one magnitude to the other perfectly definite, it has been agreed that the technical sixth-magnitude stars shall be those which are exactly $\frac{1}{100}$ as bright as the technical first-magnitude stars. The problem arises of finding what the ratio is for successive magnitudes.

Let r be the ratio of light received from a star of one magnitude to that received from a star of the next fainter magnitude. Then stars of the fifth magnitude are r times brighter than those of the sixth, and those of the fourth are r times brighter than those of the fifth, and they are therefore r^2 times brighter than those of the sixth. By a repetition of this process it is found that the first-magnitude stars are r^5 times brighter than those of the sixth magnitude. Therefore $r^5 = 100$, from which it is found that $r = 2.512\ldots.$

Since the amount of light received from different stars varies almost continuously from the faintest to the brightest, it is necessary to introduce fractional magnitudes. For example, if a star is brighter than the second magnitude and fainter than the first, its magnitude is between 1 and 2. A step of one tenth of a magnitude is such a ratio that, when repeated ten times, it gives the value $2.512\ldots.$ It is found by computation, which can easily be carried out by logarithms, that a first-magnitude star is 1.097 times as bright as a star of magnitude 1.1. The ratio of brightness of a star of magnitude 1.1 to that of a star of 1.2 is likewise 1.097; and, consequently, a star of magnitude 1 is $1.097 \times 1.097 = 1.202$ times as bright as a star of magnitude 1.2.

A star which is 2.512 times as bright as a first-magnitude star is of magnitude 0, and still brighter stars have negative magnitudes. For example, Sirius, the brightest star in the sky, has a magnitude of -1.58, and the magnitude of the full moon on the same system is about -12, while that of the sun is -26.7.

78. The First-magnitude Stars.—As first-magnitude stars are conspicuous and relatively rare objects, they serve as guideposts in the study of the constellations. All of those which are visible in the latitude of the observer should be identified and learned. They will, of course, be recognized partly by their relations to neighboring stars.

In Table I the first column contains the names of the first-magnitude stars; the second, the constellations in which they are found; the third, their magnitudes according to the Harvard determination; the fourth, their right ascensions; the fifth, their declinations; the sixth, the dates on which they cross the meridian at 8 P.M.; and the seventh, the velocity toward or from the earth in miles per second, the negative sign indicating approach and the positive, recession. Their apparent positions at any time can be determined from their right ascensions and declinations by the principles explained in Art. 69.

79. Number of Stars in the First Six Magnitudes.—The number of stars in each of the first six magnitudes is given in Table II. The sum of the numbers is 5000. There are, therefore, in the whole sky only about 5000 stars which are visible to the unaided eye. At any one time only half the sky is above the horizon, and those stars which are near the horizon are largely extinguished by the absorption of light by the earth's atmosphere. Therefore one never sees at one time more than about 2000 stars, although the general impression is that they are countless.

It is seen from the Table II that the number of stars in each magnitude is about three times as great as the number in the preceding magnitude. This ratio holds approximately down to the ninth magnitude, and in the first nine magnitudes there are in all nearly 200,000 stars. Since a telescope 3 inches in aperture will show objects as faint as the ninth magnitude, it is seen what enormous aid is obtained from optical instruments. Only a rough guess can be made respecting the number of stars which are still fainter, but there are probably more than 300,000,000 of them within the range of present visual and photographic instruments.

80. The Motions of the Stars.—The stars have motions with respect to one another which, in the course of immense ages, appreciably change the outlines of the constellations, but which have not made important alterations in the visible sky during historic times. Nevertheless, they are so large that they must be taken into account when using star catalogues in work of precision.

One result of the motions of the stars is that they drift with respect to fixed reference points and lines. The yearly change in the position of a star with respect to fixed reference points and lines is called its proper motion. The largest known proper motion is that of an eighth-magnitude star in the southern heavens, whose annual displacement on the sky is about 8.7 seconds of arc. The slight extent to which the proper motions of the stars can change the appearance of the constellations is shown by the fact that even this star, whose proper motion is more than 100 times the average proper motion of the brighter stars, will not move over an apparent distance as great as the diameter of the moon in less than 220 years.

TABLE I

NAME	CONSTELLATION	MAGNITUDE	RIGHT ASCENSION	DECLINATION	ON MERIDIAN AT 8 P.M.	RADIAL VELOCITY
Sirius	Canis Major	−1.6	6h 41m	−16°36′	Feb. 28	− 5.6
Canopus	Carina	−0.9	6 22	−52 39	Feb. 23	+12.7
Alpha Centauri	Centaurus	0.1	14 34	−60 29	June 29	−13.8
Vega	Lyra	0.1	18 34	+38 42	Aug. 30	− 8.5
Capella	Auriga	0.2	5 10	+45 55	Feb. 5	+19.7
Arcturus	Boötes	0.2	14 12	+19 37	June 24	− 2.4
Rigel	Orion	0.3	5 11	−8 17	Feb. 5	+13.6
Procyon	Canis Minor	0.5	7 35	+5 26	Mar. 14	− 2.5
Achernar	Eridanus	0.6	1 35	−57 40	Dec. 16	+10.0
Beta Centauri	Centaurus	0.9	13 58	−59 58	June 21	?
Betelgeuze	Orion	0.9	5 51	+7 24	Feb. 15	+13.0
Altair	Aquila	0.9	19 47	+8 39	Sept. 19	−20.5
Alpha Crucis	Crux	1.1	12 22	−62 38	May 29	+ 4.3
Aldebaran	Taurus	1.1	4 31	+16 21	Jan. 26	+34.2
Pollux	Gemini	1.2	7 40	+28 14	Mar. 15	+ 2.4
Spica	Virgo	1.2	13 21	−10 44	June 12	+ 1.2
Antares	Scorpius	1.2	16 24	−26 15	July 27	− 1.9
Fomalhaut	Piscis Australis	1.3	22 53	−30 4	Nov. 8	+ 4.2
Deneb	Cygnus	1.3	20 39	+44 59	Oct. 4	− 2.5
Regulus	Leo	1.3	10 4	+12 23	Apr. 23	− 5.0

TABLE II

First Magnitude	20	Fourth Magnitude	425
Second Magnitude	65	Fifth Magnitude	1100
Third Magnitude	190	Sixth Magnitude	3200

Another component of the motion of a star is that which is in the line joining it with the earth. This component can be measured by the spectroscope (Art. 222), and is found to range all the way from a velocity of approach of 40 miles per second to one of recession with the same speed; and in some cases even higher velocities are encountered. In the course of immense time the changes in the distances of the stars will alter their magnitudes appreciably; but the distances of the stars are so great that there is probably no case in which the motion of a star toward or from the earth will sensibly change its magnitude in 20,000 years.

81. The Milky Way, or Galaxy.—The Milky Way is a hazy band of light giving indications to the unaided eye of being made up of faint stars; it is on the average about 20° in width and stretches in nearly a great circle entirely around the sky. The telescope shows that it is made up of millions of small stars which can be distinguished separately only with optical aid. It is clear that because of its irregular form and great width its position cannot be precisely described, but in a general way its location is defined by the fact that it intersects the celestial equator at two places whose right ascensions are approximately 6 hours 40 minutes and 18 hours 40 minutes, and it has an inclination to the equator of about 62°. Or, in other terms, the north pole of the Milky Way is at right ascension about 12 hours 40 minutes and at declination about +28°. For a long distance it is divided more or less completely into two parts, and at one place in the southern heavens it is cut entirely across by a dark streak. A very interesting feature for observers in northern latitudes is a singular dark region north of the star Deneb.

82. The Constellations and Their Positions.—The work on reference points and lines in the preceding chapter together with the discussions so far given in this chapter are sufficient to prepare for the study of the constellations with interest and profit, and the student should not stop short of an actual acquaintance with all the first-magnitude stars and the principal constellations that are visible in his latitude. Table III contains a list of the constellations and gives their positions. The numbers at the top show the degrees of declination between which the constellations lie, the numerals at the left show their right ascensions, and the numbers placed in connection with the names of the constellations give the number of stars in them which are easily visible to the unaided eye. The constellations which lie on the ecliptic, or the so-called zodiacal constellations, are printed in italics.

The following maps show the constellations from the north pole to $-50°$ declination. When Map I is held up toward the sky, facing north, with its center in the line joining the eye with the north pole, and with the hour circle having the right ascension of the meridian placed directly above its center, it shows the circumpolar constellations in their true relations to one another and to the horizon and pole. The other maps are to be used, facing south, with their centers held on a line joining the eye to the celestial equator, and with the hour circle having the right ascension of the meridian held in the plane of the eye and the meridian. When they are placed in this way, they show the constellations to the south of the observer in their true relationships. In order to apply the maps according to these instructions, it is necessary to know the right ascension of the meridian for the day and hour in question, and it can be computed with sufficient approximation by the method of Art. 69.

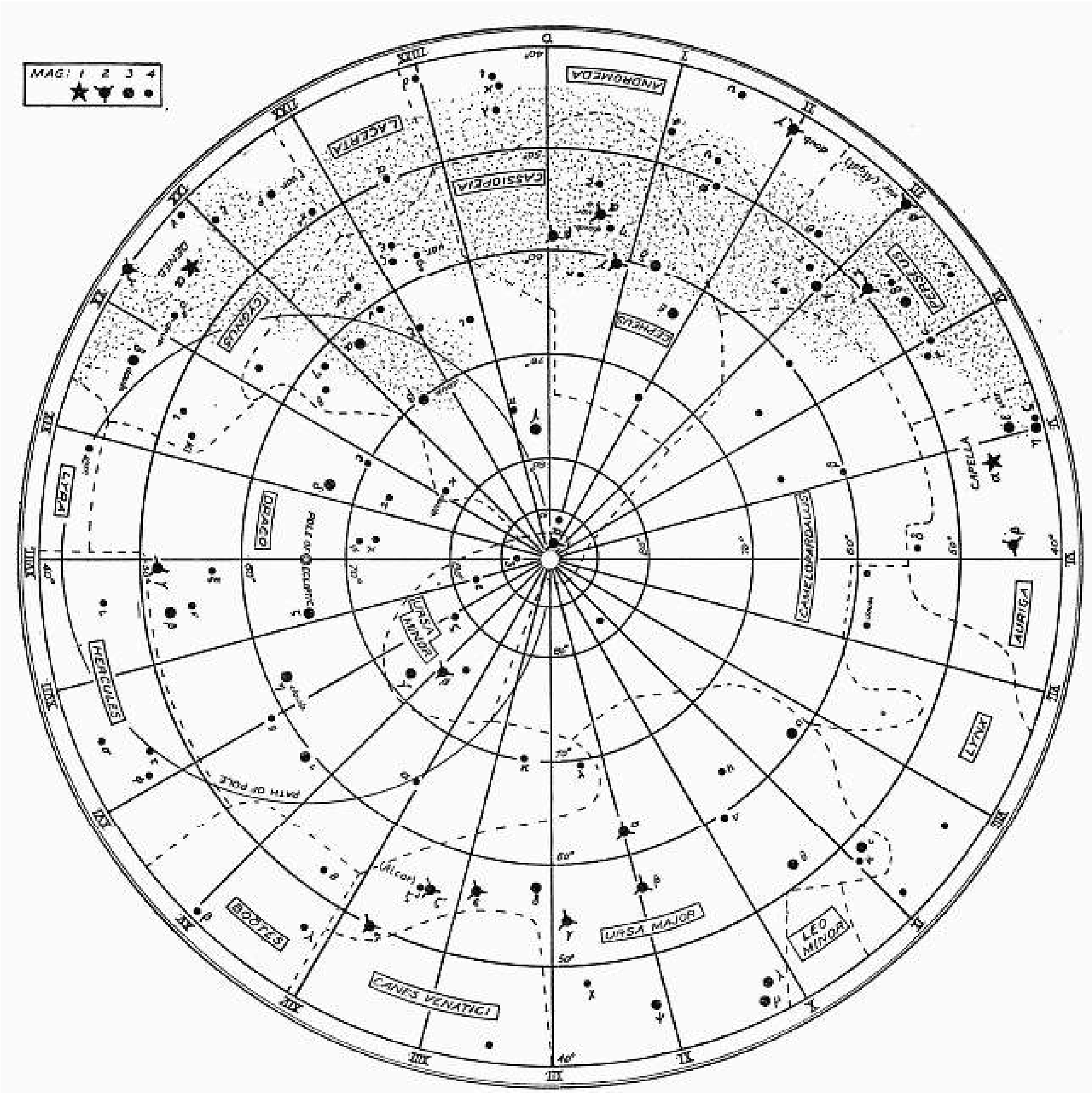
83. Finding the Pole Star.—The first step to be taken in finding the constellations, either from their right ascensions and declinations or from star maps, is to determine the north-and-south line. It is defined closely enough for present purposes by the position of the pole star.

The Big Dipper is the best known and one of the most conspicuous groups of stars in the northern heavens. It is always above the horizon for an observer in latitude 40° north, and, because of its definite shape, it can never be mistaken for any other group of stars. It is made up of 7 stars of the second magnitude which form the outline of a great dipper in the sky. Figure 56 is a photograph of this group of stars distinctly showing the dipper. The stars Alpha and Beta are called The Pointers because they are almost directly in a line with the pole star Polaris. In order to find the pole star, start with Beta, Fig. 55, go through Alpha, and continue about five times the distance from Beta to Alpha. At the point reached there will be found the second-magnitude star Polaris with no other one so bright anywhere in the neighborhood.

TABLE III

R.A. \ DEC.	+90° TO +50°	+50° TO +25°	+25° TO 0°	0° TO −25°	−25° TO −50°	−50° TO −90°
I–II	Cassiopeia, 46.	Andromeda, 18; Triangulum, 5.	*Pisces*, 18; *Aries*, 17.	Cetus, 37.	Phœnix, 32; Apparatus Sculptoris, 13.	(Phœnix); Hydrus, 18.
III–IV	—	Perseus, 46.	*Taurus*, 58.	Eridanus, 64.	(Eridanus.)	Horologium, 11; Reticulum, 9.
V–VI	Camelopardalis, 36.	Auriga, 35.	Orion, 58; *Gemini*, 33.	Lepus, 18.	Columba, 15.	Dorado, 16; Pictor, 14; Mons Mensa, 12.
VII–VIII	—	Lynx, 28.	Canis Minor, 8; *Cancer*, 15.	Canis Major, 27; Monoceros, 12.	Argo-Navis, 149.	(Argo-Navis, Puppis); Piscis Volans, 9.
IX–X	—	Leo Minor, 15.	*Leo*, 47.	Hydra, 49; Sextans, 5.	—	(Argo-Navis, Vela.)
XI–XII	Ursa Major, 53.	—	Coma Berenices, 20.	Crater, 15; Corvus, 8.	Centaurus, 56.	(Argo-Navis, Carina); Chameleon, 13.
XIII–XIV	—	Canes Venatici, 15; Boötes, 36.	—	*Virgo*, 39.	Lupus, 34.	(Centaurus); Crux, 13; Musca, 15.
XV–XVI	Ursa Minor, 23.	Corona Borealis, 19; Hercules, 65.	Serpens, 25.	*Libra*, 23.	Norma, 14.	Circinus, 10.
XVII–XVIII	Draco, 80.	Lyra, 18.	Aquila, 37; Sagitta, 5.	*Scorpius*, 34; Ophiuchus, 46.	Ara, 15.	Triangulum Australe, 11; Apus, 8.
XIX–XX	—	Cygnus, 67.	Vulpecula, 23; Delphinus, 10.	*Sagittarius*, 48.	Corona Australis, 8.	Telescopium, 16; Pavo, 37; Octans, 22.
XXI–XXII	Cepheus, 44.	Lacerta, 16.	Equuleus, 5.	*Capricornus*, 22.	Piscis Australis, 16.	Indus, 15; (Octans).
XXIII–XXIV	—	—	Pegasus, 43.	*Aquarius*, 36.	Grus, 30.	(Octans); Tucana, 22.

Map I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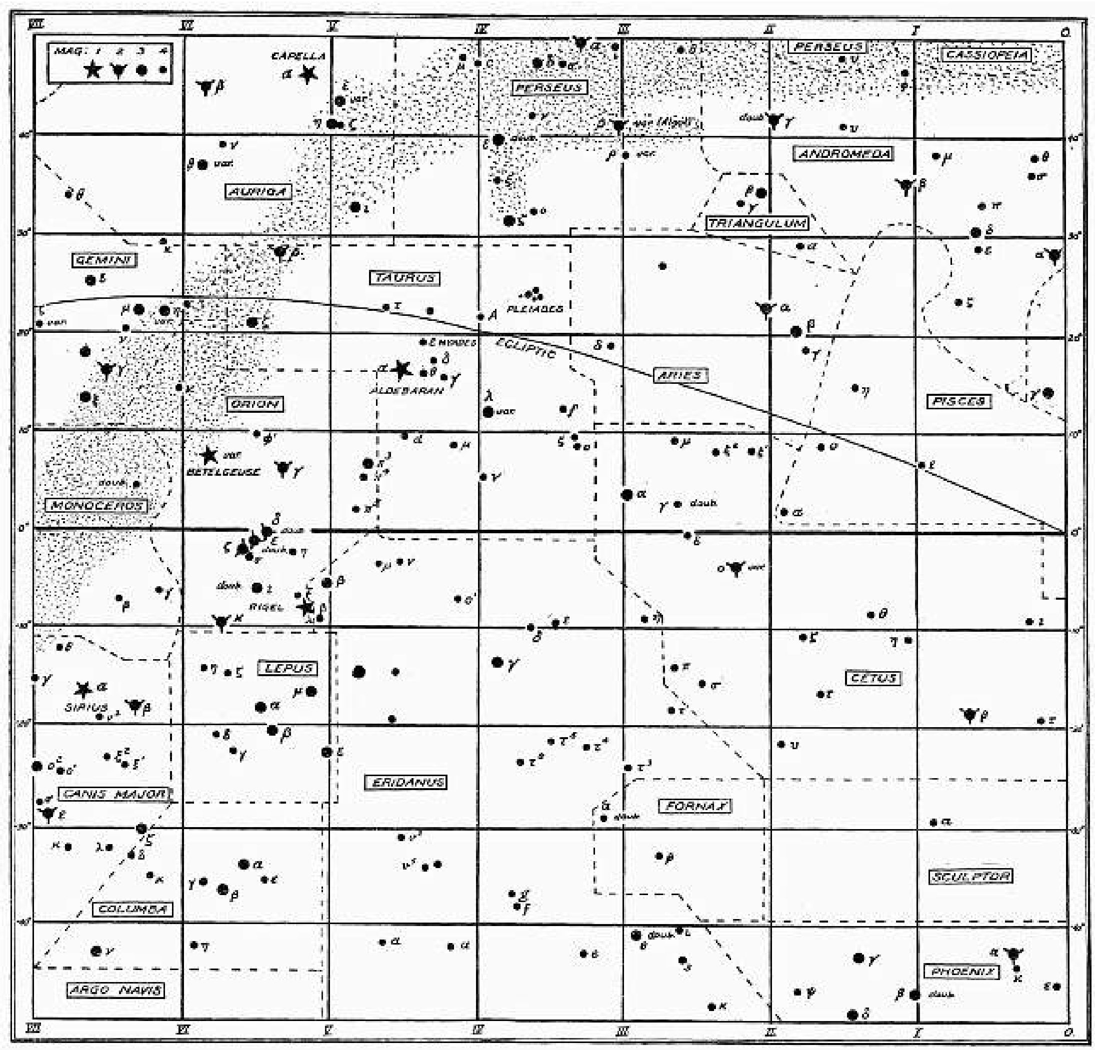
CANES VENATICI
BOOTES
LEO MINOR
LYNX
AURIGA
CAPELLA
CAMELOPARDALIS
URSA MINOR
DRACO
HERCULES
LYRA
CYGNUS
DENEB
CEPHEUS
CASSIOPEIA
LACERTA
ANDROMEDA
PERSEUS
POLE OF ECLIPTIC
PATH OF POLE

MAP II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TRIANGULUM
GEMINI
TAURUS
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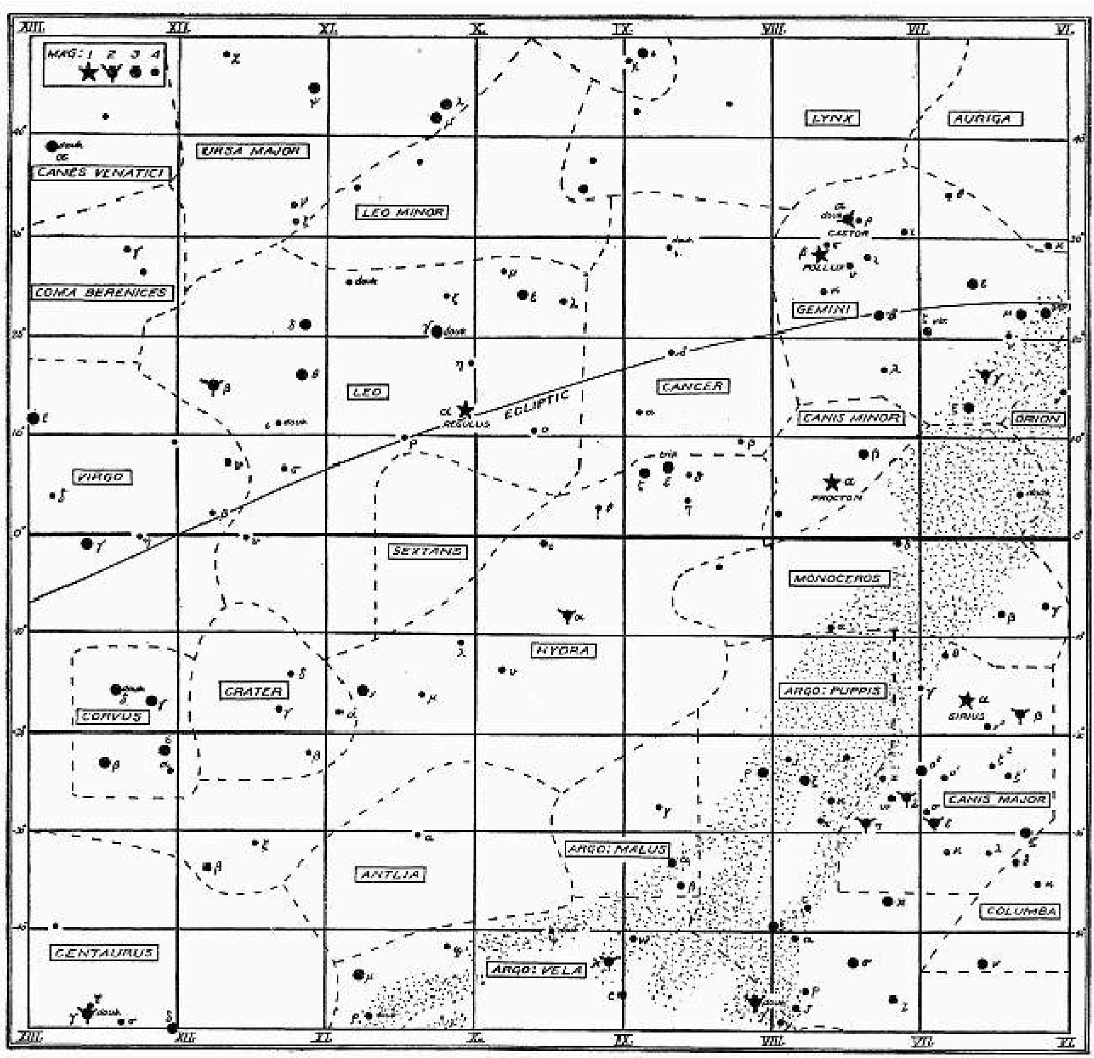
HYADES
ECLIPTIC
ALDEBARAN
ARIES
PISCES
ORION
BETELGEUSE
MONOCEROS
RIGEL
LEPUS
CETUS
SIRIUS
CANIS MAJOR
ERIDANUS
FORNAX
COLUMBA
SCULPTOR
PHOENIX
ARGO NAVIS

MAP III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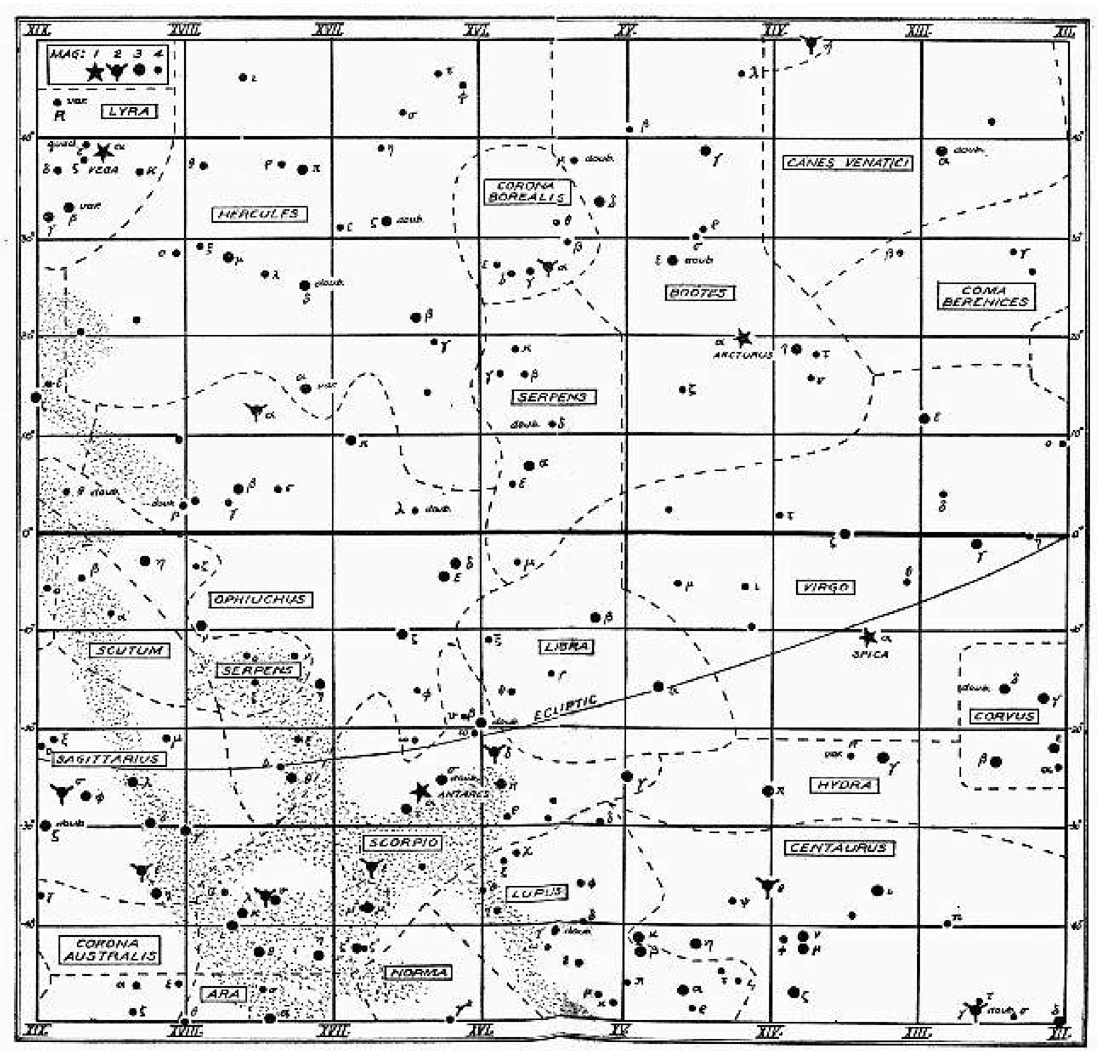
LYNX
AURIGA
CANES VENATICI
LEO MINOR
CASTOR
POLLUX
COMA BERENICES
GEMINI
LEO
CANCER
REGULUS
ECLIPTIC
CANIS MINOR
ORION
VIRGO
PROCYON
SEXTANS
MONOCEROS
HYDRA
CRATER
ARGO: PUPPIS
CORVUS
SIRIUS
CANIS MAJOR
ARGO: MALUS
ANTLIA
COLUMBA
CENTAURUS
ARGO: VELA

MAP IV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LYRA
VEGA
HERCULES
CORONA BOREALIS
CANES VENATICI
BOOTES
COMA BERENICES
ARCTURUS
SERPENS
OPHIUCHUS
VIRGO
SCUTUM
SERPENS
LIBRA
SPICA
CORVUS
ECLIPTIC
SAGITTARIUS
ANTARES
HYDRA
SCORPIO
CENTAURUS
LUPUS
CORONA AUSTRALIS
NORMA
ARA

Map V

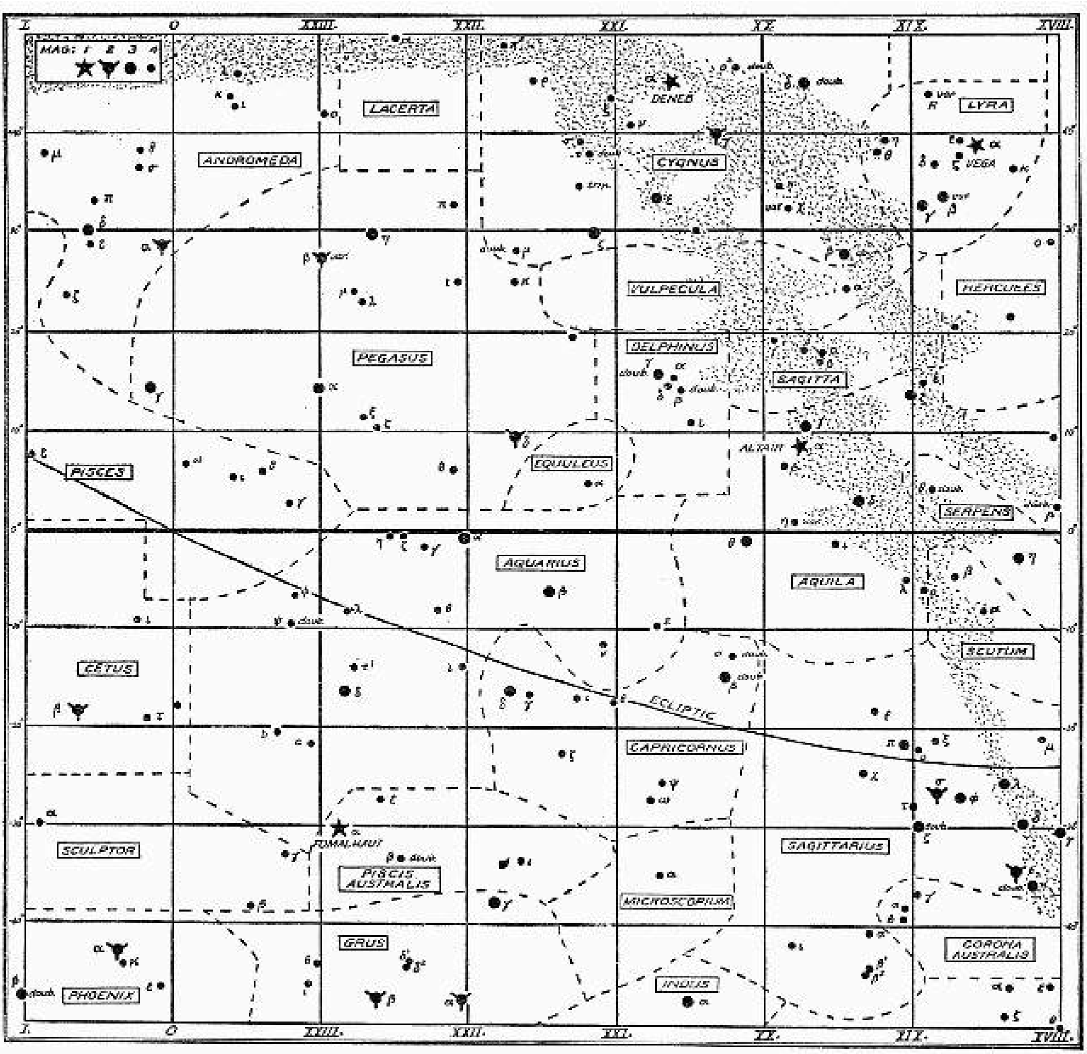
MAG: 1 2 3 4
I
O
XXIII
XXII
XXI
XX
XIX
XVIII
LACERTA
ANDROMEDA
CYGNUS
DENEB
LYRA
VEGA
HERCULES
VULPECULA
DELPHINUS
SAGITTA
PEGASUS
PISCES
EQUULEUS
ALTAIR
SERPENS
AQUARIUS
AQUILA
CETUS
SCUTUM
ECLIPTIC
CAPRICORNUS
SCULPTOR
FOMALHAUT
PISCIS AUSTRALIS
SAGITTARIUS
MICROSCOPIUM
GRUS
CORONA AUSTRALIS
INDUS
PHOENIX

FIG. 56.—The Big Dipper.

Besides defining the north-and-south line and serving as a guide for a study of the constellations in the northern heavens, the pole star is an interesting object in several other respects. It has a faint companion of the ninth magnitude, distant from it about 18.5 seconds of arc. This faint companion cannot be seen with the unaided eye because, in order that two stars may be seen as separate objects without a telescope, they must be distant from each other at least 3 minutes of arc, and, besides, they must not be too bright or too faint. The brighter of the two components of Polaris is also a double star, a fact which was discovered by means of the spectroscope in 1899. Indeed, it has turned out on more recent study at the Lick Observatory that the principal star of this system is really a triple sun.

84. Units for Estimating Angular Distances.—The distances between stars, as seen projected on the celestial sphere, are always given in degrees. There is, in fact, no definite content to the statement that two stars seem to be a yard apart. In order to estimate angular distances, it is important to have a few units of known length which can always be seen.

It is 90° from the horizon to the zenith, and one would suppose that it would be a simple matter to estimate half of this distance. As a matter of fact, few people place the zenith high enough. In order to test the accuracy with which one locates it, he should face the north and fix his attention on the star which he judges to be at the zenith, and then, keeping it in view, turn slowly around until he faces the south. The first trial is apt to furnish a surprise.

The altitude of the pole star is equal to the latitude of the observer which, in the United States, is from 25° to 50°. This unit is not so satisfactory as some others because it depends upon the position of the observer and also because it is more difficult to estimate from the horizon to a star than it is between two stars. Another large unit which can always be observed from northern latitudes is the distance between

Alpha Ursæ Majoris and Polaris, which is 28°. For a smaller unit the distance between The Pointers in the Big Dipper, which is $5° 20'$, is convenient.

85. Ursa Major (The Greater Bear).—The Big Dipper, to which reference has already been made, and which is one of the most conspicuous configurations in the northern heavens, is in the eastern part[1] of the constellation Ursa Major and serves to locate the position of this constellation. The outline of the Bear extends north, south, and west of the bowl of the Dipper for more than 10°; but all the stars in this part of the sky are of the third magnitude or fainter.

According to the Greek legend, Zeus changed the nymph Callisto into a bear in order to protect her from the jealousy of his wife Hera. While the transformed Callisto was wandering in the forest, she met her son Arcas, who was about to slay her when Zeus intervened and saved her by placing them both among the stars, where they became the Greater and the Smaller Bears. Hera was still unsatisfied and prevailed on Oceanus and Thetis to cause them to pursue forever their courses around the pole without resting beneath the ocean waves. Thus was explained the circumpolar motions of those stars which are always above the horizon.

The Pawnee Indians call the stars of the bowl of the Dipper a stretcher on which a sick man is being carried, and the first one in the handle is the medicine man.

The star at the bend of the handle of the Dipper, called Mizar by the Arabs, has a faint one near it which is known as Alcor. Mizar is of the second magnitude, and Alcor is of the fifth. Any one with reasonably good eyes can see the two stars as distinct objects, without optical aid. It is probable that this was the first double star that was discovered. The distance of $11'.5$ between them is so great, astronomically speaking, that it is no longer regarded as a true double star. It has been supposed by some writers that the word Alcor is derived from an Arabic word meaning the test, and the Arabs are said to have tested their eyesight on it. The Pawnee Indians call it the Medicine Man's Wife's Dog.

[1] East and west on the sky must be understood to be measured along declination circles. Consequently, near the pole east may have any direction with respect to the horizon. Above the pole, east on the sky is toward the eastern part of the horizon, while below the pole it is toward the western part of the horizon. All statements of direction in descriptions of the constellations refer to directions on the sky unless otherwise indicated, and care must be taken not to understand them in any other sense.

The star Mizar itself is a fine telescopic double, the first one ever discovered; the two components are distant from each other 14″.6 and can be seen separately with a 3-inch telescope. The distance from the earth to Mizar, according to the work of Ludendorff, is 4,800,000 times as far as from the earth to the sun, and about 75 years are required for light to come from it to us. The star appears to be faint only because of its immense distance, for, as a matter of fact, it radiates 115 times as much light as is given out by the sun. The actual distance even from Mizar to Alcor, which is barely discernible with the unaided eye, is 16,000 times as far as from the earth to the sun.

The first of a series of very important discoveries was made by E. C. Pickering, in 1889, by spectroscopic observations of the brighter component of Mizar. It was found by methods which will be discussed in Arts. 285 and 286 that this star is itself a double in which the components are so close together that they cannot be distinguished separately with the aid of any existing telescope. Such a star is called a spectroscopic binary. The complete discussion showed that the brighter component of Mizar is composed of two great suns whose combined mass is many times that of our sun, and that they revolve about their common center of gravity at a distance of 25,000,000 miles from each other in a period of 20.5 days.

86. Cassiopeia (The Woman in the Chair).—To find Cassiopeia go from the middle of the handle of the Big Dipper through Polaris and about 30° beyond. The constellation will be recognized because the principal stars of which it is composed, ranging in magnitude from the second to the fourth, form a zigzag, or letter *W*. When it is tilted in a particular way as it moves around the pole in its diurnal motion, it has some resemblance to the outline of a chair. The brightest of the 7 stars in the *W* is the one at the bottom of its second part, and a 2-inch telescope will show that it is a double star whose colors are described as rose and blue.

One of the most interesting objects in this constellation is the star Eta Cassiopeiæ, which is near the middle of the third stroke of the *W* and about 2° from Alpha. It is a fine double which can be separated with a 3-inch telescope. The two stars are not only apparently close together, but actually form a physical system, revolving around their common center of gravity in a period of about 200 years. If there are planets revolving around either of these stars, their phenomena of night and day and their seasons must be very complicated.

In 1572 a new star suddenly blazed forth in Cassiopeia and became brighter than any other one in the sky. It caught the attention of Tycho Brahe, who was then a young man, and did much to stimulate his interest in astronomy.

87. How to Locate the Equinoxes.—It is advantageous to know how to locate the equinoxes when the positions of objects are defined by their right ascensions and declinations. To find the vernal equinox, draw a line from Polaris through the most westerly star in the *W* of Cassiopeia, and continue it 90°. The point where it crosses the equator is the vernal equinox which, unfortunately, has no bright stars in its neighborhood.

If the vernal equinox is below the horizon, the autumnal equinox may be conveniently used. One or the other of them is, of course, always above the horizon. To find the autumnal equinox, draw a line from Polaris through Delta Ursæ Majoris, or the star where the handle of the Big Dipper joins the dipper, and continue it 90° to the equator. The autumnal equinox is in Virgo. This constellation contains the first-magnitude star Spica, which is about 10° south and 20° east of the autumnal equinox.

88. Lyra (The Lyre, or Harp).—Lyra is a small but very interesting constellation whose right ascension is about 18.7 hours and whose declination is about 40° north. It is, therefore, about 50° from the pole, and its position can easily be determined by using the directions for finding the vernal and autumnal equinoxes. Or, its distance east or west of the meridian can be determined by the methods of Art. 69. With an approximate idea of its location, it can always be found because it contains the brilliant bluish-white, first-magnitude star Vega. If there should be any doubt in regard to the identification of Vega, it can always be dispelled by the fact that this star, together with two fourth-magnitude stars, Epsilon and Zeta Lyræ, form an equilateral triangle whose sides are about 2° in length. There are no other stars so near Vega, and there is no other configuration of this character in the whole heavens.

As was stated in Art. 47, the attractions of the moon and sun for the equatorial bulge of the earth cause a precession of the earth's equator, and therefore a change in the location of the pole of the sky. About 12,000 years from now the north pole will be very close to Vega. What a splendid pole star it will make! It is approaching us at the rate of 8.5 miles per second, but its distance is so enormous that even this high velocity will make no appreciable change in its brightness in the next 12,000 years. The distance of Vega is not very accurately known, but it is probably more than 8,000,000 times as far from the earth as the earth is from the sun. At its enormous distance the sun would appear without a telescope as a faint star nearly at the limits of visibility. Another point of interest is that the sun with all its planets is moving nearly in the direction of Vega at the rate of about 400,000,000 miles a year.

The star Epsilon Lyræ, which is about 2° northeast of Vega, is an object which should be carefully observed. It is a double star in which the apparent distance between the two components is 207″. They are barely distinguishable as separate objects with the unaided eye even by persons of perfect eyesight. It is a noteworthy fact that, so far as is known, this star was not seen to be a double by the Arabs, the early Greeks, or any primitive peoples. A century ago astronomers gave their ability to separate this pair without the use of the telescope as proof of their having exceptionally keen sight. Perhaps with the more exacting use to which the eyes of the human race are being subjected, they are actually improving instead of deteriorating as is commonly supposed.

Although the angular distance between the two components of Epsilon Lyræ seems small, astronomers regularly measure one two-thousandth of this angle. The discovery of Neptune was based on the fact that in 60 years it had pulled Uranus from its predicted place, as seen from the earth, only a little more than half of the angular distance between the components of this double star. When Epsilon Lyræ is viewed through a telescope of 5 or 6 inches' aperture, it presents a great surprise. The two components are found to be so far apart in the telescope that they can hardly be seen at the same time, and a little close attention shows that each of them also is a double. That is, the faint object Epsilon Lyræ is a magnificent system of four suns.

About 5°.5 south of Vega and 3° east is the third-magnitude star Beta Lyræ. It is a very remarkable variable whose brightness changes by nearly a magnitude in a period of 12 days and 22 hours. The variability of this star is due to the fact that it is a double whose plane of motion passes nearly through the earth so that twice in each complete revolution one star eclipses the other. A detailed study of the way in which the light of this star varies shows that the components are stars whose average density is approximately that of the earth's atmosphere at sea level.

About 2°.5 southeast of Beta Lyræ is the third-magnitude star Gamma Lyræ. On a line joining these two stars and about one third of the distance from Beta is a ring, or annular, nebula, the only one of the few that are known that can be seen with a small telescope. It takes a large telescope, however, to show much of its detail.

89. Hercules (The Kneeling Hero).—Hercules is a very large constellation lying west and southwest of Lyra. It contains no stars brighter than the third magnitude, but it can be recognized from a trapezoidal figure of 5 stars which are about 20° west of Vega. The base of the trapezoid, which is turned to the north and slightly to the east, is about 6° long and contains two stars in the northeast corner which are of the third and fourth magnitudes. The star in the southeast corner is of the fourth magnitude, and the others are of the third magnitude. On the west side of the trapezoid, about one third of the distance from the north end, is one of the finest star clusters in the whole heavens, known as Messier 13. It is barely visible to the unaided eye on a clear dark night, appearing as a little hazy star; but through a good telescope it is seen to be a wonderful object, containing more than 5000 stars (Fig. 171) which are probably comparable to our own sun in dimensions and brilliancy. The cluster was discovered by Halley (1656–1742), but derives its present name from the French comet hunter Messier (1730–1817), who did all of his work with an instrument of only 2.5 inches' aperture.

90. Scorpius (The Scorpion).—There are 12 constellations, one for each month, which lie along the ecliptic and constitute the zodiac. Scorpius is the ninth of these and the most brilliant one of all. In fact, it is one of the finest group of stars that can be seen from our latitude. It is 60° straight south of Hercules and can always be easily recognized by its fiery red first-magnitude star Antares, which, in light-giving power, is equal to at least 200 suns such as ours. The word Antares means opposed to, or rivaling, Mars, the red planet associated with the god of war. Antares is represented as occupying the position of the heart of a scorpion. About 7″ west of Antares is a faint green star of the sixth magnitude which can be seen through a 5- or 6-inch telescope under good atmospheric conditions. About 5° northwest of Antares is a very compact and fine cluster, Messier 80. Scorpius lies in one of the richest and most varied parts of the Milky Way.

According to the Greek legend, Scorpius is the monster that killed Orion and frightened the horses of the sun so that Phaëton was thrown from his chariot when he attempted to drive them.

91. Corona Borealis (The Northern Crown).—Just west of the great Hercules lies the little constellation Corona Borealis. It is easily recognized by the semicircle, or crown, of stars of the fourth and fifth magnitudes which opens toward the northeast. The Pawnee Indians called it the camp circle, and it is not difficult to imagine that the stars represent warriors sitting in a semicircle around a central campfire.

92. Boötes (The Hunter).—Boötes is a large constellation lying west of Corona Borealis, in right ascension about 14 hours, and extending from near the equator to within 35° of the pole. It always can be easily recognized by its bright first-magnitude star Arcturus, which is about 20° southwest of Corona Borealis. This star is a deep orange in color and is one of the finest stars in the northern sky. It is so far away that 100 years are required for its light to come to the earth, and in radiating power it is equivalent to more than 500 suns like our own.

In mythology Boötes is represented as leading his hunting dogs in their pursuit of the bear across the sky.

FIG. 57.—The sickle in Leo, as seen when it is on the meridian.

93. Leo (The Lion).—Leo lies about 60° west of Arcturus and is the sixth zodiacal constellation. It is easily recognized by the fact that it contains 7 stars which form the outline of a sickle. In the photograph, Fig. 57, only the 5 brightest stars are shown. The most southerly star of the sickle is Regulus, at the end of the handle. The blade of the sickle opens out toward the southwest. One of the most interesting things in connection with this constellation is that the meteors of the shower which occurs about November 14 seem to radiate from a point within the blade of the sickle (Art. 204).

The star Regulus is at the heart of the Nemean lion which, according to classic legends, was killed by Hercules as the first of his twelve great labors.

94. Andromeda (The Woman Chained).—Andromeda is a large constellation just south of Cassiopeia. It contains no first-magnitude stars, but it can be recognized from a line of 3 second-magnitude stars extending northeast and southwest. The most interesting object in this constellation is the Great Andromeda Nebula, Fig. 58, the brightest nebula in the sky. It is about 15° directly south of Alpha Cassiopeiæ, and it can be seen without difficulty on a clear, moonless night as a hazy patch of light. When viewed through a telescope it fills a part of the sky nearly 2° long and 1° wide. In its center is a star which is

FIG. 58.—The Great Andromeda Nebula. *Photographed by Ritchey with the two-foot reflector of the Yerkes Observatory.*

probably variable. The analysis of its light with the spectroscope seems to indicate that it is composed of solid or liquid material surrounded by cooler gases. It has been suggested that, instead of being a nebula, it may be an aggregation of millions of suns comparable to the Galaxy, but so distant from us that it apparently covers an insignificant part of the sky.

95. Perseus (The Champion).—Perseus is a large constellation in the Milky Way directly east of Andromeda. Its brightest star, Alpha, is in the midst of a star field which presents the finest spectacle through field glasses or a small telescope in the whole sky. The second brightest star in this constellation is the earliest known variable star, Algol (the Demon). Algol is about 9° south and a little west of Alpha Persei, and varies in magnitude from 2.2 to 3.4 in a period of 2.867 days. That is, at its minimum it loses more than two thirds of its light. There is also a remarkable double cluster in this constellation about 10° east of Alpha Cassiopeiæ.

Algol, together with the little stars near it, is the Medusa's head which Perseus is supposed to carry in his hand and which he used in the rescue of Andromeda. He is said to have stirred up the dust in heaven in his haste, and it now appears as the Milky Way.

96. Auriga (The Charioteer).—The next constellation east of Perseus is Auriga, which contains the great first-magnitude star Capella. Capella is about 40° from the Big Dipper and nearly in a line from Delta through Alpha Ursæ Majoris. It is also distinguished by the fact that near it are 3 stars known as The Kids, the name Capella meaning The She-goat. It is receding from us at the rate of nearly 20 miles per second and its distance is 2,600,000 times that of the earth from the sun. It was found at the Lick Observatory, in 1889, to be a spectroscopic binary with a period of 104.2 days. The computations of Maunder show that it radiates about 200 times as much light as is given out by the sun.

97. Taurus (The Bull).—Taurus is southwest of Auriga and contains two conspicuous groups of stars, the Pleiades and the Hyades, besides the brilliant red star Aldebaran.

Among the many mythical stories regarding this constellation there is one which describes the bull as charging down on Orion. According to a Greek legend, Zeus took the form of a bull when he captured Europa, the daughter of Agenor. While playing in the meadows with her friends, she leaped upon the back of a beautiful white bull, which was Zeus himself in disguise. He dashed into the sea and bore her away to Crete. Only his head and shoulders are visible in the sky because, when he swims, the rest of his body is covered with water.

The Pleiades group, Fig. 59, consists of 7 stars in the form of a little dipper about 30° southwest of Capella and nearly 20° south of, and a little east of, Algol. Six of them, which are of the fourth magnitude, are easily visible without optical aid; but the seventh, which is near the one at the end of the handle in the dipper, is more difficult. There seems to have been considerable difficulty in seeing the faintest one in ancient times, for it was frequently spoken of as having been lost. There is no

FIG. 59.—The Pleiades. *Photographed by Wallace at the Yerkes Observatory.*

difficulty now, however, for people with good eyes to see it, while those with exceptionally keen sight can see 10 or 11 stars.

No group of stars in all the sky seems to have attracted greater popular attention than the Pleiades, nor to have been mentioned more frequently, not only in the classic writings of the ancients, but also in the stories of primitive peoples. They were The Seven Sisters of the Greeks, The Many Little Ones of the ancient Babylonians, The Hen and Chickens of the peoples of many parts of Europe, The Little Eyes of the savage tribes of the South Pacific Islands, and The Seven Brothers of some of the tribes of North American Indians. They cross the meridian at midnight in November, and many primitive peoples began their year at that time. It is said that on the exact date, November 17, no petition was ever presented in vain to the kings of ancient Persia. These stars had an important relation to the religious ceremonies of the Aztecs, and certain of the Australian tribes held dances in their honor.

Besides the 7 stars which make up the Pleiades as observed without a telescope, there are at least 100 others in the group which can be seen with a small instrument. While their distance from the earth is not known, it can scarcely be less than 10,000,000 times that of the sun. It follows that these stars are apparently small only because they are so remote. A star among them equal to the sun in brilliancy would appear to us as a telescopic object of the ninth magnitude. The larger stars of the group are at least from 100 to 200 times as great in light-giving power as the sun.

About 8° southeast of the Pleiades is the Hyades group, a cluster of small stars scarcely less celebrated in mythology. They have been found recently to constitute a cluster of stars, occupying an enormous space, all of which move in the same direction with almost exactly equal speeds (Art. 277). The magnificent scale of this group of stars is quite beyond imagination. Individually they range in luminosity from 5 to 100 times that of the sun, and the diameter of the space which they occupy is more than 2,000,000 times the distance from the earth to the sun.

98. Orion (The Warrior).—Southeast of Taurus and directly south of Auriga is the constellation Orion, lying across the equator between the fifth and sixth hours of right ascension. This is the finest region of the whole sky for observation without a telescope.

The legends regarding Orion are many and in their details conflicting. But in all of them he was a giant and a mighty hunter who, in the sky, stands facing the bull (Taurus) with a club in his right hand and a lion's skin in his left.

About 7° north of the equator and 15° southeast of Aldebaran is the ruddy Betelgeuze. About 20° southwest of Betelgeuze is the first-magnitude star Rigel, a magnificent object which is at least 2000 times as luminous as the sun. About midway between Betelgeuze and Rigel and almost on the equator is a row of second-magnitude stars running northwest and southeast, which constitute the Belt of Orion, Fig. 60. From its southern end another row of fainter stars reaches off to the southwest, nearly in the direction of Rigel. These stars constitute the Sword of Orion. The central one of them appears a little fuzzy without

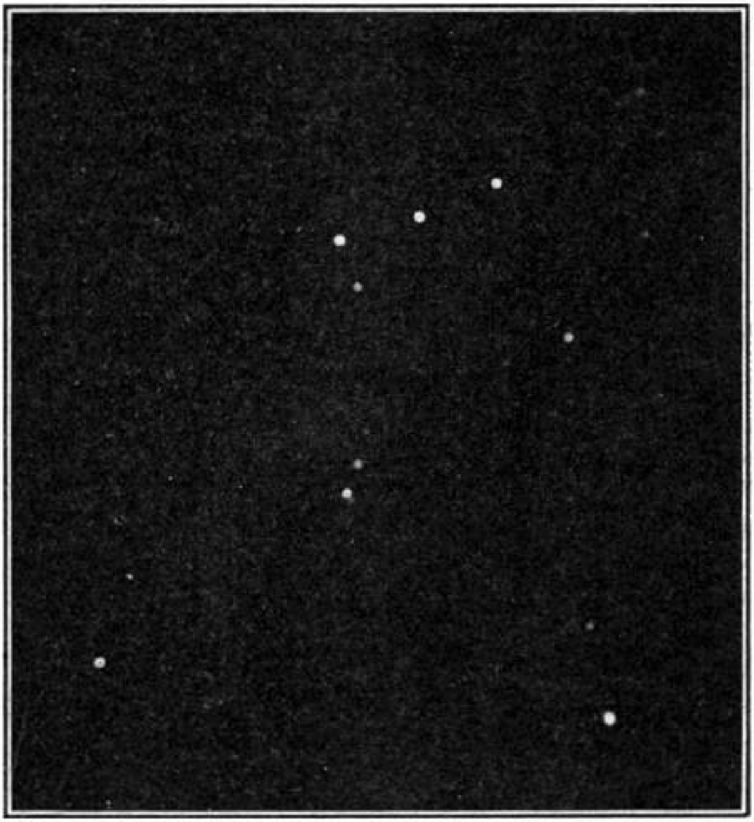

FIG. 60.—Orion. *Photographed at the Yerkes Observatory* (*Hughes*).

a telescope, and with a telescope is found to be a magnificent nebula, Fig. 61. In fact, the Great Orion Nebula impresses many observers as being the most magnificent object in the whole heavens. It covers more than a square degree in the sky, and the spectroscope shows it to be a mass of glowing gas whose distance is probably several million times as great as that to the sun, and whose diameter is probably as great as the distance from the earth to the nearest star. The stars in this region of the sky are generally supposed by astronomers to be in an early stage of their development; most of them are of great luminosity, and a considerable fraction of them are variable or double.

99. Canis Major (The Greater Dog).—The constellation Canis Major is southeast of Orion and is marked by Sirius, the brightest star in the whole sky. Sirius is almost in a line with the Belt of Orion and a little more than 20° from it. It is bluish white in color and is supposed to be in an early stage of its evolution, though it has advanced

FIG. 61.—The Great Orion Nebula. *Photographed by Ritchey with the two-foot reflector of the Yerkes Observatory.*

somewhat from the condition of the Orion stars. Sirius is comparatively near to us, being the third star in distance from the sun. Nevertheless, 8.4 years are required for its light to come to us, and its distance is 47,000,000,000,000 miles. It is approaching us at the rate of 5.6 miles per second; or, rather, it is overtaking the sun, for the solar system is moving in nearly the opposite direction.

The history of Sirius during the last two centuries is very interesting, and furnishes a good illustration of the value of the deductive method in making discoveries. First, Halley found, in 1718, that Sirius has a motion with respect to fixed reference points and lines; then, a little more than a century later, Bessel found that this motion is slightly variable. He inferred from this, on the basis of the laws of motion, that Sirius and an unseen companion were traveling around their common center of gravity which was moving with uniform speed in a straight line. This companion actually was discovered by Alvan G. Clark, in 1862, while adjusting the 18-inch telescope now of the Dearborn Observatory, at Evanston, Ill. The distance of the two stars from each other is 1,800,000,000 miles, and they complete a revolution in 48.8 years. The combined mass of the two stars is about 3.4 times that of the sun. The larger star is only about twice as massive as its companion but is 20,000 times brighter; together they radiate 48 times as much light as is emitted by the sun.

100. Canis Minor (The Lesser Dog).—Canis Minor is directly east of Orion and is of particular interest in the present connection because of its first-magnitude star Procyon, which is about 25° east and just a little south of Betelgeuze. The history of this star is much the same as that of Sirius, the fainter companion having been discovered in 1896 by Schaeberle at the Lick Observatory. The period of revolution of Procyon and its companion is 39 years, its distance is a little greater than that of Sirius, its combined mass is about 1.3 that of the sun, and its luminosity is about 10 times that of the sun. If the orbits of such systems as Sirius and Procyon and their fainter companions were edgewise to the earth, the brighter components would be regularly eclipsed and they would be variable stars of the Algol type (Art. 288), though with such long periods and short times of eclipse that their variability would probably not be discovered.

101. Gemini (The Twins).—Gemini is the fourth zodiacal constellation and lies directly north of Canis Minor. It has been known as "The Twins" from the most ancient times because its two principal stars, Castor and Pollux, are almost alike and only 4°.5 apart. These stars are about 25° north of Procyon, and Castor is the more northerly of the two. Castor is a double star which can be separated by a small telescope. In 1900 Bélopolsky, of Pulkowa, found that its fainter companion is a spectroscopic binary with a period of 2.9 days. In 1906 Curtis, of the Lick Observatory, found that the brighter companion is also a spectroscopic binary with a period of 9.2 days. Thus this star, instead of being a single object as it appears to be without telescopic and spectroscopic aid, is a system of four suns. The two pairs revolve about the common center of gravity of the four stars in a long period which probably lies between 250 and 2000 years.

Castor is called Alpha Geminorum, because probably in ancient times it was a little brighter than, or at least as bright as, Pollux. Now Pollux is a little brighter than Castor.

About 10° southeast of Pollux is the large open Præsepe (The Beehive) star cluster which can be seen on a clear, moonless night without a telescope.

102. On Becoming Familiar with the Stars.—The discussion of the constellations will be closed here, not because all have been described, or, indeed, any one of them adequately, but because enough has been said to show that the sky is full of objects of interest which can be found and enjoyed with very little optical aid. The reader is expected to observe all the objects which have been described, so far as the time of year and the instrumental help at his command will permit. If he does this, the whole subject will have a deeper and more lively interest, and it will be a pleasure to make constant appeals to the sky to verify statements and descriptions.

The general features of the constellations are very simple, but the whole subject cannot be mastered in an evening. One should go over it several times with no greater optical aid than that furnished by a field glass.

VIII. QUESTIONS

1. Show why about 22,000 plates will be required to photograph the whole sky as described in Art. 76.

2. Find the brightness of the stars in Table I compared to that of a first-magnitude star.

3. Find the amount of light received from the sun compared to that received from a first-magnitude star.

4. Take the amount of light received from a first-magnitude star as unity, and compute the amount of light received from each of the first six magnitudes (Table II).

5. If the ratio of the number of stars from one magnitude to the next continued the same as it is in Table II, how many stars would there be in the first 20 magnitudes?

6. At what time of the year is the most northerly part of the Milky Way on the meridian at 8 p.m.? What are its altitude and azimuth at that time?

7. What constellations are within two hours of the meridian at 8 p.m. to-night? Identify them.

8. If Lyra is visible at a convenient hour, test your eyes on Epsilon Lyræ.

9. If Leo is visible at a convenient hour, test your eyes by finding which star in the sickle has a very faint star near it.

10. If Andromeda is visible at a convenient hour, find the great nebula.

11. How many stars can you see in the bowl of the Big Dipper?

12. If Perseus is visible at a convenient hour, identify Algol and verify its variability.

13. How many of the Pleiades can you see?

14. If Orion is visible at a convenient hour, identify the Belt and Sword and notice that the great nebula looks like a fuzzy star.

CHAPTER VI

TIME

103. Definitions of equal Intervals of Time.—It is impossible to give a definition of time in terms which are simpler and better understood than the word itself; but it is profitable to consider what it is that determines the length of an interval of time. The subject may be considered from the standpoint of the intellectual experience of the individual, which varies greatly from time to time and which may differ much from that of another person, or it may be treated with reference to independent physical phenomena.

Consider first the definition of the length of an interval of time or, rather, the equality of two intervals of time, from the psychological point of view. If a person has had a number of intellectual experiences, he is not only conscious that they were distinct, but he has them arranged in his memory in a perfectly definite order. When he recalls them and notes their distinctness, number, and order, he feels that they have occurred in time; that is, he has the perception of time. An interval in which a person has had many and acute intellectual experiences seems long; and two intervals of time are of equal length, psychologically, when the individual has had in them an equal number of equally intense intellectual experiences. For example, in youth when most of life's experiences are new and wonderful, the months and the years seem to pass slowly; on the other hand, with increasing age when life reduces largely to routine, the years slip away quickly. Or, to take an illustration within the range of the experience of many who are still young, a month of travel, or the first month in college, seems longer than a whole year in the accustomed routine of preparatory school life. It follows from these considerations that the true measure of the length of the life of an individual from the psychological point of view, which is the one in which he has greatest interest as a thinking being, is the number, variety, and intensity of his intellectual experiences. A man whose life has been full, who has become acquainted with the world's history, who is familiar with the wonders of the universe, who has read and experienced again the finest thoughts of the best minds of all ages, who has seen many places and come into contact with many men, and who has originated ideas and initiated intellectual movements of his own, has lived a long life, however few may have been the number of revolutions of the earth around the sun since he was born.

But since men must deal with one another, it is important to have some definition of the equality of intervals of time that will be independent of their varying intellectual life. The definition, or at least its consequences, must be capable of being applied at any time or place, and it must not disagree too radically with the psychological definition. Such a definition is given by the first law of motion (Art. 40), or rather a part of it, which for present purposes will be reworded as follows:

Two intervals of time are equal, by definition, if a moving body which is subject to no forces passes over equal distances in them. It is established by experience that it makes no difference what moving body is used or at what rate it moves, for they all give the same result.

104. The Practical Measure of Time.—A difficulty with the first law of motion and the resulting definition of equal intervals of time arises from the fact that it is impossible to find a body which is absolutely uninfluenced by exterior forces. Therefore, instead of using the law itself, one of its indirect consequences is employed. It follows from this law, together with the other laws of motion, that a solid, rotating sphere which is subject to no exterior forces turns at a uniform rate. There is no rotating body which is not subject to at least the attraction of other bodies; but the simple attraction of an exterior body has no influence on the rate of rotation of a sphere which is perfectly solid. Therefore the earth rotates at a uniform rate, according to the definition of uniformity implied in the first law of motion, except for the slight and altogether negligible modifying influences which were enumerated in Art. 45, and hence can be used for the measurement of time.

If the rotation of the earth is to be used in the measurement of time, it is only necessary to determine in some way the angle through which it turns in any interval under consideration. This can be done by observations of the position of the meridian with reference to the stars. Since the stars are extremely far away and do not move appreciably with respect to one another in so short an interval as a day, the rotation of the earth can be measured by reference to any of them. Let it be remembered that, though the rate of the rotation of the earth is subject to some possible slight modifications, its uniformity is far beyond that of any clock ever made.

105. Sidereal Time.—Sidereal time is the time defined by the rotation of the earth with respect to the stars. A sidereal day is the interval between the passage of the meridian, in its eastward motion, across a star and its next succeeding passage across the same star. Since the earth rotates at a uniform rate, all sidereal days are of the same length. The sidereal day is divided into 24 sidereal hours, which are numbered from 1 to 24, the hours are divided into 60 minutes, and the minutes into 60 seconds. The sidereal time of a given place on the earth is zero when its meridian crosses the vernal equinox.

Since the definition of sidereal time depends upon the meridian of the observer, it follows that all places on the earth having the same longitude have the same sidereal time, and that those having different longitudes have different sidereal time. It follows from the uniformity of the earth's rotation that equal intervals of sidereal time are equal according to the first law of motion.

106. Solar Time.—Solar time is defined by the rotation of the earth with respect to the sun. A solar day is the interval of time between the passage of a meridian across the center of the sun and its next succeeding passage across the center of the sun. Since the sun apparently moves eastward among the stars, a solar day is longer than the sidereal day. The sun makes an apparent revolution of the heavens in 365 days, and therefore, since the circuit of the heavens is 360°, it moves eastward on the average a little less than 1° a day. The earth turns 15° in 1 hour, and 1° in 4 minutes, from which it follows that the solar day is nearly 4 minutes longer on the average than the sidereal day.

107. Variations in the Lengths of Solar Days.—If the apparent motion of the sun eastward among the stars were uniform, each solar day would be longer than the sidereal day by the same amount; and since the sidereal days are all of equal length, the solar days also would all be of equal length. But the eastward apparent motion of the sun is somewhat variable because of two principal reasons, which will now be explained.

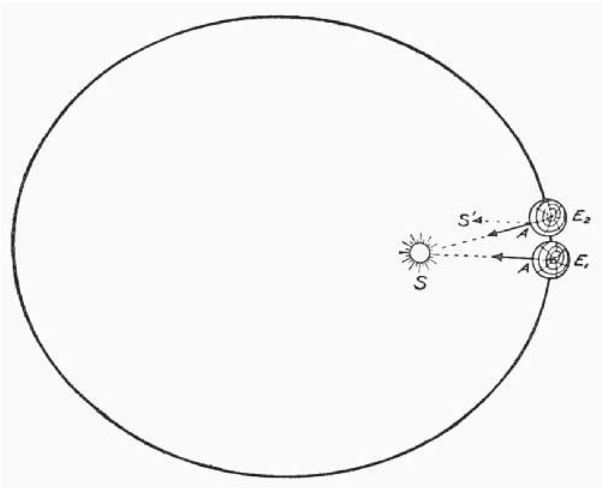

FIG. 62.—Solar days are longer than sidereal days.

The earth moves in

its elliptical orbit around the sun in such a way that the law of areas is fulfilled. The angular distance the sun appears to move eastward among the stars equals the angular distance the earth moves forward in its orbit. This is made evident from Fig. 62, in which E_1 represents the position of the earth when it is noon at A. At the next noon at A, solar time, the earth has moved forward in its orbit through the angle E_1SE_2 (of course the distance is greatly exaggerated). Suppose that when the earth is at E_1 the direction of a star is E_1S. When the earth is at E_2, the same direction is E_2S'. The sun has apparently moved through the angle $S'E_2S$, which equals E_2SE_1.

Since the earth moves in its orbit in accordance with the law of areas, its angular motion is fastest when it is nearest the sun. Consequently,

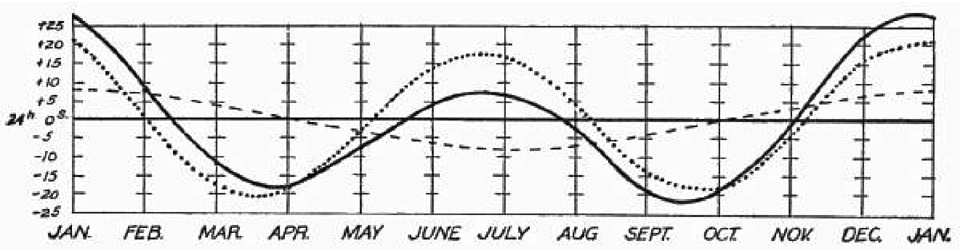

FIG. 63.—Length of solar days. Broken line gives effects of eccentricity; dotted line, the inclination; full line, the combined effects.

when the earth is at its perihelion the sun's apparent motion eastward is fastest, and the solar days, so far as this factor alone is concerned, are then the longest. The earth is at its perihelion point about the first of January and at its aphelion point about the first of July. Consequently, the time from noon to noon, so far as it depends upon the eccentricity of the earth's orbit, is longest about the first of January and shortest about the first of July. The lengths of the solar days, so far as they depend upon the eccentricity of the earth's orbit, are shown by the broken line in Fig. 63.

The second important reason why the solar days vary in length is that the sun moves eastward along the ecliptic and not along the equator. For simplicity, neglect the eccentricity of the earth's orbit and the lack of uniformity of the angular motion of the sun along the ecliptic. Consider the time when the sun is near the vernal equinox. Since the ecliptic intersects the equator at an angle of 23°.5, only one component of the sun's motion is directly eastward. However, the reduction is somewhat less than might be imagined for so large an inclination and amounts to only about 10 per cent. When the sun is near the autumnal equinox the situation is the same except that, at this time, one component of the sun's motion is toward the south. At these two times in the year the sun's apparent motion eastward is less than it would otherwise be, and, consequently, the solar days are shorter than the average. At the solstices, midway between these two periods, the sun is moving approximately along the arcs of small circles 23°.5 from the equator, and its angular motion eastward is correspondingly faster than the average. Therefore, so far as the inclination of the ecliptic is concerned, the solar days are longest about December 21 and June 21, and shortest about March 21 and September 23. The lengths of the solar days, so far as they depend upon the inclination of the ecliptic, are shown by the dotted curve in Fig. 63.

Now consider the combined effects of the eccentricity of the earth's orbit and the inclination of the ecliptic on the lengths of the solar days. Of these two influences, the inclination of the ecliptic is considerably the more important. On the first of January they both make the solar day longer than the average. At the vernal equinox the eccentricity has only a slight effect on the length of the solar day, while the obliquity of the ecliptic makes it shorter than the average. On June 21 the effect of the eccentricity is to make the solar day shorter than the average, while the effect of the obliquity of the ecliptic is to make it longer than the average. At the autumnal equinox the eccentricity has only a slight importance and the obliquity of the ecliptic makes the solar day shorter than the average.

The two influences together give the following result: The longest day in the year, from noon to noon by the sun, is about December 22, after which the solar day decreases continually in length until about the 26th of March; it then increases in length until about June 21; then it decreases in length until the shortest day in the year is reached on September 17; and then it increases in length continually until December 22. On December 22 the solar day is about 4 minutes and 26 seconds of mean solar time (Art. 108) longer than the sidereal; on March 26 it is 3 minutes and 38 seconds longer; on June 21 it is 4 minutes 9 seconds longer; and on September 17 it is 3 minutes and 35 seconds longer. The combined results are shown by the full line in Fig. 63. The difference in length between the longest and the shortest day in the year is, therefore, about 51 seconds of mean solar time. While this difference for most purposes is not important in a single day, it accumulates and gives rise to what is known as the equation of time (Art. 109).

It might seem that it would be sensible for astronomers to neglect the differences in the lengths of the solar days, especially as the change in length from one day to the next is very small. Only an accurate clock would show the disparity in their lengths, and their slight differences would be of no importance in ordinary affairs. But if astronomers should use the rotation of the earth with respect to the sun as defining equal intervals of time, they would be employing a varying standard and they would find apparent irregularities in the revolution of the earth and in all other celestial motions which they could not bring under any fixed laws. This illustrates the extreme sensitiveness of astronomical theories to even slight errors.

108. Mean Solar Time.—Since the ordinary activities of mankind are dependent largely upon the period of daylight, it is desirable for practical purposes to have a unit of time based in some way upon the rotation of the earth with respect to the sun. On the other hand, it is undesirable to have a unit of variable length. Consequently, the *mean solar day*, which has the average length of all the solar days of the year, is introduced. In sidereal time its length is 24 hours, 3 minutes, and 56.555 seconds.

The mean solar day is divided into 24 mean solar hours, the hours into 60 mean solar minutes, and the minutes into 60 mean solar seconds. These are the hours, minutes, and seconds in common use, and ordinary timepieces are made to keep mean solar time as accurately as possible. It would be very difficult, if not impossible, to construct a clock that would keep true solar time with any high degree of precision.

109. The Equation of Time.—The difference between the true solar time and the mean solar time of a place is called *the equation of time*. It is taken with such an algebraic sign that, when it is added to the mean solar time, the true solar time is obtained.[1]

The date on which noon by mean solar time and true solar time shall coincide is arbitrary, but it is so chosen that the differences between the times in the two systems shall be as small as possible. On the 24th of December the equation of time is zero. It then becomes negative and increases numerically until February 11, when it amounts to about −14 minutes and 25 seconds; it then increases and passes through zero about April 15, after which it becomes positive and reaches a value of

[1]This is the present practice of the American Ephemeris and Nautical Almanac; it was formerly the opposite.

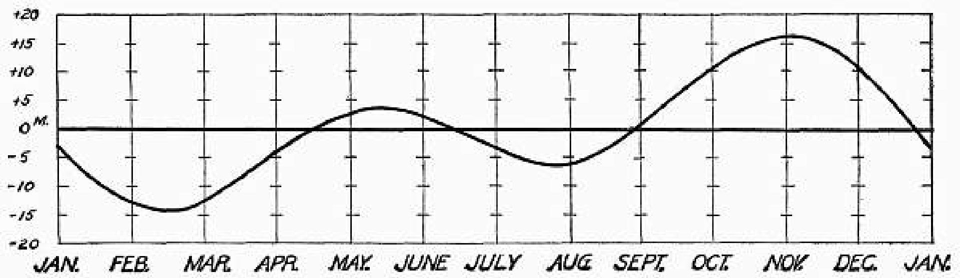

FIG. 64.—The equation of time.

3 minutes 48 seconds on May 14; it then decreases and passes through zero on June 14 and becomes −6 minutes and 20 seconds on July 26; it then increases and passes through zero on September 1 and becomes 16 minutes and 21 seconds on November 2, after which it continually decreases until December 24. The results are given graphically in Fig. 64. The dates may vary a day or two from those given because of the leap year, and the amounts by a few seconds because of the shifting of the dates.

Some interesting results follow from the equation of time. For example, on December 24 the equation of time is zero, but the solar day is about 30 seconds longer than the mean solar day. Consequently, the next day the sun will be about 30 seconds slow; that is, noon by the mean solar clock has shifted about 30 seconds with respect to the sun. As the sun has just passed the winter solstice, the period from sunrise to sunset for the northern hemisphere of the earth is slowly increasing, the exact amount depending upon the latitude. For latitude 40° N. the gain in the forenoon resulting from the earlier rising of the sun is less than the loss from the shifting of the time of the noon. Consequently, almanacs will show that the forenoons are getting shorter at this time of the year, although the whole period between sunrise and sunset is increasing. The difference in the lengths of the forenoons and afternoons may accumulate until it amounts to nearly half an hour.

110. Standard Time.—The mean solar time of a place is called its *local time.* All places having the same longitude have the same local time, but places having different longitudes have different local times. The circumference of the earth is nearly 25,000 miles and 15° correspond to a difference of one hour in local time. Consequently, at the earth's equator, 17 miles in longitude give a difference of about one minute in local time. In latitudes 40° to 45° north or south 13 to

12 miles in longitude give a difference of one minute in local time.

If every place along a railroad extending east and west should keep its own local time, there would be endless confusion and great danger in running trains. In order to avoid these difficulties, it has been agreed that all places whose local times do not differ more than half an hour from that of some convenient meridian shall use the local time of that meridian. Thus, while the extreme difference in local time of places using the local time of the same meridian may be about an hour, neither of them differs more than about half an hour from its standard time. In this manner a strip of country about 750 miles wide in latitudes 35° to 45° uses the same time, and the next strip of the same width an hour different, and so on. The local time of the standard meridian of each strip is the *standard time* of that strip.

At present standard time is in use in nearly every civilized part of the earth. The United States and British America are of such great extent in longitude that it is necessary to use four hours of standard time. The eastern portion uses what is called Eastern Time. It is the local time of the meridian 5 hours west of Greenwich. This meridian runs through Philadelphia, and in this city local time and standard time are identical. At places east of this meridian it is later by local time than by standard time, the difference being one minute for 12 or 13 miles. At places west of this meridian, but in the Eastern Time division, it is earlier by local time than by standard time. The next division to the westward is called Central Time. It is the local time of the meridian 6 hours west of Greenwich, which passes through St. Louis. The next time division is called Mountain Time. It is the local time of the meridian 7 hours west of Greenwich. This meridian passes through Denver. The last time division is called Pacific Time. It is the local time of the meridian 8 hours west of Greenwich. This meridian passes about 100 miles east of San Francisco.

If the exact divisions were used, the boundaries between one time division and the next would be 7°.5 east and west of the standard meridian. As a matter of fact, the boundaries are quite irregular, depending upon the convenience of railroads, and they are frequently somewhat altered. The change in time is nearly always made at the end of a railway division; for, obviously, it would be unwise to have railroad time change during the run of a given train crew. As a result the actual boundaries of the several time divisions are quite irregular and vary in many cases radically from the ideal standard divisions. Moreover, many towns near the borders of the time zones do not use standard

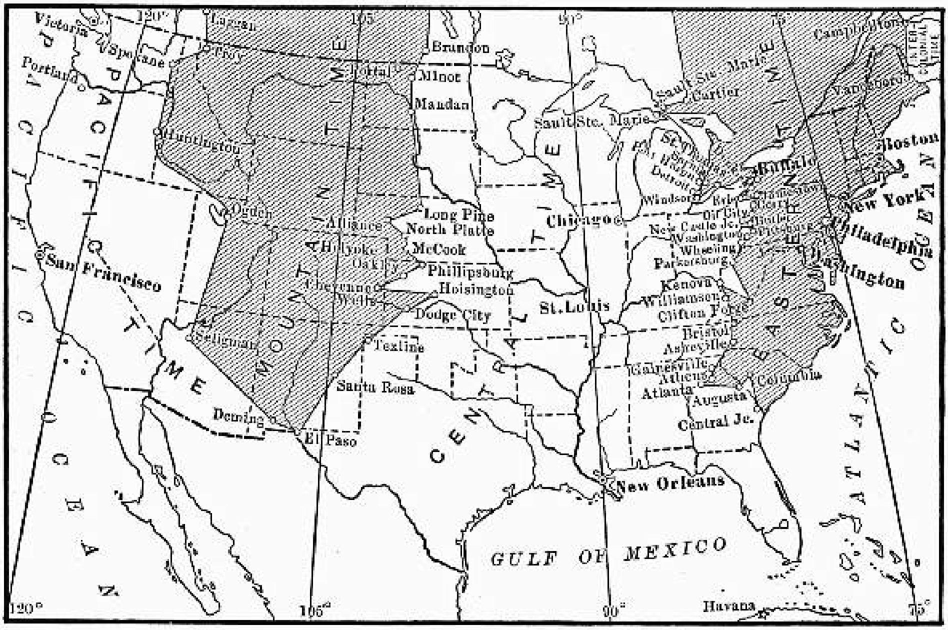

FIG. 65.—Standard time divisions in the United States.

time.

111. The Distribution of Time.—The accurate determination of time and its distribution are of much importance. There are several methods by which time may be determined, but the one in common use is to observe the transits of stars across the meridian and thus to obtain the sidereal time. From the mathematical theory of the earth's motion it is then possible to compute the mean solar time. It might be supposed that it would be easier to find mean solar time by observing the transit of the sun across the meridian, but this is not true. In the first place, it is much more difficult to determine the exact time of the transit of the sun's center than it is to determine the time of the transit of a star; and, in the second place, the sun crosses the meridian but once in 24 hours, while many stars may be observed. In the third place, observations of the sun give true solar time instead of mean solar time, and the computation necessary to reduce from one to the other is as difficult as it is to change from sidereal time to mean solar time.

It remains to explain how time is distributed from the places where the observations are made. In most countries the time service is under the control of the government, and the time signals are sent out from the national observatory. For example, in the United States, the chief source of time for railroads and commercial purposes is the Naval Observatory, at Georgetown Heights, Washington, D.C. There are three high-grade clocks keeping standard time at this observatory. Their errors are found from observations of the stars; and after applying corrections for these errors, the mean of the three clocks is taken as giving the true standard time for the successive 24 hours. At 5 minutes before noon, Eastern Time, the Western Union Telegraph Company and the Postal Telegraph Company suspend their ordinary business and throw their lines into electrical connection with the standard clock at the Naval Observatory. The connection is arranged so that the sounding key makes a stroke every second during the 5 minutes preceding noon except the twenty-ninth second of each minute, the last 5 seconds of the fourth minute, and the last 10 seconds of the fifth minute. This gives many opportunities of determining the error of a clock. To simplify matters, clocks are connected so as to be automatically regulated by these signals, and there are at present more than 30,000 of them in use in this country. The time signals are sent out from the Naval Observatory with an error usually less than 0.2 of a second; but frequently this is considerably increased when a system of relays must be used to reach great distances.

These noon signals also operate time balls in 18 ports in the United States. This device for furnishing time, chiefly to boat captains, consists of a large ball which is dropped at noon, Eastern Time, from a considerable height at conspicuous points, by means of electrical connection with the Naval Observatory.

Time for the extreme western part of the United States is distributed from the Mare Island Navy Yard in California; and besides, a number of college observatories have been furnishing time to particular railroad systems. Naturally most observatories regularly determine time for their own use, though with the accurate distribution of time from Washington the need for this work is disappearing except in certain special problems of star positions.

112. Civil and Astronomical Days.—The civil day begins at midnight, for then business is ordinarily suspended and the date can be changed with least inconvenience. The astronomical day of the same date begins at noon, 12 hours later; because, if the change were made at midnight, astronomers might find it necessary to change the date in the midst of a set of observations. It is true that many observations of the sun and some other bodies are made in the daytime, but of course most observational work is done at night. The hours of the astronomical day are numbered up to 24, just as in the case of sidereal time.

113. Place of Change of Date.—If one should start at any point on the earth and go entirely around it westward, the number of times the sun would cross his meridian would be one less than it would have been if he had stayed at home. Since it would be very inconvenient for him to use fractional dates, he would count his day from midnight to midnight, whatever his longitude, and correct the increasing difference from the time of his starting point by arbitrarily changing his date one day forward at some point in his journey. That is, he would omit one date and day of the week from his reckoning. On the other hand, if he were going around the earth eastward, he would give two days the same date and day of the week. The change is usually made at the 180th meridian from Greenwich. This is a particularly fortunate selection, for the 180th meridian scarcely passes through any land surface at all, and then only small islands. One can easily see how troublesome matters would be if the change were made at a meridian passing through a thickly populated region, say the meridian of Greenwich. On one side of it people would have a certain day and date, for example, Monday, December 24, and on the other side of it a day later, Tuesday, December 25.

The place of actual change of date does not strictly follow the 180th meridian from Greenwich, for travelers, going eastward from Europe, lose half a day, while those going westward from Europe and America arrive in the same longitude with a gain of half a day; hence their dates differ by one day. The change-of-date line is shown in Fig. 66.

114. The Sidereal Year.—The sidereal year is the time required for the sun apparently to move from any position with respect to the stars, as seen from the earth, around to the same position again. Perhaps it is better to say that it is the time required for the earth to make a complete revolution around the sun, directions from the sun being determined by the positions of the stars. Its length in mean solar time is 365 days, 6 hours, 9 minutes, 9.54 seconds, or just a little more than 365.25 days.

115. The Anomalistic Year.—The anomalistic year is the time required for the earth to move from the perihelion of its orbit around to the perihelion again. If the perihelion point were fixed, this period would equal the sidereal year. But the attraction of the other planets causes the perihelion point to move forward at such a rate that it completes a revolution in about 108,000 years; and the consequence

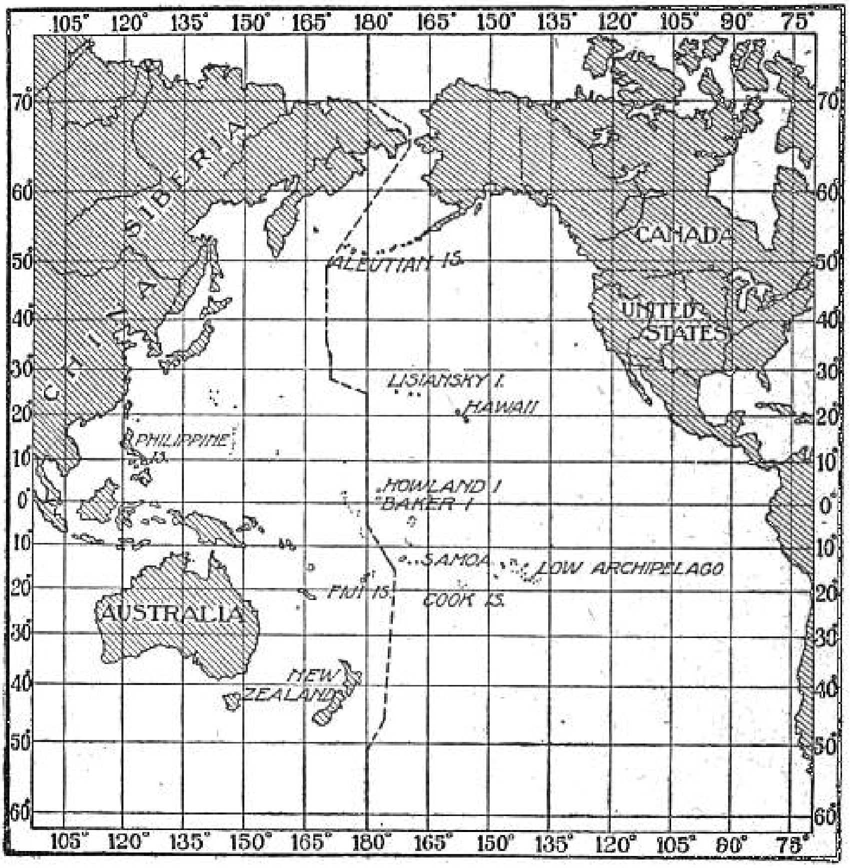

FIG. 66.—The change-of-date line.

is that the anomalistic year is a little longer than the sidereal year. It follows from the period of its revolution that the perihelion point advances about 12″ annually. Since the earth moves, on the average, about a degree daily, it takes it about 4 minutes and 40 seconds of time to move 12″. The actual length of the anomalistic year in mean solar time is 365 days, 6 hours, 13 minutes, 53.01 seconds.

116. The Tropical Year.—The tropical year is the time required for the sun to move from a tropic around to the same tropic again; or, better for practical determination, from an equinox to the same equinox again. Since the equinoxes regress about 50″.2 annually, the tropical year is about 20 minutes shorter than the sidereal year. Its actual length in mean solar time is 365 days, 5 hours, 48 minutes, 45.92 seconds.

The seasons depend upon the sun's place with respect to the equinoxes. Consequently, if the seasons are always to occur at the same time according to the calendar, the tropical year must be used. This is, indeed, the year in common use and, unless otherwise specified, the term *year* means the tropical year.

117. The Calendar.–In very ancient times the calendar was based largely on the motions of the moon, whose phases determined the times of religious ceremonies. The moon does not make an integral number of revolutions in a year, and hence it was occasionally necessary to interpolate a month in order to keep the year in harmony with the seasons.

The week was another division of time used in antiquity. The number of days in this period was undoubtedly based upon the number of moving celestial bodies which were then known. Thus, Sunday was the sun's day; Monday, the moon's day; Tuesday, Mars' day; Wednesday, Mercury's day; Thursday, Jupiter's day; Friday, Venus's day; and Saturday, Saturn's day. The names of the days of the week, when traced back to the tongues from which English has been derived, show that these were their origins.

In the year 46 B.C. the Roman calendar, which had fallen into a state of great confusion, was reformed by Julius Cæsar under the advice of an Alexandrian astronomer, Sosigenes. The new system, called the Julian Calendar, was entirely independent of the moon; in it there were 3 years of 365 days each and then one year, the leap year, of 366 days. This mode of reckoning, which makes the average year consist of 365.25 days, was put into effect at the beginning of the year 45 B.C.

It is seen from the length of the tropical year, which was given in Art. 116, that this system of calculation involves a small error, averaging 11 minutes and 14 seconds yearly. In the course of 128 years the Julian Calendar gets one day behind. To remedy this small error, in 1582, Pope Gregory XIII introduced a slight change. Ten days were omitted from that year by making October 15 follow immediately after October 4, and it was decreed that 3 leap years out of every 4 centuries should henceforth be omitted. This again is not quite exact, for the Julian Calendar gets behind 3 days in $3 \times 128 = 384$ years instead of 400 years; yet the error does not amount to a day until after more than 3300 years have elapsed.

To simplify the application, every year whose date number is exactly divisible by 4 is a leap year, unless it is exactly divisible by 100. Those years whose date numbers are divisible by 100 are not leap years unless they are exactly divisible by 400, when they are leap years. Of course, the error which still remains could be further reduced by a rule for the leap years when the date number is exactly divisible by 1000, but there is no immediate need for it.

The calendar originated and introduced by Pope Gregory XIII in 1582, and known as the Gregorian Calendar, is now in use in all civilized countries except Russia and Greece, although it was not adopted in England until 1752. At that time 11 days had to be omitted from the year, causing considerable disturbance, for many people imagined they were in some way being cheated out of that much time. The Julian Calendar is now 13 days behind the Gregorian Calendar. The Julian Calendar is called Old Style (O.S.), and the Gregorian, New Style (N.S.).

In certain astronomical work, such as the discussion of the observations of variable stars, where the difference in time of the occurrence of phenomena is a point of much interest, the Julian Day is used. The Julian Day is simply the number of the day counting forward from January 1, 4713 B.C. This particular date from which to count time was chosen because that year was the first year in several subsidiary cycles, which will not be considered here.

118. Finding the Day of the Week on Any Date.–An ordinary year of 365 days consists of 52 weeks and one day, and a leap year consists of 52 weeks and 2 days. Consequently, in succeeding years the same date falls one day later in the week except when a twenty-ninth of February intervenes; and in this case it is two days later. These facts give the basis for determining the day of the week on which any date falls, after it has been given in a particular year.

Consider first the problem of finding the day of the week on which January 1 falls. In the year 1900 January 1 fell on Monday. To fix the ideas, consider the question for 1915. If every year had been an ordinary year, January 1 coming one day later in the week in each succeeding year, it would have fallen, in 1915, 15 days, or 2 weeks and one day, after Monday; that is, on Tuesday. But, in the meantime there were 3 leap years, namely, 1904, 1908, and 1912, which put the date 3 additional days forward in the week, or on Friday. Similarly, it is seen in general that the rule for finding the day of the week on which January 1 falls in any year of the present century is to take the number of the year in the century (15 in the example just treated), add to it the number of leap years which have passed (which is given by dividing the number of the year by 4 and neglecting the remainder), divide the result by 7 to eliminate the number of weeks which have passed, and finally, count forward from Monday the number of days given by the remainder. For example, in 1935 the number of the year is 35, the number of leap years is 8, the sum of 35 and 8 is 43, and 43 divided by 7 equals 6 with the remainder of 1. Hence, in 1935, January 1 will be one day later than Monday; that is, it will fall on Tuesday.

In order to find the day of the week on which any date of any year falls, find first the day of the week on which January 1 falls; then take the day of the year, which can be obtained by adding the number of days in the year up to the date in question, and divide this by 7; the remainder is the number of days that must be added to that on which January 1 falls in order to obtain the day of the week. For example, consider March 21, 1935. It has been found that January 1 of this year falls on Tuesday. There are 79 days from January 1 to March 21 in ordinary years. If 79 is divided by 7, the quotient is 11 with the remainder of 2. Consequently, March 21, 1935, falls 2 days after Tuesday, that is, on Thursday.

IX. QUESTIONS

1. Give three examples where intervals of time in which you have had many and varied intellectual experiences now seem longer than ordinary intervals of the same length. Have you had any contradictory experiences?

2. If the sky were always covered with clouds, how should we measure time?

3. What is your sidereal time to-day at 8 P.M.?

4. What would be the relations of solar time to sidereal time if the earth rotated in the opposite direction?

5. What is the length of a sidereal day expressed in mean solar time?

6. What is the standard time of a place whose longitude is 85° west of Greenwich when its local time is 11 A.M.?

7. What is the local time of a place whose longitude is 112° west of Greenwich when its standard time is 8 P.M.?

8. Suppose a person were able to travel around the earth in 2 days; what would be the lengths of his days and nights if he traveled from east to west? From west to east?

9. If the sidereal year were in ordinary use, how long before Christmas would occur when the sun is at the vernal equinox?

10. On what days of the week will your birthday fall for the next 12 years?

CHAPTER VII
THE MOON

119. The Moon's apparent Motion among the Stars.—The apparent motion of the moon can be determined by observation without any particular reference to its actual motion. In fact, the ancient Greeks observed the moon with great care and learned most of the important peculiarities of its apparent motion, but they did not know its distance from the earth and had no accurate ideas of the character of its orbit. The natural method of procedure is first to find what the appearances are, and from them to infer the actual facts.

The moon has a diurnal motion westward which is produced, of course, by the eastward rotation of the earth. Every one is familiar with the fact that it rises in the east, goes across the sky westward, and sets in the west. Those who have observed it except in the most casual way, have noticed that it rises at various points on the eastern horizon, crosses the meridian at various altitudes, and sets at various points on the western horizon. They have also noticed that the interval between its successive passages across the meridian is somewhat more than 24 hours.

Observations of the moon for two or three hours will show that it is moving eastward among the stars. When its path is carefully traced out during a whole revolution, it is found that its apparent orbit is a great circle which is inclined to the ecliptic at an angle of $5^\circ\, 9'$. The point at which the moon, in its motion eastward, crosses the ecliptic from south to north is called the *ascending node* of its orbit, and the point where it crosses the ecliptic from north to south is called the *descending node* of its orbit. The attraction of the sun for the moon causes the nodes continually to regress on the ecliptic; that is, in successive revolutions the moon crosses the ecliptic farther and farther to the west. The period of revolution of the line of nodes is 18.6 years.

120. The Moon's Synodical and Sidereal Periods.–The synodical period of the moon is the time required for it to move from any apparent position with respect to the sun back to the same position again. The most accurate means of determining this period is by comparing ancient and modern eclipses of the sun; for, at the time of a solar eclipse, the moon is exactly between the earth and the sun. The advantages of this method are that, in the first place, at the epochs used the exact positions of the moon with respect to the sun are known; and, in the second place, in a long interval during which the moon has made hundreds or even thousands of revolutions around the earth, the errors in the determinations of the exact times of the eclipses are relatively unimportant because they are divided by the number of revolutions the moon has performed. It has been found in this way that the synodical period of the moon is 29 days, 12 hours, 44 minutes, and 2.8 seconds; or 29.530588 days, with an uncertainty of less than one tenth of a second.

The sidereal period of the moon is the time required for it to move from any apparent position with respect to the stars back to the same position again. This period can be found by direct observations; or, it can be computed from the synodical period and the period of the earth's revolution around the sun. Let S represent the moon's synodical period, M its sidereal period, and E the period of the earth's revolution around the sun, all expressed in the same units as, for example, days. Then $1/M$ is the fraction of a revolution that the moon moves eastward in one day, $1/E$ is the fraction of a revolution that the sun moves eastward in one day, and $1/M - 1/E$ is, therefore, the fraction of a revolution that the moon gains on the sun in its eastward motion in one day. Since the moon gains one complete revolution on the sun in S days, $1/S$ is also the fraction of a revolution the moon gains on the sun in one day. Hence it follows that

$$\frac{1}{S} = \frac{1}{M} - \frac{1}{E},$$

from which M can be computed when S and E are known.

It is easy to see that the synodical period is longer than the sidereal. Suppose the sun, moon, and certain stars are at a given instant in the same straight line as seen from the earth. After a certain number of days the moon will have made a sidereal revolution and the sun will have moved eastward among the stars a certain number of degrees. Since additional time is required for the moon to overtake it, the synodical period is longer than the sidereal.

It has been found by direct observations, and also by the equation above, that the moon's sidereal period is 27 days, 7 hours, 43 minutes, and 11.5 seconds, or 27.32166 days. When the period of the moon is referred to, the sidereal period is meant unless otherwise stated.

The periods which have been given are averages, for the moon departs somewhat from its elliptical orbit, primarily because of the attraction of the sun, and to a lesser extent because of the oblateness of the earth and the attractions of the planets. The variations from the average are sometimes quite appreciable, for the perturbations, as they are called, may cause the moon to depart from its undisturbed orbit about 1°.5, and may cause its period of revolution to vary by as much as 2 hours.

121. The Phases of the Moon.—The moon shines entirely by reflected sunlight, and consequently its appearance as seen from the earth depends upon its position relative to the sun. Figure 67 shows eight positions of the moon in its orbit with the sun's rays coming from the right in lines which are essentially parallel because of the great distance of the sun. The left-hand side of the earth is the night side, and similarly the left side of the moon is the dark side.

The small circles whose centers are on the large circle around the earth as a center show the illuminated and unilluminated parts of the moon as they actually are; the accompanying small circles just outside of the large circle show the moon as it is seen from the earth. For example, when the moon is at M_1 between the earth and sun, its dark side is toward the earth. In this position it is said to be *in conjunction*, and the phase is *new*. At M_2 half of the illuminated part of the moon can be seen from the earth, and it is in the *first quarter*. In this position the moon is said to be *in quadrature*. Between the new moon and the first quarter the illuminated part of the moon as seen from the earth is of crescent shape, and its convex side is turned toward the sun.

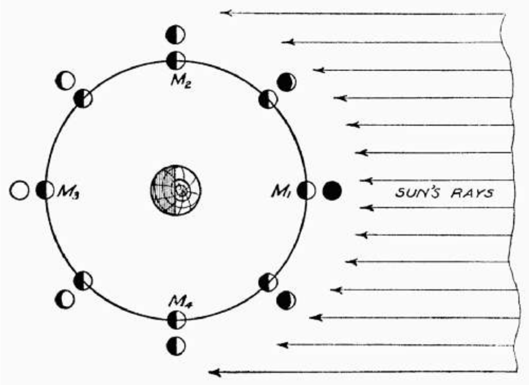

FIG. 67.—Explanation of the moon's phases.

When the moon is at M_3 the illuminated side is toward the earth. It is then *in opposition*, and the phase is *full*. If an observer were at the sunset point on the earth, the sun would be setting in the west and the full moon would be rising in the east. At M_4 the moon is again in quadrature, and the phase is *third quarter*.

To summarize: The moon is new when it has the same right ascension as the sun; it is at the first quarter when its right ascension is 6 hours greater than that of the sun; it is full when its right ascension is 12 hours greater than that of the sun; and it is at the third quarter when its right ascension is 18 hours greater than that of the sun.

It is observed from the diagram that the earth would have phases if seen from the moon. When the moon is new, as seen from the earth, the earth would be full as seen from the moon. The phases of the earth corresponding to every other position of the moon can be inferred from the diagram. The phases of the moon and earth are supplementary; that is, the illuminated portion of the moon as seen from the earth plus the illuminated portion of the earth as seen from the moon always equals 180°. When the moon is nearly new, and, consequently, the earth nearly full as seen from the moon, the dark side of the moon is somewhat illuminated by sunlight reflected from the earth, as is shown in Fig. 68.

FIG. 68.—The moon partially illuminated by light reflected from the earth. *Photographed by Barnard at the Yerkes Observatory.*

122. The diurnal Circles of the Moon.—Suppose first that the moon moves along the ecliptic and consider its diurnal circles. Since they are parallel to the celestial equator (if the motion of the moon in declination between rising and setting is neglected), it is sufficient, in view of the discussion of the sun's diurnal circles (Art. 58), to give the places where the moon crosses the meridian. Let VAV, Fig. 69, represent the celestial equator spread out on a plane, and $VSAWV$ the

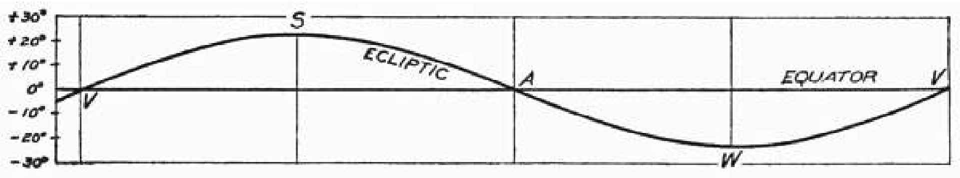

FIG. 69.—The equator and ecliptic.

ecliptic. Suppose, for example, that the time of the year is March 21. Then the sun is at V. If the moon is new, it is also at V, because at this phase it has the same right ascension as the sun. Since V is on the celestial equator, the moon crosses the meridian at an altitude equal to 90° minus the latitude of the observer. In this case it rises in the east and sets in the west. But if the moon is at first quarter on March 21, it is at S, because at this phase it is 6 hours east of the sun. It is then 23°.5 north of the equator, and, consequently, it crosses the meridian 23°.5 above the equator. In this case it rises north of east and sets north of west. If the moon is full, it is at A, and if it is in the third quarter, it is at W. In the former case it is on the equator and in the latter 23°.5 south of it.

Suppose the sun is at the summer solstice, S. Then it rises in the northeast, crosses the meridian 23°.5 north of the equator, and sets in the northwest. At the same time the full moon is at W, it rises in the southeast, crosses the meridian 23°.5 south of the equator, and sets in the southwest. That is, when sunshine is most abundant, the light from the full moon is the least. On the other hand, when the sun is at the winter solstice W, the full moon is at S and gives the most light. The other positions of the sun and moon can be treated similarly.

Suppose the ascending node of the moon's orbit is at the vernal equinox (Fig. 70), and consider the altitude at which the moon crosses

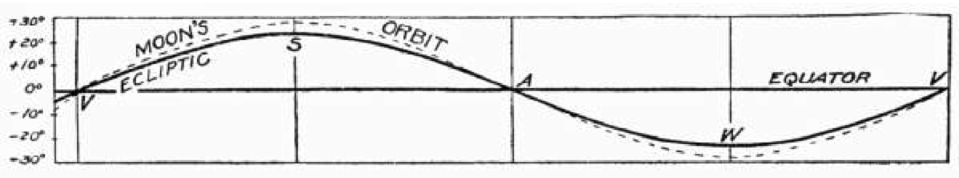

FIG. 70.—Ascending node of the moon's orbit at the vernal equinox.

the meridian when full at the time of the winter solstice. The sun is at W and the full moon is in its orbit 5° 9′ north of S. If the latitude of the observer is 40°, the moon then crosses his meridian at an altitude of $50° + 23°.5 + 5° = 78°.5$. That is, under these circumstances the full moon crosses the meridian higher in the winter time than it would if its orbit were coincident with the ecliptic. On the other hand, in the summer time, when the sun is at S and the full moon is at W, the moon crosses the equator farther south than it would if it were on the ecliptic. Under these circumstances there is more moonlight in the winter and less in the summer than there would be if the moon were always on the ecliptic.

Now suppose the descending node is at V and the ascending node is at A, Fig. 71. For this position of its orbit the moon crosses the meridian

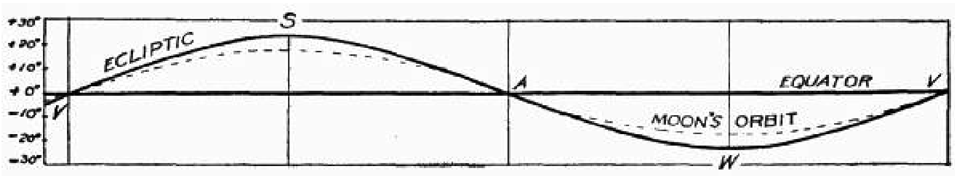

FIG. 71.—Ascending node of the moon's orbit at the autumnal equinox.

lower in the winter than it would if it moved along the ecliptic. The opposite is true when the sun is at S in the summer. Of course, the ascending node of the moon's orbit might be at any other point on the ecliptic.

It is clear from this discussion that when the sun is on the part of the ecliptic south of the equator, the full moon is near the part of the ecliptic which is north of the equator, and *vice versa*. Therefore, when there is least sunlight there is most moonlight, and there is the greatest amount of moonlight when the moon's ascending node is at the vernal equinox. When it is continuous night at a pole of the earth, the gloom is partly dispelled by the moon which is above the horizon that half of the month in which it passes from its first to its third quarter.

123. The Distance of the Moon.—One method of determining the distance of the moon is by observing the difference in its directions as seen from two points on the earth's surface, as O_1 and O_2 in Fig. 72. Suppose, for simplicity, that O_1 and O_2 are on the same meridian, and that the moon is in the plane of that meridian. The observer at O_1 finds that the moon is the angular distance Z_1O_1M south of his zenith; and the observer at O_2 finds that it is the angular distance Z_2O_2M north of his zenith. Since the two observers know their latitudes, they know the angle O_1EO_2, and consequently, the angles EO_1O_2 and EO_2O_1. By subtracting Z_1O_1M plus EO_1O_2 and Z_2O_2M plus EO_2O_1 from 180°, the angles MO_1O_2 and MO_2O_1 are obtained. Since the size of the earth is known, the distance O_1O_2 can be found. Then, in the triangle O_1MO_2 two angles and the included side are known, and all the other parts of the triangle can be computed by trigonometry. Suppose O_1M has been found; then, in the triangle EO_1M two sides and the

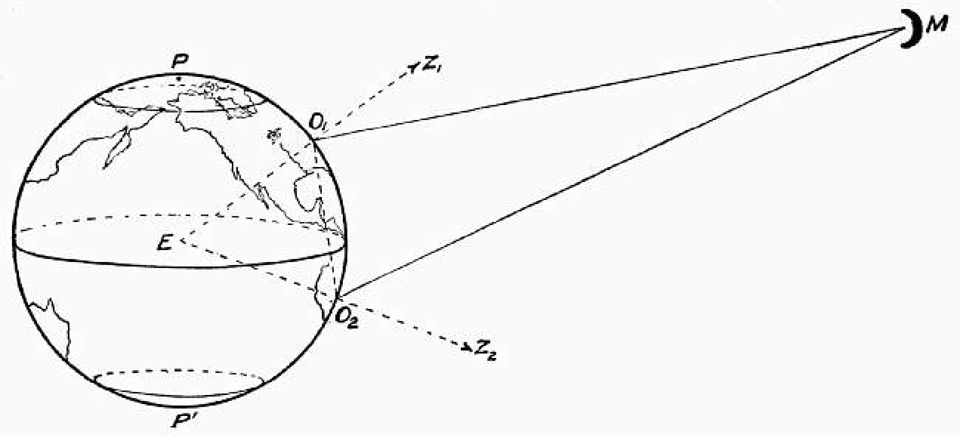

FIG. 72.—Measuring the distance to the moon.

included angle are known, and the distance EM can be computed. In general, the relations and observations will not be so simple as those assumed here, but in no case are serious mathematical or observational difficulties encountered. It is to be noted that the result obtained is not guesswork, but that it is based on measurements, and that it is in reality given by measurements in the same sense that a distance on the surface of the earth may be obtained by measurement. The percentage of error in the determination of the moon's distance is actually much less than that in most of the ordinary distances on the surface of the earth.

The mean distance from the center of the earth to the center of the moon has been found to be 238,862 miles, and the circumference of its orbit is therefore 1,500,818 miles. On dividing the circumference by the moon's sidereal period expressed in hours, it is found that its orbital velocity averages 2288.8 miles per hour, or about 3357 feet per second.

A body at the surface of the earth falls about 16 feet the first second; at the distance of the moon, which is approximately 60 times the radius of the earth, it would, therefore, fall $16 \div 60^2 = 0.0044$ feet, because the earth's attraction varies inversely as the square of the distance from its center. Therefore, in going 3357 feet, or nearly two thirds of a mile, the moon deviates from a straight-line path only about $\frac{1}{20}$ of an inch.

124. The Dimensions of the Moon.—The mean apparent diameter of the moon is $31' \; 5''.2$. Since its distance is known, its actual diameter can be computed. It is found that the distance straight through the moon is 2160 miles, or a little greater than one fourth the diameter of the earth. Since the surfaces of spheres are to each other as the squares of their diameters, it is found that the surface area of the earth is 13.4 times that of the moon; and since the volumes of spheres are to each other as the cubes of their diameters, it is found that the volume of the earth is 49.3 times that of the moon.

It has been stated that the mean apparent diameter of the moon is $31' 5''.2$. The apparent diameter of the moon varies both because its distance from the center of the earth varies, and also because when the moon is on the observer's meridian, he is nearly 4000 miles nearer to it than when it is on his horizon. In the observations of other celestial objects the small distance of 4000 miles makes no appreciable difference in their appearance; but, since the distance from the earth to the moon is, in round numbers, only 240,000 miles, the radius of the earth is $\frac{1}{60}$ of the whole amount.

In spite of the fact that the moon is nearer the observer when it is on his meridian than when it is on his horizon, every one has noticed that it appears largest when near the horizon and smallest when near the meridian. The reason that the moon appears to us to be larger when it is near the horizon is that then intervening objects give us the impression that it is very distant, and this influences our unconscious estimate of its size.

125. The Moon's Orbit with Respect to the Earth.—The moon's distance from the earth varies from about 225,746 miles to 251,978 miles, causing a corresponding variation in its apparent diameter. Its orbit is an ellipse, having an eccentricity of 0.0549, except for slight deviations due to the attractions of the sun, planets, and the equatorial bulge of the earth. The moon moves around the earth, which is at one of the foci of its elliptical orbit, in such a manner that the line joining it to the earth sweeps over equal areas in equal intervals of time. This statement requires a slight correction because of the perturbations produced by the attractions of the sun and planets. The point in the moon's orbit which is nearest the earth is called its *perigee*, and the farthest point is called its *apogee.*

126. The Moon's Orbit with Respect to the Sun.—The distance from the earth to the sun is about 400 times that from the earth to the moon. Consequently, the oscillations of the moon back and forth across the earth's orbit as the two bodies pursue their motion around the sun are so small that they can hardly be represented to scale in a diagram. As a consequence of the relative nearness of the moon and its comparatively long period, its orbit is always concave toward the sun. If the orbit of the moon were at any time convex toward the sun, it would be when it is moving from a position between the earth and sun to opposition, that is, from A to B, Fig. 73. It takes 14 days for the moon to move from the former position to the latter, and during this time its distance from the sun increases by about 480,000 miles; but, in the meantime, the earth moves forward about 14° in its orbit from P to Q, and it, therefore, is drawn by the sun away from the straight line PT in which it was originally moving by a distance of about 3,000,000 miles. That is, in the 14 days the moon actually

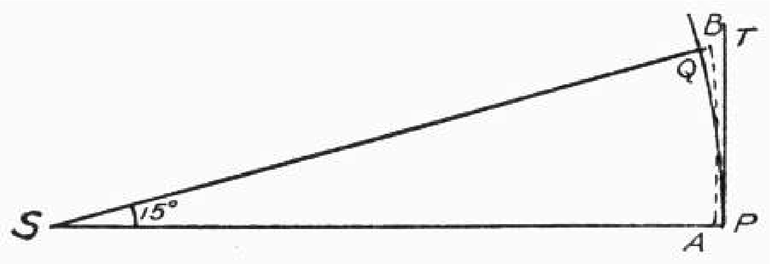

FIG. 73.—The orbit of the moon is concave to the sun.

moves in toward the sun away from the original line of the earth's motion $3,000,000 - 480,000 = 2,520,000$ miles, and its orbit, which is represented by the broken line, is, therefore, concave toward the sun at every point.

As a matter of fact, it is the center of gravity of the earth and moon which describes what is called the earth's elliptical orbit around the sun, and the earth and moon both describe ellipses around this point as it moves on in its elliptical path around the sun. Since the earth's mass is very large compared to that of the moon, as will be seen in Art. 127, the center of the earth is always very near the center of gravity of the two bodies.

127. The Mass of the Moon.—Although the moon is comparatively near the earth, its mass cannot be obtained so easily as that of many other objects farther away.

One of the best methods of finding the mass of the moon depends upon the fact that the center of gravity of the earth and moon describes an elliptical orbit around the sun in accordance with the law of areas. Sometimes the earth is ahead of the center of gravity, and at other times behind it. When the earth is ahead of the center of gravity the sun will be seen behind the position it would apparently occupy if it were not for the moon. On the other hand, when the earth is behind the center of gravity, the sun will be displaced correspondingly ahead of the position it would otherwise apparently occupy. That is, the sun's apparent motion eastward among the stars is not strictly in accordance with the law of areas, for it sometimes is a little ahead of, and at others a little behind, the position it would have except for the moon. From very delicate observations it has been found that the sun is displaced in this way about $6''.4$. Since the distance of the sun is known, the amount of displacement of the earth in miles necessary to produce this apparent displacement of the sun can be computed. It has been found in this way that the distance of the center of gravity of the earth and moon from the center of the earth is 2886 miles.

Now consider the problem of finding the ratio of the mass of the earth to that of the moon. In Fig. 74 let E represent the earth, C the center of gravity of the earth and moon, and M the moon. Let the distance EC be represented by x, and the distance EM, which is $238,862$ miles, by r. Since the mass of the earth multiplied by the distance of its center from the center of gravity of the earth and moon equals the mass of the moon multiplied by its distance from the center of gravity of the earth and moon, it follows that

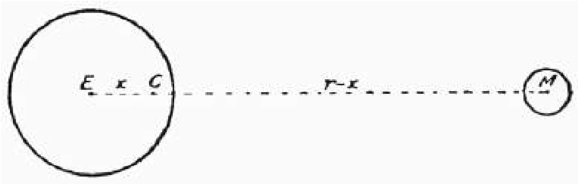

FIG. 74.—Center of gravity of the earth and moon.

$$x \times E = (r - x)\,M.$$

Since $x = 2886$ miles and $r = 238,862$ miles, it is found that

$$E = 81.8\,M.$$

In round numbers the mass of the earth is 80 times that of the moon.

Since the orbit of the moon is inclined $5^\circ\,9'$ to the plane of the ecliptic, the earth is sometimes above and sometimes below this plane. This causes an apparent displacement of the sun from the ecliptic in the opposite direction. From the amount of the apparent displacement of the sun in latitude, as determined by observations, and from the inclination of the moon's orbit and the distance of the sun, it is possible to compute, just as from the sun's apparent displacement in longitude, the mass of the moon relative to that of the earth.

128. The Rotation of the Moon.—The moon always presents the same side toward the earth, and therefore, as seen from some point other than the earth or moon, it rotates on its axis once in a sidereal month. For, in Fig. 67, when the moon is at M_1 a certain part is on the left toward the earth, but when it has moved to M_3 the same side is on the right still toward the earth. Its direction of rotation is the same as that of its revolution, or from west to east. The plane of its equator is inclined about $1^\circ\,32'$ to the plane of the ecliptic, and the two planes always intersect in the line of nodes of the moon's orbit.

It follows from what has been stated that the moon's sidereal day is the same as its sidereal month, or 27.32166 mean solar days. Its solar day is of the same length as its synodical month, or 29.530588 mean solar days, because its synodical month is defined by its position with respect to the earth and sun. Other things being equal, the temperature changes from day to night on the moon would be much greater than on the earth because its period of rotation is so much longer; but the seasonal changes would be very slight because of the small inclination of the plane of its equator to the plane of its orbit.

It is a most remarkable fact that the moon rotates at precisely such a rate that it always keeps the same face toward the earth. It is infinitely improbable that it was started exactly in this way; and, if it were not so started, there must have been forces at work which have brought about this peculiar relationship. It has been suggested that the explanation lies in the tidal reaction between the earth and moon. Since the moon raises tides on the earth, it is obvious that the earth also raises tides on the moon unless it is absolutely rigid. Since the mass of the earth is more than 80 times that of the moon, the tides generated by the earth on the moon, other things being equal, would be much greater than those generated by the moon on the earth. If a body is rotating faster than it revolves, and in the same direction, one of the effects of the tides is to slow up its rotation and to tend to bring the periods of rotation and revolution to an equality. It has been generally believed that the tides raised by the earth on the moon during millions of years, part of which time it may have been in a plastic state, have brought about the condition which now exists. There are, however, serious difficulties with this explanation (Art. 265), and it seems probable that the earth and moon are connected by forces not yet understood.

129. The Librations of the Moon.—The statement that the moon always has the same side toward the earth is not true in the strictest sense. It would be true if the planes of its orbit and of its equator were the same, and if it moved at a perfectly uniform angular velocity in its orbit.

The inclination of the moon's orbit to the ecliptic averages about $5^\circ\, 9'$, and the inclination of the moon's equator to the ecliptic is about $1^\circ\, 32'$. The three planes are so related that the inclination of the moon's equator to the plane of its orbit is $5^\circ\, 9' + 1^\circ\, 32' = 6^\circ\, 41'$. The sun shines alternately over the two poles of the earth because of the inclination of the plane of the equator to the plane of the ecliptic. In a similar manner, if the earth were a luminous body it would shine $6^\circ\, 41'$ over the moon's poles. Instead of shining on them (except by reflected light), the tilting of the moon's axis of rotation enables us to see $6^\circ\, 41'$ over the poles. This is the *libration* in latitude.

The moon rotates at a uniform rate,—at least the departures from a uniform rate are absolutely insensible. It would take inconceivably great forces to make perceptible short changes in its rate of rotation. On the other hand, the moon revolves around the earth at a non-uniform rate, for it moves in such a way that the law of areas is fulfilled. Consider the moon starting from the perigee. It takes about 6.5 days, or considerably less than one quarter of its period, for the moon to revolve through 90°; and, therefore, the angle of rotation is considerably less than 90°. The result is that the part of the moon on the side toward the perigee, that is, the western edge, is brought partially into view. On the opposite side of the orbit, the eastern edge of the moon is brought partially into view. This is the libration in longitude.

In addition to this, the moon is not viewed from the earth's center. When it is on the horizon, the line from the observer to the moon makes an angle of nearly 1° (the parallax of the moon) with that from the earth's center to the moon. This enables the observer to see nearly 1° farther around its side than he could if it were on his meridian.

The result of the moon's librations is that there is only 41 per cent of its surface which is never seen, while 41 per cent is always in sight, and 18 per cent of it is sometimes visible and sometimes invisible.

130. The Density and Surface Gravity of the Moon.—The volume of the earth is about 50 times that of the moon and its mass is 81.8 times that of the moon. Therefore the density of the moon is somewhat less than that of the earth. It is found from the relative volumes and masses of the earth and moon that the density of the moon on the water standard is about 3.4.

If the radius of the moon were the same as that of the earth, gravity at its surface would be less than $\frac{1}{80}$ that at the surface of the earth;

but the small radius of the moon tends to increase the attraction at its surface. If its mass were the same as that of the earth, its surface gravity would be nearly 16 times that of the earth. On taking the two factors together, it is found that the surface gravity of the moon is about $\frac{1}{6}$ that of the earth. That is, a body on the earth weighs by spring balances about 6 times as much as it would weigh on the moon.

If a body were thrown up from the surface of the moon with a given velocity, it would ascend 6 times as high as it would if thrown up from the surface of the earth with the same velocity. Perhaps this is the reason why the forces to which both the earth and moon have been subjected have produced relatively higher elevations on the moon than on the earth. Also it would be possible for mountains of a given material to be 6 times as high on the moon as on the earth before the rock of which they are composed would be crushed at the bottom.

131. The Question of the Moon's Atmosphere.—The moon has no atmosphere, or at the most, an excessively rare one. Its absence is proved by the fact that, at the time of an eclipse of the sun, the moon's limb is perfectly dark and sharp, with no apparent distortion of the sun due to refraction. Similarly, when a star is occulted by the moon, it disappears suddenly and not somewhat gradually as it would if its light were being more and more extinguished by an atmosphere.

Besides this, if the moon had an atmosphere, its refraction would keep a star visible for a little time after it had been occulted, just as the earth's atmosphere keeps the sun visible about 2 minutes after it has actually set. In a similar way, the star would become visible a short time before the moon had passed out of line with it. The whole effect would be to make the time of occultation shorter than it would be if there were no atmosphere.

If the moon had an atmosphere of any considerable extent, there would be the effects of erosion on its surface; but so far as can be determined, there is no evidence of such action. Its surface consists of a barren waste, and it is, perhaps, much cracked up because of the extremes of heat and cold to which it is subject. But there is nothing resembling soil except, possibly, volcanic ashes. There can be no water on the moon; for, if there were, it would be at least partly evaporated, especially in the long day, and form an atmosphere.

One cannot refrain from asking why the moon has no atmosphere. It may be that it never had any. But the evidence of great surface disturbances makes it not altogether improbable that vast quantities of vapors have been emitted from its interior. If this is true, they seem to have disappeared. There are two ways in which their disappearance can be explained. One is that they have united chemically with other elements on the moon. As a possible example of such action it may be mentioned that there are vast quantities of oxygen in the rocks of the earth's crust, which may, perhaps, have been largely derived from the atmosphere. The second explanation is that, according to the kinetic theory of gases, the moon may have lost its atmosphere by the escape of molecule after molecule from its gravitative control. This might be a relatively rapid process in the case of a body having the low velocity of escape of 1.5 miles per second (Art. 33), especially if its days were so long that its surface became highly heated.

It seems probable, therefore, that the moon could not retain an atmosphere if it had one, and that whatever gases it may ever have acquired from volcanoes or other sources were speedily lost.

132. The Light and Heat received by the Earth from the Moon.—The average distances of the earth and the moon from the sun are about the same; and, consequently, the earth and the moon receive about equal amounts of light and heat per unit area. The amount of light and heat that the earth receives from the moon depends upon how much the moon receives from the sun, what fraction it reflects, its distance from the earth, and its phase. It is easy to see that, if all the light the moon receives were reflected, the amount which strikes the earth could be computed for any phase as, for example, when the moon is full. It is found by taking into account all the factors involved that, if the moon were a perfect mirror, it would give the earth, when it is full, about $\frac{1}{100{,}000}$ as much light as the earth receives from the sun. As a matter of fact, the moon is by no means a perfect reflector, and the amount of light it sends to the earth is very much less than this quantity.

It is not easy to compare moonlight with sunlight by direct measurements, and the results obtained by different observers are somewhat divergent. The measurements of Zöllner, which are commonly accepted, show that sunlight is 618,000 times greater than the light received from the full moon. Sir John Herschel's observations gave the notably smaller ratio of 465,000. At other phases the moon gives not only correspondingly less light, but less than would be expected on the basis of the part of the moon illuminated. For example, at first quarter the illuminated area is half that at full moon, but the amount of light received is less than one eighth that at full moon. This phenomenon is doubtless due to the roughness of the moon's surface. Moreover, the amount of light received from the moon near first quarter is somewhat greater than that received at the corresponding phase at third quarter, the difference being due to the dark spots on the eastern limb of the moon. On taking into consideration the whole month, the average amount of light and heat which the moon furnishes the earth cannot exceed $\frac{1}{2{,}500{,}000}$ of that received from the sun. In other terms, the earth receives as much light and heat from the sun in 13 seconds as it receives from the moon in the course of a whole year.

133. The Temperature of the Moon.—The temperature of the moon depends upon the amount of heat it receives, the amount it reflects, and its rate of radiation. About 7 per cent of the heat which falls on the moon is directly reflected, and this has no effect upon its temperature. The remaining 93 per cent is absorbed and raises the temperature of its surface. The rate of radiation of the moon's surface materials for a given temperature is not known because of the uncertainties of their composition and physical condition. Nevertheless, it can be determined, at least roughly, at the time of a total eclipse of the moon.

Consider the moon when it is nearly full and just before it is eclipsed by passing into the earth's shadow, as at N, Fig. 81. The side toward the earth is subject to the perpendicular rays of the sun and has a higher temperature than any other part of its surface. It is easy to measure with some approximation the amount of heat received from the moon, but it is not easy to determine what part of it is reflected and what part is radiated. Now suppose the moon passes on into the earth's shadow so that the direct rays of the sun are cut off. Then all the heat received from the moon is that radiated from a surface recently exposed to the sun's rays. This can be measured; and, from the amount received and the rate at which it decreases as the eclipse continues, it is possible to determine approximately the rate at which the moon loses heat by radiation, and from this the temperature to which it has been raised. The observations show that the amount of heat received from the moon diminishes very rapidly after the moon passes into the earth's shadow. This means that its radiation is very rapid and that probably its temperature does not rise very high. It doubtless is safe to state that at its maximum it is between the freezing and the boiling points. The recent work of Very leads to the conclusion that the surface is heated at its highest to a temperature of 200° Fahrenheit.

It is now possible to get a more or less satisfactory idea of the temperature conditions of the moon. It must be remembered, in the first place, that its day is 28.5 times as long as that of the earth. In the second place, it has no atmospheric envelope to keep out the heat in the daytime and to retain it at night. Consequently, when the sun rises for a point on the moon, its rays continue to beat down upon the surface, which is entirely unprotected by clouds or air, for more than 14 of our days. During this time the temperature rises above the freezing point and it may even go up to the boiling point. When the sun sets, the darkness of midnight immediately follows because there is no atmosphere to produce twilight, and the heat rapidly escapes into space. In the course of an hour or two the temperature of the surface probably falls below the freezing point, and in the course of a day or two it may descend to 100° below zero. It will either remain there or descend still lower until the sun rises again 14 days after it has set.

The climatic conditions on the moon illustrate in the most striking manner the effects of the earth's atmosphere and the consequences of the earth's short period of rotation.

134. General surface Conditions on the Moon.—On the whole, the surface of the moon is extremely rough, showing no effects of weathering by air or water. It is broken by several mountain chains, by numerous isolated mountain peaks, and by more than 30,000 observed craters. There are several large, comparatively smooth and level areas, which were called *maria* (seas) by Galileo and other early observers, and the names are still retained though modern instruments show that they not only contain no water but are often rather rough. The smooth places are the areas which are relatively dark as seen with the unaided eye or through a small telescope. For example, the dark patch near the bottom of Fig. 75 and a little to the left of the center with a rather sharply defined lower edge is known as *Mare Serenitatis* (The Serene Sea). The light line running out from the right of it and just under the big crater Copernicus is the Apennine range of mountains. The most conspicuous features which are visible with an ordinary inverting telescope are shown on the map, Fig. 76.

135. The Mountains on the Moon.—There are ten ranges of mountains on the part of the moon which is visible from the earth. The mountains are often extremely slender and lofty, in some cases attaining an altitude of more than 20,000 feet above the plains on which they stand. If the mountains on the earth were relatively as large, they would be more than 15 miles high. The height of the lunar mountains is undoubtedly due, at least in part, to the low surface gravity on the moon, and to the fact that there has been no erosion by air and water.

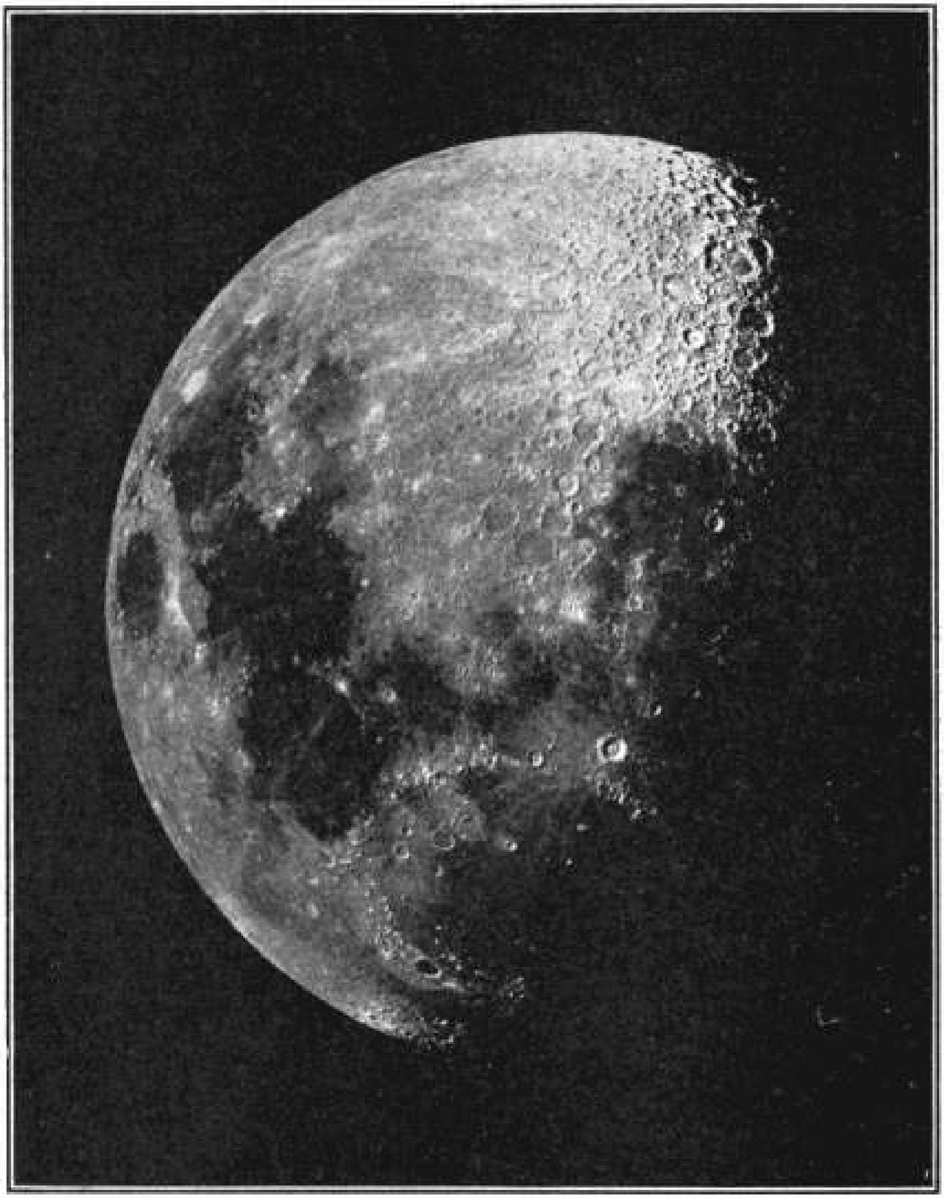

FIG. 75.—The moon at $9\frac{3}{4}$ days. *Photographed at the Yerkes Observatory.*

The height of a lunar mountain is determined from the length of its shadow when the sun's rays strike it obliquely. For example, in Fig. 77 the crater Theophilus is a little below the center, and in its interior are three lofty mountains whose sharp, spirelike shadows stretch off to

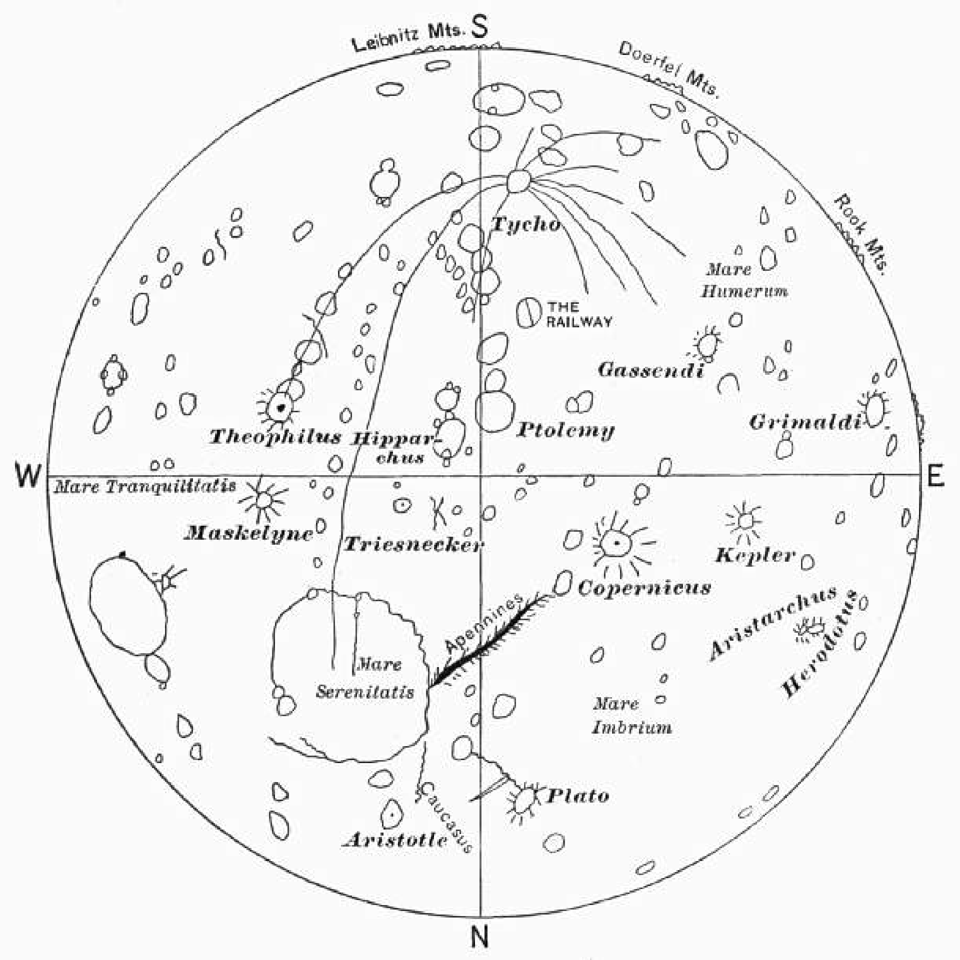

FIG. 76.—Outline map of the moon.

the left. Since the size of the moon and the scale of the photograph are both known, the lengths of the shadows can easily be determined. There is also no difficulty in finding the height of the sun in the sky as seen from this position on the moon when the picture was taken. Consequently, it is possible from these data to compute the height of the mountains. In the particular case of Theophilus, the mountains in its interior are more than 16,000 feet above its floor. On the earth the heights of mountains are counted from the sea level, which, in most cases, is far away. For example, Pike's Peak is about 14,000 feet above the level of the ocean, which is more than 1000 miles away, but only about half that height above the plateau on which it rests. The shadows

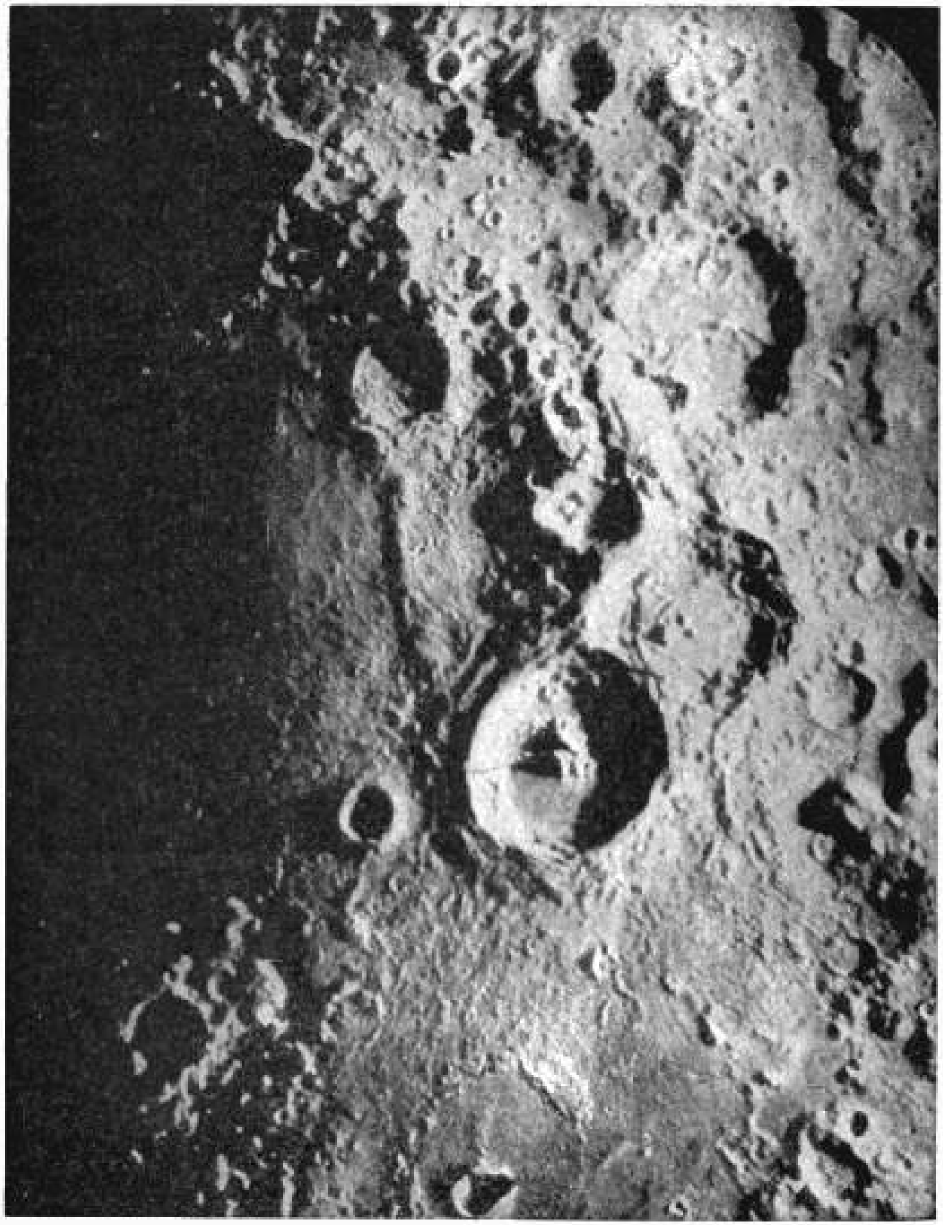

FIG. 77.—The crater Theophilus and surrounding region (Ritchey).

of the lunar mountains are black and sharp because the moon has no atmosphere, and they are therefore well suited for use in measuring the heights of objects on its surface.

136. Lunar Craters.—The most remarkable and the most conspicuous objects of the lunar topography are the craters, of which more than 30,000 have been mapped. There have been successive stages in their formation, for new ones in many places have broken through and encroached upon the old, as shown in Fig. 78. Sometimes the newer ones are precisely on the rims of the older, and sometimes they are entirely in their interiors. The newer craters have deeper floors and steeper and higher rims than the older, and one of the most interesting things about them is that very often they have near their centers lofty and spirelike peaks.

The term crater at once carries the impression to the mind that these objects on the moon are analogous to the volcanic craters on the earth. There is at least an immense difference in their dimensions. Many lunar craters are from 50 to 60 miles in diameter, and, in a number of cases, their diameters exceed 100 miles. Ptolemy is 115 miles across, while Theophilus is 64 miles in diameter and 19,000 feet deep. The lofty peak in the great crater Copernicus towers 11,000 feet above the plains from which it rises. Some of these craters are on such an enormous scale that their rims would not be visible from their centers because of the curvature of the surface of the moon.

The explanation of the craters is by no means easy, and universal agreement has not been reached. If they are of volcanic origin, the activity which was present on the moon enormously surpassed anything now known on the earth. In view of the fact that there are no lava flows, and that in most cases the material around a crater would not fill it, the volcanic theory of their origin seems very improbable and has been abandoned. Another suggestion is that the craters have been formed by the bursting out of great masses of gas which gathered under the surface of the moon and became heated and subject to great tension because of its contraction. According to this theory, the escaping gas threw out large masses of the material which covered it and thus made the rims of the craters. But it is hard to account for the mountains which are so often seen in the interiors of craters.

Gilbert suggested that the lunar craters may have been formed by the impacts of huge meteorites, in some cases many miles across. It is certain that such bodies, weighing hundreds of pounds and even tons, now fall upon the earth occasionally. It is supposed that millions of years ago the collisions of these wandering masses with the earth and moon were much more frequent than they are at the present time. When they strike the earth, their energy is largely taken up by the cushion of the earth's atmosphere; when they strike the moon, they

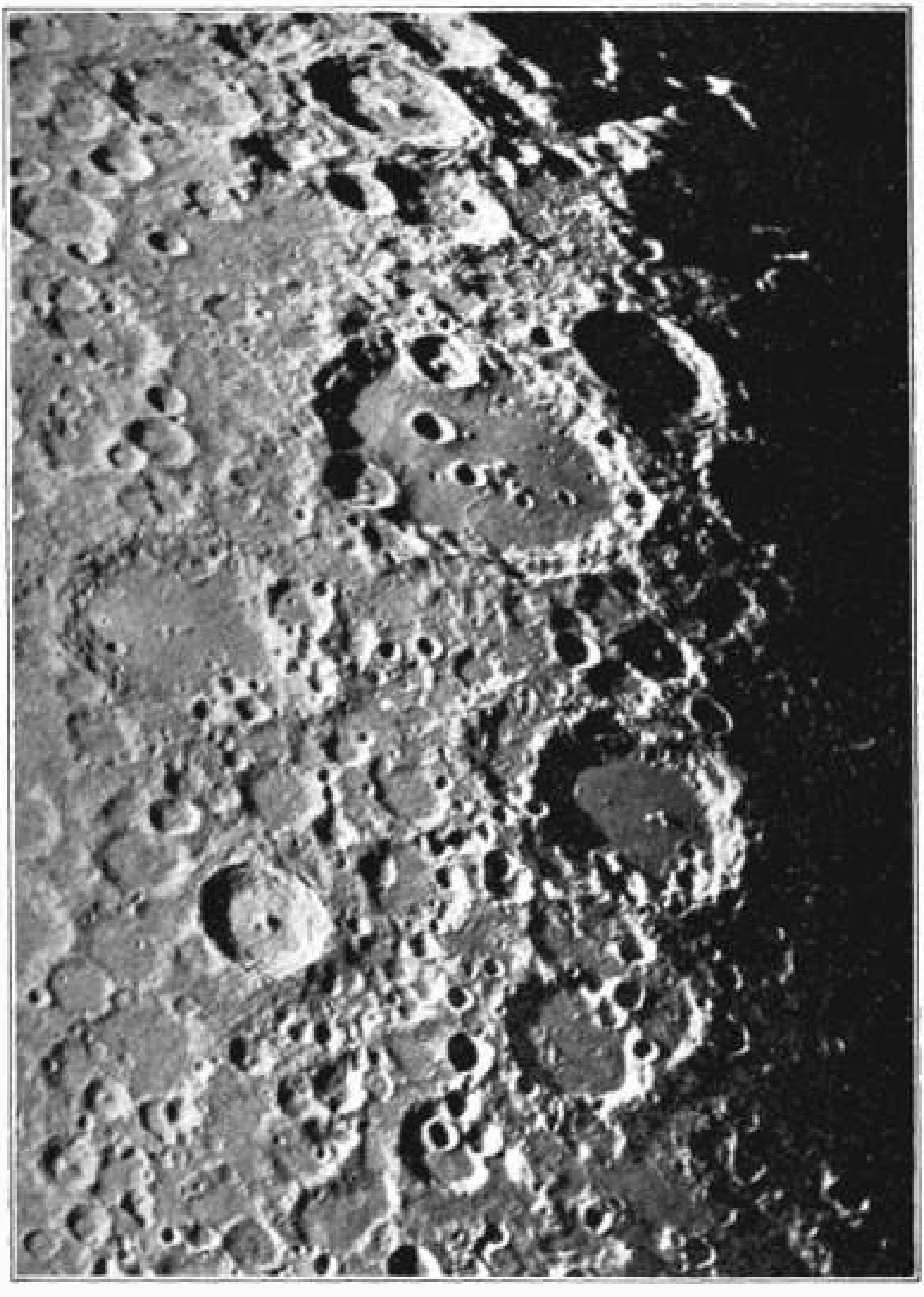

FIG. 78. — The great crater Clavius with smaller craters on its rim and in its interior. *Photographed by Ritchey with the* 40*-inch telescope of the Yerkes Observatory.*

plunge in upon its surface with a speed from 50 to 100 times that of a cannon ball. It does not seem improbable that masses many miles across and weighing millions of tons might produce splashes in the surface of the moon, even though it be solid rock, analogous to the craters which are now observed. The heat generated by the impacts would be sufficient to liquefy the materials immediately under the place where the meteorites struck, and might even cause very great explosions. The mountains in the centers might be due to a sort of reaction from the original splash, or from the heat produced by the collision. At any rate, numerous experiments with projectiles on a variety of substances have shown that pits closely resembling the lunar craters are very often obtained. This view as to the cause of the craters is in harmony with the theory that the earth and moon grew up by the accretion of widely scattered material around nuclei which were originally of much smaller dimensions (Art. 250).

An obvious objection to the theory that the craters on the moon were produced by meteorites is that the earth has no similar formations. Since the earth and moon are closely associated in their revolution around the sun, it is clear that the earth would have been bombarded at least as violently as the moon. The answer to this objection is that, for millions of years, the rains and snows and atmosphere have disintegrated the craters and mountains on the earth, and their powdered remains have been carried away into the valleys. Whatever irregularities of this character the earth's surface may have had in its early stages, all traces of them disappeared millions of years ago. On the other hand, since air and water are altogether absent from the moon, this nearest celestial body has preserved for us the records of the forces to which it, and probably also the earth, were subject in the early stages of their development.

Probably the most serious objection to the impact theory of the craters on the moon is that they nearly all appear to have been made by bodies falling straight toward the moon's center. It is obvious that a sphere circulating in space would in a majority of cases be struck glancing blows by wandering meteorites. The attraction of the moon would of course tend to draw them toward its center, but their velocities are so great that this factor cannot seriously have modified their motions. The only escape from this objection, so far as suggested, is that the heat generated by the impacts may have been sufficient to liquefy the material in the neighborhood of the places where the meteorites struck, and thus to destroy all evidences of the directions of the blows.

137. Rays and Rills.—Some of the large craters, particularly Tycho and Copernicus, have long light streaks, called *rays*, radiating from them like spokes from the axle of a wheel. They are not interfered with by hill or valley, and they often extend a distance of several hundred miles. They cast no shadows, which proves that they are at the same level as the adjacent surface, and they are most conspicuous at

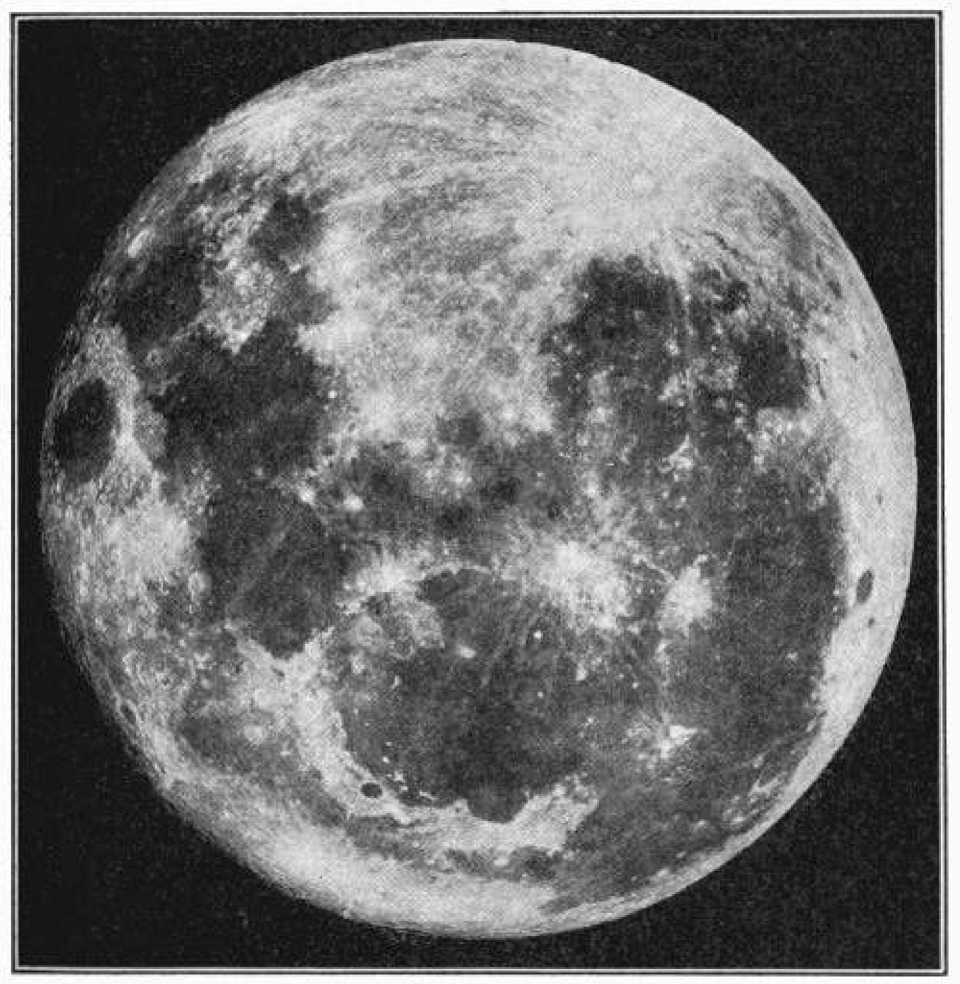

FIG. 79.—The full moon. *Photographed at the Yerkes Observatory* (*Wallace*).

the time of full moon. They are easily seen in Fig. 79. It has been supposed by some that they are lava streams and by others that they were great cracks in the surface, formed at the time when the craters were produced, which have since filled up with lighter colored material from below.

The rills are cracks in the moon's surface, a mile or so wide, a quarter of a mile deep, and sometimes as much as 150 miles in length. They are very numerous, more than 1000 having been so far mapped. The only things at all like them on the earth are such chasms as the Grand Canyon of the Colorado and the cut below Niagara Falls. But these gorges are the work of erosion, which has probably been entirely absent from the surface of the moon. At any rate, it is incredible that the rills have been produced by erosion. The most plausible theory is that they are cracks which have been caused by violent volcanic action, or by the rapid cooling and shrinking of the moon.

The rays and rills are very puzzling lunar features which seem to be fundamentally unlike anything in terrestrial topography. Even our nearest neighbor thus differs very radically from the earth.

138. The Question of Changes on the Moon.—There have been no observed changes in the larger features of the lunar topography, although, from time to time, minor alterations have been suspected. The most probable change of any natural physical feature is in the small crater Linné, in Mare Serenitatis. It was mapped about a century ago, but in 1866 was said by Schmidt to be entirely invisible. It is now visible as on the original maps. It is generally believed that the differences in appearance at various times have been due to slightly different conditions of illumination.

Since the moon's orbit is constantly shifting because of the attraction of the sun, and since the month does not contain an integral number of days, it follows that an observer never gets at two different times exactly the same view of the moon. W. H. Pickering has noticed changes in some small craters, depending upon the phase of the moon, which he interprets as possibly being due to some kind of vegetation which flourishes in the valleys where he supposes heavier gases, such as carbon dioxide, might collect. Some of his observations have been verified by other astronomers, but his rather bold speculations as to their meaning have not been accepted.

It is altogether probable that the moon long ago arrived at the stage where surface changes practically ceased. The only known influences which could now disturb its surface are the feeble tidal strains to which it is subject, and the extremes of temperature between night and day. While it would be too much to say that slight disintegration of the surface rocks may not still be taking place, yet it is certain that, on the whole, the moon is a body whose evolution is essentially finished. The seasonal changes are unimportant, but there is alternately for two weeks the blinding glare of the sunlight, never tempered by passing clouds or even an atmosphere, and the blackness and frigidity of the long lunar night. Month succeeds month, age after age, with no important variations in these phenomena.

139. The Effects of the Moon on the Earth.—The moon reflects a relatively small amount of sunlight and heat to the earth, and in conjunction with the sun it produces the tides. These are the only influences of the moon on the earth that can be observed by the ordinary person. It has a number of very minor effects, such as causing minute variations in the magnetic needle, the precession of the equinoxes, and slight changes in the motion of the earth; but they are all so small that they can be detected only by refined scientific methods.

There are a great many ideas popularly entertained, such as that it is more liable to rain at the time of a change of the moon, or that crops grow best when planted in certain phases, which have no scientific foundation whatever. It follows from the fact that more light and heat are received from the sun in 13 seconds than from the moon in a whole year, that its heating effects on the earth cannot be important. The passing of a fleecy cloud, or the haze of Indian summer, cuts off more heat from the sun than the moon sends to the earth in a year. Consequently, it is entirely unreasonable to suppose that the moon has any important climatic effects on the earth. Besides this, recorded observations of temperature, the amount of rain, and the velocity of the wind, in many places, for more than 100 years, fail to show with certainty any relation between the weather and phases of the moon.

The phenomena of storms themselves show the essential independence of the weather and the phases of the moon. Storm centers move across the country in a northeasterly direction at the rate of 400 to 500 miles per day, and sometimes they can be followed entirely around the earth. Consequently, if a storm should pass one place at a certain phase of the moon, it would pass another a few thousand miles eastward at quite a different phase. The theory that a storm occurred at a certain phase of the moon would then be verified for one longitude and would fail of verification at all the others.

140. Eclipses of the Moon.—The moon is eclipsed whenever it passes into the earth's shadow so that it does not receive the direct light of the sun. In Fig. 80, E represents the earth and PQR the earth's shadow, which comes to a point at a distance of $870,000$ miles from the

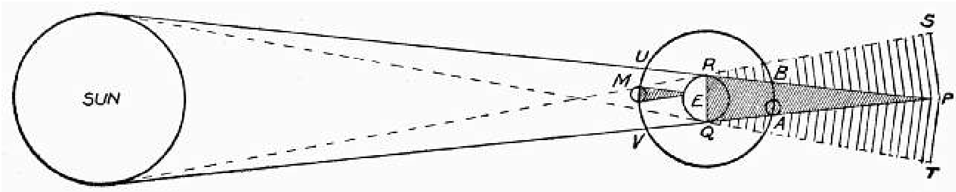

FIG. 80.—The condition for eclipses of the moon and sun.

earth's center. The only light received from the sun within this cone is that small amount which is refracted into it by the earth's atmosphere in the zone QR. In the regions TQP and SRP the sun is partially eclipsed, the light being cut off more and more as the shadow cone is approached. The shadow cone PQR is called the *umbra*, and the parts TQP and SRP, the *penumbra*.

When the moon is about to be eclipsed, it passes from full illumination by the sun gradually into the penumbra, where at first only a small part of the sun is obscured, and it then proceeds steadily across the shadow of increasing density until it arrives at A, where the sun's light is entirely cut off. The distance across the earth's shadow is so great that the moon is totally eclipsed for nearly 2 hours while it is passing through the umbra, and the time from the first contact with the umbra until the last is about 3 hours and 45 minutes.

It appears from Fig. 80 that the moon would be eclipsed every time it is in opposition to the sun, but this figure is drawn to show the relations as one looks perpendicularly on the plane of the ecliptic, neglecting the inclination of the moon's orbit. Figure 81 shows another section in which the plane of the moon's orbit, represented by MN, is

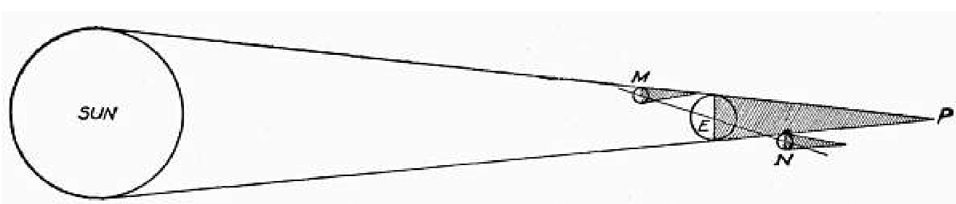

FIG. 81.—Condition in which eclipses of the moon and sun fail.

perpendicular to the page. It is obvious from this that, when the moon is in the neighborhood of N, it will pass south of the earth's shadow instead of through it. The proportions in the figure are by no means true to scale, but a detailed discussion of the numbers involved shows that usually the moon will pass through opposition to the sun without encountering the earth's shadow. But when the earth is 90° in its orbit from the position shown in the figure, that is, when the earth as seen from the sun is at a node of the moon's orbit, the plane of the moon's orbit will pass through the sun, and consequently the moon will be eclipsed. At least, the moon will be eclipsed if it is full when the earth is at or near the node. The earth is at a node of the moon's orbit at two times in the year separated by an interval of six months. Consequently, there may be two eclipses of the moon a year; but because the moon may not be full when the earth is at one of these positions, one or both of the eclipses may be missed.

Since the sun apparently travels along the ecliptic in the sky, the earth's shadow is on the ecliptic 180° from the sun. The places where the moon crosses the ecliptic are the nodes of its orbit, and, consequently, there can be an eclipse of the moon only when it is near one of its nodes. Since the nodes continually regress as a consequence of the sun's attraction for the moon, the eclipses occur earlier year after year, completing a cycle in 18.6 years.

One scientific use of eclipses of the moon is that when they occur, the heat radiated by the moon after it has just been exposed to the perpendicular rays of the sun gives an opportunity, as was explained in Art. 133, of determining its temperature. Also, at the time of a lunar eclipse, the stars in the neighborhood of the moon can easily be observed, and it is a simple matter to determine the exact instant at which the moon passes in front of a star and cuts off its light. Since the positions of the stars are well known, such an observation locates the moon with great exactness at the time the observation is made. It is imaginable that the moon may be attended by a small satellite. If the moon is not eclipsed, its own light or that of the sun will make it impossible to see a very minute body in its neighborhood; but at the time of an eclipse, a satellite may be exposed to the rays of the sun while the neighboring sky will not be lighted up by the moon. Only at such a time would there be any hope of discovering a small body revolving around the moon. A search for such an attendant has been made, but has so far proved fruitless.

141. Eclipses of the Sun.—The sun is eclipsed when the moon is so situated as to cut off the sun's light from at least a portion of the earth. The apparent diameter of the moon is only a little greater than that of the sun, and, consequently, eclipses of the sun last for a very short time. This statement is equivalent to saying that the shadow cone of the moon comes to a point near the surface of the earth, as is shown in Fig. 80. It is also obvious from this diagram that the sun is eclipsed as seen from only a small part of the earth. As the moon moves around the earth in its orbit and the earth rotates on its axis, the shadow cone of the moon describes a streak across the earth which may be somewhat curved.

It follows from the fact that the path of the moon's shadow across the earth is very narrow, as shown in Fig. 82, that a total eclipse of

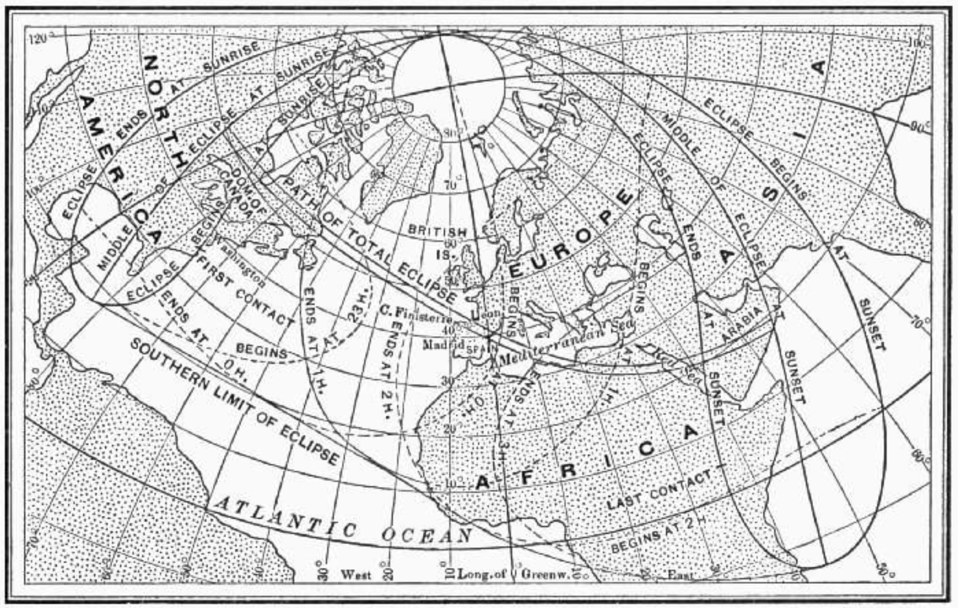

FIG. 82. — Path of the total eclipse of the sun, August 29–30, 1905.

the sun will be observed very infrequently at any given place. On this account, as well as because it is a startling phenomenon for the sun to become dark in the daytime, eclipses have always been very noteworthy occurrences. Repeatedly in ancient times, in which the chronology was very uncertain, writers referred to eclipses in connection with certain historical events, and astronomers, calculating back across the centuries, have been able to identify the eclipses and thus fix the dates for historians in the present system of counting time. The infrequency of eclipses at any particular place is evident from Fig. 83, which gives the paths of all the total eclipses of the sun from 1894–1973. In this long period the greater part of the world is not touched by them at all.

So far the discussion has referred only to total eclipses of the sun; but in the regions on the earth's surface which are near the path of totality,

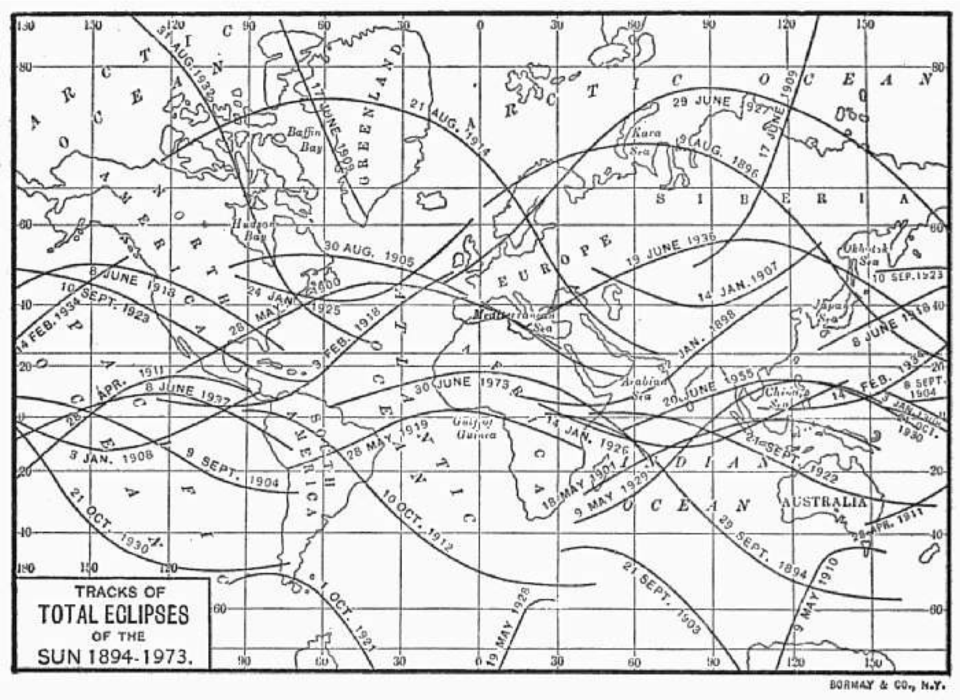

FIG. 83.—Paths of total eclipses of the sun. (From Todd's Total Eclipses.)

or in the penumbra of the moon's shadow, which is entirely analogous to that of the earth, there are partial eclipses of the sun. The region covered by the penumbra is many times that where an eclipse is total; and, consequently, partial eclipses of the sun are not very infrequent phenomena.

There is not an eclipse of the sun every time the moon is in conjunction with the sun because of the inclination of its orbit. For example, when it is near M, Fig. 81, its shadow passes north of the earth. In fact, eclipses of the sun occur only when the sun is near one of the moon's nodes, just as eclipses of the moon occur only when the earth's shadow is near one of the moon's nodes. Consequently, eclipses occur twice a year at intervals separated by 6 synodical months. Since the moon's nodes regress, making a revolution in 18.6 years, eclipses occur, on the average, about 20 days earlier each year than on the preceding year.

The distance UV, Fig. 80, within which an eclipse of the sun can occur is greater than AB, within which an eclipse of the moon can occur. Therefore it is not necessary that the sun shall be as near the moon's node in order that an eclipse of the sun may result as it is in order that there may be an eclipse of the moon. When the relations are worked out fully, it is found that there will be at least one solar eclipse each time the sun passes the moon's node, and that there may be two of them. Consequently, in a year, there may be two, three, or four eclipses of the sun. If there are only two eclipses, the moon's shadow is likely to strike somewhere near the center of the earth and give a total eclipse. On the other hand, if there are two eclipses while the sun is passing a single node of the moon's orbit, they must occur, one when the sun is some distance from the node on one side, and the other when it is some distance from the node on the other side. In this case the moon's shadow, or at least its penumbra, strikes first near one pole of the earth and then near the other. These eclipses are generally only partial.

142. Phenomena of Total Solar Eclipses.—A total eclipse of the sun is a startling phenomenon. It always occurs precisely at new moon, and consequently the moon is invisible until it begins to obscure the sun. The first indication of a solar eclipse is a black slit or section cut out of the western edge of the sun by the moon which is passing in front of it from west to east. For some time the sunlight is not diminished enough to be noticeable. Steadily the moon moves over the sun's disk; and, as the instant of totality draws near, the light rapidly fails, animals become restless, and everything takes on a weird appearance. Suddenly a shadow rushes across the surface of the earth at the rate of more than 1300 miles an hour, the sun is covered, the stars flash out, around the apparent edge of the moon are rose-colored prominences (Art. 236) of vaporous material forced up from the sun's surface to a height of perhaps 200,000 miles, and all around the sun, extending out as far as half its diameter, are the streamers of pearly light which constitute the sun's corona (Art. 238). After about 7 minutes, at the very most, the western edge of the sun is uncovered, daylight suddenly reappears, and the phenomena of a partial eclipse take place in the reverse order.

Total eclipses of the sun afford the most favorable conditions for searching for small planets within the orbit of Mercury, and it is only during them that the sun's corona can be observed.

X. QUESTIONS

1. Verify by observations the motion of the moon eastward among the stars, and its change in declination during a month.

2. For an observer on the moon describe, (a) the apparent motions of the stars; (b) the motion of the sun with respect to the stars; (c) the diurnal motion of the sun; (d) the motion of the earth with respect to the stars; (e) the motion of the earth with respect to the sun; (f) the diurnal motion of the earth; (g) the librations of the earth.

3. Describe the phases the moon would have throughout the year if the plane of its orbit were perpendicular to the plane of the ecliptic.

4. What would be the moon's synodical period if it revolved around the earth from east to west in the same sidereal period?

5. Show by a diagram that, if the moon always presents the same face toward the earth, it rotates on its axis and its period of rotation equals the sidereal month.

6. Is it possible that the moon has an atmosphere and water on the side remote from the earth?

7. Suppose you could go to the moon and live there a month. Give details regarding what you would observe and the experiences you would have.

8. What are the objections to the theory that lunar craters are of volcanic origin? That they were produced by meteorites?

9. How do you interpret rays and rills under the hypothesis that lunar craters were produced by meteorites?

10. If the earth's reflecting power is 4 times that of the moon, how does earthshine on the moon compare with moonshine on the earth?

CHAPTER VIII
THE SOLAR SYSTEM

I. The Law of Gravitation

143. The Members of the Solar System.—The members of the solar system are the sun, the planets and their satellites, the planetoids, the comets, and the meteors. It may possibly be that some of the comets and meteors, coming in toward the sun from great distances and passing on again, are only temporary members of the system. The sun is the one preëminent body. Its volume is nearly a thousand times that of all the other bodies combined, its mass is so great that it controls all their motions, and its rays illuminate and warm then. It is impossible to treat of the planets without taking into account their relations to the sun, but the constitution and evolution of the sun are quite independent of the planets.

The eight known planets are, in the order of their distance from the sun, Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune. The first six are conspicuous objects to the unaided eye when they are favorably located, and they have been known from prehistoric times; Uranus and Neptune were discovered in 1781 and 1846, respectively. The planetoids (often called the small planets and sometimes the asteroids) are small planets which, with a few exceptions, revolve around the sun between the orbits of Mars and Jupiter. The comets are bizarre objects whose orbits are very elongated and lie in every position with respect to the orbits of the planets. Probably at least a part of the meteors are the remains of disintegrated comets; they are visible only when they strike into the earth's atmosphere.

144. The Relative Dimensions of the Planetary Orbits.—The distance from the earth to the sun is called the *astronomical unit.* The distances from the planets to the sun can be determined in terms of the astronomical unit without knowing its value in miles.

Consider first the planets whose orbits are interior to that of the earth. They are called the *inferior planets.* In Fig. 84 let S represent the sun, V the planet Venus, and E the earth. The angle SEV is called the *elongation* of the planet, and may vary from zero up to a maximum which depends upon the size of the orbit of V. When the elongation is greatest, the angle at V is a right angle. Suppose the elongation of V is determined by observation day after day until it reaches its maximum. Then, since the elongation is measured and the angle at V is 90°, the shape of the triangle is determined, and SV can be computed by trigonometry in terms of SE.

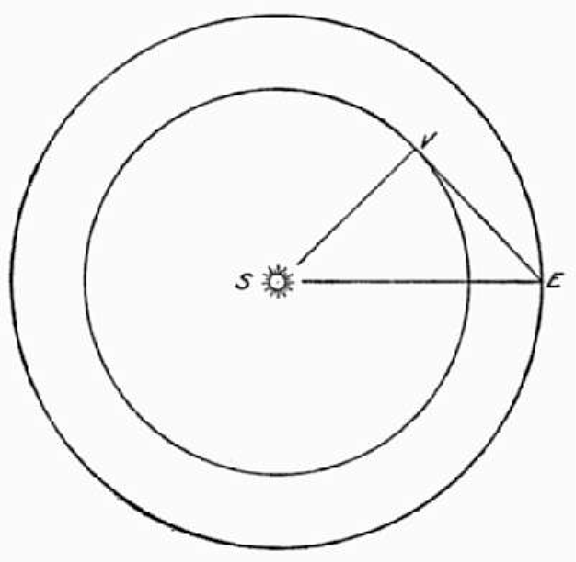

FIG. 84.—Finding the distance of an inferior planet.

Now consider the planets whose orbits are outside that of the earth. They are called the *superior planets*. Suppose the periods of revolution of the earth and Mars, for example, have been determined from long series of observations. This can be done without knowing anything about their actual or relative distances. For, in the first place, the earth's period can be obtained from observations of the apparent position of the sun with respect to the stars; and then the period of Mars can be found from the time required for it to move from a certain position with respect to the sun back to the same position again. For example, when a planet is exactly 180° from the sun in the sky, as seen from the earth, it is said to be in *opposition*. The period from opposition to opposition is called the *synodical period* (compare Art. 120). Let the sidereal period of the earth be represented by E, the sidereal period of the planet by P, and its synodical period by S. Then, analogous to the case of the moon in Art. 120, P is defined by

$$\frac{1}{P} = \frac{1}{E} - \frac{1}{S}.$$

Now return to the problem of finding the distance of a superior planet in terms of the astronomical unit. In Fig. 85, let S represent the sun, and E_1 and M_1 the positions of the earth and Mars when Mars is in opposition. Let E_2 and M_2 represent the positions of the earth and Mars when the angle at E_2 is, for example, a right angle. Mars is then said to be in *quadrature*, and the time when it has this position can be determined by observation. The angles M_1SE_2 and M_1SM_2 can be determined from the periods of the earth and Mars and the interval of time required for the earth and Mars to move from E_1 and M_1 respectively to E_2 and M_2. The difference of these two angles is M_2SE_2, from which, together with the right angle at E_2, the distance SM_2 in terms of SE_2 can be computed by trigonometry.

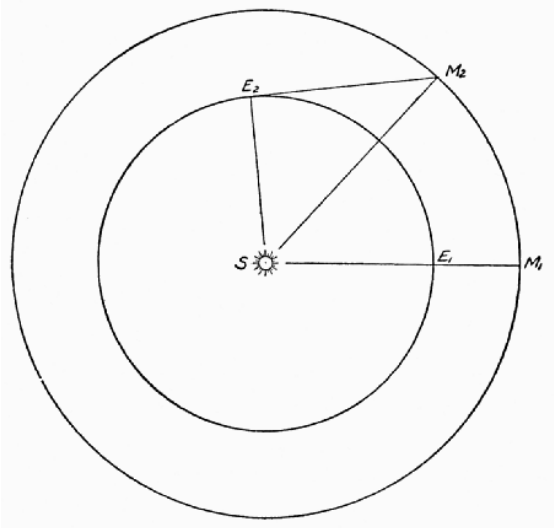

FIG. 85.—Finding the distance of a superior planet.

A little complication in the processes which have been described arises from the fact that the orbit of the earth is not a circle. But the manner in which the distance of the earth from the sun varies can easily be determined from observations of the apparent diameter of the sun, for the apparent diameter of an object varies inversely as its distance. After the variations in the earth's distance have been found, the results can all be reduced without difficulty to a single unit. The unit adopted is half the length of the earth's orbit, and is called its *mean distance*, though it is a little less than the average distance to the sun.

145. Kepler's Laws of Planetary Motion.—The last great observer before the invention of the telescope was the Danish astronomer Tycho Brahe (1546–1601). He was an energetic and most painstaking worker. He not only catalogued many stars, but he also observed comets, proving they are beyond the earth's atmosphere, and obtained an almost continuous record for many years of the positions and motions of the sun, moon, and planets.

Tycho Brahe's successor was his pupil Kepler (1571–1630), who spent more than 20 years in attempting to find from the observations of his master the manner in which the planets actually move. The results of an enormous amount of calculation on his part are contained in the following three laws of planetary motions:

I. *Every planet moves so that the line joining it to the sun sweeps over equal areas in equal intervals of time, whatever their length.* This is known as the law of areas.

II. *The orbit of every planet is an ellipse with the sun at one of its foci.*

III. *The squares of the periods of any two planets are proportional to the cubes of their mean distances from the sun.*

All the complexities of the apparent motions of the planets are explained by Kepler's three simple laws when taken in connection with the periods of the planets and the positions of their orbits.

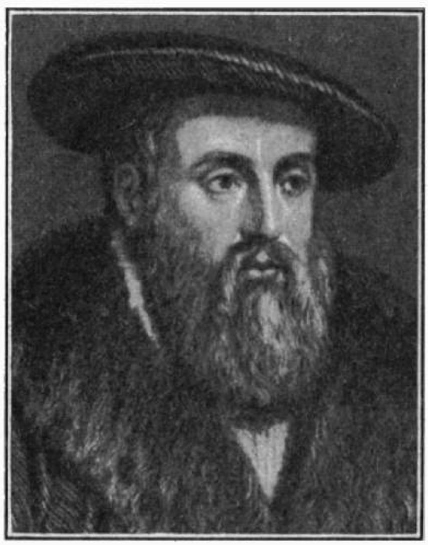

FIG. 86. — Johann Kepler.

146. The Law of Gravitation.—Newton based his greatest discovery, the law of gravitation, on Kepler's laws. From each one of them he drew an important conclusion.

Newton proved by a suitable mathematical discussion, based on his laws of motion, that it follows from Kepler's first law that *every planet is acted on by a force which is directed toward the sun.* This was the first time that the sun and planets were shown to be connected dynamically. Before Newton's time it was generally supposed that there was some force acting on the planets in the direction of their motion which kept them going in their orbits.

The first law of Kepler led to the conclusion that the planets are acted on by forces directed toward the sun, but gave no information whatever regarding the manner in which the forces depend upon the position of the planet. The second law furnishes a basis for the answer to this question, and from it Newton proved that the *force acting on each planet varies inversely as the square of its distance from the sun.*

The law of the inverse squares is encountered in many phenomena besides gravitation. For example, it holds for magnetic and electric forces, the intensity of light and of sound, and the magnitudes of water and earthquake waves. The reason it holds for the radiation of light is easily understood. The area of the spherical surface which the rays cross in proceeding from a point is proportional to the square of its radius. Since the intensity of illumination is inversely proportional to the illuminated area, it is inversely as the square of the distance. If gravitation in some way depended on lines of force extending out from matter radially, it would vary inversely as the square of the distance, but nothing is positively known as to its nature.

Another interesting question remains, and that is whether the gravitation of a body is strictly proportional to its inertia, regardless of its constitution and condition, or whether it depends upon its composition, temperature, and other characteristics. All other known forces, such as magnetism, depend upon other things than mass, and it might be expected the same would be true of gravitation. But it follows from Kepler's third law that the sun's attraction for the several planets is independent of their different constitutions, motions, and physical conditions. Since the same law holds for the 800 planetoids as well, in which there is opportunity for great diversities, it is concluded that gravitation depends upon nothing whatever except the masses and the distances of the attracting bodies.

Suppose the attraction between unit masses at unit distance is taken as unity, and consider the attraction of a body composed of many units for another of many units. To fix the ideas, suppose one body has 5 units of mass and the other 4 units; the problem is to find the number of units of force between them at distance unity. Each of the 5 units exerts a unit of force on each of the 4 units. That is, each of the 5 units exerts all together 4 units of force on the second body. Therefore, the entire first body exerts $5 \times 4 = 20$ units of force on the second body; or, the whole force is proportional to the products of the masses.

On uniting the results obtained from Kepler's three laws and assuming that they hold always and everywhere, the universal law of gravitation is obtained:

Every particle of matter in the universe attracts every other particle with a force which is proportional to the product of their masses, and which varies inversely as the square of the distance between them.

147. The Importance of the Law of Gravitation.—The importance of a physical law depends upon the number of phenomena it coördinates and upon the power it gives the scientist of making predictions. Consider the law of gravitation in these respects. In his great work, *Philosophiæ Naturalis Principia Mathematica* (The Mathematical Principles of Natural Philosophy), commonly called simply the *Principia*, Newton showed how every known phenomenon of the motions, shapes, and tides of the solar system could be explained by the law of gravitation. That is, the elliptical paths of the planets and the moon, the slow changes in their orbits produced by their slight mutual attractions, the oblateness of rotating bodies, the precession of

FIG. 87.—Isaac Newton.

the equinoxes, and the countless small irregularities in planetary and satellite motions that can be detected by powerful telescopes, are all harmonious under the law of gravitation, and what once seemed to be a hopeless tangle has been found to be an orderly system. All the discoveries in this direction for more than 200 years have confirmed the exactness of the law of gravitation until it is now by far the most certainly established physical law.

Not only is the law of gravitation operative in the great phenomena where its effects are easy to detect, but also in everything in which the motion of matter is involved. It is found on reflection that all phenomena depend either directly or indirectly upon the motion of matter, for even changes of the mental state of an individual are accompanied by corresponding changes in the structure of his brain. When a person moves, his changed relation to the remainder of the universe causes a corresponding change in the gravitational stress by which he is connected with it; indeed, when he thinks, the alterations in his brain at once cause alterations in the gravitational forces between it and matter even in the remotest parts of space. These effects are certainly real, though there is no known means of detecting them.

The law of gravitation became in the hands of the successors of Newton one of the most valuable means of discovery. Time after time such great mathematicians as Laplace and Lagrange, using it as a basis, predicted things which had not then been observed, but which invariably were found later to be true. But scientific men are not contented with simply making predictions and finding that they come true. On the basis of their established laws they seek to foresee what will happen in the almost indefinite future, even beyond the time when the human race shall have become extinct, and, similarly, what the conditions were back before the time when life on the earth began.

The law of gravitation was undoubtedly Newton's greatest discovery, and the importance of it and his other scientific work is indicated by the statements of competent judges. The brilliant German scholar, Leibnitz (1646–1716), a contemporary of Newton and his greatest rival, said, "Taking mathematics from the beginning of the world to the time when Newton lived, what he had done was much the better half." The French mathematician, Lagrange (1736–1813), one of the greatest masters of celestial mechanics, wrote, "Newton was the greatest genius that ever existed, and the most fortunate, for we cannot find more than once a system of the world to establish." The English writer on the history of science, Whewell, said, "It [the law of gravitation] is indisputably and incomparably the greatest scientific discovery ever made, whether we look at the advance which it involved, the extent of the truth disclosed, or the fundamental and satisfactory nature of this truth." Compare these splendid and deserved eulogies with Newton's own estimate of his efforts to find the truth: "I do not know what I may appear to the world; but to myself I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, while the great ocean of truth lay all undiscovered before me." There is every reason to believe that this is the sincere and unaffected expression of a great mind which realized the magnitude of the unknown as compared to the known.

In Westminster Abbey, in London, Newton lies buried among the noblest and the greatest English dead, and over his tomb on a tablet they have justly engraved, "Mortals, congratulate yourselves that so great a man has lived for the honor of the human race."

148. The Conic Sections.—After having found that, if the orbit of a body is an ellipse with the center of force at a focus, then the force to which it is subject varies inversely as the square of its distance, Newton took up the converse problem. Under the assumption that the attractive force varies inversely as the square of the distance, he proved that the orbit must be what is called a *conic section*, an example of which is the ellipse.

The conic sections are highly interesting curves first studied by the ancient Greeks. They derive their name from the fact that they can be obtained by cutting a circular cone with planes. In Fig. 88 is shown a double circular cone whose vertex is at V. A plane section perpendicular to the axis of the cone gives a circle C. An oblique section gives an ellipse E; however, the plane must cut both sides of the cone. When the plane is parallel to one side, or element, of the cone, a parabola P is obtained. When the plane cuts the two branches of the double cone, the two branches of an hyperbola HH are obtained. There are in addition to these figures certain limiting cases. One is that in which the intersecting plane passes only through the vertex V giving a simple point; another is that in which the intersecting plane touches only one element of the cone, giving a single straight line; and the last is that in which the intersecting plane passes through the vertex V and cuts both branches of the cone, giving two intersecting straight lines.

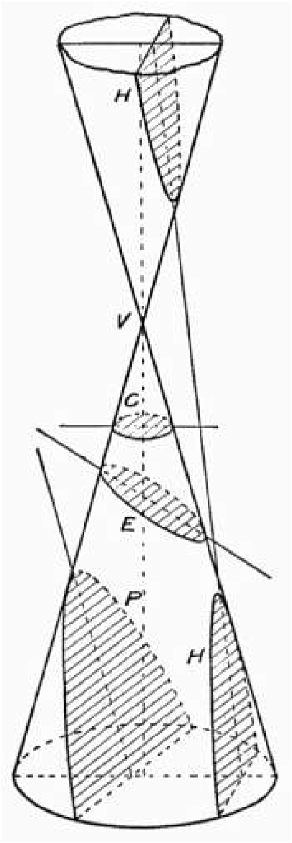

FIG. 88.—The conic sections.

The character of the conic described depends entirely upon the central force and the way in which the body is started. For example, suppose a body is started from O, Fig. 89, in the direction OT, perpendicular to OS. If the initial velocity of the body is zero, it will fall straight to S. If the initial velocity is not too great, it will describe the ellipse E, and O will be the aphelion point. If the initial velocity is just great enough so that the centrifugal acceleration balances the attraction, the orbit will be the circle C. If the initial velocity is a little greater than that in the circle, the body will describe the ellipse E', and O will be the perihelion point. If the initial velocity is exactly $\sqrt{2}$ times that for the circular orbit, the body will move in the parabola P. If the initial velocity is still greater, the orbit will be an hyperbola H. And finally, if the initial velocity is infinite, the path will be the straight line whose direction is OT. If the initial direction of motion is not perpendicular to OS, the results are analogous, except that there is then no initial velocity which will give a circular orbit.

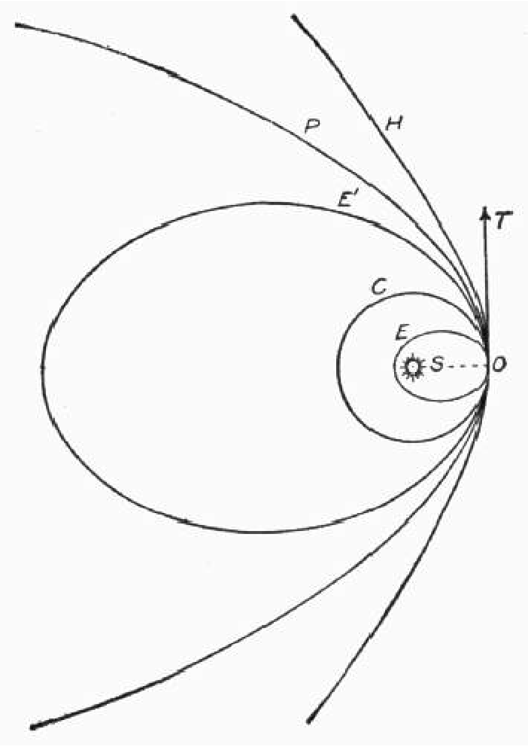

FIG. 89.—Different conics depending on the initial velocity.

It is seen from this discussion that it is as natural for a body to move in one conic section as in another. Some of the satellites move in orbits which are very nearly circular; the planets move in ellipses with varying degrees of elongation; many comets move in orbits which are sensible parabolas; and there may possibly be comets which move in hyperbolas.

149. The Question of other Laws of Force.—Many other laws of force than that of the inverse squares are conceivable. For example, the intensity of a force might vary inversely as the third power of the distance. The character of the curve described by a body moving subject to any such force can be determined by mathematical processes. It is found that, if the force varied according to any other power of the distance than the inverse square, except directly as the first power, then (save in special initial conditions) the orbits would be curves leading either into the center of force or out to infinity. Such a law would of course be fatal to the permanence of the planetary system.

If the force varied directly as the distance, the orbits would all be exactly ellipses, in spite of the mutual attractions of the planets, the sun would be at the center of all the orbits, and all the periods would be the same. This would imply an enormous speed for the remote bodies.

150. Perturbations.—If the planets were subject to no forces except the attraction of the sun, their orbits would be strictly ellipses. But, according to the law of gravitation, every planet attracts every other planet. Their mutual attractions are small compared to that of the sun because of their relatively small masses, but they cause sensible, though small, deviations from strict elliptical motion, which are called *perturbations*.

The mutual perturbations of the planets are sometimes regarded as blemishes on what would be otherwise a perfect system. Such a point of view is quite unjustified. Each body is subject to certain forces, and its motion is the result of its initial position and velocity and these forces. If the masses of the planets were not so small compared to that of the sun, their orbits would not even resemble ellipses.

The problems of the mutual perturbations of the planets and those of the perturbations of the moon are exceedingly difficult, and have taxed to the utmost the powers of mathematicians. In order to obtain some idea of their nature consider the case of only two planets, P_1 and P_2. The forces that P_1 and P_2 would exert upon each other if they both moved in their unperturbed elliptical orbits can be computed without excessive difficulty, and the results of these forces can be determined. But the resulting departures from elliptical motion cause corresponding alterations in the forces, which produce new perturbations. These new perturbations in turn change the forces again. The forces give rise to new perturbations, and the perturbations to new perturbing forces, and so on in an unending sequence. In the solar system where the masses of the planets are small compared to that of the sun, the perturbations of the series decrease very rapidly in importance. If the masses of the planets were large compared to the sun so that Kepler's laws would not have been even approximately true, it is doubtful if even the genius of Newton could have extracted from the intricate tangle of phenomena the master principle of the celestial motions, the law of gravitation.

Although the perturbations may be small, the question arises whether they may not be extremely important in the long run. The subject was treated by Lagrange and Laplace toward the end of the eighteenth century. They proved that the mean distances, the eccentricities, and the inclinations of the planetary orbits oscillate through relatively narrow ranges, at least for a long time. If these results were not true, the stability of the system would be imperiled, for with extreme variation of especially the first two of these quantities the characteristics of the planetary orbits would be entirely changed. On the other hand, the perihelion points and the places where the planes of the orbits of the planets intersect a fixed plane not only have small oscillations, but they involve terms which continually change in one direction. Examples of perturbations of precisely this sort already encountered are the precession of the equinoxes (Art. 47) and the revolution of the moon's line of nodes (Art. 119).

151. The Discovery of Neptune.—Not only can the perturbations be computed when the positions, initial motions, and the masses of the planets are given, but the converse problem can be treated with some success. That is, if the perturbations are furnished by the observations, the nature of the forces which produce them can be inferred. The most celebrated example of this converse problem led to the discovery of the planet Neptune.

In 1781 William Herschel discovered the planet Uranus while carrying out his project of examining every object in the heavens within reach of his telescope. After it had been observed for some time its orbit was computed. In order to predict its position exactly it was necessary to compute the perturbations due to all known bodies. This was done

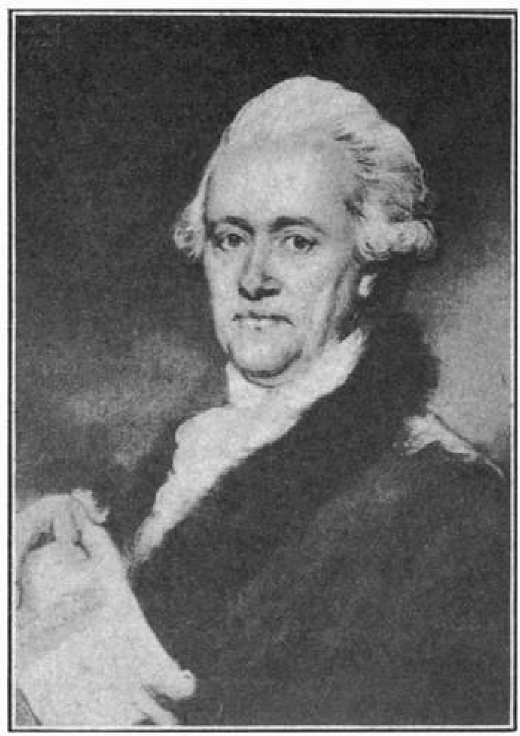

FIG. 90.—William Herschel.

by Bouvard on the basis of the mathematical theory of Laplace. But by 1820 there were unmistakable discordances between theory and observation; by 1830, they were still more serious; by 1840, they had become intolerable. This does not mean that prediction assigned the planet to one part of the sky and observation found it in a far different one; for, in 1840, its departure from its calculated position amounted to only two thirds the apparent distance between the two components of Epsilon Lyræ (Art. 88), a quantity invisible to the unaided eye. It seems incredible that so slight a discordance between theory and observation after 60 years of accumulation could have led to any valuable results.

By 1820 it began to be suggested that the discrepancies in the motion of Uranus might be due to the attraction of a more remote unknown planet. The problem was to find the unknown planet. Such excessive mathematical difficulties were involved that it seemed insoluble. In fact, Sir George Airy, Astronomer Royal of England, expressed himself later than 1840 as not believing the problem could be solved. However, a young Englishman, Adams, and a young Frenchman, Leverrier, with all the enthusiasm of youth, quite independently took up the problem about 1845. Adams finished his work first and communicated his results both to Challis, at Cambridge, and to Airy, at Greenwich. To say the least, they took no very active interest in the matter and allowed the search for the supposed body to be postponed. Adams continued his work and made five separate and very laborious computations. In the meantime Leverrier completed his work and sent the results to a young German astronomer, Galle. Impatiently Galle waited for the night and the stars. On the first evening after receiving Leverrier's letter, September 23, 1846, he looked for the unknown body, and found it within half a degree of the position assigned to it by Leverrier, which agreed substantially with that indicated by Adams.

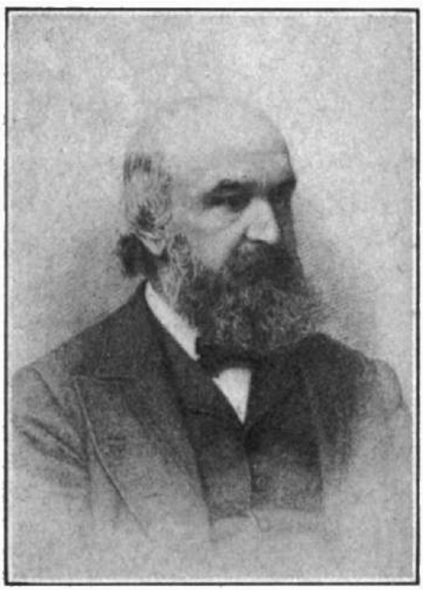

FIG. 91.—John Couch Adams.

Neptune is nearly three thousand millions of miles from the earth, beyond the reach of all our senses except that of sight, and it can be seen only with telescopic aid; its distance is so great that more than four hours are required for its light to come to us, yet it is bound to the remainder of the system by the invisible bonds of gravitation. But its attraction slightly influenced the motions of Uranus, and from these slight disturbances its existence and position were inferred. Notwithstanding the fact that both Adams and Leverrier made assumptions respecting the distance of the unknown body which were somewhat in error, their work stands as a monument to the reasoning powers of the human mind, and to the perfection of the theory of the motions of the heavenly bodies.

FIG. 92. — Joseph Leverrier.

152. The Problem of Three Bodies.—While the problem of two mutually attracting bodies presents no serious mathematical troubles, because the motion is always in some kind of a conic section, that of three bodies is one of the most formidable difficulty. It is often supposed that it has not been, and perhaps that it cannot be, solved. Such an idea is incorrect, as will now be explained.

The theory of the perturbations of the planets is really a problem of three, or rather of eight, bodies, and has been completely solved for an interval of time not too great. That is, while the orbits of the bodies cannot be described for an indefinite interval of time because they are not closed curves but wind about in a very complicated fashion, nevertheless it is possible to compute their positions with any desired degree of precision for any time not too remote. Therefore, in a perfectly real and just sense the problem has been solved.

There are particular solutions of the problem of three bodies in which the motion can be described for any period of time, however long. The first of these were discovered by Lagrange, who found two special cases. In one of them the bodies move so as to remain always in a straight line, and in the other so as to be always at the vertices of an equilateral triangle. In both cases the orbits are conic sections. In

1878 an American astronomer, Hill, in connection with his work on the motion of the moon, discovered some less simple but immensely more important special cases. Since 1890 Poincaré, universally regarded as the greatest mathematician of recent times, has proved the existence of an infinite number of these special cases called periodic solutions. In all of them the problem is exactly solved. Still more recently Sundman, of Helsingfors, Finland, has in an important mathematical sense solved the general case. However, in spite of all the results that have been achieved, the problem still presents to the mathematician unsolved questions of almost infinite variety.

153. The Cause of the Tides.—So far in the present discussion only the effect of one body on the motion of another, taken as a whole, has been considered. There remains to be considered the distortion of one body by the attraction of another. These deformations give rise to the tides.

Before proceeding to a direct discussion of the tidal problem it is necessary to state an important principle, namely, *if two bodies are subject to equal parallel accelerations, their relative positions are not changed.* The truth of this proposition follows from the laws of motion, but it is better understood from an illustration. Suppose two bodies of the same or different dimensions are dropped from the top of a high tower. They have initially a certain relation to each other and they are subject to equal parallel accelerations, namely, those produced by the earth's attraction. In their descent they fall faster and faster; but, neglecting the effects of the resistance of the air, they preserve the same relations to each other.

Let E, Fig. 93, represent the earth, and O and O' two points on its surface. Consider the tendency of the moon M to displace O on the surface of the earth. The moon attracts the center of the earth E in the direction EM. Let its acceleration be represented by EP. In the same units OA represents the acceleration of M on O in direction and amount. The line OA is greater than EP because the moon is nearer to O than it is to E. Now resolve OA into two components, one of which, OB, shall be equal and parallel to EP. The other component is OC. Since OB and EP are equal and parallel, it follows from the principle stated at the beginning of this article that they do not change the relative positions of E and O. Therefore OC, the outstanding component, represents the tide-raising acceleration both in direction and amount.

The results for O' are analogous, and the tide-raising force $O'C'$ is

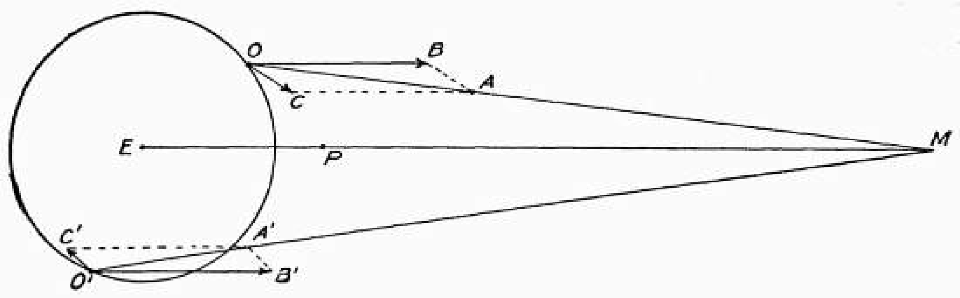

FIG. 93.—Resolution of the tide-raising forces.

directed away from the moon because $O'A'$ is shorter than EP. Figure 94 shows the tide-raising accelerations around the whole circum-

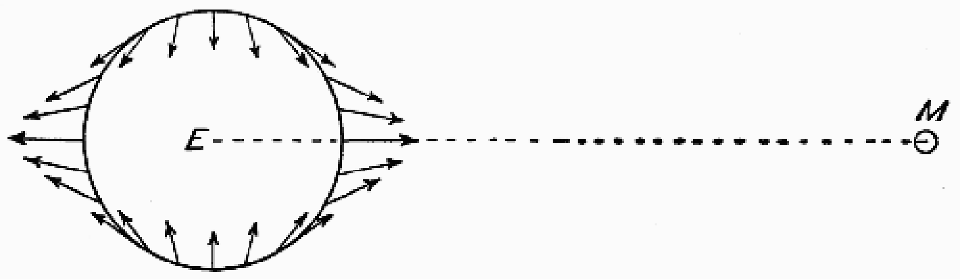

FIG. 94.—The tide-raising forces.

ference of the earth. This method of deriving the tide-raising forces is the elementary geometrical counterpart of the rigorous mathematical treatment,[1] and it can be relied on to give correctly all that there is in this part of the subject.

A more detailed discussion than can be entered into here shows that the tide-raising forces are about 5 per cent greater on the side of the earth which is toward the moon than on the side away from the moon. The forces outward from the surface of the earth in the line of the moon are about twice as great as those which are directed inward 90° from this line. The tidal forces due to the sun are a little less than half as

[1] An analytical discussion proves that the tide-raising force is proportional to the product of the mass of the disturbing body and the radius of the disturbed body, and inversely proportional to the cube of the distance between the disturbing and disturbed bodies.

great as those due to the moon; no other bodies have sensible tidal effects on the earth.

154. The Masses of Celestial Bodies.—The masses of celestial bodies are determined from their attractions for other bodies. Suppose a satellite revolves around a planet in an orbit of measured dimensions in an observed period. From these data it is possible to compute the acceleration of the planet for the satellite because the attraction balances the centrifugal acceleration. It is possible to determine what the earth's attraction would be at the same distance, and, consequently, the relation of its mass to that of the other planet. There has been much difficulty in finding the masses of Mercury and Venus because they have no known satellites. Their masses have been determined with considerable reliability from their perturbations of each other and of the earth, and from their perturbations of certain comets that have passed near them.

A useful formula for the sum of the masses of any two bodies m_1 and m_2 which attract each other according to the law of gravitation, for example, the two components of a double star, is

$$m_1 + m_2 = \frac{a^3}{P^2},$$

where a is the distance between the bodies expressed in terms of the earth's distance from the sun as unity, and where P is the period expressed in years. The sum of the masses is expressed in terms of the sun's mass as unity.

155. The Surface Gravity of Celestial Bodies.—The surface gravity of a celestial body is an important factor in the determination of its surface conditions, and is fundamental in the question of its retaining an atmosphere. The surface gravity of a spherical body depends only upon its mass and dimensions.

Let m represent the mass of the earth, g its surface gravity, and r its radius. Then by the law of gravitation

$$g = k^2 \frac{m}{r^2},$$

where k^2 is a constant depending on the units employed. Let M, G, and R represent in the same units the corresponding quantities for another body. Then

$$G = k^2 \frac{M}{R^2}.$$

On dividing the second equation by the first, it is found that

$$\frac{G}{g} = \frac{M}{m}\left(\frac{r}{R}\right)^2,$$

from which the surface gravity G can be found in terms of that of the earth when the mass and radius of M are given.

It is sometimes convenient to have the expression for the ratio of the gravities of two bodies in terms of their densities and dimensions. Let d and D represent the densities of the earth and the other body respectively. Then, since $m = \frac{4}{3}\pi d r^3$ and $M = \frac{4}{3}\pi D R^3$, it is found that

$$\frac{G}{g} = \frac{D}{d}\frac{R}{r}.$$

That is, the surface gravities of celestial bodies are proportional to the products of their densities and radii. A small density may be more than counterbalanced by a large radius, as, for example, in the case of the sun, whose density is only one fourth that of the earth but whose surface gravity is about 27.6 times that of the earth.

XI. QUESTIONS

1. If the sidereal period of a planet were half that of the earth, what would be its period from greatest eastern elongation to its next succeeding greatest eastern elongation?

2. If the sidereal period of a planet were twice that of the earth, what would be its period from opposition to its next succeeding opposition?

3. What would be the period of a planet if its mean distance from the sun were twice that of the earth?

4. What would be the mean distance of a planet if its period were twice that of the earth?

5. The motion of the moon around the earth satisfies (nearly) Kepler's first two laws. What are the respective conclusions which follow from them?

6. The force of gravitation varies directly as the product of the masses. Show that the acceleration of one body with respect to another, both being free to move, is proportional to the sum of their masses. *Hint.* Use both the second and third laws of motion.

7. In Lagrange's two special solutions of the problem of three bodies the law of areas is satisfied for each body separately with respect to the center of gravity of the three. What conclusion follows from this fact? How does the force toward the center of gravity vary?

II. The Orbits, Dimensions, and Masses of The Planets

156. Finding the actual Scale of the Solar System.—It was seen in Art. 144 that the relative dimensions of the solar system can be determined without knowing any actual distance. It follows from this that if the distance between any two bodies can be found, all the other distances can be computed.

The problem of finding the actual scale of the solar system is of great importance, because the determination of the dimensions of all its members depends upon its solution, and the distance from the earth to the sun is involved in measuring the distances to the stars. Not until after the year 1700 had it been solved with any considerable degree of approximation, but the distance from the earth to the sun is now known with an error probably not exceeding one part in a thousand.

The direct method of measuring the distance to the sun, analogous to that used in case of the moon (Art. 123), is of no value because the apparent displacement to be measured is very small, the sun is a body with no permanent surface markings, and its heat seriously disturbs the instruments. But, as has been seen (Art. 144), the distance from the earth to any other member of the system is equally useful, and in some cases the measurement of the distances to the other bodies is feasible.

Gill, at the Cape of Good Hope, measured the distance of Mars with considerable success, but its disk and red color introduced difficulties. These difficulties do not arise in the case of the smaller planetoids, which appear as starlike points of light, but their great distances decrease the accuracy of the results by reducing the magnitude of the quantity to be measured. However, in 1898, Witt, of Berlin, discovered a planetoid whose orbit lies largely within the orbit of Mars and which approaches closer to the earth than any other celestial body save the moon. Its nearness, its minuteness, and its absence of marked color all unite to make it the most advantageous known body for getting the scale of the solar system by the direct method. Hinks, of Cambridge, England, made measurements and reductions of photographs secured at many observatories, and found that the parallax of the sun, or the angle subtended by the earth's radius at the mean distance of the sun, is $8''.8$, corresponding to a distance of $92,897,000$ miles from the earth to the sun.

The distance of the earth from the sun can also be found from the aberration of light. The amount of the aberration depends upon the velocity of light and the speed with which the observer moves across the line of its rays. The velocity of light has been found with great accuracy from experiments on the surface of the earth. The amount of the aberration has been determined by observations of the stars. From the two sets of data the velocity of the observer can be computed. Since the length of the year is known, the length of the earth's orbit can be obtained. Then it is an easy matter, making use of the shape of the orbit, to compute the mean distance from the earth to the sun. The results obtained in this way agree with those furnished by the direct method.

Another and closely related method depends upon the determination of the earth's motion in the line of sight (Art. 226) by means of the spectroscope. Spectroscopic technique has been so highly perfected that when stars best suited for the purpose are used the results obtained give the earth's speed with a high degree of accuracy. Its velocity and period furnish the distance to the sun, as in the method depending upon the aberration, and the results are about as accurate as those furnished by any other method.

There are several other methods for finding the distance to the sun which have been employed with more or less success. One of them depends upon transits of Venus across the sun's disk. Another involves the attraction of the sun for the moon. But none of them is so accurate as those which have been described.

157. The Elements of the Orbits of the Planets.—The position of a planet at any time depends upon the size, shape, and position of its orbit, together with the time when it was at some particular position, as the perihelion point. These quantities are called the *elements of an orbit*, and when they are given it is possible to compute the position of the planet at any time.

The size of an orbit is determined by the length of its major axis. It is an interesting and important fact that the period of revolution of a planet depends only upon the major axis of its orbit, and not upon its eccentricity or any other element. The shape of an orbit is defined by its eccentricity. The position of a planet's orbit is determined by its orientation in its plane and the relation of its plane to some standard plane of reference. The longitude of the perihelion point defines the orientation of an orbit in its plane. The plane of reference in common use is the plane of the ecliptic. The position of the plane of the orbit is defined by the location of the line of its intersection with the plane of the ecliptic and the angle between the two planes. The distance from the vernal equinox eastward to the point where the orbit of the body crosses the ecliptic from south to north is called the longitude of the ascending node, and the angle between the plane of the ecliptic and the plane of the orbit is called the inclination.

In Fig. 95, VNQ represents the plane of the ecliptic and SNP the plane of the orbit. The vernal equinox is at V, the angle VSN is the longitude of the ascending node, the angle $VSN + NSP$ is the longitude of the perihelion, and the angle QNP is the inclination of the orbit.

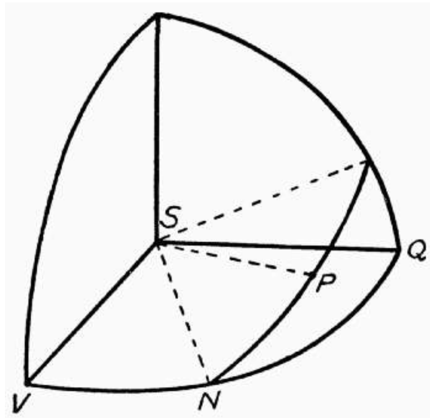

FIG. 95.—Elements of the orbit of a planet.

The elements of the orbits of the planets, which change very slowly, are given for January 1, 1916, in Table IV.

TABLE IV

PLANET	DISTANCE, MILLIONS OF MILES	PERIOD IN YEARS	ECCENTRICITY	INCLINATION TO ECLIPTIC	LONGITUDE OF NODE	LONGITUDE OF PERIHELION	LONGITUDE ON JAN. 1, 1916
Mercury	36.0	0.241	0.20562	7° 0′	47° 20′	76° 9′	334° 2′
Venus	67.2	0.615	0.00681	3 24	75 55	130 23	345 50
Earth	92.9	1.000	0.01674	0 00	——	101 30	99 49
Mars	141.5	1.881	0.09332	1 51	48 55	334 31	116 25
Jupiter	483.3	11.862	0.04836	1 18	99 36	12 58	3 51
Saturn	886.0	29.458	0.05583	2 30	112 55	91 24	102 20
Uranus	1781.9	84.015	0.04709	0 46	73 34	169 18	312 9
Neptune	2791.6	164.788	0.00854	1 47	130 51	43 54	120 12

To the elements of the orbits of the planets must be added the direction of their motion in order to be altogether complete. The result is very simple, for they all revolve in the same direction, namely, eastward.

The most interesting and important element of the planetary orbits is the mean distance. The distance of Neptune from the sun is 30 times that of the earth and nearly 80 times that of Mercury. Since the amount of light and heat received per unit area by a planet varies inversely as the square of its distance from the sun, it follows that if the units are chosen so that the amount received by the earth is unity, then the respective amounts received by the several planets are: Mercury, 6.66; Venus, 1.91; Earth, 1.00; Mars, 0.43; Jupiter, 0.037; Saturn, 0.011; Uranus, 0.0027; Neptune, 0.0011. It is seen that the earth receives more than 900 times as much light and heat per unit area as Neptune, and that in the case of Mercury and Neptune the ratio is more than 6000. Obviously, other things being equal, the climatic conditions on planets differing so greatly in distance from the sun would be enormous.

As seen from Neptune the sun presents a smaller disk than Venus does to us when nearest to the earth. It is sometimes supposed that Neptune is far away in the night of space where the sun looks simply like a bright star. This is far from the truth, for, since the sunlight received by the earth is $600,000$ times full moonlight, and Neptune gets $\frac{1}{900}$ as much light as the earth, it follows that the illumination of Neptune by the sun is nearly 700 times that of the earth by the brightest full moon. Another erroneous idea frequently held is that Neptune is so far away from the sun that it gets a considerable fraction of its light from other suns. The nearest known star is more than 9000 times as distant from Neptune as Neptune is from the sun, and, consequently, Neptune receives more than $80,000,000$ times as much light and heat as it would if the sun were at the distance of the nearest star.

It is almost impossible to get a correct mental picture of the enormous dimensions of the solar system, and there are often misconceptions in regard to the relative dimensions of the orbits of the various planets. To assist in grasping these distances; suppose one has traveled sufficiently to have obtained some comprehension of the great size of the earth. Then he is in a position to attempt to appreciate the distance to the moon, which is so far that in spite of the fact it is more than 2000 miles in diameter, it is apparently covered by a one-cent piece held at the distance of 6.5 feet. In terms of the earth's dimensions, its distance is about 10 times the circumference of the earth. It is so remote that about 14 days would be required for sound to come from it to the earth if there were an atmosphere the whole distance to transmit it at the rate of a mile in 5 seconds.

Now consider the distance to the sun; it is 400 times that to the moon. If the earth and sun were put 4 inches apart on such a diagram as could be printed in this book, on the same scale the distance from the earth to the moon would be $\frac{1}{100}$ of an inch. If sound could come from the sun to the earth with the speed at which it travels in air, 15 years would be required for it to cross the 92,900,000 of miles between the earth and sun. Some one, having found out at what rate sensations travel along the nerve fibers from the hand to the brain, proved by calculation that if a small boy with a sufficiently long arm should reach out to the sun and burn his hand off, the sensation would not arrive at his brain so that he would be aware of his loss unless he lived to be more than 100 years of age.

The relative dimensions of the orbits of the planets can be best understood from diagrams. Unfortunately, it is not possible to represent them to scale all on the same diagram. Figure 96 shows the orbits of the first four planets, together with that of Eros, which occupies a unique position, and which has been used in getting the scale of the system. Figure 97 shows the orbits of the planets from Mars to Neptune on a scale which is about $\frac{1}{20}$ that of the preceding figure. The most noteworthy fact is the relative nearness of the four inner planets and the enormous distances that separate the outer ones.

158. The Dimensions, Masses, and Rotation Periods of the Planets.—The planets Mercury and Venus have no known satellites and their masses are subject to some uncertainties. The rotation periods of Mercury and Venus are very much in doubt because of their unfavorable positions for observation, while the distances of Uranus and Neptune are so great that so far it has been impossible to see clearly any markings on their surfaces. There is some uncertainty in the diameters of the planets on account of what is called irradiation, which makes a luminous object appear larger than it actually is.

The data given in Table V are based partly on Barnard's many measures at the Lick Observatory, and partly on those adopted for the American Ephemeris and Nautical Almanac.

Some interesting facts are revealed by this table. The first four planets are very small compared to the outer four, and since their volumes are as the cubes of their diameters, the latter average more than a thousand times greater in volume than the former. The inner planets are much denser than the outer ones and, so far as known, rotate on their axes more slowly.

Figure 98 shows an arc of the sun's circumference and the eight planets to the same scale. It is apparent from this diagram how insignificant the earth is in comparison with the larger planets, and how small they are all together in comparison with the sun.

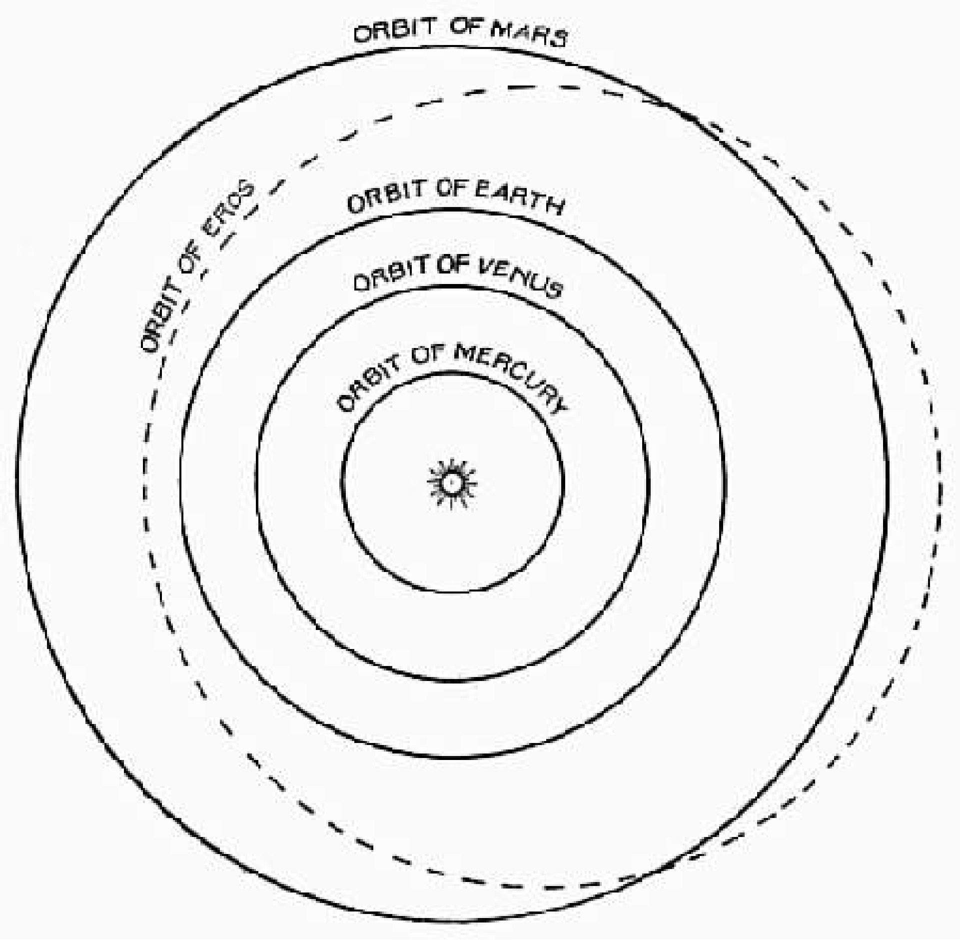

FIG. 96.—Orbits of the four inner planets.

159. The Times for Observing the Planets.—Mercury and Venus are most conveniently situated for observation when they are near their greatest elongations, for then they are not dimmed by the more brilliant rays of the sun. When they are east of the sun they can be seen in the evening, and when they are west of the sun they are observable only in the morning. Ordinarily the evening is more convenient for making observations than the morning, and therefore the results will be given only for this time.

Those planets which are farther from the sun than the earth can be observed best when they are in opposition, or 180° from the sun, for then they are nearest the earth and their illuminated sides are toward the earth. When a planet is in opposition it crosses the meridian at

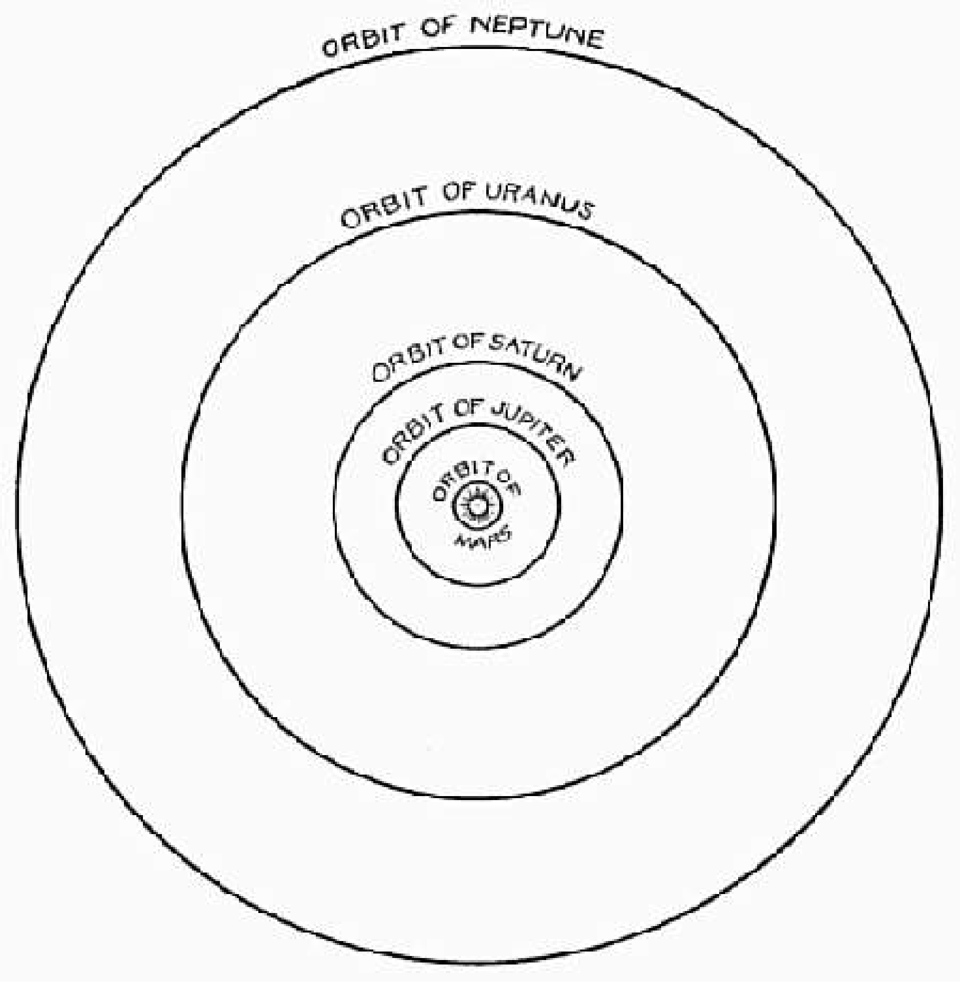

FIG. 97.—Orbit of the outer planets.

midnight, and it can be observed late in the evening in the eastern or southeastern sky.

The problem arises of determining at what times Mercury and Venus are at greatest eastern elongation, and at what times the other planets are in opposition. If the time at which a planet has its greatest eastern elongation is once given, the dates of all succeeding eastern elongations can be obtained by adding to the original one multiples of its synodical period. If S represents the synodical period of an inferior planet, P its sidereal period, and E the earth's period, the synodical period is given

TABLE V

BODY	MEAN DIAMETER	MASS (EARTH = 1)	DENSITY (WATER = 1)	SURFACE GRAVITY ($g = 1$)	PERIOD OF ROTATION	INCLINATION OF EQUATOR TO ORBIT
Sun	864,392	329,390	1.40	27.64	25 d. 8 h.	7° 15′
Moon	2,160	0.0122	3.34	0.16	27 d. 7.7 h.	6° 41′
Mercury	3,009	0.045(?)	4.48(?)	0.31(?)	?	?
Venus	7,701	0.807(?)	4.85(?)	0.85	?	?
Earth	7,918	1.0000	5.53	1.00	23 h. 56 m.	23° 27′
Mars	4,339	0.1065	3.58	0.36	24 h. 37 m.	23° 59′
Jupiter	88,392	314.50	1.25	2.52	9 h. 55 m.	3°
Saturn	74,163	94.07	0.63	1.07	10 h. 14 m.	27°
Uranus	30,193	14.40	1.44	0.99	?	?
Neptune	34,823	16.72	1.09	0.86	?	?

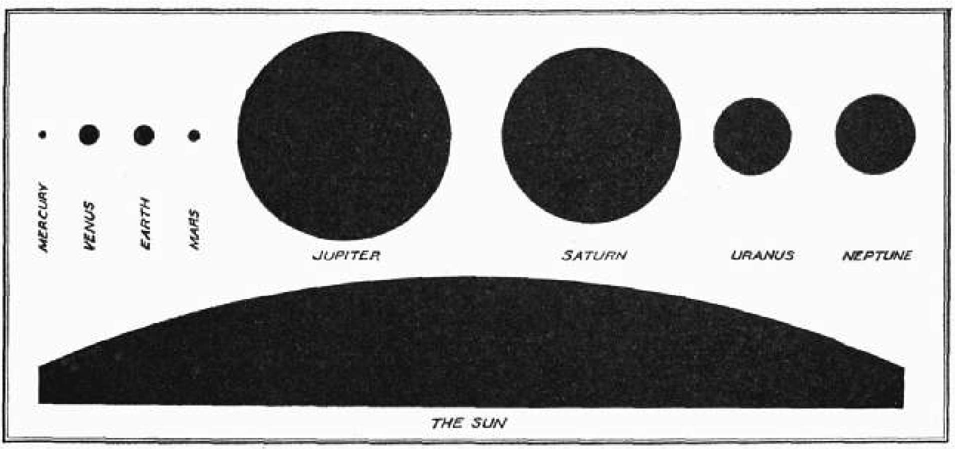

FIG. 98.—Relative dimensions of sun and planets.

by (Arts. 120, 144)

$$\frac{1}{S} = \frac{1}{P} - \frac{1}{E};$$

and in the case of a superior planet the corresponding formula for the synodical period is

$$\frac{1}{S} = \frac{1}{E} - \frac{1}{P}.$$

On the basis of the sidereal periods given in Table IV, these formulas, and data from the American Ephemeris and Nautical Almanac, the following table has been constructed:[1]

TABLE VI

PLANET	EASTERN ELONGATION OR OPPOSITION	SYNODICAL PERIOD
Mercury . . .	Sept. 9, 1916	0 yr. 3 mo. 24.2 d. = 0.31726 yr.
Venus	April 23, 1916	1 yr. 7 mo. 5.7 d. = 1.59882 yr.
Mars	Feb. 9, 1916	2 yr. 1 mo. 18.7 d. = 2.13523 yr.
Jupiter	Oct. 23, 1916	1 yr. 1 mo. 3.1 d. = 1.09206 yr.
Saturn	Jan. 4, 1916	1 yr. 0 mo. 12.6 d. = 1.03514 yr.
Uranus	Aug. 10, 1916	1 yr. 0 mo. 4.3 d. = 1.01205 yr.
Neptune . . .	Jan. 22, 1916	1 yr. 0 mo. 2.2 d. = 1.00611 yr.

The superior planets are most brilliant when they are in opposition; the inferior planets are brightest some time after their greatest eastern elongation because they are then relatively approaching the earth and their decrease in distance more than offsets their diminishing phase. For example, in 1916 Venus was at its greatest eastern elongation April 23, but kept getting brighter until May 27.

Mercury is so much nearer the sun than the earth that its greatest elongation averages only 23°, though it varies from 18° to 28° because of the eccentricity of the orbit of the planet. Consequently, it can be observed only for a very short time after the sun is far enough below the horizon for the brightest stars to be visible. Mercury at its brightest is somewhat brighter than a first-magnitude star. There is no difficulty in observing any of the other planets except Uranus and Neptune, Uranus being near the limits of visibility without optical aid, and Neptune being quite beyond them. Venus is brilliantly white and at its brightest quite surpasses every other celestial object except the sun and moon. Mars is of the first magnitude and decidedly red. Jupiter is white and next to Venus in brilliance. Saturn is of the first magnitude and slightly yellowish.

160. The Planetoids.—On examination it is found that the distance of each planet from the sun is roughly twice that of the preceding,

[1] In this table the tropical year is used and 30 days are taken as constituting a month.

with the exception of Jupiter, whose distance is about 3.5 times that of Mars. In 1772 Titius derived a series of numbers by a simple law which gave the distances of the planets (Uranus and Neptune were not known then) with considerable accuracy, except that there was a number for the vacant space between Mars and Jupiter. The law is that if 4 is added to each of the numbers 0, 3, 6, 12, 24, 48, the sums thus obtained are nearly proportional to the distances of the planets from the sun. This law, commonly called Bode's law, because the writings of Bode made it widely known, rests on no scientific basis and entirely breaks down for Neptune, but it played an important rôle in two discoveries. One of these was that both Adams and Leverrier assigned distances to the planet Neptune on the basis of this law, and computed the other elements of its orbit from its perturbations of Uranus (Art. 151). The other discovery to which Bode's law contributed was that of the planetoids.

Toward the end of the eighteenth century the idea became widespread among astronomers that there was probably an undiscovered planet between Mars and Jupiter whose distance would agree with the fifth number of the Bode series. In 1800 a number of German astronomers laid plans to search for it, but before their work was actually begun Piazzi, at Palermo, on January 1, 1801, the first day of the nineteenth century, made the discovery when he noticed an object (apparently a star) where none had previously been seen. Piazzi called the new planet, which was of small dimensions, Ceres.

After the discovery of Ceres had been made, but before the news of it had reached Germany by the slow processes of communication of those days, the philosopher Hegel published a paper in which he claimed to have proved by the most certain and conclusive philosophical reasoning that there were no new planets, and he ridiculed his astronomical colleagues for their folly in searching for them.

Piazzi observed Ceres for a short time and then he was taken ill. By the time he had recovered, the earth had moved forward in its orbit to a position from which the planetoid could no longer be seen. In a little less than a year the earth was again in a favorable position for observations of Ceres, but the problem of picking it up out of the countless stars that fill the sky, and from which it could not be distinguished except by its motions, was almost as difficult as that of making the original discovery. The difficulty was entirely overcome by Gauss, then a young man of 24, but later one of the greatest mathematicians of his time, for, under the stimulus of this special problem, he devised a practical method of determining the elements of the orbit of a planet from only three observations. After the elements of the orbit of a body are known, its position can be computed at any time. Gauss determined the elements of the orbit of Ceres, and his calculation of its position led to its rediscovery on the last day of the year.

On March 28, 1802, Olbers discovered a second planetoid, which he named Pallas; on September 2, 1804, Harding found Juno; and on March 29, 1807, Olbers picked up a fourth, Vesta. No other was found until 1845, when Hencke discovered Astræa, after a long search of 15 years. In 1847 three more were discovered, and every year since that time at least one has been discovered.

In 1891 a new epoch was started by Wolf, of Heidelberg, who discovered a planetoid by photography. The method is simply to expose a plate two or three hours with the telescope following the stars. The star images are points, but the planetoids leave short trails, or streaks, Fig. 99, because they are moving among the stars. There are now all together more than 800 known planetoids.

After the first two planetoids had been discovered it was supposed that they might be simply the fragments of an original large planet which had been torn to pieces by an explosion. If such were the case, the different parts in their orbits around the sun would all pass through the position occupied by the planet at the time of the explosion; therefore, for some time the search for new planetoids was largely confined to the regions about the points where the orbits of Ceres and Pallas intersect. But this theory of their origin has been completely abandoned. The orbits of Eros and two other planetoids are interior to the orbit of Mars, while many are within 75,000,000 miles of this planet; on the other hand, many others are nearly 300,000,000 miles farther out, and the aphelia of four are even beyond the orbit of Jupiter. Their orbits vary in shape from almost perfect circles to elongated ellipses whose eccentricities are 0.35 to 0.40. The average eccentricity of their orbits is about 0.14, or approximately twice that of the orbits of the planets. Their inclinations to the ecliptic range all the way from zero to 35°, with an average of about 9°.

The orbits of the planetoids are distributed by no means uniformly over the belt which they occupy. Kirkwood long ago called attention to the fact that the planetoids are infrequent, or entirely lacking, at the distances at which their periods would be $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, ... of Jupiter's period. The numerous discoveries since the application of photography have still further emphasized the existence of these remarkable gaps.

FIG. 99.—Photograph of stars showing a planetoid (Egeria) trail in the center of the plate. *Photographed by Parkhurst at the Yerkes Observatory.*

It is supposed that the perturbations by Jupiter during indefinite ages have cleared these regions of the bodies that may once have been circulating in them, but the question has not received rigorous mathematical treatment.

The diameters of Ceres, Pallas, Vesta, and Juno were measured by Barnard with the 36-inch telescope of the Lick Observatory, and he found that they are respectively 485, 304, 243, and 118 miles. There are probably a few more whose diameters exceed 100 miles, but the great majority are undoubtedly much smaller. Probably the diameters of the faintest of those which have been photographed do not exceed 5 miles.

By 1898 the known planetoids were so numerous and their orbits caused so much trouble, because of their large perturbations by Jupiter, that astronomers were on the point of neglecting them, when Witt, of Berlin, found one within the orbit of Mars, which he named Eros. At once great interest was aroused. On examining photographs which had been taken at the Harvard College Observatory in 1893, 1894, and 1896, the image of Eros was found several times, and from these positions a very accurate orbit was computed by Chandler. The mean distance of Eros from the sun is 135,500,000 miles, but its distance varies considerably because its orbit has the high eccentricity of 0.22; its inclination to the ecliptic is about 11°. At its nearest, Eros is about 13,500,000 miles from the earth, and then conditions are particularly favorable for getting the scale of the solar system (Art. 156); and at its aphelion it is 24,000,000 miles beyond the orbit of Mars (Fig. 96).

Not only is Eros remarkable because of the position of its orbit, but in February and March of 1901 it varied in brightness both extensively and rapidly. The period was 2 hr. 38 min., and at minimum its light was less than one third that at maximum. By May the variability ceased. Several suggestions were made for explaining this remarkable phenomenon, such as that the planetoid is very different in reflecting power on different parts, or that it is really composed of two bodies very near together, revolving so that the plane of their orbit at certain times passes through the earth, but the cause of this remarkable variation in brightness is as yet uncertain.

161. The Question of Undiscovered Planets.—The great planets Uranus and Neptune have been discovered in modern times, and the question arises if there may not be others at present unknown. Obviously any unknown planets must be either very small, or very near the sun, or beyond the orbit of Neptune, for otherwise they already would have been seen.

The perihelion of the orbit of Mercury moves somewhat faster than it would if this planet were acted on only by known forces. One explanation offered for this peculiarity of its motion is that it may be disturbed by the attraction of a planet whose orbit lies between it and the sun. A planet in this position would be observed only with difficulty because its elongation from the sun would always be small. Half a century ago there was considerable belief in the existence of an intra-Mercurian planet, and several times it was supposed one had been observed. But photographs have been taken of the region around the sun at all recent total eclipses, and in no case has any object within the orbit of Mercury been found. It is reasonably certain that there is no object in this region more than 20 miles in diameter.

The question of the existence of trans-Neptunian planets is even more interesting and much more difficult to answer. There is no reason to suppose that Neptune is the most remote planet, and the gravitative control of the sun extends enormously beyond it. There are two lines of evidence, besides direct observations, that bear on the question. If there is a planet of considerable mass beyond the orbit of Neptune, it will in time make its presence felt by its perturbations of Neptune.

Since Neptune was discovered it has made less than half a revolution, and the fact that its observed motion so far agrees with theory is not conclusive evidence against the existence of a planet beyond. In fact, there are some very slight residual errors in the theory of the motion of Uranus, and from them Todd inferred that there is probably a planet revolving at the distance of about 50 astronomical units in a period of about 350 years. The conclusion is uncertain, though it may be correct. A much more elaborate investigation has been made by Lowell, who finds that the slight discrepancies in the motion of Uranus are notably reduced by the assumption of the existence of a planet at the distance of 44 astronomical units (period 290 years) whose mass is greater than that of the earth and less than that of Neptune.

It will be seen (Art. 196) that planets sometimes capture comets and reduce their orbits so that their aphelia are near the orbits of their captors. Jupiter has a large family of comets, and Saturn and Uranus have smaller ones. As far back as 1880, Forbes, of Edinburgh, inferred from a study of the orbits of those comets whose aphelia are beyond the orbit of Neptune that there are two remote members of the solar family revolving at the distances of 100 and 300 astronomical units in the immense periods of 1000 and 5000 years. W. H. Pickering has made an extensive statistical study of the orbits of comets and infers the probable existence of three or four trans-Neptunian planets. The data are so uncertain that the correctness of the conclusion is much in doubt.

162. The Zodiacal Light and the Gegenschein.—The zodiacal light is a soft, hazy wedge of light stretching up from the horizon along the ecliptic just as twilight is ending or as dawn is beginning. Its base is 20° or 30° wide and it generally can be followed 90° from the sun, and sometimes it can be seen as a narrow, very faint band 3° or 4° wide entirely around the sky. It is very difficult to decide precisely what its limits are, for it shades very gradually from an illumination perhaps a little brighter than the Milky Way into the dark sky.

The best time to observe the zodiacal light is when the ecliptic is nearly perpendicular to the horizon, for then it is less interfered with by the dense lower air. In the spring the sun is very near the vernal equinox. At this time of the year the ecliptic comes up after sunset from the western horizon north of the equator, and makes a large angle with the horizon. Consequently, the spring months are most favorable for observing the zodiacal light in the evening, and for analogous reasons the autumn months are most favorable for observing it in the morning.

It cannot be seen in strong moonlight.

The *gegenschein*, or counterglow, is a very faint patch of light in the sky on the ecliptic exactly opposite to the sun. It is oval in shape, from 10° to 20° in length along the ecliptic, and about half as wide. It was first discovered by Brorsen in 1854, and later it was found independently by Backhouse and Barnard. It is so excessively faint that it has been observed by only a few people.

The cause of the gegenschein is not certainly known. It has been suggested that it is a sort of swelling in the zodiacal band which appears to be a continuation of the zodiacal light. This explanation calls for an explanation of the zodiacal light, which, of course, might well be independently asked for. The zodiacal light is almost certainly due to the reflection of light from a great number of small particles circulating around the sun in the plane of the earth's orbit, and extending a little beyond the orbit of the earth. An observer at O, Fig. 100, would see a considerable number of these illuminated particles above his horizon H; and with the conditions as represented in the diagram, the zodiacal band would extend faintly beyond the zenith and across the sky.

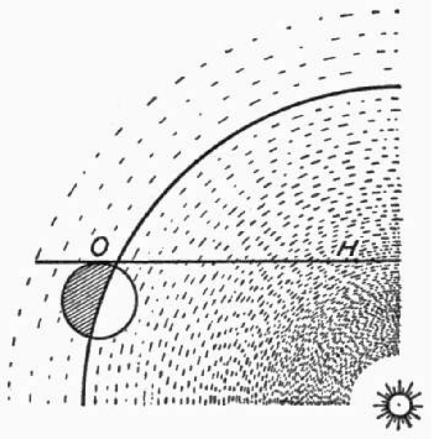

FIG. 100.—Explanation of the zodiacal light.

It is not clear from this theory of the zodiacal light why there should be a condensation exactly opposite the sun. But at a point 930,000 miles from the earth, which is beyond the apex of its shadow, there is a region where, in consequence of the combined forces of the earth and sun, wandering particles tend to circulate in a sort of dynamic whirlpool. It has been suggested that the circulating particles which produce the zodiacal light are caught in this whirl and are virtually condensed enough to produce the observed phenomenon of the gegenschein.

XII. QUESTIONS

1. Which of the methods of measuring the distance from the earth to the sun depend upon our knowledge of the size of the earth, and which are

independent of it?

2. Make a single drawing showing the orbits of all the planets to the same scale. On this scale, what are the diameters of the earth and of the moon's orbit?

3. If the sun is represented by a globe 1 foot in diameter, what would be the diameters and distances of the planets on the same scale?

4. How long would it take to travel a distance equal to that from the sun to the earth at the rate of 60 miles per hour? How much would it cost at 2 cents per mile?

5. The magnitude of the sun as seen from the earth is -26.7. What is its magnitude as seen from Neptune? As seen from Neptune, how many times brighter is the sun than Sirius?

6. If Jupiter were twice as far from the sun, how much fainter would it be when it is in opposition?

7. How great are the variations in the distances of the planets from the sun which are due to the eccentricities of their orbits?

8. Suppose the earth and Neptune were in a line between the sun and the nearest star; how much brighter would the star appear from Neptune than from the earth?

9. In what respects are all the planets similar? In what respects are the four inner planets similar and different from the four outer planets? In what respects are the four outer planets similar and different from the four inner planets?

10. Find the velocities with which the planets move, assuming their orbits are circles.

11. Find the next dates at which Mercury and Venus will have their greatest eastern elongations, and at which Mars, Jupiter, and Saturn will be in opposition.

12. If possible, observe the zodiacal light and describe its location and characteristics.

CHAPTER IX
THE PLANETS

I. Mercury and Venus

163. The Phases of Mercury and Venus.—The inferior planets Mercury and Venus are alike in several respects and may conveniently be treated together. They both have phases somewhat analogous to those of the moon. When they are in inferior conjunction, that is, at *A*, Fig. 101, their dark side is toward the earth and their phase is new. Since the orbits of these planets are inclined somewhat to the plane of the ecliptic, they do not in general pass across the sun's disk. If they do not make a transit, they present an extremely thin crescent when they have the same longitude as the sun. As they move out from *A* toward *B* their crescents increase, and their disks, as seen from the earth, are half illuminated when they have their greatest elongation at *B*. During their motion from inferior conjunction at *A* to their greatest elongation at *B*, and on to their superior conjunction at *C*, their distances from the earth constantly increase, and this increase of distance to a considerable extent offsets the advantage arising from the fact that a larger part of their illuminated areas are visible. In order that an inferior planet may be seen, not only must its illuminated side be at least partly toward the earth, but it must not be too nearly in a line with the sun. For example, a planet at *C*, Fig. 101, has its illuminated side toward the earth, but it is invisible because it is almost exactly in the same direction as the sun.

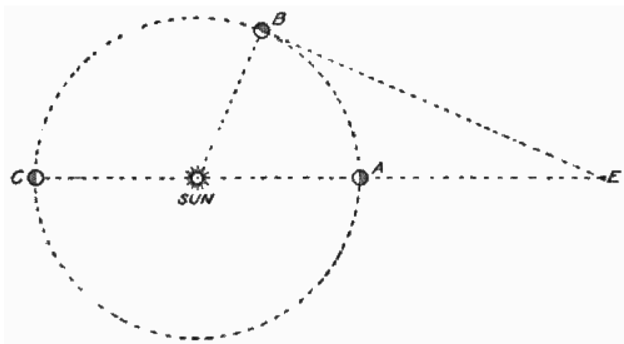

Fig. 101.—Phases of an inferior planet.

The variations in the apparent dimensions of Venus are greater than those of Mercury because, when Venus is nearest the earth, it is much nearer than the closest approach of Mercury, and when it is farthest from the earth, it is much farther than the most remote point in Mercury's orbit. At the time of inferior conjunction the distance of Venus is 25,700,000 miles, while that of Mercury is 56,900,000 miles; and at superior conjunction their respective distances are 160,100,000 and 128,900,000 miles. These numbers are modified somewhat by the eccentricities of the orbits of these three bodies, and especially by the large eccentricity of the orbit of Mercury.

Mercury and Venus transit across the sun's disk only when they pass through inferior conjunction with the sun near one of the nodes of their orbits. The sun is near the nodes of Mercury's orbit in May and November, and consequently this planet transits the sun only if it is in inferior conjunction at one of these times. Since there is no simple relation between the period of Mercury and that of the earth, the transits of Mercury do not occur very frequently. A transit of Mercury is followed by another at the same node of its orbit after an interval of 7, 13, or 46 years, according to circumstances, for these periods are respectively very nearly 22, 41, and 145 synodical revolutions of the planet. Moreover, there may be transits also when Mercury is near the other node of its orbit. The next transits of Mercury will occur on May 7, 1924, and on November 8, 1927. Mercury is so small that its transits can be observed only with a telescope.

The transits of Venus, which occur in June and December, are even more infrequent than those of Mercury. The transits of Venus occur in cycles whose intervals are, starting with a June transit, 8, 105.5, 8, and 121.5 years. The last two transits of Venus occurred on December 8, 1874, and on December 6, 1882. The next two will occur on June 8, 2004, and on June 5, 2012.

The chief scientific uses of the transits of Mercury and Venus are that they give a means of determining the positions of these planets, they make it possible to investigate their atmospheres, and they were formerly used indirectly for determining the scale of the solar system (Art. 156).

164. The Albedoes and Atmospheres of Mercury and Venus.—The albedo of a body is the ratio of the light which it reflects to that which it receives. The amount of light reflected depends to a considerable extent upon whether or not the body is surrounded by a cloud-filled atmosphere. A body which has no atmosphere and a rough and broken surface, like the moon, has a low albedo, while one covered with an atmosphere, especially if it is filled with partially condensed water vapor, has a higher reflecting power. Every one is familiar with the fact that the thunderheads which often appear in the summer sky shine as white as snow when illuminated fully by the sun's rays. It was found by Abbott that their albedo is about 0.65. If an observer could see the earth from the outside, its brightest parts would undoubtedly be those portions of its surface which are covered either by clouds or by snow.

The albedo of Mercury, according to the careful work of Müller, of Potsdam, is about 0.07, while that of Venus is 0.60. This is presumptive evidence that the atmosphere of Mercury is either very thin or entirely absent, and that that of Venus is abundant.

It follows from the kinetic theory of gases (Art. 32) and the low surface gravity of Mercury (Art. 158, Table V) that Mercury probably does not have sufficient gravitative control to retain a very extensive atmospheric envelope. This inference is confirmed by the fact that, when Mercury transits the sun, no bright ring is seen around it such as would be observed if it were surrounded by any considerable atmosphere. Moreover, Müller found that the amount of light received from Mercury at its various phases proves that it is reflected from a solid, uneven surface. Therefore there is abundant justification for the conclusion that Mercury has an extremely tenuous atmosphere, or perhaps none at all.

The evidence regarding the atmosphere of Venus is just the opposite of that encountered in the case of Mercury. Its considerable mass and surface gravity, approximating those of the earth, naturally lead to the conclusion that it can retain an atmosphere comparable to our own. But the conclusions do not rest alone upon such general arguments; for, when Venus transits the sun, its disk is seen to be surrounded by an illuminated atmospheric ring. Besides this, when it is not in transit, but near inferior conjunction, the illuminated ring of its atmosphere is sometimes seen to extend considerably beyond the horns of the crescent. Also, the brilliancy of Venus decreases somewhat from the center toward the margin of its disk where the absorption of light would naturally be the greatest. Spectroscopic observations, which are as yet somewhat doubtful, point to the conclusion that the atmosphere of Venus contains water vapor. Taking all the evidence together, we are justified in the conclusion that Venus has an atmospheric envelope corresponding in extent, and possibly in composition, to that of the earth.

165. The Surface Markings and Rotation of Mercury.— The first astronomer to observe systematically and continuously the surface markings of the sun, moon, and planets was Schröter (1745–

1816). He was an astronomer of rare enthusiasm and great patience, but seems sometimes to have been led by his lively imagination to erroneous conclusions.

Schröter concluded from observations of Mercury made in 1800, that the period of rotation of this planet is 24 hours and 4 minutes. This result was quite generally accepted until after Schiaparelli took up his systematic observations of the planets, at Milan, about 1880. Schiaparelli found that Mercury could be much better seen in the daytime, when it was near the meridian, than in the evening, because the illumination of the sky was found to be a much less serious obstacle than the absorption and irregularities of refraction which were encountered when Mercury was near the horizon. His experience in this matter has been confirmed by later astronomers.

Schiaparelli came to the conclusion, from elusive and vague markings on the planet, that its axis is essentially perpendicular to the plane of its orbit, and that its periods of rotation and revolution are the same. These results are agreed to by Lowell, who has carefully observed the planet with an excellent 24-inch telescope at Flagstaff, Ariz. Although the observations are very difficult, we are perhaps entitled to conclude that the same face of Mercury is always toward the sun.

166. The Seasons of Mercury.—If the period of rotation of Mercury is the same as that of its revolution, its seasons are due entirely to its varying distance from the sun and the varying rates at which it moves in its orbit in harmony with the law of areas. The eccentricity of the orbit of Mercury is so great that at perihelion its distance from the sun is only two thirds of that at aphelion. Since the amount of light and heat the planet receives varies inversely as the square of its distance from the sun, it follows that the illumination of Mercury at aphelion is only four ninths of that at perihelion. It is obvious that this factor alone would make an important seasonal change.

Whatever the period of rotation of Mercury may be, its rate of rotation must be essentially uniform. Since it moves in its orbit so as to fulfill the law of areas, its motion of revolution is sometimes faster and sometimes slower than the average. The result of this is that not exactly the same side of Mercury is always toward the sun, *even if its periods of revolution and rotation are the same.* The mathematical discussion shows that, at its greatest, it is 23°.7 ahead of its mean position in its orbit, and consequently, at such a time, the sun shines around the surface of Mercury 23°.7 beyond the point its rays would reach if its orbit were strictly a circle. Similarly, the planet at times gets 23°.7 behind its mean position. That is, Mercury has a libration (Art. 129) of 23°.7. If Mercury's period of rotation equals its period of revolution, there are, therefore, 132°.6 of longitude on the planet on which the sun always shines, an equal amount on which it never shines, and two zones 47°.4 wide in which there is alternating day and night with a period equal to the period of the planet's revolution around the sun.

If the periods of rotation and revolution of Mercury are the same, the side toward the sun is perpetually subject to its burning rays, which are approximately ten times as intense as they are at the distance of the earth, and, moreover, they are never cut off by clouds or reduced by an appreciable atmosphere. The only possible conclusion is that the temperature of this portion of the planet's surface is very high. On the side on which the sun never shines the temperature must be extremely low, for there is no atmosphere to carry heat to it from the warm side or to hold in that which may be conducted to the surface from the interior of the planet. The intermediate zones are subject to alternations of heat and cold with a period equal to the period of revolution of the planet, and every temperature between the two extremes is found in some zone.

167. The Surface Markings and Rotation of Venus.—The history of the observations of Venus and the conclusions regarding its rotation are almost the same as in the case of Mercury. As early as 1740 J. J. Cassini inferred from the observations of his predecessors that Venus rotates on its axis in 23 hours and 20 minutes. About 1790 Schröter concluded that its rotation period is about 23 hours and 21 minutes, and that the inclination of the plane of its equator to that of its orbit is 53°. These results were generally accepted until 1880, when Schiaparelli announced that Venus, like Mercury, always has the same face toward the sun.

The observations of Schiaparelli were verified by himself in 1895, and they have been more or less definitely confirmed by Perrotin, Tacchini, Mascari, Cerulli, Lowell, and others. However, it must be remarked that the atmosphere interferes with seeing the surface of Venus and that the observations are very doubtful. Moreover, recent direct observations by a number of experienced astronomers point to a period of rotation of about 23 or 24 hours.

The spectroscope can also be applied under favorable conditions to determine the rate at which a body rotates. In 1900 Bélopolsky concluded from observations of this sort that the period of rotation of Venus is short. More accurate observations by Slipher, at the Lowell Observatory, show no evidence of a short period of rotation. The preponderance of evidence seems to be in favor of the long period of rotation, but the conclusion is at present very uncertain.

168. The Seasons of Venus.—The character of the seasons of Venus depends very much upon whether the planet's period of rotation is approximately 24 hours or is equal to its period of revolution. If the planet rotates in the shorter period and if its equator is somewhat inclined to the plane of its orbit, the seasons must be quite similar to those of the earth, though the temperature is probably somewhat higher because the planet is nearer to the sun. On the other hand, if the same face of Venus is always toward the sun, the conditions must be more like those on Mercury, though the range of temperatures cannot be so extreme because its atmosphere reduces the temperature on the side toward the sun and raises it on the opposite side by carrying heat from the warmer side to the cooler.

Suppose the periods of rotation and revolution of Venus are equal. Since the orbit of Venus is very nearly circular, it is subject to only a small libration and only a very narrow zone around it has alternately day and night. The position of the sun in its sky is nearly fixed and the climate at every place on its surface is remarkably uniform. There must be a system of atmospheric currents of a regularity not known on the earth, and it has been suggested that all the water on the planet was long ago carried to the dark side in clouds and precipitated there as snow. This conclusion is not necessarily true, for it seems likely that the air would ascend on the heated side and lose its moisture by precipitation before the high currents which would go to the dark side had proceeded far on their way.

Considered as a whole, Venus is more like the earth than any other planet; and, so far as can be determined, it is in a condition in which life can flourish. In fact, if any other planet than the earth is inhabited, that one is probably Venus. It must be added, however, that there is no direct evidence whatever to support the supposition that there is life upon its surface.

II. Mars

169. The Satellites of Mars.—In August, 1877, Asaph Hall, at Washington, discovered two very small satellites revolving eastward around Mars, sensibly in the plane of its equator. They are so minute and so near the bright planet that they can be seen only with a large telescope, and usually it is advantageous, when observing them, to obscure Mars by a small screen placed in the focal plane. These satellites are called Phobos and Deimos. The only way of determining their dimensions is from the amount of light they reflect to the earth. Though Phobos is considerably brighter than Deimos, its diameter probably does not exceed 10 miles.

Not only are the satellites of Mars very small, but in other respects they present only a rough analogy to the moon revolving around the earth. The distance of Phobos from the center of Mars is only 5850 miles, while that of Deimos is 14,650 miles. That is, Phobos is only 3680 miles from the surface of the planet. The curvature of the planet's surface is such that Phobos could not be seen by an observer from latitudes greater than 68° 15′ north or south of the planet's equator. The relative dimensions of Mars and the orbits of its satellites are shown in Fig. 102.

As was seen in Art. 154, the period of a satellite depends upon the mass of the planet around which it revolves and upon its distance from the planet's center. Notwithstanding the small mass of Mars, its satellites are so close that their periods of revolution are very short, the period of Phobos being 7 hrs. 39 m. and that of Deimos being 30 hrs. 18 m. Since Mars rotates on its axis in 24 hrs. and 37 m., Phobos makes more than 3 revolutions while Mars is making one rotation. It therefore rises in the west, passes eastward across the sky, and sets in the east. Here is an example in which the direction of apparent motion and actual motion are the same. The period of Phobos from meridian to meridian is 11 hrs. and 7 m. On the other hand, Deimos rises in the east and sets in the west with a period from

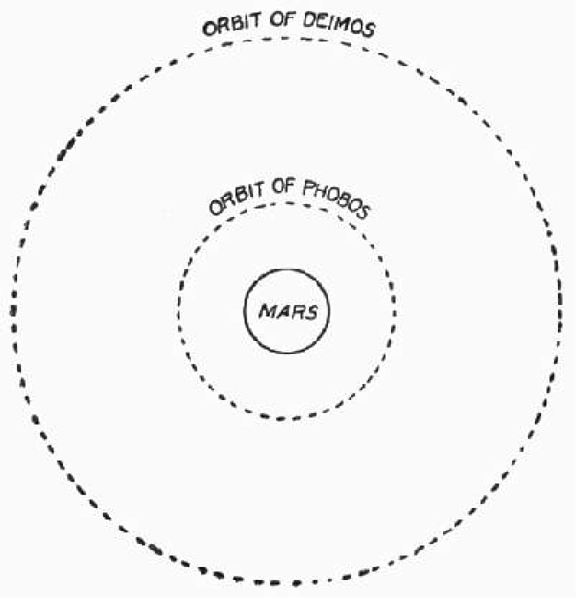

FIG. 102.—Mars and the orbits of its satellites.

meridian to meridian of 131 hrs. and 14 m.

170. The Rotation of Mars.—In 1666 Hooke, an English observer, and Cassini, at Paris, saw dark streaks on the ruddy disk of Mars, and these features of the planet's surface are so definite and permanent that even to-day astronomers can recognize the objects which these men observed and drew. Some of them are shown in Fig. 103, which is a series of 9 photographs, taken one after the other at short intervals, by Barnard, at the Yerkes Observatory. By comparing observations at one time with those made at a later date the period of rotation of the planet can be found. In fact, considerable rotation is observable in the short interval covered by the photographs in Fig. 103. Hooke and Cassini soon discovered that Mars turns on its axis in a

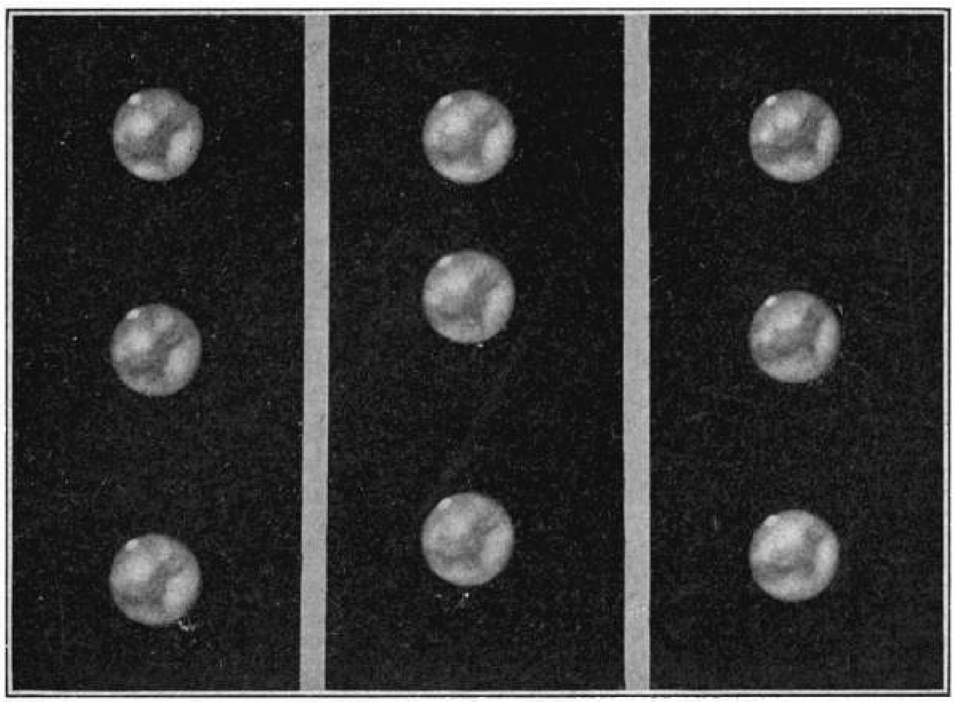

FIG. 103.—Mars. *Photographed by Barnard with the* 40-*inch telescope of the Yerkes Observatory, Sept. 28, 1909.*

period of a little more than 24 hrs. By comparing their observations with those of the present day it is found that its period of rotation is 24 hrs. 37 m. 22.7 secs. The high order of accuracy of this result is a consequence of the fact that the importance of the errors of the observations diminishes as the time over which they extend increases.

The inclination of the plane of the equator of Mars to the plane of its orbit is between 23° and 24°. The inclination cannot be determined as accurately as the period of rotation because the only advantage of a long series of observations consists in their number. But, in spite of its uncertainty, the obliquity of the ecliptic of Mars to its equator is certainly approximately equal to that of the earth, and, consequently, the seasonal changes are qualitatively much like those of the earth. One important difference is that the period of Mars is about 23 months, and, therefore, while its day is only a little longer than that of the earth, its year is nearly twice as long. It is not meant to imply by these statements that the climate of Mars is similar to that of the earth. Its distance from the sun is so much greater that the amount of light and heat it receives per unit area is only about 0.43 of that which the earth receives.

171. The Albedo and Atmosphere of Mars.—According to the observations of Müller, the albedo of Mars is 0.15, which indicates probably a thin atmosphere on the planet.

The surface gravity of Mars is only 0.36 that of the earth, and, consequently, it would be expected on the basis of the kinetic theory of gases that it might retain some atmosphere, though not a very extensive one. Direct observations of the planet confirm this conclusion. In the first place, its surface can nearly always be seen without appreciable interference from atmospheric phenomena. If the earth were seen from a distant planet, such as Venus, not only would the clouds now and then entirely shut off its surface from view, but the reflection and absorption of light in regions where there were no clouds would probably make it impossible to see distinctly anything on its surface.

The fact that Mars has a rare atmosphere is also proved by the suddenness with which it cuts off the light from stars when it passes between them and the earth. Those planets which have extensive atmospheres, such as Jupiter, extinguish the light from the stars more gradually. If the atmosphere of Mars, relatively to its mass, were of the same density as that of the earth, it would be rarer at the surface of the planet than our atmosphere is at the top of the loftiest mountains.

A number of lines of evidence have been given for the conclusion that the atmosphere of Mars is not extensive. The question occasionally arises whether it has any atmosphere at all. The answer to this must be in the affirmative, because faint clouds, possibly of dust or mist, have often been observed on its surface. They are very common along the borders of the bright caps which cover its poles. Another related phenomenon which is very remarkable and not easy to explain is that, sometimes for considerable periods, the planet's whole disk is dim and obscure as though covered by a thin mist. While the cause of this obscuration is not known, it is supposed that it is a phenomenon of the atmosphere of the planet. Besides this, Mars undergoes seasonal changes, not only in the polar caps, which will be considered in the next article, but also even in conspicuous markings of other types. Figure 104 gives three drawings of the same side of Mars by Barnard, on September 23, October 22, and October 28, 1894, showing notable temporary changes in its appearance.

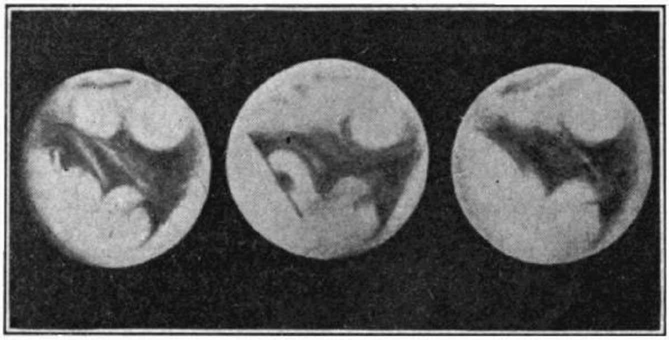

FIG. 104.—Barnard's drawings of Mars.

172. The Polar Caps and the Temperature of Mars.—The surface of Mars on the whole is dull brick-red in color, but its polar regions during their winter seasons are covered with snow-white mantles. One of these so-called polar caps sometimes develops in the course of two or three days over an area reaching down from the pole 25° to 35°; it remains undiminished in brilliancy during the long winter of the planet; and, as the spring advances, it gradually diminishes in size, contracting first around the edges; it then breaks up more or less, and it sometimes entirely disappears in the late summer.

After the southern polar cap has shrunk to the dimensions given by Barnard's observation of August 13, 1894, Fig. 105, an elongated white patch is found to be left behind the retreating white sheet. The same thing was observed in the same place at the corresponding Martian season in 1892, and also at later oppositions. This means that the phenomenon is not an accident, but that it depends upon the nature of the surface of Mars. Barnard has suggested that there may be an elevated region in the place on which the spot is observed where the snow or frost remains until after it has entirely disappeared in the valleys. At any rate, this phenomenon strongly points to the conclusion that there are considerable irregularities in the surface of Mars, though on the whole it is probably much smoother than the earth. This is an important point which must be borne in mind in interpreting other things observed upon the surface of the planet.

The polar cap around the south pole of Mars has been more thoroughly studied than the one at the north pole because the south pole is turned toward the earth when Mars is in opposition near the perihelion point of its orbit. The eccentricity of the orbit of this planet is so great that its distance from the orbit of the earth when it is at its perihelion (which is near the aphelion of the earth's orbit) is more than 23,000,000 miles less than when it is at its aphelion. However, in the course of immense time the mutual perturbations of the planets will so change the orbit of Mars that its northern polar region will be more favorably situated for observations from the earth than its southern.

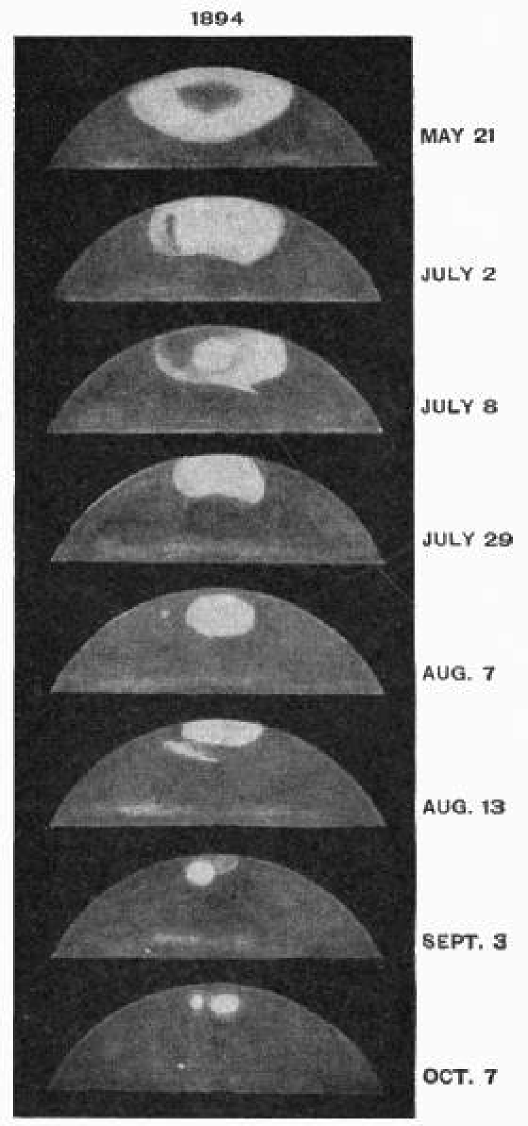

FIG. 105.—Disappearance of polar cap of Mars (Barnard).

If the polar caps of Mars are due to snow, there must be water vapor in its atmosphere. The spectroscope is an instrument which under suitable conditions enables the astronomer to determine the constitution of the atmosphere of a celestial body from which he receives light. Mars is not well adapted to the purpose because, in the first place, the light received from it is only reflected sunlight which may have traversed more or less of its shallow and tenuous atmosphere; and, in the second place, the atmosphere of the earth itself contains usually a large amount of water vapor. It is not easy to make sure that the absorption spectral lines (Art. 225) may not be due altogether to the water vapor in the earth's atmosphere.

The early spectroscopic investigations of Huggins and Vogel pointed toward the existence of water on Mars; the later ones by Keeler and Campbell, with much more powerful instruments and under better atmospheric conditions, gave the opposite result; but the spectrograms obtained by Slipher at the Lowell Observatory, under exceptionally favorable instrumental and climatic conditions, again indicate water on Mars. In view of the difficulties of the problem, a negative result could scarcely be regarded as being conclusive evidence of the entire absence of water on Mars, while evidence of a small amount of water vapor in its atmosphere is not unreasonable and is quite in harmony with the phenomena of its polar caps.

The distance of Mars from the sun is so great that it receives only about 0.43 as much light and heat per unit area as is received by the earth. The question then arises how its polar caps can nearly, or entirely, disappear, while the poles of the earth are perpetually buried in ice and snow. The responses to this question have been various, many of them ignoring the fundamental physical principles on which a correct answer must be based.

In the first place, consider the problem of determining what the average temperature of Mars would be if its atmosphere and surface structure were exactly like those of the earth. That is, let us find what the temperature of the earth would be if its distance from the sun were equal to that of Mars. The amount of heat a planet radiates into space on the average equals that which it receives, for otherwise its temperature would continually increase or diminish. Therefore, the amount of heat Mars radiates per unit area is on the average 0.43 of that radiated per unit area by the earth. Now the amount of heat a body radiates depends on the character of its surface and on its temperature. In this calculation the surfaces of Mars and the earth are assumed to be alike. According to Stefan's law, which has been verified both theoretically and experimentally, the radiation of a black body varies as the fourth power of its absolute temperature. Or, the absolute temperatures of two black bodies are as the fourth roots of their rates of radiation.

Now apply this proportion to the case of Mars and the earth. On the Fahrenheit scale the mean annual surface temperature of the whole earth is about 60°, or 28° above freezing. The absolute zero on the Fahrenheit scale is 491° below freezing. Therefore, the mean temperature of the earth on the Fahrenheit scale counted from the absolute zero is about $491° + 28° = 519°$. Let x represent the absolute temperature of Mars; then, under the assumption that its surface is like that of the earth, the proportion becomes

$$x : 519 = \sqrt[4]{0.43} : \sqrt[4]{1},$$

from which it is found that $x = 420°$. That is, under these hypotheses, the mean surface temperature of Mars comes out $491° - 420° = 71°$ below freezing, or $71° - 32° = 39°$ below zero Fahrenheit.

The results just obtained can lay no claim to any particular degree of accuracy because of the uncertain hypotheses on which they rest. But it does not seem that the hypothesis that the surfaces of Mars and the earth radiate similarly can be enough in error to change the results by very many degrees. If the atmosphere of Mars were of the same constitution as that of the earth, but simply more tenuous, its actual temperature would be lower than that found by the computation. On the other hand, if the atmosphere of Mars contained an abundance of gases which strongly absorb and retain heat, such as water vapor and carbon dioxide, its mean temperature might be considerably above $-39°$. But, taking all these possibilities into consideration, it seems reasonably certain that the mean temperature of Mars is considerably below zero Fahrenheit. The question then arises how its polar caps can almost, or entirely, disappear each summer.

The fundamental principles on which precipitation and evaporation depend can be understood by considering these phenomena in ordinary meteorology. If there is a large quantity of water vapor in the air and the temperature falls, there is precipitation before the freezing point is reached, and the result is rain. On the other hand, if the amount of water vapor in the air is small, there is no precipitation until after the temperature has descended below the freezing point of water. In this case when precipitation occurs it is in the form of snow or hoar frost.

The reverse process is similar. Suppose the temperature of snow is slowly being increased. If there is only a very little water vapor in the air surrounding it, the snow evaporates into water vapor without first melting. On the other hand, if the atmosphere contains an abundance of water vapor, the snow does not evaporate until after its temperature has risen above the freezing point. But at the freezing point the snow turns into water.

The gist of the whole matter is this: If the water vapor in the atmosphere is abundant, precipitation and evaporation take place above the freezing point; and if it is scarce, precipitation and evaporation take place below the freezing point. The temperature at which these processes begin depends only on the density of water vapor present and not at all upon the constitution and density of the remainder of the atmosphere. For example, snow evaporates (or sublimes) at −35° Fahrenheit when the density of the water vapor surrounding it is such that its pressure is less than 0.00018 of ordinary atmospheric pressure; or, if this is the water-vapor pressure and the temperature falls below −35°, snow is precipitated. Similarly, water evaporates at 80° Fahrenheit if the pressure of the water above it is less than 0.034 of atmospheric pressure; or, with this pressure of water vapor, precipitation occurs if the temperature falls below 80°. This explains why the earth's atmosphere on the whole is much dryer in the winter than it is in the summer.

The application to Mars is simple. Suppose its polar caps are actually due to snow or hoar frost, as they appear to be. The fact that they nearly or entirely disappear in the long summers means only that the atmosphere is dry enough for evaporation to take place at the temperature which prevails on the planet. If the temperature of Mars were known, the amount of water vapor in its atmosphere could be computed from the phenomena of the polar caps; and conversely, if the amount of water vapor in the atmosphere of Mars were known, its temperature could be computed.

Some direct considerations on the possible temperature of Mars have been given, and reference has been made to the possibility of determining the water content of its atmosphere by means of the spectroscope. There is an additional line of evidence which bears on the question in hand. The surface of the planet is largely of a brick-red color, and is interpreted as being in a desert condition. While there are dark regions which have been supposed possibly to be marshes, there are certainly no large bodies of water on its surface comparable to the oceans and seas upon the earth. These things confirm the conclusion that water is not abundant on Mars and that its mean temperature may be below zero; but, in the equatorial regions in the long summers, the temperature may rise for a considerable time even above the freezing point.

173. The Canals of Mars.—In 1877, Schiaparelli, an Italian observer of Milan, made the first of a series of important discoveries respecting the surface markings of Mars. Although he had only a modest telescope of 8.75 inches' aperture, he found that the brick-red regions, which had been supposed to be continental areas, were crossed and recrossed by many straight, dark, greenish streaks which always ended in the darker regions known as "seas." These streaks were of great length, extending in uniform width from a few hundred miles up to three or four thousand miles. While they appeared to be very narrow, they must have been at least 20 miles across. Schiaparelli called them "canali" (channels), which was translated as "canals," a designation unfortunately too suggestive, for they have no analogy to anything on

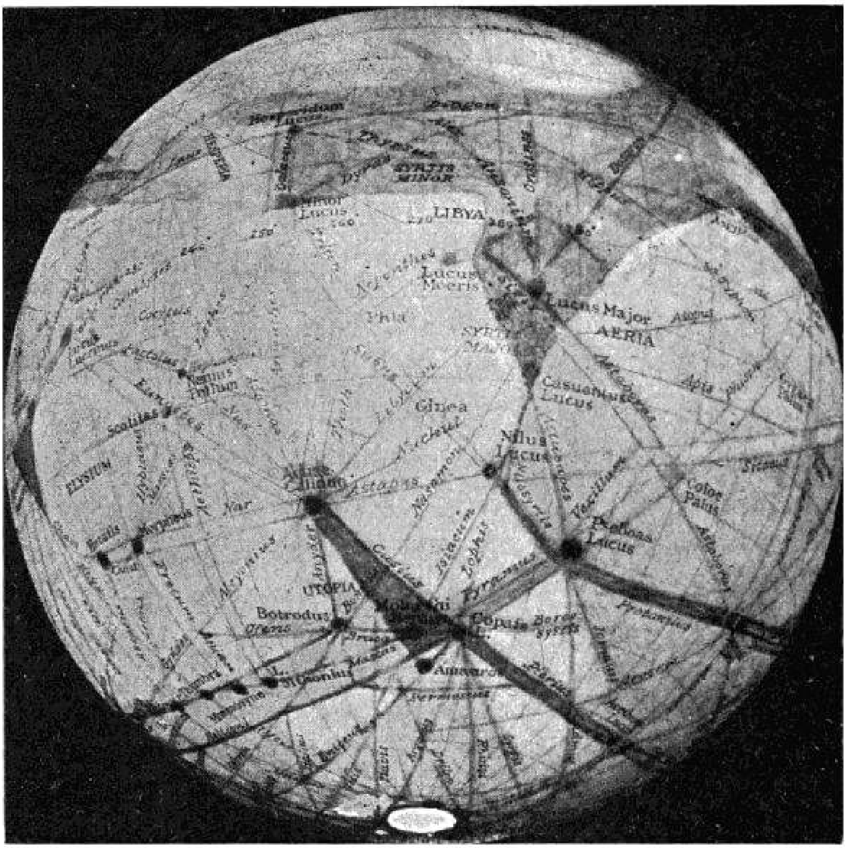

FIG. 106.—Lowell's map of Mars.

the earth. When a number of them intersect, there is generally a dark spot at the point of intersection which is called a "lake." Sometimes a number of them intersect at a single point; and, according to Lowell, the junctions of canals are always surrounded by lakes, while lakes are found at no other places.

In the winter of 1881–82 Mars was again in opposition, though not so near the earth as in 1877. Schiaparelli not only verified his earlier observations, but he also found the remarkable fact that a number of the canals had doubled; that is, that, in a number of cases, two canals ran parallel to each other at a distance of from 200 to 400 miles, as shown on Lowell's map in Fig. 106, which is a photograph of a globe on which he had drawn all the markings he had observed. The doubling was found to depend upon the seasons and to develop with great rapidity when the sun was at the Martian equinox.

The history of the observations of the markings of Mars since the time of Schiaparelli is filled with the most remarkable contradictions. The observations of the keen-eyed Italian have been confirmed by a number of other astronomers, among whom may be mentioned Perrotin and Thollon, of Nice, Williams, of England, W. H. Pickering, of Harvard, and especially Lowell, who has a large 24-inch telescope favorably located at Flagstaff, Arizona. On the other hand, many of the foremost observers working with the very largest telescopes, such as Antoniadi, with the 32.75-inch Meudon refractor, the Lick observers, with the great 36-inch telescope, Barnard, with the 40-inch Yerkes telescope, and Hale, with the 60-inch reflector of the Solar Observatory at Mt. Wilson, California, have been entirely unable to see the canals. This does not mean that they have not seen markings on Mars, for they have observed many of them; but they do not find the narrow, straight lines observed by Schiaparelli, Lowell, and others. In Fig. 107 four views of Mars are shown as seen by Barnard with the great telescope of the Lick Observatory, and Fig. 108 is a photograph made with the 60-inch reflecting telescope of the Mt. Wilson Solar Observatory. In the midst of these conflicting results it is difficult

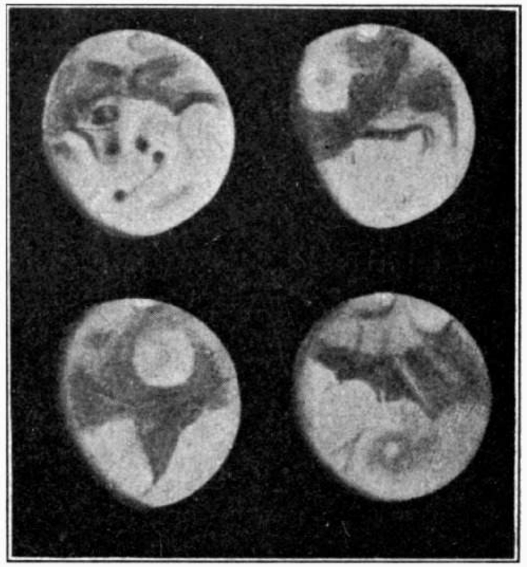

FIG. 107.—Drawings of Mars in 1894 by Barnard at the Lick Observatory.

to draw any certain conclusion; but it must be remembered in considering such a subject that reliable positive evidence ought to outweigh a large amount of negative evidence.

174. Explanations of the Canals of Mars.—The explanations of the canals of Mars have been extremely varied. Many astronomers believe they are illusions of some sort. They think the eye in some way integrates the numerous faint markings which certainly exist on Mars into straight lines and geometrical figures. The experiments of Maunder

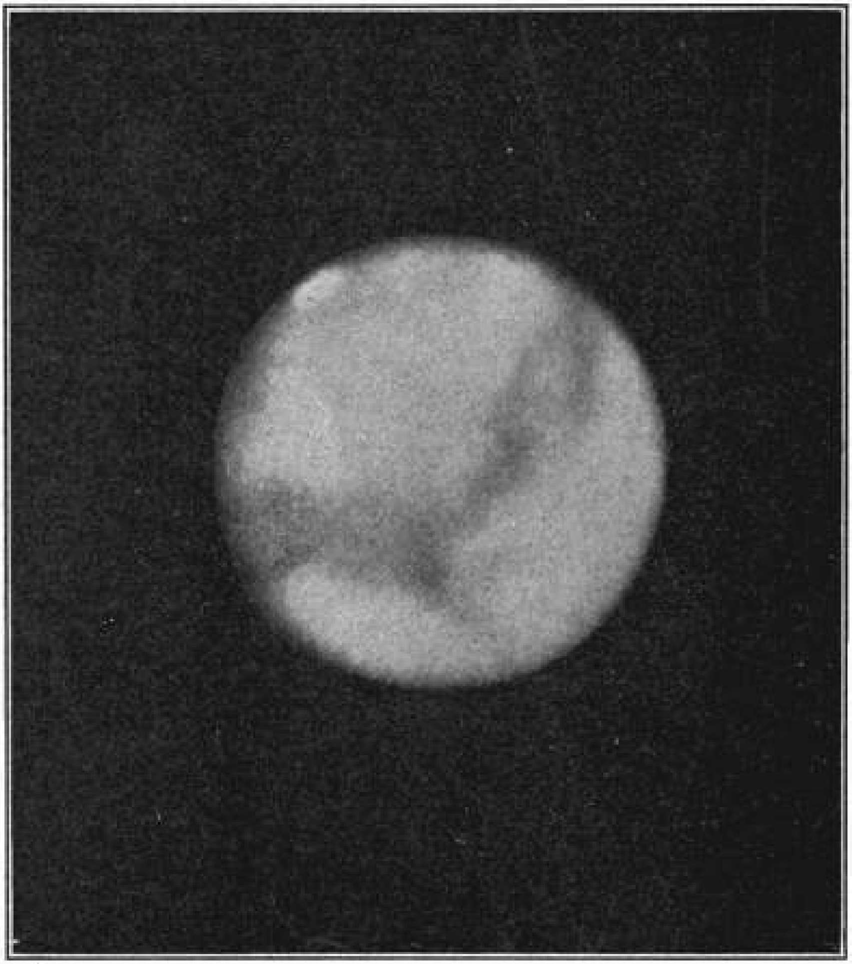

FIG. 108.—Photograph of Mars (the 60-inch reflector of the Mt. Wilson Solar Observatory).

and Evans and the more recent ones of Newcomb of having a number of persons make drawings of what they could see on a disk covered with irregular marks and held slightly beyond the limits of distinct vision, strikingly confirm this conclusion. Antoniadi states in the most unequivocal terms that the observations of Mars at the opposition of 1909 give to the theory of the objective existence of canals on Mars an unanswerable confutation. Other astronomers hold that such a network of markings on a planet whose surface is certainly somewhat uneven is inherently improbable, and should not be accepted without the most conclusive evidence.

At the other extreme stands Lowell, who maintains that not only are the canals real but that they prove the existence on the planet of highly intelligent beings. He argues for the reality of the canals on the ground that they always appear at well-defined positions on the planet and that they change in a systematic way with the seasons. He argues that they are artificial because they always run along the arcs of great circles, because several of them sometimes cross at a point with the utmost precision, and because in many cases two of them run perfectly parallel for more than a thousand miles. Obviously this remarkable regularity could not be the result of such processes as the erosion of rivers or the cracking of the surface.

W. H. Pickering first suggested that the canals may be due to vegetation, and Lowell's theory is an elaboration of this idea. Lowell believes the streaks, known as canals, are strips of vegetation 20 or more miles wide, which grow on a region irrigated by lateral ditches from a large central canal. This explains their seasonal character. Moreover, he finds the streaks first developing near the dark (marshy?) regions and extending gradually out from them even across the equator of the planet to regions having the opposite season. The explanation given for this phenomenon is that when the snow of the polar caps melts, the resulting water first collects in the marshes and is led thence out into the waterways which extend through the centers of the canals. The observations of Lowell show that, according to his explanation, water must flow along the canals at the rate of 2.1 miles per hour. He infers from the elaborate system of irrigated regions that Mars is inhabited by creatures possessing a high order of intelligence.

Although Lowell's theory seems highly improbable and may be altogether wrong, life may nevertheless exist upon Mars. But if there is life on this planet, the creatures which inhabit it must be very different physically from those on the earth, because it would be necessary for them to be adapted to an entirely different environment. On Mars the surface gravity is less than on the earth, the light and heat received from the sun are less and the temperature is probably far lower, the atmosphere is much less abundant, and it may be quite different in constitution, and the seasonal changes are nearly twice as long. The plants and animals which inhabit the earth are more or less perfectly adapted to the conditions existing on its surface, and the conditions have not been made to fit them, as was once generally believed. Similarly, life on other planets must be adapted to the environment in which it is placed or it would shortly perish.

Further, if Mars or any other world is inhabited, there is no reason to suppose that its highest intelligence has reached the precise stage attained by the human race. The most intelligent creatures on another planet may be in the condition corresponding to that in which our ancestors were when they lived in caves and ate uncooked food; or, millions of years ago they may have passed through the stage of strife and deadly competition in which the human race is to-day.

It is a curious fact that those who know but little about astronomy are nearly always very much interested in the question whether other worlds are inhabited, while as a rule astronomers who devote their whole lives to the subject scarcely give the question of the habitability of other planets a thought. Astronomers are doubtless influenced by the knowledge that such speculations can scarcely lead to certainty, and they are deeply impressed by the fundamental laws which they find operating in the universe. Nevertheless, there seems to be no good reason why we should not now and then consider the question of the existence of life, not only on the other planets of the solar system, but also on the millions of planets that possibly circulate around other suns. Such speculations help to enlarge our mental horizon and to give us a better perspective in contemplating the origin and destiny of the human race, but we should never forget that they are speculations.

III. Jupiter

175. Jupiter's Satellite System.—The first objects discovered by Galileo when he pointed his little telescope to the sky in 1610 were the four brightest moons of Jupiter. They are barely beyond the limits of visibility without optical aid and, indeed, could be seen with the unaided eye if they were not obscured by the dazzling rays of Jupiter. No other satellite of Jupiter was discovered until 1892, when Barnard, then at the Lick Observatory, caught a glimpse of a fifth one very close to the planet. It is so small and so buried in the rays of the neighboring brilliant planet that it can be seen only by experienced observers with the aid of the most powerful telescopes in the world.

Early in 1905 Perrine found by photography that Jupiter has still two more satellites which are more remote from the planet than those previously known. Their distances from Jupiter are both about 7,000,000 miles and their periods of revolution are about 0.75 of a year. The eccentricities of their orbits are considerable and their paths actually loop through one another. The mutual inclination of their orbits is 28° and they do not pass nearer than 2,000,000 miles of each other.

The seven satellites so far enumerated revolve around Jupiter from west to east, but two more have been discovered whose motion is in the opposite direction. The eighth was found by Melotte, at Greenwich, England, in January, 1908. It revolves around Jupiter at a mean distance of approximately 14,000,000 miles in a period of about 740 days. Its orbit is inclined to Jupiter's equator by about 28°. The ninth was discovered by S. B. Nicholson, in July, 1914, at the Lick Observatory. Its mean distance from Jupiter is about 15,400,000 miles and its period is nearly 3 years. These remote satellites are very small and faint, the ninth being of the nineteenth magnitude, and the eighth about one magnitude brighter.

TABLE VII

SATELLITE	DISTANCE FROM CENTER OF JUPITER	PERIOD OF REVOLUTION	DIAMETER
V (Unnamed)	112,500 mi.	0d. 11h. 57m.	about 100 mi.
I (Io)	261,000 mi.	1d. 18h. 28m.	2452 mi.
II (Europa)	415,000 mi.	3d. 13h. 14m.	2045 mi.
III (Ganymede)	664,000 mi.	7d. 3h. 43m.	3558 mi.
IV (Callisto)	1,167,000 mi.	16d. 16h. 32m.	3345 mi.
VI (Unnamed)	7,300,000 mi.	about 266 days	small
VII (Unnamed)	7,500,000 mi.	about 277 days	small
VIII (Unnamed)	14,000,000 ± mi.	about 740 days	very small
IX (Unnamed)	15,400,000 ± mi.	nearly 3 years	very small

The first four satellites discovered are numbered I, II, III, IV in the order of their distance from Jupiter. The fifth, although it is very close to Jupiter, was given the number V. The orbits of these five satellites, shown in Fig. 109, are nearly circular and lie in the plane of Jupiter's equator. The four larger satellites are of considerable dimensions and their diameters have been determined by Barnard, the results being given in the following table.

176. Markings on Jupiter's Satellites.—The great distance of Jupiter makes it difficult to detect any but large and distinctly colored markings on its satellites. In 1890 Barnard found satellite I to be elongated parallel to the equator of Jupiter when transiting its darker portions and elongated, or double, in the opposite direction when passing over its brighter parts. He interpreted this as meaning that the poles of the satellite are dark and that the equatorial belt is light colored. The accompanying drawing, Fig. 110, showing the satellite transiting a light region above and a dark one below, exhibits the observed appearance at the left and the probable actual condition at the right. When held at some distance from the eye, the two appear the same.

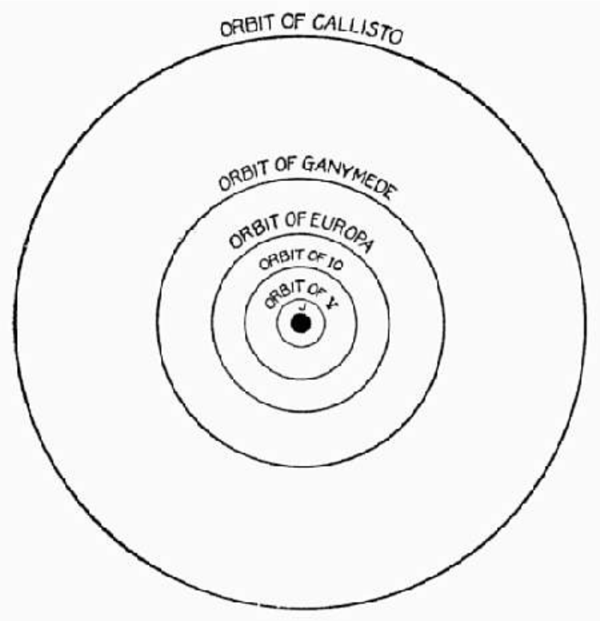

FIG. 109.—Orbits of first four satellites of Jupiter.

Some observers have thought that satellites III and IV are somewhat elliptical in shape, but Barnard has observed them repeatedly with the great Lick and Yerkes telescopes and has been quite unable to detect in them any departures from strict sphericity. Various markings have been at times observed on the satellites, and Douglas inferred from his observations of satellite III that its period of rotation is about 7 hours. At present these are matters of speculation.

177. Discovery of the Finite Velocity of Light.—A very important discovery was made in connection with observations of Jupiter's satellites. The periods of revolution of the four largest satellites naturally were determined when Jupiter was in opposition, and therefore nearest the earth. Since the satellites are in the plane of Jupiter's equator, which is only slightly inclined to the ecliptic, they are eclipsed when they pass behind Jupiter. From their periods of revolution the times at which they will be eclipsed can be predicted.

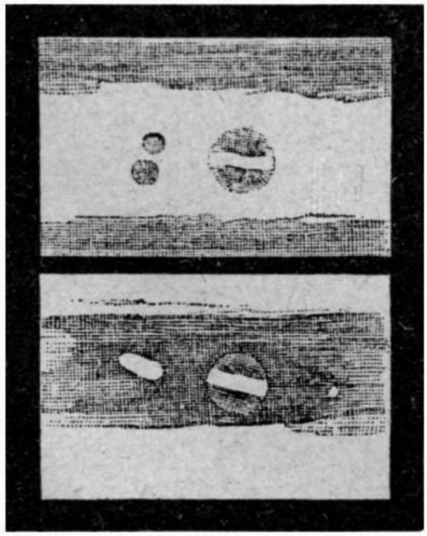

FIG. 110.—Barnard's drawings of Jupiter's satellite I.

Suppose the periods of revolution of the satellites and the times at which they are eclipsed are determined when the earth is in the vicinity of E_1, Fig. 111. Six months later, when the earth has arrived at E_2, its distance from Jupiter is greater by approximately the diameter of the earth's orbit, and then the eclipses of the satellites are found to be behind their predicted times by the time required for light to travel across the earth's orbit. From such observations, in 1675, the Danish astronomer Römer inferred that it takes light 600 seconds to travel a distance equal to that from the sun to the earth. Later observations have shown that the correct time is 498.58 seconds. When the distance from the earth to the sun has been determined by

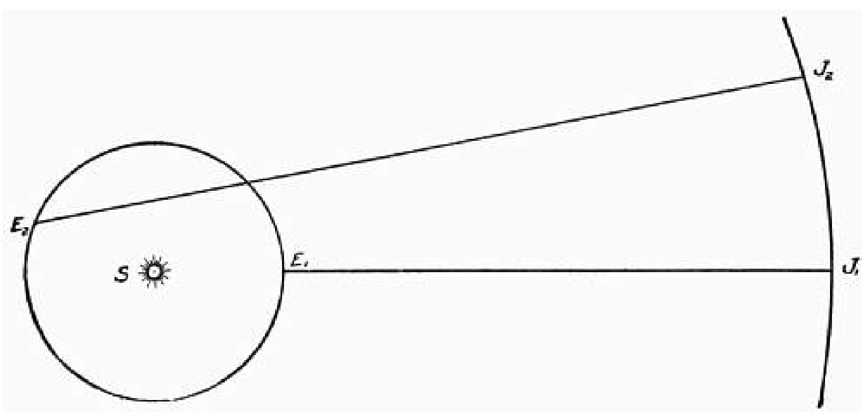

FIG. 111.—Discovery of velocity of light from eclipses of Jupiter's satellites.

independent means, the velocity of light can be found from this interval, which is called the light equation.

At the present time the velocity of light can be determined much more accurately by physical experiments on the surface of the earth than it can from observations of Jupiter's satellites. The work of Fizeau, Michelson, and Newcomb shows that it is very approximately $186,324$ miles per second. From this velocity and the light equation of 498.58 seconds, the distance to the sun can be computed.

178. The Rotation of Jupiter.—The surface of Jupiter is covered with a great number of semi-permanent markings from which its rotation can be determined. The period of rotation for spots near the equator has been found to be about 9 hrs. and 50 m., and for those in higher latitudes about 9 hrs. and 57 m., with an average of 9 hrs. and 54 m.; that is, between the equatorial zone and high latitudes there is a difference in the period of about $\frac{1}{85}$ of the whole period. In 85 rotations the equator gains a rotation on the higher latitudes. Moreover, as Barnard has found, the rates of rotation in corresponding northern and southern latitudes are quite different in several zones.

The circumference of Jupiter is nearly $300,000$ miles, and it follows from this and its rate of rotation that the motion at its equator is about $30,000$ miles per hour. Consequently, if two spots whose periods of rotation differ by 7 minutes were both near the equator, they would pass each other with the relative speed of $30,000 \div 85 = 353$ miles per hour. Though spots whose periods differ by 7 minutes are probably in no case in approximately the same latitude, yet they must have large relative motions. Compare these results with the speed of from 70 to 100 miles per hour with which tornadoes sweep along the surface of the earth.

The fact that the equatorial belt of Jupiter rotates in a shorter period than its higher latitudes is a most remarkable phenomenon. If it were an isolated case, one would naturally suppose that the peculiarity was due to irregularities of motion inherited from the time of its origin. Such currents in a body in a fluid condition would be destroyed by friction only very slowly; but the same phenomenon is also found in the case of Saturn and the sun. It can hardly be supposed that the three are mere coincidences. If they are not, the implication is that these peculiarities of rotation have been produced by similar causes. It has been suggested, as will be explained in Arts. 253, 254, that the cause may be the impacts of circulating meteors or other material.

179. Surface Markings of Jupiter.—The characteristic markings of Jupiter are a series of conspicuous dark and bright belts which stretch around the planet parallel to its equator as shown in Figs. 112, 113, and 114. The central equatorial belt is usually very light and about 10,000 miles wide; on each side is a belt of reddish-brown color generally of about the same width. Several other alternately light and dark belts can be made out in higher latitudes, though not as distinctly as the equatorial belts, partly, at least, because they are observed obliquely. The belts vary considerably in width from year to year as the drawings, Fig. 114, by Hough show. On the whole, the southern dark belt of Jupiter is wider and more conspicuous than the northern one.

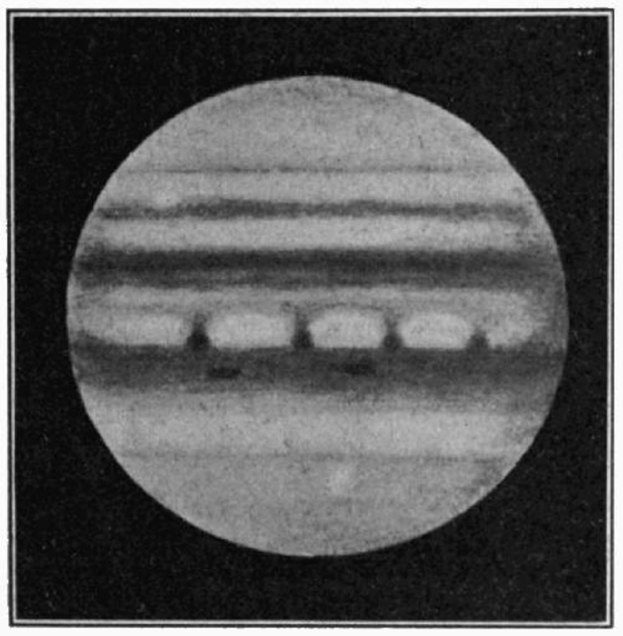

FIG. 112.—Jupiter, Sept. 7, 1913 (Barnard).

A good telescope under favorable atmospheric conditions reveals in the belts many details which continually change as though what we see is cloud-like in structure. In fact, it follows from the low mean density of the planet and the almost certain central condensation that its exterior parts, to a depth of many thousands of miles, must have a very low density; and it is improbable that anything which is visible from the earth approaches the solid state.

Dark spots often appear on Jupiter, especially in the northern hemisphere, which gradually turn red and finally vanish. The most remarkable and permanent spot so far known appeared in 1878 just beneath the southern red belt. When first discovered it was a pinkish oval about 7000 miles across in the direction perpendicular to the equator, and about 30,000 miles long parallel to the equator. In a year it had changed to a bright red color and was by far the most conspicuous object on the planet. It has since then been known as "the great red spot," but it has undergone many changes, both in color and brightness. At

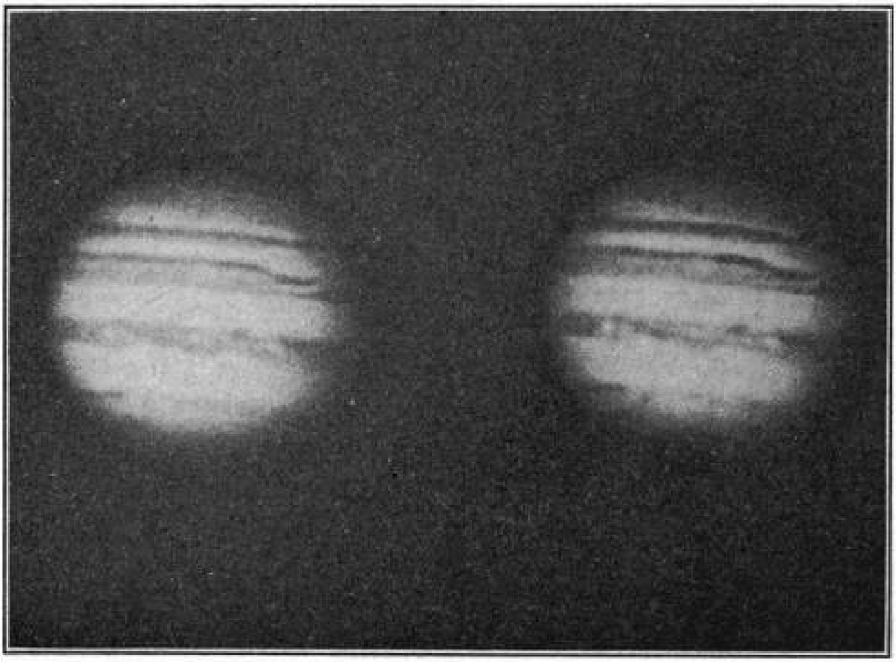

FIG. 113.—Photographs of Jupiter (E. C. Slipher, Lowell Observatory).

the present time it has become rather inconspicuous, and the material of which it is composed seems to be sinking back beneath the vapors which surround the planet.

A very remarkable thing in connection with the red spot was that its period of rotation increased 7 seconds the first eight years following its discovery, but it has remained essentially constant since that time. Possibly the increase in period of rotation of the red spot, which was somewhat longer than that of the surrounding material which continually flowed by it, was due to its being elevated so that its distance from the axis of rotation of the planet was increased. Under these conditions the rate of rotation would be reduced in harmony with the principle of the conservation of moment of momentum (Art. 45). At any rate, changes in rotation are always accompanied by considerable changes in color and visibility of the parts affected.

180. The Physical Condition and Seasonal Changes of Jupiter.—In considering the physical condition of Jupiter it should be remembered that it has the low average density of 1.25 on the water standard, that its surface markings are not permanent, and that there are violent relative motions of its visible parts. All these things indicate that Jupiter is largely gaseous near its surface.

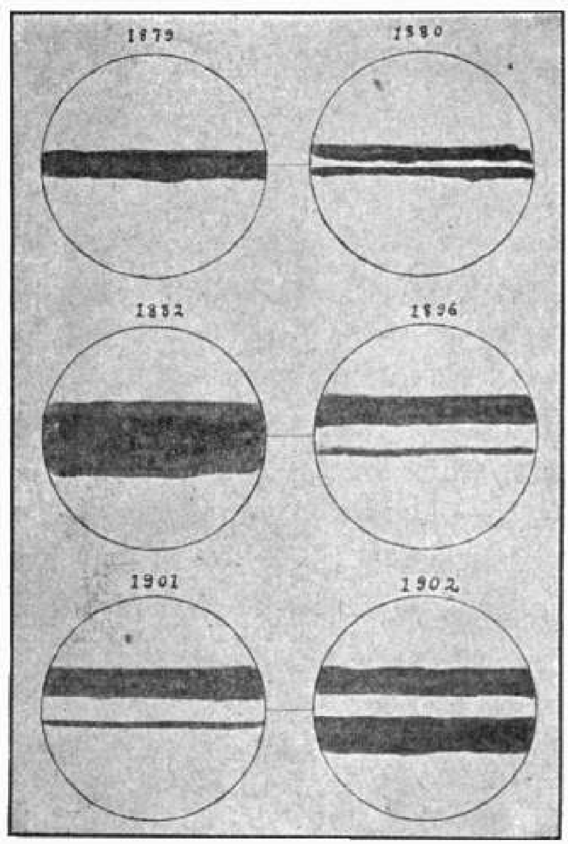

FIG. 114.—Drawings of Jupiter showing variations in widths of dark belts (Hough).

The surface gravity of Jupiter is 2.52 times that of the earth, and this produces great pressures in its atmosphere at moderate depths. These pressures are sustained by the expansive tendencies of the interior gases which may be composed of light elements, or which may have high temperatures. It has sometimes been supposed that the surface of Jupiter is very hot and that it is self-luminous, but such cannot be the case, for the shadows cast on the planet by the satellites are perfectly black, and when a satellite passes into the shadow of Jupiter it becomes absolutely invisible.

In conclusion, we shall probably not be far from the truth if we infer that Jupiter is still in an early stage of its evolution, rather than far advanced like the terrestrial planets, that it contains enormous volumes of gases which are in rapid circulation both along and perpendicular to its surface, and that possibly the energy of its internal fires gives rise to violent motions.

The eccentricity of Jupiter's orbit is very small and the plane of its equator is inclined only $3° 5'$ to the plane of its orbit. The factors which produce seasonal changes are, therefore, unimportant in the case of this planet. Its distance from the sun is so great that it receives per unit area only $\frac{1}{27}$ as much light and heat as is received by the earth; and, consequently, its surface must be cold unless it is warmed by internal heat.

IV. Saturn

181. Saturn's Satellite System.—Saturn, like Jupiter, has 9 satellites. The largest one was discovered by Huyghens in 1655, then four more were found by J. D. Cassini between 1671 and 1684, two by William Herschel in 1789, one by G. P. Bond and Lassell in 1848; and the ninth by W. H. Pickering in 1899. Pickering suspected the existence of a tenth in 1905, but the supposed discovery has not been confirmed.

Saturn is so remote that the dimensions of its satellites are only roughly known from their apparent brightness. All their masses are unknown except that of Titan, which, from its perturbation of its neighboring satellite Hyperion, was found by Hill to be $\frac{1}{4714}$ that of Saturn. The 7 satellites which are nearest to Saturn revolve sensibly in the plane of its equator, while the orbit of the eighth, Japetus, is inclined about 10°, and that of the ninth about 20°.

FIG. 115.—Orbit of Saturn's satellites.

When the eighth satellite, Japetus, is on the western side of Saturn it always appears considerably brighter than when it is on the eastern side. This difference in brightness is undoubtedly due to the fact that this satellite, like the moon, always has the same side toward the planet around which it revolves, and that its two sides reflect light very unequally. Similar, but less marked, phenomena have been observed by Lowell and E. C. Slipher in connection with the first two satellites, and the explanation is the same as in the case of Japetus.

Table VIII gives the list of Saturn's satellites, together with their mean distances from its center, their periods, and their approximate diameters. It will be observed that an enormous gap separates the first eight from the ninth.

Figure 115 gives to scale the orbits of Saturn's satellites, with the exception of the ninth, which is too remote to be shown. The eight satellites revolve around Saturn from west to east, the direction in which it rotates, but the ninth, like the eighth and ninth satellites of Jupiter, revolves in the retrograde direction. This satellite was the first object discovered in the solar system having retrograde motion, and it aroused great interest. These retrograde revolutions have a fundamental bearing on the question of the origin of the satellite systems.

TABLE VIII

SATELLITE	DISTANCE FROM CENTER OF SATURN	PERIOD OF REVOLUTION	DIAMETER
I (Mimas)	117,000 mi.	0d. 22h. 37m.	about 600 mi.
II (Enceladus)	157,000 mi.	1 8 53	about 800 mi.
III (Tethys)	186,000 mi.	1 21 18	about 1200 mi.
IV (Dione)	238,000 mi.	2 17 41	about 1100 mi.
V (Rhea)	332,000 mi.	4 12 25	about 1500 mi.
VI (Titan)	771,000 mi.	15 22 41	about 3000 mi.
VII (Hyperion)	934,000 mi.	21 6 39	about 500 mi.
VIII (Japetus)	2,225,000 mi.	79 7 54	about 2000 mi.
IX (Phœbe)	7,996,000 mi.	546 12 0	about 200 mi.

The question may be asked why the remote satellites of both Jupiter and Saturn revolve in the retrograde direction. This question cannot be answered with certainty at the present time. But it is certain that the farther a satellite is from a planet, the less securely is it held under the gravitative control of its primary; and there is a distance beyond which a satellite cannot permanently revolve because it would abandon the planet in obedience to the greater attraction of the sun. A mathematical discussion of the problem shows that, at a given distance from a planet, motion in the retrograde direction is much more stable than in the forward direction; and consequently, out near the region where instability begins, it would be expected that only retrograde satellites would be found. The orbit of the ninth satellite of Saturn is in the region of stability even for direct motion; but Jupiter's eighth and ninth satellites would both have unstable orbits if they revolved in the forward direction at the same distances from Jupiter.

182. Saturn's Ring System.—Saturn is distinguished from all the other planets by three wide, thin rings which extend around it in the plane of its equator. They were first seen by Galileo in 1610, but their true character was not known until the observations of Huyghens in 1655. The dimensions of Saturn and its ring system according to the extensive measurements of Barnard are given in Table IX.

TABLE IX

Equatorial radius of Saturn	38,235 miles
Center of Saturn to inner edge of crape ring	44,100 miles
Center of Saturn to inner edge of bright ring	55,000 miles
Center of Saturn to outer edge of bright ring	73,000 miles
Center of Saturn to inner edge of outer ring	75,240 miles
Center of Saturn to outer edge of outer ring	86,300 miles

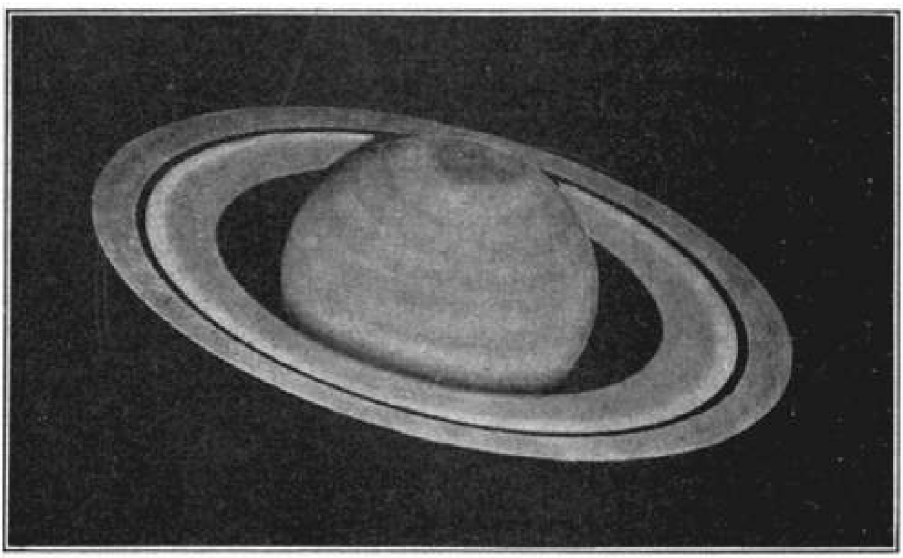

FIG. 116.—Saturn with rings tilted at greatest angle (drawing by Barnard).

The distance from the surface of Saturn to the inner edge of the thin, faint ring, known as "the crape ring," is nearly 6000 miles. The width of the crape ring is about 11,000 miles. Outside of the crape ring is the main bright ring, whose width is about 18,000 miles. Its brightness increases from its junction with the crape ring outward nearly to its outer margin. At its brightest place it is as luminous as the planet itself. Beyond the main bright ring there is a dark gap about 2200 miles across. It is known as "Cassini's Division" because it was first observed by Cassini. Outside of this dark space is the outer bright ring with a width of about 11,000 miles. The distance across the whole ring system from one side to the other is about 172,600 miles.

The rings of Saturn are inclined about 27° to the plane of the planet's orbit and about 28° to the plane of the ecliptic. Consequently, they are observed from the earth at a great variety of angles. When their inclination is high, Saturn and its ring system present through a good telescope one of the finest sights in the heavens, as is evident from Figs. 116 and 117. When their plane passes through the earth,

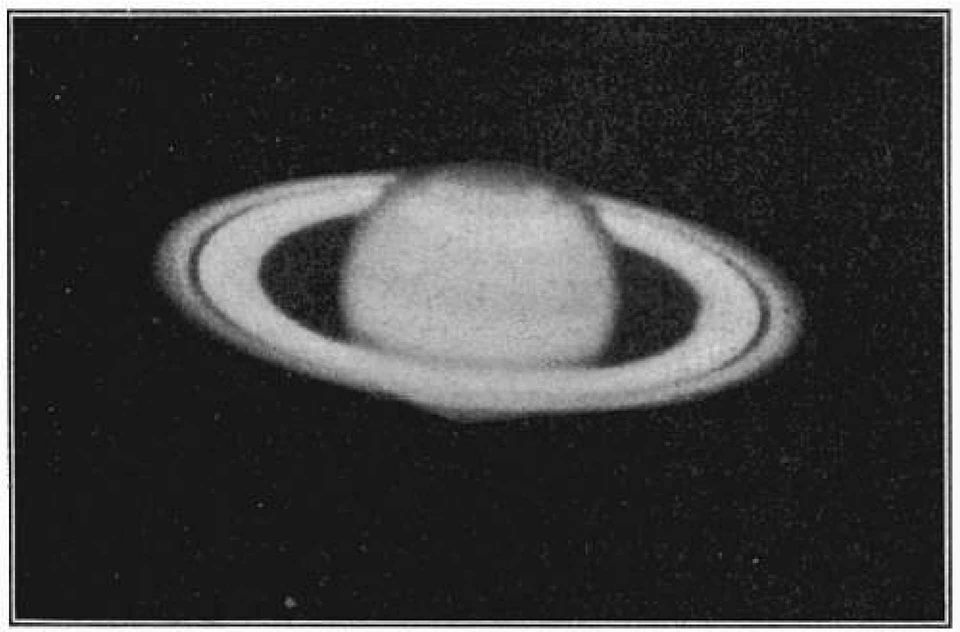

FIG. 117.—Saturn. *Photographed Nov. 19, 1911, with the* 60*-inch telescope of the Mount Wilson Solar Observatory.*

they appear to be a very thin line and even entirely disappear from view for a few hours, as Barnard found when observing them with the great 40-inch telescope in 1907. It follows that the rings must be very thin, their thickness probably not exceeding 50 miles. When the rings were nearly edgewise to the earth, Barnard could see them faintly; but the places which are entirely vacant when they are highly inclined to the earth, were found to be brighter than the places where the rings are really brilliant (Fig. 118). Barnard made the suggestion that this appearance is due to the fact that light shining from the sun through the open regions is reflected back from the interior edges of the denser parts of the rings.

183. The Constitution of Saturn's Rings.—The bright rings of Saturn have the same appearance of solidity and continuity as the planet itself. It was generally believed until about a century ago that they were solid or fluid. Yet since 1715, when J. Cassini first mentioned the possibility, it has frequently been suggested that the rings may be simply swarms of meteors, or exceedingly minute satellites, revolving

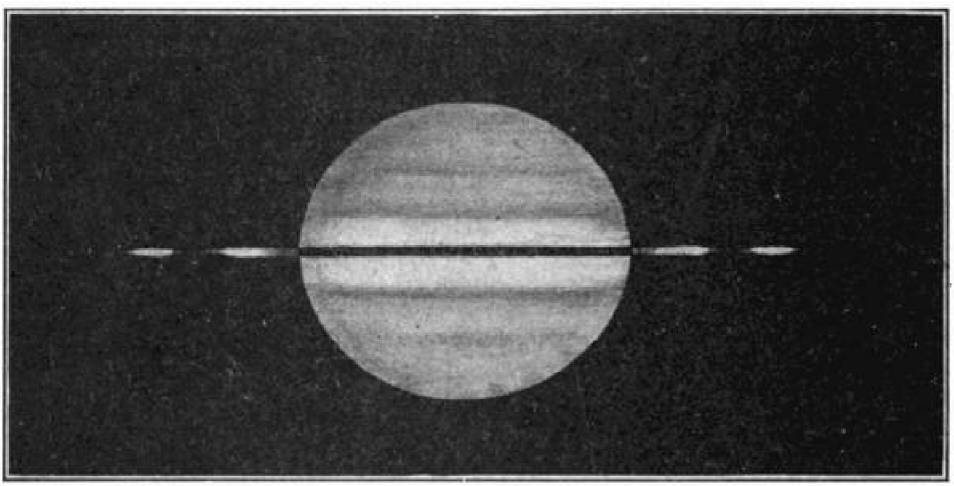

FIG. 118.—Rings of Saturn, December 12, 1907 (drawing by Barnard).

around the planet in the plane of its equator. Such small bodies would exert only negligible gravitational influences upon one another, and their orbits would be sensibly independent of one another except for collisions.

The meteoric theory of the constitution of Saturn's rings was first rendered probable by Laplace, who showed that a symmetrical, solid ring would be dynamically unstable. That is, solid rings would be something like spans of enormous bridges, whose ends do not rest upon the planet but upon other portions of the rings. They would have to be composed of inconceivably strong material to withstand the strains due to their motion and the gravitational forces to which they would be subjected. In 1857, Clerk-Maxwell proved from dynamical considerations that the rings could be neither solid nor fluid, and that they were, therefore, composed of small independent particles. Now, if they are meteoric, those parts which are nearest the planet must move fastest, just as those planets which are nearest the sun move fastest; while, if they are solid, the opposite must be the case. In 1895, Keeler showed by line-of-sight observations with the spectroscope (Art. 226) that the inner parts not only move fastest, but that all parts move precisely as they would move if they were made up of totally disconnected particles, the innermost particles of the crape ring performing their revolution in about 5 hours, while the outermost particles of the outer bright ring require 137 hours to complete a revolution. Moreover, Barnard found that they do not cast perfectly black shadows, for he saw Japetus faintly illuminated by the rays of the sun which filtered through the ring. Hence it may be considered as firmly established that the rings of Saturn are swarms of meteors.

Rings are strange substitutes for satellites, but a probable explanation of their existence in place of satellites is at hand. A planet exerts tidal strains upon satellites in its vicinity, and these tendencies to rupture increase very rapidly as the distance of the satellite decreases. In 1848, Roche proved that these tidal forces would break up a fluid satellite of the same density as the planet around which it revolved if its distance were less than 2.44... radii of the planet. The limit would be less for denser satellites, and a little less for solid satellites, but not much less if they were of large dimensions. It is seen from the numbers in Table IX, or from Fig. 116, that the rings are within this limit. It is not supposed that they are the pulverized remains of satellites that ever did actually exist, but rather that the material of which they are composed is subject to such forces that the mutual gravitation of the separate particles can never draw them together into a single body. If they should unite into a satellite, it would probably be small, for they are not massive enough to have produced by their attraction any disturbance of the motions of the satellites which can so far be observed.

One more interesting thing remains to be mentioned. If a meteor were to revolve in the vacant space between the rings known as Cassini's division, its period would be nearly commensurable with the periods of four of the satellites, and would be one half that of Mimas. Kirkwood called attention to this relation, which is entirely analogous to that found in the case of the planetoids (Art. 160). Encke and other astronomers have suspected that there is a narrow division between the crape ring and the inner edge of the bright ring, where the period of a revolving meteor would be one third that of Mimas. More recently Lowell has been convinced by his observations at Flagstaff of the existence of several other very narrow divisions at places where the periods of revolving particles would be simply commensurable with the periods of Mimas or Enceladus. But in order to secure perfect commensurability he was led to the conclusion that Saturn is composed of layers of different densities, and that the inner ones are more oblate, and, therefore, rotate faster, than the outer ones.

184. On the Permanency of Saturn's Rings.—The question at once arises whether the meteoric constitution of the rings, in which there is abundant opportunity for collisions, is a permanent one. The fact that the rings exist and are separated from the planet by a number of thousands of miles, while beyond them there are 9 satellites, indicates that they are not transitory in character. The only circumstance that distinguishes them dynamically from the satellites is the possibility of their collisions. If a collision occurred, at least some heat would be generated at the expense of their energy of motion. When the revolutionary energy of a body is decreased, its orbit diminishes in size. Therefore, when two of the small bodies of which Saturn's ring is composed collide, the orbit of at least one of them must be diminished in size. These collisions with the accompanying degradation of energy are probably taking place at a very slow rate. If so, the rings of Saturn are slowly shrinking down on the planet. It may be that the crape ring is the result of particles whose orbits have been reduced from the larger dimensions of the bright ring by collisions with other particles.

185. The Surface Markings and the Rotation of Saturn.—The surface markings of Saturn are much like those of Jupiter, though, of course, they are not seen so well because of the great distance of this planet. There are a bright equatorial belt and a number of darker and broader belts in the higher latitudes, though they are less conspicuous than the belts on Jupiter.

It has been rather difficult for observers to find spots on Saturn conspicuous and lasting enough to enable them to determine the period of its rotation. From observations made in 1794 Herschel concluded that its period of rotation is 10 hrs. and 16 m.; Hall's observation of a bright equatorial spot in 1876 gave for this spot a period of 10 hrs. and 14 m. This was generally adopted as the period of Saturn's rotation, particularly after it had been verified by a number of other observers. But, in 1903, Barnard discovered some bright spots in northern latitudes, and his observations of them, together with those of several other astronomers, showed that these spots were passing around Saturn in 10 hrs. and 38 m. This difference in period means that there is a relative drift between the material of Saturn's equatorial belt and that of its higher latitudes of 800 or 900 miles per hour.

In sharp contrast to the planet Jupiter, the plane of the equator of Saturn is inclined to the plane of its orbit by an angle of 27°. This is a still higher inclination than those found in the case of the earth and Mars, and would hardly be expected in so large a planet as Saturn after finding that the axis of Jupiter is almost exactly perpendicular to the plane of its orbit.

186. The Physical Condition and Seasonal Changes of Saturn.—The density of Saturn is about 0.63 on the water standard. Consequently, it must be largely in a gaseous condition. Probably no considerable portion of it is purely gaseous, for it seems more likely, in view of the fact that it is opaque, that the gases of which it is composed are filled with minute liquid particles, just as our own atmosphere becomes charged with globules of water, forming clouds.

The remarkable relative motions of the different parts of the surface of Saturn show that it is at least in a fluid state and that it is a place of the wildest turmoil. Doubtless it is a world whose evolution has not yet sufficiently advanced to give it any permanent markings, much less to fit it as a place in any way suitable for the abode of even the lowest forms of life.

The high inclination of the plane of Saturn's equator to that of its orbit gives it marked seasonal changes. Moreover, its orbit is rather more eccentric than the orbits of the other large planets. But it is so far from the sun that it receives only $\frac{1}{90}$ as much light and heat per unit area as the earth receives; and it follows that its surface is very cold unless it has an atmosphere of remarkable properties, or unless a large amount of heat is conveyed to it from a hot interior.

A consequence of the rapid rate of rotation and low density of Saturn is that it is very oblate. The difference between its equatorial and polar diameters is nearly 6700 miles, or about 10 per cent of its whole diameter. Its oblateness is so great that it is conspicuous even through a telescope of 6 inches' aperture.

V. Uranus and Neptune

187. The Satellite Systems of Uranus and Neptune.—Uranus has four known satellites, two of which were discovered by William Herschel, in 1787, and the other two by Lassell, in 1851. Their distances are respectively 120,000, 167,000, 273,000 and 365,000 miles, and their periods of revolution are respectively 2.5, 4.1, 8.7, and 13.5 days. Their diameters probably range between 500 and 1000 miles. They all move sensibly in the same plane, but this plane is inclined about 98° to the plane of the planet's orbit; that is, if the plane of the orbits of the satellites is thought of as having been turned up from that of the planet's orbit, the rotation has been continued 8° beyond perpendicularity, and the satellites revolve in the retrograde direction.

Neptune has one known satellite which was discovered by Lassell, in 1846. It revolves at a distance of 221,500 miles in a period of 5 days 21 hours. Its diameter is probably about 2000 miles. The plane of its orbit is inclined about 145° to that of the planet's orbit; that is, the inclination between the two planes is about 35° and the satellite revolves in the retrograde direction.

188. Atmospheres and Albedoes of Uranus and Neptune.—Very little is known directly respecting the atmospheres of Uranus and Neptune. Their low mean densities imply that their exterior parts are largely in the gaseous state. As confirmatory of this conclusion, the spectroscope shows that the light which we receive from them must have passed through an extensive absorbing medium in addition to the sun's atmosphere and that of the earth, through which the light from all planets passes. The absorbing effects of the element hydrogen and water vapor are shown in the spectra of both planets, but, according to the recent results of Slipher, more strongly in the case of Neptune than in that of Uranus. A number of the other absorption bands are due to unknown substances.

The albedo of Uranus is 0.63, and that of Neptune, 0.73.

189. The Periods of Rotation of Uranus and Neptune.—Surface markings have been seen on Uranus by Buffham, Young, the Andre brothers, Perrotin, Holden, Keeler, and other observers, but they have been so indefinite and fleeting that it has not been possible to draw any certain conclusions from them. Nevertheless, so far as they go, they indicate that the period of rotation of Uranus is 10 or 12 hours, and that the plane of its equator is inclined something like 10° to 30° to the plane of the orbits of the satellites. In 1894, Barnard detected a slight flattening of the disk, with the equatorial diameter inclined 28° to the plane of the orbits of the satellites. Finally, in 1912, V. M. Slipher, at the Lowell Observatory, found by spectroscopic means that Uranus rotates in the direction of revolution of its satellites in a period of 10 hrs. 50 m. This result is entitled to considerable confidence.

No certain markings have been seen on Neptune, and, consequently, its rate of rotation has not been found by direct means. But by indirect processes both the position of the plane of its equator and its rate of rotation have been found, at least approximately. The dimensions and mass of Neptune are known with considerable accuracy. Now, if the rate of rotation were known, the equatorial bulging could be computed. Suppose the plane of the orbit of the satellite were inclined to that of the planet's equator. Then the equatorial bulge would perturb the motion of the satellite; in particular, it would cause a revolution of its nodes, and the rate could be computed.

The problem of determining the rate of rotation of Neptune is about the converse of that which has just been described. The nodes of the orbit of its satellite revolve, and the manner of their motion shows the existence of a certain equatorial bulge inclined about 20° to the plane of the satellite's orbit. The bulging, or ellipticity, of the planet is $\frac{1}{85}$, indicating, according to the work of Tisserand and Newcomb, a rather slow rotation as compared to the rates of rotation of Jupiter and Saturn.

190. Physical Condition of Uranus and Neptune.—We can infer the physical conditions of Uranus and Neptune only from that of other planets which are more favorably situated for observation. They are probably in much the same state as Jupiter and Saturn, though, possibly, somewhat further advanced in their evolution because of their smaller dimensions. One thing to be noticed is that they receive relatively little light and heat from the sun. The amounts per unit area are about $\frac{1}{368}$ and $\frac{1}{904}$ that received by the earth. If their capacity for absorbing and radiating heat were the same as that of the earth, their temperatures (Art. 172) would be respectively about $-340°$ and $-364°$ Fahrenheit. Nevertheless, it must not be imagined that even Neptune would receive only feeble illumination from the sun. Although the sun, as seen from that vast distance, would subtend a smaller angle than Venus does to us when nearest the earth, the noonday illumination would be equal to 700 times our brightest moonlight.

XIII. QUESTIONS

1. Find by the method of Art. 172 what the mean temperatures of the earth would be at the distances of Mercury and Venus.

2. If the earth always presented the same face toward the sun, and if there were no distribution of heat by the atmosphere, what would be the mean temperature of its illuminated side? What would be the result if the earth were at the distance of Venus from the sun?

3. If the mean temperature of the equatorial zone of the earth is 85°, and if it receives, per unit area, 2.5 times as much heat as the polar regions, what is the mean temperature of the polar regions, neglecting the transfer of heat by the atmosphere?

4. What would be the mean temperature of the equatorial zone of the earth at the mean distance of Mars?

5. Suppose the mean temperature of the Thibetan plateau at a height of 15,000 feet above sea level is 40°; what would it be if the earth were at the distance of Mars from the sun?

6. Suppose the atmosphere which a planet can hold is proportional to its surface gravity; how does the atmosphere of Mars compare with that of the earth at an altitude of 15,000 feet above sea level?

7. Waiving the temperature difficulties in the hypothesis regarding the habitability of Mars, what reasonable explanation can you give for the fact that the canals are always along the arcs of great circles?

8. Try the experiment of Maunder and Evans.

9. What would be the total area of 400 canals having an average width of 20 miles and an average length of 300 miles? Suppose to irrigate this area for a season a foot of water is required; how much would this water weigh on the earth? On Mars? Suppose a fall of four feet per mile is required to get a flow in the canals at the necessary rate; suppose it is necessary to pump the water out of the "marshes" to a higher level to get the fall; suppose the pumps work 10 hours a day for 300 days; how many horsepower of work must they deliver?

CHAPTER X
COMETS AND METEORS

I. Comets

191. General Appearance of Comets.—The planets are characterized by the invariability of their form, the simplicity of their motions, and their general similarity to one another. In strong contrast to these relatively stable bodies are the comets, whose bizarre appearance, complex motions, and temporary visibility have led astronomers to devote to them a great amount of attention. Until the last two centuries they were objects of superstitious terror which were supposed to portend calamities. At least so far as their motions are concerned, they are now known to be as lawful as the other members of the solar system.

The typical comet is composed of a head, or *coma*, a brighter nucleus within the head which is often starlike in appearance, and a tail streaming out in the direction opposite to the sun. The apparent size of the head may be anywhere from almost starlike smallness to the angular dimensions of the sun. The nucleus is usually very small and bright, but the tail often extends many degrees from the head before it gradually fades out into the darkness of the sky. The head is the most distinctive part of the comet, for it is always present and looks much like a circular nebula. Either the nucleus or tail, or both, may be absent, especially if the comet is a small one. Comets vary in brightness from those which are so faint that they are barely visible through large telescopes to those which are so bright that they may be observed in full daylight, even when almost in the direction of the sun. In spite of their being sometimes very bright, they are so nearly transparent that faint stars are visible through them without the slightest appreciable diminution of their light.

There are records of about 400 comets having been seen before the invention of the telescope, in 1609, and more than the same number have been observed since that date. Astronomers now keep a close watch of the sky, and only very faint ones can escape their notice. From 3 to 10 are found yearly. They are lettered for each year a, b, $c, \ldots$ in the order of their discovery, and are numbered I, II, III, ... in the order that they pass their perihelia. Besides this, they are generally named after their discoverers.

192. The Orbits of Comets.—In ancient times it was supposed

FIG. 119.—Brooks' Comet, Oct. 19, 1911. *Photographed by Barnard at the Yerkes Observatory.*

that comets were malevolent visitors prowling through the earth's atmosphere, bent on mischief. Kepler supposed they moved in straight lines, but Doerfel showed that the comet of 1681 moved in a parabola around the sun as a focus. In 1686 Newton invented a graphical method of computing comets' orbits from three or more observations of their apparent positions. Better methods have been devised by Lambert, Laplace, Gauss, and later astronomers, and now there is usually no difficulty in determining the elements of an orbit from three complete observations which are separated by a few days.

The orbits of about 400 comets have been computed, and as nearly as can be determined from the imperfect observations on which the computations of many of them are based, the orbits of about 300 of them are essentially parabolic. In fact, they are so generally parabolic, or, at least, extremely elongated, that it has been customary in the preliminary computations to assume they are parabolas. Of the remaining cometary orbits, nearly 100 have been shown to be distinctly elliptical in shape.

A conic section is an ellipse if its eccentricity is less than unity, a parabola if its eccentricity equals unity, and an hyperbola if its eccentricity exceeds unity. Since a body moving subject to gravitation may describe any one of these three classes of orbits, and since the eccentricity of a parabola is the limiting case between that of an ellipse and that of an hyperbola, it is infinitely improbable that the orbit of any comet is exactly parabolic.

It is important to determine whether the eccentricities of the orbits of comets are slightly less than unity or slightly greater than unity. In the former case comets are permanent members of the solar system; in the latter, they are only temporary visitors. The difficulty in answering the question is not theoretical, but practical. In the first place, comets are more or less fuzzy bodies and it is difficult to locate the exact positions of their centers of gravity. In the second place, they are observed during only a very small part of their whole periods while they are in the neighborhood of the earth's orbit. Generally they are not seen much beyond the orbit of Mars and very rarely at the distance of Jupiter. For such a small arc the motion is sensibly the same in a very elongated ellipse, in a parabola, and in an hyperbola whose eccentricity is near unity, as is evident from Fig. 120.

More than 80 comets move in orbits whose major axes are so short that they will certainly return to the sun. The remainder move in exceedingly elongated orbits, and the character of their motion is less certain. But it is significant that the recent computations of Strömgren show that in all cases in which comets have been sufficiently observed to give accurate results respecting their orbits, they were moving in ellipses when they entered the solar system. At the present time there is no known case of a comet which was well observed for a long time whose orbit was hyperbolic, and astronomers are becoming united in the opinion that they are permanent members of the solar system.

The orbits of all the planets are nearly in the same plane; on the other hand, the planes of the orbits of the comets lie in every possible direction and exhibit no tendency to parallelism. The perihelia of the orbits of comets are distributed all around the sun, but show a slight tendency to cluster in the direction in which the sun is moving among the stars, a fact which probably has some connection with the sun's motion.

Some comets have perihelion points only a few hundred thousand miles from the surface of the sun, and when nearest the sun they actually pass through its corona (Art. 238). About 25 comets pass within the orbit of Mercury; nearly three fourths of those which have been observed come within the orbit of the earth; very few so far seen are permanently without the orbit of Mars, and all known comets come within the orbit of Jupiter. This does not mean that there are no comets with great perihelion distances, or even that those with perihelion distances greater than the distance from the earth to the sun are not very numerous. Comets are relatively inconspicuous objects until they come considerably within the orbit of Mars. Sometimes their brightness increases a hundred thousandfold while they move from the orbit of Mars to that of Mercury. Consequently, even if comets whose perihelia are beyond the orbit of Mars were very numerous, not many of them would be observed.

193. The Dimensions of Comets.—After the orbits of comets have been computed so that their distances from the earth are known, their actual dimensions can be determined from their apparent dimensions. It has been found that the head of a comet may have any diameter from $10,000$ miles up to more than $1,000,000$ miles. The most remarkable thing about the head of a comet is that it nearly always contracts as the comet approaches the sun, and expands again when the comet recedes. The variation in volume is very great, the ratio of the largest to the smallest sometimes being as great as $100,000$ to 1. John Herschel suggested that the contraction may be only apparent, the outer layers of the comet becoming transparent as it approaches

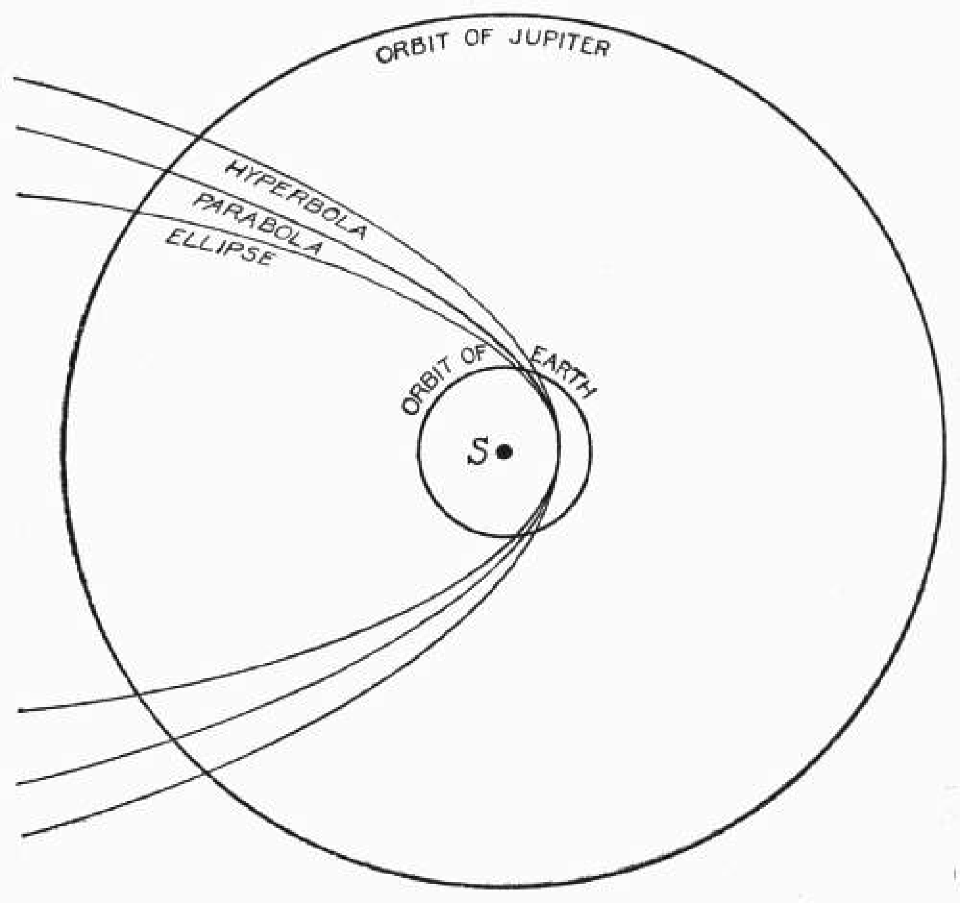

FIG. 120. — Similarity of elongated ellipses, parabolas, and hyperbolas in the vicinity of the orbit of the earth.

the sun. This suggestion contradicts the appearances and seems to be extremely improbable.

The nucleus of a comet may be so small as to be scarcely visible, say 100 miles in diameter, or it may be as large as the earth. For example, William Herschel observed the great comet of 1811 when its head was more than 500,000 miles in diameter, while its nucleus measured only 428 miles across. The nuclei vary in size during the motion of comets, but the change is quite irregular and no law of variation has been discovered.

The tails of comets are inconceivably large. Their diameters are counted by thousands and tens of thousands of miles where they leave the heads of comets, and by tens of thousands or hundreds of thousands of miles in their more remote parts. They vary in length from a few million miles, or even less, up to more than a hundred million of miles. In volume, the tails of comets are thousands of times greater than the sun and all the planets together. The strangest thing about them is that they point almost directly away from the sun whichever way the comet may be going. That is, when the comet is approaching the sun, the tails trail behind like the smoke from a locomotive; when the comet is receding, they project ahead like the rays from the head light on a misty night. When a comet is far from the sun, its tail is small, or may be entirely absent; as it approaches the sun, the tail develops in dimensions and splendor, and then diminishes again on its recession from the sun.

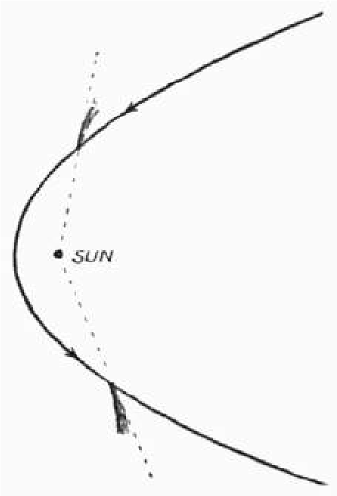

FIG. 121.—The tails of comets are always directed away from the sun.

194. The Masses of Comets.—Comets give visible evidence of remarkable tenuity, but their volumes are so great that, if their densities were one ten-thousandth of that of air at the surface of the earth, their masses in many cases would be comparable to the masses of the planets.

The masses of comets are determined from their attractions for other bodies (Arts. 19, 154). Or, rather, their lack of appreciable mass is shown by the fact that they do not produce observable disturbing effects in the motions of bodies near which they pass. Many comets have had their orbits entirely changed by planets without producing any sensible effects in return. Since, according to the third law of motion, action between two bodies is equal and opposite, it follows that the masses of comets are very small, probably not exceeding one millionth that of the earth.

One of the most striking examples of the feeble gravitational power of comets was furnished by the one discovered by Brooks in 1889. It had passed through Jupiter's satellite system in 1886 without interfering sensibly with the motions of these bodies, although its own orbit was so transformed that its period was reduced from 27 years to 7 years.

195. Families of Comets.—Notwithstanding the great diversities in the orbits of comets, there are a few groups whose members seem to have some intimate relation to one another, or to the planets. There are two types of these groups, and they are known as *comet families.*

Families of the first type are made up of comets which pursue nearly identical paths. The most celebrated family of this type is composed of the great comets of 1668, 1843, 1880, and 1882. A much smaller one seen in 1887 probably should be added to this list. Their orbits were not only nearly identical, but the comets themselves were very similar in every respect. They came to the sun from the direction of Sirius—that is, from the direction away from which the sun is moving with respect to the stars—and escaped the notice of observers in the northern hemisphere until they were near perihelion. They passed half way around the sun in a few hours at a distance of less than 200,000 miles from its surface, moving at the enormous velocity of more than 350 miles per second. Their tails extended out in dazzling splendor 100,000,000 miles from their heads.

One might think that the various members of a comet family are but the successive appearances of the same comet; but such is not the case, for the observations show that though their orbits may be ellipses, their periods are at least 600 or 800 years. This means that they recede to something like five times the distance of Neptune from the sun. The most plausible theory seems to be that they are the separate parts of a great comet which at an earlier visit to the sun was broken up by tidal disturbances.

Families of the second type are made up of comets whose orbits have their aphelion points and the ascending and descending nodes of their orbits near the orbits of the planets. About 30 comets have their aphelia near Jupiter's orbit, and are known as Jupiter's family of comets, Fig. 122. Their orbits are, of course, all elliptic, and their periods are from 3 to 8 years. They move around the sun in the same direction that the planets revolve. Half of them have been seen at two or more perihelion passages. These comets are all inconspicuous objects and entirely invisible to us except when they are near the earth.

Saturn has a family of 2 comets, Uranus a family of 3, and Neptune a family of 6 members. The terrestrial planets do not possess comet families. There are, according to the statistical study of W. H. Pickering, two or three groups of comets whose aphelia are several times the distance of Neptune from the sun, suggesting, possibly, the existence of planets at these respective distances.

196. The Capture of Comets.—A very great majority of comets move in sensibly parabolic orbits whose positions have no special relations to the positions of the orbits of the planets. But the orbits of

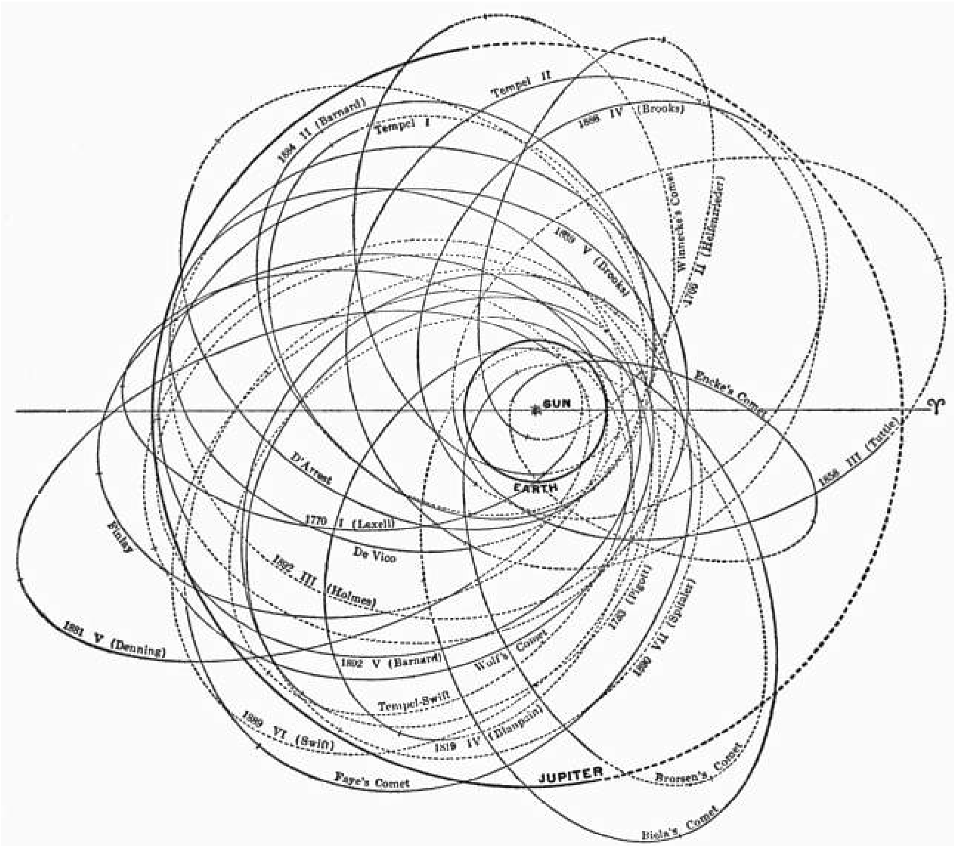

FIG. 122. — Jupiter's family of comets (Popular Astronomy).

nearly all those comets which are elliptical and not exceedingly elongated lie near the plane of the planetary orbits and have their aphelia near the orbits of the planets. These facts suggest that the orbits of comets moving in these ellipses have been changed from parabolas or very elongated ellipses by the disturbing action of the planet near whose orbit their aphelion points lie. This question of the transformation of orbits of comets was first discussed by Laplace, who found that if a comet which is approaching the sun on a parabolic or elongated elliptical orbit passes closely in front of a planet, its motion will be retarded so that it will subsequently move in a shortened elliptical orbit, at least until it is disturbed again.

Suppose a comet approaches the sun in a sensibly parabolic orbit and passes closely in front of a planet so that its orbit is reduced to an ellipse. It is then said to have been *captured.* It will in the course of time pass near the planet again, when its orbit may be still further reduced; or, its orbit may be elongated and it may possibly be driven from the solar system on a parabola or an hyperbola.

It is a generally accepted theory that the members of the comet families of the various planets have been captured by the method described. Jupiter has a larger family of comets than any other planet because of its greater mass and also because, if a comet were captured originally by any planet beyond the orbit of Jupiter, it would yet be possible for Jupiter to reduce its orbit still further. On the other hand, when Jupiter has captured a comet and made it a member of its own family, it is far within the orbit of the remoter planets and is no longer subject to capture by them. The planets beyond the orbit of Jupiter have a few comets each, and the clustering of the aphelia of comets at still more remote distances has suggested the existence of planets as yet undiscovered (Art. 161). The terrestrial planets have no comet families partly because their masses are small compared to that of the sun, and partly because comets cross their orbits at very great speed.

The masses of the planets are not great enough to reduce a parabolic comet to membership in their own families at one disturbance. The matter is illustrated by Brooks' comet, 1889-V, whose period, according to the computations of Chandler, was reduced by Jupiter, in 1886, from 27 years to 7 years. Lexell's comet, of 1770, furnishes an example of a disturbance of the opposite character. In 1770 it was moving in an elliptical orbit with a period of 5.5 years; but in 1779 it approached near to Jupiter, its orbit was enlarged, and it has never been seen again.

When a planet captures a comet, the former reduces the dimensions of the orbit of the latter, but the latter still revolves around the sun. The question arises whether a planet might not capture a comet in a more fundamental sense; that is, reduce its orbit so that it would become a satellite of the planet. It has been repeatedly suggested that the planets may have captured their satellites in this manner. The answer to this suggestion is that a planet cannot capture a comet and make it into a satellite simply by its own gravitation and that of the sun. The only possibility is that the comet should encounter resistance in a very special manner, and even then the problem presents serious difficulties. No small resistance would be sufficient because the motion of a comet around the sun in a parabolic orbit is much greater than it would be in a satellite orbit; and, in order that resistance should reduce the velocity by the required amount, it would be necessary for the comet to encounter so much material that its mass would grow several fold.

197. On the Origin of Comets.—The similarities of the motions of the various planets point to the conclusion that they had a common origin, and the agreement of the direction of the rotation of the sun with their direction of revolution indicates that they have been associated with the sun throughout their whole evolution. This line of reasoning does not lead to the inference that the comets belong to the planetary family. They may have had quite a different origin; at any rate, most of them recede from the sun to regions several times as remote as the planet Neptune.

It was formerly supposed that comets are merely small, wandering masses which pass from star to star, visiting our sun but once. The intervals of time required for such excursions are enormously greater than has generally been supposed. For example, the great comet of 1882 came almost exactly from the direction of Sirius and returned again in the same direction. Suppose the comet moved under the attraction of Sirius until it had passed over half of the distance from Sirius to the sun, and that it then moved sensibly under the attraction of the sun. Although Sirius is one of the nearest known stars in all the sky, it is found that it would take $70,000,000$ years to describe this part of its orbit. About twice this period of time would be required for it to come from Sirius to the sun, and eight times this immense interval for a comet to come from a star four times as far away. These figures do not disprove the theory that comets wander from star to star, but they show that if this hypothesis is true, then comets spend most of their time in traveling and but little in visiting.

If the comets moved from star to star, their orbits with respect to the sun would never be elliptical until after they had been captured; they would, indeed, nearly always be strongly hyperbolic because the stars are moving with respect to one another with velocities which correspond to hyperbolic speed for comets at such great distances. The fact that no comet out of the hundreds whose orbits have been computed has moved in a sensibly hyperbolic orbit points strongly to the conclusion that comets have been permanent members of the solar system. They are possibly the remains of the far outlying masses of a nebula from which the solar system may have been developed. With increasing proof that they are actually permanent members of the solar system, their importance in connection with the question of its origin and evolution continually increases.

198. Theories of Comets' Tails.—The fact that the tails of comets usually project almost directly away from the sun indicates that they are in some way acted upon by a repelling force emanating from the sun. The intensity of this repulsion has been computed in a number of cases by Barnard and others from the accelerations which masses have undergone which were receding from the heads of comets along their tails. These accelerations have been determined by comparing photographs of the comets taken at different times separated by short intervals.

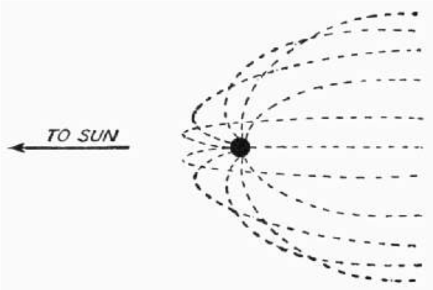

FIG. 123.—The repulsion theory of the origin of comets' tails.

It was suggested by Olbers as early as 1812 that the repulsive force which apparently produces the tails of comets may be electrical in character. This theory has been taken up and systematically developed by Bredichin, of Moscow. According to it, the sun and comet nuclei both repel the material of which the tails of comets are composed. Those particles which leave the nuclei in the direction away from the sun continue on in straight lines; those which leave in other directions are gradually bent back by the force from the sun and form the outer parts of the tails, as shown in Fig. 123. The resulting tails, especially if they are very long, are slightly curved because the motion of the comet is somewhat athwart the line along which the repelled particles move, that is, the line from the sun through the nucleus (see Fig. 121).

Electrical repulsion acts on the surfaces of particles, while gravitation depends on their masses. Therefore, while large masses are attracted by the sun more than they are electrically repelled, the opposite may be true for small particles, and the electrical repulsion is relatively stronger the smaller they are. Consequently, the tails which are produced out of small particles will be more nearly straight than those which are composed of larger particles. Bredichin advanced the theory that the long, straight tails are due to hydrogen gas, the ordinary slightly curved tails to hydrocarbon gases, and the short, stubby, and much curved tails to vapors of metals. Spectroscopic observations have to a considerable extent confirmed these conclusions. Some comets have tails of more than one type, as for example Delavan's comet (Fig. 124).

If the electrical repulsion theory is adopted, the question at once arises why the sun and the materials of which the tails of comets are composed are similarly electrified. A plausible answer to this question can be given. At least the hydrogen in the sun's atmosphere seems to be negatively electrified. Suppose a comet approaches the sun from a remote part of space without an electrical charge. Laboratory experiments show that the ultra-violet rays from the sun, striking on the nucleus of the comet, will probably drive off negatively charged particles which will be repelled by the negative charge of the sun, and they will thus form a tail for the comet. The repulsion will depend upon the size of the particles and the electrical potential of the sun. After the negatively electrified particles have been driven off, the nucleus will be positively charged and, consequently, will be electrically attracted by the sun. But since the particles driven off will be only an exceedingly small part of the whole comet, this attraction will not be great enough sensibly to alter the comet's motion.

Another theory which merits careful attention is that the particles which constitute comets' tails are driven off by the pressure of the sun's light. According to Clerk-Maxwell's electromagnetic theory, light exerts a pressure upon bodies upon which it falls which is proportional to the light energy in a unit of space. For bodies of considerable magnitude the pressure is relatively very small, though it has been detected by Nichols and Hull; but for minute bodies, say a ten-thousandth of an inch in diameter, the light pressure may greatly exceed the sun's attraction. For still smaller bodies the light pressure becomes relatively larger until their diameters are approximately equal to a wave length of light, say, one fifty-thousandth of an inch. Then, as Schwarzschild has shown, the light pressure decreases relatively to the force of gravitation. Consequently, if the particles are very small the attraction will more than equal the repulsion.

But it has been shown more recently by Lebedew that there is light pressure upon gases, in which the diameters of the molecules are always a very small fraction of a wave length of light, and that the pressure is proportional to the amount of energy which the gas absorbs. Consequently, it is not necessary to assume that the particles of which the tails of comets are composed are larger than molecules.

It is generally supposed by astronomers that both electrical repulsion and light pressure are factors in the production of comets' tails. Nevertheless, there are outstanding phenomena which these theories do not explain. In the first place, there is no adequate explanation of the luminosity of comets' tails. As comets approach the sun, their tails increase in brightness much more rapidly than they should if they were

FIG. 124.—Delavan's comet, Sept. 28, 1914, showing a long, straight tail and one having considerable curvature (Barnard).

shining only by reflected light. The luminosity of such exceedingly tenuous bodies whose density is doubtless far less than that in the best vacuum tubes of the present time can scarcely be explained as a temperature effect. And still more embarrassing to these theories are the facts that comets' tails do not always point directly away from the sun, and that sometimes they change their direction by a number of degrees in a very short time. For example, Barnard took photographs of Brooks's comet, 1893-IV, on November 2 and November 3. In this interval the comet moved forward in its orbit about 1°; and, consequently, according to these theories, the direction of its tail should have changed about 1°. But there was an actual change of direction of the tail of 16° which has not been explained. There are also sudden and great changes in the character and luminosity of comets' tails which no theory explains. Sometimes secondary tails are developed with great rapidity, making an angle of as much as 45° with the line joining the comet with the sun. Obviously much remains to be learned in connection with the tails of comets.

199. The Disintegration of Comets.—The particles that leave the head of a comet to form its tail never unite with it again. In this way, at each reappearance of a comet, that part of the material which goes to form its tail is dispersed into space; and, as the quantity remaining becomes reduced, the comet becomes less and less conspicuous. Possibly this is one of the reasons why Halley's comet in 1910 was not such a remarkable object as it seems to have been in some of its earlier apparitions.

There is another way in which comets disintegrate. Since their masses are very small, the mutual attractions of their parts are not sufficient to hold them together if they are subject to strong disturbing forces. When they pass near the sun, they are elongated by enormous tides. In fact, if they pass within Roche's limit (Art. 183), the tidal forces exceed their self gravitation unless they are as dense as the sun. Comets have such exceedingly low density that the limits of tidal disintegration for them must be very great. Consequently, when a comet passes near the sun, the tidal forces to which it is subject tend to tear it into fragments, which, of course, may be assembled again by their mutual gravitation after they have receded far from the sun. But on their way out they may pass near a planet which will exert analogous forces, and may so disorganize them that they will never again be united into a single body.

The theory which has just been outlined is clear. Now what have been the observed facts? Biela's comet was broken into two parts by some unknown forces, and the two components subsequently traveled in independent paths. The great comet of 1882 was seen to have a number of outlying fragments when it was in the vicinity of the sun, and many other comets have exhibited analogous phenomena.

Another source of disturbance to which comets are subject is the scattered meteoric material which may more or less fill the space among the planets. The phenomenon of the zodiacal light gives an almost certain proof of its extensive existence. Such scattered particles would have little effect on a dense body like a planet, but might cause serious disturbances in a tenuous comet. In fact, there are many instances in which comets and comets' tails seem to have been subjected to unknown exterior forces. They are now and then more or less broken up, and occasionally the tails of comets have been apparently cut off and brushed aside.

Many comets which have been observed at two or three perihelion passages have been found to be fainter at each successive return than they were at the preceding, and some have eventually entirely disappeared. It seems to be a safe conclusion that comets are slowly disintegrated under the disturbing forces of the sun and planets and the resisting meteoric material which they may encounter. As confirmatory of this view, it may be noted that the members of Jupiter's family have small tails or none at all; that this comet family does not contain as many members as might be expected; and that a number of comets have totally disappeared, presumably by disintegration.

200. Historical Comets.—In this article some of those comets will be briefly described which have exhibited phenomena of unusual interest. The enumeration of their peculiarities will illustrate the general statements which have preceded, and will give additional information respecting these remarkable objects.

The Comet of 1680.—The comet of 1680 was the first one whose orbit was computed on the basis of the law of gravitation. Newton made the calculations and found that its period of revolution was about 600 years. It is one of the family of comets mentioned in Art. 195. At its perihelion it passed through the sun's corona at a distance of only 140,000 miles from its surface. It flew along this part of its orbit at the rate of 370 miles per second, and its tail, 100,000,000 miles long, changed its direction to correspond with the motion of the comet in its orbit.

The Great Comet of 1811.—The great comet of 1811 was visible from March 26, 1811, until August 17, 1812, and was carefully observed by William Herschel. He discovered from the changes in its brightness, that it shone partly by its own light; for its brilliance increased as it approached the sun more rapidly than it would have done if it had been shining entirely by reflected light. At one time its tail was 100, 000, 000 miles long and 15, 000, 000 miles in diameter. The phenomena connected with it suggested to Olbers the electrical repulsion theory of comets' tails.

Encke's Comet (1819).—Encke's comet was the first member of Jupiter's family to be discovered, and it has a shorter period (3.3 years) than any other known comet. At its brightest it was an inconspicuous telescopic object (Fig. 125), but it is noted for the fact that its period was shortened, presumably by encountering some resistance, about 2.5 hours at each revolution until 1868; since that time the change in the period of revolution has been only one half as great. The change in volume of Encke's comet at times was extraordinary. On October 28, 1828, it was 135, 000, 000 miles from the sun and had a diameter of 312, 000 miles; on December 24, its distance was 50, 000, 000 miles from the sun, and its diameter was only 14, 000 miles; while at its perihelion passage, on December 17, 1838, at a distance of 32, 000, 000 miles, its diameter was only 3000 miles. That is, at one time its volume was more than a million times greater than it was at another.

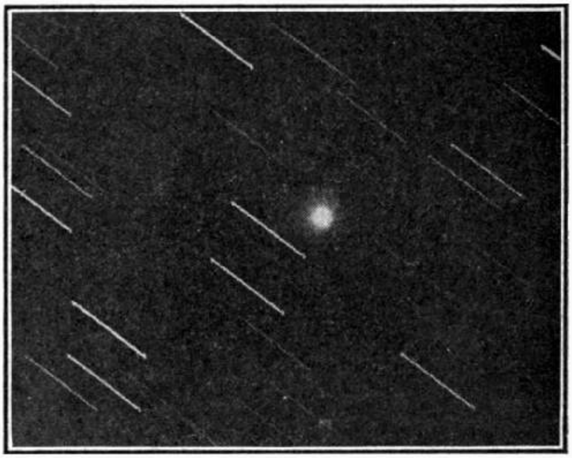

FIG. 125.—Encke's comet (Barnard).

Biela's Comet (1826).—Biela's comet is also a small member of Jupiter's family and has a period of about 6.6 years. At its appearance in 1846, it presented no unusual phenomena until about the 20th of December, when it was considerably elongated. By the first of January it had separated into two distinct parts which traveled along in parallel orbits at a distance of about 160, 000 miles from each other. At this time the two parts were undergoing considerable changes in brightness, usually alternately, and sometimes they were connected by a faint stream of light. At their appearance in 1852 the two components were 1,500,000 miles apart, and they have never been seen again, although searched for very carefully. De Vico's comet, of 1844, and Brorsen's comet, of 1846, are also comets which have disappeared, the former having been observed but once, and the latter but four times after its discovery.

Donati's Comet (1858).—Donati's comet was one of the greatest comets of the nineteenth century. It was visible with the unaided eye for 112 days, and through a telescope for more than 9 months. Its tail, which was more than 54,000,000 miles long, at one time subtended an angle of more than 30° as seen from the earth. It moved in the retrograde direction in an orbit with a period of more than 2000 years, and at its aphelion its distance from the sun was more than 5.3 times that of Neptune.

Tebbutt's Comet (1861).—Tebbutt's comet was of great dimensions, but is noteworthy chiefly because the earth passed through its tail. As could have been anticipated from the excessive tenuity of comets' tails, the earth experienced no sensible effects from the encounter. The earth must have passed through the tails of comets many times in geological history, and there is no evidence whatever that it has ever been disturbed by them. In fact, if a comet should strike the earth, head on, it is probable that the result would not be disastrous to the earth.

The Great Comets of 1880 *and* 1882.—The comets of 1880 and 1882 were two splendid members of the most remarkable known family of comets which travel in the same orbit. Both of these comets, as well as the earlier members of the same family, are noteworthy for their vast dimensions, their great brilliancy, and their close approach to the sun. The comet of 1882 was observed both before and after perihelion passage. Although it swept through several hundred thousand miles of the sun's corona, its orbit was not sensibly altered. Yet it gave evidence of having been subject to violent disrupting forces. After perihelion passage it was observed to have as many as 5 nuclei, while Barnard and other observers saw in the immediate vicinity as many as 6 or 8 small comet-like masses, apparently broken from the large body, traveling in orbits parallel to it.

Morehouse's Comet (1908).—On September 1, 1908, Morehouse, at the Yerkes Observatory, discovered the third comet of the year. It was found on photographic plates taken for other purposes, and is one of the few examples in which comets have been discovered by photography. This comet was never bright, but was one of the most remarkable comets ever observed in the extent and variety of its activities. It was well situated for observation, and Barnard obtained 239 photographs of it on 47 different nights. The material which went into the tail of the comet was often evolved with the most startling rapidity. For example, on the 30th of September, in the early part of the night, the comet presented an almost normal appearance. Before the night was over, the tail had become cyclonic in form and was attached to the head, which then was small and starlike, by a very slender, curved, tapering neck. On the succeeding night the material that then constituted the tail was entirely detached from the head. On October 15, there was another large outbreak of material which was shown by successive photographs to be swiftly receding from the comet (Fig. 126).

Not only was Morehouse's comet noteworthy for the extraordinary activities exhibited by its tail, but it changed in brightness in a very remarkable manner. It was generally considerably below the limits of visibility with the unaided eye, but now and then it would flash up, without apparent reason, for a day or so until it could be seen very faintly without a telescope. While a number of larger comets have been observed in recent years, no other has given evidence of such remarkable changes in the forces that produce comets' tails, and no other has exhibited such mysterious variations in brightness.

201. Halley's Comet.—Halley's comet is the most celebrated one in all the history of these objects. It is named after Halley, not because he discovered it, but because he computed its orbit from observations made in 1682 by the methods which had been developed by his friend Newton. Halley found that the orbit of this comet was almost identical with the orbits of the comets of 1607 and 1531. He came to the conclusion that these various comets were only different appearances of the same one which was revolving around the sun in a period of about 75 years. The records of comets in 1456, 1301, 1145, and 1066 confirmed this view because these dates differ from 1682 by nearly integral multiples of 75 or 76 years. From his computations Halley predicted that the comet would appear again and pass its perihelion point on March 13, 1759.

Many of Halley's contemporaries were very skeptical regarding this prediction. The law of gravitation had only recently been discovered and the certainty with which it had been established was not yet fully comprehended. Halley was accused by skeptics of seeking notoriety by making a prophecy and cleverly putting forward the date of its fulfill-

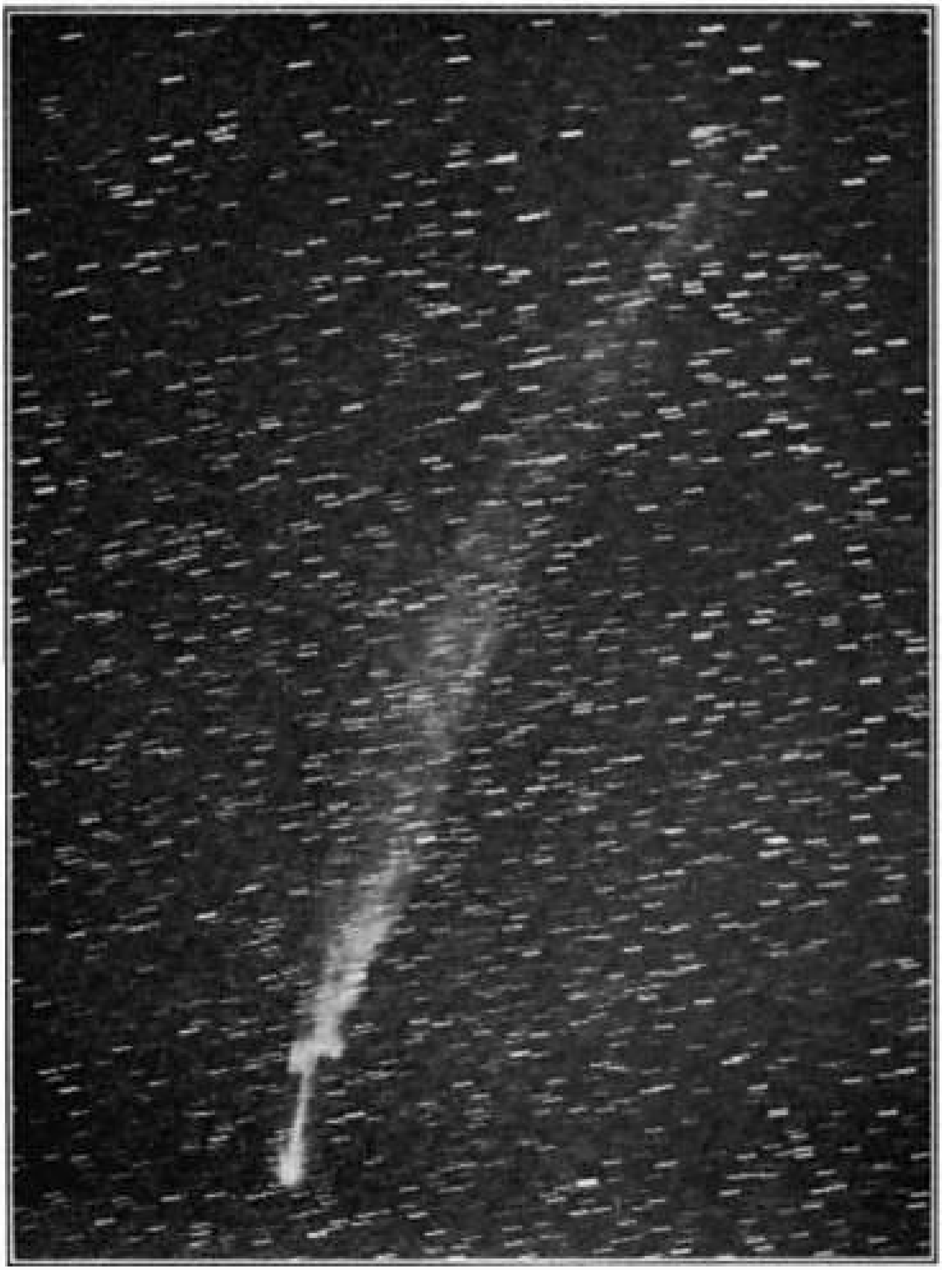

FIG. 126. — Morehouse's comet, Oct. 15, 1908. *Photographed by Barnard at the Yerkes Observatory.*

ment so far that he would be dead before his failure became known. However, before the 75 years had passed away, the law of gravitation had become so firmly established, and the mathematical processes employed in astronomical work had become so well understood, that astronomers, at least, had implicit faith in the correctness of Halley's

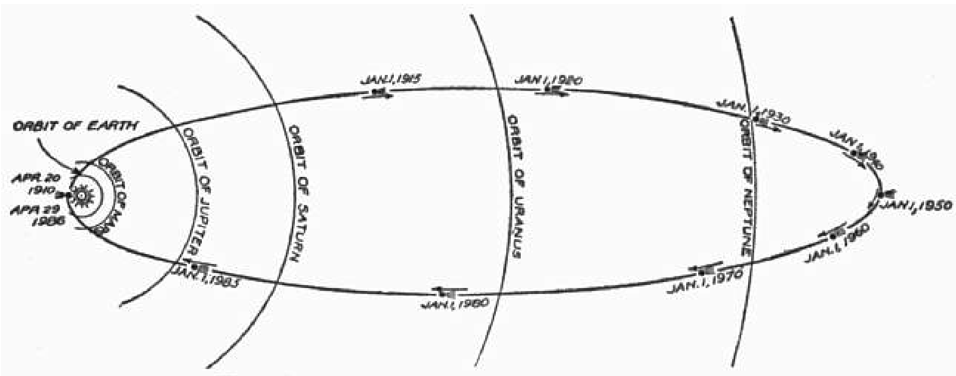

FIG. 127. — The orbit of Halley's comet.

prediction, although since its last appearance the comet had been invisible for the lifetime of a man and had gone out 3,000,000,000 miles from the sun to beyond the orbit of Neptune. There was great popular interest in the comet as the date for its return approached. It actually passed its perihelion within one month of the time predicted by Halley. The slight error in the prediction was due to the imperfect observations of its positions in 1682, and to the perturbations by planets which were then unknown. This was the first verification of such a prediction; and the definiteness and completeness with which it was fulfilled had been entirely unapproached in the case of all the prophecies which the world had known up to that time.

Halley's comet passed the sun again in 1835. At this time it was so accurately observed that its subsequent orbit could be computed with a high degree of precision. If it had made its next revolution in the same period as the one ending in 1835, it would have passed its perihelion in July, 1912. Instead of this, it passed its perihelion on April 19, 1910. The perturbations of the remote planets reduced its period by more than two years. The most accurate computations of its orbit and predictions of the time of its return were made by Cowell and Cromellin, of Greenwich, who missed the time of perihelion passage by only 2.7 days. Their computations were so accurate that even this

FIG. 128.—Halley's comet, May 29, 1910. *Photographed by Barnard at the Yerkes Observatory.*

small discrepancy could not be the result of accumulated errors, and they believe that the comet has been subject to some unknown forces. Its next return will be about 1985, and Fig. 127 shows the position in its orbit for various epochs during this interval. In order to get the precise time of its return, it will be necessary to take into account the

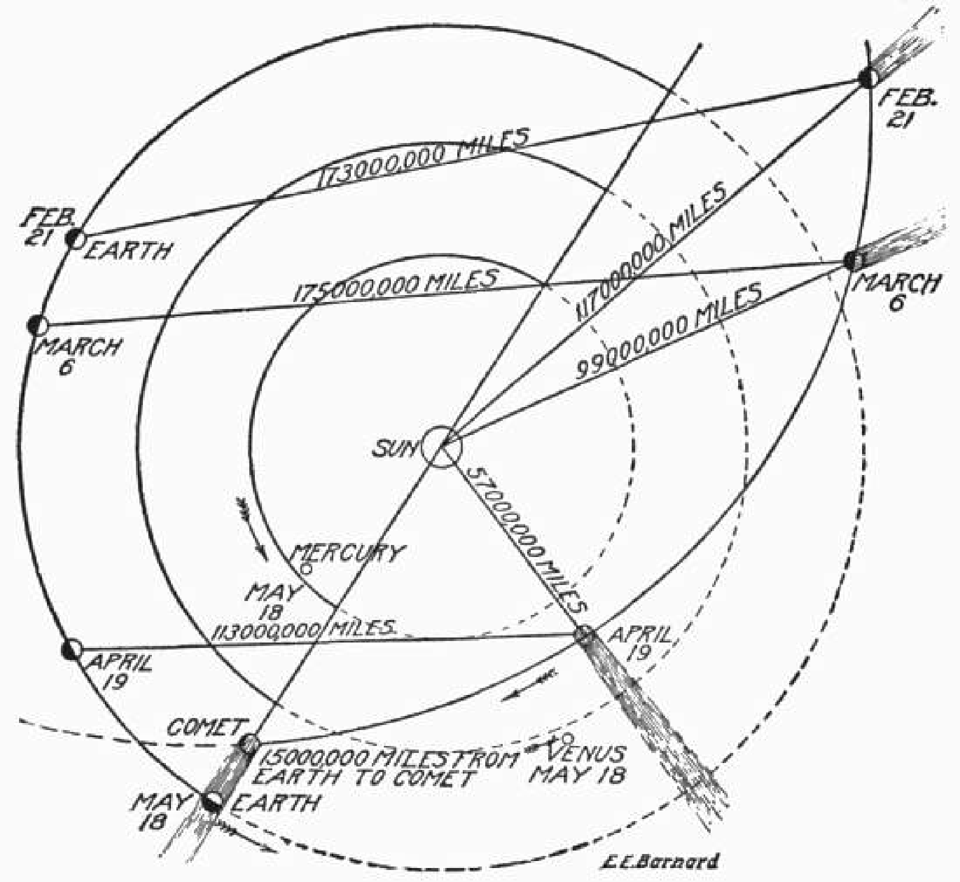

FIG. 129. — The relations of the sun, earth, and Halley's comet in 1910.

perturbations of the planets.

While Halley's comet is a very large one (Fig. 128), its latest appearance was somewhat disappointing, especially to the general public, who had been led to expect that it would rival the sun in brightness. One of the reasons for the disappointment was that the earth was not very near the comet when it was at its perihelion where it was brightest and had the longest tail. The relations of the earth, comet, and sun in this part of its orbit are shown in Fig. 129, drawn by Barnard. On May 5, the length of the comet's tail was 37, 000, 000 miles. On May 18 the comet passed between the earth and the sun and was entirely invisible when projected on the sun's disk. This shows that even its nucleus was extremely tenuous and transparent. At this time the earth passed through at least the outlying part of its tail. Neither at this time nor at any other did the comet have any sensible influence upon the earth. On the whole, it was altogether devoid of interesting features.

II. Meteors

202. Meteors, or Shooting Stars.—An attentive watch of the sky on almost any clear, moonless night will show one or more so-called "shooting stars." They are little flashes of light which have the appearance of a star darting across the sky and disappearing. Instead of being actual stars, which are great bodies like our sun, they are, as a matter of fact, tiny masses so small that a person could hold one in his hand. Under certain circumstances of motion and position, they dash into the earth's atmosphere at a speed of from 10 to 40 miles per second, and the heat generated by the friction with the upper air vaporizes or burns them. The products of the combustion and pulverization slowly fall to the earth if they are solid, or are added to the atmosphere if they are gaseous. Since it is misleading to call them "shooting stars," they will always be called "meteors" hereafter.

The distances of meteors were first determined in 1798 by Brandes and Benzenberg, at Göttingen. They made simultaneous observations of them from positions separated by a few miles, and from the differences in their apparent directions they computed their altitudes above the surface of the earth (Art. 29). Their observations and those of many succeeding astronomers, among whom may be mentioned Denning, of England, and Olivier, of Virginia, have shown that meteors rarely, if ever, become visible at altitudes as great as 100 miles, and nearly all of them disappear before they have descended to within 30 miles of the earth's surface.

The velocity with which a meteor enters the atmosphere can be found by determining the point at which it becomes visible, the point at which it disappears, and the interval of time during which it is visible. The total amount of light energy given out by a meteor can be determined from its apparent brightness, its distance from the observer, and the time during which it is radiant. The energy radiated by a meteor has its source in the heat generated by the friction of the meteor with the earth's atmosphere, and it cannot exceed the kinetic energy of the meteor when it entered the atmosphere. Suppose all the kinetic energy of a meteor is transformed into light. This assumption is not strictly true, but it will be approximately true for matter moving with the high speed of a meteor. Then, since the energy of motion of a body is one half its mass multiplied by the square of its velocity, the mass of the meteor can be computed because its light energy and velocity can be determined directly from observations by the methods which have just been described. By such means it has been found that ordinarily the masses of meteors do not exceed a few tenths of an ounce. However, the observational data are difficult to determine and the subject has received relatively less attention than it deserves. Consequently, no great reliance should be placed on the precise numerical results.

203. The Number of Meteors.—If a person scans the sky an hour or so and finds that he can see only a few meteors, he is tempted to draw the conclusion that the number of them which strike the earth's atmosphere daily is not very large. He bases his conclusion mostly on the fact that half of the celestial sphere is within his range of vision, but a diagram representing the earth and its atmosphere to scale will show him that he can see by no means half the meteors which strike the earth's atmosphere. As a matter of fact, he can see the atmosphere over only a few square miles of the earth's surface.

From very many counts of the number of meteors which can be seen from a single place during a given time, it has been computed that between 10 and 20 millions of them strike into the earth's atmosphere daily. There are probably several times this number which are so small that they escape observation. Often when astronomers are working with telescopes they see faint meteors dart across the field of vision which would be quite invisible with the unaided eye.

Meteors enter the earth's atmosphere from every direction. The places where they strike the earth and the velocities of their encounter depend both upon their own velocities and also upon that of the earth around the sun. The side of the earth which is ahead in its motion encounters more meteors than the opposite, for it receives not only those which it meets, but also those which it overtakes, while the part of the earth which is behind receives only those which overtake it. The meridian is on the forward side of the earth in the morning and on the rearward side in the evening. It is found by observation that more meteors are seen in the morning than in the evening, and that the relative velocities of impact are greater.

204. Meteoric Showers.—Occasionally unusual numbers of meteors are seen, and then it is said that there is a meteoric shower. There have been a few instances in which meteors were so numerous that they could not be counted, but usually not more than one or two appear in a minute.

At the time of a meteoric shower the meteors are not only more numerous than usual, but a majority of them move so that when their apparent paths are projected backward, they pass through, or very near, a point in the sky. This point is called the *radiant point* of the shower, for the meteors all appear to radiate from it. A number of meteor trails which clearly define a radiant point are shown in Fig. 130.

The most conspicuous meteoric showers occur on November 15 and November 24 yearly. The former have their radiant in Leo, within the sickle, and are called the *Leonids*. From the position of this constella-

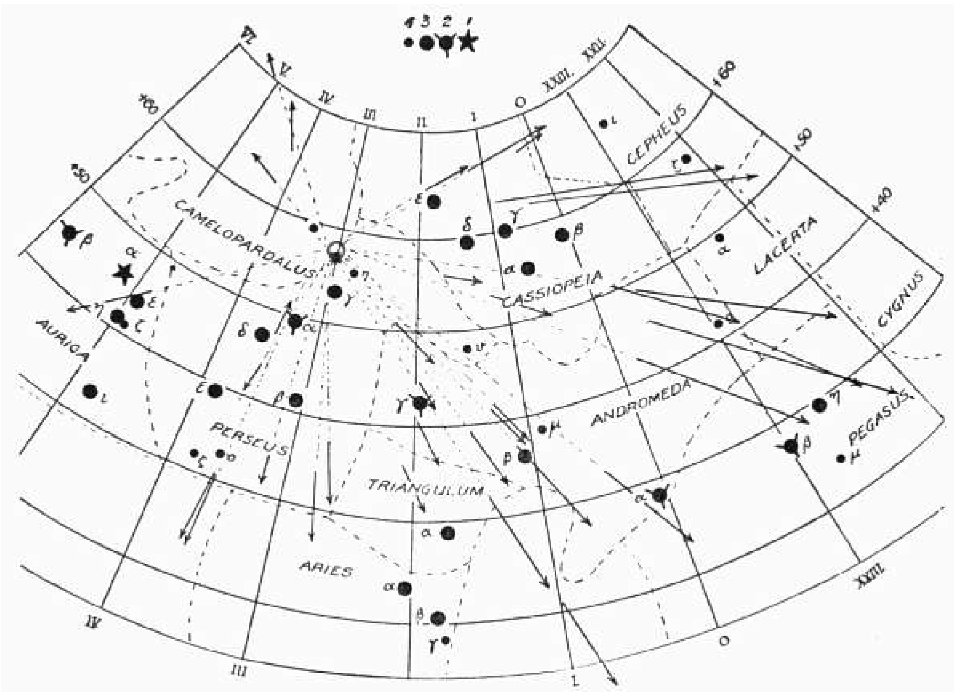

FIG. 130.—Meteor trails defining a radiant point (Olivier).

tion (Arts. 82, 93), it follows that they can be seen only in the early morning hours. The latter have their radiant in Andromeda, and are called the *Andromids*. They can be seen only in the early part of the night. The Leonids and Andromids are not equally numerous every year. Great showers of the Leonids occurred in 1833 and 1866, and less remarkable ones, though greater than the ordinary, from 1898 to 1901. The Andromids appear in unusual numbers every thirteen years.

Besides these meteoric showers, according to Denning, nearly 3000 other less conspicuous ones have been found. The Perseids appear for a week or more near the middle of August, the Lyrids on or about April 20, the Orionids on or about October 20, etc.

205. Explanation of the Radiant Point.—In 1834 Olmsted showed that the apparent radiation of meteors from a point is due to the fact that they move in parallel lines, and that we see only the projection of their motion on the celestial sphere. Thus, in Fig. 131, the actual paths of the meteors are AB, but their apparent paths as seen by an observer at O are AC. When these lines are all continued backward, they meet in the point which is in the direction from which

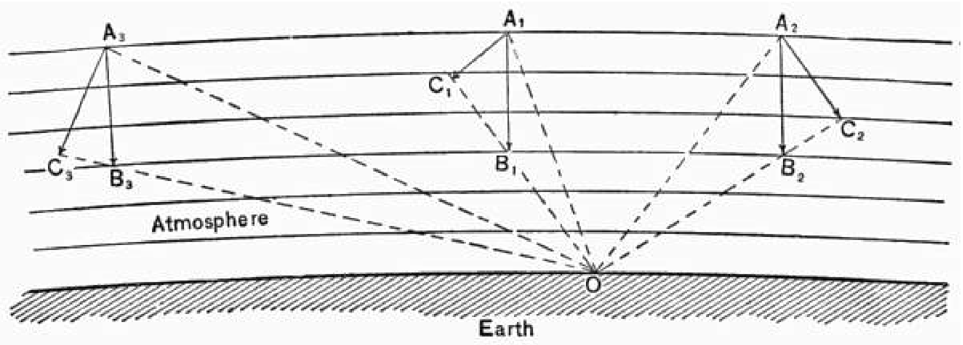

FIG. 131.—Explanation of the radiant point of meteors.

the meteors come.

It follows that the meteors which give rise to the meteoric showers are moving in vast swarms along orbits which intersect the orbit of the earth. When the earth passes through the point of intersection, it encounters the meteors and a shower occurs. Thus, the orbit of the Leonids touches the orbit of the earth at the point which the earth occupies on November 14. In this case the earth meets the meteors (Fig. 132), while the Andromids overtake the earth.

206. Connection between Comets and Meteors.—The fact that the volatile material of which comets' tails are composed gradually becomes exhausted, after which the comets themselves become invisible, and the fact that meteoric showers are due to wandering swarms of small particles which revolve around the sun in elongated elliptical orbits, suggest the hypothesis that comets and meteors are related. The hypothesis is confirmed and virtually proved by the identity of the orbits of certain meteoric swarms and comets.

In 1866 Schiaparelli showed that the August meteors move in the same orbit as Tuttle's comet of 1862. That is, in addition to the comet, which is a member of Saturn's family, there are many other small bodies (meteors) traveling in the same orbit. In 1867 Leverrier found that the Leonids move in the same orbit as Tempel's comet of 1866, while Weiss showed that the meteors of April 20 and the comet of 1861 move in

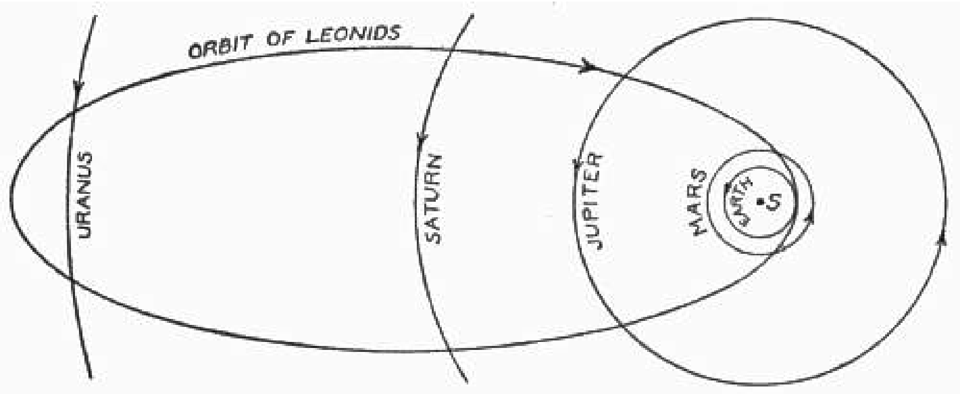

FIG. 132. — Orbit of the Leonid meteors.

the same orbit, and that the paths of the Andromids and Biela's comet were likewise the same. It has recently been claimed that the Aquarid meteors of early May have an orbit almost identical with that of Halley's comet.

While it is not possible to be certain as to the origin of comets, the history of their later evolution and final end is tolerably clear. The elongated orbits in which they may have originally moved are reduced when they are captured by the planets. Their periods of revolution are subsequently shorter, their volatile material wastes away in the form of tails, and the remaining material is scattered along their orbits by the dispersive forces to which they are subject. If these orbits cross the orbit of a planet, the remains of the comets are gradually swept up by the larger body. If an orbit of a comet does not originally cross the orbit of a planet, the perturbations of the planets will, in general, in the course of time, cause it to do so. The result will be that the planets sweep up more and more of the remains of disintegrated comets and undergo a gradual growth in this manner.

207. Effects of Meteors on the Solar System.—The most obvious effect of the numerous meteors which swarm in the solar system is a resistance both to the rotations and the revolutions of all the bodies. As was stated in Art. 45, the effects of meteors upon the rotation of the earth are at present exceedingly slight, and it is very probable that their influences upon the rotations of the other members of the system are also inappreciable. A retardation in the translatory motion of a body causes its orbit to decrease in size. Hence, so far as the meteors affect the planets in this way, they cause them continually to approach the sun.

Another effect of meteors upon the members of the solar system is to increase their masses by the accretion of matter which may have come originally from far beyond the orbit of Neptune. As the masses of the sun and planets increase, their mutual attractions increase and the orbits of the planets become smaller. Looking backward in time, we are struck by the possibility that the accretion of meteoric matter may have been more rapid in former times, and that it may have been an important factor in the growth of the planets from much smaller bodies.

208. Meteorites.—Sometimes bodies weighing from a few pounds up to several hundred pounds, or even a few tons, dash into the earth's atmosphere, glow brilliantly from the heat generated by the friction, roar like a waterfall, occasionally produce violent detonations, and end by falling on the earth. Such bodies are called *meteorites*, *siderites*, or *aërolites*.

About two or three meteorites are seen to fall yearly; but, since a large part of the earth is covered with water or is uninhabited for other reasons, it is probable that in all at least 100 strike the earth annually. The outside of a meteorite during its passage through the air is subject to intense and sudden heating, and the rapid expansion of its surface layers often breaks it into many fragments. The surface is fused and on striking cools rapidly. The result is that it has a black, glossy structure, usually with many small pits where the less refractive material has been melted out. Since meteors pass entirely through the atmosphere in a few seconds, only their surfaces give evidence of the extremes of heat and pressure to which they have been subjected in their final flight.

Most meteors are composed of stone, though it is often mixed with some metallic iron. Even where pure iron is not present, some of its compounds are usually found. About three or four out of every hundred are nearly pure iron with a little nickel. All together about 30 elements which occur elsewhere on the earth have been found in meteorites, but no strange ones. Yet in some respects their structure is quite different from that of terrestrial substances. They have peculiar crystals, they show but little oxidation and no action of water, and they con-

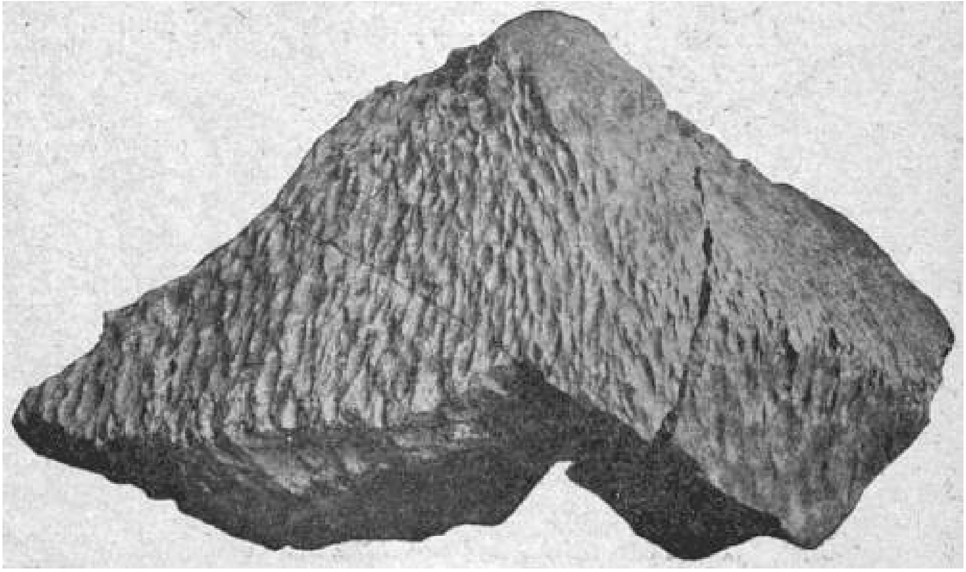

FIG. 133. — Stony meteorite which fell at Long Island, Kansas; weight, 700 pounds (Farrington).

tain in their interstices relatively large quantities of occluded gases, some of which are combustible. According to Farrington, some meteors

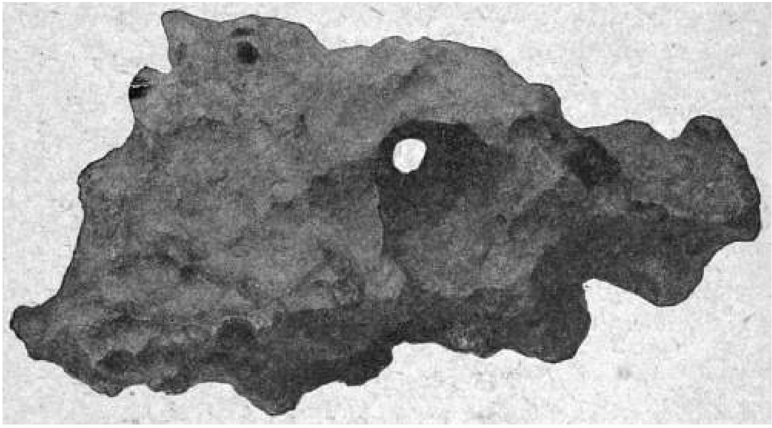

FIG. 134. — Iron meteorite from Cañon Diablo, Arizona; weight, 265 pounds (Farrington).

give evidence of fragmentation and recementation, others show faulting (fracture and sliding of one surface on another) with recementation, and others, veins where foreign material has been slowly deposited.

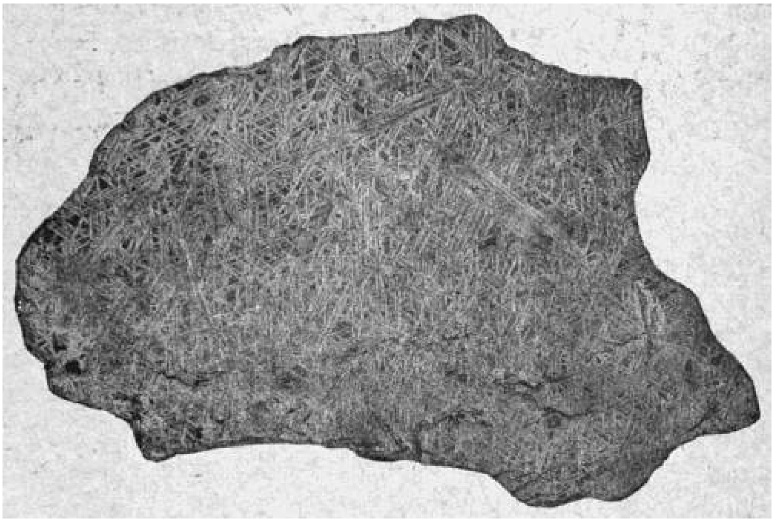

FIG. 135. — Durango, Mexico. Meteorite showing peculiar crystallization characteristic of certain meteorites (Farrington).

209. Theories respecting the Origin of Meteorites.—If it were known that meteorites are but meteors which are so large that they reach the earth before they are completely oxidized and pulverized, we might justly conclude that they are probably the remains of disintegrated comets. This would enable us to learn certain things about comets which cannot be settled yet. But no meteorite is known certainly to have been a member of any meteoric swarm. However, two meteorites have fallen during the time of meteoric showers, one in France, at the time of the Lyrids in 1905, and the other in Mexico, just before the Andromids in 1885.

The structure of some meteorites is more like that of lava from deep volcanoes than anything else found on the earth. An old theory was that they have been ejected by volcanic explosions from the moon, planets, or perhaps the sun. This theory would account for some of their characteristics, and would explain why they contain only familiar elements, at least if the other bodies of the solar system contain only those found on the earth; but it does not at all explain the fragmentation, faulting, and veins, for forces great enough to produce ejections would scarcely be found without heat enough to produce at least fusion.

Chamberlin has maintained that meteorites may be the débris of bodies, perhaps of planetary dimensions, which have been broken up by tidal strains when they have passed some larger mass within Roche's limit. When suns pass by other suns, it is probable that at rare intervals they pass so near each other that their planets (if they have any) are broken up. More rarely, the suns themselves may be disintegrated. Indeed, this may be the origin of all cometary and meteoric matter. Whether it is or not, there is here a possibility of disintegration which must be taken into account in any theory of cosmical evolution.

The present desiderata are more accurate determinations of comets' orbits to find whether any of them are really hyperbolic, more accurate determinations of the velocities of meteors to find whether they ever come into our system on parabolic or hyperbolic orbits, and finally the answer to the question whether meteors and meteorites are really related.

The suggestion that a meteorite may be a fragment of a world which was disrupted before the origin of the earth makes some demands on the imagination, but it seems no more incredible to us than seemed the suggestion to our predecessors a century ago that great mountains have been utterly destroyed by the rains and snows and winds.

XIV. QUESTIONS

1. What observations would prove that comets are not in the earth's atmosphere, as the ancients supposed they were?

2. Suppose two small masses are moving around the sun in the same elongated orbit, but that one is somewhat ahead of the other. How will their distance apart vary with their position in their orbit (use the law of areas)? Does this suggest an explanation of the variations in the dimensions of comets' heads?

3. The velocity of a comet moving in a parabolic orbit is inversely as the square root of its distance from the sun. At the distance of the earth a comet has a velocity of about 25 miles per second. What is the distance between the comets of 1843 and 1882 when they are $100,000$ astronomical units from the sun?

4. Suppose the particles of which a comet is composed have almost exactly the same perihelion point but somewhat different aphelion points. How would the dimensions of the comet vary with its position in its orbit?

5. By means of Kepler's third law compute the period of a comet whose aphelion point is at a distance of $140,000$ astronomical units, which is about half the distance of the nearest known star.

6. What objections are there to the theory that originally all comets had an aphelion distance equal to that of Neptune, and that the orbits of some have been increased and others diminished by the action of the planets?

7. On the repulsion theory should a comet's tail be equally long when it is approaching the sun and when it is receding?

8. Draw the diagram mentioned in the first paragraph of Art. 203.

9. Count the number of meteors you can observe in an hour on some clear, moonless night.

10. If possible, observe the Leonid or Andromid meteors.

11. Make a list of the fairly well-explained cometary phenomena, and of those for which no satisfactory theory exists.

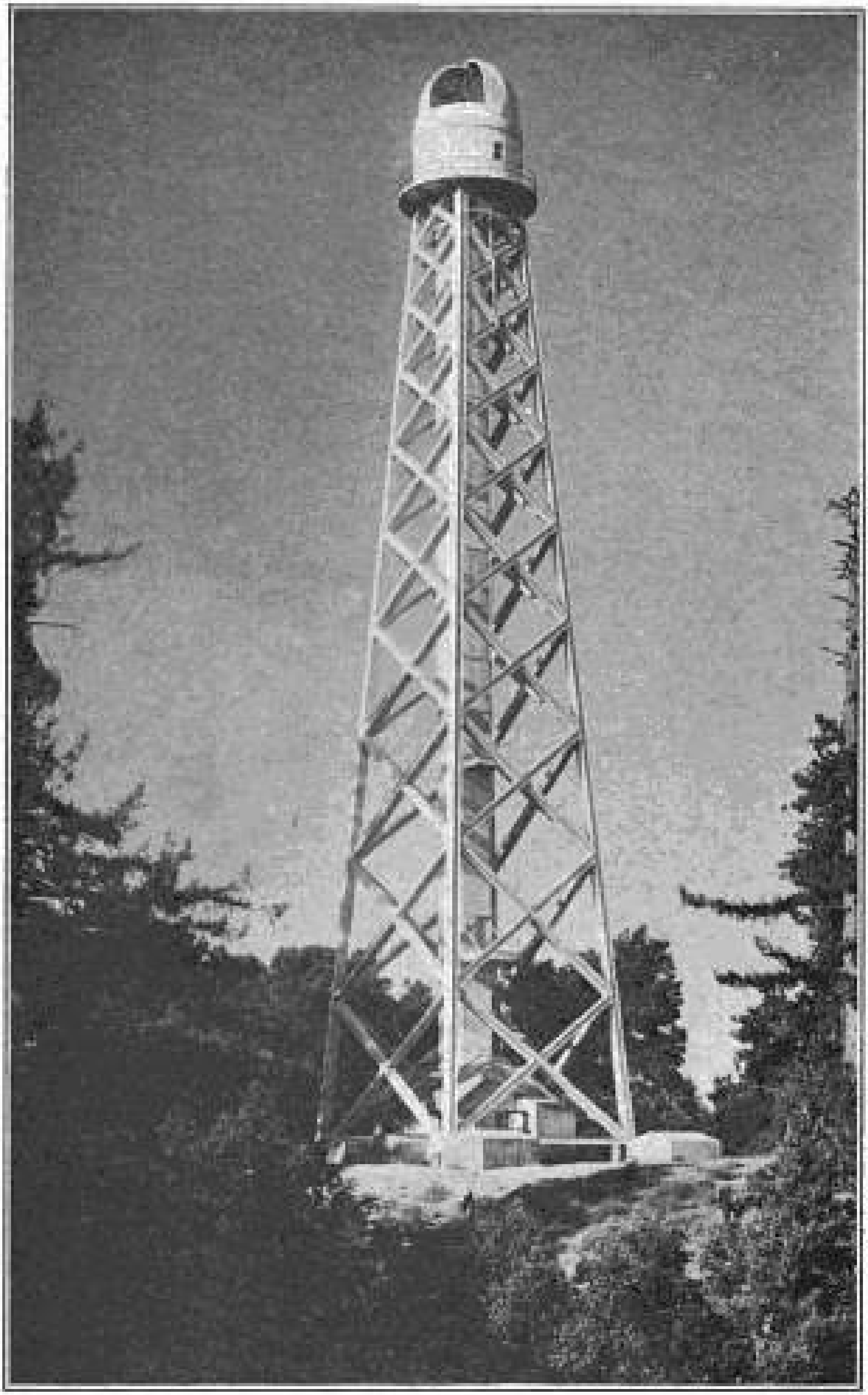

FIG. 136. — The tower telescope of the solar observatory of the Carnegie Institution of Washington, Pasadena, California.

CHAPTER XI
THE SUN

I. The Sun's Heat

210. The Problem of the Sun's Heat.—The light and heat radiated by the sun are essential for the existence of life on the earth, and consequently the question of the source of the sun's energy, how long it has been supplied, and how long it will last are of vital interest. Not only are these questions of importance because the sun is the dominant member of the solar system, governing the motions of the planets and illuminating and heating them with its abundant rays, but also because the sun is a star, and the only one of the hundreds of millions in the sky which is so near that its surface can be studied in detail.

Obviously the first thing to do in studying the heat of the sun is to measure the amount received from it by the earth; then, the amount which the sun radiates can be computed. The amount of heat given out by the sun gives the basis for determining its temperature. Then naturally follows the question of the origin of the sun's heat. The answers to these questions are of great importance in considering the the evolution of the solar system and the stars.

211. The Amount of radiant Energy received by the Earth from the Sun.—Light is a wave motion in the ether whose wave lengths vary from about $\frac{1}{65{,}000}$ of an inch, in the violet, to about $\frac{1}{40{,}000}$ of an inch, in the red. Radiant heat differs from light physically only in that its waves are longer. The circumstance that human eyes are sensitive to ether waves of certain lengths and not to those that are longer or shorter is, of course, of no importance in discussing the physical question of the sun's heat. Consequently, in the problem of solar radiation rays of all wave lengths are included, and together they constitute the radiant energy emitted by the sun.

Physicists have devised various methods of measuring the amount of energy received from a radiating source. In applying them to the problem of determining the amount of energy received from the sun the chief difficulty consists in making correct allowance for the absorption of light and heat by the earth's atmosphere. The best results have been obtained by making simultaneous measurements from near sea level, from the summits of lofty mountains, and from balloons. Langley measured the intensity of solar radiation at the top of Mount Whitney, 14, 887 feet above the sea, and at its base. He arrived at the conclusion that 40 per cent of the rays striking the atmosphere perpendicularly, when it is free from clouds, are absorbed before they reach the surface of the earth; later investigations have reduced this estimate to 35 per cent. The work initiated by Langley has been continued most successfully by Abbott, Fowle, and Aldrich, and they find that the rate at which radiant energy of all wave lengths is received by the earth from the sun at the outer surface of our atmosphere when the sun is at its mean distance is, in terms of mechanical work, 1.51 horse power per square yard.

The earth intercepts a cylinder of rays from the sun whose cross section is equal to a circle whose diameter equals the diameter of the earth. The area of this circle is, therefore, πr^2, where r equals $3955 \times 1760 = 6,960,800$ yards.[1] Hence the rate at which solar energy is intercepted by the whole earth is in round numbers 230, 000, 000, 000, 000 horse power.

In the evolution of life upon the earth the sun has been as important a factor as the earth itself. Consequently, geologists and biologists have a deep interest in the sun, and particularly in the question whether or not its rate of radiation is constant. It has long been supposed that probably the sun is slowly cooling off and that the light and heat received from it are gradually diminishing, but it was a distinct surprise when Langley and Abbott found that its rate of radiation sometimes varies in a few days by as much as 10 per cent. If a change of this amount in the rate of radiation of the sun were to persist indefinitely, the mean temperature of the earth would be changed about 13° Fahrenheit; but a variation of 10 per cent for only a few days has no important effect on the climate. Abbott, Fowle, and Aldrich have continued the investigation of this question, and by making observations simultaneously in Algiers, in Washington, and in California, so as to eliminate the effects of local and transitory atmospheric conditions, they have firmly established the reality of small and rapid variations in the sun's rate of radiation.

The question of variation in the amount of energy received from the sun can also be considered in the light of geological evidence. The fossils preserved in the rocks of all geological ages prove that there has been an unbroken life chain upon the earth for many tens of millions of years. This means that during all this vast period of time the temperature of the earth has been neither so high nor so low as to destroy all

[1] The mean radius of the earth is 3955 miles and there are 1760 yards in a mile.

life. Moreover, the record is clear that, in spite of glacial epochs and intervening warmer eras, the temperature changes have not been very great, and there is no evidence of a progressive cooling of the sun.

212. Sources of the Energy used by Man.—One of the earliest extensive sources of energy for mechanical work used by man was the wind. It has turned, and still turns, millions of windmills for driving machinery or pumping water. Until the last few decades it moved nearly all of the ocean-borne commerce of the whole world, and it is still an important factor in shipping. But that part of the energy of the wind which is used is an insignificant fraction of all that exists. For example, if, in a breeze blowing at the rate of 20 miles an hour, all the energy in the air crossing an area 100 feet square perpendicular to its direction of motion were used, it would do about 560 horse power of work.

What is the origin of the energy in the wind? The sun warms the atmosphere over the equatorial regions of the earth more than that over the higher latitudes, and the resulting convection currents constitute the wind. Consequently, all the energy in every wind that blows came originally from the sun.

Another source of energy which has been of great practical value is water power. The source of this energy is also the sun, because the sun's heat evaporates the water and raises it into the air a half mile or more, the winds carry part of it out over the land, where it falls as rain or snow, and in descending again to the ocean it may now and then plunge over a precipice, where its energy can be utilized by men. Amazing as are the figures for such great waterfalls as Niagara, they give but a faint idea of the enormous work the sun has done in raising water into the sky, and the equally great amount of work the water does in falling back to the earth. During a heavy rain an inch of water may fall. An inch of water on a square mile weighs over 60,000 tons. In the eastern half of the United States, where the annual rainfall is about 35 inches, every year over 2,000,000 tons of water fall on each square mile from a height of half a mile or more.

The great modern source of energy for mechanical work is coal. The coal has formed from vegetable matter which accumulated in peat beds ages and ages ago. Consequently, the immediate source of its energy is the plants out of which it has developed. But the plants obtained their energy from the sun. In millions of tiny cells the sun's energy broke up the carbon dioxide which they inhaled from the atmosphere; then the oxygen was exhaled and the carbon was stored up in their tissues. When a plant is burned, as much energy is developed and given up again as the sun put into it when it grew.

Thus it is seen that all the great sources of energy can be traced back to the sun; it is true of the minor ones also. One naturally inquires whether these sources of energy are perpetual. The winds will certainly continue to blow and the rains to descend as long as the earth and sun exist in their present conditions, but the coal and petroleum will eventually be exhausted. They will last several centuries and perhaps a few thousand years. This period seems long compared to the lifetime of an individual, or perhaps of a nation, but it is only a minute fraction of the time during which our successors will probably occupy the earth. It follows that they will be compelled to depend upon sources of energy at present but little utilized. Perhaps some great benefactor of mankind will discover a means of putting to direct use the enormous quantities of energy which the sun is now sending to the earth. At present we are depending on that infinitesimal residue of the energy which the earth received in earlier geological times and which has been stored up and preserved in petroleum and coal.

213. The Amount of Energy radiated by the Sun.—The earth as seen from the sun subtends an angle of only $17''.6$. That is, its apparent area is about $\frac{1}{15}$ the greatest apparent area of Venus as seen from the earth. A glance at Venus will show that this is an exceedingly small part of the whole celestial sphere. Since the little earth at a distance of $93,000,000$ of miles receives the enormous quantity of heat given in Art. 211, it follows that the amount which is radiated by the sun must be inconceivable. It can be brought within the range of our understanding only by contemplating some of the things it might do.

The energy radiated per square yard from the sun's surface is equivalent to $70,000$ horse power. This amount of heat energy would melt a layer of ice 2200 feet thick every hour all over the surface of the sun; and it would melt a globe of ice as large as the earth in 2 hours and 40 minutes. Less than one two-billionth of the energy poured forth by the sun is intercepted by the earth, and less than ten times this amount by all the planets together; the remainder travels on through the ether to the regions of the stars at the rate of $186,000$ miles per second.

214. The Temperature of the Sun.—Stefan's law (Art. 172) that a black body radiates as the fourth power of its absolute temperature, gives a basis for determining the temperature of a body whose rate of radiation is known. While the sun is probably not an ideal radiator, such as is contemplated in the statement of Stefan's law, and while it radiates from layers at various depths below its surface, with the upper layers absorbing part of the energy coming from the lower, yet an approximate idea of the temperature of its radiating layers can be obtained from its rate of radiation. On using Stefan's law as a basis for computation, it is found that the temperature of the radiating layers of the sun is at least $10,000°$ Fahrenheit. Or, it would be more accurate to say that an ideal radiating surface at this temperature would have the same rate of radiation as the sun, and since the sun is not a perfect radiator, its temperature is probably still higher. This temperature is several thousand degrees higher than has been obtained in the most efficient electrical furnaces, and is far beyond that required to melt or vaporize any known terrestrial substance; yet, the temperature of the interior of the sun is undoubtedly far higher.

Another method of determining the temperature of the sun is from the proportion of energy of different wave lengths which it radiates. A body of low temperature radiates relatively a large amount of red light and a small amount of blue light. As the temperature rises the relative proportion of blue light increases. The uncertainties in the results obtained by this method of determining the temperature of the sun arise, in the first place, from the fact that, at the best, it is not very precise, and, in the second place, from the fact that both the sun's and the earth's atmospheres absorb very unequally radiant energy of various wave lengths. After making the necessary allowances for the absorption, the results obtained by this method confirm those found by the other.

There have been a number of other methods of obtaining the temperature of the sun from the time of Newton, but most of them have rested on physical principles which are unsound, and in some cases they have led to most extravagant results.

215. The Principle of the Conservation of Energy.—Before taking up the question of the origin of the sun's heat, it is advisable to consider the principle of the conservation of energy. It is comparable in importance and generality to the principle of the conservation (indestructibility) of matter. It was once supposed that when inflammable material, as wood, is burned, it is utterly annihilated. But it has been known for about 150 years that if the ashes, the smoke, and the gases produced by the combustion were all gathered up and weighed in a vacuum, their weight would exactly equal that of the original wood together with the oxygen which united with it in burning.

Similarly, it was supposed until after 1840 that energy might be destroyed as well as transformed. For example, it was supposed that the energy lost by friction ceased to exist. But it had been noted that friction produced heat, and heat was known to be equivalent to mechanical energy, for it had been turned into work, for example, by means of the steam engine. It does not seem now to have been a large step to have conjectured that the heat produced by the friction is exactly equivalent to the energy lost. But many elaborate experiments were required (made mostly by Mayer and Joule) to prove the correctness of this conjecture and to lead to the generalization, now universally accepted, that *the total amount of energy in the universe is always the same*. This is one of the most far-reaching principles of science, and, like the law of gravitation, is involved in every phenomenon in which there is motion of matter.

The energy of a body as used in the principle of the conservation of energy means both its energy of motion (kinetic energy) and also its energy of position, or the power it may have of doing work because of its position (potential energy). It is the sum of the potential and kinetic energies of the universe which is constant. Since energy may be in a radiant form and in transit from one body to another, or from a body out into endless space, the principle holds only when the energy which is in the ether is also included.

216. The Contraction Theory of the Sun's Heat.—The mutual attractions of the particles of which the sun is composed tend to cause it to contract. A contraction of the sun would be equivalent to a fall of all of its particles toward its center. If they should fall the whole distance one at a time, they would generate a certain amount of heat upon their impacts. If they should fall simultaneously, first a fraction of the distance and then another, the same total amount of heat would be generated. It might be supposed without computation that an enormous contraction would be necessary in order to produce enough heat to change appreciably the temperature of the sun.

The effect of the sun's contraction can be considered more exactly in terms of energy. The sun in an expanded condition would have more potential energy with respect to the force of gravitation than if it were contracted, because work would be done on it by gravitation in changing it from the first state to the second. Therefore the kinetic energy, or temperature, of the sun must rise on its contraction. It is analogous to a falling body. The higher it is above the surface of the earth, the greater its potential energy; the farther it falls, the more potential energy it loses and the more kinetic energy it acquires.

The problem is to determine whether the contraction of the sun might not supply it with heat to take the place of that which it radiates so lavishly. With the insight of genius, Helmholtz saw the nature of the question and foresaw its probable answer. In 1854, at a celebration in commemoration of the philosopher Kant, he gave a solution of the problem under the assumption that the sun contracts in such a way as to remain always homogeneous. With our present data regarding its rate of radiation, its volume, and its mass, it is found by the methods of Helmholtz that, under the assumption that it is homogeneous and remains homogeneous during its shrinking, a contraction of its radius of 120 feet per year would produce as much heat as it radiates annually. This contraction is so small that it could not be detected from the distance of the earth with our most powerful telescopes in less than 10,000 years.

So far in this discussion it has been assumed that the sun contracts, and the consequences of the contraction have been deduced. It remains to consider the question whether under the conditions which prevail it actually does contract. The reason it does not at once shrink under the mutual gravitation of its parts is that its high temperature gives it a great tendency to expand. As it radiates energy into space its temperature doubtless falls a little; the decrease in temperature permits it to contract a little; the contraction produces heat which momentarily restores the equilibrium; and so on in an endless cycle. This conclusion is certainly correct, as Ritter and Lane proved about 1870, provided the sun behaves as a monatomic gaseous body. Moreover, Lane established the fact, known as Lane's paradox, that so long as a purely gaseous body cools and contracts, its temperature rises, because, with decreasing volume and greater concentration of matter, the gravitational forces can withstand stronger expansive tendencies due to high temperature. If, with increasing concentration, the laws of gases fail because the deep interior becomes liquid or solid, the temperature might no longer increase.

The question of the variation in the rate of radiation of a contracting sun with increasing age is an important one. Lane showed that, so long as the sun obeys the law of gases, its temperature is inversely as its radius. By Stefan's law the rate of radiation is proportional to the fourth power of the absolute temperature. Consequently the rate of radiation, per unit area, of a contracting gaseous sphere is inversely as the fourth power of its radius. But the whole radiating surface is proportional to the square of the radius. Therefore the rate of radiation of the entire surface of a contracting gaseous sphere is inversely as the square of its radius. That is, according to this theory, the earth received continually more and more heat until the sun ceased to be perfectly gaseous, if, indeed, it has yet reached that stage. When the sun's radius was twice as great as it is at present it gave the earth one fourth as much heat, and the theoretical temperature of the earth (Art. 172) was about 200 degrees lower than at present.

217. Other Theories of the Sun's Heat.—A number of other hypotheses as to the source of the sun's energy have been advanced, but they are all inadequate. They will be enumerated here in order that the reader may not suppose that they are important, and that astronomers have failed to consider them.

The most obvious suggestion is that the sun started hot and is simply cooling. If it had the very high specific heat of water, at its present rate of radiation its mean temperature would fall 2.57 degrees annually. On referring to its present temperature, it is seen that its radiation could not continue more than a few thousand years, and that a few thousand years ago its rate of radiation must have been several times that at present. These results are absurd and show the falsity of the suggestion.

It is natural to associate heat with something burning, and one naturally inquires whether the heat of the sun cannot be accounted for by the combustion of the material of which it is composed. In considering this hypothesis the first thing to be noted is that the same material will burn only once. It is found from the amount of heat produced by coal that if the sun were entirely made up of the best anthracite coal and oxygen in such proportion that when the combustion was completed there would be no residue of either, the heat generated would supply the present rate of radiation less than 1500 years. If none of the heat produced by the combustion were radiated away, and if the specific heat of the sun were unity, the temperature of the sun would rise to only about one third of its present value. Consequently this theory is even less satisfactory than the preceding.

Shortly after the discovery of the law of the conservation of energy the large amount of heat generated by the impact of meteors was established. The heat generated by a meteor striking into the earth's atmosphere at the average rate of 25 miles per second is about 100 times as great as would be produced by its combustion if it were oxygen and anthracite coal. A meteor would fall into the sun from the distance of the earth with a velocity of about 380 miles per second, and since the energy is proportional to the square of the velocity, the heat generated would be about 23,000 times that produced by the combustion of an equal amount of carbon and oxygen. Lord Kelvin supposed that possibly enough meteors strike into the sun to replenish the energy it loses by radiation.

A complete answer to the meteoric theory of the sun's heat is that it requires an impossibly large total mass for the meteors. They could not possibly exist in sufficient numbers within the earth's orbit; and, if they came from without, they would strike the earth in enormously greater numbers than are observed. In fact, computation shows that if the heat of the sun were due to meteors coming into it from all directions and from beyond the earth's orbit, the earth would receive $\frac{1}{236}$ as much heat directly from the meteors as it receives from the sun. This is millions of times more heat than the earth receives from meteors, and, consequently, the theory that the sun's heat is maintained by the impact of meteors is untenable.

218. The Past and the Future of the Sun on the Basis of the Contraction Theory.—The contraction theory of the sun's heat is the only one of those considered which even begins to satisfy the conditions a successful theory must meet. If it is the only important source of the sun's heat, it is possible to determine, at least roughly, how long the sun can have been radiating at its present rate, and how long it can continue to radiate in the future.

Computation shows that if the sun had contracted from infinite expansion, the widest possible dispersion, the total amount of heat generated would have been less than 20,000,000 times the amount now radiated annually. If it had contracted only from the distance of the earth's orbit, the amount of heat that would have been generated would have been about one half of one per cent less. Therefore, according to the contraction theory, the earth can have received heat from the sun at its present rate only about 20,000,000 years. If the sun is strongly condensed at its center, this time limit should be increased about 5,000,000 years.

In the future, according to this theory, the sun will contract more and more until it ceases to be gaseous. Probably by the time its mean density equals 5 its temperature will begin to fall. A contraction to this density will produce enough heat to supply the present rate of radiation only 10,000,000 years. Then, *if the sun's contraction is the only important source of its energy*, its temperature will begin to fall, its rate of radiation will diminish, the temperature of the earth will gradually decline, and all life on the earth will eventually become extinct. The sun, a dead and invisible mass, will speed on through space with its retinue of lifeless planets.

219. The Age of the Earth.—After the development of the contraction theory of the sun's heat, physicists, among whom Lord Kelvin was especially prominent, informed the geologists and biologists in rather arbitrary terms that the earth was not more than 25,000,000 years of age, and that all the great series of changes with which their sciences had made them familiar must have taken place within this time. But no one science or theory should be placed above all others, and other lines of evidence as to the age of the earth are entitled to a full hearing. If they should unmistakably agree that the earth is much more than 25,000,000 years of age, the inevitable conclusion would be that the contraction theory is not the whole truth. This is a matter of the greatest importance, for not only is it at the foundation of the interpretation of geological and biological evolution, but it bears vitally on the question of the age of the stars and on the past and the future of the sidereal universe.

One of the simplest methods employed by geologists for determining the age of the earth is that of computing the time necessary for the oceans to acquire their salinity. The rivers that flow into the oceans carry to them various kinds of salts in solution; the water that is evaporated from them leaves these minerals behind. Consequently the salinity of the oceans continually increases. It is clear that it is possible to compute the age of the oceans from the present amount of salt in them and the rate at which it is being carried into them. Of course, it is necessary to make some assumptions regarding the rate at which salt was carried to the sea in earlier geological ages. The last factor is somewhat uncertain, but this method has led to the conclusion that the interval which has elapsed since the oceans were formed and salt began to be carried down into them is more than 60,000,000 years, and that it is probably from 90,000,000 to 140,000,000 years.

Nearly all the rocks that are exposed on the surface of the earth are stratified. This means that, on the whole, they have been formed from silt carried by the wind and water and deposited on the bottoms of lakes or oceans. These stratified deposits are in many places of enormous thickness. When it is remembered that the present rocks are usually not the result of the simple disintegration and deposition of the original earth material, but that most of them have been repeatedly broken up and redeposited, it is evident that the time required for the great stratification which is now observed is enormous. There is obviously much chance for divergence of views regarding the rates at which these processes have gone on, but nearly every calculation on this basis has led to the conclusion that the time since the disintegration and stratification of the earth's rocks began is at least 100,000,000 years, and most of them have reached much larger figures. The disintegration and total destruction of mountains and plateaus is a closely related process and leads to the same results.

The rocks of the earliest geological formations contain only a few fossils, and they are of primitive forms of life. Later rocks contain the remains of higher forms of plants and animals, until finally the vertebrates and the highest types existing at the present time are found. Obviously an enormous interval of time has been required for all this great series of changes in life forms to have taken place, but it is difficult to make a numerical estimate. Huxley gave the question much attention and thought a billion years would be necessary for the evolution. The recent discovery of mutation has shown that the process of evolution, at least in plants, may be more rapid than he supposed; but, on the whole, biologists feel that the contraction theory of the sun's heat sets much too restricted limits for the age of the earth.

The most recent, and possibly the best, method of arriving at the age of the earth has followed the discovery of radioactive substances. Uranium degenerates by a slow breaking up of its atoms in which radium, lead, and helium are evolved. From the relative proportions of these products in certain rocks it is possible to compute the time during which degeneration has been going on in them. This method has led to a greater age for the earth than any other. Strutt, in England, Boltwood, of Yale, and many others have given this method a large amount of study, and have obtained figures reaching up into several hundreds of millions of years. Boltwood, especially, has found that the geologically older rocks show greater antiquity by this method of determining their age, and he reaches the conclusion that some of them are nearly 2,000,000,000 years old.

It is difficult to reach a positive conclusion regarding the age of the earth from this conflicting evidence. The geological methods point to an age for the earth since erosion began of at least 100,000,000 years. Geologists do not see how the facts in any of their lines of attacking the problem can be brought into harmony with the theory that the sun has been furnishing light and heat to the earth for only 25,000,000 years. This discrepancy between their figures and those given by the contraction theory cannot be ignored, and therefore we are forced to the conclusion that the sun has other important sources of heat energy besides its contraction. Aside from this, the fact that a contracting gaseous mass radiates inversely as the square of its radius gives a distribution of the radiation of solar energy altogether at variance with geological evidence.

A possible source of energy for the sun which has not been considered here as yet is that liberated in the degeneration of radioactive elements. It is not certain that uranium and radium exist in the sun, but helium, which is one of the products of the disintegration of these elements, exists there in abundance; in fact, it is called helium because it was first discovered in the sun (Greek, *helios* = sun), and gives presumptive evidence of uranium and radium being there, too. The disintegration of uranium and radium is accompanied by the evolution of an enormous quantity of heat, the energy liberated by radium being about $260,000$ times that produced by the combustion of an equal weight of coal and oxygen. These results are startling, and at first it seems that if a small fraction of the sun were radium or uranium, its radiation of energy would be almost indefinitely prolonged.

If one part in $800,000$ of the sun were radium, heat would be produced from this source alone as fast as it is now being radiated, but in less than 2000 years half of the radium would be gone and the production of heat would correspondingly diminish. Or, to go backward in time, only 2000 years ago the amount of radium would have been twice as great as at present, and the production of heat would have been twice as rapid. Since this conclusion is not in harmony with the facts, the hypothesis that the sun's heat is largely due to the disintegration of radium is untenable.

Now consider uranium, which degenerates $3,000,000$ times more slowly than radium. In the case of this element the slowness of the rate of degeneration presents a difficulty. If the sun were entirely uranium, heat would not be produced more than one third as fast as it is now being radiated. But in the deep interior of the sun where the temperature and pressure are inconceivably high, the release of the subatomic energies may possibly be much more rapid than under laboratory conditions, and the process may not be confined to the elements which are radioactive at the surface of the earth. There is no laboratory experience to support this suggestion because within the range of experiment the rates of the radioactive processes have been found to be independent of temperature and other physical conditions. But, if there is something in the suggestion, and especially if under the conditions prevailing in the sun the subatomic energies of all elements are released, the amount of energy may be sufficient for hundreds and even thousands of millions of years. But at once the question regarding the origin of the subatomic energies arises, and, at present, there is no answer to it.

XV. QUESTIONS

1. How many horse power of energy per inhabitant is received by the earth from the sun?

2. What is the average amount of energy per square yard received by the whole earth from the sun?

3. Does the energy which is manifested in the tides come from the sun? What becomes of the energy in the tides?

4. What becomes of that part of the sun's energy which is absorbed by the earth's atmosphere?

5. If the earth's atmosphere absorbs 35 per cent of the energy which comes to it from the sun, how can the atmosphere cause the temperature of the earth's surface to be higher than it would otherwise be?

6. Show from the rate at which the earth receives energy from the sun, the size of the sun, and the earth's distance from the sun, that the sun radiates 70,000 horse power of energy per square yard.

7. Taking the earth's mean temperature as 60° F. and the rates of radiation of the earth (see question 2) and of the sun, compute the temperature of the sun on the basis of Stefan's law.

8. All scientists agree that the earth is more than 5,000,000 years old. On the hypothesis that the contraction of the sun is its only source of heat, and that during the last 5,000,000 years it has radiated at its present rate, what were its radius and density at the beginning of this period? On the basis of Lane's law, what was its temperature? On the basis of Stefan's law, what was its rate of radiation per unit area and as a whole? On the basis of the method of Art. 172, what was the mean temperature of the earth?

II. Spectrum Analysis

220. The Nature of Light.—In order to comprehend the principles of spectrum analysis it is necessary to understand the nature of light. A profound study of the fundamental properties of light was begun by Newton, but, unfortunately, some of his basal conclusions were quite erroneous. Thomas Young (1773–1829) laid the foundation of the modern undulatory theory of light. That is, he established the fact that light consists of waves in an all-pervading medium known as the *ether*, by showing that when two similar rays of light meet they destroy each other where their phases are different, and add where their phases are the same. These phenomena, which are analogous to those exhibited by waves in water, would not be observed if Newton's idea were correct that light consisted of minute particles shot out from a radiating body.

Physical experiments prove that light waves in the ether are at right angles to the line of their propagation, like the up-and-down waves which travel along a steel beam when it is struck with a hammer, or the torsional waves that are transmitted along a solid elastic body when one of its ends is suddenly twisted. In an ordinary beam of light the vibrations are in every direction perpendicular to the line of propagation. If the vibrations in one direction are destroyed while those at right angles to it remain, the light is said to be *polarized*. Many substances have the property of polarizing light which passes through them.

The distance from one wave to the next for red light is about $\frac{1}{40{,}000}$ of an inch, and for violet light about $\frac{1}{70{,}000}$ of an inch. There are vibrations both of smaller and greater wave lengths. The range beyond the violet[1] is not very great, for, even though very short waves are emitted by a body, they are absorbed and scattered by the earth's atmosphere before reaching the observer; but there is no limit in the other direction to the lengths of rays. Langley explored the so-called heat rays of the sun with his bolometer far beyond those which are visible to the human eye. The waves used in wireless telegraphy, which differ from light waves only in their length, are often hundreds of yards long.

221. On the Production of Light.—A definite conception of the way in which matter emits radiant energy is important for an understanding of the principles of spectrum analysis, but, unfortunately, the fundamental properties of matter are involved, and physicists are not yet in agreement on the subject. However, the theory that radiant energy is due to accelerated electrons is in good standing and gives a correct representation of the principal facts.

The molecules of which substances are composed are themselves made up of atoms. The atoms were generally supposed to be indivisible until the year 1895, when the cathode and X-rays prepared the way for the recent discoveries in radioactivity and subatomic units. In connection with these discoveries it was found that the atoms are made up of numerous still smaller particles, called *electrons* or *corpuscles*. An atom, according to the hypothesis of Rutherford, is composed of a

[1]Excepting the so-called X-rays, which are much shorter.

small central nucleus, carrying a positive charge of electricity, and one or more rings of electrons carrying (or perhaps consisting of) negative

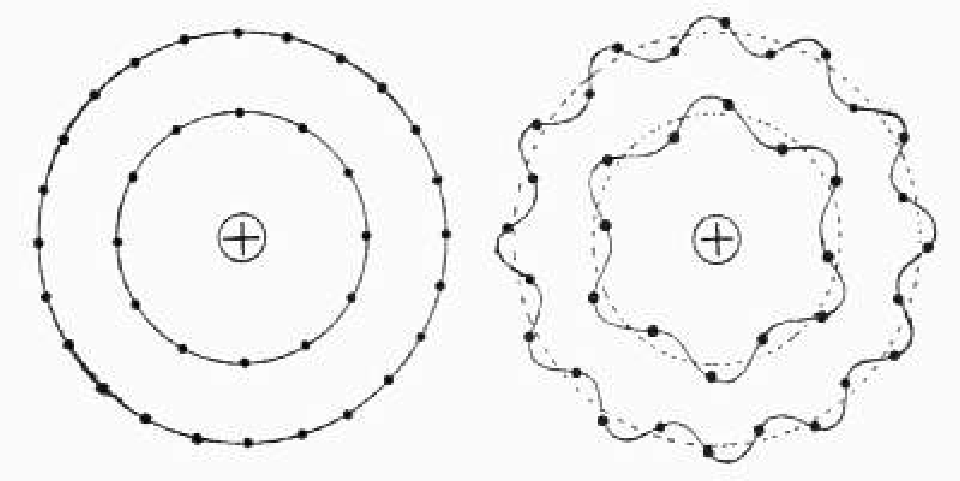

FIG. 137.—Model of atom, non-radiating at left and radiating at right.

charges of electricity, which revolve around the positive nucleus at great speed. Under ordinary circumstances the electrons revolve in circular paths with uniform speed, all those of a given ring traveling in the same circle. Under these circumstances, represented in the left of Fig. 137, the atom is not radiating.

When a body is highly heated the molecules and atoms of which it is composed are in very rapid motion and jostle against one another with great frequency. These impacts disturb the motions of the electrons and cause them to describe wavy paths in and out across the circles in which they ordinarily move. This condition is shown at the right in Fig. 137. These small vibrations, which are periodic in character, produce light waves in the ether; and light waves are also produced by the impacts themselves, but they are not periodic.

The character of the motions of the corpuscles can be understood by considering a bell. Suppose it is suspended by a twisted cord which is rapidly untwisting. A ring of particles around the bell corresponds to a ring of corpuscles in an atom. If the bell is simply rotating, it gives out no sound. Suppose it strikes something. The particles of which it is composed vibrate rapidly in and out; this, combined with its rotation, causes them to describe wavy paths across their former circular orbits. These waves produce the sound. Of course, it is not necessary that the bell should be rotating in order to produce sound, and in this respect the analogy is imperfect.

The frequency of the vibrations of a corpuscle in an atom is astounding. The length of a light wave of yellow light is in round numbers $\frac{1}{50,000}$ of an inch. In a second of time enough waves are emitted to make a line of them $186,000$ miles along. Therefore, the number of oscillations per second of the corpuscles in an atom is in round numbers $600,000,000,000,000$.

It has often been suggested that the atoms of all the chemical elements are made out of exactly the same kind of electrons. Certainly there is as yet no evidence to the contrary. If the electrons are not composite structures themselves, the idea is reasonable enough; but if they are made up of still smaller units, the hypothesis seems improbable.

The dynamics of an atom, according to the corpuscular theory, is of much interest. The positive nucleus attracts the revolving negative corpuscles. They are kept from falling in on the nucleus both by the centrifugal force due to their rapid revolution, and also by their mutual repulsions which result from their being similarly electrified. If the number of corpuscles in a ring is small, the atom is stable. With an increasing number of corpuscles the stability of the atom diminishes. Finally, the atom is stable only if the corpuscles revolve in two or more rings. The regions of instability which separate atoms having a certain number of rings from those having other numbers possibly give a clue to the celebrated periodic law of the chemical elements discovered by Mendeléeff.

222. Spectroscopes and the Spectrum.—The energy which a body radiates is completely characterized by the wave lengths which it includes and their respective intensities. The spectroscope is an instrument which enables us to analyze light into its parts of different wave lengths, and to study each one separately.

There are three principal types of spectroscopes. In the first and oldest type the light passes through one or more prisms; in the second, perfected by Rowland and Michelson, the light is reflected from a surface on which are ruled many parallel equidistant lines; and in the third, invented by Michelson, the light passes through a pile of equally thick plane pieces of glass piled up like a stairway. The first type is most advantageous when the source of light is faint, like a small star, comet, or nebula. Its chief fault is that the scale of the spectrum is not the same in all parts. The second type is advantageous for bright sources of light like the sun or the electric arc in the laboratory. It gives the same scale for all parts of the spectrum, but uses only a small part of the incident light. The third type, known as the *echelon*, gives high dispersion without great loss of light. Only the first type, which is most used in astronomy, will be more fully described here.

The basis of the prism spectroscope is the refraction and the dispersion of light when it passes through a prism. Let L, Fig. 138, represent a beam of white light which passes through the prism P. As it enters at A from a rarer to a denser medium, it is bent *toward* the perpendicular to the surface; and as it emerges at B from a denser to a rarer medium, it is bent *from* the perpendicular to the surface. This change in the direction of the beam of light is its *refraction*.

Not only is the beam of light refracted, but it is also spread out into its colors. As it enters the prism the violet light is refracted the most and the red the least, and the same thing is true when it emerges. Consequently, instead of a beam of white light falling on the screen S there is found a band of colors which, in order from the most refracted to the least refracted, are violet, indigo, blue, green, yellow, orange, and red. This separation of light into its colors is called *dispersion*.

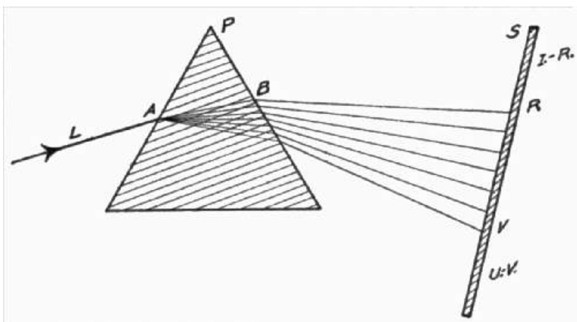

FIG. 138.—Refraction and dispersion of light by a prism.

In the diagram only the visible part of the spectrum is indicated. Beyond the red are the infra-red, or heat, rays *I-R*, and beyond the violet are the ultra-violet rays *U-V*. The colors are not separated by sharp boundaries, but shade from one to another by insensible gradations. The ultra-violet part of the spectrum is several times as long as the visible part, and the infra-red part is several times as long as the ultra-violet part.

While Fig. 138 shows exactly the way in which a spectrum might be formed, it would be too faint to be of any value in practice. In order to obtain a bright spectrum the apparatus is arranged as sketched in Fig. 139, though in practice several prisms, one after the other, are often employed. The rays which pass through the screen at O are made parallel by the lens L_1. They strike the prism P in parallel lines, and those of a given color continue through P and to the lens L_2 in parallel lines (the dispersion is not indicated in the diagram). The lens L_2 brings the rays to a focus at F, and the eyepiece E sends all those of each color out in a small bundle of parallel lines (only one color is represented in the diagram). The eye is placed just to the right of E, and all the parallel rays of each bundle are brought to a focus at a point on the retina. In this way many rays of each color are brought to a focus at the same place in the observer's eye.

While strictly white light gives all colors, it is not necessary that a luminous body should emit all kinds of light, or that all colors emitted should be given out in equal intensity. In fact, it is well known that if

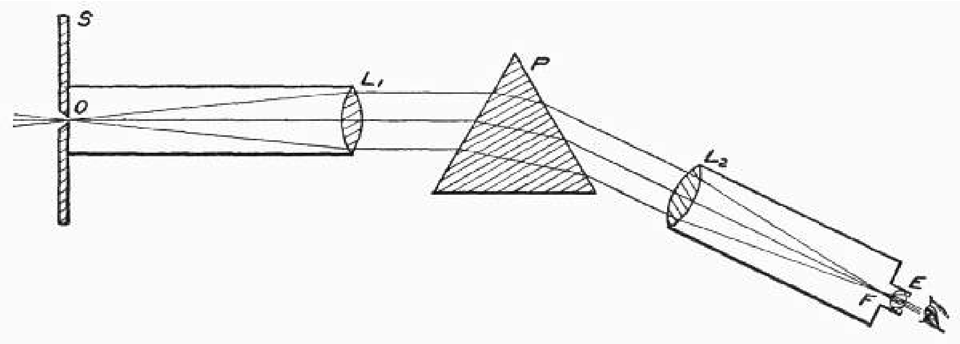

FIG. 139. — A spectroscope having only one prism.

a body is simply warm but not self-luminous, it gives out in sensible quantities only infra-red rays. If it is extremely hot, it may radiate mostly ultra-violet rays.

223. The First Law of Spectrum Analysis.—The first theoretical discussion of the principles of spectrum analysis which reached approximately correct conclusions was made by Ångström in 1853. The work of Bunsen, and especially of Kirchhoff in 1859, put the subject on essentially its present basis. The laws of spectrum analysis as formulated here are consequences of a general law due to Kirchhoff, and of certain experimental facts. After they have been stated, they will be seen to be simple consequences of the mode of production of radiant energy.

The first law of spectrum analysis is: *A radiating solid, liquid, or gas under high pressure gives a continuous spectrum whose position of maximum intensity depends upon the temperature of the source; and conversely, if a spectrum is continuous, the source of light is a solid,* *liquid, or gas under high pressure, and the position of radiation of maximum intensity determines the temperature of the source.*

This law means, in the first place, that a radiating solid, liquid, or gas under high pressure gives out light, or more generally radiant energy, of all wave lengths; and, in the second place, the wave length at which the radiation is most intense depends upon the temperature of the source. It is clear from the way in which light is produced that the first part of the law should be true. When a body is in a solid or liquid state, or when it is a gas under high pressure, the molecules are so close together that they continually interfere with one another. Under these circumstances the oscillations of the corpuscles cannot take place in their natural periods, but they are altered in all possible manners. This results in vibrations of all periods, and therefore the spectra are continuous.

The way in which the wave length of maximum radiation depends upon the temperature is given by Wien's law[1]—

$$\lambda = \frac{0.2076}{T},$$

where λ is the wave length in inches and T is the absolute temperature on the Fahrenheit scale. For example, if the temperature of the sun is $10,000°$, its wave length of maximum radiation is about $\frac{1}{50{,}000}$ of an inch.

224. The Second Law of Spectrum Analysis.—The second law of spectrum analysis is: *A radiating gas under low pressure gives a spectrum which consists of bright lines whose relations to one another and whose positions in the spectrum*[2] *depend upon the nature of the gas (and in some cases to some extent upon its temperature, density, electrical and magnetic condition); and conversely, if a spectrum consists of bright lines, then the source is a radiating gas (or gases) under low pressure, and the composition of the gas (or gases) can be determined from the relations of the lines to one another and from their positions in the spectrum.*

When molecules are free from all restraints the oscillations of their electrons take place in fixed periods which depend upon the internal

[1]Experiments show that this law does not give good results for low temperatures, but the applications in astronomy are to high temperatures.

[2]The positions of lines in a spectrum determine, of course, their relations to one another; but in practice the lines of an element are usually identified by their relations to one another, just as a constellation is recognized by the relative positions of its stars.

forces involved, just as free bells of given structure vibrate in definite ways and give forth sounds of definite pitch. Consequently, free radiating molecules emit light of one or more definite wave lengths depending on the structure of the molecules, and there are bright lines at corre-

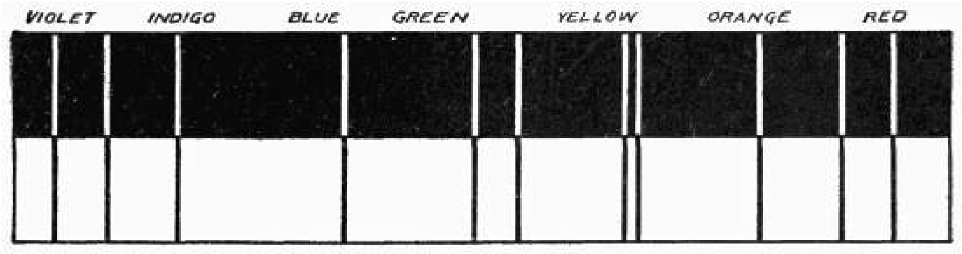

FIG. 140.—A bright-line spectrum above and a reversed spectrum below.

sponding places in the spectrum and no light whatever at other places. A bright-line spectrum is shown in the top part of Fig. 140. Some elements give only a few lines and others a great many. For example, sodium has but two lines, both in the yellow, and iron more than 2000 lines. It is needless to say that all these facts are established by laboratory experiments.

It may be objected that in a gas, even under low pressure, the molecules are not free from outside interference, for they collide with one another many millions of times per second. But the intervals during which they are in collision are very short compared with the intervals between collisions. Consequently, while there will be some light of all wave lengths, it will be inappreciable compared to that which is characteristic of the radiating gas, and the spectrum will seem to consist of bright lines of various colors on a perfectly black background.

225. The Third Law of Spectrum Analysis.—The third law of spectrum analysis is: *If light from a solid, liquid, or gas under great pressure passes through a cooler gas (or gases), then the result is a bright spectrum which is continuous except where it is crossed by dark lines, and the dark lines have the positions which would be occupied by bright lines if the intervening cooler gas were the source of light, and conversely, if a bright spectrum is continuous except where it is crossed by dark lines, then the source of light is a solid, liquid, or gas under great pressure, and the light has passed through a cooler intervening gas (or gases) whose constitution can be determined from the relations of the dark lines to one another and from their positions in the spectrum.*

In a word, a cool gas absorbs the same kinds of rays it would give out if it were incandescent, and no others. Similarly, a musical instrument absorbs tones of the same pitch as those which it can produce. For example, if the key for middle C on a piano is held down and this tone is produced near by, the piano will respond with the same tone; but if D is produced, the piano will give no response. This phenomenon occurs in many branches of physics and is very important. For example, it is at the basis of wireless telegraphy. The receiving instrument and the sending instrument are tuned together, and only in this way do the effects of the feeble waves which reach to great distances become sensible. The fact that the sending and receiving instruments must be tuned the same explains how it is that many different wireless instruments can be working at the same time without sensible interference.

When the intervening cooler gas absorbs certain parts of the energy which passes through it, it becomes heated and its rate of radiation is increased. It might be supposed that this new radiation would make up for the energy which has been absorbed. That which has been absorbed and that which is radiated are, indeed, exactly equal, but the radiated energy is sent out in every direction and not alone in the direction of the original light passing through the gas. That is, certain parts of the original energy are taken out and scattered in every direction. Therefore, in a spectrum crossed by dark lines the dark lines are not absolutely black, but only black relatively to the remainder of the spectrum. A spectrum of this sort is called an *absorption*, or *dark-line*, or *reversed* spectrum. The reverse of the bright-line spectrum given in the top of Fig. 140 is shown in the bottom part of the figure.

226. The Fourth Law of Spectrum Analysis.—The fourth law of spectrum analysis was first discovered by Doppler and was experimentally established by Fizeau. It is commonly called the Doppler principle, or the Doppler-Fizeau law. It is: *If the source (radiating gas in the case of a spectrum of bright lines, and an intervening cooler gas in case of an absorption spectrum) and receiver are relatively approaching toward, or receding from, each other, then the lines of the spectrum are displaced respectively in the direction of the violet or the red by an amount which is proportional to the relative speed of approach or recession; and conversely, if the lines of a spectrum are displaced toward the violet or the red, the source and receiver are respectively approaching toward, or receding from, each other, and the relative speed of approach or recession can be determined from the amount of the displacement.*

The explanation of the shift of the lines of the spectrum when there is relative motion of the source and the receiver is very simple. If the source is stationary, it sends out wave after wave separated by a given interval; if it is moving toward the receiver, it follows up the waves which it emits and the intervals between them are diminished. That is, the wave lengths have become shorter, which is only another way of stating that the corresponding spectral lines have been shifted toward the violet. Of course, for motion in the opposite direction the spectral lines are shifted toward the red.

If the receiver moves toward the source, he receives not only the waves which would reach him if he were stationary, but also those which he meets as a consequence of his motion. The distances between the waves are diminished and the spectral lines are shifted toward the violet. Motion in the opposite direction produces the opposite results.

The formula for the shift in the spectral lines is

$$\Delta\lambda = \frac{v}{V}\lambda,$$

where $\Delta\lambda$ is the amount of the shift, λ is the wave length of the line in question, v the relative velocity of the source and receiver, and V the velocity of light. Suppose v is 18.6 miles per second; then, since V is $186,000$ miles per second and the greatest wave length in the visible spectrum is nearly twice that of the shortest, the displacement is about $\frac{1}{10{,}000}$ of the distance between the ends of the visible spectrum. It follows that for the velocities with which the planets move the displacements of the spectral lines are very small, and that refined means must be employed in order to determine them accurately. The usual method is to photograph the spectrum of the distant object and at the same time to send through the spectroscope beside it the light from some suitable laboratory source. The lines of the latter will of course have their normal positions. The displacements of the lines of the celestial object with respect to them are measured with the aid of a microscope.

When the spectral lines of an object are well defined, displacement results of astonishing precision can be obtained. In the case of stars of certain types the relative velocities toward or from the earth, called *radial velocities*, can be determined to within one tenth of a mile per second.

XVI. QUESTIONS

1. What problems can be solved approximately for the sun and stars by the first principle of spectrum analysis?

2. What would be the character of the spectrum of moonlight?

3. Comets have continuous bright spectra crossed by still brighter lines; what interpretation is to be made of these facts, remembering that comets shine partly by reflected light?

4. The spectra of Uranus and Neptune contain dark lines and bands of great intensity at the positions of the less intense hydrogen lines of the solar spectrum; what interpretation is to be placed on these phenomena?

5. Can the motion of the earth with respect to the sun and moon be determined by spectroscopic means? The motion of the earth with respect to the planets?

6. If an observer were approaching a deep red star with the velocity of light, what color would the star appear to have? If he were receding with the velocity of light?

7. What effect would the rapid rotation of a star have on its spectral lines?

8. Suppose an observer examines the spectra of the eastern and western limbs of the sun; how would the spectral lines be related? Could they be distinguished from lines due to absorption by the earth's atmosphere?

III. The Constitution of the Sun

227. Outline of the Sun's Constitution.—The apparent surface of the sun is called the *photosphere* (light sphere). It has the appearance of being rather sharply defined, Fig. 141, and it is the boundary used to define the size of the sun, but the sun is disturbed by such violent vertical motions that it is probably very broken in outline. At the distance of the sun from the earth an object 500 miles across subtends an angle of only one second of arc, and, therefore, irregularities in the photosphere would not be visible unless they amounted to several hundred miles. The part of the sun interior to the photosphere is always invisible.

Above the photosphere lies a sheet of gas, probably from 500 to 1000 miles thick, which is called the *reversing layer* because, as will be seen (Art. 233), it produces a reversed, or absorption, spectrum. It contains many terrestrial substances, such as calcium and iron, in a vaporous state.

Outside of the reversing layer is another layer of gas, from 5000 to 10,000 miles deep, called the *chromosphere* (color sphere). At the time of a total eclipse of the sun it is seen as a brilliant scarlet fringe whose outer surface seems to be covered with leaping flames. There are often

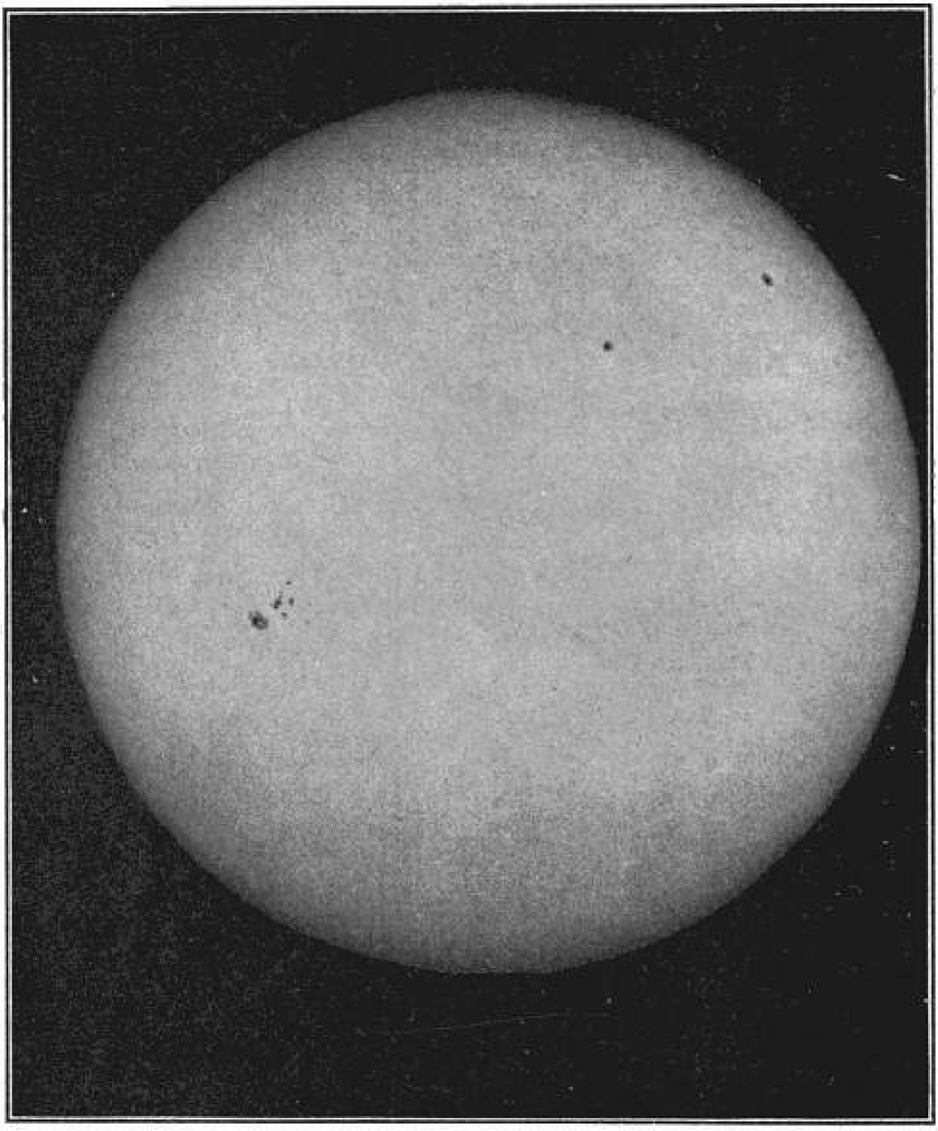

FIG. 141. — The Sun. *Photographed by Fox with the 40-inch telescope of the Yerkes Observatory.*

eruptions, called *prominences*, which break up into it and ascend to great heights.

The outermost portion of the sun is the *corona* (crown), a halo of pearly light which is so much fainter than the illumination of the earth's atmosphere that it can be seen only at the time of a total solar eclipse.

It is irregular in form and gradually fades out into the blackness of the sky at the distance of from 1,000,000 to 3,000,000 miles from the surface of the sun.

Figure 142 shows an ideal section through the sun. The upper surface of the invisible interior is the photosphere, R is the reversing

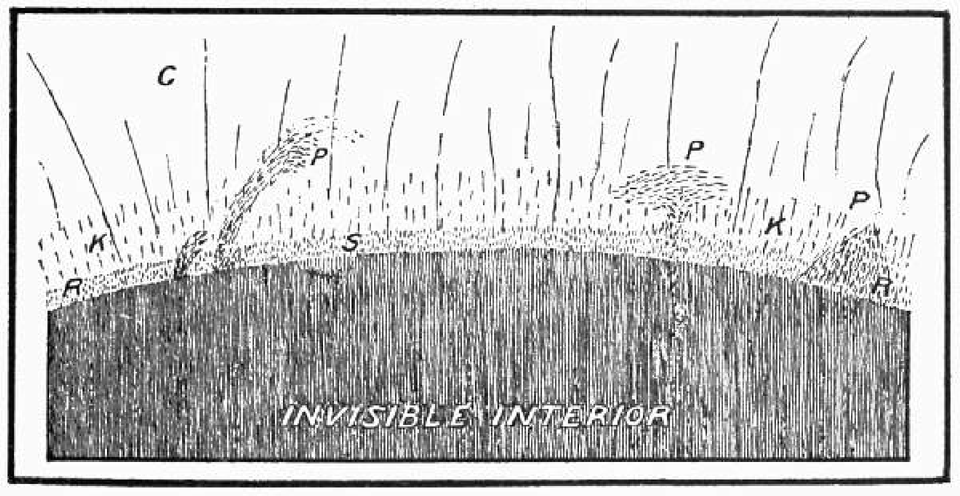

FIG. 142.—Ideal section of the sun.

layer, S is a spot, K is the chromosphere, P is a prominence, and C is the corona.

228. The Photosphere.—When the sun is examined through a good telescope it presents a finely mottled appearance instead of the uniform luster which might be expected. The brighter parts are intensely luminous nodules, somewhat irregular in form, 500 or 600 miles across. These "rice grains," as they are sometimes called, have been resolved into smaller elements having a diameter of not over 100 miles; and although all these granules together do not constitute over one fifth of the sun's surface, yet, according to Langley's estimates, they radiate about three fourths of the light. A small portion of the sun's surface highly magnified is shown in Fig. 143.

The photosphere of the sun gives a continuous spectrum. Therefore, according to the first law of spectrum analysis, it is a solid, liquid, or gas under great pressure. Since the photosphere is not transparent there is a strong inclination to infer that it is liquid, or at least consists of clouds of liquid particles (carbon, iron, calcium, etc.) floating in a vapor of similar substances. But the temperature of the sun is so high that this conclusion is not certain.

In considering the sun it must be remembered that its surface gravity is nearly 28 times that of the earth, and that the pressure under equal masses of atmosphere is correspondingly greater. Hence, it is not unreasonable to suppose that the pressure down under the corona, chromosphere, and reversing layer is great enough to produce a continuous spectrum. The conclusion that the photosphere is almost entirely, if not altogether, gaseous is supported by the fact that the cooler, overlying reversing layer is gaseous and contains some of the most refractory known substances. The "rice-grain" structure of the photosphere is explained by Abbott as being due to relative motions of layers at different levels analogous to those which produce a mackerel sky in the earth's atmosphere. He supposes that the dark places between the "rice-grains" correspond to those places where clouds form in our own atmosphere, and that they are regions where the temperature has fallen somewhat below that of the remainder of the photosphere. There are other astronomers, however, who believe that the bright nodules are the summits of ascending convection currents, which, by expansion and cooling, are reduced to the state where the most refractory substances partially condense and radiate most brilliantly, while the darker spaces between are where the cooler currents descend.

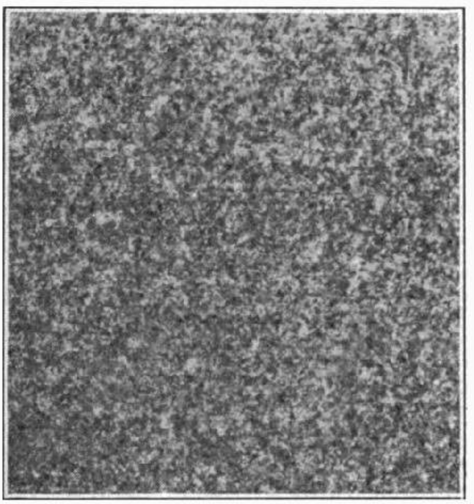

FIG. 143. — Small portion of the sun's surface, highly magnified.

The photosphere is the region from which the sun loses energy by radiation. This energy must be supplied from the interior. There are three processes by which heat may be transferred from one position to another, viz., by conduction, by convection, and by radiation. Conduction is entirely too slow to be quantitatively adequate for bringing heat to the surface of the sun. Convection currents might be violent enough and might reach deep enough to bring to the surface the requisite amount of heat. In order to get a quantitative idea of the requirements suppose that essentially all of the sun's radiation is from a layer of the photosphere, of average density one tenth, 500 miles thick. Suppose its specific heat is unity. At the rate at which the sun radiates, the temperature of this layer would decrease one degree Fahrenheit in 1.6 hours if fresh energy were not supplied from below. Hence the requirements do not seem to be unreasonably severe.

In a body as nearly opaque as the sun seems to be, radiation probably is of no importance in the escape of heat from the deep interior to the surface layers.

229. Sun Spots.—The most conspicuous markings ever seen on the sun are relatively dark spots which occasionally appear in the photosphere and last from a few days up to several months, with an average duration of a month or two. The typical spot consists of a round, relatively black nucleus, called the *umbra*, and a surrounding less dark belt called the *penumbra*, Fig. 144. The penumbra is made up of converging filaments, or "willow leaves," of brighter material, which look as though the intensely luminous photospheric columns were tipped over so as to make their sides visible. The umbra and penumbra do not gradually merge into each other, and likewise the penumbra and surrounding photosphere have a fairly definite line of separation.

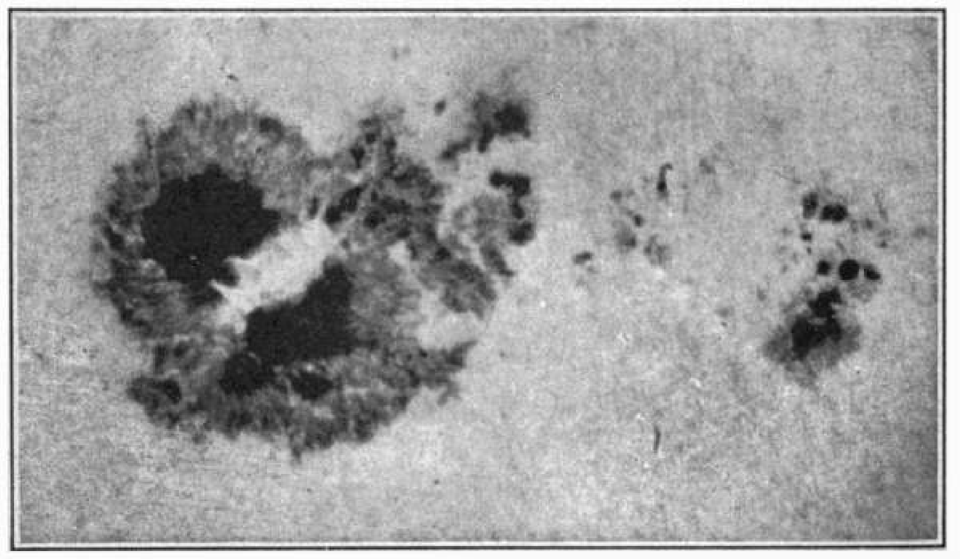

FIG. 144. — Great sun spot of July 17, 1905. *Photographed by Fox with the 40-inch telescope of the Yerkes Observatory.*

The umbra of a sun spot may be anywhere from 500 to 50,000 miles across; the diameter of the penumbra may be as great as 200,000 miles. When the spots are of these dimensions they can be seen simply with the aid of a smoked glass to reduce the glare of the sun. The Chinese claim to have records of observations of sun spots made centuries before their discovery by Galileo in 1610.

The umbra of a sun spot is dark only in comparison with the glowing photosphere which surrounds it, for a calcium light projected on it appears black. In fact, it sometimes shows many details of darker spots and brighter streaks which most often appear shortly before it breaks up. In the neighborhood of spots the brightness of the photosphere is usually above the average, and there are nearly always in their vicinity very bright elevated masses of calcium which constitute the *faculæ*. These faculæ are especially conspicuous when near the sun's apparent margin, or limb, as it is called, for in these regions the photosphere is greatly dimmed by the extensive absorbing material through which its rays must pass, while on the other hand the faculæ project out through the absorbing material and shine with but slightly diminished luster.

230. The Distribution and Periodicity of Sun Spots.—Sun spots are rarely seen except in two belts extending from latitude 6° to latitude 35° on each side of the sun's equator. Moreover, they are not always equally numerous. For three or four years they appear with great frequency, then they become less numerous and decline to a minimum for three or four years, after which they are more numerous again. The interval from maximum number to maximum number averages about 11.11 years, though the period varies from about 7 years to more than 16 years. When a period is short the maximum which follows it is very marked, as though a large amount of sun-spot activity had been crowded into a short interval; on the other hand, when a maximum is delayed it is below normal in activity.

There is a connection between the positions of sun spots and their numbers, first pointed out by Schwabe in 1852. After a sun-spot maximum has passed, the spots appear year after year for about five years, on the average, in successively lower latitudes, and they are continually less numerous. At the sixth year a few are still visible in about latitudes $\pm 6°$, and a new cycle starts in latitudes about $\pm 35°$. After this the spots in the low latitudes disappear, those in the higher latitudes increase in numbers, but gradually descend in latitude until the maximum activity is reached in latitudes $\pm 16°$. The areas covered by spots in years of maximum activity are from 15 to 45 times those covered in years of minimum activity. The results from 1876 to 1902 are shown in Fig. 145.

Since accurate records of the numbers and dimensions of sun spots have been kept, the sun's southern hemisphere has been somewhat more active than the northern. For the period from 1874 to 1902,

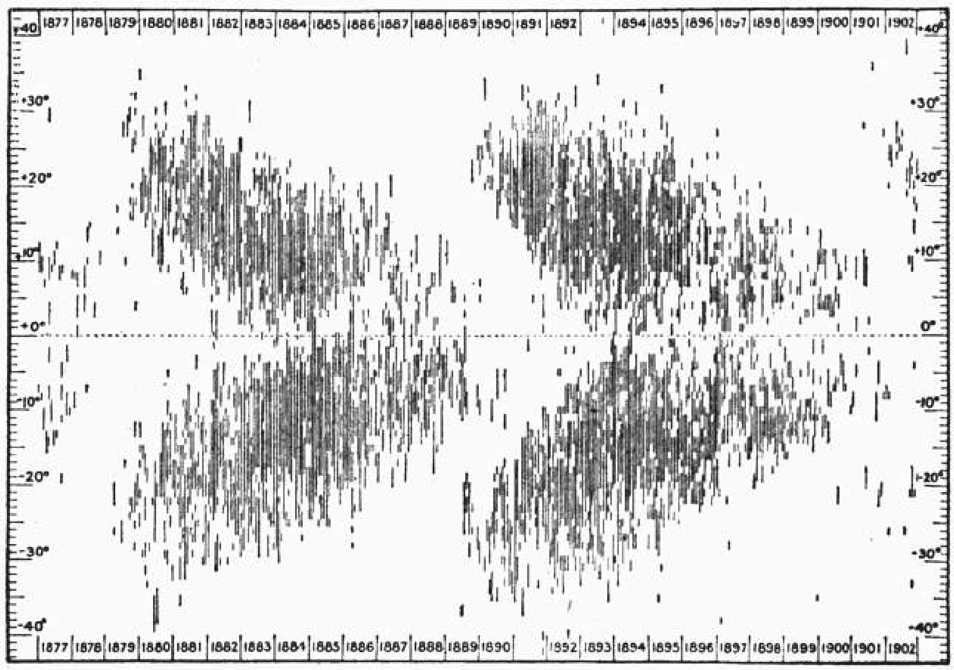

FIG. 145.—Distribution and magnitudes of sun spots for the period from 1876 to 1902 (Maunder).

57 per cent of the total spot area was in the southern hemisphere of the sun and only 43 per cent in the northern. That is, the activity in the southern hemisphere was about one third greater than that in the northern. Whether this difference is permanent and what it means cannot at present be determined.

231. The Motions of Sun Spots.—Individual sun spots may drift both in latitude and in longitude, and they often have complicated and violent internal motions. As a rule, those spots whose latitudes are less than 20° drift slowly toward the sun's equator, and those which are in higher latitudes drift away from it. When two spots are near together they are generally on the same parallel of latitude. The spot which is ahead usually moves forward with respect to the sun's surface, while the one which is behind lags continually farther in the rear. If a large spot divides into two components, they generally recede from each other, sometimes at the rate of 1000 miles an hour.

Sun spots sometimes have spiral motions, but until recently the phenomenon was thought to be hardly characteristic because it was observed in only a small percentage of cases. Hale's invention of the spectroheliograph (Art. 237) furnished a new and powerful means of studying solar phenomena, and it has led in recent years to a discovery of great interest and importance in this connection.

In 1908 Hale proved the existence of magnetic fields in the high levels of sun spots. One may well wonder how such a result could be established, since we receive only light and heat from the sun. Naturally it must be done from the characteristics of the radiant energy which the sun sends to the earth. About 1896 Zeeman found that most spectral lines are doubled, or at least widened, when observed along the lines of force of a magnet, and that the two components are circularly polarized in opposite directions. Hale examined the widened spectral lines belonging to sun spots and proved that they have the properties of spectral lines in a magnetic field. Then he took up the question of the origin of the magnetic fields. It was shown by Rowland in 1876 that static electric charges in revolution produce electromagnetic effects like those produced by electric currents. Consequently Hale concluded that the magnetic fields in sun spots are due to vortical motions of particles carrying static electric charges, and the explanation is almost certainly correct.

More recently the whole sun has been found to be involved in a magnetic field whose poles agree approximately with its poles of rotation; it may be analogous to that which envelopes the earth. Schuster has suggested that the magnetic states of the earth and sun may be a consequence of their rotations, and that all rotating bodies must be magnets.

Hale's discovery is a proof of cyclonic motion in the upper parts of sun spots. Unlike cyclones on the earth, the direction of motion in a hemisphere is not always the same. Hale found numerous examples where two spots seemed to be connected, one having one polarity and the other the opposite (Fig. 146). It has been suggested they are the two ends of a cylindrical whirl. This idea is confirmed, at least to some extent, by the fact that, so far as observational evidence goes at present, when two spots are near together, they always have opposite polarity. Another remarkable fact is that if two neighboring spots are

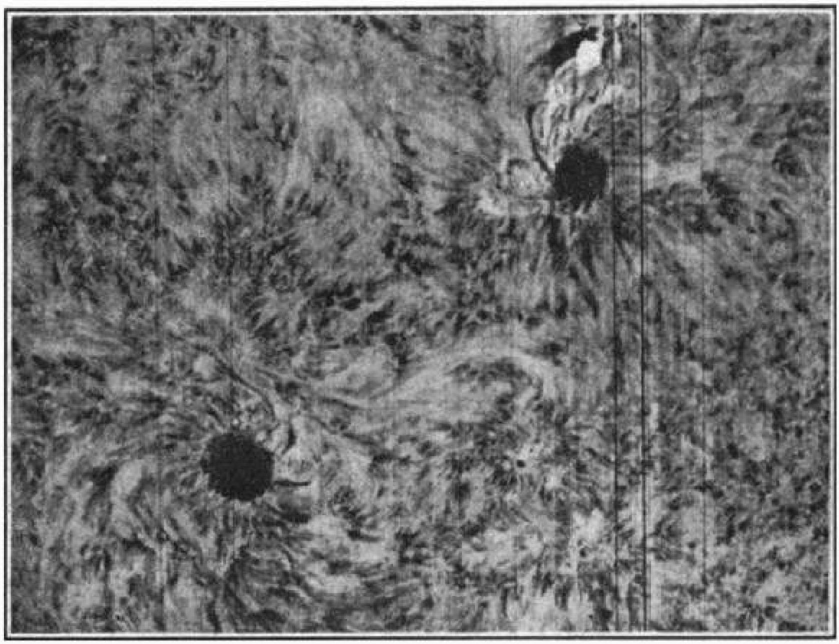

FIG. 146.—Sun spots having opposite polarity. *Photographed at the Mt. Wilson Solar Observatory with the spectroheliograph* (Hale).

in the northern hemisphere of the sun, the one which is ahead has a counter-clockwise vortical motion, while the motion in the other is in the opposite direction. The conditions are the opposite in the sun's southern hemisphere.

Evershed, in India, announced in 1909 that at the lowest visible levels there is radial motion outward from spots parallel to the surface of the sun. More recently St. John, at the Mt. Wilson Solar Observatory (Fig. 147), has made extensive studies of the motions in sun spots with the advantage of most powerful instruments, and he concludes that at the lower levels there is motion radially outward from spot centers, at levels about 2500 miles higher there is no horizontal motion, and in the high levels of the chromosphere (10,000 to 15,000 miles) the motion is inward toward the centers of the spots. This suggests that spots are produced by cooler gases from high levels rushing in toward a center, descending some thousands of miles, and then spreading out at lower levels, but the consideration of the quality and quantity of the materials involved in the two movements, together with their kinetic energies, led St. John to the conclusion that the material flowing inward and downward by no means equals that flowing outward at lower levels

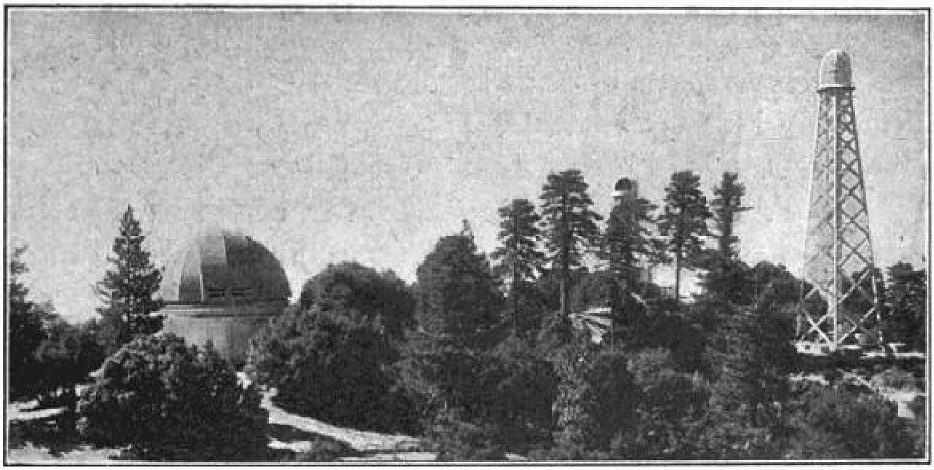

FIG. 147.—The Mt. Wilson Solar Observatory of the Carnegie Institution of Washington. Pasadena, California.

from the axes of spots. He believes, rather, that a spot is formed by currents ascending from the sun's interior and spreading out just above the photosphere. The in-rushing and descending chromospheric material is a secondary result of the primary currents. The spots are dark because the expanding gases of which they are composed are cooler than those which constitute the photosphere.

Independent evidence of a conclusive character shows that spots are cooler than the ordinary photosphere. There is evidence from the so-called enhanced spectral lines which has been brought out by Hale, Adams, and Gale; the lines in the spectrum of spots are related to those in the spectrum of the remainder of the sun just as the spectra with low temperatures in the electric furnace are related to those with high temperatures; and finally, the spectra of spots contain flutings, or bands, which are believed to be due to absorption by chemical compounds which would be broken up into their constituent elements in the higher temperatures of the photosphere.

232. The Rotation of the Sun.—The rotation of at least that part of the sun in which the spots occur can be found from their apparent transits across its disk. The first systematic investigation of the sun's rotation was made by Carrington and Spoerer about the middle of the nineteenth century. They found that the sun rotates from west to east about an axis inclined 7° to the perpendicular to the plane of the ecliptic. The sun's axis points to a position whose right ascension and declination are respectively 18 h. 44 m. and +46°, which is almost exactly midway between Vega and Polaris. The period of the solar rotation depends upon the latitude. Spots near the sun's equator complete a revolution in about 25 days; in latitude 30°, in about 26.5 days; in latitude 45°, in about 27.5 days; in higher latitudes spots are not seen.

Reference has already been made to the faculæ, or bright clouds, which are especially abundant in the neighborhood of sun spots. The positions of the faculæ are easily determined on photographs of the sun, and from photographs made at sufficiently short intervals the rotation of the sun can be found. This method has given results in accord with those obtained from observations of spots.

The remarkable developments of spectroscopic methods which followed Hale's invention of the spectroheliograph have furnished a third method of measuring the rotation of the sun. By its use bright clouds of calcium vapor, called *flocculi* by Hale, and both bright and dark flocculi of hydrogen have been photographed. The rotation of the sun has been determined by Hale and Fox from photographs of flocculi.

Finally, the rotation of the sun has been determined by the Doppler-Fizeau effect. One limb of the sun at the equator approaches the earth at the rate of 1.3 miles per second, while the other recedes at the same velocity. The spectroscopic method is so highly developed that it not only gives the rate of rotation of the sun approximately, but it shows that the period is shorter at the equator than it is in higher latitudes.

The results for the periods of rotation of the sun by the various methods are given in the following table, in which the results are expressed in mean solar days.

By the Doppler-Fizeau method Adams found the periods of rotation of the sun in latitudes 45°, 60°, and 74°, to be respectively 28.1, 31.3, and 32.2 days.

The reason that the sun rotates in its peculiar manner is not certainly known, though Elliott Smith has attempted to show that the more rapid rotation of the equatorial zone is an inevitable consequence of the contraction of a rotating mass of gas. The question deserves further quantitative examination.

Under the hypothesis that the sun is a mixture of fluids in equilibrium, Wilsing, Sampson, and Wilczynski have reached the conclusion from hydrodynamical considerations that cylindrical layers of it rotate with the same speed. According to this view the outermost cylinder, which includes only the equatorial zone, rotates fastest, and successive

TABLE X

LATITUDE	SUN SPOTS	FACULÆ	CALCIUM FLOCCULI	HYDROGEN FLOCCULI	DOPPLER-FIZEAU METHOD
0° to 5°	25.00	24.73	24.76	25.7	24.67
5 to 10	25.09	24.79	24.98	25.0	24.86
10 to 15	25.26	25.12	25.17	24.7	25.12
15 to 20	25.48	25.33	25.48	24.8	25.44
20 to 25	25.75	25.37	25.73	24.5	25.81
25 to 30	26.09	25.64	25.77	24.5	26.20
30 to 35	26.47	26.47	26.18	24.2	26.67

cylinders toward the axis rotate more and more slowly. It is supposed that this condition is inherited from some primitive state and that friction has not yet reduced the rotation to uniformity. Wilczynski showed that friction between the different layers would not wear down the differences of motion appreciably in many millions of years. But he neglected the convection currents which must certainly exist to great depths and which would quickly destroy the supposed different rotations in different cylinders. Notwithstanding these difficulties, no other theory at present is more satisfactory than that the sun's peculiar rotation has been inherited from more extreme conditions which prevailed in the remote past.

233. The Reversing Layer.—Newton began the analysis of light by passing it through a small circular opening. In 1802 Wollaston passed the light from the sun through a narrow slit, instead of a pinhole, and found that the solar spectrum was crossed by 7 dark lines. In a few years the subject was taken up by Fraunhofer, who soon found that the spectrum was crossed by an immense number of dark lines. In 1815 he mapped 324 of them, and they have since been known as "Fraunhofer lines." A greatly improved map of these lines was made by Kirchhoff in 1862, and still another by Ångström in 1868. In 1886 Langley mapped the solar spectrum with the aid of his bolometer far into the infra-red region, and in 1886, 1889, and 1893 Rowland published extensive and very accurate maps from measurements of the positions and characteristics obtained with his powerful grating spectroscope. In 1895 Rowland published his great "Preliminary Table of Solar Spectrum Wave Lengths," containing the results for about 14,000 spectral lines.

A portion of the solar spectrum is shown in Fig. 148 with a bright-line comparison spectrum above.

The spectrum of the sun is continuous except for the very numerous dark lines which cross it. Therefore, in accordance with the third law of spectrum analysis, there is between the photosphere and the observer cooler gas, and its constitution can be determined from the relations among the dark lines and from their positions. The lines prove the existence of sodium, iron, and other heavy metals in this intervening gas, and since they cannot remain in the gaseous state in our own atmosphere they must be in that of the sun. This absorbing material

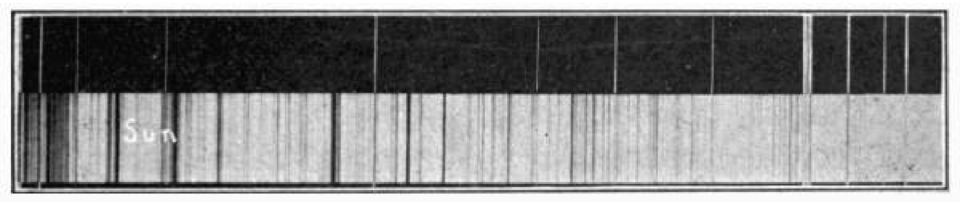

FIG. 148.—Portion of solar spectrum below with a Titanium comparison spectrum above.

which overlies the photosphere constitutes the *reversing layer*.

If the reversing layer could be viewed not projected against the brilliant photosphere, it would give a spectrum of bright lines exactly at the places occupied by the dark lines under the conditions as they normally exist. At the total eclipse of the sun in 1870, Young placed the slit of his spectroscope tangent to the limb of the sun. Just as the moon cut off the last of the photosphere the spectrum suddenly flashed out in bright lines where an instant before the dark ones had been. Since 1895, during nearly every total eclipse of the sun, this "flash spectrum" has been photographed, and there is no doubt that the positions of its lines are identical with those of the corresponding dark Fraunhofer lines. From the duration of their appearance as bright lines and the known rate at which the moon apparently passes across the disk of the sun, it has been found that the reversing layer is 500 or 600 miles deep.

As a rule the effect of pressure on an absorbing gas is to cause the dark lines to shift slightly toward the red end of the spectrum. Extensive studies by various astronomers of the displacements of the Fraunhofer lines have led to the conclusion that the pressure of the reversing layer, even at its lower levels, does not exceed 5 or 6 times that of the earth's atmosphere at sea level. This is a very remarkable result in view of the great extent of the sun's atmosphere and the fact that gravity at the surface of the sun is nearly 28 times as great as it is at the surface of the earth. Possibly electrical repulsion from the sun and light pressure partly offset the great surface gravity of the sun.

234. Chemical Constitution of the Reversing Layer.—Of the 14,000 lines in Rowland's spectrum about one third are due to the absorption by the earth's atmosphere, and the remainder are produced by the sun's reversing layer and chromosphere. By comparing the positions of the lines of the sun's spectrum with those given by the various elements in laboratory experiments, it is possible to infer the chemical constitution of the material which produces the absorption. In this manner 38 elements are known certainly to exist in the sun, but more than 6000 of the lines mapped by Rowland have not as yet been identified as belonging to any element.

The presence of iron is established by more than 2000 line coincidences, carbon by more than 200, calcium by more than 75, magnesium by 20, sodium by 11, copper by 2, and lead by 1. It will be noticed that nearly all the elements in the table which follows are metals, the exceptions being hydrogen, helium, carbon, and oxygen. On the other hand, a number of heavy metals, such as gold and mercury, are missing. The following table gives the elements found in the sun and their atomic weights.

While the presence of the spectral lines of an element proves its existence, their absence does not show that it is not present. In the first place, heavy elements, like gold, mercury, and platinum, would probably sink far below the level of the reversing layer, and consequently would give no lines in the solar spectrum. Then, again, the characteristic spectra of some of the elements, particularly non-metals, are suppressed by the presence of some other elements, particularly metals. Sometimes the spectrum of an element is entirely obliterated by the presence of a small percentage of another element. This may be the explanation of the fact that the spectra of fluorine, chlorine, bromine, iodine, sulphur, selenium, tellurium, nitrogen, phosphorus, arsenic, antimony, and boron are not found in the sun, although most of these elements occur abundantly in the earth. Some elements have spectra that change radically with alterations in their conditions of temperature, pressure, and electrical excitation. One of these elements is oxygen, which was long sought for in the sun before it was certainly found. Of course, the proof of its existence was complicated by the fact that it occurs in abundance in the earth's atmosphere. Finally,

Table XI

Element	Atomic Weight	Element	Atomic Weight
Hydrogen	1	Copper	64
Helium	4	Zinc	65
Glucinum	9	Germanium . . .	72
Carbon	12	Strontium	88
Oxygen	16	Yttrium	89
Sodium	23	Zirconium	91
Magnesium . . .	24	Niobium	93
Aluminum . . .	27	Molybdenum . .	96
Silicon	28	Rhodium	103
Potassium	39	Palladium	107
Calcium	40	Silver	108
Scandium	44	Cadmium	112
Titanium	48	Tin	119
Vanadium	51	Barium	137
Chromium . . .	52	Lanthanum . . .	139
Manganese . . .	55	Cerium	140
Iron	56	Neodymium . . .	144
Nickel	59	Erbium	168
Cobalt	59	Lead	207

as Lockyer suggested, some of the so-called elements may be in reality compounds which are broken up under the extreme conditions of temperature prevailing in the sun, and their characteristic spectra may be in this manner destroyed.

The reversing layer is undoubtedly constantly receiving material from below and above, and therefore it is safe to conclude that its composition is not qualitatively different from that of the remainder of the sun. It is interesting that nearly 40 terrestrial elements are found, for it points strongly to the conclusion that the sun and the earth have had a common origin.

The distribution of the elements in distance above the sun's photosphere was determined by Mitchell from excellent photographs of the flash spectrum which he secured in the eclipse of 1905, and by St. John from considerations of the Doppler-Fizeau effect. On the whole the lighter elements extend to high altitudes while the heavier elements are confined to the lower levels. A peculiar exception is that calcium, whose atomic weight is 40, extends in abundance up into the chromosphere

10,000 miles, even as high as hydrogen. Iron and the heavier metals are found only down in the reversing layer.

235. The Chromosphere.—As has been stated, the chromosphere is a gaseous envelope around the sun above the reversing layer whose depth is from 5000 to 10,000 miles. It gets its scarlet color from the incandescent hydrogen and calcium of which it is largely composed.

The spectrum of the chromosphere consists of many lines, some of which are permanent while others come and go. The permanent lines are due mostly to hydrogen, helium, and calcium; the intermittent lines are due to many elements which seem to have been temporarily thrown up into it through the reversing layer.

The existence of the element helium was first inferred from the presence of a bright yellow line in the solar spectrum near the two yellow lines of sodium. It is universally prevalent in the chromosphere, giving a bright line when the sun is eclipsed, or at any time when the slit of the spectroscope is put on the chromosphere parallel to the sun's limb. For some unknown reason helium does not give a dark-line absorption spectrum when the light from the photosphere passes through it. This seems to be a direct contradiction to the third law of spectrum analysis, which holds true in all other known cases. But helium is a very remarkable element in several other respects. Next to hydrogen, it has the lowest atomic weight, it is very inactive, and enters into no known chemical combinations with other elements, it has the lowest known refractive index, it is an excellent conductor of electricity, its rate of diffusion is 15 times its theoretical value, its solubility in water is nearly zero, and it is liquefied only with the utmost difficulty. It has already been explained that helium is one of the products of the disintegration of uranium, radium, and other radioactive substances. It was not discovered on the earth until 1895, when Ramsay, on examining the spectrum of the mineral clevite, found the yellow spectral line of helium.

236. Prominences.—Vast eruptions, called *prominences*, shoot up from the sun's photosphere through its chromosphere to heights ranging from 20,000 miles up to 300,000 miles, or even to greater elevations in extreme cases. One 80,000 miles in height is shown in Fig. 149. They usually occur in the neighborhood of sun spots and are never seen near the sun's poles. They leap up in jets and flames, often changing their appearance greatly in the course of 10 or 15 minutes, as is shown in Fig. 150. Their velocity of ascent is frequently 100 miles per second and sometimes exceeds 200 miles per second.

If eruptive prominences should leave the photosphere with a velocity of more than 380 miles per second, and if they should suffer no resistance from the reversing layer and chromosphere, they would escape entirely from the sun and pass out beyond the planets to the distances of the stars. It is very difficult to account for their great velocities. No satisfactory theory has been developed for explaining how such violent explosive forces are long held in restraint and then suddenly released.

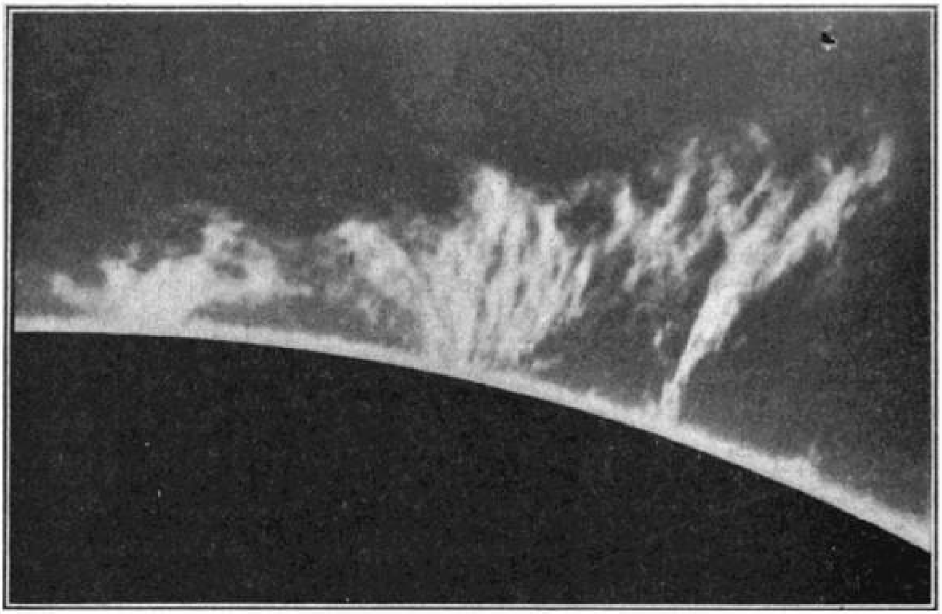

FIG. 149. — Solar prominence, August 21, 1909, reaching to a height of 80,000 miles. *Photographed at the Mt. Wilson Solar Observatory.*

Perhaps under the extreme conditions of temperature and pressure prevailing in the interior of the sun, all elements, like radium under terrestrial conditions, explode because of their subatomic energies. Julius has maintained that the prominences may be mirage-like appearances due to unusual refraction, and that they are not actual eruptions from the sun as they seem to be. But their velocities are determined both from their motion perpendicular to the line of sight when they are seen on the sun's limb, and also from spectral line displacements in accordance with the Doppler-Fizeau principle, and it seems very improbable that they are not real.

The spectra of eruptive prominences show many lines, especially in the lower levels. In them the bright lines of sodium, magnesium, iron, and titanium are conspicuous, while those of calcium, chromium, and manganese are generally found. In the higher levels calcium is the

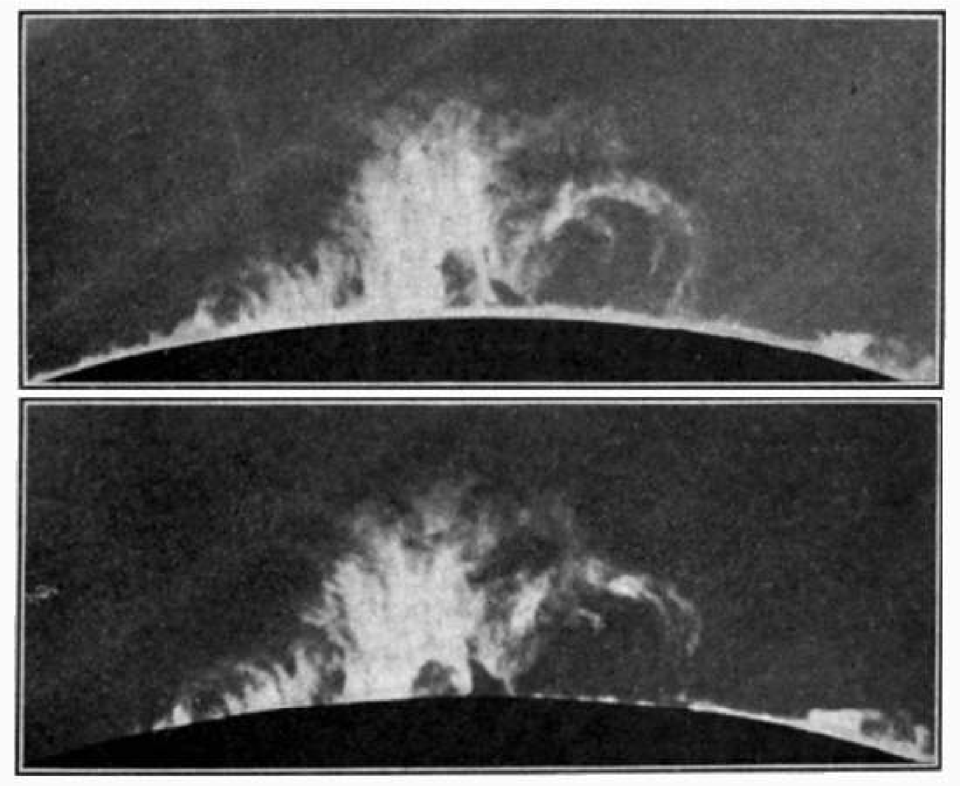

FIG. 150. — Changes in a solar prominence in an interval of ten minutes. *Photographed by Slocum at the Yerkes Observatory.*

predominating element, a remarkable fact in view of its atomic weight of 40.

Prominences were formerly observed only when the sun was totally eclipsed, for at other times the illumination of the sky made them altogether invisible. But since the development of the spectroscope they can be observed at any time. If the light from the limb of the sun is passed through the spectroscope, the continuous illumination of the earth's atmosphere is spread out and correspondingly enfeebled; on the other hand, the light from the prominences consists of single colors and is not diminished in intensity by passing through the spectroscope. Consequently, if the dispersion is sufficient, the atmospheric illumination is reduced until the prominences become visible.

Not all the prominences are eruptive. Besides those which burst out suddenly, rising to great heights and soon disappearing or subsiding again, there are others, called *quiescent* prominences, which spread out, like the tops of banyan trees, with here and there a stem reaching below. They often develop far above the surface of the sun, without apparent connections with it, and seem to be due to material which for some mysterious reason suddenly becomes visible. They rest quietly at great altitudes, somewhat like terrestrial clouds, often for many days, notwithstanding the sun's gravity. They are made up of hydrogen, helium, and calcium.

237. The Spectroheliograph.—The photosphere radiates a continuous spectrum, while above it is the reversing layer which produces the dark absorption lines. Some of the lines, as the K-line due to calcium, are broad because of the great extent of the absorbing layer. Now, calcium is abundant in the prominences, and, moreover, it shines with an intensity greater than that of the reversing layer. The result is that the reversing layer makes a broad, dark line, say the K-line, and above it is more luminous calcium in a rarer state which produces a narrow bright line in the midst of the dark one. The line is said to be "doubly reversed."

The spectroheliograph is an instrument invented and perfected by Hale in 1891 for the purpose of photographing the sun with the light from a single element. The ideas on which it depends were almost simultaneously developed and applied by Deslandres. In this instrument, or rather combination of instruments, the sunlight is passed through a spectroscope and is spread out into a spectrum. The K-line, which is most frequently used, is doubly reversed in the regions of faculæ and prominences. All the spectrum is cut off by an opaque screen except the bright part of the K-line which passes through a second narrow slit. That is, the only light which passes through both slits is the calcium light from that portion of the sun's image which falls on the first slit of the spectroscope. In Fig. 151, S is the image of the sun at the focal plane of the telescope, A is the slit of the spectroscope (the prisms are not shown), T is the spectrum which falls on the screen B, R is a slit in the screen B which is adjusted so that it admits the bright center of the doubly reversed K-line, and P is a photographic plate on which the K-line falls. The apparatus is so made that the slit A may be moved across the image of the sun S, and the slit R simultaneously moved so that the K-line falls on successively different parts of the photographic plate P. In this manner a photograph of the hot calcium vapors which

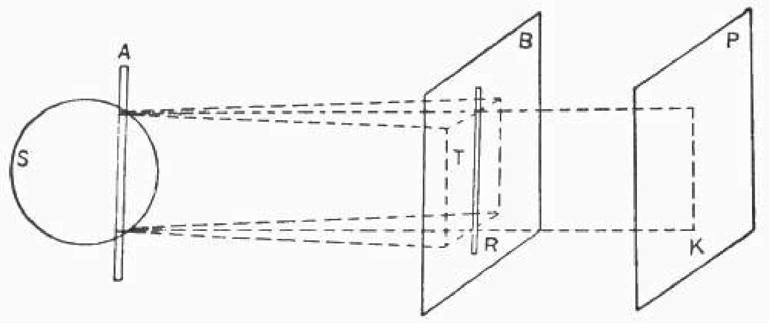

FIG. 151.—The spectroheliograph.

lie above the reversing layer may be obtained; such a photograph is shown in Fig. 152. Some other spectral lines have also been used in this way. For example, two photographs of a spot with the so-called *H*-line are shown in Fig. 153.

The width of a spectral line depends upon the density of the gas which is emitting the light. Suppose a thick layer of calcium gas which is rare at the top and denser at the bottom gives a bright *K*-line. The central part will be due to light coming from all depths, particularly from the higher layers where absorption is unimportant. On the other hand, the marginal parts of the line will be due to light coming from the lower levels where the gas is denser. Following out these principles, and using a very narrow slit, Hale first obtained photographs of different levels of the solar atmosphere.

238. The Corona.—During total eclipses the sun is seen to be surrounded by a halo of pearly light, called the *corona*, extending out 200,000 or 300,000 miles, while some of the streamers reach out at least 5,000,000 miles. So far it has not been possible to find any observational evidence of the corona except at the times of total eclipses of the sun. One of the reasons that eclipses are of great scientific interest is that they afford an opportunity of studying this remarkable solar appendage. The brief duration of total eclipses and their infrequency have made progress in the researches on the corona rather slow. The corona is not arranged in concentric layers like an atmosphere, but is made up of complicated systems of streamers (Fig. 154), in general stretching out radially from the sun, but often simply and doubly curved, and somewhat resembling auroræ. Many observers have declared that its finely detailed structure resembles the Orion nebula.

The coronal streamers often, perhaps generally, have their bases in the regions of active prominences, but exceptions have been noted.

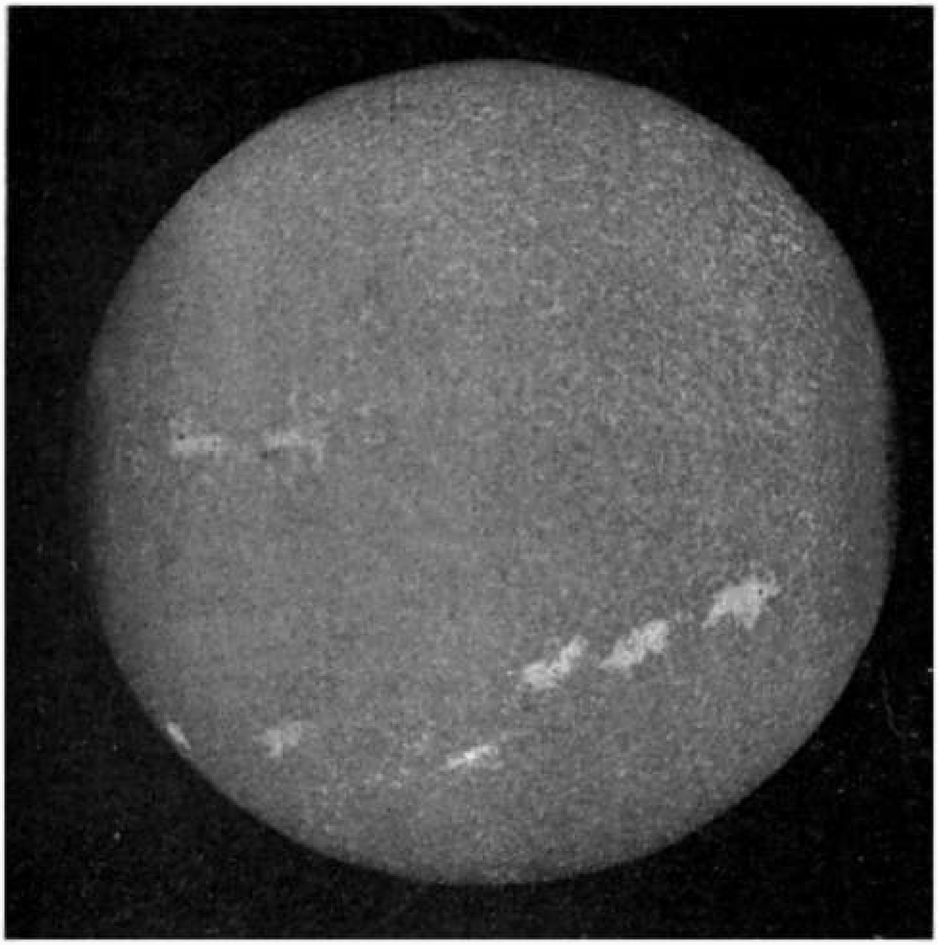

FIG. 152.—Spectroheliogram of the sun taken with the doubly reversed calcium line. *Photographed by Hale and Ellerman at Yerkes Observatory.*

That they are in some way connected with activity on the sun is shown by the fact that the form of the corona changes in a cycle of about eleven years, the same as that of sun-spot activity. At sun-spot maxima the coronal streamers radiate from all latitudes nearly equally. As the maxima pass, the coronal streamers gradually withdraw from the poles of the sun and extend out to greater distances in the sun-spot zones. At the sun-spot minima, the corona consists of short rays in the polar regions, curved away from the solar axis, and long streamers extending out in the equatorial plane.

The spectroscope shows that the corona emits three kinds of light.

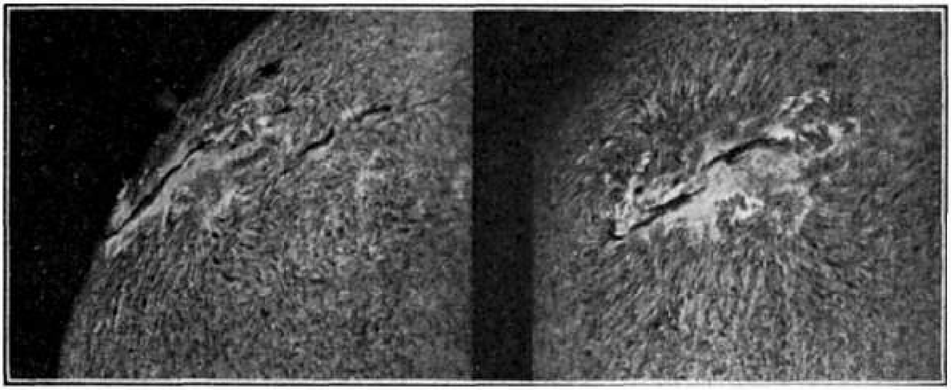

FIG. 153. — Spectroheliograms of a sun spot with the doubly reversed H-line of calcium. *Hale and Ellerman, Solar Observatory, Aug. 7 and 9, 1915.*

First, there is a small quantity which is known to be reflected sunlight, for it gives, though faintly, the Fraunhofer absorption lines, and it is polarized. Second, there is white light whose source, according to the first law of spectrum analysis, must be incandescent solid or liquid particles. Lastly, there is a bright-line spectrum whose source, according to the second law of spectrum analysis, must be an incandescent gas. The most conspicuous line is in the green and is emitted by an element, called *coronium*, which is not yet known on the earth. There seems to be at least one other substance present, but no known elements.

According to present ideas, the corona consists of dust particles, liquid globules, and small masses of gas which are widely scattered. From the amount of light and heat radiated, and from the temperature which masses so near the sun must have, Arrhenius computed that there is one dust particle, on the average, in every 14 cubic yards of the corona. The excessive rarity of the corona is shown by the fact that comets have plunged through hundreds of thousands of miles of it without being sensibly retarded. The dust particles and liquid globules give the reflected light; the liquid, the continuous spectrum; and the gases, the bright-line spectrum. The form of the corona shows that its condition of equilibrium is not at all similar to that of an atmosphere like the one surrounding the earth. Its increase of density toward the sun is inexplicably slow, though doubtless light pressure and electrical forces are opposed to gravity. Its radial structure and periodical variation in general form are without satisfactory explanation.

239. The Eleven-Year Cycle.—It has been explained that sun

Fig. 154.—Photograph of the corona at the total eclipse of the sun, May 28, 1900 (Barnard and Ritchey).

spots vary in frequency and distribution on the sun's surface in a period averaging a little more than 11 years. There are a number of other phenomena which undergo changes in the same period.

The faculæ are most numerous in the sun-spot zones, although they occur all over the sun. Both their number and the positions of the zones where they are most numerous vary periodically with the sun-spot period. This is quite to be expected, for the sun spots and the faculæ are both photospheric phenomena.

The eruptive prominences are frequent in the sun-spot belts, and vary in position with them. The evidence so far also shows periodic variations in their numbers. The quiescent prominences, on the other hand, cluster in the polar regions.

The coronal types clearly vary in the eleven-year cycle, as was explained in the preceding article. Doubtless the total solar radiation varies to some extent in the same period, though this has not been verified observationally, but the time is now ripe for the investigation.

The spectra of sun spots vary with the period of the spots, but the Fraunhofer lines are singularly invariable.

The great vibrations which so powerfully agitate the sun extend to the earth and probably to the whole solar system. It has long been known that both the horizontal and vertical components of the earth's magnetism vary in the sun-spot period, and that magnetic disturbances ("storms") are most frequent at the times when sun spots are most numerous. Likewise, auroræ occur most frequently at the epochs of great sun-spot activity. In fact, magnetic storms and auroræ never occur except when there is great activity in the sun in the form of sun spots or prominences; but there are frequent disturbances on the sun without accompanying terrestrial phenomena. The correlation of these phenomena is shown in Fig. 155.

The first suggested explanation of magnetic storms on the earth was that the sun induces changes in the earth's magnetic state by sending out electromagnetic waves. Lord Kelvin raised the objection that if the sun were sending out these waves in every direction, it would give out as much energy in 8 hours of an ordinary electric storm as it radiates in light and heat in 4 months. A recent exhaustive discussion of the data has led Maunder to the conclusion that the source of the periodic magnetic storms is in the sun, that the magnetic disturbances are confined to restricted areas on the sun, and that their influences are propagated out from the sun in cones which rotate with the sun; that when these

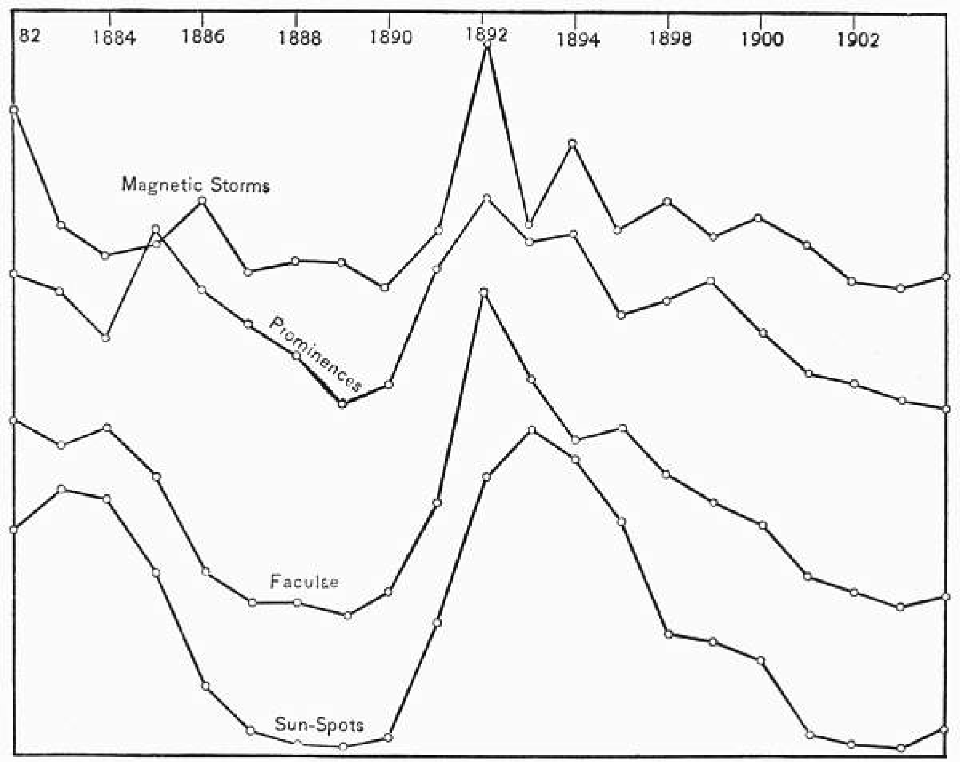

FIG. 155. — Curves of magnetic storms, prominences, faculæ, and sun spots from 1882 to 1904.

cones of magnetic disturbances strike the earth, magnetic storms are induced, and that these magnetic storms have intimate, though unknown, relations with sun spots. The most important contribution of this investigation was that there is much observational evidence to show that the sun is not to be regarded as surrounded by a polarized magnetic sphere, but that there are definite and intense stream lines of magnetic influence, probably connected with the coronal rays, reaching out principally from the spot zones in directions which are not necessarily exactly radial. It is a little too early to formulate a precise theory as to whether these streams are electrified particles driven off by magnetic forces and light pressure, or whether they involve the minute corpuscles of which atoms are composed, or whether they are phenomena of matter and energy of a character and in a state not yet recognized by science.

XVII. QUESTIONS

1. The apparent diameter of the sun as seen from the earth is about $32'$; what are the apparent thicknesses of the corona, chromosphere, and reversing layer?

2. The sun's disk is considerably brighter at its center than near its margin (Fig. 141); can this phenomenon be explained by the absorption of light by the reversing layer? By small solid or liquid particles somewhere above the photosphere?

3. If the smallest spot that can be seen subtends an angle of $1'$, what is the diameter of the smallest sun spot that can be seen simply through a smoked glass?

4. In what direction do sun spots appear to cross the sun's disk as a consequence of its rotation?

5. Why cannot the corona be observed with the aid of the spectroscope at any time, just as the prominences are observed?

CHAPTER XII

EVOLUTION OF THE SOLAR SYSTEM

I. General Considerations on Evolution

240. The Essence of the Doctrine of Evolution.—The fundamental basis on which science rests is the orderliness of the universe. That it is not a chaos has been confirmed by an enormous amount of experience, and the principle that it is orderly is now universally accepted. This principle is completed in a fundamental respect by the doctrine of evolution.

According to the fundamental principle of science the universe was orderly yesterday, is orderly to-day, and will be orderly to-morrow; according to the doctrine of evolution, the order of yesterday changed into that of to-day in a continuous and lawful manner, and the order of to-day will go over into that of to-morrow continuously and systematically. That is, the universe is not only systematic and orderly in space, but also in time. The real essence of the doctrine of evolution is that it maintains the orderliness of the universe in time as well as in space.

Evolution may be from the simple and relatively unorganized to the complex and highly organized, or it may be in the opposite direction. In fact, evolution generally involves the two types of changes. For example, the minerals of the soil and the elements of the atmosphere sometimes combine and produce a tree having foliage, flowers, and fruit. But the tree grows, at least partly, on the disintegrating products of other trees or plants, and in its own trunk the processes of decay are active. Or, to take a less commonplace example, with the advancement of civilization men have become more sensitive to discords and more and more capable of appreciating certain types of harmony. There is almost certainly a corresponding improvement in the structure of their nervous system. On the other hand, there is degeneration in the quality of their teeth and hair. The changes in the two directions are both examples of evolution.

As knowledge increases it is found that everything is continually changing. Individuals change, institutions change, languages change, and even the "eternal hills" are broken up and washed away by the elements in a moment of geological time. Moreover, all these changes are found to be perfectly orderly. The doctrine of evolution, as defined here, is so fundamentally sensible and is confirmed by so much experience that scientists, the world over, accept it with absolute confidence. There have been, and there doubtless will continue to be, differences of opinion regarding what the precise processes of certain particular evolutions may have been, but there is no disagreement whatever regarding the fundamental principles.

241. The Value of a Theory of Evolution.—The importance of a general principle is proportional to the number of known facts it correlates. This is a general proposition with special applications in science. Since a theory of evolution is concerned largely with the relations among the data established by experience, it naturally forces an attempt at their correlation. Moreover, the relations are examined in a critical spirit, so that any errors in the data or misconceptions regarding their relations are apt to be revealed. Therefore, an attempt to construct a theory of evolution is of value because it leads to a better understanding of the material upon which it is being based.

A theory of evolution invariably demands a knowledge of facts in addition to those upon which it is based. In this way it stimulates and directs investigation. A great majority of the investigations which scientific men make are for the purpose of proving or disproving some theory they have tentatively formulated. The true scientist often has pre-conceived notions as to what is true, but he conscientiously follows the results of experience.

A broad scientific theory involves many secondary theories depending upon special groups of phenomena. For example, a theory of the origin and development of the solar system will involve theories of the sun's heat, of the revolution of the planets, of the rotation of the planets, of the planetoids, of the zodiacal light, etc. In the construction of a general theory of evolution the secondary theories are related to the whole, and in this way they are subjected to a searching examination. The criticism of secondary theories, whether the result is favorable or adverse, constitutes another important value of the development of a theory of evolution.

The activities of men are largely directed toward satisfying their intellectual wants, though this fact might be easily overlooked. For example, they do not ordinarily visit foreign countries to get more to eat or wear, but to acquire broader views of the world. The important thing in traveling is not that a person goes physically to any particular place, but that he gets the intellectual experiences that result from going there. Astronomers cannot travel through the vast regions of space which they explore, but the long arms of their analysis reach out and gather up the facts and bring them to their consciousness with a vividness scarcely surpassed in any experience. As their powerful instruments and mathematical processes extend their experience in space, so a theory of evolution, to the extent that it is complete and sound, extends their experience in time.

Finally, a theory which gives unity to a great variety of observational data is of rare æsthetic value. It is related to the catalogue of imperfectly correlated facts upon which it is based as a finished and magnificent cathedral is to the unsightly heaps of stone, brick, and wood from which it was built. In some reflections along this line, near the close of his popular work on astronomy, Laplace said, "Contemplated as one grand whole, astronomy is the most beautiful monument of the human mind, the noblest record of its intelligence."

In view of these considerations it is evident that the evolution of the solar system is a subject to which the astronomer naturally gives serious attention. The foremost authorities of the present time have treated the question in lectures, in essays, and in books. When new discoveries are made their bearings on evolutionary theories are at once examined. Astronomers are rapidly approaching the point of view of the biologists, who interpret all of their phenomena in terms of evolutionary doctrines. Yet scarcely a generation ago many astronomers regarded the consideration of the evolution of the solar system as a dangerous speculation.

242. Outline of the Growth of the Doctrine of Evolution.—Every great discovery doubtless has been the culmination of a long period of preliminary work, and before final success has been attained generally many men have approximated to the truth. So it has been with the doctrine of evolution. The ancient Greeks developed theories that everything had evolved from fire, or from air and water. These theories contained the germ of the idea of evolution, but their authors had not laid securely enough the foundations of science to enable them to treat successfully the development of the universe. After the decline of their intellectual activity the subject of evolution was not considered seriously for many centuries.

In the eighteenth century geologists were groping for a satisfactory theory regarding the succession of the life forms whose fossils were found in the rocks. They seem to have concluded on the whole that the earth had been subject to a number of great cataclysms in which all life was destroyed. They supposed that following each destruction of life there had been a new creation in which higher forms were produced.

The prevalence of such ideas as these shows with what difficulty the doctrine of evolution was developed.

In 1750 Thomas Wright, of Durham, England, published a theory of the evolution, not only of the solar system, but also of all the stars that fill the sky. The chief merit of this work was that indirectly it gave a straightforward exposition of the doctrine of evolution. Its chief influence seems to have been on the young philosopher Kant, into whose hands it fell. Kant at once turned his brilliant mind to the contemplation of the problems of cosmogony, or the evolution of the celestial bodies, and in 1755 he published a remarkable book on the subject. But the world seems not to have been ripe for the idea of evolution, because neither the work of Wright nor that of Kant had any important influence upon science.

In 1796 the great French astronomer and mathematician Laplace published his celebrated "Nebular Hypothesis." It was supported by the great name of its author, and it was relatively simple and easily understood. Moreover, during the French Revolution the world had acquired a new point of view and had become more receptive of new ideas. For these reasons the theory of Laplace soon obtained wide acceptance among scientific men. It made a profound impression on geologists because it furnished them with an account of the early history of the earth. It gave them astronomical authority for an originally hot and molten earth which had solidified on cooling. It encouraged them to interpret geological phenomena by geological principles. In the early decades of the nineteenth century geologists largely abandoned the idea that the earth had necessarily been visited by destructive cataclysms, and adopted the view that it had undergone a continuous series of great changes at a roughly uniform rate.

The work of the geologists led naturally to the extension of the doctrine of evolution to the biological sciences. In the first place, the belief that the earth was enormously old had become current. In the second place, there were unmistakable evidences that the surface of the earth had undergone extensive changes. In the third place, the early rocks contained only fossils of low forms of life, while the later rocks contained fossils of higher forms of life. In addition, there were many direct evidences of a purely biological character that there was an almost continuous series of life forms from the lowest to the highest.

The principle of biological evolution seems to have been taking definite shape simultaneously in the minds of Charles Darwin, Alfred Russel Wallace, and Herbert Spencer. Darwin and Wallace were naturalists and Spencer was a philosopher. In 1858 Darwin published his *Origin of Species*, in which he brought together the results of almost a lifetime of keen observations and profound reflections. He gave unanswerable evidence for his conclusion that during the geological ages, as a consequence of changing environment, natural selection, survival of the fittest, etc., one species of animals gradually changed into another, and that at the present time all the higher types of animals, including man, are more or less closely related.

In spite of the fact that the doctrine of evolution is full of hope for the future progress of the human race, Darwin's book aroused the bitterest antagonism. While biologists do not now fully agree with him as to the relative importance of the various factors involved in biological evolution, nevertheless they universally accept his fundamental conclusions. Moreover, the changes in political, social, and religious institutions are now considered in the light of the same ideas. That is, the condition of the whole universe at one time evolves continuously and in obedience to all the factors operating on it into that which exists at another time.

In brief, the development of the modern doctrine of evolution is as follows: In the middle of the eighteenth century its first beginnings were laid in astronomy by Wright and Kant. At the end of the century it was given an enormous impulse by the astronomer and mathematician, Laplace. His theory of the origin of the earth stimulated geologists to adopt it in the early decades of the nineteenth century. By the middle of the century it was being definitely applied in the biological sciences. In 1858 Darwin published his great masterpiece, *The Origin of Species*, which gave the whole world a new point of view and revolutionized its methods of thought. The development and adoption of the doctrine of evolution was the greatest achievement of the nineteenth century.

XVIII. QUESTIONS

1. Is the erosion of the chasm below Niagara Falls an example of an evolution? Is the clearing away of the forests and the preparation of the land for cultivation? Is an explosion of dynamite?

2. Would the direct creation of men and lower animals be an example of evolution?

3. Do the changes in scientific ideas constitute an evolution?

4. Are religious ideas undergoing an evolution?

5. Will the doctrine of evolution undergo an evolution?

6. The universe in our vicinity at the present time is believed to be orderly; is it reasonable to suppose that in remote regions or at remote times it was not orderly?

7. Why was the doctrine of evolution first clearly understood in astronomy?

8. According to the doctrine of evolution, will two identical conditions of the universe lead to identical results? Is it probable that the universe is twice in exactly the same state?

II. Data of the Problem of Evolution of the Solar System

243. General Evidences of Orderly Development.—There are certain obvious evidences that the solar system has undergone an orderly evolution. For example, the planets all revolve around the sun in nearly the same plane and in the same direction. There are in addition over 800 planetoids which have similar motions. Moreover, the sun and the four planets whose surface markings are distinctly visible rotate in the same direction. So great a uniformity can scarcely be the result of chance.

In order to treat the matter numerically, suppose there are 800 bodies whose planes of motion do not differ from the plane of the earth's orbit by more than 18°, and whose directions of motion are the same as that of the earth. Since the inclination of an orbit could be anything from 0° to 180°, the chance that it would lie between 0° and 18° is $\frac{1}{10}$. The probability that the planes of the orbits of two bodies would be less than 18° is $\left(\frac{1}{10}\right)^2$. And the probability that the same would be true for 800 bodies is only $\left(\frac{1}{10}\right)^{800}$, or unity divided by 1 followed by 800 ciphers. This probability is so small that we are forced to the conclusion that the arrangement of the planets in the solar system is not accidental. Both Kant and Laplace made use of this line of reasoning.

A planet may revolve around the sun in an orbit of any eccentricity from 0 to 1. Of the more than 800 planets and planetoids, the orbits of 624 have eccentricities less than 0.2, the orbits of all except 26 have eccentricities less than 0.3, and the orbit of only one has an eccentricity greater than 0.5. These remarkable facts imply that some systematic cause has been at work which has produced planetary orbits of low eccentricity. And both the positions of the planes and the small eccentricities of the orbits of the planets prove conclusively that the solar system, in all its history, has not been subject to any important external disturbance, such as a closely passing star.

244. Distribution of Mass in the Solar System.—Nearly all the matter of the solar system is concentrated in the sun. In fact, all the planets together contain less than one seventh of one per cent of the mass of the entire system. Although the mass of Jupiter is more than 2.5 times that of all the other planets combined, it is less than one thousandth that of the sun.

It is important to know whether the masses of the sun and planets are now changing. There is certainly at present no appreciable transfer of matter from one body to another. The sun may be losing some particles by ejecting them from its surface in an electrified condition, and a very small percentage of the ejected particles may strike the planets, but it is very improbable that the process has had important effects on the distribution of mass in the solar system, even in the enormous intervals of time required for its evolution.

The mass of the earth is slowly increasing by the meteoric material which it sweeps up in its journey around the sun. It is not unreasonable to suppose that the other planets, and possibly the sun, are growing similarly. This growth, at least in the case of the earth, is too slow at present to have a very important bearing on the evolution of the whole system. But if the meteors are permanent members of the solar system, the more they are swept up by the planets the more infrequent they become and the smaller the number a planet encounters in a day. Consequently, the acquisition of meteoric material by collision may once have been a much more important factor in the evolution of the planets than it is at the present time. In fact, so far as general considerations go, appreciable fractions of the masses of the planets may have been obtained from meteoric material. But it is improbable that the great sun has grown sensibly in this way.

It follows from this discussion that probably the remote antecedent of the solar system consisted of an overwhelming central mass and a very small quantity of matter distributed somewhat irregularly out from it to an enormous distance. At any rate, if this were not the original distribution of matter, the conditions must have been such that the central condensation resulted in harmony with the laws of dynamics. The ever-increasing distances between the planets is shown in Figs. 96 and 97. The relatively small masses of the planets and their enormous distances from one another are among the most remarkable facts that need to be taken into account when considering their origin and evolution.

An additional fact which must be noted is that the terrestrial planets contain the heaviest known substances. The sun also contains heavy elements (Art. 234), though the spectral lines of the very heaviest have not been found. The constitution of the large planets is not so well known, though it may be inferred from their low densities and moderate temperatures that they contain largely only the light elements. Any hypothesis as to the origin of the planets, in order to be satisfactory, must make provision for this distribution of the elements.

245. Distribution of Moment of Momentum.—In attempting to go back to the origin of the solar system it is natural to consider its mass and distribution of mass because matter is indestructible. For a similar reason, the distribution of the moment of momentum of the system among its various members is of fundamental importance. That is, if the solar system has undergone its evolution free from exterior disturbances, its total moment of momentum is now exactly equal to what it was at the beginning and at every stage of its development.

As has been stated, the small mutual inclinations of the orbits of the planets and the small eccentricities of their orbits both prove that the solar system has been subject to no important exterior influences since the planets were formed. Hence any hypothetical antecedent of the system must be assigned the quantity of moment of momentum it now possesses. Although this fact is perfectly clear, it was overlooked by Kant and was not given adequate consideration by Laplace and his followers.

In Table XII the mass and moment of momentum is given for the sun and each of the eight planets in such units that the sums are unity. The moment of momentum of the sun depends upon its law of density. In the computation it was assumed that the mass is concentrated toward the interior according to a law of increase of density formulated by Laplace. The rotations of the planets contribute so little to the final results that it is not important what law of density is used for them.

It is seen from this table that although the mass of the sun is 700 times as great as that of all the planets combined, its moment of momentum is only a little over $\frac{1}{40}$ that of the planets. Or, considering the material interior to the orbit of Saturn, it is found that while Jupiter contains only $\frac{1}{10}$ of one per cent, or $\frac{1}{1000}$, of the entire mass, it possesses more than 95 per cent of the moment of momentum.

One at once inquires whether the distribution of moment of momentum is now being changed. The mutual attractions of the planets

TABLE XII

BODY	MASS	MOMENT OF MOMENTUM
Sun	0.9986590	0.027423
Mercury	0.0000001	0.000017
Venus	0.0000025	0.000576
Earth	0.0000030	0.000827
Mars	0.0000003	0.000112
Jupiter	0.0009558	0.599273
Saturn	0.0002852	0.241924
Uranus	0.0000430	0.052845
Neptune	0.0000511	0.077003
Total	1.0000000	1.000000

produce some changes in the distribution of moment of momentum, but they are of no importance whatever in connection with the problem under consideration. The tides which a planet generates in the sun reduce the moment of momentum of the sun and increase that of the planet. But here again the results are inappreciable even for thousands of millions of years. The earth encounters meteoric matter in its revolution around the sun, and it is probable that the other planets are subject to similar disturbances. The result of the resistance by meteors is to reduce the moment of momentum of the planets. At present the effects of meteors on the motion of the earth are inappreciable, but it is not certain that they were not once important. However, whether or not they have ever been of importance, they cannot relieve the inequalities in the table, for they are decreasing the moment of momentum of the planets, which are still relatively very large. In fact, there have been no known influences at work which could have sensibly modified the distribution of the moment of momentum of the system since the sun and planets have been separate bodies.

It remains to inquire whether the sun and planets may not once have been parts of one mass with a distribution of moment of momentum quite different from that found at present. Since the planets are not receding from the sun, the only possibility is that the sun and the planets were formerly so expanded that the material of which they are composed was more or less intermingled.

According to the contraction theory of the heat of the sun, the sun's dimensions were formerly greater than they are at present. Indeed, the sun has been supposed to have once filled all the space now occupied by the planets. Followed backward in time, the sun is found to be larger and larger, rotating more and more slowly because its moment of momentum remained constant during contraction, and more and more nearly spherical because a rotating body becomes more oblate with contraction. It follows from the table that if the planets which are interior to Jupiter were added to the sun they would not have an important effect on its moment of momentum.

Now suppose the sun was once expanded out to the orbit of Jupiter; its radius was more than 1000 times its present radius, its volume was more than $1000^3 = 1,000,000,000$ times its present volume, and its density was correspondingly less. Even if it was not condensed toward the center, the density at its periphery was then less than one millionth of that of the earth's atmosphere at sea level. It follows from the fact that the moment of momentum was necessarily constant, that its period of rotation must have been about $70,000$ years. But Jupiter's period of revolution is about 12 years. Now, therefore, either Jupiter was then quite independent of the general solar mass; or, if not, in some unknown way this extremely tenuous material must have imparted to that minute fraction of itself which later became Jupiter enough moment of momentum to reduce the period of this part from $70,000$ years to 12 years. More specifically, it is seen from the table that Jupiter, which contains one tenth of one per cent of the mass of the solar system within the orbit of Saturn, carries over 95 per cent of the moment of momentum. It is incredible that this extreme distribution of moment of momentum could have developed from an approximately uniform distribution, especially in a mass of such low density, and no one has been able to formulate a plausible explanation of it. Consequently, it must be concluded that the distribution of moment of momentum in the solar system has not changed appreciably since it has been free from important exterior forces.

246. The Energy of the Solar System.—In considering the energy of the solar system, the discussion must include its kinetic energy, heat energy, potential energy, and subatomic energy.

The kinetic energy of a body is its energy of motion including translation, rotation, and internal currents. The kinetic energy of the solar system consists of its energy of translation and of the internal motions of its parts. The former cannot have changed except by the action of exterior forces. Moreover, its value is not accurately known, and it has no relation to the remaining energy of the system so long as no other celestial body is encountered. Therefore it will be given no further consideration in this connection. The mutual attractions of the planets change their translatory motions, but in such a way that the sum of their kinetic and potential energies remains constant.

The sun, planets, and satellites raise tides in one another. In these tides there is some friction in which kinetic energy degenerates into heat energy, which is radiated away into space. In this way the solar system is losing energy. The heat energy from all other sources is likewise being lost by radiation.

The potential energy of a system is equal to the work which may be done upon it, in virtue of the relative positions of its parts, by the forces to which it is subject. For example, a body 100 feet above the surface of the earth is subject to the attraction of the earth. The earth would do a certain amount of work upon the body in causing it to fall from an altitude of 100 feet to its surface. This work equals the potential energy of the body in its original position. In the case of the translations of the planets, as has been stated, the sum of their kinetic and potential energies is constant. But if the sun or a planet contracts, the potential energy of its expanded condition is transformed into heat (Art. 216), which is at least partly lost by radiation. In this way the total energy of the system decreases, and the diminution may be large in amount.

There is certainly a large amount of subatomic energy in uranium, radium, and probably in all other elements. In the case of the radioactive substances this energy is slowly transformed into heat, which is dissipated by radiation. As has been suggested (Art. 219), the subatomic energies may be liberated in great quantities under the extreme conditions of pressure and temperature which prevail in the interior of the sun.

Since the solar system is losing energy in several ways and acquiring only inappreciable amounts from the outside, as, for example, the radiant energy received from the stars, it originally had more energy than at present, and this condition must be satisfied by all hypotheses respecting its evolution.

XIX. QUESTIONS

1. What is the probability that when 3 coins are tossed up they will all fall heads up? What is the probability that in a throw of 4 dice there will be 4 aces up? If 100 coins were found heads up, could it reasonably be supposed that the arrangement was accidental? How would its probability compare with that that the positions of the orbits of the planets and planetoids are accidental?

2. Suppose a star should pass near the solar system in the plane of the orbits of the planets; would it disturb the positions of the planes, or the eccentricities, of their orbits?

3. How many tons of meteors would have to strike the earth daily in order to double its mass in 200,000,000 years? How many would daily strike each square mile of its surface?

4. What is the definition of moment of momentum? How does it differ from momentum? Is it manifested in various forms like energy? Does the loss of energy of a body by radiation change its moment of momentum?

5. The mass of the earth is 1.2 times that of Venus (Table XII); why is its moment of momentum more than 1.2 times that of Venus?

6. Could the total energy of the solar system have been infinite at the start? Can the system have existed in approximately its present condition for an infinite time?

7. When carbon and oxygen unite chemically, heat is produced; is this heat energy developed at the expense of the kinetic, potential, heat, or subatomic energies of the original materials?

III. THE PLANETESIMAL HYPOTHESIS[1]

247. Brief Outline of the Planetesimal Hypothesis.—The fundamental conditions imposed by the distribution of mass and moment of momentum in the solar system, together with many supplementary considerations, have led to the planetesimal hypothesis. According to this hypothesis, the remote ancestor of the solar system was a more or less condensed and well-defined central sun, having slow rotation, surrounded by a vast swarm of somewhat irregularly scattered secondary bodies, or planetesimals (little planets), which all revolved in elliptical orbits about the central mass in the same general direction. This organization evidently satisfies the data of the problem. Moreover, the

[1]The Planetesimal Hypothesis was developed by Professor T. C. Chamberlin and the author in 1900 and the following years.

spiral nebulæ (Art. 302) offer numerous examples of matter which is apparently in this state.

According to the planetesimal hypothesis, our present sun developed from the central parent mass and possibly some outlying parts which fell in upon it because they had small motions of translation. The revolving scattered material contained nuclei of various dimensions which, in their motions about the central sun, swept up the remaining scattered material and gradually grew into planets whose masses depend upon the original masses of the nuclei and the amount of matter in the regions through which they passed. The angles between the planes of the orbits were gradually reduced by the collisions, and at the same time the eccentric orbits became more nearly circular. In the process of growth the planetary nuclei acquired their forward rotations.

248. Examples of Planetesimal Organization.—The planetoids afford a trace of the former planetesimal condition of the solar system. The average inclination and the average eccentricity of their orbits are considerably larger than the corresponding quantities for the planets. If the region which they occupy had been swept by a dominating nucleus, they would have combined with it in a planet occupying approximately the mean position of the planes of their orbits and having a small eccentricity (Art. 252).

Another example of planetesimal organization is furnished by the particles of which the rings of Saturn are composed. One might at first thought conclude that they would have formed one or more satellites if dominating nuclei had been revolving around the planet in the zone which they occupy. But they are very close to Saturn, and a satellite revolving at their distance would be subject to the strains of the tides produced by the planet. As has been stated (Art. 183), Roche showed that a fluid satellite could not revolve within 2.44 radii of a planet without being broken up, unless its density were greater than that of the planet. Since the rings of Saturn are within this limit, it follows that they could not have formed a satellite, and that a large nucleus revolving among them, instead of sweeping them up, would itself have been reduced to the planetesimal condition, unless it was solid and strong enough to withstand great tidal strains.

The examples of planetesimal organization which have been given may not be very convincing. But we may inquire whether there are not numerous examples in the heavens, beyond the solar system, confirmatory of the planetesimal theory. The answer is in the affirmative. There are tens of thousands of spiral nebulæ that are almost certainly in the planetesimal condition, though on a tremendous scale. They consist of central sunlike nuclei which are generally well defined, and arms of widespreading, scattered material. Their arms in most cases probably contain large masses, but they are small in comparison with the central suns. Their great numbers imply that they are in general semi-permanent in character. Consequently, the material of which the arms are composed cannot in general be moving along them, either in toward or out from the central nucleus, for under these circumstances they would condense into suns or dissipate into space, and in either case lose their peculiar characteristics. Besides this, matter subject to the law of gravitation could not move along the arms of spirals. It is therefore believed that in a spiral nebula the arms are composed of material which, instead of proceeding along them, moves across them around the central nucleus as a focus. The spirals owe their coils to the fact that the inner parts revolve faster than the outer parts. As a rule they radiate white light, which indicates that they are at least partly in a solid or liquid state. When a spiral is seen edgewise to the earth there is a dark band through its center, doubtless produced by dark, opaque material revolving at its periphery.

While a few spiral nebulæ have been known for a long time, their great numbers were not suspected until Keeler began to photograph them with the Crossley reflector at the Lick Observatory. In a paper published in 1900 shortly before his death, he said:

"1. Many thousands of unrecorded nebulæ exist in the sky. A conservative estimate places the number within the reach of the Crossley reflector at about 120,000. The number of nebulæ in our catalogues is but a small fraction of this.

"2. These nebulæ exhibit all gradations of apparent size from the great nebula in Andromeda down to an object which is hardly distinguishable from a faint star disk.

"3. Most of these nebulæ have a spiral structure.... While I must leave to others an estimate of the importance of these conclusions, it seems to me that they have a very direct bearing on many, if not all, questions concerning the cosmogony. If, for example, the spiral is the form normally assumed by a contracting nebulous mass, the idea at once suggests itself that the solar system has been evolved from a spiral nebula, while the photographs show that the spiral is not, as a rule, characterized by the simplicity attributed to the contracting mass in the nebular (Laplacian) hypothesis. This is a question which has already been taken up by Chamberlin and Moulton of the University of

Chicago."

While the spirals are almost certainly examples of planetesimal organization, those which have been photographed are enormously larger than the parent of the solar system unless, indeed, there are many undiscovered planets beyond the orbit of Neptune. But, as Keeler remarked, there is no lower limit to the apparent dimensions of the spiral nebulæ, and it is possible that many of them are actually of very moderate size.

249. Suggested Origin of Spiral Nebulæ.—Although the validity of the planetesimal theory does not hang upon any hypothesis as to the origin of spiral nebulæ, yet, if the solar system has evolved from a spiral nebula, the theory of its origin will not be regarded as complete and fully satisfactory until the mode of generation of these nebulæ has been explained. The best suggestion regarding their genesis, which is due primarily to Chamberlin, is as follows:

There are several hundreds of millions of stars in the heavens and they are moving with respect to one another with an average velocity of about 600,000,000 miles per year. While their motions are by no means entirely at random, yet there are millions of them moving in essentially every direction. It is inevitable that in the course of time every star will pass near some other star. If two stars should collide, the energy of their motion would largely be changed into heat and the combined mass would be transformed into a gaseous nebula. If they should simply pass near one another without striking, an event which would occur many times more frequently than a collision, a spiral nebula would probably be formed, as will now be shown.

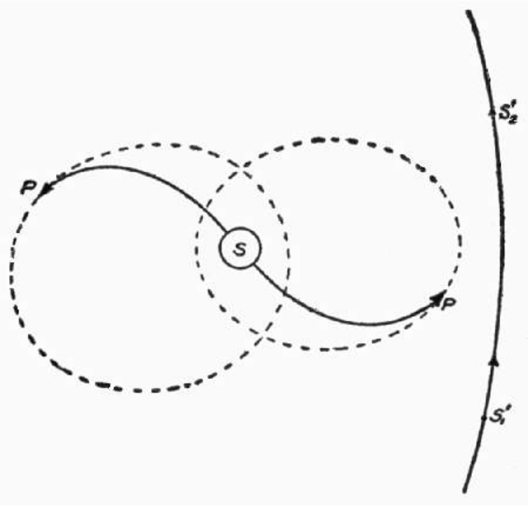

FIG. 156. — Deflection of ejected material by a passing star.

Consider two stars passing near each other. They both move about their center of gravity, but no error will be committed in representing one of them as being at rest and the other as passing by it. If the stars are equal, their effects on each other are the same, but in order not to divide the attention, only the action of S' on S will be considered.

Consider S' when it is at the position S'_1, Fig. 156. It raises tides on S, one on the side toward S' and the other on the opposite side. The heights of the tides depend upon the relative masses of the two suns and their distance apart compared to the radius of S. An approach within 10,000,000 miles is more than 100 times as probable as even a grazing collision. At this distance the tide-raising force of S' on S compared to the surface gravity of S is more than 2000 times the tide-raising force of the moon on the earth compared to the surface gravity of the earth. The tide-raising force varies directly as the radius of the disturbed body and inversely as the cube of the distance of the disturbing body (Art. 153). Hence, if the nearest approach were 5,000,000 miles, the tide-raising force would be more than 16,000 times greater, relatively to surface gravity, than that of the moon on the earth. This force would raise tides approximately 500 miles high if the sun were a homogeneous fluid, and there would be a corresponding slight constriction of the sun in a belt midway between the tidal cones. The tides on a highly heated gaseous body would probably be much higher.

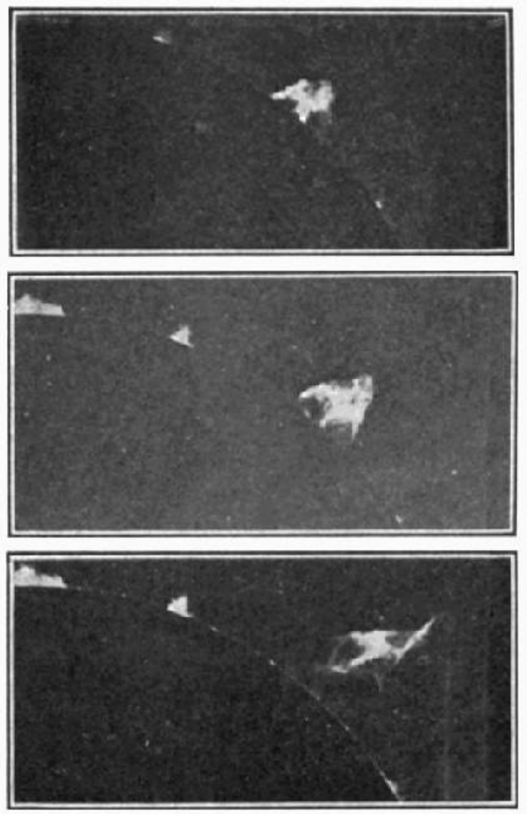

FIG. 157. — Eruptive prominence at three altitudes. *Photographed by Slocum at the Yerkes Observatory.*

The sun is the seat of violent explosive forces which now often eject matter in the eruptive prominences to distances of several hundred thousand miles (Fig. 157). If the sun were tidally distorted, the eruptions would be mostly toward and from the disturbing sun; certainly the ejections would reach to greater distances in these directions. Besides this, after the ejected material had once left the sun, its distance would be increased still further by the attraction of S'. Consequently, if S' were not moving along its orbit, the ejections toward and from it would be to more remote distances than they would be in any other direction. In fact, those toward S' might even strike it. But S' would be moving along in its orbit, and, in a short time, it would have a component of attraction at right angles to the original direction of motion of the ejected matter. Consequently, by the time S' had arrived at S'_2, the paths of the ejected masses would be curved somewhat like those shown in Fig. 156. It is easy to see that, for the mass ejected toward S', the curvature is in the right direction; a discussion based on the resolution of the forces involved (Art. 153) proves that, for the mass ejected in the other direction, the indicated curvature is also correct. Eventually S' would move on in its orbit so far that it would no longer have sensible attraction for the ejected masses, and they would be left revolving around S in elliptical orbits. If the initial speed of the ejected material were very great, it might leave S never to return.

The critical question is whether matter would be ejected far enough to produce the large orbits required by the theory. In order to throw light on this question the following table has been computed, giving the surface velocities necessary to cause undisturbed ejected matter to recede various distances from the surface of the sun.

The most remarkable thing shown in the table is that after a velocity is reached sufficient to cause the ejected matter to recede a few millions of miles, a small change in the initial speed produces radically different final results. Since prominences now ascend to a height of half a million of miles without the disturbing influence of a visiting sun, it is seen that the numerical requirements of the hypothesis are not excessive. Moreover, numerous actual computations of hypothetical cases have shown that, on the recession of S', the ejected material is usually left revolving around S in elliptical orbits.

As one star passes another the ejection of material is more or less continuous. When the visiting star is far away, the ejections are to moderate distances and the matter returns to the sun. As the visiting star approaches, the ejected materials recede farther and their paths become more curved. At a certain time the lateral disturbance of S' becomes so great that the ejected material revolves around S instead of falling back upon it. Let the orbits for this case be those marked 1 and $1'$ in Fig. 158, the former being toward S', and the latter away from it. At a later time the ejections will be to greater distances and the

TABLE XIII

HEIGHT OF ASCENT	INITIAL VELOCITY	HEIGHT OF ASCENT	INITIAL VELOCITY
100,000 mi.	72 mi. per sec.	5,000,000 mi.	353 mi. per sec.
200,000 mi.	121 mi. per sec.	10,000,000 mi.	368 mi. per sec.
300,000 mi.	157 mi. per sec.	20,000,000 mi.	376 mi. per sec.
400,000 mi.	184 mi. per sec.	50,000,000 mi.	380 mi. per sec.
500,000 mi.	206 mi. per sec.	100,000,000 mi.	382 mi. per sec.
1,000,000 mi.	268 mi. per sec.	500,000,000 mi.	383 mi. per sec.
2,000,000 mi.	316 mi. per sec.	Infinite	384 mi. per sec.

materials will have greater lateral motions. Suppose they are 2 and $2'$, and so on for still later ejections until S' recedes from S.

Now consider the location of all of the ejected material at a given time after S' has passed its nearest point to S. If it has been sent out from S continuously, it will lie along two continuous curves, represented by the full lines in Fig. 158. These are the arms of the spiral nebula whose individual particles move across them in the dotted lines. The diagram shows an ideal simple case, and Fig. 159 an actual photograph. But if the approach of S' were close, or if there were a partial collision, and if the ejected material should go beyond S', a very complicated structure would result. The arms of the spiral might be very irregular (Fig. 160), the particles might cross them at a great variety of angles, and some of them might continue to recede indefinitely.

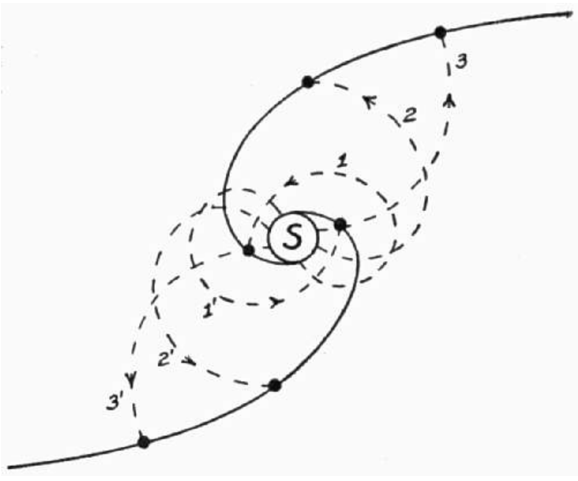

FIG. 158.—The origin of a spiral nebula.

Thus, the suggested explanation of the origin of the spiral nebulæ rests upon the existence of a great number of stars, their rapid and somewhat heterogeneous motions which imply near approaches now

FIG. 159. — The great spiral nebula in Canes Venatici (M. 51), showing the two arms. *Photographed by Ritchey at the Yerkes Observatory.*

and then, their eruptive activities, and the disturbance of one star by another passing near it. All the factors involved are well established—the only question is that of their quantitative efficiency. Here some doubts remain. It follows from the number of stars, the space they occupy, and their motions that, if they were moving at random, an individual sun would pass near some other one, on the average, only once in many thousands of millions of years. Perhaps the mutual gravitation of the stars is important out on the borders of the great clusters of suns of which the Milky Way is composed, where it may reasonably be supposed that their relative velocities are small, and it may be that in these regions close approaches are for this reason much more frequent. But in any case the demands of time are very formidable. Besides this,

FIG. 160. — The great spiral nebula in Triangulum (M. 33). *Photographed by Ritchey at the Yerkes Observatory.*

many of the spiral nebulæ are of such enormous dimensions that it is difficult to suppose they have been produced by the encounter or near approach of ordinary suns. It may be stated, however, that, in the first place, there is no positive knowledge whatever respecting the masses of spiral nebulæ; and that, in the second place, near approaches are not confined to single stars, but may involve multiple stars, clusters, and systems of stars. The observed spirals may be simply the larger examples originating from several or many suns.

It should be remembered that, whatever doubts may remain respecting the validity of this or any other hypothesis, the spiral nebulæ certainly exist in great numbers, and they apparently have, on an enormous scale, an organization similar to that which we have inferred must have been the antecedent of the solar system. And it may be stated again that the planetesimal hypothesis rests primarily upon the evidence now furnished by the solar system, and that it does not stand or fall with any theory respecting spiral nebulæ.

250. The Origin of Planets.—According to the planetesimal hypothesis, the parent of the solar system consisted of a central sun surrounded by a vast swarm of planetesimals which moved approximately in the same plane in essentially independent elliptic orbits. Among these planetesimals there were nuclei, or local centers of condensation, which, in their revolutions, swept up the smaller planetesimals and grew into planets. It is not to be understood that the original nuclei were solid or even continuous masses. It is much more probable that in their early stages they were swarms of smaller masses having about the same motion with respect to the central sun, and that, under their mutual attractions and collisions, they gradually condensed into continuous bodies. Indeed, the condensation may have been very slow and may have been dependent to an important extent upon the impacts of other planetesimals.

It seems to be impossible to determine the probable masses of the original nuclei. If they were less than that of the moon at present, they could not have retained any atmospheres under their gravitative control. But as the nuclei grew, their surface gravities increased, and a time came when those which have become the larger planets possessed sufficient gravitative power to prevent the escape of atmospheric particles. The acquisition of atmospheres was then inevitable because, in the first place, the materials grinding together and settling under the weight of accumulating planetesimals would squeeze out the lighter elements; in the second place, the pulverizing and heating effects of the impacts of meteors would liberate gases; and, in the third place, the growing planets in their courses around the sun would sweep up directly great numbers of atmospheric molecules. The extent of the atmospheres of the planets at all stages of their growth depended primarily on their surface gravities.

The rate at which the nuclei swept up the planetesimals must have been excessively slow. This conclusion follows from the fact that if all the matter in the largest planet were scattered around the sun in a zone reaching halfway to the adjacent planets, the resulting planetesimals would be very far apart, and also from the fact that the orbits of only a fraction of them would at any one time intersect the orbit of the nucleus. It must be remembered that the orbits of the planetesimals were continually changed by their mutual attractions and especially by the attractions of the nuclei. Moreover, the orbits of the nuclei were continually altered by collisions with the planetesimals and by their perturbations of one another. Consequently, if the orbits of the nuclei and certain planetesimals did not originally intersect, they might very well have done so later. But it does not follow that they have all been swept up yet, or, indeed, that they all ever will be swept up. Possibly some of the meteors which the earth now encounters are the straggling remains of the original planetesimals.

If the planetesimal theory is correct, the earth is very old and the sun must have important sources of energy besides its contraction. Most of the geological processes did not begin until it became large enough to retain water and an atmosphere. These same conditions were necessary for even the beginnings of the development of life, which may have had a continuous existence from the time the earth was half its present size.

251. The Planes of the Planetary Orbits.—If the planetesimal hypothesis is true, it must explain the important features of the solar system. The most striking thing about the motions of the planets is that they all go around the sun in the same direction, and the mutual inclinations of the planes of their orbits are small. However, some deviations exist, and in general they are greatest in case of the small masses like Mercury and the planetoids.

It is assumed that the planetesimals all revolved around the sun in the same direction. This would certainly have been true if they originated by the close approach of two suns, as explained in Art. 249. But the planes of their orbits would not be exactly coincident. The plane of motion of an ejected particle would depend upon its direction of ejection and the forces to which it was subject. The ejections would be nearly toward or directly away from the visiting sun, but slight deviations would be expected because the ejecting body might be rotating in any direction, and the direction of ejection would depend to some extent upon its rotation.

Consider, therefore, a central body surrounded by an enormous swarm of planetesimals which move in intersecting elliptical orbits, some close to the sun and others far away. The system of planetoids now in the solar system gives a fair picture of the hypothetical situation, especially if, as seems very probable, there are countless numbers of small ones which are invisible from the earth. Suppose, also, that there exist a number of nuclei revolving at various distances. They gradually sweep up the smaller masses, and the problem is to determine what happens to the planes of their orbits.

Consider a nucleus and all the planetesimals which it will later sweep up. All together they have what may be called in a rough way an average plane of revolution. This is a perfectly definite dynamical quantity which Laplace treated and which he called the "invariable plane."

When all the masses have united, the resulting body will inevitably revolve in this plane. If the nucleus originally moved in some other plane, the plane of its orbit would continually change as its mass increased. The same would be true for every other nucleus. There would be also an average plane for the whole system. Those nuclei which moved in regions that were richest in planetesimals, and that grew the most, would, in general, have final orbits most nearly coincident with this average plane. It is clear that so far as the planes of the orbits of the planets are concerned (see Table IV), the consequences of the planetesimal theory are in perfect harmony with the facts established by observation.

252. The Eccentricities of the Planetary Orbits.—The orbits of the original planetesimals probably had a considerable range of eccentricities. This view is supported by the fact that the eccentricities of the orbits of the planetoids vary from nearly zero to about 0.5. It is also supported by the computations of orbits of particles which were assumed to be ejected from one sun when another was passing it. The problem is to find whether nearly circular planetary orbits would be evolved from such a system of planetesimals.

When a nucleus sweeps up a planetesimal, the impact on the larger body may be in any direction. If the nucleus overtakes the planetesimals so that they act like a resisting medium, the eccentricity of its orbit is in general diminished, as was proved by Euler more than 150 years ago. But many other kinds of encounters can occur between bodies all moving in the same direction around the sun. Collisions will obviously be most numerous between bodies whose orbits are approximately of the same dimensions; if the orbits of two bodies differ greatly in size, collision between them is impossible unless the orbits are very elongated. It is a remarkable general proposition that if two bodies are moving in orbits of the same size and shape, but differently placed, and if they collide in any way, the eccentricity of the orbit of the combined mass will be smaller than the common eccentricity of the orbits of the separate parts.[1]

Of course, if two orbits were of exactly the same size, the periods of the bodies would be the same and collisions would result either at the first revolution or only after their mutual attractions had modified their motions. But if they were of nearly the same size, the conditions for collisions would be favorable, and in nearly all cases the eccentricity would be reduced.

It follows from this discussion that, in general, collisions between planetesimals cause the eccentricities of their orbits to decrease. Consequently, the more a nucleus grows by sweeping up planetesimals, the more nearly circular, in general, its orbit will be. If a nucleus revolves in a region rich in planetesimals, the result is likely to be a large planet whose orbit has small eccentricity. These conclusions agree precisely with what is found in the solar system, for the orbits of all the large planets are nearly circular, while the orbits of some of the smaller plan-

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[1]To prove this, suppose a nucleus M and a planetesimal m are moving in orbits whose major semi-axis and eccentricity are a_0 and e_0. Let their velocities at the instant preceding collision be V_0 and v_0, and their combined velocity after collision be V. The kinetic energy of the two bodies at the instant preceding collision is $\frac{1}{2}(MV_0^2 + mv_0^2)$. Their kinetic energy after their union is $\frac{1}{2}(M+m)V^2$. The latter will be smaller than the former because some energy will have been transformed into heat by the impact of the two parts. Therefore $MV_0^2 + mv_0^2 > (M+m)V^2$.

It is shown in celestial mechanics in the problem of two bodies that in elliptic orbits $V^2 = \frac{2}{r} - \frac{1}{a}$. Hence, the inequality becomes

$$M\left(\frac{2}{r} - \frac{1}{a_0}\right) + m\left(\frac{2}{r} - \frac{1}{a_0}\right) > (M+m)\left(\frac{2}{r} - \frac{1}{a}\right),$$

where a is the major semi-axis of the combined mass. It follows from this inequality that $\frac{M+m}{a_0} < \frac{M+m}{a}$, whence $a < a_0$. That is, under the circumstances of the problem a collision always reduces the major semi-axis of the orbit.

Another principle established in celestial mechanics is that the moment of momentum is constant whether there are collisions or not. The orbital moment of momentum of a mass m is $m\sqrt{a(1-e^2)}$, where e is the eccentricity. The condition that the moment of momentum before collision shall equal that after collision is, therefore,

$$M\sqrt{a_0(1-e_0^2)} + m\sqrt{a_0(1-e_0^2)} = (M+m)\sqrt{a(1-e^2)}, \text{ or}$$
$$\sqrt{a_0(1-e_0^2)} = \sqrt{a(1-e^2)}.$$

Since $a_0 > a$, it follows that $\sqrt{(1-e_0^2)} < \sqrt{1-e^2}$, and therefore that $e < e_0$.

ets and many of the planetoids are considerably eccentric.

253. The Rotation of the Sun.—If the central body in the planetesimal system rotates in the direction of the motion of the outlying parts, the final result will be a sun rotating in the direction of revolution of its planets. But if the planetesimal organization is the result of the close approach of two suns, the central mass might originally have been rotating in any direction. In this case the final outcome is not quite so obvious.

The only planetesimals which could sensibly affect the rotation of the central mass are those which fall back upon it. If the planetesimals originated by the close approach of two suns, there would certainly be many which would return to the central mass. They would not fall straight in towards its center, but would have a small forward motion similar in character to that of the remainder of the planetesimals. The result of the collision would be that the sun would acquire their moment of momentum. It does not seem unreasonable that the mass of the central sun might grow in this way by as much as 10 per cent. Since the planetesimals would have enormously more moment of momentum than equal masses in the central body, they would substantially determine its direction of rotation. In fact, if they were moving in orbits whose eccentricity was 0.9 and if they just grazed the sun at their perihelion, the mass necessary to account for the present rotation of the sun, if it had no rotation originally, would be one fifth of one per cent of the sun's mass.

Another interesting result remains to be mentioned. The planetesimals would strike the equatorial region of the sun in greatest abundance and would give it the most rapid motion. Unless the inequalities in motion were worn down by friction the equatorial zone would be rotating fastest, as is the case with our own sun.

254. The Rotations of the Planets.—The earth, Mars, Jupiter, and Saturn rotate in the direction in which the planets revolve; the surfaces of the other planets have not been observed well enough to enable astronomers to determine how they rotate. It has been generally supposed that the equators of Uranus and Neptune coincide with the planes of the orbits of their satellites, but the evidence in support of the supposition is as yet inconclusive.

The earlier theories regarding the origin of the planets all fail to explain their forward rotations.

Chamberlin has shown that if a planet develops from a planetesimal system it will in general rotate in the direction of its revolution.

Consider a nucleus N, Fig. 161, which, in its early stages, will probably be simply an immense swarm of planetesimals. For simplicity, suppose its orbit is a circle C around the sun as a center (if this assumption were not made, the discussion would not be essentially modified). The planetesimals which can encounter N are divided into three classes: (a) those whose aphelion points are inside the circle C; (b) those whose perihelion points are inside C and whose aphelion points are outside of C; and (c) those whose perihelion points are outside of C. They are designated by (a), (b), and (c) respectively in Fig. 161.

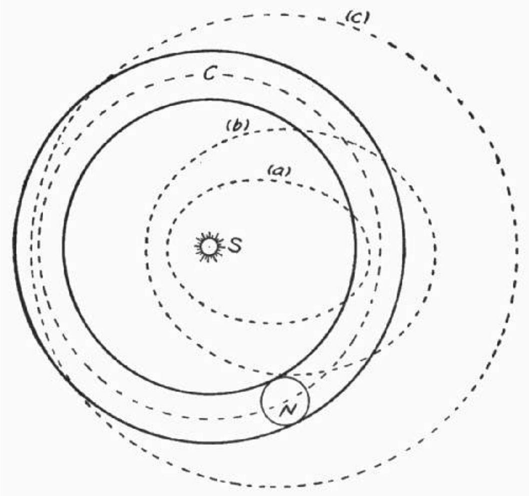

FIG. 161.—Development of the forward rotation of a planet nucleus by the accretion of planetesimals.

Consider collisions of the planetesimals of class (a) with the nucleus N. A collision can occur only when a planetesimal is near its aphelion point. At and near this point the planetesimal is moving slower than the nucleus.[1]

Hence the nucleus will overtake the planetesimal, and the collision will be a blow backward on the inner side of the nucleus. That is, planetesimals of class (a) tend to give the nucleus a forward rotation.

Planetesimals of class (b) can strike the nucleus so as to tend to give it a rotation in either direction, or so as not to have any effect on its rotation. If they are not distributed in some special way, the collective result of the collision of many of them will be very small.

Planetesimals of class (c) move faster than the nucleus at the time of collision. Therefore they overtake the nucleus and tend to give it a forward rotation.

[1]Let V and v represent the velocity of the nucleus and planetesimal respectively, and A and a the semi-axes of their orbits. It is shown in celestial mechanics that $V^2 = \frac{2}{r} - \frac{1}{A}$, and $v^2 = \frac{2}{r} - \frac{1}{a}$. Since $a < A$ and r is the same in the two equations, it follows that $V^2 > v^2$.

It follows from this discussion that two of the three classes of planetesimals tend to give the nucleus a forward rotation. The effects are most important at the equator of the planet, for there they strike farthest from its axis. Hence, the impacts of planetesimals on the whole tend to make the equators of fluid planets rotate faster than the higher latitudes, as is the case with Jupiter and Saturn. The precise final result depends upon the initial rotation of the nucleus and upon the distribution of the planetesimals among the three classes.

Obviously the relative numbers of planetesimals in classes (a) and (c) would in general be small. In order to get some idea of the numbers required to account for the observed rotations, a numerical example has been treated. It was assumed that the original earth nucleus had no rotation and that the planetesimals of class (b) gave it none. It was assumed that all the planetesimals of classes (a) and (c) moved in orbits having the eccentricity 0.2 and that they struck the nucleus 4000 miles from the center. Then, in order to account for the present rotation of the earth, it was found that their total mass must have been about 5.7 per cent of that of the whole earth. Whether or not these results are reasonable cannot be determined without further quantitative investigations. But it must be insisted that the results are qualitatively correct, and that not even this much can be said for any earlier hypothesis regarding the origin of the planets.

In the preceding discussion the effects of the rotations of the original nuclei, or swarms of planetesimals out of which the nuclei condensed, have been ignored. As a matter of fact, they were probably in rotation around axes essentially perpendicular to the plane of the system. There seems to be no conclusive reason why the original rotations should have been in one direction rather than in the other. The observed rotations of the planets seem to indicate that, for some reason at present unknown, the original nuclei rotated in the forward direction.

255. The Origin of Satellites.—According to the planetesimal theory, the satellites developed either from small secondary nuclei which were associated with the larger planetary nuclei from the beginning, or from neighboring secondary nuclei which became entangled at a later time in the outlying parts of the swarms of planetesimals constituting the nuclei. If the satellites originated in the former way, their directions of revolution would be the same as those of rotation of their respective primaries; if in the latter way, they might revolve originally in any directions around their primaries.

With the exception of the eighth and ninth satellites of Jupiter and the ninth satellite of Saturn (and possibly the satellites of Uranus and Neptune), all the known satellites revolve in the directions in which their primaries rotate. This seems to indicate that at least most of the satellites originated from secondary nuclei which were associated with their respective primary nuclei from the beginning and partook of their common motion of rotation. The satellite nuclei, like the planetary nuclei, swept up the planetesimals and grew in mass. The craters on the moon may have been produced by the impact of planetesimals.

With the growth in mass of a planet its attraction for its satellites increases and this results in a reduction in the dimensions of their orbits. Suppose the most remote direct satellites were originally revolving at the greatest distances at which their primaries could hold them in gravitative control, and that their orbits have been reduced to their present dimensions by the growth of the planets. The amount of reduction in the size of the orbit of a satellite depends upon the amount of growth of the planet around which it revolves, and furnishes the basis for computing the increase in the mass of the planet.

The three retrograde satellites revolve at great distances from their respective primaries in orbits which are rather eccentric and considerably inclined to their respective systems. Their origin is evidently different from that of the direct satellites. They may have been neighboring planetesimals which became entangled in the remote parts of the planetary swarm. The question arises why they revolve in the retrograde direction. The answer probably depends upon the fact that, at a given distance, a retrograde satellite is much more stable, so far as the disturbance of the sun is concerned, than a direct one. Consequently, a retrograde satellite would not be driven by collisions away from the control of its planet so easily as a direct one. Also, the effects of collisions with planetesimals and satellitesimals (planetesimals revolving around planetary nuclei) must be considered.

256. The Rings of Saturn.—The rings of Saturn are swarms of particles revolving in the plane of the planet's equator. According to the planetesimal theory, they are the remains of outlying masses in the original nucleus which were moving so fast that they did not fall toward the center. Of course, they were subject to encounters with infalling planetesimals. These collisions transformed some of their energy of motion into heat and some of them fell toward, or perhaps on to, the growing planetary nucleus. It may be that only a small part of the original ring material now remains. But when they fell, they retained at least a portion of their motion of revolution, and the result was that they struck the planet obliquely in the direction in which it rotated. This increased its rotation, especially in the plane of its equator.

There may be and probably are collisions even now among the particles which constitute the rings of Saturn. If there are collisions, the energy of motion is being transformed into heat, and this comes from the energy of the orbital motions, with the result that the dimensions of the rings are being decreased. They may ultimately disappear for this reason, and it is not impossible that other planets also once had ring systems.

257. The Planetoids.—The planetoids occupy a zone in which there was no predominating nucleus. They probably have not grown so much relatively as the planets by the accretion of planetesimals. Hence the ranges in the eccentricities and inclinations of their orbits give a better idea of the character of the orbits of the original planetesimals.

Besides the known planetoids, there are probably thousands of others which are so small that they have not been seen. There may be others also between the orbits of Jupiter and Saturn and beyond the orbit of Saturn. At those vast distances none but large bodies would be visible, both because they would not be strongly illuminated by the sun and also because they would always be very remote from the earth. The planetoid Eros has escaped collision with Mars only because of the inclination of its orbit. It is not unreasonable to suppose that there are many other planetesimals between the orbits of the earth and Mars which are too small to be visible.

258. The Zodiacal Light.—It is universally agreed that the zodiacal light is due to a great swarm of small bodies, or particles, revolving around the sun near the plane of the earth's orbit. These small bodies are in reality planetesimals which have not been swept up by the planets either because of the high inclination of their orbits, or more probably because their orbits are so nearly circular that they do not cross the orbits of any of the planets.

259. The Comets.—Recent investigations have shown that it is very probable that comets are permanent members of the solar system. As they have no intimate relationship to the planets, the question of their origin presents new problems and difficulties.

According to the planetesimal theory, the comets are possibly only the outlying and tenuous fragments of the original nebula which did not partake of the general rotation of the system. If the planetesimal system was produced by the near approach of two suns, they may have had their origin, as Chamberlin has suggested, in the dispersion and scattering of earlier planetesimals which revolved in different planes; or there may have been explosions of lighter gases in various directions, which, under the disturbing action of the visiting sun, did not fall back upon our own; or the comets may be matter which was ejected from the visiting sun. The differences in the lengths of their orbits and in the positions of the planes of their orbits may originally have been much less than at present, for the planets may have disturbed their motions to almost any extent. The planets may have captured some comets and greatly enlarged the orbits of an equal number of others, and they may have entirely changed the positions of the planes of the cometary orbits.

260. The Future of the Solar System.—The theory has been developed that the planets have grown up from nuclei by the accretion of scattered planetesimals. They acquired and retained atmospheres when their masses became great enough to prevent the escape of gases, molecule by molecule. Their masses are still increasing, but the process of growth seems to be essentially finished. Those planets which are dense and solid like the earth will retain all their essential characteristics as long as the sun continues to furnish radiant energy at its present rate. The large rare planets will lose heat from their interiors and may contract appreciably. The reason that loss of heat may be important for them and not for the solid planets is that it can be carried to the surface rapidly by convection in a gaseous or liquid body, while in a solid body it is transferred from the interior only by the excessively slow process of conduction.

The duration of the sun is a very important factor in the future of the planets. There is no known source of energy which could supply its present rate of radiation many tens of millions of years. Yet it is not safe to conclude that the sun will cool off in a few millions of years because the earth gives indisputable evidences (Art. 219) that the sun has radiated more energy than could have been supplied by any known source. The existence of hundreds of millions of stars blazing in full glory also suggests strongly that the lifetime of a sun is very long, for it is not reasonable to suppose that, if they endured only a comparatively short time, so many of them would now have such great brilliancy. In view of these uncertainties it is not safe to set any definite limit on the future duration of the sun, however probable its final extinction may be.

If the sun cools off before something destroys the planets, they will revolve around it cold, lifeless, and invisible, while it pursues its journey through the trackless infinity of space. If the radiation of the sun does not sensibly diminish, the earth, and possibly some of the other planets, will continue to be suited for the abode of life until they are in some way destroyed. Whether or not the sun becomes cold, the planets will be broken into fragments when our sun passes sufficiently near another star. Their remains may then be dispersed among the revolving masses of a new planetesimal system, to become in time parts of new planets. Indeed, the meteorites which now strike the earth often give evidence of having once been in the interior of masses of planetary dimensions, and Chamberlin has suggested that they may be the remains of a family of planets antedating our own. To such dizzy heights are we led in sober scientific pursuits!

The question of the purpose of all the wonderful things in the universe is one which ever arises in the human mind. With sublime egotism men have usually answered that everything was created for their pleasure and benefit. The sun was made to give them light by day, and the moon and the myriads of stars to illuminate their way by night. The numberless plants and animals of forest and prairie and sea were supposed to exist primarily for the profit of the human race. But with the increase of knowledge this conclusion is seen to be absurd. How infinitesimal a part of the solar system and its energy man can use, to say nothing of that in the hundreds of millions of other systems which are found in the sky!

How many billions of creatures were born, lived, and died before man appeared! The precise time of the beginning of life on the earth and the manner of its origin are lost in the distant past. In the oldest rocks laid down as sediments tens of millions of years ago in the Archeozoic era there are indications of the existence of low forms of life on the earth. In the Cambrian period trilobites and other lowly creatures lived in great abundance. In the Ordovician period the types of low forms greatly increased and the vertebrates began to appear; in the Silurian, they were firmly established; in the Devonian, they were still further developed. And after many other geological periods had passed, the higher forms of life, including man, appeared. Now man finds himself a part of this great life stream, not something fundamentally different from the rest and that for which it exists. If the earth shall last some millions or tens of millions of years in the future, as seems likely, the physical and mental changes which the human race will undergo may be as great as those through which the animal kingdom has passed during the long periods of geological time. This is especially probable if men learn how to direct the processes of their own evolution. But if they do not, the human race may become extinct just as many other species of animals have become extinct. However this may be, it seems certain that its end will come, for eventually the light of the sun will go out, or the earth and the other planets will be wrecked by a passing star, and the question of the purpose of it all, if indeed there is any purpose in it, still remains unanswered.

XX. QUESTIONS

1. Are the particles which produce the zodiacal light an example of the planetesimal organization?

2. In the case of one star passing by another, why would their ejections of material be largely toward or from each other?

3. Show by a resolution of the forces that the material ejected both toward and from S' will describe curves around S in the same direction.

4. Will the orbit of S' be changed if it changes the moment of momentum of the system S? What will be the result in the very special case where the orbit of S' relatively to S is originally a parabola?

5. In view of Table XIII, what fraction of the material ejected from S would reasonably be expected (a) to fall back on S, (b) to revolve around it in the planetesimal state, (c) to escape from its gravitative control? On the basis of these figures, find what fraction of S would need to be ejected altogether in order to provide material for the planets.

6. Would the eccentricities of the orbits of the material which fell back upon S have been large or small? Would most of the collisions have been grazing, as was assumed in the discussion in Art. 253?

7. In view of the kinetic theory of gases, would a gaseous nucleus as massive as the moon concentrate or dissipate? Would a nucleus of the materials found in the sun remain gaseous on cooling?

IV. Historical Cosmogonies

261. The Hypothesis of Kant.—The work of Thomas Wright, which preceded that of Kant by five years, was concerned chiefly with the evolution of the whole sidereal universe. Wright supposed the Milky Way is made up of a great number of mutually attracting systems which are spread out in a great double ring rotating about an axis perpendicular to its plane. Kant was the first one to undertake the development of a detailed theory of the evolution of the solar system on the basis of the law of gravitation.

Kant's interest in cosmogony was aroused by the book of Wright, which fell into his hands in 1751. He at once turned his keen and penetrating mind to the question of the origin of the planets, and wrote a brilliant work on the subject. On almost every page he gave proof of the intellectual power which later made him the foremost philosopher of his time, yet his theories were not without serious imperfections.

Kant postulated that the parent of the solar system was a vast aggregation of simple elements, without motion and subject only to gravitational and chemical forces and the repulsion of molecules in a gaseous state. Nothing could have been simpler for a start. The problem was to show how such a system could develop into a central sun and a family of widely separated planets.

Kant reasoned that motions among the molecules would be set up by their chemical affinities and mutual attractions. He stated that the large molecules would draw to themselves the smaller ones in their immediate neighborhood, and that with growth their power of growing would continually increase. He believed that not only would aggregations of molecules be formed, but that these masses would acquire motions both because of the attraction of the system as a whole and also because of their mutual attractions. Kant called attention to the fact that attraction would be opposed by gaseous expansion, and he supposed that these repulsive forces in some obscure way would generate lateral motions in the small nuclei. At first the nuclei would be moving in every possible direction, but he assumed that successive collisions would eliminate all except a few moving in the same direction in nearly circular orbits.

The beauty and generality of Kant's theory are enticing, but it involves some obvious and fatal difficulties. In the first place, the attractive and repulsive forces would not be competent to set up a general revolution of a system which was originally at rest. His conclusion in this matter squarely violates the principle that the moment of momentum of an isolated system remains constant.

Notwithstanding clear statements by Kant, some writers have modified his theory by supposing that there was heterogeneous motion of the original chaos with a predominance in the direction in which the planets now revolve. But with this concession to the theory, which makes it dynamically a different theory, difficulties still remain. It is not at all clear that in a system of such enormous extent the orbits of all bodies except those having motion in the dominant direction would be destroyed by collisions. There is, indeed, no apparent reason why, if this were the true history of the origin of the planets, some planets should not now be found revolving at right angles to the general plane of the system, or even in the retrograde direction. This is not impossible, as is proved by the motions of the comets. Thus it is seen that if Kant's hypothesis is taken strictly as he gave it, the condition that the moment of momentum of the system should have its present value is violated, and that if the postulates are changed so as to relieve this difficulty, others still remain.

Kant's theory has also secondary difficulties of a serious nature. For example, in a gas the mutual attractions of the molecules could not draw them together into small nuclei. Even the moon could not now add to its mass if it should pass through a gas. To avoid this difficulty one might assume that there was first condensation into the liquid or solid state. So many molecules would be involved in the formation of even the minutest drop that, by an averaging process, their lateral motions would essentially destroy one another, the particle would fall toward the center of the whole system, and no planets would be formed. In order to avoid this difficulty it is necessary to depart from Kant's ideas and to assume either that the whole gaseous mass was rotating with considerable velocity, or that the matter was not in a gaseous state. If the first of the two assumptions is made, it is found by a mathematical treatment that the moment of momentum of the system would be more than 200 times what it is at present. Since the moment of momentum would remain unaltered, the second alternative must be adopted. But this is directly contrary to the fundamental assumptions of Kant, and it is hardly permissible to regard a theory as having preserved its identity after having been modified to this extent. The condition to which one is forced, viz., that of discrete particles in orbital revolution in the same direction, is actually the planetesimal organization.

In successive chapters Kant considered the densities and ratios of the masses of the planets, the eccentricities of the planetary orbits and the origin of comets, the origin of satellites and the rotation of the planets, etc. He even claimed to have proved without observational evidence the existence of life on other planets. In spite of the keenness of his intellect and his remarkable powers of generalization, his theory has not had much influence on speculations in cosmogony, because it is marred by so many serious errors in the application of physical and dynamical laws.

262. The Hypothesis of Laplace.—The hypothesis of Laplace appeared near the end of a splendid popular work on astronomy which he published in 1796. He advanced it "with that distrust which everything ought to inspire that is not a result of observation or of calculation." How great an advance over Kant this one sentence indicates!

In outline, the hypothesis of Laplace was that originally the solar atmosphere (in later editions a nebulous envelope), in an intensely heated state, extended out beyond the orbit of the farthest planet; the whole mass rotated as a solid in the direction in which the planets now revolve; the dimensions of the solar atmosphere were maintained mostly by gaseous expansion of the highly heated vapors, and only slightly by the centrifugal acceleration due to the rotation; as the mass lost heat by radiation, it contracted under the mutual gravitation of its parts; simultaneously with its contraction, its rate of rotation necessarily increased because the moment of momentum remained constant; after the rotating mass had contracted to certain dimensions the centrifugal acceleration at the equator equaled the attraction by the interior parts and an equatorial ring was left behind, the remainder continuing to contract; a ring was abandoned at the distance of each planet; a ring could scarcely have had absolute uniformity, and, separating at some point, it united at some other because of the mutual attractions of its parts and formed a planet; and, finally, the satellites were formed from rings which were left off by the contracting planets, Saturn's rings being the only examples still remaining.

FIG. 162.—Laplace.

The contraction theory of the sun's heat, which was developed by Helmholtz in 1854, fitted in very well with the Laplacian hypothesis and was considered as supporting it. Some objections to the Laplacian theory, however, began to appear. In 1873 Roche, the author of the theorem that a satellite would be broken up by tidal strains if its distance from its primary should become less than 2.44 radii of the latter, seriously undertook to modify the hypothesis of Laplace so as to relieve it of the difficulties with which it was beset. His modifications were for the most part improbable and do not in the least meet later objections. Kirkwood, an American astronomer, criticized the Laplacian hypothesis and pointed out that the direct rotation of the planets might be due to the effect of the sun's tides on them when they were expanded in the gaseous state. In 1884 Faye made very radical modifications of the hypothesis of Laplace for the purpose of avoiding the difficulties in which it was becoming involved. He supposed that the planets were formed in the depths of the solar nebula and that those nearer the sun are actually older than those which are more remote. About 1878 Darwin began his great work on the tides which he regarded as supplementing and strengthening the hypothesis of Laplace.

It is now generally recognized that the Laplacian hypothesis fails because it does not meet the most fundamental requirements of the problem. For example, the density of the hypothetical solar atmosphere must have varied in harmony with the laws of gases. With this distribution of density, which can be theoretically determined, and the rotation which is given by the revolution of the planets, it is an easy matter to compute the moment of momentum possessed by the hypothetical system when it extended out to the orbit of Neptune. It turns out to be more than 200 times that of the system at present. If the hypothesis of Laplace were correct, the two would be equal; the discrepancy is so enormous that the hypothesis must be radically wrong.

The details of the Laplacian hypothesis are subject to equally serious difficulties. For example, it would be impossible for successive rings to be left off. Kirkwood long ago pointed out that if instability in the equatorial zone once set in, it would persist, and Chamberlin has shown that the result would be a continuous disk of particles describing nearly circular orbits. Further, if a ring were left off, it could not even begin to condense into a planet because both gaseous expansion and the tidal forces due to the sun would more than offset the mutual gravitation of its parts. It has been seen how large and dense[1] a planet must be in order to hold an atmosphere; while the ring would be large, its density would be extremely low and it could not control the lighter elements. And it has been shown that even if a circular ring had in some way largely condensed into a planet, the process could not have completed itself. In order that a nucleus may gather up scattered materials, it is

[1]The power of control of a planet on an atmosphere is proportional to the product of its density and radius.

necessary that they shall be moving in considerably eccentric orbits.

Since the Laplacian hypothesis fails in the fundamental requirement of moment of momentum, as well as in a number of other essential respects, it will be sufficient simply to enumerate some of the phenomena which are obviously not in harmony with it:

(1) It does not provide for the planetoids with their interlacing orbits, some having high inclinations or eccentricities.

(2) It does not permit of the existence of an object having such an orbit as that of Eros, which reaches from near that of the earth out beyond that of Mars.

(3) It implies that a continuous disk of particles, such as that producing the zodiacal light, cannot exist.

(4) It does not anticipate the considerable eccentricity and inclination of Mercury's orbit.

(5) It does not agree with the fact that the terrestrial planets seem to be at least as old as the more remote ones.

(6) It does not permit of there being any retrograde satellites because the rings abandoned by a contracting nebula would necessarily all rotate in the same direction.[1]

(7) It implies that the rotation period of each planet shall be shorter than the shortest period of revolution of its satellites. This condition is not only violated in the case of the inner satellite of Mars, but the particles of the inner ring of Saturn revolve in half the period of the planet's rotation.

263. Tidal Forces.—The sun and moon generate tides in the oceans that cover the earth. Tides are raised also in the atmosphere and in the solid earth itself. Similarly, every celestial body raises tides in every other celestial body. The first problem which will be considered here will be the character of the tide-raising forces, and the second will be the effects of the tides on the rotations and revolution of the two bodies.

Consider the tide-raising effects of m on M, Fig. 163. For simplicity, consider the effects of m on P and P', two particles on the surface of M. The problem of the resolution of the forces is that which was treated in Art. 153. Let MA represent the acceleration of m on M in direction and amount. Then the acceleration of m on P and P' will be represented by PB and $P'B'$ respectively. The former is greater

[1] Attempts have been made, though not successfully, to avoid this difficulty by invoking tidal friction (Art. 264).

than MA because the acceleration varies inversely as the square of the distance, and Mm is greater than Pm. For a similar reason $P'B'$ is less than MA. Now resolve PB into two components, PC and PD, in such a way that PC shall be equal and parallel to MA. Similarly, resolve $P'B'$ into $P'C'$, equal and parallel to MA, and $P'D'$. Since PC and $P'C'$ are equal and parallel to MA, they have no tendency to displace P and P' respectively with respect to M. The remaining components, PD and $P'D'$, are the tide-raising accelerations.

Now consider the tide-raising forces all around M. They are as indicated in Fig. 94. The forces toward m are slightly greater than

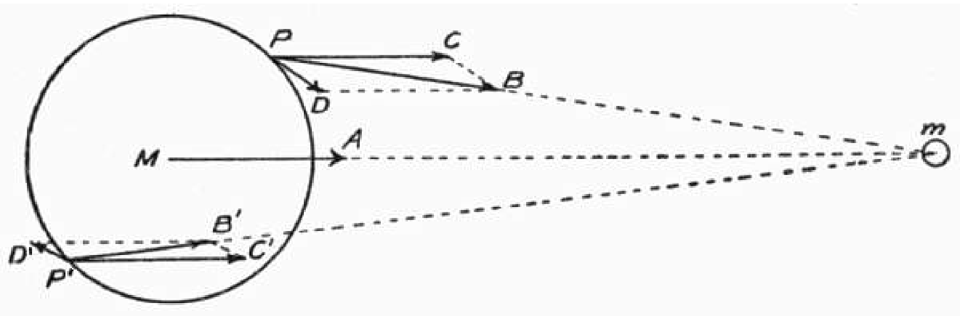

FIG. 163.—The tide-raising force.

those in the opposite direction, while the compressional forces at 90° from these directions are half as great. It is clear from this figure that if M were not rotating and m were not revolving around it, there would be a tide on the side of M towards m, and a nearly equal tide on the side of M away from m (compare Art. 153). The motions of the bodies produce important modifications.

Suppose the rotation of M and the revolution of m are in the same direction and that the period of rotation of M is shorter than that of the revolution of m. This is the case in the earth-moon system. Under these circumstances the tides T_1 and T_2 are carried somewhat ahead of the line Mm, as represented in Fig. 164. The more nearly equal the rates of rotation of M and revolution of m, the more nearly will the tides T_1 and T_2 be in the line Mm.

Consider a point on the rotating body M. It will first pass the line Mm, and then somewhat later it will pass the tide T_1. The interval is the lag of the tide. In the case of the earth-moon system a meridian passes eastward across the moon (the moon seems to pass westward across the meridian), and somewhat later the meridian passes the tidal cone and has high tide. The problem is enormously complicated in the case of the earth by the addition of the tides due to the sun, by the varying distances of the moon and sun north or south of the celestial equator, by their changing distances from the earth, and especially by the irregular contours of the continents and the varying depths of the oceans. These modifying factors are so numerous and in some cases so poorly known that at present it is not possible to predict entirely in advance of observation either the times or heights of the tides; but, after a few observational data have established the way in which the tides at a station depend upon the unknown factors, predictions become thoroughly reliable, for the tides vary in perfect harmony with the tidal forces.

264. Tidal Evolution.—The tides are not fixed on the surface of M, Fig. 164, unless the period of its rotation equals the period of revolution of m. When the periods are unequal so that the tides move around the rotating body, some energy is changed to heat by friction. This energy comes from the kinetic and potential energies of the system; and, in accordance with the laws of dynamics, the transformation of energy takes place in such a way that the total moment of momentum of the system remains unchanged. Of course, in general there will be tides on both of the mutually attracting bodies.

The character of the transformation of energy that takes place under tidal friction depends upon the dynamical properties of the system. Suppose that the motions of rotation and revolution are in the same direction and that the period of M is shorter than that of m. It can be shown that under these circumstances the periods of both M and m and their distance apart are increased. The reason that the period of rotation of M is increased is that m has a component of attraction back on both T_1 and T_2, Fig. 164, as can be shown by resolving the forces as they were resolved in Fig. 163. If m pulls T_1 and T_2 backward, it follows from the reaction of forces that T_1 and T_2 pull m forward. The result of a forward component on m is to increase the size of its orbit and to lengthen its period.

If m is near M, the effect of the tides on the period of revolution of m is greater than their effect on the period of rotation of M. If the bodies are far apart, the result is the opposite.

Suppose the two bodies are initially close together and that the period of rotation of M is only a little shorter than the period of revolution of m. The friction of the tides will lengthen both periods and increase the difference between them. If nothing else interferes, this will

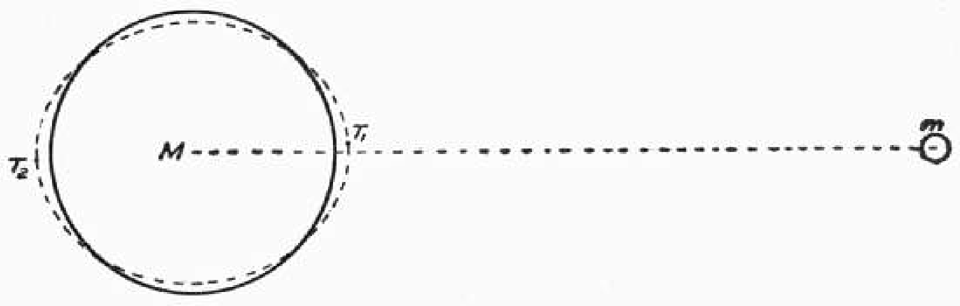

FIG. 164.—Tidal cones and the lag of the tides.

continue until a certain distance between the bodies has been reached. After that, the effect on the period of rotation of M will be greater than that on the period of revolution of m. Consequently, although both periods will continue to increase in length, they will approach equality. Eventually, the periods of rotation and revolution will be equal, the tides will remain fixed on M, and there will be no further tidal friction or tidal evolution.

The most important case from a practical point of view has been considered, but there are two others. In the first the bodies move in the same direction, but the period of rotation of M is longer than that of revolution of m. Under these circumstances both periods are decreased, the relative amounts depending on the distance of the bodies from each other. If the bodies are initially far apart, the effect will be greater on the period of rotation of M than on the period of revolution of m, and the two periods will approach equality. But if the bodies are near together, the effect will be relatively greater on the period of m, the periods will not approach equality, and the bodies will ultimately collide. In the second case the rotation of M is in the direction opposite to that of the revolution of m. Under these circumstances M rotates faster and faster, the distance of m continually decreases, and the inevitable outcome is the collision and union of the two bodies.

The rate at which tidal friction takes place depends upon the physical properties of the bodies. If they are perfect fluids so that there is no degeneration of energy, there is no tidal evolution. Likewise if they are perfectly elastic, there is no tidal friction.

The rate of tidal friction also depends upon the difference in the periods of the two bodies. If the difference between the periods is small, the tides T_1 and T_2, Fig. 164, are almost in the line Mm, and it is obvious that the backward components are small and the rate of tidal friction is very slow. Suppose the periods are approaching equality. The smaller their difference the slower is their rate of change, and they never become exactly equal but approach equality as the time becomes infinitely great.

265. Effects of the Tides on the Motions of the Moon.—The most striking thing in the earth-moon system is that the moon's periods of rotation and revolution are equal. It is extremely improbable that this unique relation is accidental. The only explanation of it heretofore advanced is that the moon's period of rotation has been brought into equality with its period of revolution by the tides generated in it by the earth.

The tidal force exerted by the earth on the moon is about 20 times the tidal force exerted by the moon on the earth. The amount of tidal friction is proportional to the square of the tidal force. Therefore, if the physical properties of the earth and moon were the same and if their periods of rotation were equal, the effectiveness of the tides generated by the earth on the moon in changing the moment of momentum of the moon would be 400 times that of the tides generated by the moon on the earth in changing the moment of momentum of the earth. Since the moment of momentum of a body is proportional to the product of its mass and the square of its radius, a given change in the moment of momentum of the moon alters its rate of rotation 1200 times as much as the same change in moment of momentum alters the rate of rotation of the earth. Consequently, taking the two factors together, if the earth and moon were physically alike and had the same period of rotation, tidal friction would change the period of rotation of the moon $400 \times 1200 = 480,000$ times as fast as it would change the period of rotation of the earth.

The results which have been obtained seem to be very favorable to the theory that the tides have caused the moon to present one side toward the earth, but some serious difficulties remain. It can be shown that, considering the tidal interactions of the earth and moon and the effect of the sun's tides on the moon, the present condition of the earth-moon system is not one of equilibrium. The tides raised by the earth on the moon have no effect under present circumstances on the rotation and revolution of the moon. The tides raised by the moon on the earth increase the length of the month but do not affect the rotation of the moon. The tides raised by the sun on the moon increase the moon's period of rotation but do not affect its revolution. Consequently the moon's periods of rotation and revolution are both increasing, and it is infinitely improbable that all the factors on which these effects depend are so related that they are exactly equal. Even if they were equal at one time, they would become unequal with a changed distance of the moon from the earth. That is, the present is not a fixed state of equilibrium, and the consideration of the tides does not remove the difficulties. It seems probable from this line of thought that some influence so far not considered has caused the moon always to present the same face toward the earth.

266. Effects of the Tides on the Motions of the Earth.—The theory of the tidal evolution of the earth-moon system, on the basis of certain assumptions regarding the physical condition of the earth, was elaborated by Sir George Darwin in a splendid series of investigations. While the experiment of Michelson and Gale (Art. 25) proves that his assumptions are not satisfied, at least at the present time, the possible sequence of events which he worked out is interesting.

Since the tides are increasing the lengths of both the day and the month, both of these periods were formerly shorter and the moon was nearer the earth. On the basis of his assumptions, Darwin traced the day back until it was only four or five of our present hours. At that time the moon was revolving close to the earth in a period almost equally short. This led him to the conclusion that at an earlier stage the earth and moon were one body, that they divided into two parts because of the rapid rotation of the combined mass, and that they have attained their present state as a consequence of tidal friction. The same reasoning leads to the conclusion that in the future they will continue to separate and that the day will continually increase in length.

The critical question is whether the physical properties of the earth are such that the rate at which tidal evolution takes place makes it an appreciable factor in the history of the earth. Darwin supposed the main effects were due to bodily tides in the earth which he assumed to be viscous. Since it is highly elastic, they cannot at present be important, but it has generally been assumed that, whatever its present condition may be, it was formerly viscous. There is absolutely no evidence to support the assumption, and if its present properties of solidity and elasticity are a consequence of the pressure in its interior, the assumption seems very improbable. As Poisson and Lord Kelvin showed, the temperature of the interior of the earth cannot have fallen appreciably in several hundreds of millions of years by the conduction of heat to its surface. Since the temperature of the interior of the earth cannot have changed appreciably, there seems to be no good ground for assuming that the earth was once viscous.

Since there cannot now be an important degeneration of energy in the bodily tides of the earth, tidal evolution must depend at present almost entirely upon the tides in the ocean and the atmosphere. The latter may be neglected without important error. The oceanic tides are so irregular that it is difficult to determine their effects on the rotation of the earth. But MacMillan has made liberal estimates of the unknown factors, and has found that at present the length of the day cannot be increasing at a rate of more than one minute in $30,000,000$ years.

It is possible to determine observationally the present rate of tidal evolution. The day and the month are increasing in length, but the effect on the day is the greater. Consequently, if the length of the month is measured in days, as is done practically, it will seem to be decreasing in length. It is found from observations that the moon is getting ahead of its predicted place from 4 to 6 seconds of arc in 100 years. That is, in 1240 revolutions the moon gets ahead of its predicted place about $\frac{1}{400}$ of its diameter. On the basis of these facts and the assumption that the increase in the length of the month is due to the tidal interactions of the earth and moon, the proper discussion shows that at the present time the length of the day is increasing as a consequence of all the factors affecting the rotation of the earth at the rate of one minute in $900,000,000$ years.

It is evident that tidal evolution is not an important factor in the earth-moon system for any period short of several hundred millions of years. Either the theory of tidal evolution as elaborated by Darwin must be abandoned as not representing what has actually taken place, or a condition for the earth's interior totally different from that which exists at present must be arbitrarily assumed.

267. Tidal Evolution of the Planets.—There is perhaps some slight evidence that Mercury and Venus always keep the same side toward the sun, and this condition has been ascribed to the effects of tides which the sun may have raised in them. The tidal force exerted by the sun on Mercury is about 2.5 times as great as that of the moon on the earth. In view of the fact that the moon's tides on the earth certainly do not have appreciable effects, it does not seem probable that the sun's tides have radically changed the rotations of Mercury and Venus. Besides this, it must be remembered that the condition of equality of periods of rotation and revolution would be attained in any case only after an infinite time.

The tidal action of the sun on the more remote planets is much less than that on the earth. No other satellite has relatively as great effects on its primary as the moon has on the earth. Consequently, we are forced to the conclusion that in the solar system tidal evolution has not been an important factor for a period of at least several hundreds of millions of years.

XXI. QUESTIONS

1. According to Kant's theory, why should the sun rotate in the direction the planets revolve?

2. Is the assumption of Laplace that the original nebula was highly heated in harmony with the present temperature of the sun and Lane's law? Why did Laplace make the assumption?

3. Why did Laplace assume that the original nebula was rotating as a solid?

4. To what extent does the contraction theory of the sun's heat support the Laplacian hypothesis? Is it opposed to the planetesimal hypothesis and Kant's hypothesis?

5. In what way does the Laplacian hypothesis fail to meet the requirements of moment of momentum?

6. On the basis of Lane's law, what was the temperature of the surface of the sun if it extended to the orbit of the earth? How do you account for the presence of refractory materials in the earth, under the Laplacian hypothesis?

7. Explain carefully in what respects the seven things mentioned at the end of Art. 262 are opposed to the Laplacian hypothesis.

8. What should be the present shape of the sun if the Laplacian hypothesis were true?

9. In the case of the earth and moon, what is the magnitude of the tidal force at the point on the side of the earth toward the moon compared to the whole attraction of the moon? Compared to the attraction of the earth?

10. The tides produced on the earth by the moon decrease the earth's moment of momentum; what becomes of that which the earth loses, and what changes in the system does it cause?

11. Show that when M rotates faster than m revolves, the attractions of m for both T_1 and T_2 tend to decrease the rate of rotation of M.

12. Suppose the rate of rotation of the earth is constant and that in a century the moon gets $6''$ ahead of the place it would occupy if its rate of revolution were constant. How long would be required for its period to decrease 1 per cent?

FIG. 165. — Milky Way in Aquila. *Photographed by Barnard at the Yerkes Observatory, August 27, 1905.*

CHAPTER XIII
THE SIDEREAL UNIVERSE

I. The Apparent Distribution of the Stars

268. On the Problems of the Sidereal Universe.—The invention of the telescope and the discovery of the law of gravitation were followed by a long period of successes in unraveling the mysteries of the solar system. The positions of the sun, moon, and planets were measured with extraordinary precision, and the law of gravitation accounted for the numerous peculiarities of their motions. What had been mysterious and inexplicable became familiar and thoroughly understood. Telescopes of continually increasing power enabled astronomers to measure accurately the distances and diameters of these bodies and to learn much of their surface conditions. At last the invention of the spectroscope enabled them to determine the chemical constitution of the sun.

There is great pleasure now in working in a science whose data are exact and whose laws are firmly established. The certainty of the results satisfies the human instinct for final truth. But there was also pleasure of a different kind for those pioneers who first explored the planetary system with good instruments and showed by mathematical processes that its members are obedient to law. For them every observation and every calculation was an adventure. They were continually in fear that their dreams of knowing the order prevailing in the universe would be shattered; they were continually elated by having their theories confirmed.

The pioneer days of discovery in the solar system are past. Not that great discoveries do not remain to be made, but they will henceforth fit into a large body of organized facts. From now on the romance and excitement of astronomical adventure will be largely furnished by the explorations of the sidereal universe. Astronomers have become accustomed to the fact that the sun is a million times as large as the earth, and familiarity has dulled their amazement at its terrific activities; from now on they must deal with millions of stars, some of them much larger and thousands of times more luminous than the sun. They have measured and at least partially grasped the enormous dimensions of the solar system; from now on they must conceive of and deal with distances millions of times as great. They have observed the differences in characteristics exhibited by eight planets; from now on they will be face to face with the infinite diversity presented by the stars. They have definitely accepted the doctrine that the solar system has undergone a great evolution whose details are yet much in doubt; the corresponding question for hundreds of millions of other systems is looming up more indistinctly through the fogs of uncertainties which still surround them. It might be supposed that astronomers would be tempted to lay down the arms of their understanding before the transcendental and appallingly difficult problems presented by the sidereal system. Instead, with all the weapons at their command, they are making more vigorous, persistent, and successful attacks than ever before. Astronomers of all the leading countries are united and coöperate in this campaign; they employ telescopes of many kinds, spectroscopes, photographic plates, measuring machines, and powerful mathematical processes in their attempts to penetrate the unknown.

269. The Number of Stars of Various Magnitudes.—The simplest and most easily determined thing about the stars is their number. Of course the number that can be seen depends upon the power of the instrument with which the observations are made. If the stars extend infinitely in every direction with approximately equal distances from one another, the number of them revealed by a telescope will be proportional to the space it brings within visual range. On the other hand, if the stars are less densely distributed at a great distance in any direction, then the number of faint distant stars seen in that direction will fall short of being proportional to the space penetrated by the instrument. For this reason it is important to find the number of stars of each magnitude down to the limits of range of the most powerful telescopes.

Consider first what the apparent distribution in magnitude would be if stars of every actual size and luminosity were scattered uniformly throughout space. Take a large enough volume so that accidental groupings will not sensibly affect the results. For example, suppose there are 5000 stars in the first six magnitudes and compute the number there should be, under the hypothesis, in the first seven magnitudes. The sixth-magnitude stars are $2.512\ldots$ times as bright as the seventh-magnitude stars. Since the magnitudes of stars of any given absolute brightness are directly proportional to the squares of their distances, it follows that stars of the seventh magnitude are $\sqrt{2.512}\ldots = 1.585\ldots$ times as distant as corresponding stars of the sixth magnitude. Therefore the volume occupied by stars out to the seventh magnitude, inclusive, is $(1.585\ldots)^3 = 3.98\ldots$ times that occupied by the first six magnitudes. Hence, if the stars were uniformly distributed and the light of the remote ones were in no way obstructed, there would be 3.98 ... times as many stars in the first seven magnitudes as in the first six magnitudes, or nearly 20,000 stars. The ratio is the same for the total number of stars up to any two successive magnitudes because the particular magnitudes do not enter into its computation. And it can be shown easily that the ratio of the number of stars of any magnitude to the number of the next magnitude brighter is also 3.98 ....

It remains to examine the results furnished by the observations. The stars are so extremely numerous that only a small fraction of the total number within reach of modern instruments has been counted. But an approximation to the solution of the problem of determining the number of stars has been obtained by counting sample regions of known size in many parts of the sky, and then multiplying the result by the number necessary to include the whole celestial sphere. By far the most extensive work of this kind has been carried out by Chapman and Melotte of the Royal Observatory at Greenwich. They made use of stars down to magnitude 17.5, where 4,000,000 of them send to the earth only a little more light than one star of the first magnitude. Their results are given in the following table.[1]

TABLE XIV

MAGNITUDE	NUMBER OF STARS	MAGNITUDE	NUMBER OF STARS
5 to 6	2,026	11 to 12	961,000
6 to 7	7,095	12 to 13	2,023,000
7 to 8	22,550	13 to 14	3,964,000
8 to 9	65,040	14 to 15	7,824,000
9 to 10	172,400	15 to 16	14,040,000
10 to 11	426,200	16 to 17	25,390,000

The ratio of the number of stars of a given magnitude to the number of stars one magnitude fainter is 3.5 at the beginning of the table, and it

[1]The numbers in the first of this table disagree with those in Table II because here, in the first line, for example, the number is that of stars from magnitude 5.0 to 6.0, while in Table II the corresponding number is that of stars whose magnitudes are 4.5 to 5.5.

continually decreases to 1.8 at the end. Therefore, not only is the ratio for every interval of one magnitude less than the 3.98 corresponding to uniform distribution of the stars, but it falls off about 50 per cent in 12 magnitudes.

What conclusions can be drawn from the facts given by the table? It is certain that the stars cannot be uniformly distributed to indefinite distances unless there is something which prevents their light from coming to us. If there were a sufficient number of dark stars and planets, the light from remote luminous stars would be shut off; but the number of non-luminous bodies required to account for the black sky would be millions of times the number of bright ones. In spite of the fact that certain variable stars (Art. 288) prove the existence of relatively dark bodies, and that analogy with the planets would lead to the conclusion that there are many non-luminous bodies of secondary dimensions, it seems extremely improbable that they are sufficiently numerous to explain the observed phenomena. But if the obscure matter were finely divided, as in meteoric dust, a given mass of it would be a much more effective screen,[1] and the total mass requirements would not be so severe. Finely divided material would not only absorb light, but it would scatter the blue light and cause distant stars to appear redder than nearer stars of the same character.

There are certain phenomena which give slight support to the hypothesis that there is some scattering of light of this nature, but they are not conclusive. One of them is directly related to the question in hand. Kapteyn found from an investigation of stars down to the fourteenth magnitude, part of the data being furnished by the visual observations of Sir John Herschel, that the number of stars of the fainter magnitudes is much greater than is given in the table of Chapman and Melotte. The faintest stars used in the construction of their table are obtained from the Franklin-Adams photographic charts of Greenwich. Turner has suggested that, because of the scattering of light, the remote faint stars may be deficient in the blue end of the spectrum, to which photographic plates are most sensitive, and consequently that a considerable part of the stars belonging visually to a certain magnitude belong photographically to a fainter magnitude. In spite of these possible indications of scattered particles, it seems extremely improbable that the falling off of the star ratio from 3.98 to 1.8 is due appreciably

[1]The effectiveness of opaque matter of given total mass in cutting off light is inversely proportional to the radius of its separate parts.

to this cause.

The most obvious, though not necessary, conclusion which has generally been drawn from the table is that the stars are limited in number and that they occupy a limited portion of space. In the first seventeen magnitudes there are in round numbers 55,000,000 stars. Chapman and Melotte derived a simple formula which represented the numbers closely for these magnitudes, and then, under the assumption that the same formula holds indefinitely beyond, they determined the magnitude for which there are as many stars brighter as there are fainter, and computed the total number of stars altogether. By this process they concluded that the median magnitude lies between 22.5 and 24.3, which are several magnitudes beyond the reach of existing instruments, and that the number of stars of all magnitudes is between 770,000,000 and 1,800,000,000. It is obvious that such an extrapolation is hazardous, and they did not lay any particular stress on the results. In fact, the data given by the observations can be as exactly represented by many other less simple formulæ which will give totally different results for the fainter magnitudes.

There is an even simpler line of reasoning which has led many astronomers to the conclusion that the material universe is limited. Since the stars of any magnitude are 2.512 times fainter than those of the next preceding magnitude, and, under the hypothesis of uniform distribution, 3.98 times more numerous, it follows that if the star density did not diminish as the distance increases, the stars of each magnitude would give us $3.98 \div 2.512 = 1.58$ times as much light as those of the next magnitude brighter. Consequently, the first 20 magnitudes would give 17,000 times as much light as the first-magnitude stars, the first 100 magnitudes would give 168,000,000,000,000,000,000 times as much light, and so on. If there were no limit to the number of magnitudes and no absorbing material, there would be no limit, except for the mutual eclipsing of the stars, to the amount of light received from all of them. The sky would be everywhere ablaze with the average brightness of a star, perhaps equal to that of the sun. The stars in one hemisphere would give us more than 90,000 times as much light as the sun, but actually the sun gives us 15,000,000 times as much light as all the stars together. Therefore, unless much light is absorbed, the hypothesis of uniform distribution of the stars to infinity is radically false.

Is it necessary, therefore, to conclude that the number of stars is limited and that they occupy only a finite part of space? By no means;

simply that they cannot be distributed with approximate uniformity throughout infinite space. It was pointed out by Lambert long ago that, just as the solar system is a single unit in a galaxy of several hundred million stars, so the Galaxy may be but a single one out of an enormous number of galaxies separated by distances which are very great in comparison with their dimensions, and that these galaxies may form larger units, or super-galaxies, and so on without limit. There is nothing in such an organization which is inconsistent with the facts established by observation, for it is possible to build up infinite systems of stars in this way which would give us only a finite amount of light. Hence the conclusion to be adopted is that the sun is in the midst of an aggregation of at least several hundred millions of stars which form a sort of system, and that beyond and far distant from this system there may be other somewhat similar systems in great numbers, which may be units in larger systems, and so on without limit.

It is conceivable that the ether is not infinitely extensive, but that it surrounds the stars of the sidereal system (and other stellar systems if there are such) as the atmospheres surround the planets. Light could not come to us from beyond its borders, however many stars might exist there, as sound cannot come to the earth from other bodies beyond the limits of its atmosphere. It must be understood that this is merely a suggestion entirely without any observational basis.

270. The Apparent Distribution of the Stars.—The brighter stars are quite irregularly distributed over the sky, but a careful examination of the fainter of even those which can be seen with the unaided eye shows that they are considerably more numerous in and near the Milky Way than elsewhere. When those stars which can be seen only with the help of a telescope are included, the condensation toward the Milky Way is still more pronounced.

Precise numbers for all the stars are known only to the ninth magnitude; but the star counts of the Herschels, and especially the work of Chapman and Melotte, go much further and give what are very probably approximately correct results down to the seventeenth magnitude. Since the stars are apparently condensed toward the Milky Way, it is natural to use its plane as the fundamental plane of reference. According to E. C. Pickering the north pole of the Galaxy is in right ascension 190° and its declination is +28°. The Milky Way is very irregular in outline, and it is difficult to locate its center; but its median line is possibly not quite a great circle, from which it follows that the sun is somewhat out of the plane near which the stars are congregated.

Let the center of the Milky Way be the circle from which galactic latitudes are counted. Chapman and Melotte divided the sky up into eight zones, the first including the belt of galactic latitude 0° to ±10°, the second the two belts from ±10° to ±20°, the third the two belts from ±20° to ±30°, the fourth from ±30° to ±40°, the fifth from ±40° to ±50°, the sixth from ±50° to ±60°, the seventh from ±60° to ±70°, and the eighth the regions from ±70° to ±90° around the galactic poles. With the belts numbered in this order they found for the average number of stars in each magnitude in 10 square degrees the results given in Table XV.

TABLE XV

ZONE	I	II	III	IV	V	VI	VII	VIII
Galactic Latitude / MAG.	0 to ±10°	±10° to ±20°	±20° to ±30°	±30° to ±40°	±40° to ±50°	±50° to ±60°	±60° to ±70°	±70° to ±90°
1 to 5	0.27	0.23	0.15	0.11	0.11	0.11	0.13	0.13
6	0.7	0.7	0.5	0.4	0.3	0.3	0.3	0.3
7	2.6	2.3	1.8	1.5	1.2	1.1	1.1	1.1
8	8.0	7.0	6.1	4.8	3.8	3.4	3.2	3.1
9	24	21	18	14	10	10	9	8
10	62	55	50	38	28	26	22	20
11	157	135	123	93	63	62	52	47
12	363	311	280	199	136	141	115	100
13	798	658	569	409	276	295	240	205
14	1,642	1,354	1,142	770	531	572	482	392
15	3,253	2,650	2,080	1,390	940	1,050	916	773
16	6,150	4,936	3,680	2,340	1,680	1,830	1,630	1,400
17	11,540	9,170	6,350	3,980	2,870	3,100	2,990	2,610
Total	24,000	19,300	14,300	9,240	6,540	7,090	6,460	5,560

Three things follow from this table: (*a*) Stars of all magnitudes down to the seventeenth are more numerous in the plane of the Milky Way than near its poles. Since the only reasonable supposition is that the nearer stars are distributed more or less uniformly with no special relations to the Milky Way, it follows from the fact the bright stars are condensed near the Milky Way that some of them are very distant. That is, the stars differ greatly in absolute luminosity, a conclusion confirmed by direct evidence. (*b*) The decrease in the number of stars is on the average gradual from the Milky Way to its poles, showing that the sun is actually in the midst of the clouds of stars on which the table is based. (*c*) The relative condensation in the plane of the Milky Way is greater, the fainter the stars. This proves that the stars are not only much more numerous near the plane of the Milky Way, but also that they extend to much greater distances in this plane than in the direction of its poles. The counts of stars by Kapteyn, based in part on the visual observations of Sir John Herschel, give still greater relative condensation in the plane of the Milky Way, and still more strongly confirm this conclusion.

271. The Form and Structure of the Milky Way.—Before attempting to arrive at a more precise conclusion regarding the distribution of the stars in space, it is desirable to obtain a better idea of the form and properties of the Milky Way.

As has been stated, the center of the Milky Way is nearly a great circle around the celestial sphere. Its greatest northerly declination (45° to 65°) is at right ascension zero in the constellation Cassiopeia, where it is about 20° wide. It extends from this point southeastward across Perseus with very irregular outlines (Map I, Art. 82), and narrows down where it crosses the borders of Taurus to a width of about 5°. It then bulges wider in Monoceros and across the northeast corner of Canis Major. Farther south in Argo, with its several divisions, it becomes as much as 30° wide, but its borders are irregular, it is broken through by vacant lanes, one of which in its center is called the "coal sack," and at right ascension about 9 hours and declination 45° south a dark gap stretches almost across it. After reaching its most southerly point in Crux it stretches out in irregular outline through Centaurus, part of Musca, Circinus, Norma, and then north again into Ara, Lupus, and Scorpius. In Scorpius and in Sagittarius to the east are some of the most remarkable star clouds in the heavens, Fig. 166. Barnard's photographs of these regions show countless suns massed in banks, with intervening dark lanes, the whole often enveloped by a soft nebulous haze (see Fig. 167). Northeast of Scorpius lie Ophiuchus, Serpens, and Aquila. From Aquila and Ophiuchus northward through Vulpecula and Cygnus to Cepheus, the Milky Way is divided longitudinally by a rift of varying width and form. This bifurcation, which extends through more than 50° of its length, is one of its most remarkable features. In Cepheus the two branches join and reach on into Cassiopeia, where the description of the Milky Way began.

FIG. 166. — Great star clouds in Sagittarius. *Photographed by Barnard at the Yerkes Observatory.*

It is obvious that the stars do not form any simple system. It seems probable that the Galaxy is composed of a large number of star clouds,

FIG. 167.—The region of Rho Ophiuchi. *Photographed by Barnard.*

each with peculiarities of its own, but having relations to the whole mass of stars. Since the Milky Way is roughly in the form of a great discus, or "grindstone" as Herschel called it, the prevailing motions must be in its plane in order to have preserved its shape. This does not mean that the relative velocities would need to be great enough to be easily observed; they would, in fact, be very slight as seen from the enormous distances separating the stars from the earth.

XXII. QUESTIONS

1. Prove that the magnitudes of stars of equal absolute brightness are proportional to the squares of their distances.

2. Prove that, under the hypothesis of the second paragraph of Art. 269, the ratio of the number of stars of any magnitude to the number of the next magnitude brighter is 3.98.

3. If there are 2000 stars of magnitude 5 to 6, and if the ratio for successive magnitudes were 3.98, how many stars would there be of magnitude 16 to 17?

4. Prove that the effectiveness of a given mass in screening off light is inversely proportional to the radius of the particles into which it is divided.

5. Show in detail how it follows from Table XV and the assumption under (*a*) that some of the bright stars are very distant. How many of the 20 first-magnitude stars have parallaxes greater than $0''.2$ (see Table XVI)?

6. At what distance, expressed in parsecs (Art. 272), would the sun be a first-magnitude star? A sixth-magnitude star? If Canopus has a parallax of $0''.005$, how does its absolute brightness compare with that of the sun?

7. Prove that the area of one hemisphere of the sky is $92,000$ times the apparent area of the sun.

8. Prove in detail that conclusion (*b*) of Art. 270 follows from Table XV.

9. At what time of the year does the portion of the Milky Way which is divided by a longitudinal rift pass the meridian at 8 P.M.? If possible, observe it.

10. Draw a diagram and show that the fact that the central line of the Milky Way is not quite a great circle proves that the solar system is not in the center of the disk of stars of which the Milky Way is composed.

11. The fact that the Milky Way is very oblate implies that it has large moment of momentum about an axis perpendicular to its plane. What inference do you draw respecting the general motions of stars in exactly opposite parts of the Milky Way?

12. If all visible objects belong to the Galaxy, is it possible to prove the rotation of the Milky Way by observations of the stars?

13. What observational evidence disproves the hypothesis that there are infinitely many galaxies distributed with approximate uniformity, but separated from one another by distances which are enormous compared to their dimensions?

II. Distances and Motions of the Stars

272. Direct Parallaxes of the Nearest Stars.—One of the proofs that the earth revolves around the sun is that the apparent directions of the nearest stars vary with the position of the earth in its orbit (Art. 51). The difference in direction of a star as seen from two points separated from each other by the mean distance from the earth to the sun is the parallax of the star; or, in other terms, the parallax of the star is the angle subtended by the mean radius of the earth's orbit as seen from the star (Fig. 35). If the parallax were one second of arc, the distance of the star would be 206, 265 times[1] the mean distance from the earth to the sun. This distance, which is a very convenient unit in discussing the distances of the stars, is called the *parsec*, and for most practical purposes it may be taken equal to 200, 000 astronomical units, or 20, 000, 000, 000, 000 miles. It is the distance that light travels in about 3.3 years.

The stars are so remote that the problem of measuring their parallaxes is one of great practical difficulty. Alpha Centauri, the nearest known star, has a parallax of only $0''.75$. That is, its difference in direction as seen from two points on the earth's orbit, separated by the distance from the earth to the sun, is the same as the difference in direction of an object at the distance of 10.8 miles when seen first with one eye and then with the other. Not only is the difference in the apparent position of a star very small as seen from different parts of the earth's orbit, but it can be determined only from observations separated by a number of months during which climatic conditions and the instruments may have appreciably changed.

The best direct means of determining the parallax of a star is by comparing, at various times of the year, its apparent position with the positions of more distant stars. Let S, Fig. 168, represent a star whose parallax is required, and S' a much more distant star. When the earth is at E_1 the angular distance between them is $\angle SE_1S'$; when the earth is at E_2, it is SE_2S'. The parallax of S is $\angle E_1SE_2$; the parallax of S' is $E_1S'E_2$, which will be negligible if S' is sufficiently remote. It easily follows from the geometry of the figure that the parallax of S minus the parallax of S' equals the difference of the measured angles SE_1S' and SE_2S'. Hence, if the parallax of S' is inappreciable, the parallax of S can be found.

[1]This number is the number of seconds in the arc of a circle which equals its radius in length.

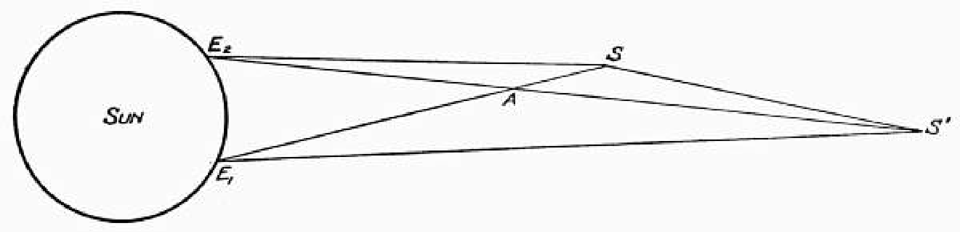

FIG. 168.—Determination of parallax from apparent changes in relative positions of stars.

In practice the position of S is measured with respect to a number of comparison stars. At present the work is done almost entirely by photography. Plates of a star and the surrounding region are secured at different times of the year, and the distances between the stars are measured under a microscope on a machine designed for the purpose. The scale of the photograph is proportional to the focal length of the telescope, and consequently for this purpose only large and excellent instruments are of value.

With present means of measurement, a parallax of $0''.02$ or less cannot be determined with sufficient accuracy to be of much value; in fact, the probable error in one of $0''.05$ is large. The great distances of the stars can be inferred from the fact that only about 100 are known whose parallaxes come within the wider of these limits.

The distances of stars whose parallaxes are $0''.2$ or greater can be measured with an error not exceeding about 25 per cent of the quantity to be determined. There are at present 19 such stars known, 9 of which are too faint to be seen without optical aid. These stars are given in Table XVI. When the distance of a star of known magnitude has been determined, the total amount of light it radiates, or its luminosity, as compared with the sun can be computed. The luminosity of each of the nineteen stars is given in the fifth column.

The 19 stars of Table XVI together with our sun occupy a sphere whose radius is 5 parsecs. If they were uniformly distributed in this space, the distance between adjacent stars would be about 3.7 parsecs, or 12.2 light years. In view of the fact that a number of stars in the list are far below the limits of visibility without optical aid, it is reasonable to suppose that there may be a considerable number of others within 5 parsecs of the sun which are as yet undiscovered.

It should not be supposed that attempts have been made to measure

TABLE XVI

STAR	MAG-NITUDE	PARAL-LAX	DISTANCE (PARSECS)	LUMINOSITY (SUN = 1)	MASS (SUN = 1)	VELOCITY (MI. PER SEC.)
		″				
α Centauri	0.3	0.76	1.32	2.0	1.9	20
Lalande 21,185	7.6	0.40	2.50	0.009	?	35+
Sirius	−1.6	0.38	2.63	48.0	3.4	11
τ Ceti	3.6	0.33	3.00	0.50	?	20
Procyon	0.5	0.32	3.13	9.7	1.3	12
C. Z. 5^h 243	8.3	0.32	3.13	0.007	?	170
ϵ Eridani	3.3	0.31	3.23	0.79	?	14
61 Cygni	5.6	0.31	3.23	0.10	?	63
Lacaille 9352	7.4	0.29	3.45	0.019	?	72
Pos. Med. 2164	8.8	0.29	3.45	0.006	?	23+
ϵ Indi	4.7	0.28	3.57	0.25	?	54
Groombridge 34	8.2	0.28	3.57	0.010	?	30+
OA(N.) 17,415	9.3	0.27	3.70	0.004	?	14+
Krueger 60	9.2	0.26	3.85	0.005	?	11+
Altair	0.9	0.24	4.17	12.3	?	22
η Cassiopeiæ	3.6	0.20	5.00	1.4	1.0	20
σ Draconis	4.8	0.20	5.00	0.5	?	30
Lalande 21,258	8.9	0.20	5.00	0.011	?	66+
OA(N.) 11,677	9.2	0.20	5.00	0.008	?	45+

the parallaxes of all stars brighter than the ninth, or even the sixth, magnitude. The process is excessively laborious, and only those stars are selected which are believed to be within measurable distance, or which are objects of especial interest for other reasons. A star with a given motion across the line of sight will apparently move faster the nearer it is to the observer. Consequently, those stars will be nearest on the average whose *proper motions*, as they are called, are greatest. As a rule only those stars are examined for parallax which have been found to have large proper motions.

Under the hypotheses that the stars are uniformly distributed throughout the space occupied by the Galaxy and that their density is the same as it is in the vicinity of the sun, the extent of the stellar universe can be computed. Suppose the space occupied by the stars is spherical in shape and that there are 500,000,000 of them. Then it turns out that, under the hypotheses adopted, the radius of this sphere is 1500 parsecs, or 5000 light-years. Since the Galaxy is very much flattened, the distance to its poles is probably only a few hundred parsecs while the borders of its periphery are probably several thousand parsecs from its center.

One very interesting and important conclusion follows from Table XVI, and that is that the luminosities of the stars vary enormously. For example, Sirius radiates 12,000 times as much light as OA(N.) 17,415. These differences in luminosity may be due to the fact that some stars are larger than others, or at least partly to the fact that some are intrinsically more brilliant than others. Probably both factors are important. Some stars are certainly much more massive than others, and the table gives examples of stars whose masses differ very much less than their luminosities. For example, while the mass of Sirius is only 3.4 times that of the sun, its luminosity is 48 times as great. But Sirius is a double star and presents in its own system a still more remarkable contrast. The mass of the brighter component is approximately twice that of the fainter one, but in luminosity it is at least 5000 times greater. There are other stars, such as Rigel and Canopus, which, though they are so remote that no evidence of their having measurable parallaxes has been found, shine with the greatest brilliancy. Their luminosity must be at least several thousand times that of the sun. In fact, the average luminosity of the stars visible to the unaided eye probably exceeds that of the sun several hundred fold. It must not be assumed from this that the luminosity of the sun is below the average, for it is exceeded in luminosity by only five of the 19 stars in the table.

In order to determine the velocity of a star its motion both along and across the line of sight must be found. The proper motions of all the stars in Table XVI are known, but the radial velocities of six of them are unknown; in these cases a plus sign is placed after the number giving the velocity because the radial component is not known. It follows from the table that the less luminous stars move with much higher velocities than the brighter ones. The average speed of those five stars whose luminosities exceed the sun is 17 miles per second, while the average speed of the six whose luminosities are less than 0.01 that of the sun is more than 50 miles per second. Since the more luminous stars are almost certainly the more massive, it follows that the more massive stars move more slowly than the smaller ones.

One may inquire to what extent reliance can be put in conclusions based on only 19 stars. When compared to hundreds of millions the number is ridiculously small, but all the conclusions which have been stated are strongly supported by the evidence furnished by the much more numerous stars having smaller and less accurately determined parallaxes.

273. Distances of the Stars from Proper Motions and Radial Velocities.—The parallaxes of possibly 100 stars have been determined by direct means with considerable accuracy. Probably not over 1000 are within reach of present instruments and methods. Are astronomers doomed to remain in ignorance as to the distances of all the other stars which fill the sky? By no means. There are several indirect methods of finding the average distances of classes of stars.

Consider all the stars of a large class, say the stars of the sixth magnitude. Suppose they are moving at random; that is, that they do not tend to move in any particular direction, or with any particular speed. Suppose both their proper motions and their radial velocities have been determined by observation. Under these hypotheses as many stars will be approaching as receding, and the velocities of approach will average the same as those of recession. Also, the proper motions will be as numerous and as large in one direction as in the opposite. The extent to which these conditions are fulfilled is a measure of the accuracy of the assumptions.

Whatever the individual motions of the class of stars under consideration, they will have an average speed of motion which may be represented by V. The average component of motion toward or from the observer will be $\frac{1}{2}V$, as can be shown by a mathematical discussion. This is the average radial velocity as determined by the spectroscope, and is therefore known. The average component at right angles to the line of sight is found by a mathematical discussion to be $0.7854V$. This quantity is therefore also known because V has been given by spectroscopic observations.

Now consider the proper motions. They are expressed in angle, and they depend upon the distances of the stars and the speed with which they move across the line of sight. Since both the linear speed across the line of sight and the angular velocity, or proper motion, have been found, the distances of the stars can be computed.

The hypotheses on which this discussion has been made are not exactly fulfilled, and the necessary modifications of the proposed method must now be considered.

274. Motion of the Sun with Respect to the Stars.—Since the stars are in motion, it is reasonable to suppose that the sun is moving among them. Such was found to be the case by Sir William Herschel more than a century ago. He proved by observations extending over many years that the apparent distances between the stars in the direction of the constellation Hercules are increasing, on the average, and that they are decreasing in the exactly opposite part of the sky. He interpreted this as meaning that the sun is moving toward the constellation Hercules, and it is obvious that this would explain the observed phenomena; for, as objects are approached, they subtend larger angles. While Herschel's observations gave the direction of motion of the sun, they did not give its speed, which could be found by this method only if the distances of the stars were known. Since the distances of only a few stars can be measured directly, there is little hope of determining the motion of the sun in this way with any considerable degree of accuracy.

The spectroscope has been used to determine both the direction of the sun's motion and also the rate at which it moves. Instead of finding as many stars approaching as receding in every part of the sky, as was assumed in the discussion in Art. 273, it has been found that the stars in the direction of the constellation Hercules on the whole are relatively approaching the sun, while those in the opposite direction are relatively receding. This means that with respect to the stars which were observed the sun is moving toward Hercules.

The best determination of the direction of the sun's motion from proper motions of the stars is by Lewis Boss, who based his discussion on the 6188 stars in his catalogue. The best spectroscopic determination is by W. W. Campbell, who based his discussion on the radial velocities of 1193 stars measured at the Lick Observatory and its branch in South America. The results of these determinations are as follows:

	Right Ascension	Declination	Speed
Solar Apex (Boss)	$270°.5 \pm 1°.5$	$+34°.3 \pm 1°.3$	?
Solar Apex (Campbell)	$268°.5 \pm 2°.0$	$+25°.3 \pm 1°.8$	12 mi. per sec.

The agreement of these results in right ascension is remarkable, and the disagreement in declination is small considering the difference in the methods and the stars used.

The number of stars used by Boss in his determination of the direction of the motion of the sun is so great that he could divide them up into separate groups and make the discussion for each one separately. He took the stars of various galactic latitudes and obtained essentially the same result for each group. Dyson and Thackeray found from another (the Groombridge) list of 3707 stars that the declination of the apex of the sun's way increases from +16° for the brightest stars to +43° for those from magnitude 8.0 to 8.9. This was confirmed by Comstock, who found even a greater declination for the apex of the sun's way as determined from still fainter stars, but the result must be accepted with reserve until it is confirmed by a discussion depending on a much larger and better distributed list of stars. The spectra of the stars are divided into a number of classes (Art. 295), and it was found both by Boss and by Dyson and Thackeray that the declination of the apex of the sun's way is about 12° greater when determined from stars of Secchi's second type than it is when determined from stars of the first type. But the results altogether indicate that the sun is moving, relatively to the few thousand brightest stars, toward a point whose right ascension is about 270° and whose declination is about 34°, and that the speed of relative motion is about 12 miles per second.

The motion of the sun with respect to the stars evidently requires some modification of the process described in Art. 273. There is, however, no real difficulty, because the effect of the sun's motion can be avoided by considering only those components of the proper motions of the stars which are at right angles to the line of the sun's way.

Campbell made a determination of the mean parallaxes of the stars down to magnitude 5.5 by the method of this article. The brighter stars were not sufficiently numerous to give very reliable results. He found that the mean parallax of stars of magnitudes 4.51 to 5.50 is $0''.0125$, corresponding to a distance of 80 parsecs. This volume is 4096 times that occupied by the 20 nearest stars, and if the stars were uniformly distributed throughout it, the total number of them down to magnitude 5.50 would be $81,920$, which is much in excess of the number actually observed.

275. Distances of the Stars from the Motion of the Sun.—The parallaxes of only a comparatively small number of stars can be measured directly because their distances are so enormously great compared to the diameter of the earth's orbit. If the orbit of the earth were as large as that of Neptune, the problem would be much easier because of the larger base line which could be used. But the sun's motion can be made to afford an indefinitely large base line in statistical discussions, as will now be shown.

Suppose first that all of the stars of the observable sidereal universe except the sun are relatively at rest. The motion of the sun among them will give them an apparent displacement, or proper motion, in the direction opposite to that in which it is moving. The farther a star is away the smaller this proper motion will be. If a star is so far away that no displacement due to the sun's motion can be observed in one year, then 10 years, 100 years, or any other necessary number of years may be used. Eventually the effect of the sun's motion will be observable. Since the sun travels about 4 astronomical units per year, it follows that the parallax of a star is one fourth of that part of its annual proper motion which is due to the motion of the sun.

The false hypothesis that all the stars except the sun are relatively at rest has greatly simplified the problem. As a matter of fact, the stars are moving with respect to one another in various directions and with various speeds, and the proper motion of a star is due both to its own motion and also to the motion of the sun with respect to the system. Since the actual motion of any particular star is in general unknown, it is necessary to take the average motions of many, and then the results will be consistent, for the motion of the sun is defined with respect to the many. For any class of stars the average proper motion perpendicular to the direction of the sun's motion will be zero, while the average proper motion in the direction of the sun's motion will depend only on their distance and the speed of the sun.

This statistical study of the stars was taken up about 20 years ago by Kapteyn, of Groningen, who pursued it with rare skill and great industry. A number of other astronomers have also made important contributions to the subject. It is interesting to note the different kinds of work which contribute to the final results. In the first place, the proper motions of the stars are involved. They are obtained from two or more determinations of apparent position separated by considerable intervals. In fact, the longer the intervals the more accurately are the proper motions determined. In the second place, the spectroscope is of fundamental importance because it furnishes the motion of the sun with respect to the stars. Since certain classes of stars may be moving as a whole with respect to other classes (Art. 278), it follows that the spectroscopic determination of the motion of the sun should depend upon all those stars whose distances are sought from their proper motions. At present the radial velocities of stars fainter than the sixth magnitude can be obtained only by costly long exposures, and the practical limits do not reach beyond the eighth magnitude. On the other hand, the determination of the proper motions of stars many magnitudes fainter offers no observational difficulties.

276. Kapteyn's Results Regarding the Distances of the Stars.—As will be seen in Art. 295, most of the stars are of two principal spectral types. Type I, of which Sirius and Vega are conspicuous examples, are white or bluish white. Their spectra are characterized by absorption lines due to hydrogen in their atmospheres. They are intensely hot and probably always of large mass. Type II are the yellowish stars, of which the sun, Capella, and Arcturus are examples. The atmospheres of these stars contain many metals.

Kapteyn derived formulæ giving the mean parallaxes of all stars of each magnitude, and also the mean distances of stars of each spectral type separately. Table XVII gives Kapteyn's results transformed from parallax to parsecs and using Campbell's more recent determination of the rate of motion of the sun.

TABLE XVII

MAGNITUDE	ALL STARS	SPECTRAL TYPE I	SPECTRAL TYPE II
1	24.2	39.4	16.8
2	31.0	50.5	21.6
3	39.7	64.7	27.6
4	50.9	82.9	35.4
5	65.3	106.3	45.4
6	83.7	136.3	58.2
7	107.3	174.7	74.7
8	137.5	224.0	95.7
9	176.3	287.2	122.7
10	226.1	368.3	157.4
11	289.8	472.1	201.7
12	371.6	605.3	258.6
13	476.4	776.0	331.6
14	610.8	994.9	425.2
15	783.0	1275.5	545.0

It must be remembered that Table XVII gives mean results derived from the proper motions and radial velocities of many stars. The results may be in error for the first few magnitudes because there are not enough bright stars to make the statistical method reliable. They may

[1]One parsec equals 200,000 astronomical units, or in round numbers 20,000,000,000,000 miles.

also be in error for the fainter stars because these stars were not used in deriving the formulæ by which the computations were made.

If the table is correct, the sun is far below the average of the stars in brilliancy. According to the measures of Wollaston, Bond, and Zöllner its magnitude on the stellar basis is -26.7, or it gives us $120,000,000,000$ times as much light as a first-magnitude star. Since the light received from a body varies inversely as the square of its distance, at the mean distance of the first-magnitude stars the sun would send us only 0.005 as much light as comes from a first-magnitude star. That is, the first-magnitude stars average about 200 times as brilliant as the sun. It must not be concluded from this that the stars of all magnitudes average so much more brilliant than the sun, for those of the first magnitude are a group selected because of their great brilliancy.

277. Distances of Moving Groups of Stars.—If the two components of a double star are found to be moving in the same direction and with the same apparent speed, the conclusion to be drawn is that they are relatively close together in space and that they are physically connected; for, if they were simply in the same direction from the earth without being related, their apparent motions would almost certainly differ either in speed or direction. While the conclusion might be erroneous in the case of only two stars, it could hardly fail to be true if many stars were involved.

The study of the proper motions of the stars has shown that there are several groups which have sensibly identical proper motions; or rather, as the result of perspective, there are many stars which apparently move with the same speed toward a common point in the sky. These groups are widely scattered and many of their members would not be suspected of being associated with the others except for the equality of their motions. For example, Sirius belongs to a group which includes five of the stars in the Big Dipper.

The best-known group of stars of the type under consideration comprises part of the Hyades cluster, in the constellation Taurus, and some neighboring stars scattered over an area about 15° in diameter. This group, which includes 39 known stars, was exhaustively discussed by Lewis Boss. The stars, in their proper motions, all seem to move along the arcs of great circles. Boss found that the great circles of all the stars of the Taurus stream intersect in a common point whose right ascension and declination are, for the position of the equinox in 1875, 6 h. 7.2 m. and $+6° 56'$. It can be shown that this means that the stars of the group are moving in lines parallel to the line from the observer to the point of intersection of the circles. That is, their direction of motion is defined in this way, and since the stars cover a considerable area in the sky the point toward which they are moving is very well determined.

It will now be shown that if, in addition to the data already in hand, the radial velocity of one of the stars of the group can be obtained, then the actual motions, the distances, and the luminosities of all of them can be determined. Let O, Fig. 169, be the position of the observer and OP the direction of motion of the stars of the group. Let S be one of the stars which is moving in the known direction SA with an unknown speed. Suppose the component SB is measured by the spectroscope. Then, since the angle ASB, which equals the angle POS, is known, the whole component SA and the proper-motion component SC can be computed. That is, the actual distance SC is found and the proper motion to which it gives rise was already known. Therefore the distance OS can be computed. Since all the stars of the group must have the same total motion SA, for otherwise they would not remain long associated, the distances of all the members can be determined from their respective proper motions. Of course, it is practically advantageous to measure the radial velocities of many, or all, of the members of the group. When the distance of a star of known magnitude has been found, its absolute luminosity can be computed.

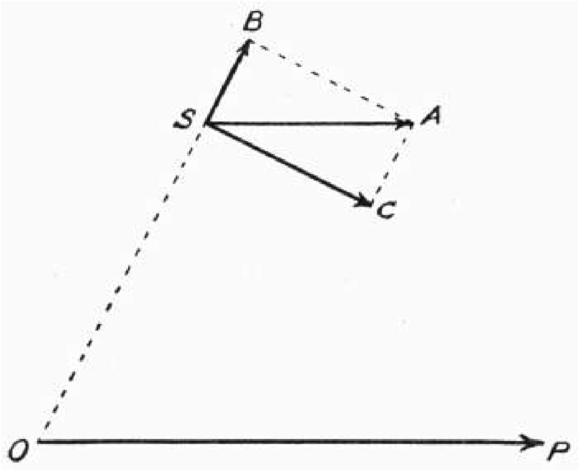

FIG. 169.—Components of motion in moving groups of stars.

By these methods Boss found that the Taurus group is a globular cluster whose center is distant about 40 parsecs from the earth. Since its apparent diameter is about 15°, its actual diameter is about 10 parsecs. There is a slight condensation toward the center of the cluster, but in the group as a whole the star density is only a little greater than it is in the vicinity of the sun. The distances between the stars of the group are so great that foreign stars could pass through it without having their motions appreciably disturbed. In fact, in the motion of the cluster it certainly sweeps past other stars and there are probably several strangers now within its borders. Boss found that 800,000 years ago the cluster was half its present distance and its apparent size was twice that at present. In 65,000,000 years it will have receded until it will appear from the earth to be a compact group one third of a degree in diameter, made up of stars of the ninth magnitude and fainter.

All the 39 stars of the Taurus cluster are much greater in light-giving power than the sun. The luminosities of even the five smallest are from five to ten times that of the sun, while the largest are 100 times greater in light-giving power than our own luminary. Their masses are probably much greater than that of the sun.

The Ursa Major group of 13 stars is another wonderful system. It is in the form of a disk whose thickness is only 4 or 5 parsecs while its diameter is 50 parsecs. The distances of the members of this group from the sun vary from 2.6 parsecs, in the case of Sirius, to 22 parsecs for the stars of the Big Dipper, and over 40 parsecs in the case of Beta Aurigæ. The luminosities of the stars vary from 7 to more than 400 times that of the sun.

There is another fairly well-established group in Perseus which was discovered almost simultaneously by Kapteyn, Benjamin Boss, and Eddington. There are several other probable groups in which the proper motions are so small that the results have not been established beyond all question. In a universe of many stars it is inevitable that there should be many accidental parallelisms and equalities of motion. Stars are at present regarded as forming a related group only if there is something quite distinctive about their positions or motions.

278. Star-Streams.—In 1904 Kapteyn announced a very important discovery respecting the motions of the stars. He found that, instead of moving at random, most of the stars belong to two great streams having well-defined directions of motion. Stars in all parts of the sky, of all magnitudes so far as the proper motions are known, and of all spectral types, partake of these motions. The phenomena do not seem to be local, so to speak, as was true in case of the groups considered in Art. 277. Yet it would be going too far to conclude that all the stars in the clouds which make up the Milky Way belong to these streams, for the discussion was based on only a few thousands of stars, while there are hundreds of millions in the sky. It seems probable that the Galaxy is made up of a great many of these streams. There is, in fact, some reason to believe that there is a third drift containing stars of the so-called Orion type. But the evidence for the existence of the two streams discovered by Kapteyn is conclusive, and his results have been verified by several other astronomers. And in connection with the larger problems of the Milky Way, it is interesting to note that both streams are moving parallel to its plane.

With respect to the sun as an origin the points toward which the stars are moving are:

Apex of Drift I:	Right Ascension, 90°;
	Declination, −15°.
Apex of Drift II:	Right Ascension, 288°;
	Declination, −64°.

If the motion of the sun is eliminated and the stars are considered only with reference to one another, the two streams necessarily move in opposite directions. With this reference, the vertices of the two drifts according to Eddington's discussion of the stars in Boss's catalogue are:

Right Ascensions, 94°, 274°;
Declinations, +12°, −12°.

About 60 per cent of the stars on which the discussion was based belong to Drift I and 40 per cent to Drift II. They are intermingled in space so that one set of stars is passing through the other. Their relative velocity is about 24 miles per second, or about 8 astronomical units per year.

279. On the Dynamics of the Stellar System.—The stars are at least several hundred millions in number, they occupy an enormous space, and they are moving with respect to one another with velocities averaging about 20 miles per second. In the two centuries during which their proper motions have been observed, they have in all cases moved in sensibly straight lines with uniform velocities. Likewise, spectroscopic determinations of motion in the line of sight give no evidence of anything but uniform rectilinear motion. These statements require modification, however, in the case of the binary stars (Art. 283).

There is no doubt that the paths of the stars eventually curve, but the time covered by our observations is as yet far too short for us to detect these deviations. It compares with the vast intervals required for the stars to move across the sidereal universe as one tenth of a second compares with the period of the earth's revolution around the sun.

The first question that springs to the mind is whether the stars travel in sensibly fixed and closed orbits similar to those of the planets, or move on indefinitely throughout the region occupied by the stars without ever retracing any parts of their paths. Since observations cannot at present answer this question, the reply must be based on dynamical considerations. There is clearly no central mass among the stars and there is no center about which they seem to be distributed with anything approaching symmetry. Moreover, their motions give no hint that they are moving, even temporarily, around some central mass or point.

The conclusion is inevitable that the stars describe more or less irregular paths, in the course of time probably extending into all parts of the sidereal system. In fact, the Galaxy was likened by Kelvin to a great gas in which the stars correspond to the molecules. When they are far apart their mutual attractions are inappreciable, just as molecules do not interfere with the motions of one another except at the times of collisions. If two stars should collide they would probably coalesce, the heat generated by their impact changing them into the nebulous state. This would be quite different from an elastic rebound of molecules. But actual collisions would be excessively rare and near approaches would be relatively much more frequent. A near approach is dynamically equivalent to an oblique impact of perfectly elastic bodies, as is illustrated in Fig. 170. In this figure C is the center of gravity around which as a focus the two masses (assumed equal) describe hyperbolas. It is easy to see that the motion before and after near approach is similar to that of two elastic spheres colliding a little to the right of their respective centers. Consequently there are some good grounds for comparing the sidereal system to a vast mass of gas.

There are, however, fundamental differences between a gas and the stellar system. In a gas the collisions are the important events in the history of a molecule, and are the only appreciable factors which influence its motion. In the stellar system the near approaches of a given star to some other one are excessively rare, and the attraction of the whole system is the primary factor determining the motion of the individual star. Or, more particularly, a molecule in a vessel of ordinary gas has thousands of millions of collisions with other molecules per second, while the attraction of the whole mass has no appreciable effect on its motion. But in the sidereal system, a star will in general travel several times from one of its visible borders to the opposite one without once passing near enough to another star to have its motion radically altered by the latter, while its motion is controlled by the attraction of the whole mass of stars.

It is difficult to realize the great distances which separate the stars and how feeble are the forces with which they attract one another. If the earth were at rest, it would fall toward the sun less than one eighth of an inch the first second. The distance of the relatively near star Sirius is 500,000 times as great; and in spite of the fact that its mass is 3.4 times that of the sun, in a whole year it would give the sun a velocity of only 0.00007 of an inch per second. Only after 900,000,000 years at the present distance would the relative velocity of the two amount to one mile per second. Long before such an immense time shall have elapsed the sun and Sirius will be far separated in space.

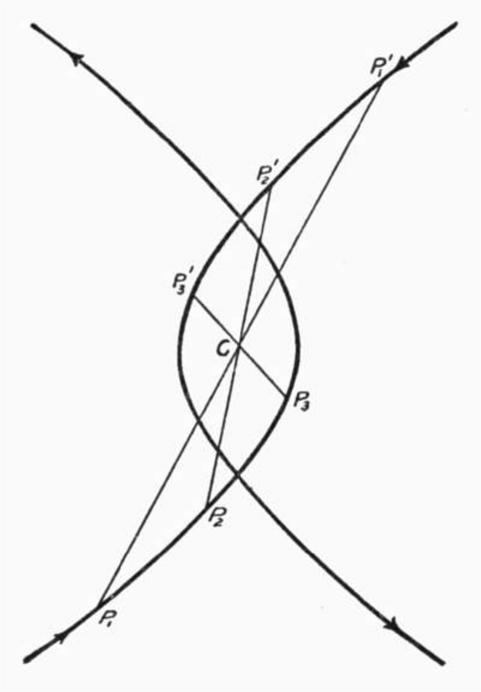

FIG. 170.—Near approach of two stars is similar to an oblique collision of elastic bodies.

Now consider a group of stars, such as the cluster in Taurus, traveling through the stellar system. So far as their mutual interactions on one another are concerned the result is the same as though they were not moving with respect to the other stars. In their motion through space they are subject as a whole to the changing attractions of the other stars, and individually to possible close approaches. These factors may be considered separately.

The Taurus cluster consists of 39 (possibly more) stars which occupy a space whose diameter is roughly 10 parsecs. From the high luminosity of the individual members of the group it is reasonable to suppose that they have large masses, and it will be supposed that they average 10 times the sun in mass. It will be assumed that their motions are such that they are neither simply falling together nor scattering more widely in space, and that they are distributed uniformly throughout the volume which they occupy. That is, it is assumed that there is a balance (speaking roughly) between the gravitational forces among them and the centrifugal forces due to their relative motions. With these data and assumptions their maximum velocities with respect to the center of gravity of the group, and the time required for one of them to move from one border of the group to the opposite, can be computed.

It is found that the velocities of the stars of the group with respect to their center of gravity will always be less than 0.4 of a mile per second, and this maximum will be approached only very infrequently. If their masses are comparable to that of the sun instead of being 10 times as great, the velocities relative to their center of mass will always be less than 0.13 of a mile per second. Consequently, the internal motions of the group due to the mutual attractions of its members will always be small, and the fact that at present the stars are moving in sensibly parallel lines with the same speed does not in the least justify the conclusion that the members of the cluster are in any sense young. It is also found that the time required for a star to move from one side of the group to the other under the attraction of all the stars in it is 25,000,000 years. At present it does not seem safe to put any time limits on the life of a star, and consequently it may be supposed, at least tentatively, that the cluster has been in existence long enough for the stars of which it is composed to have made many excursions across it. The mutual interactions of the stars have a tendency to make the cluster uniformly spherical with the stars of greatest mass somewhat condensed toward the center. The approximate sphericity of the group is in harmony with the hypothesis that it is very old.

It remains to consider the effect on the cluster of its passage through star-strewn space. The result depends, of course, upon the star density of the region which it traverses. It has been seen that there are 20 known stars within 5 parsecs of the earth. It is not unreasonable to suppose that there are 10 other stars within the same distance of the earth which are at present unknown. Under the assumption that the stars are scattered uniformly with a density such that there are 30 within a sphere whose radius is 5 parsecs, it is found that, on the average, the cluster will have to pass over a distance of 5700 parsecs in order that at least one of its 39 members shall pass another star within 1000 times the distance from the earth to the sun. Since the cluster moves at the rate of about 16 miles per second with respect to the stars now surrounding it, about 40,000 years will be required for it to describe one parsec; and to pass over 5700 parsecs will require more than 200 million years. But 5700 parsecs is probably far beyond the limits of the visible universe, and before the cluster shall have traversed any considerable fraction of this distance the attraction of the great mass of stars in the Galaxy will have radically altered, and possibly reversed, its motion.

While the stars of the cluster pass close to other stars only after very long intervals, they are continually subject to slight disturbing forces which affect them somewhat unequally. This results in a slight tendency to scatter the members of the group. One might be tempted to conclude from the fact that it is still very coherent that its age should be counted in hundreds of millions of years at the most. But it is impossible to determine how many stars once belonging to it have been torn from it by near approaches to other stars, or how many of the smaller original stars have been thrown to its borders by its internal interactions and then removed by the differential attractions of exterior bodies, or how much more condensed it may formerly have been. In short, no certain conclusions respecting the age of one of these moving clusters can be drawn from the properties of the motion of their members at present.

It is now possible to pass to the consideration of the whole sidereal system. The star-streams discovered by Kapteyn and the form of the Galaxy suggest that it is made up largely of many vast star clouds which move at least approximately in the plane of the Milky Way. There is a general tendency for the mutual interactions of the members of each star cloud to reduce it to the spherical or symmetrically oblate form. Moreover, the stars of smaller mass gradually acquire greater velocities at the expense of the larger stars, just as in a mixture of gases of molecules of different weights the lighter ones on the average move faster than the heavier ones. The fact that the individual star clouds are not spherical would argue that they have not had time to acquire the symmetrical form of equilibrium, if it were not for the fact that their passage through and near to other star clouds may occasionally introduce great irregularities.

But all the star clouds which together constitute the Milky Way may be considered as being simply a much larger system. If it remains isolated from all other systems, it will similarly tend toward a symmetrical form. Its irregularities point toward the conclusion that its age is not indefinitely great; and this would be a necessary conclusion if there were not the possibility, or perhaps even probability, of the existence of other galaxies beyond our own near which, or through which, ours passes after intervals of time of a higher order of magnitude than any so far considered. These families of galaxies may be units in still larger systems, and so on without limit. Therefore it is impossible to conclude from the irregularities in the star clouds or galaxies that they have not been of infinite duration. It should be added at once that most astronomers believe, chiefly on the basis of the finite amount of energy of the stars, that they have not existed for an infinite time.

While it has not been possible to answer the more ambitious questions which have been raised, there remain others which are not without interest. For example, suppose that throughout the whole region occupied by the stars they are as numerous as they are near the sun; that is, that there are 20 or 30 in a sphere whose radius is 5 parsecs. Suppose, further, that there is equilibrium between the attractive and centrifugal forces. So far as these assumptions approximate the truth, there is a relation between the dimensions of the whole stellar system and the mean velocity of stars at its center, for the velocities depend upon the star density and the extent of the region which they occupy. Inasmuch as the star density in the neighborhood of the sun and the velocities of the stars have been determined by observations, the extent of the whole system can be computed.

The solar system, which is far from the borders of the Galaxy, will be supposed to be approximately at its center. The mean velocity of the stars near the sun is about 22 miles per second. This fact and the assumptions which have been made imply that the radius of the Galaxy is about 1100 parsecs and that the total number of stars in it is 260, 000, 000. Although the assumptions are not in exact harmony with the facts, it is believed that these results are of the correct order of magnitude. And under the same assumptions the time required for a star to pass from one side of the system to the opposite is approximately 200, 000, 000 years. Since this is probably less than the age of the earth, our sun may have traveled in geological times more than once far toward the boundaries of the stellar system.

Whatever may have been the history of any particular star, these results, though they may be appreciably in error numerically, imply that the stars have undergone considerable mixing. So far as can be determined at present this process will continue in the future, the star clouds which form the Milky Way will become more and more uniform and the motions of the stars more and more chaotic, the stars of smaller mass will acquire higher velocities than the larger ones, at rare intervals every star will pass near some other star, and possibly at intervals of time of a higher order our Galaxy will encounter other galaxies and again be deformed and made irregular by them.

280. Runaway Stars.—Since the average radial velocity of a large group of stars is one half the average of their entire motions, the spectroscope furnishes the average speed with which the stars move. The average velocity of the stars near the sun is about 1.8 times the velocity of the sun, or 22 miles per second. This is 7.5 astronomical units per year, or one parsec in about 27,000 years.

The stars, however, do not all move with even approximately the same velocity. The variations in their speeds are evidenced both by their proper motions and by their radial velocities. The star having the largest known proper motion,[1] namely, 8″.7 per year, is the sixth in Table XVI, and by astronomers is known as C. Z. 5 h. 243, or No. 243 in the fifth hour of right ascension in the Cordoba Zone Catalogue. It was discovered by Kapteyn in 1897 from the measurement of plates taken by Gill and Innes at the Cape Observatory, in South Africa. Its actual velocity is 170 miles per second, or nearly 8 times the average velocity of the stars. The star known as 1830 Groombridge has a proper motion of 7″ per year. Its parallax, which is not yet accurately known, can scarcely exceed 0″.1 and its velocity probably exceeds 200 miles per second. The star 61 Cygni is another one in Table XVI which moves at a high speed, though its velocity is exceeded by the velocities of quite a number of other known stars.

The stars having high velocities are called "runaway stars" because, unless they pass very near other stars in their journey through space, they will escape, like molecules from a planet, from the gravitative control of the stars which constitute the Galaxy, and will recede from them forever. This conclusion is inevitable unless the total mass of the sidereal system is much greater than has hitherto been supposed. Even if the extravagant assumption is made that there are 1,000,000,000 stars, each as massive as the sun, in a spherical space whose radius is 1000 parsecs, it is found that a star moving through its center with a speed exceeding 72 miles per second will entirely escape from the system unless, in its journey toward the surface, it passes near at least one other star in a particularly favorable way so that its velocity is much reduced. Since the probability of such a near approach is very small, we are forced to the conclusion that these stars with high velocities are only temporary members of our Galaxy. The only alternative is that the mass of the system is at least 10 times as great as has been estimated.

[1] Professor Barnard has just (June, 1916) found an eleventh-magnitude star in Ophiuchus whose annual proper motion is over 10″; its parallax has not yet been measured.

If the total mass of the stellar system is greatly in excess of the estimates which have been made, the resulting attractive forces are greater than the centrifugal forces due to the average motions of the stars, and, therefore, the stars must be on the whole falling together. That is, either the runaway stars will actually escape from the Galaxy entirely, or the stellar system will necessarily become more and more concentrated under the mutual gravitation of its parts.

The question of the origin of runaway stars at once arises. Either they have come in from beyond our Galaxy, perhaps from a distant one, or their high velocities have been developed within our stellar system. The first alternative is certainly possible though it may appear at first to be improbable, especially in view of the enormous time required for a star to go from one sidereal system to another. But these stars will, in most cases, permanently leave our Galaxy, and there is no apparent reason why stars might not equally well leave other galaxies.

The second alternative is also possible, for if a large star and a small star pass near each other the velocity of the small one may be greatly increased. A series of favorable close approaches might easily produce the high velocities which are observed. The process is closely analogous to the development of high velocities in exceptional cases in a mixture of gases, the light molecules acquiring the highest velocities. The difficulty in the case of the stars is that the intervals between close approaches are so long that the process demands startling lengths of time. Perhaps astronomers in the remote future will be able to determine from their greater knowledge regarding the masses and the velocities of the stars something respecting the length of time during which the stars of the stellar system have been subject to their mutual attractions.

281. Globular Star Clusters.—Perhaps the most wonderful objects in the heavens are the dense globular star clusters. They cover portions of the sky generally less than $30'$ in diameter, that is, less than the apparent diameter of the moon. The brightest of them appear to the unaided eye as faint fuzzy stars, but a large telescope shows that they are made up of thousands of stars. The most splendid of these objects in the northern sky is the great Hercules cluster (Fig. 171), also known to astronomers as Messier 13, in which Ritchey's photograph, taken with the great 60-inch reflector of the Mt. Wilson Solar Observatory, shows more than $50,000$ stars. The great cluster Omega Centauri, in the southern heavens, is even a more wonderful aggregation of suns.

The individual stars in most of the globular clusters are very faint, ranging from about the twelfth magnitude down to the limits of visibil-

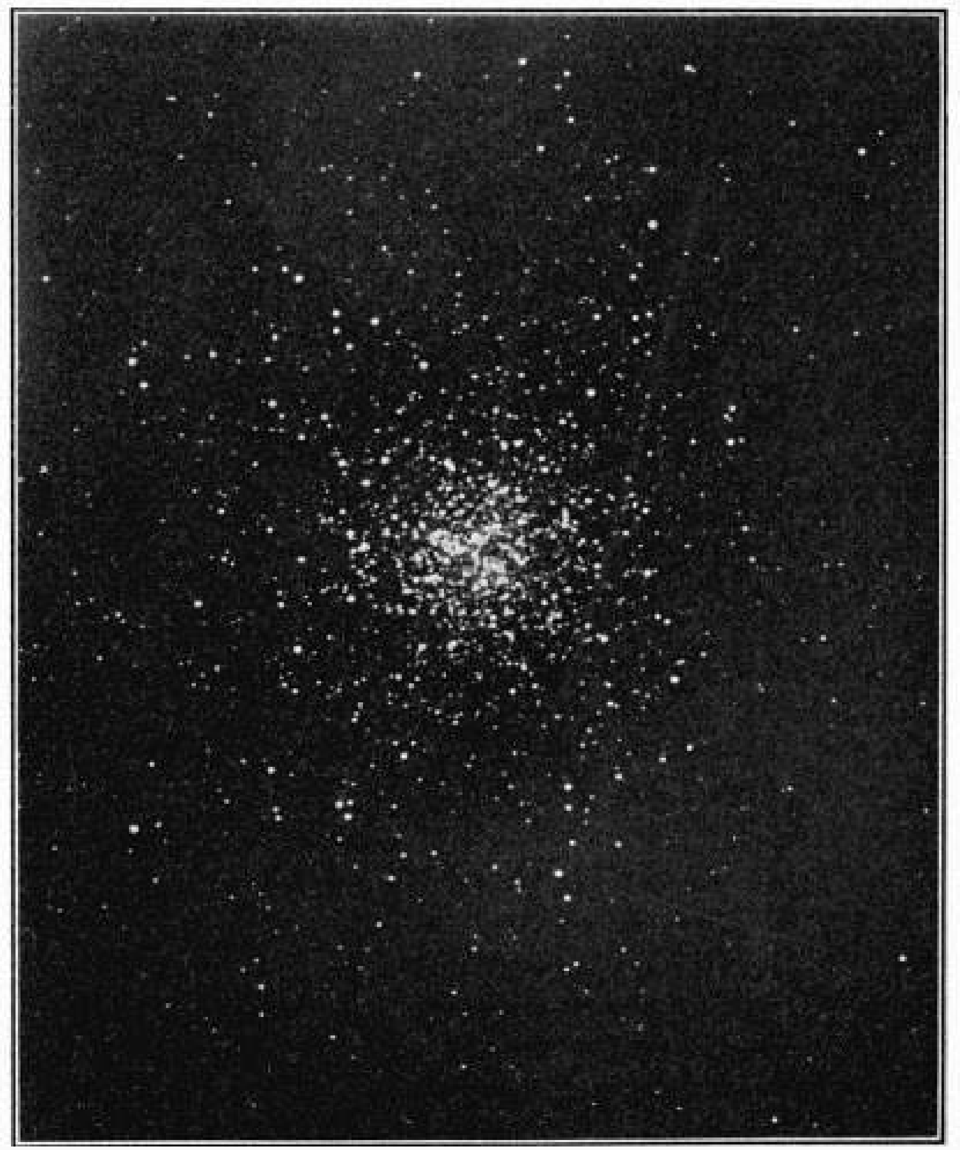

FIG. 171. — The great globular star cluster in Hercules (M. 13). *Photographed by Ritchey with the 40-inch telescope of the Yerkes Observatory.*

ity with the instrument employed. If we knew the distance of a cluster, we could determine the luminosity of its members compared to the sun. Then we could answer the question whether the stars in the clusters are great suns like our own, but which appear faint and crowded together only because of their immense distance from us, or whether they are examples of an evolution in which the mass is distributed among a very large number of relatively small bodies. It is not possible to measure directly the parallaxes of the globular clusters, and their probable distances can be inferred only from their proper motions. Unfortunately, we do not yet have any positive data bearing on the problem except that their positions in the sky are sensibly fixed. This can only mean that they are very distant, for there are more than 100 clusters known, and it is improbable that all of them should be moving in the same direction as the sun and with the same speed. It seems to be clear from their apparent fixity on the sky that their distance is at least 100 parsecs and it is much more probable that it is 1000 parsecs. At the distance of 100 parsecs the sun would be a ninth-magnitude star, while at 1000 parsecs it would be of the fourteenth magnitude. If the clusters are at the smaller distance, their members are much less luminous than the sun; if at the greater, they are comparable with the sun.

The problem may also be considered in the reverse order. That is, if there are any reasons for assuming that the individual stars in the clusters are comparable to the sun in luminosity, or related to it in any definite way, then their distances can be computed. The stars in the clusters are individually so faint that their spectra cannot be studied; but valuable information concerning the character of the light they radiate can be obtained by photographing them first with plates sensitive to the blue and then to the red end of the spectrum. Such work has been carried out at the Solar Observatory and Shapley finds evidence that the stars in the Hercules cluster are like the giant red and yellow stars, such as Antares and Arcturus, which are enormously more luminous than the sun. If this conclusion is correct, the distance of the Hercules cluster is of the order of $10,000$ parsecs. Perhaps a reasonable summary of present information would be that globular clusters are almost certainly distant much more than 100 parsecs, and that their distances probably range from 1000 to $10,000$ parsecs.

The actual dimensions of the clusters are appalling. The distance across one whose apparent diameter is $30'$ is $\frac{1}{115}$ of its distance from the earth, or probably of the order of at least 10 parsecs. If $50,000$ stars were distributed uniformly throughout a sphere of these dimensions, the average distance between adjacent stars would be more than 0.4 parsec, or more than $80,000$ times the distance from the earth to the sun. It is seen from this that, although the globular clusters are somewhat condensed toward their centers, the actual distances between the stars of which they are composed are enormous. There is abundance of room in them for almost indefinite motion without collision, and there is no apparent reason why the individual stars should not have planets revolving around them.

Dynamically, the globular clusters are much simpler than the Galaxy. They seem to have arrived at an approximately fixed state of symmetrical distribution, though, of course, the individual stars are in ceaseless motion through them. The regularity of their arrangement implies that the process of mixing has been in operation an enormous time, unless indeed they started in this remarkable state. It is not difficult to get at least an approximate idea of the time required for a star to move from the borders to the center of a globular cluster. The distribution of mass in a cluster is somewhere between condensation entirely at the center and uniform density. In the first case the force varies inversely as the square of the distance from the center, and in the second, it varies directly as the distance from the center. In a cluster whose radius is 5 parsecs and which contains 50,000 stars, each having the mass of the sun, the time required for a star to move from the surface to the center in the first case is nearly 800,000 years, and in the second is 1,100,000 years. The actual time is of the order of 1,000,000 years. Since thousands of these excursions would be necessary to reduce a group of stars with considerable irregularities in distribution to the symmetrical forms observed, the age of these systems must be enormous. Only a thousand excursions from the periphery to the center and back would require 1,000,000,000 years. It is improbable that this number is too large (it may be many times too small), and it follows that either the stars exist an enormous time as luminous bodies, or much of the dynamical evolution of the clusters was completed before the star stage, if, indeed, there has been such a preceding stage. And it follows further from the symmetry of the clusters that for at least hundreds of millions of years they have not passed near other clusters.

No rapid motions of stars in the globular clusters are to be expected. With 50,000 stars, each equal to the sun in mass, distributed uniformly throughout a sphere whose radius is 5 parsecs, the velocity of a permanent member of the group at its center would be only about 4 miles per second. Since the actual clusters have strong central condensations, the velocity for the ideal case would be considerably exceeded by stars near their centers. Suppose they move at 10 miles per second at right angles to the line of sight. At a distance of 1000 parsecs they would move with respect to the center of the cluster only one second of arc in 300 years. Of course, if the assumptions as to the distance or masses are wrong, the result will be wrong, and, besides, a certain small number of the stars, especially those of smallest mass, will have motions in excess of the mean velocities. But it is improbable that relative motions of the members of star clusters will be large enough in any case to be observable inside of several decades.

XXIII. QUESTIONS

1. Prove that, in Fig. 168, $\angle E_1SE_2 - \angle E_1S'E_2 = \angle SE_1S' - \angle SE_2S'$.

2. Suppose there are 30 stars within 5 parsecs of the sun; what is the average distance between adjacent stars?

3. Draw a diagram to prove that Herschel's observations, Art. 274, are explained by the conclusion which he drew. If this conclusion is denied, what other must be accepted?

4. If an angle of $1''.0$ can be measured with an error not exceeding 10 per cent, how small a parallax can be determined with this degree of accuracy by the method of Art. 275 in 100 years?

5. Show by a diagram that if two stars are moving in parallel lines, then the great circles in which they apparently move, as seen from the earth, intersect in a point whose direction from the earth is the direction in which the stars move (Art. 277).

6. Since the velocity of our sun is somewhat below the average of the velocities so far measured, what are the probabilities of the relation of its mass to the masses of the observed stars?

7. If the radius of the Galaxy is 1100 parsecs (end of Art. 279), how long would it take the sun at its present speed to pass from the center of the sidereal system to its borders?

8. If the velocity of the star 1830 Groombridge is 200 miles per second and remains constant, how long will be required for it to recede to a distance from which our Galaxy will appear as a hazy patch of light 1° in diameter?

9. If there are many galaxies, and if the distances between them compare to their dimensions like the distances between the stars compare to the dimensions of the stars, how long will be required for 1830 Groombridge to go from our Galaxy to another?

III. The Stars

282. Double Stars.—A few double stars have been known almost since the invention of the telescope, but William Herschel was the first astronomer to search for them systematically and to measure the distances and the directions of their components from one another. His purpose in measuring them was to determine the parallax of the nearest ones (Art. 272), for he assumed, perhaps unconsciously, that the sun is a typical star, and that when two stars are apparently in about the same direction from the earth, one is simply farther away than the other.

Herschel found a large number of double stars whose components were apparently separated by a few seconds of arc at the most. A discussion of the probability of there being such a large number of stars so nearly in lines passing through the earth would have shown him that their apparent proximity could not be accidental. He reached the same result in a few years, for his observations showed him in a considerable number of cases that the two components were revolving around their center of gravity. That is, instead of all stars consisting of single primary bodies accompanied by families of planets, there are many which are twin suns of approximately equal mass and dimension. So far as we know, they may or may not have planetary attendants, for such small objects shining entirely by reflected light would be beyond the range of our telescopes even if they were a thousand times more powerful than any yet constructed.

The names that stand out most prominently in the double-star astronomy of the nineteenth century are William Struve, Dawes, John Herschel, and Burnham. In Burnham's great catalogue of double stars the observations and descriptions of about 13,000 of these objects are given. New ones are constantly being discovered, though the northern heavens have now been very thoroughly examined with powerful telescopes. At the Lick Observatory a survey of the whole heavens to at least $-14°$ declination was begun by Hussey and Aitken and completed by Aitken. All old pairs with a separation not exceeding $5''$ of arc were observed, and 4300 new pairs were discovered within the same limits. On using a definition of double star which excludes all wider pairs except in the case of bright stars, Aitken found that there are 5400 of these objects not fainter than the ninth magnitude north of the celestial equator. This means that at least one star in 18 of those not fainter than the ninth magnitude is a double which is visible with the 36-inch telescope of the Lick Observatory. Of these stars, 2206 have an apparent angular separation not greater than $1''$, and only 200 are separated by more than $5''$. A very interesting fact is that, compared to the whole number of stars of the same brightness, double stars seem to be somewhat more numerous in the Milky Way than near its poles. Moreover, the average separation of the stars of the spectral class to which the sun belongs is considerably greater than in those of the so-called earlier types which include the blue stars.

There are doubtless some cases in which the components of a double star are at different distances and simply in nearly the same direction from the observer. But in general they form physical systems which revolve around their centers of gravity in harmony with the law of gravitation, and these pairs are called *binaries*. According to the law of probability, essentially all of the 5400 double stars in Aitken's list must be binaries, for only very rarely would two stars be accidentally so nearly in the same direction from us.

283. The Orbits of Binary Stars.—The stars in all cases are so remote from us that the components of a binary system cannot be seen as separate stars unless they are a great distance apart. But when the components of a binary pair are far from each other, their period of revolution is long, and observations must therefore extend over many years in order to furnish data for the computation of their orbits. Those binary stars which were first discovered and which have been longest under observation are not very close together, and, while in many cases it is now certain from direct observational evidence that they form physical systems, there are only 40 or 50 in which the observed arcs are long enough to define the orbits with any degree of precision. In 1896 See published the orbits of 40 of the best-known binary stars.

The periods of known visual binary stars range from 5.7 years, for Delta Aquilæ, to hundreds and probably thousands of years. The planes of their orbits are inclined at all angles to the line joining them with the earth, so that, as a rule, we see their orbits in projection. Indeed, the orbit of 42 Comæ Berenices is sensibly edgewise to us. One of the most interesting things about the orbits of binaries is that they are generally considerably eccentric. In the 40 orbits in See's list the average eccentricity was 0.48, or twelve times that of the planetary orbits. The orbit of the binary star Gamma Virginis has an eccentricity of 0.9, and therefore the greatest distance of the two members of this pair from each other is 19 times their least distance.

284. Masses of Binary Stars.—The masses of those planets which have satellites are found from the periods and distances of their respective satellites (Art. 154). The masses of Mercury and Venus are found from their attractions for other bodies, especially comets. The masses of celestial bodies are found only from their attraction for other bodies. It is evident, therefore, that the mass of a single star remote from all other visible bodies cannot be found. But when the dimensions of the orbit and the period of revolution of a binary pair are known, their combined mass can be computed just as the mass of a planet is computed.

The periods of binary stars are determined by direct observations of their apparent positions. The dimensions of the orbit of a binary pair can be determined from their apparent distance apart and their distance from the earth. The chief difficulty lies in the problem of finding their parallax, for only a small number of stars are within measurable distance from the sun.

Those binary stars whose periods and distances are known with sufficient approximation to make the mass determinations of value are given in Table XVIII. The masses of all those whose parallaxes are less than $0''.2$ are subject to some uncertainty, and the probable error is great if the parallaxes are less than $0''.1$. In this table the periods are

TABLE XVIII

STAR	PARALLAX	PERIOD	SEMI-AXIS	COMBINED MASS	LUMINOSITY
	$''$				
α Centauri	0.76	81.2	23.3	1.9	2.0
Sirius	0.38	48.8	20.0	3.4	48.0
Procyon	0.32	39.0	10.4	0.7	9.7
η Cassiopeiæ	0.20	300.(?)	47.4	1.2	1.4
70 Ophiuchi	0.17	88.4	26.8	2.5	1.2
o_2 Eridani	0.17	180.0	28.2	0.7	0.8
Bradley 2388	0.13	45.8	8.2	0.3	1.0
85 Pegasi	0.11	26.3	7.7	0.7	0.8
ζ Herculis	0.10	34.5	13.5	2.1	11.4
κ Pegasi	0.08	11.4	3.7	0.4	3.1
μ_2 Boötis	0.05	200.(?)	21.5	0.2	0.7

given in years, the semi-axes in terms of the earth's distance from the sun, the combined mass in terms of the sun's mass, and the luminosity in terms of the sun's luminosity at the same distance.

Perhaps the most interesting thing brought out by the table is that the masses of all of these stars are comparable to that of the sun, and, with the exception of Sirius, their luminosities do not differ greatly from that of the sun. But there are not enough pairs of stars in the table to justify any very positive general conclusion.

If the orbits of each of the two components of a binary star with respect to their center of gravity are known, their separate masses can be computed. The problem of determining the orbits of two stars with respect to the center of mass of their system is very difficult because their motions with respect to neighboring stars, or fixed reference lines, must be measured. In only a few cases are the results at present reliable. The discussions of Lewis Boss led him to the conclusion that probably in all cases the brighter star is the more massive, a result which is contrary to that which was sometimes found in earlier investigations.

285. Spectroscopic Binary Stars.—The spectroscope has contributed very important results to the study of binary stars. Its application depends upon the fact that it enables the observer to determine whether a source of light is approaching or receding (Art. 226). Suppose the plane of motion of a binary system passes through the earth, as is represented in Fig. 172. When the stars are in the positions *A* and *B*, one is receding from, and the other is approaching toward, the earth. If they have similar spectra, the spectrum of the combined pair will consist of double lines (Fig. 173), for the lines from one will be shifted

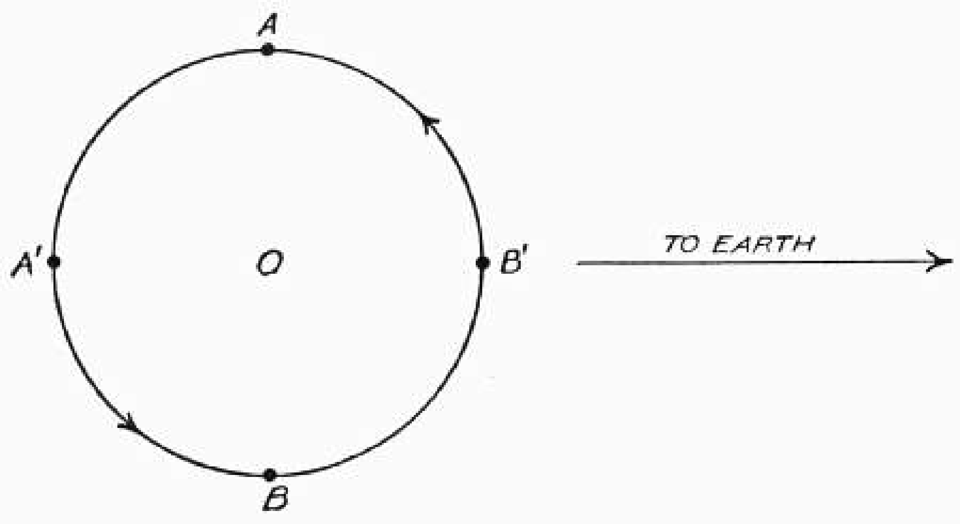

FIG. 172.—Orbit of a spectroscopic binary star.

toward the red while the lines from the other will be displaced toward the violet. When the stars have made a quarter of a revolution around their center of gravity O and have arrived at A' and B', the lines will not be displaced because the stars are neither approaching toward nor receding from the observer. After another quarter of a revolution they will be double again because A will be approaching and B receding.

The data furnished in this way by the spectroscope are very important because, in the first place, the separation of the lines determines the relative velocity of the stars in their orbits. This is true whether the system as a whole is stationary with respect to the earth, as has so far been tacitly assumed, or is moving in the line of sight. The period is also given. The period and velocity furnish the dimensions of the orbit and consequently the total mass of the binary system.

If the two stars of the binary are very unequal in luminosity, the spectrum of the fainter one will not be obtained, but the spectral lines of the brighter one will be shifted alternately toward the red and violet ends of the spectrum. The period is given in this case, but only the

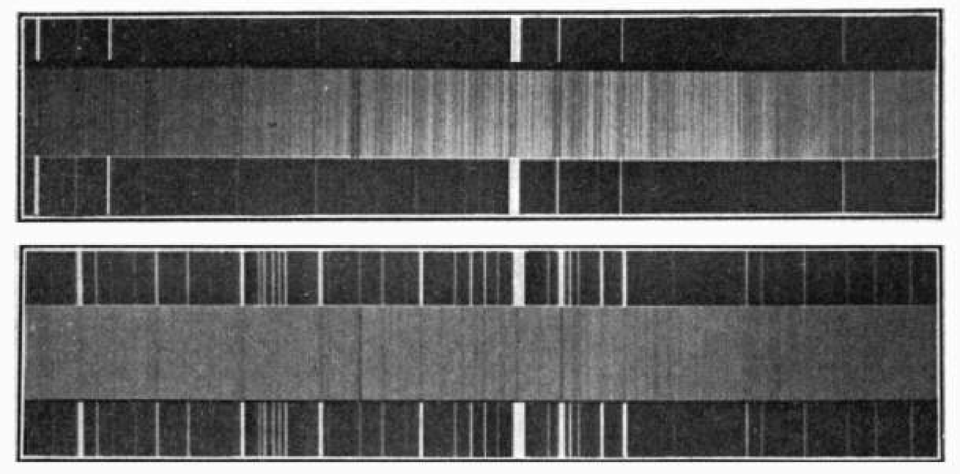

FIG. 173. — Spectrum of Mizar, showing double lines above and single lines below (period 20.5 days). (*Frost; Yerkes Observatory.*)

velocity of the brighter star with respect to the center of gravity of the system is known. Since the orbit of one star with respect to the other is necessarily larger than the orbit of the brighter one with respect to the center of gravity of the two, the mass computed in this case will always be too small.

It has so far been assumed that the plane of motion of the binary star passes through the earth. This condition is realized only very exceptionally, and indeed is not necessary for the application of the method. If the plane of motion does not pass exactly through the earth, the measured radial velocity is only a fraction of the whole velocity, and the size of the orbit and mass of the system based on it are both too small. Since the planes of the orbits of binary stars may have any relation to the observer, the measured radial velocities are in general smaller than the actual velocities; on the average the former are 0.63 of the latter. On the average the calculated masses are about 60 per cent of the true masses.

The spectroscope is particularly valuable in the study of binary stars because it is not necessary that they should be near enough to appear as visual binaries. The only requisite is that they shall be bright enough (above the eighth magnitude with present instruments) to enable astronomers to photograph their spectra in a reasonable time. With very few exceptions the spectroscopic binaries so far known are not also visual binaries. A second advantage of the spectroscope is that it furnishes at the same time lower limits for the orbital dimensions and masses of the stars.

The first known spectroscopic binary was discovered by E. C. Pickering at the Harvard Observatory, in 1889, when it was found that the spectrum of Mizar (ζ Ursæ Majoris) consisted of alternately double and single lines (Fig. 173). Mizar is a visual double star, but the double lines belong to a single component of the visual pair. The visual pair probably are revolving around their center of gravity, but their distance apart is so great that their period of revolution is very long and their motions are too slow to be measured with the spectroscope.

The first spectroscopic binary in which one of the components is dark was discovered by Vogel, at Potsdam, in 1889. He found that the lines in the spectrum of Algol, the well-known variable star, shift alternately toward the red and blue ends of the spectrum with the same period as that of its variability (2 d. 20 h. 49 m.). This confirmed the theory that this star varies in brightness because a relatively dark one revolves around it and partially eclipses it at each revolution. The star Mu Orionis has the short period of 4.45 days, and the displacements of its spectral lines are considerable (Fig. 174).

In 1898 only 13 spectroscopic binary stars were known. By 1905 the number had increased to 140 pairs, 6 of which were also visual binaries. When Campbell published his second catalogue of spectroscopic binaries in 1910, there were 306 known pairs. In 19 cases the spectra of both stars had been measured, and from the absolute displacements of each set of lines their relative masses had been determined. With one possible exception the brighter stars of the systems are the more

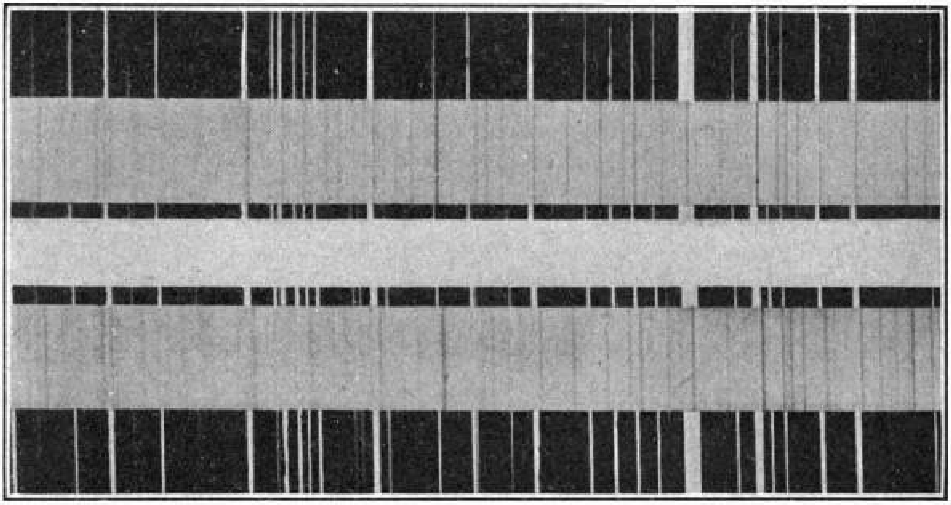

FIG. 174.—Spectra of Mu Orionis (*Frost; Yerkes Observatory*).

massive. The larger stars are generally less than twice as massive as the smaller. Of course, the difference is probably much greater in those cases where the spectrum of the smaller star is too faint to be observed.

286. Interesting Spectroscopic Binaries.—*Mizar.* As has been stated, the brighter component of Mizar was the first spectroscopic binary discovered. The later work of Vogel showed that its period is about 20.5 days, from which it follows in connection with the dimensions of its orbit (22,000,000 miles between the two components) that the mass of the system is at least four times that of the sun. The spectra of both stars are present, and their equal displacement proves that the masses of the two components are sensibly equal. The center of gravity of the system is approaching the solar system at the rate of about 9 miles per second. In 1908 Frost and Lee found that the other component of Mizar is also a spectroscopic binary of the type in which the spectrum of only one star of the pair is visible. In 1908 Frost announced that Alcor is a spectroscopic binary of short period in which both spectra are observable. Therefore Mizar is a visual double each of whose components is a spectroscopic binary, and the neighboring Alcor is also a binary.

Spica. One of the earliest known spectroscopic binaries is the first-magnitude star Spica whose spectral lines were found to vary by Vogel in 1890. The spectrum of the fainter component has also been observed. The period of the pair is 4 days, their mean distance from each other is about 11,000,000 miles, and their masses (neglecting the possible reduction due to the inclination of their orbit) are respectively 9.6 and 5.8 times that of the sun. This system is receding from the sun at about 1.2 miles per second.

Capella. The first-magnitude star Capella is a spectroscopic binary, the spectra of both stars being visible, in which the period is 104 days and the mean distance (possibly much reduced by the inclination of the plane of the orbit) about 50,000,000 miles. With these data the masses of this pair are found to be at least 1.2 and 0.9 that of the sun. This orbit has a very small eccentricity. These stars are receding from the solar system at the rate of nearly 20 miles per second. The parallax of Capella has been investigated with the utmost care by Elkin, who found for it $0''.09$, corresponding to a distance of 11 parsecs. At that distance the sun would be only $\frac{1}{70}$ as bright as Capella, or approximately of the fifth magnitude. Since the spectrum of Capella is almost exactly the same as that of the sun, which naturally leads to the conclusion that the temperature and surface brightness of Capella are approximately equal to those of the sun, it seems probable that the orbit of the pair is so inclined that the computed masses are much too small.

Polaris. The pole star has two darker companions discovered spectroscopically by Campbell in 1889. One is very close to the bright star and revolves around it in a period of a little less than 4 days, while the second companion is much more distant and requires about 12 years to complete a revolution. These stars are all quite distinct from the faint telescopic companion to Polaris.

Alpha Centauri. Alpha Centauri is at the same time a visual and a spectroscopic binary. Moreover, its parallax has been very accurately determined by direct means, so that the actual distance of the components from each other and their masses can be determined (Table XVIII). Since the same results can be determined spectroscopically, their comparison affords a valuable check on the accuracy of the results. The spectroscopic data were obtained by Wright at the branch of the Lick Observatory in South America, and the results obtained from them agree almost exactly with those based on other methods. But the spectroscope gives the additional fact, which cannot be determined otherwise, that Alpha Centauri is approaching the sun at the rate of 13.8 miles per second.

287. Variable Stars.—A star whose brightness changes is said to be a variable. The first known variable, Omicron Ceti, was discovered by Fabricius in 1596. The variability of Algol was definitely announced by Goodricke in 1783, though it seems to have been noticed a century earlier. The following year he recorded the variability of Beta Lyræ. But variable stars were not discovered in any considerable numbers until toward the close of the nineteenth century. Now more than 3000 of these objects are known in addition to those which have been found in considerable numbers in some of the globular star clusters. Some of them vary regularly and periodically, with periods ranging from less than a day to more than two years; others vary irregularly without any apparent rule or order. Some flash out brilliantly for a short time and then sink back more slowly into permanent oblivion. It is certain that the brightness of every star varies slowly because of its changing distance from the sun, if for no other reason, but there is no observational evidence of a change for this reason.

Variable stars are classified according to the peculiarities of their light changes, and the principal types are enumerated in the following articles. It must be remembered, however, that variable stars are strange objects which present numerous exceptions to all rules.

288. Eclipsing Variables.—If the plane of the orbit of a binary pair passes very nearly through the earth, the stars partially or totally eclipse each other every time they are in a line with the earth. If one of the two is a dark star and nearly as large as the bright one, it is clear that the light received from the pair will remain constant except when the brighter star is eclipsed. As the dark star begins to eclipse the brighter one, the light diminishes very rapidly until the time of greatest obscuration, after which as a rule the star rapidly regains its normal brightness. However, in some cases the dark star is very large so that the eclipse

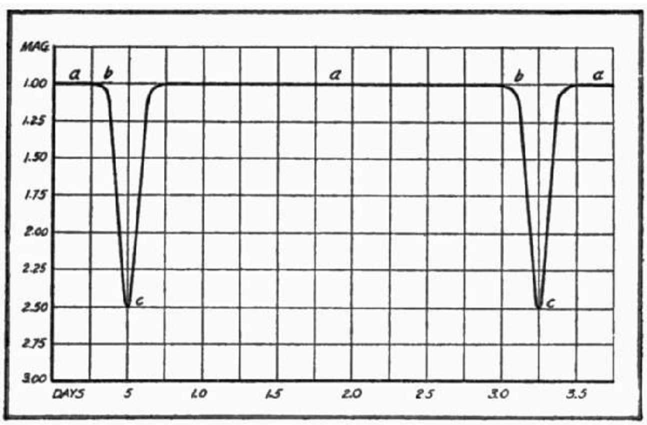

FIG. 175. — Light curve of typical eclipsing variable star.

persists for a considerable time, and then the variable remains at minimum for a few minutes or possibly a few hours.

The variability in the brightness of a star is represented by a curve. In Fig. 175 the curve for a typical eclipsing variable is given. The time is marked off along the horizontal axis and the brightness of the star is proportional to the distance of the curve above this axis. The parts marked a give the brightness when the star shines undimmed by an eclipse, the points b are where the light begins to wane as the eclipse commences, and the points c indicate the brightness at the moment of greatest obscuration. If the fainter star is somewhat luminous instead of being entirely dark, there will be a secondary and less pronounced minimum.

The typical eclipsing variable in which one component is dark is Algol (Beta Persei), whose light curve is essentially the same as that given in Fig. 175. About 100 stars of this type are known, and they are often called Algol variables. They are characterized by the shortness of their periods, many of which are less than 5 days and only 12 of which are longer than 10 days, and by the regularity of their light curves. Doubtless the explanation of their short periods is that when the two stars are far apart they do not eclipse one another, even partially, unless the plane of their motion passes very exactly through the earth.

Eclipsing variables are necessarily spectroscopic binary stars. It increases our confidence in both the methods and the interpretations to find that the data obtained in the two distinct ways are perfectly in accord. It is not to be inferred from this that the data are coextensive. The spectroscope furnishes the velocity and therefore the dimensions and mass of the system, especially when both stars are luminous. From the duration of the eclipses the dimensions of the stars can be found. Since their masses are known, their densities can then be computed. It has been found by Russell, Shapley, and other astronomers that the mean density of the variable stars for which there are sufficient observational data is about one eighth that of the sun. This is a remarkable result in view of the fact that usually one of the pair is very dark, and, according to current doctrine, in a condensed state approaching extinction. It should be added that in the case where there is a single minimum the result depends upon an assumption as to the relative densities of the components, and consequently may be considerably in error.

The period of Algol is 2 d. 20 h. 48 m. 55 s. It is normally a star of the second magnitude, but at the time of eclipse it loses five sixths of its light. In 1889 Vogel discovered that it is a spectroscopic binary. He found that the combined mass of the system is two thirds that of the sun, the bright star has twice the mass of the darker one, the distance between their centers is about 3,000,000 miles, the diameters of the stars are about 1,000,000 and 800,000 miles, and their density is about one fourth that of the sun. Schlesinger found that for the similar system Delta Libræ the density is also one fourth that of the sun.

There are several variations from the normal Algol variable. In one the stars are of unequal size and both bright. Then each eclipses the other, but the loss of light is different in the two eclipses, and the light curve has two minima of different depths. There are often irregularities which have not yet been explained. Sometimes the periods increase slightly for a number of years and then decrease again, showing possibly the presence of a third body. Sometimes the minima as determined photographically do not occur at the times found by visual observations.

289. Variable Stars of the Beta Lyræ Type.—Variable stars of the Beta Lyræ type are closely related to those which have been considered; in fact, the distinction between the two classes seems to be disappearing. Their light varies continuously from maximum to minimum and back to maximum again. The maxima are all equal, but as a rule there are two unequal minima. The standard star of this class is Beta Lyræ (Fig. 176), which is one of the earliest known variables and gives the class its name.

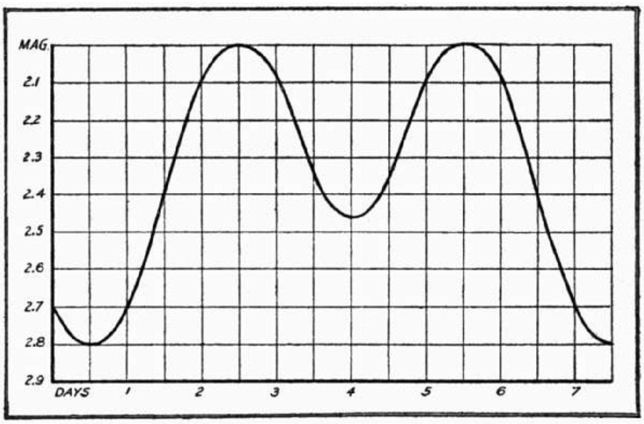

FIG. 176.—Light curve of a variable star of the Beta Lyræ type.

The explanation of the Beta Lyræ variables is that they consist of two stars revolving in such small orbits compared to their dimensions that the intervals in which neither obscures the other are very short. While this explanation satisfies the phenomena in a general way, there are many troubles in connection with the details. For example, about a dozen minor variations in the light curve of Beta Lyræ have been detected, or at least strongly suspected. Moreover, the spectroscopic data are often puzzling. But, on the whole, astronomers are satisfied that the eclipse explanation is the true one, and the gap between the light curves of Algol and Beta Lyræ is gradually being filled. In fact, Shapley includes many stars of the Beta Lyræ type among eclipsing variables of the Algol type.

290. Variable Stars of the Delta Cephei Type.—The star Delta Cephei has given its name to a third class of variables. In these stars the light curves are periodic with periods ranging from a few hours to 45 days. But that which particularly characterizes these stars is that they increase very rapidly in brightness from minimum to maximum, and then decline much more slowly with many minor irregularities modifying the gradual diminution in brightness. The characteristics of their light curves are given in Fig. 177. There are a few, however, known as the Geminids after Alpha Geminorum, whose light curves are nearly symmetrical with respect to their maxima.

The explanation of the Cepheid variables has been a very puzzling problem. Clearly their light changes are not ordinary eclipse phenomena, but their spectral lines shift periodically with the periods of their light variations. The natural conclusion has been that they are spectroscopic binaries and that the changes in light are abnormal eclipse phenomena. While the light changes and spectral shifts agree in period, they absolutely disagree in phase. That is, interpreting the spectroscopic data in the ordinary way, these stars are brightest when the principal stars are approaching the observer and faintest when they are receding, instead of having their minima when they are eclipsed. Evidently there are inconsistencies in the interpretations, and it is questionable whether eclipses have anything whatever to do with the light variations of these stars. A number of other explanations have been suggested, the most plausible of which is that the light variations are due

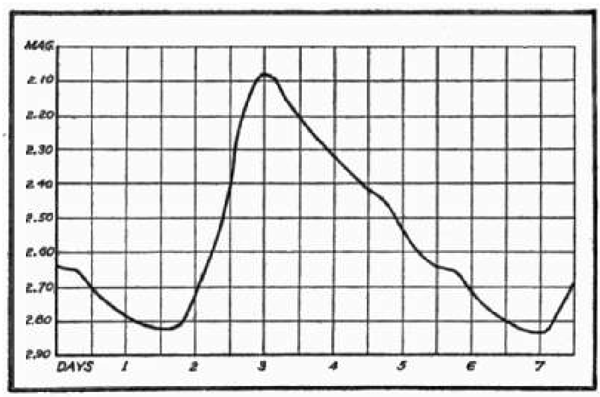

FIG. 177.—Light curve of a variable star of the Delta Cephei type.

to internal oscillations of the stars produced perhaps by collisions with masses of planetary dimensions. It has been found that very moderate oscillations would account for the variations in the rates of radiation. According to this hypothesis, the shifts of the spectral lines are produced partly by internal motions of the stars and partly by the effects of alterations in pressure of the radiating parts.

291. Variable Stars of Long Period.—A majority of variable stars belong to the class whose periods range from 50 to several hundred days. They are not periodic in the strict use of the term which is applicable to the Algol variables, yet their light varies in an approximately periodic manner. But the intervals between maxima, or between minima, are subject to some irregularities, and their luminosities at corresponding phases are by no means always the same.

The best-known star of this class is Omicron Ceti, the first known variable. It has been observed through more than 300 of its cycles, and yet it has not been found possible to formulate any law describing accurately its light variations. Its maxima and its minima are subject to as great irregularities as the intervals between corresponding phases. In 1779 William Herschel saw it when it was nearly as bright as Aldebaran, while 4 years later it was not visible even through his telescope. This means that it was at least 10,000 times as bright at its maximum as at that particular minimum. Ordinarily its maximum is much below that observed by Herschel in 1779, and its minimum is considerably above the limit of visibility with his telescope. Omicron Ceti was called *Mira*, the wonderful, and 300 years of observation have only added to the mysteries associated with its peculiar behavior.

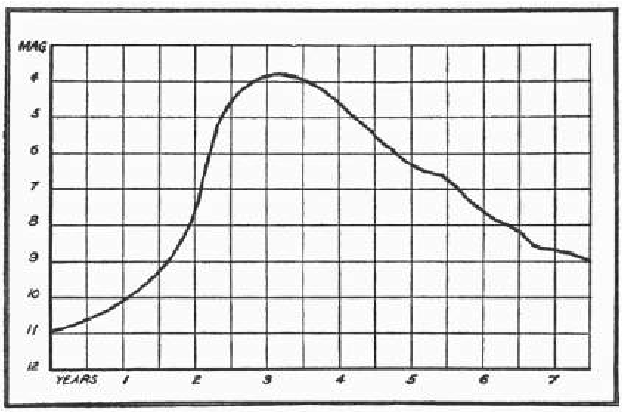

FIG. 178.—Light curve of variable star of long period.

The general characteristics of the light curves of variable stars of long period is a slow, but gradually accelerated, increase in brightness followed by a much more gradual decline. The spectroscope shows marked changes in their spectra, but no evidence of their being spectroscopic binaries. They are nearly all red and are probably of not very high temperatures. The cause of their variation seems to lie within the stars themselves, yet it is difficult to imagine any internal disturbances which would produce the remarkable fluctuations which are observed in many stars of this class.

292. Irregular Variable Stars.—In addition to the classes of variable stars so far enumerated, there are others whose variations have no semblance of periodicity. Some flash out with relatively great brilliancy after intervals usually counted in years. These stars are generally, if not always, red. Others unaccountably fade away now and then and sometimes become invisible through good telescopes, even though they had been ordinarily visible with the unaided eye. These stars are sometimes associated, at least apparently, with faint nebulous masses.

293. Cluster Variables.—A very interesting and important discovery was made in the last decade of the nineteenth century by Bailey at the South American branch of the Harvard Observatory. He found that in the great globular cluster, Omega Centauri, 125 stars were variable out of the 3000 which he examined. He and other astronomers have found similar variables in many other globular star clusters. In a given cluster the range of variability is nearly the same, usually a magnitude or two, the character of the light variation is essentially the same, and the periods are approximately the same, generally less than 24 hours. Their light curves are closely similar to those of the variables of the Delta Cephei type, and it is really a question whether the cluster variables should be considered a separate class. The brightness increases with great rapidity from their minimum to a luminosity at maximum from two to six times as great. Then they diminish in brightness much more slowly to their minimum, at which they remain nearly stationary for a few hours at most.

The approximately equal periods and range of variation of the cluster variables indicate that they are very much alike in spite of the enormous distances which separate them. Possibly they were once much more alike and now differ to some extent because of slightly different courses of evolution or present environment. Or, possibly, though not probably, there is some great common cause for their changes, a force causing pulsations in scores of stars distributed widely throughout the clusters. Although nearly 2000 of these objects have already been discovered and studied, astronomers have no idea as to the reasons for their peculiarities.

294. Temporary Stars.—Occasionally stars have been observed to blaze forth in parts of the sky (mostly in the Milky Way) where none had previously been seen, and then to sink away into obscurity in the course of a few weeks or months. They are characterized by a sudden rise to one great maximum of brilliancy which, notwithstanding later temporary increases, is never repeated. One of the most remarkable of these stars of which there are any records blazed out in Cassiopeia in 1572 and was for a time as bright as Venus. This is the star which attracted the attention of Tycho Brahe and turned him to astronomy. The interest of Kepler also was stimulated by the discovery of a temporary star in Ophiuchus in 1604. At its maximum it was as brilliant as Jupiter. It must not be supposed all temporary stars are so brilliant, for only a few rise to such splendor.

In recent times the number of temporary stars discovered has greatly increased, both because more observers are scanning the sky than ever before, and more especially because they are now recorded by photography. In the last 30 years 19 of these objects have been discovered, 15 of which were found first on the photographic record of the sky which is being secured at the Harvard College Observatory. Only 10 of these stars were discovered from 1572 to 1886, when the photography of the sky was first systematically begun at Harvard.

Temporary stars are called *novæ*, or new stars. A description of one of them will give a good idea of the characteristics of all of them. One of the most interesting and best studied novæ of recent times is the one discovered by Anderson, February 22, 1901, in Perseus. On the 23d of February it was brighter than Capella, while an examination of the photographs of the region taken by Pickering and by Stanley Williams showed that on the 19th it was not brighter than the 12th magnitude. In the short space of four days its rate of radiation had increased more than

FIG. 179.—Light curve of Nova Persei.

20,000 fold. Twenty-four hours later it lost one third of its light, and within a year it had dwindled to the 12th magnitude, or near the limits of visibility with a telescope of considerable power. Its light curve for the first three months after its maximum is shown in Fig. 179.

The changes in the spectra of the novæ are as remarkable as their changes in luminosity. Very early in their development they have (at least in case of those stars which were observed early) dark-line spectra. Shortly thereafter bright lines appear. In the case of Nova Aurigæ, discovered in 1892, and the first temporary star whose spectrum was examined in any detail, the dark lines and bright lines were both visible at one time. The displacement of the bright lines showed, on the basis of the Doppler-Fizeau principle, a velocity away from the earth of over 200 miles per second, while the dark lines showed, on the same basis, an approach toward the earth of more than 300 miles per second. There are abundant grounds for doubting the correctness of this interpretation, but no satisfactory explanation is at hand. These phenomena are characteristic of novæ in general. As they become fainter the dark lines vanish and the bright lines characteristic of nebulæ appear, except that in the novæ they are broad while they are narrow in the nebulæ.

The most interesting thing observed in connection with Nova Persei was the nebulous matter which was later found around it. Its existence was first shown on photographs by Wolf taken August 22 and 23, 1901. Later photographs by Perrine and Ritchey showed that it was gradually becoming visible at increasing distances from the star. It looked as though the star had ejected luminous matter, but it was found on computation that, if this were the correct explanation, the expelled matter must have been leaving the star with about the velocity of light. This, of course, is improbable if not impossible.

The temporary stars demand explanation. The theory was suggested by Kapteyn and W. E. Wilson, and expounded in detail by Seeliger, that there is invisible nebulous or meteoric matter lying in various parts of space, particularly in the region occupied by the Milky Way (there is confirmatory evidence of this hypothesis); that there are dark or very faint stars (confirmed by phenomena of eclipse variables); that the dark stars, rushing through the nebulæ, blaze into incandescence as meteors glow when they enter the earth's atmosphere; that the heating is only superficial and quickly dies away, to be partially revived once or twice by encounters of the stars with stray nebulous wisps; and that the nebulous ring observed around Nova Persei became visible as it was illuminated by the light from the star itself.

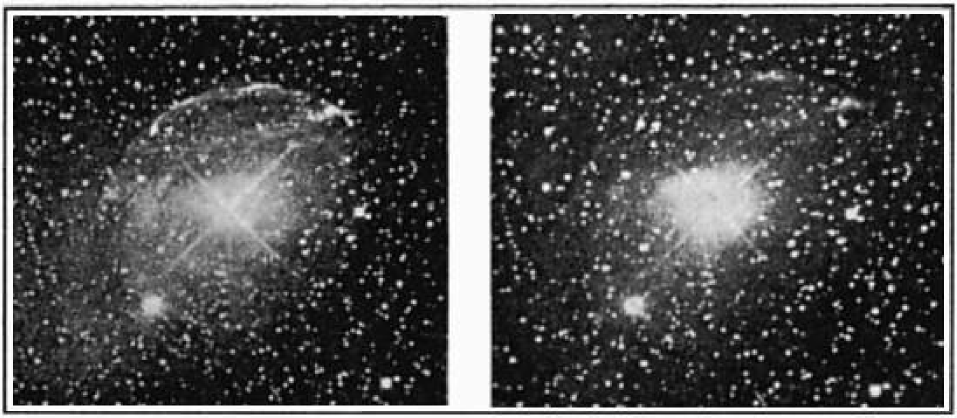

FIG. 180.—Nebulosity surrounding Nova Persei on Sept. 20 and Nov. 13, 1901. *Photographed by Ritchey at the Yerkes Observatory.*

The explanation of Kapteyn at first seems plausible, but there are serious objections to it. In the first place, the photographs of Nova Persei indicate strongly that the expanding nebulous ring surrounding it was due to something actually moving out radially from the star. In the second place, the density of the nebula demanded to account for the enormous rise in luminosity is impossibly high. In the third place, the fact that the star stays at its maximum only a very short time implies a nebula whose thickness is incredibly small.

Lindemann has developed the hypothesis that novæ are produced by collisions of stars with stars. If one star should encounter another in central collision with the great speed at which they would move as a consequence of their initial motion and mutual gravitation, the heat generated would be enormous. If they were of equal mass and started from rest, the heat developed would be five sixths of that which would be generated, according to the principles of Helmholtz, by the contraction of both of them from infinite expansion. This heat would be developed in a few hours, or days at the most, and the temperature of the combined mass would rise enormously. But with increase of temperature there would be corresponding expansion, which would result in a diminution of the temperature. If the stars were originally gaseous, the final temperature after expansion would be lower than that before collision because the conditions are the opposite of those in Lane's law (Art. 216), according to which the temperature of a gaseous star increases as it loses heat by radiation and contracts. Or, stated directly, if heat could be applied to a gaseous star by radiation or otherwise, it would expand and increase its potential energy at the expense, not only of all the heat supplied, but also partly at the expense of that which it already possessed.

While in a general way the collision theory of the origin of novæ corresponds with the observations, it is not without difficulties. Obviously, actual collisions of stars would be excessively rare phenomena. Lindemann finds that in order to account for the observed number of temporary stars there must be about 4000 times as many dark stars as there are bright ones. Such a large number of obscure masses would radically modify the dynamics of the stellar system (Art. 279); and it is generally regarded as improbable that so many of them exist.

295. The Spectra of the Stars.—The spectra of the stars differ as greatly as their colors. They were first classified in 1863, by Secchi, who divided them into four groups. While more powerful instruments

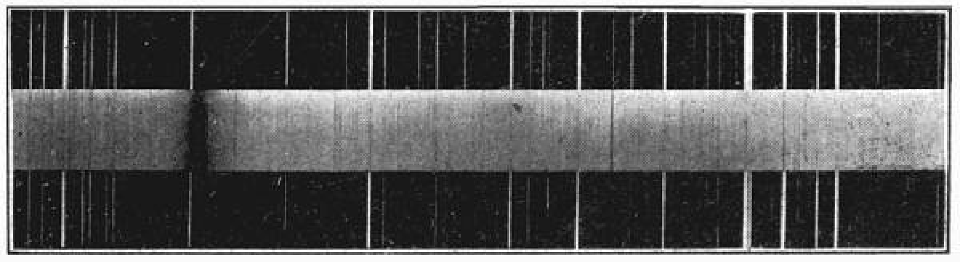

FIG. 181.—The spectrum of Sirius (Secchi's Type I).

have shown many new facts and have made it necessary to add many new subclasses, the four types described by Secchi still form a general basis for classification. A more detailed classification, which is now much used, was devised by E. C. Pickering, Miss Maury, Mrs. Fleming, and Miss Cannon in connection with the great photographic survey of stellar spectra which is being made at the Harvard College Observatory.

Type I. Stars of Secchi's first type are blue or bluish white. Examples are Sirius, Vega, and all bright stars in the Big Dipper except the first one. Nearly half of all stars examined are of this type. Their spectra are brightest toward the violet end, indicating presumably that they are at high temperatures. The spectrum of Sirius is shown in Fig. 181.

Type I, in Secchi's system, includes Types B and A of the Harvard system. Type B is often called the Orion type because of the abundance of these stars in Orion, or the helium type, because the absorption lines are due almost entirely to helium, while the metallic lines which are characteristic of the sun's spectrum are absent. The Type A, or Sirian stars, are characterized by strong hydrogen absorption lines in their spectra, and almost complete absence of metallic lines.

Type II. The stars of the second type are somewhat yellowish; they are called solar stars because their spectra are similar to that of the

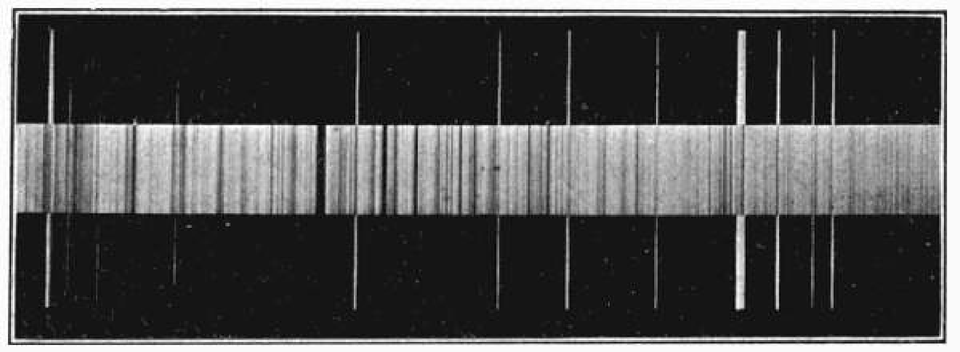

FIG. 182.—Spectrum of Beta Geminorum (Harvard Class K). *Photographed at the Yerkes Observatory.*

sun. That is, the lines of helium are absent, the lines of hydrogen are still present, and there are many fine metallic lines. The stars of the second type are about as numerous as those of the first type.

Secchi's second type includes three classes of the Harvard system. Those nearest like the Sirian stars are called Type F, or the calcium type. In their spectra the hydrogen lines are still conspicuous, though somewhat reduced in density, and two lines, known as H and K, due to calcium have become conspicuous. Following the class F is the class G, of which the sun is a typical member. Then come the stars of Type K, of which Beta Geminorum and Arcturus are examples, in which the intensity of the hydrogen lines is reduced until they are less conspicuous than some of the metallic lines. The spectra of these stars are given in Figs. 182 and 183.

Type III. Stars of the third type are red, and the two most conspicuous examples of them are Antares and Betelgeuze. Only about 500 of these stars are known, and many of them are variable. Their spectra show heavy absorption bands, due almost entirely to titanium oxide, which are sharp on their borders toward the violet and which gradually fade away toward the red. The fact that a compound exists in these stars indicates that their temperatures are lower than those of Types I and II. The same thing is indicated by their colors in accordance with the first law of spectrum analysis (Art. 223). In all known cases they have very small proper motions, which means that they are immensely remote from the sun. Hence such brilliant stars as Antares and Betel-

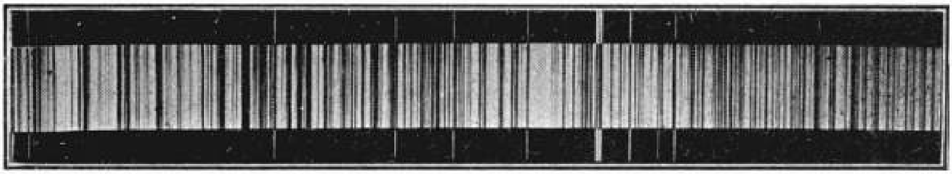

FIG. 183.—Spectrum of Arcturus (Harvard Class K). *Photographed at the Yerkes Observatory.*

geuze, whose light is largely absorbed, must be enormous objects. They are almost certainly many thousand times greater in volume than our own sun.

The stars of Secchi's third type are of Type M in the Harvard system. They are divided into two chief subclasses, Ma and Mb; a third subclass Md includes the long-period variable stars whose spectra show bright hydrogen lines in addition to the bands characteristic of the whole type.

Type IV. The 250 stars of Secchi's fourth type are all faint and of a deep red color. Their spectra have heavy absorption bands, or flutings, sharp on the red side and indefinite on the violet, being in this respect opposite to the stars of the third type. The absorption bands in this case are probably due to carbon compounds. These stars are all very remote from the sun, and nothing is known of their absolute magnitudes, or of their masses and dimensions.

The Wolf-Rayet Stars. There is another class of stars, discovered in 1867 by Wolf and Rayet at the Paris Observatory. They are Type O, having five subdivisions, in the Harvard system. Their spectra consist of fairly continuous backgrounds on which are superimposed many dark lines and bands, some few of which are due to helium and hydrogen, but most of them to unknown substances. They contain in addition many bright lines. The metallic lines of the solar spectrum are quite unknown in these stars. Of the more than 100 stars of this type so far discovered, all are situated either in the Milky Way or in the Magellanic Clouds in the southern heavens, which have most of the characteristics of the Milky Way.

296. Phenomena Associated with Spectral Types.—A large number of phenomena combine to show that the classification of stars according to their spectra is on a fundamental basis. The order of arrangement from the simplest to the most complex spectra is:

Secchi's Types:	Wolf-Rayet;	I;	II;	III;	IV.
Harvard Types:	O;	B, A;	F, G, K;	M;	N.

If the gaseous nebulæ were included, they would be put ahead of the Wolf-Rayet stars. There is a fairly continuous sequence of spectra from Type O to Type M, but there is an abrupt break between Types M and N.

The principal phenomena which are associated with the spectral types and which agree on the whole, in arranging the stars in the same order, are:

(*a*) The average radial velocities of the stars, determined largely at the Lick Observatory and its southern branch, and discussed by Campbell, are slowest for stars of Type B and increase to Type M. The results, as given by Campbell, with velocities expressed in miles per second, are:

Types:	B,	A,	F,	G,	K,	M,	Planetary Nebulæ.
Velocities:	4.0,	6.8,	8.9,	9.3,	10.4,	10.6,	15.7

(*b*) The average velocities of the stars across the line of sight, as determined by Lewis Boss, show a similar relation to the spectral type. The results are:

Types:	B,	A,	F,	G,	K,	M.
Velocities:	3.9,	6.3,	10.0,	11.5,	9.4,	10.6.

These results together with those depending on the spectroscope establish the fact that the stars of Types B and A move on the average only about half as fast as those of Types G, K, and M.

(*c*) In Kapteyn's star-stream I, the B and A stars are relatively numerous, the F, G, and K stars occur less frequently, and the red stars are very few in number. In the star-stream II, the B and A stars are not numerous, the F, G, and K stars occur in relatively great numbers, and the M stars are scarce.

(*d*) While there are two great star-streams, there are very many divergencies from them on the part of individual stars. The stars of Type B scarcely show the star-streaming tendency, those of Type A conform very closely to the two streams, and succeeding types show more and more of heterogeneity of motion.

(*e*) On considering only stars brighter than magnitude 6.5 so as not to have the results influenced by the myriads of remote stars, it is found that the B stars are 10 times as numerous in the Milky Way as near its poles, the A stars are less strongly condensed in the Milky Way, and finally, after continuous gradation through the various types, the M stars are scattered uniformly over the sky.

(*f*) For a given magnitude the stars of Type B are more remote than those of Type A, which, in turn, are more remote than those succeeding down to Type G; then, beyond Type G, the distances increase to stars of Type M, whose distances are exceeded only by the B stars. This means, of course, that the B stars are most luminous, the A stars less luminous, the G stars least luminous, while the M stars are more luminous than any except the B stars.

(*g*) The proportion of B stars which are spectroscopic binaries is large, the proportion is less for the A stars and it decreases through the list of types to M.

(*h*) Lower limits to the combined masses of spectroscopic binaries can be determined (Art. 285). The average mass of those of Type B is about 7.5 times the average mass of all other types.

(*i*) The average period of spectroscopic binaries of Type B is very short, the average is a little longer for stars of Type A, and increases through Types F, G, K, and M.

(*j*) The average eccentricity of the orbits of spectroscopic binaries is small for stars of Type B, is larger for stars of Type A, and is increasingly larger for stars of the Types F, G, and K, in order.

297. Evolution of the Stars.—All the resources of science have been taxed to the utmost in attempting to discover the present constitution and properties of the sidereal system. At the best, astronomers have barely begun to explore the wonders of that part of infinite space which is within the reach of modern instruments. Moreover, their observational experience is limited to a moment of time compared with the immense ages required for appreciable changes to take place in the heavenly bodies. Hence it may seem presumptuous for them to attempt to discover the mode, or modes, of evolution of the stars. Any theories of stellar evolution that may be developed at the present time are probably no more than first approximations, and they may be entirely wrong.

Astronomers almost universally hold that the stars have contracted from the nebulæ, and most of them believe that with increasing age they have gone, or are now going, successively and in order through the spectral types B, A, F, G, K, and M. The B stars are of very high temperature and are pouring out radiant energy at an extravagant rate. After they cool somewhat it is supposed that they become stars of Type A. Their spectra are supposed to be simple because all compounds, and possibly some elements, are broken up and dissociated at those high temperatures. With further loss of heat they are supposed to pass successively through the other spectral types until, at the M stage, compounds exist in their atmospheres. Beyond the M stage their light diminishes and they finally become, in the course of time, cold and dark, and they remain in this condition until, perhaps, they are again reduced to the nebulous state by collision with other stars. All the forms in the chain from nebulæ to relatively dark stars are known to exist from observational evidence. The many other characteristics which arrange the stars in nearly, or exactly, the same order are regarded as strongly supporting the theory.

The theory of the evolution of the stars has strong resemblances to the Laplacian theory of the development of the solar system. This is only natural in view of the general acceptance of the theory of Laplace almost up to the present time. As additional facts have been discovered they have been placed in this scheme, often without inquiring if they would not fit as well in some other theory.

Laplace started with an intensely heated and widely expanded solar nebula and he supposed that it has cooled down to its present temperature. Helmholtz supplemented and corrected this theory by proving that contraction would develop an enormous amount of heat and greatly retard the process of cooling. The conclusions of Helmholtz have been given place in the theory of the evolution of the stars. Lane made a further very important supplement to the work of Laplace when he proved that if a body in a monatomic gaseous state contracts, heat is produced in quantities not only sufficient to make up for that which had been radiated away, but also sufficient actually to increase its temperature. In spite of the fact that the results of Lane have been current for almost fifty years, they have often been ignored in their application to the evolution of the stars. If the stars of any type are in a tenuous monatomic gaseous condition and contract, their temperature will inevitably rise and continue to rise until they cease to be entirely gaseous and monatomic.

Consequently, if the stars of the types B, A, F, G, K, M are in the order of decreasing temperature and are gaseous, the logical conclusion on the basis of the supplements to Laplace's theory is that the evolution proceeded in the reverse order. Of course, the stars may not all be completely gaseous. This has given rise to the theory, proposed by Lockyer and amplified and ably supported by Russell, that the nebulæ contract into tenuous red stars of Type M which have low temperatures; with loss of heat they contract, their temperatures rise, their spectra become simpler until they reach their climax in Types A and B; after this they cease to be completely gaseous, and with increasing condensation and liquefaction, their temperatures decline and their spectra proceed back through the types F, G, and K to M. The cogency of the arguments on which these conclusions rest cannot be denied, and many observational data are quite in harmony with them. But there are also some things (for example, the high velocities of the nebulæ, Art. 301) which have been thought to be strongly opposed to them. The two theories are alike in starting from nebulæ and ending with cold and lifeless suns.

298. The Tacit Assumptions of the Theories of Stellar Evolution.—In every theory there are many more or less tacit assumptions, some of which may be of great importance. It has been found by a large amount of experience that errors more frequently enter through unexpressed hypotheses than in any other way. This has been particularly true in mathematics where it is relatively easy to determine precisely the location of the error that has been made in any course of reasoning. It follows that one of the best ways of avoiding errors is to express fully all the hypotheses on which reasoning is based. And quite aside from this, it is useful and important to know all the bases on which conclusions actually rest. Consequently, the tacit and imperfectly established assumptions on which the present theories of stellar evolution are founded will be enumerated; it will be found that at the present time most of them must remain simply assumptions.

(*a*) *It is assumed that the evolution of the stars is from nebulæ to dense bodies and not in the opposite direction.*

The best evidence in support of or against a proposition is usually observational; when observational evidence is lacking, we must resort to reasoning based as far as possible on principles which have been established by experience.

There is as yet no observational evidence that nebulæ or stars contract; observations have extended over so short a time that it could not be expected. On the other hand, in the case of the novæ, stars are observed to acquire the characteristics of the Wolf-Rayet stars, which border on the planetary nebulæ. Of course, this may be quite exceptional, but it should not be neglected. Consequently, in this matter there is no conclusive observational evidence.

The principal known force which tends to produce condensation is gravitation. In the case of the stars this force is balanced by the expansive forces due to their high temperatures. If their heat is produced only by their contraction, as they lose heat by radiation, they certainly contract. But the contraction theory is inadequate to explain the heat which the sun has radiated (Art. 219), and it seems very probable, if not altogether certain, that stars have other important sources of energy. As has been suggested, the heat of the sun is probably due in part to the disintegration of radioactive substances. Perhaps in the extreme conditions of pressure and temperature prevailing in the deep interiors of stars the process of disintegration is greatly accelerated and is going on in all elements. And probably there are very important sources of energy not now suspected, just as the subatomic energies were not suspected a few years ago.

Now suppose the amount of energy generated in a star in all these ways is greater than that radiated. Then the star will inevitably expand and its temperature will fall, because with increased dimensions gravitation cannot balance so high a temperature. If the process continues, the star will expand to a nebula, which will necessarily have a low temperature. In this case the direction of evolution would be reversed. But as the star expands, the conditions in its interior are changed, and the production of energy might be reduced so that it would only equal that radiated. In this case the star would reach a condition of equilibrium which would be indefinitely maintained unless the subatomic and other possible sources of energy were ultimately exhausted, and it seems certain that they would become exhausted. Then the star would contract if its disintegrated products still obeyed the law of gravitation, and its evolution would proceed in the direction assumed in current theories, though at a greatly retarded rate.

In reaching the conclusions which have been set forth it has been assumed that the masses of the stars are constant. It is clear that their masses probably are increased somewhat by the accretion of meteoric matter and individual molecules, but, so far as may be judged from the sun, this is not an important factor. It is quite certain that the sun is emitting electrified particles in great numbers and with high velocities. Probably the auroral displays in the earth's atmosphere are produced by such particles impinging on the molecules in the tenuous gases at great altitudes. In view of the considerable light sometimes emitted by auroræ and the earth's immense distance from the sun, it seems probable that the sun loses these particles at a rate which makes the process important. If so, the stars may possibly be disintegrating into nebulæ. For example, the nebulosities around the Pleiades (Fig. 184) may have come out from these stars instead of being gradually drawn in upon them. Besides this, comets give numerous examples of matter being dispersed in space.

It is obvious that we do not know with any high degree of certainty in which direction stellar evolution is proceeding. Sound scientific method calls for keeping both of them in mind until a decision is reached on the basis of unequivocal evidence. Whichever of the two conclusions may prevail, the result will be unsatisfactory, for it will indicate a universe evolving always in one direction, leaving the origin unexplained. Possibly there are changes in both directions, and it may be that stellar evolution in some way and on a stupendous scale is approximately cyclical like most of the changes which come entirely within the range of our experience.

(*b*) *It is assumed that all stars have approximately the same chemical constitution; or, if not, that their spectra do not depend to an important extent upon their chemical constitutions.* One or the other of these assumptions is made tacitly when it is supposed that all stars pass in one direction or the other through several identical spectral types.

The spectroscope proves that the stars contain familiar elements; it does not prove that they do not contain some unknown elements, or that the known elements occur in all stars in the same proportions. The great diversities on the earth make it natural to conclude that there are important differences in the millions of stars in the heavens. Moreover, the different dimensions, densities, and absorption spectra of the planets lead to the same conclusion. The hypothesis that the stars are of approximately identical constitution must be considered improbable until it is supported by observational evidence.

It is too bold to assume that if the stars are differently constituted they nevertheless have the same spectra at the same temperatures. But the assumption actually made is not quite so bad as it at first seems, for the stellar spectra from B to F, and even G, are classified primarily

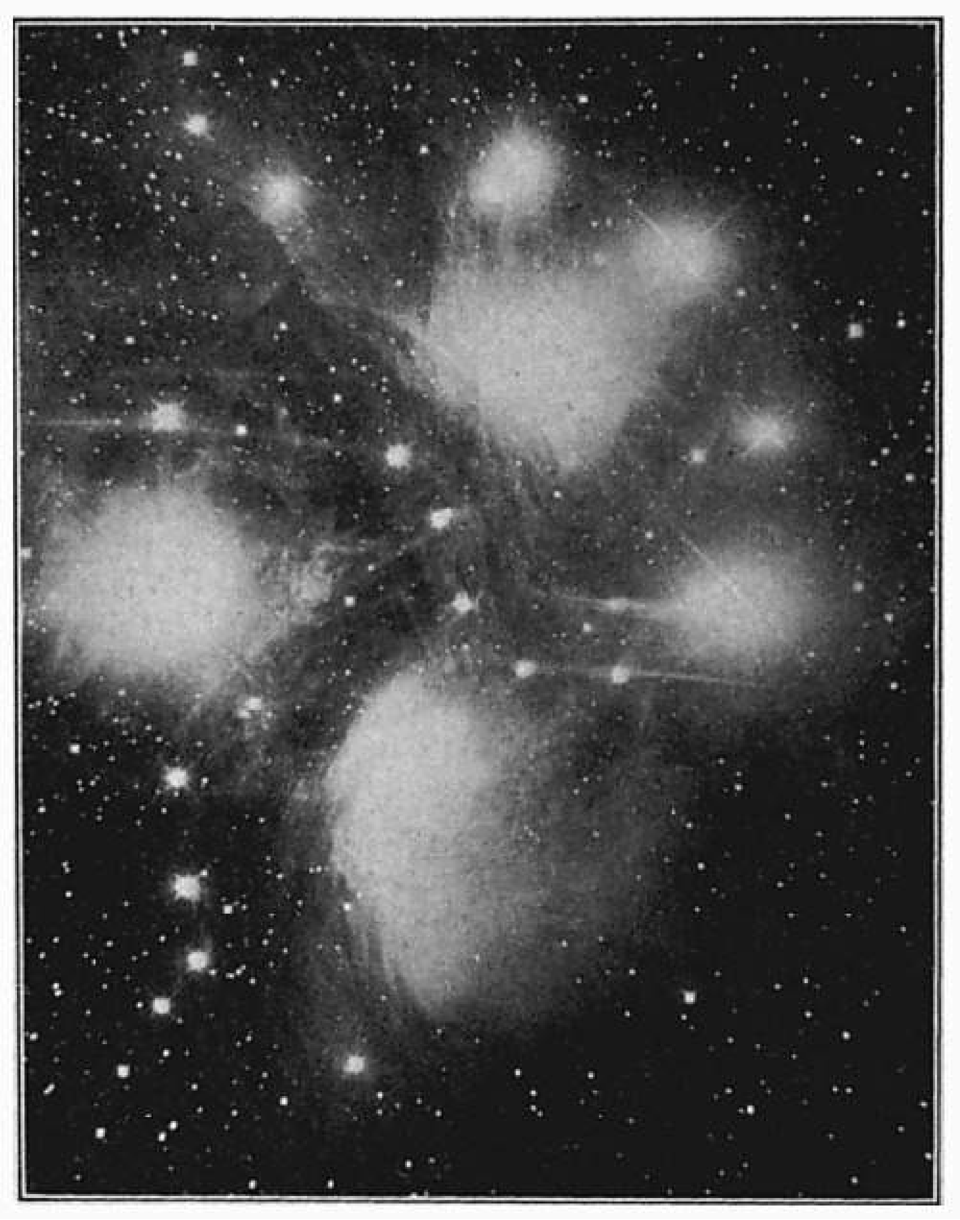

FIG. 184. — The Pleiades. These stars are surrounded by nebulous masses of enormous volume. *Photographed by Ritchey with the two-foot reflector of the Yerkes Observatory.*

on the basis of their hydrogen emission and absorption lines. Within these classes there is opportunity for great variety, and indeed variety is not wanting. There is nothing obviously unsound in supposing that the character of the hydrogen spectra of the stars depends upon their temperatures. But the question is whether a star which has only helium and hydrogen lines can ever show the strong metallic absorption lines which are characteristic of stars of Types F and G. Fortunately, there is now direct evidence on this point, for there are certain variable stars which, at their maxima, are of spectral Types B or A, while, at their minima, they are of Types F or G. There is nothing inherently improbable in ascribing these changes in luminosity and spectra to changes in

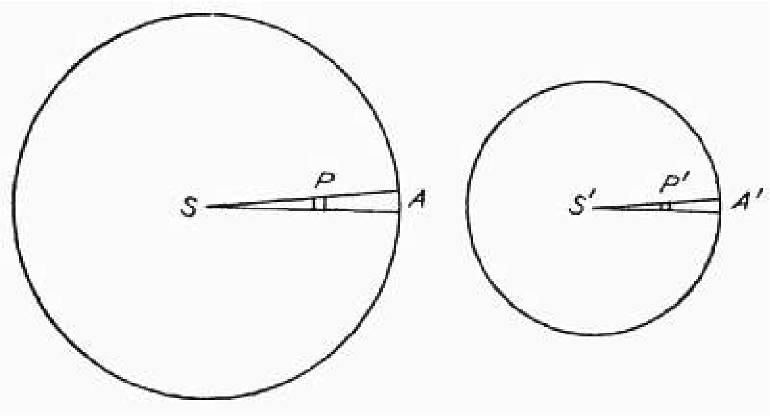

FIG. 185.—For a given density, the more massive the star the higher its temperature.

temperature, produced, perhaps, by contracting and expanding oscillations of these stars.

(*c*) *It is assumed that, aside from the rate of change, the evolution of a star does not depend on its mass.* In considering this point the assumption that the spectrum of a star depends upon the temperature of its radiating surface, or radiating layer, should constantly be borne in mind.

It should be recalled in the first place that the known masses of the stars differ considerably (Art. 284), and it is improbable that the few which are known cover anywhere nearly the whole range. Consider two stars, S and S', Fig. 185, of the same material and equal density but one having twice the mass of the other, and fasten attention on unit volumes at any corresponding points P and P' in their interiors. The pressure on the unit volume at P is greater than that on the unit volume at P', both because the column PA is longer than $P'A'$ and also because each unit mass in PA is subject to a greater attraction than that to which the corresponding mass in $P'A'$ is subject. To balance the higher pressure in the larger star the gaseous mass at P must have a higher temperature than that at P'. Consequently, if two stars of the same material are of the same density at corresponding parts and are of unequal masses, the temperature of the larger star at all points from its center to its surface is higher than that of the smaller star; and if the spectrum of a star depends primarily on its temperature, their spectra are different.

A mathematical discussion shows that if two stars are of the same material and of equal densities at corresponding points, their absolute temperatures are as the squares of their radii. On combining this result with Lane's law that the absolute temperature of a monatomic gaseous star is inversely as its radius, it is found that the absolute temperatures of stars of equal volumes and the same material are proportional to their masses.

The results which have just been reached are very important, even if they represent the physical facts only approximately, and they should not be ignored in discussions of stellar evolution. For the purposes of numerical illustration suppose the sun is gaseous and consider a star of the same material and density having a radius twice as great. Its mass is eight times that of the sun. By the first law, its temperature is four times that of the sun. Since the rate of radiation is proportional to the fourth power of the absolute temperature, its radiation per unit area is 256 times that of the sun. Since its radius is twice that of the sun, its surface is 4 times greater, and its whole radiation, or *luminosity*, is $4 \times 256 = 1024$ times that of the sun. That is, two stars of the same material and density, whose masses are in the ratio of only 8 to 1, differ in luminosity in the ratio of 1024 to 1. If a star were eight times more massive than the sun, it would have a spectrum of Type B or A, if these spectra indicate high temperatures, and it would be a star comparable to the most brilliant ones found in the heavens. On the other hand, if it were one eighth as massive as the sun, it would have a spectrum characteristic of low temperatures (Type M?), and would be a feebly luminous body.

Of course, it is not necessary that other stars should have the same density as the sun. It is known from eclipsing variables that comparatively few are as dense as the sun, and that the densities may be as small as one hundredth or even one thousandth of that of the sun. It can be shown that the temperature of a gaseous star is proportional to the cube root of the product of the square of the mass and the density. Hence, in order that a star having a density one hundredth that of the sun should be as hot as the sun, its mass must be about 10 times greater. But under these conditions its surface and luminosity would both be about 100 times as great as those of the sun. That is, a star nearly as brilliant as one of the Pleiades might be only one hundredth as dense as the sun if its mass were only 10 times greater. A star 10 times as great in mass and one tenth as dense as the sun would be 460 times as luminous.

It can be seen from this incomplete discussion that in order that a star shall have high temperature and great luminosity it must have a mass at least as great as that of the sun; for it is not probable that a much denser body would be in a gaseous condition. But the luminosity of a gaseous star is so sensitive a function of its mass that one 10 times more massive than the sun would be a brilliant object unless its density were exceedingly low; and one only one tenth as massive as the sun would be relatively faint, even if it were as dense as the sun. Therefore, it is not strange that no stars with very small masses have been found; one as small as one of the planets could not be self-luminous while in a gaseous state. On the other hand, no star many times more massive than the sun has been found. Perhaps the reason is that the data respecting masses is yet so meager; perhaps the temperatures in massive stars become so great that their atoms disintegrate and the remains fly away into space.

(*d*) *It is assumed that the contraction of nebulæ into stars began at such a time, or at such times, and that the individual nebulæ had such masses that there has resulted the present sidereal system of nebulæ and stars in all stages from hottest to coldest.* The implications of this assumption are not at once fully evident; they can be brought out only by a mathematical discussion whose results alone can be given here.

On the basis of Stefan's law of radiation and the assumption that the heat of a star is developed entirely by contraction, it is found that the change of radius is directly proportional to the product of the time and the square of the mass. If there are other important sources of heat, and if the radiation is from a layer of varying depth instead of from the surface, the law may be much in error. But on the assumption that this result applies to the sun, it is possible to compute the time required for it to have contracted from any given dimensions. According to the contraction theory its radius is now diminishing at the rate of a mile in

44 years. Consequently, on this basis it has contracted from the orbit of Mercury in 1,500,000,000 years. At first thought this would seem to give a long supply of heat to the earth to meet geological needs; but if the sun ever filled a sphere as large as the orbit of Mercury and radiated according to Stefan's law, whatever the source of heat may have been, its temperature must have been so low that its rate of radiation could have been only a little more than one seven-thousandth that at present, a quantity altogether inadequate to support life on the earth. According to this contraction theory, 4,400,000 years ago the radius of the sun was 100,000 miles greater than at present, and its rate of radiation was only two thirds that which is now observed. With this rate of radiation the theoretical mean temperature of the earth, determined by the method used for Mars in Art. 172, comes out 51° lower than at present (60° F.), or 23° below freezing.

The second part of the law gives the interesting and unforeseen result that the more massive a star, the more rapidly it contracts. Or, if the results are translated over into a relation between density and time, it is found that if a star of large mass and one of smaller mass start with the same density, the density of the large star will increase faster than that of the smaller one. The rate of change of density is proportional to the cube root of the fifth power of the mass. Therefore, if one star has 8 times the mass of another and they start contracting from the same density, it will arrive at some greater density in $\frac{1}{32}$ of the time required by the smaller star to reach the same density. As applied to the stellar system, this means that if the stars all started condensing from nebulæ at the same time, those which have the largest masses are at present by far the densest and hottest. The large stars are probably much hotter on the average than the small ones, but it is doubtful if they are denser. It must be remembered that these results depend upon the very questionable assumption that the heat of stars is due entirely to their contraction.

299. The Origin and Evolution of Binary Stars.—The great number of binary stars calls for a consideration of their origin and evolution. If the stars have condensed from nebulæ, it is natural to suppose that binary stars have developed from nebulæ which divided into two parts, or that the divisions have taken place after the condensing masses have reached the star stage. It is also conceivable that stars which originated separately have later united to form physical systems. Both of these theories will be considered.

Consider first the theory that the binary stars have originated by the fission of nebulæ or larger stars. The basis for the theory is the very reasonable assumption that the original nebulæ had more or less rotation, possibly quite irregular in character. In those cases where the amount of rotation, that is, the moment of momentum, was small, it is believed that single stars rotating slowly have resulted. In those cases where the moment of momentum was large, it is supposed that there has been separation into two parts.

There is some theoretical basis for this conclusion, though from a practical point of view it has generally been greatly overestimated. In a brilliant piece of work on figures of equilibrium of homogeneous fluids rotating as solids, Poincaré, following Maclaurin and Jacobi, showed that for slow rotation an oblate spheroid is a figure of equilibrium, for faster rotation an elongated ellipsoid is the corresponding figure, and for still faster rotations the ellipsoid has a constriction, suggesting that for still faster rotations the figure would be two very unequal masses. Now, when a nebula or a star contracts it rotates more rapidly because the moment of momentum is constant. Hence it seems reasonable to suppose that nebulæ and stars follow at least roughly the figures found by Poincaré for the homogeneous case.

There is one very important point of difference in the problem treated by Poincaré and that presented by contracting bodies. Poincaré considered masses all of the *same density*, but having different rates of rotation. In a contracting nebula or star both the density and the rate of rotation change. The increase in density tends to sphericity; the increase in rate of rotation tends to oblateness. The two effects almost balance each other, but the effect of increasing rotation prevails by a narrow margin. For example, if the sun contracts with loss of heat, it will not become so oblate as Saturn is now until its density is hundreds of times greater than that of platinum. This does not mean that a body contracting from a nebula may not divide into two parts at any stage of its development, but it shows that the tendency for fission is very much smaller than has been supposed.

Suppose a star divides into two parts. Originally the two components will be rotating so as to keep their same faces toward each other. But with further contraction they will rotate more rapidly while their period of revolution remains unchanged. Then tidal evolution begins, and under these conditions Darwin has shown that the tides will increase the periods of rotation rapidly and the period of revolution more slowly. Moreover, if the original orbit had any eccentricity it will be increased. Consequently, as the age of a binary star having originated by fission increases, its period of revolution increases and the eccentricity of its orbit increases.

From an extensive study of the orbits of spectroscopic and visual binaries, Campbell has found that stars of Types B and A have short periods and nearly circular orbits, and that both the periods and the eccentricities increase, on the average, through the spectral types F, G, K, and M. One would be tempted to infer, in accordance with the theory of the evolution of stars through the spectral types from B to M, that binaries of Type B had recently originated by fission and that with increasing age they would go through the various spectral types with periods increasing correspondingly from a few hours to an average of more than a century, and the eccentricity from near zero to an average of about 0.5.

But such an inference would be entirely unwarranted and erroneous, for an ample consideration of the dynamics involved shows that when a nebula or star divides into two equal masses, tidal friction in any time however long is not competent to make the period more than about twice its original value; if the masses are unequal but comparable, as in the case of all known binaries, the period may be lengthened several fold. But it is altogether impossible for tidal friction to increase the period of a binary star whose components have comparable masses from a few hours or days to the many years found in the case of most visual binaries.

There is a similar difficulty in the eccentricities of the orbits of binary stars. Consequently the important facts brought out in Campbell's discussion do not confirm the current theory of the evolution of the stars. So far as the periods are concerned they are in harmony with the hypothesis that the B and A stars are massive, for the greater the mass, the shorter the period for a given distance between the stars, but it is highly improbable that the great range of periods depends upon the masses alone. The dynamical conditions imply that if visual binaries originated by fission, the division took place while they were yet in the nebular stage.

The hypothesis that two independent stars can unite to form a binary remains to be considered. If two stars are drawn toward each other by their mutual gravitation, they may pass near and around each other without any contact, as a comet passes around the sun; each may collide with the outlying parts of the other; they may undergo a grazing, or partial, collision; and, in the extreme case, they may have a central collision. If they do not collide at all, they will recede to the distance from which they were drawn together, and a binary star cannot result. If they suffer a collision with outlying parts, their velocities will be reduced and they may not recede to a very great distance from each other. The character of their orbits after collision will depend upon the amount of kinetic energy which is transformed at the time of collision. This energy goes into heat, and the question arises whether, if sufficient motion is destroyed to produce a binary, the heat evolved may not reduce both stars to the nebulous state.

Consider a special example of two stars each in mass equal to the sun. At a great distance from each other their relative velocities might be anything from zero to several hundred miles per second; take the most favorable case where it is zero. Suppose that at their nearest approach their distance from each other is as great as that from the earth to the sun. Under the hypotheses adopted they will have a relative velocity of about 37 miles per second. Suppose they encounter enough resistance from outlying nebulous or planetesimal matter, or from collision with a planet, to reduce their most remote point of recession after collision to 100 astronomical units. It can be shown that their velocity must have been reduced by $\frac{1}{200}$ of its amount, or by 0.185 mile per second. This would generate as much heat as the sun radiates in about 8 years. Consequently the expansive effect of the heat generated by the collision will not be important, and after the encounter the stars will be moving in an orbit whose eccentricity is 0.98 and whose period is about 250 years. The resistance could have been produced by collision with a planet

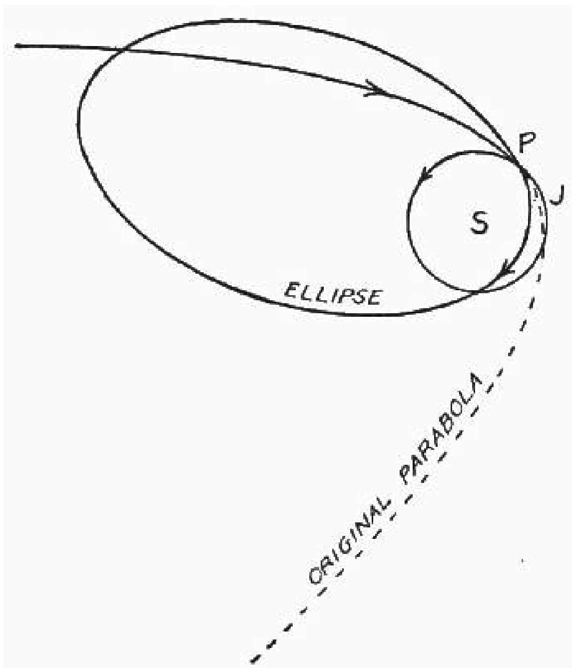

FIG. 186.—Reduction of parabolic orbit to an ellipse by collision of a sun with a planet of another sun.

whose mass was $\frac{1}{200}$ that of one of the suns. It follows that if a star passing the sun should meet Jupiter, something comparable to what has been given in the example would result. Figure 186 shows the original parabola, the point of collision P, and the elliptical orbit after collision.

Now let us follow out the history of the star after such a collision as has been described. If there are no subsequent collisions, the stars will continue to describe very elongated elliptical orbits about their center of gravity. If there are subsequent collisions with other planets or with any other material in the vicinity of the stars, their points of nearest approach will not be appreciably changed unless the collisions are far from the perihelion point, their points of most remote recession will be diminished by each collision, and the result is that both the period and the eccentricity of the orbit will be decreased as long as the process continues. If this is the correct theory of the origin of binary stars, those whose periods and eccentricities are small, are older on the average, at least as binaries, than those whose periods and eccentricities are large, and this would suggest that the B and A stars are older than the K and M stars. The only obvious difficulty with the basis of this theory of the origin of binary stars is that these near approaches and partial collisions are necessarily extremely infrequent, while binary stars are very numerous. The seriousness of this difficulty depends upon the length of time the stars endure, about which nothing certain is known.

As has been stated in Art. 294, a central collision would produce a temporary star, which would later change into a nebula.

300. The Question of the Infinity of the Physical Universe in Space and in Time.—There are transcendental questions which, from their nature, can never be answered with certainty, but which the human mind ever persists in attacking. Among such questions is that of the infinity of the physical universe in space and in time.

It has been seen in Art. 270 that the apparent distribution of the stars proves that they cannot be scattered uniformly throughout infinite space. It has also been seen that there is no observational evidence that galaxies, separated by distances of a higher order than those between the stars, may not be units in larger aggregations and so on to super-galaxies without limit. This may be adopted as a working hypothesis. We may then inquire whether there will be luminous stars through infinite time, or whether they all will ultimately become extinct.

According to physical laws as they are known at present, the stars are pouring radiant energy out into the ether at an extravagant rate and it is not being returned to them in relatively appreciable amounts. For example, the sun loses more light and heat by radiation in a second than it will receive from all the stars in the sky in a million years. It is inconceivable that a star has an unlimited store of internal energy. Therefore its energy will ultimately become exhausted unless a new supply is furnished in some way. One method by which the internal energy of a star may be increased is by collision with another star. But after collision the combined mass would lose its energy similarly until another restoration by another collision. But by this process the matter of the universe becomes aggregated in larger and larger masses, and if it is finite in amount, a stage will be reached when no more collisions will take place. Then these final stars will in the course of time radiate away all their internal energy and remain throughout eternity dark, cold, and lifeless. At least, such is the teaching of present-day science if the physical universe is finite, as has usually been assumed.

But now suppose that there are myriads of galaxies composing larger and still larger cosmic units, and remember that there are no observational facts whatever which contradict this hypothesis. Under this assumption the energy in the universe is also infinite. It does not follow from this, however, that it will last an infinite time, for there are, by hypothesis, infinitely many bodies which are subject to collisions and which are radiating energy into the ether. But, on the other hand, if the relative speed of the larger cosmic units is great enough, there will be enough energy to last the infinite universe an infinite time. This follows from the fact that infinities may be of different orders, as the mathematicians say. The actual demands in the present case are not severe. In order that the energy should last an infinite time it is sufficient that the relative speeds of the larger cosmic units of all order shall exceed some finite value.

The energy in any particular galaxy might run down, as in the finite case considered above; but, according to the present hypothesis, at immense intervals this galaxy would collide with some other one with speed sufficient to restore its internal energies if the energy of their relative motions were thus transformed. It might require only a very small fraction of the energy of the relative motions. The process would terminate, however, if there were only a finite number of galaxies, but by hypothesis the super-galaxies are units in still larger aggregations. There might be a restoration of heat energy by interactions of these larger units, and so on without limit. It is not profitable to pursue the inquiry further here, but it is not without interest to know that according to our present understanding of the laws of nature it is not necessary to conclude that the physical universe will in a finite time reach the condition of eternal night and death. This discussion also gives an answer, though perhaps not the correct one, to the question why the universe has not already attained a condition of stagnation and death. In short, it gives a picture of a universe whose life and activity are without beginning and without end.

IV. The Nebulæ

301. Irregular Nebulæ.—There are many nebulæ in the sky of enormous extent and irregular form. Among the finest examples of these objects, though by no means the most extensive, are the veil-like structures which are seen in the constellation Cygnus, one of which is shown in Fig. 187. It is altogether probable that they are at least as remote as the nearer stars. Since they extend across regions occupied by hundreds of stars, they are of inconceivable magnitude; certainly a hundred years are required for light to cross them. They are extremely faint (the long-exposure photographs being quite misleading) and they are probably very tenuous, though nothing is actually known regarding their density. If they are condensing under gravitation, the process must be going on extremely slowly.

An example of a less widely extended and apparently much denser nebula is the great nebula in Orion (Fig. 61), which is, perhaps, the most wonderful and beautiful object in the heavens. It fills a space whose apparent diameter is more than half a degree. This means it is of enormous volume, for it is as remote as certain stars which are associated with its denser parts. Its parallax can scarcely be over $0''.01$ and it probably is much smaller; if the larger value is correct, its diameter is $20,000,000$ times that of the sun and several years would be required for light to travel from one side of it to the other. The density of the Orion nebula is altogether unknown, but it is generally regarded as being very low. If it averages even $\frac{1}{100,000}$ that of the atmosphere and if it is spherical (?), its total mass is $100,000,000,000,000$ times that of the sun, and in spite of its enormous distance, its attraction for the earth is one fourth that of the sun. If the nebula is rare, it is difficult to account for its radiation, because it could not have a high temperature except possibly in its deep interior where pressure of the outlying parts would prevent expansion. The luminosity of the nebulæ, like that of the comets, has long been an unexplained phenomenon.

FIG. 187. — Irregular nebula in Cygnus (N. G. C. 6960). *Photographed by Ritchey with the two-foot reflector of the Yerkes Observatory.*

The form of the Orion nebula suggests whirling motions of its parts. Relative internal motions were found first by Bourget, Fabry, and Buisson; Frost and Maney have shown by the spectroscope that its northeastern part is receding from the solar system, while the southwestern part is approaching at the relative rate of about 6 miles per second. It is clear that unless the density is sufficiently great these motions will cause the nebula to dissipate in space. On the assumption that this is simply a motion of rotation, and neglecting gaseous expansion, it is found that the nebula is in no danger of disrupting if its average density is greater than 10^{-22} times that of water. At this limiting density its total mass would about equal that of the sun.

It was supposed in the days of Sir William Herschel that the nebulæ may be galaxies which are so remote that their individual stars are not distinguishable, even with the most powerful telescopes. This is certainly not the true explanation of the irregular nebulæ. In the first place, the spectra of the brighter ones for which the data are at hand consist of bright lines, proving on the basis of the first law of spectrum analysis that they are incandescent gases under low pressure. The bright lines belong to a hypothetical element nebulium, found only in nebulæ, and to hydrogen. In the second place, they are condensed in the zone of the Milky Way, which indicates they are in some way connected with it. Campbell and Moore have found that they show the streaming tendencies which are characteristic of the stars. For these reasons the conclusion is held that they are tenuous gaseous members of our own Galaxy.

A very interesting fact has recently been discovered in connection with the Magellanic Clouds, two masses of stars in the far southern heavens, having the appearance of two smaller galaxies which are quite independent of the Milky Way. R. E. Wilson, at the South American branch of the Lick Observatory, has found that the radial velocities of the nebulæ in the Magellanic clouds which are bright enough for measurement show rapid recession of all of these objects, the average speed being over 150 miles per second. This suggests that these aggregations of stars have velocities with respect to our own Galaxy of a higher order than the average internal velocities, in harmony with the suggestion in Art. 300.

Barnard has recently brought forward strong evidence for the conclusion that there are relatively dark and opaque masses, perhaps nebulous in character, in certain parts of the Milky Way. He has found regions in which the stars seem to be blotted out by obscure material, as is shown in Fig. 188. Probably the apparent breaks in some of the nebulæ, as, for example, the celebrated Trifid Nebula in Sagittarius (Fig. 189), are due to obscuring material which cuts off the light from certain regions. At any rate, it is difficult to see how matter could be

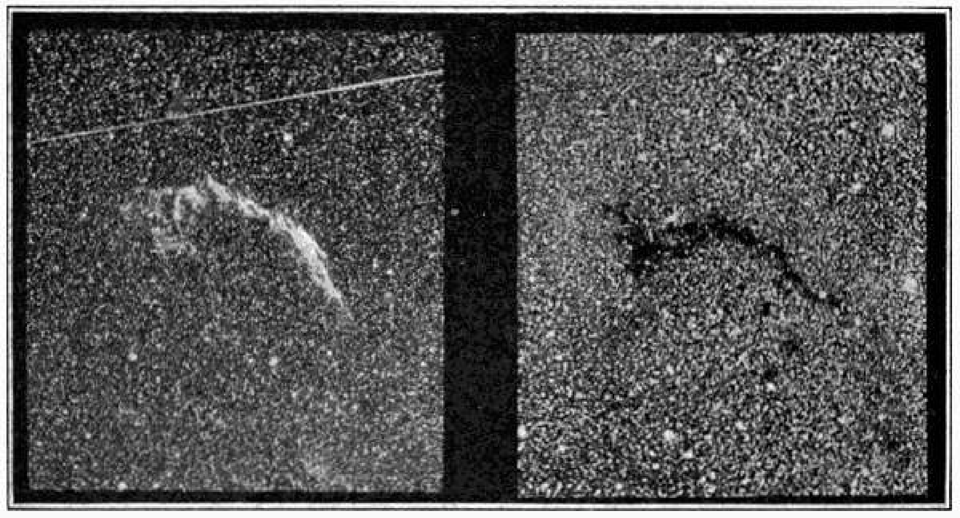

FIG. 188.—On the left a bright nebula (in Cygnus) and on the right a dark patch which is probably due to a dark nebula. *Photographed by Barnard at the Yerkes Observatory.*

in equilibrium in any such forms as the luminous matter assumes.

302. Spiral Nebulæ.—Spiral nebulæ are more numerous than all other kinds together. According to Keeler's original estimate there are at least 120,000 within the reach of the telescope which he used; there may be five or ten times the number within reach of the great reflectors of the Solar Observatory of the Carnegie Institution. They are characterized by their great extent (Fig. 190) and by irregular arms, generally two in number when they are distinctly defined, which wind out from centers. They almost invariably have well-defined centers, apparently of considerable density, and their arms usually contain a number of conspicuous local condensations, or nuclei.

The spiral nebulæ are further characterized by being white, whereas the large irregular nebulæ have a greenish tinge due to the green light from nebulium. Most of them are too faint for detailed spectroscopic study, but some of the brighter of them have been found to have spectra similar to the sun's spectrum. This leads to the inference that they are perhaps partly solid or liquid. On the other hand, Seares has photographed some of them through a screen which cuts off the blue end of the spectrum. The brightness of the arms was much more reduced than that of the central nuclei, indicating that a considerable part of their light is similar to that from gases. Moreover, their transparency implies that they are tenuous. Hence, they seem to be vast swarms of incandescent solid or liquid particles, perhaps with many larger masses, surrounded by gaseous materials. There is difficulty in explaining their luminosity, though Lockyer attempted to account for the light of all nebulæ by ascribing it to heat generated by the collisions of meteorites of which he supposed they are largely composed. The obscure material in and around nebulæ may be very abundant. This supposition is confirmed in the case of spiral nebulæ, for when one is seen edgewise the dark material at its periphery eclipses the center and causes an apparently dark rift through it (Fig. 191). Another distinguishing feature of spiral nebulæ is that they are very infrequent in or near the Milky Way.

FIG. 189.—The Trifid Nebula. The dark lanes by which it is crossed are probably due to intervening dark material. *Photographed with the Crossley reflector of the Lick Observatory.*

The spiral nebulæ range in magnitude all the way from the Great Nebula in Andromeda (Fig. 192), which is about 1°.5 long and 30′ wide, to minute, faint objects which are barely discoverable after long exposures with powerful photographic telescopes. There is no reason to believe there are not others still smaller. Since the Andromeda nebula is certainly as distant as the nearest stars, its volume is enormous; the smallest ones may be as small as the solar system, though they would wind up and lose their spiral characteristics in a short time.

FIG. 190.—Spiral nebula in Ursa Major (M. 101). *Photographed by Ritchey at the Yerkes Observatory.*

The suggestion has been made (Art. 249) that a spiral nebula may develop when a star is visited closely by another star, or when a group of stars passes near another group of stars. There is no apparent difficulty in explaining small spirals in this way, but the large ones present a more serious problem, especially if we limit ourselves to the close approach of two single stars. It is not at all necessary to do this, for in a general way the dynamical principles involved apply to aggregates of all dimensions up to galaxies, and even beyond if there are larger units in the universe. There is possibly some evidence that the Milky Way has a spiral structure.

Although the larger spirals are enormous in extent, they may have only moderate masses. However improbable this may be on the basis of their appearance, it must be remembered that there is no direct evidence whatever at present regarding their masses, and the source of their luminosity is quite unknown. It is natural to suppose that though a spiral of dimensions comparable to the solar system might be produced by the disruptive forces of a near approach of two stars, it would not be possible for one a thousand times larger to be formed in the same way. An examination of the equations involved shows that, if a certain velocity of ejection would cause matter to recede (neglecting the attraction of the passing sun) to the distance of Neptune, a velocity one twenty-four-thousandth greater would cause it to recede 1000 times farther (Table XIII). Hence the argument against very large spirals being formed by the near approach of two great suns is not so conclusive as it might at first seem. They may have been formed, however, by the passage near one another of two great groups of stars such as the globular clusters; or they may have been formed in some other way not yet considered.

FIG. 191.—Spiral nebula in Andromeda (H. V. 19) presenting edge toward the earth. Central line eclipsed by obscure material. *Photographed with the Crossley reflector of the Lick Observatory.*

The spectra of spiral nebulæ are in harmony with the suggested mode of their origin. Their distribution demands consideration. Their apparent distribution may mean that they are out on the borders of the Galaxy and that they are not seen in the Milky Way because of their great distances in these directions. It would be expected that close approaches would occur most frequently in the interior of the Galaxy where the stars move the fastest if they are making excursions to and fro through it. On the other hand, out on the borders they would move more slowly and their mutual attractions would be more efficient in bringing them together.

FIG. 192. — Great Nebula in Andromeda. *Photographed by Ritchey with the two-foot reflector of the Yerkes Observatory.*

There is one fact which is opposed to the suggested explanation of spiral nebulæ, and that is, as Slipher first found, their radial velocities average very great. For example, the Great Andromeda Nebula is approaching the solar system at the rate of 200 miles per second. Moreover, Slipher found spectroscopic evidence that it is rotating. Even if the result is in doubt for this nebula, it is altogether certain in the case of another spiral which is edgewise to the earth, and which Slipher investigated in 1913. Among the stars high velocities are on the whole associated with small masses. If this is a universal principle, which seems dynamically sound, the spirals must have smaller masses than any known class of stars. Or, perhaps, spirals have been formed on the whole only from stars which passed one another at great speed, and they of course still possess most of their kinetic energy.

It has been more than once suggested that the spiral nebulæ are not in reality nebulæ at all, but distant galaxies. If this is true, it is difficult to explain their distribution with respect to the Milky Way, or their strong central condensations, or the fact that they are crossed by dark streaks when they are presented edgewise to us. Besides, the results of Seares' photographs are opposed to this hypothesis.

FIG. 193.—The ring nebula in Lyra. *Photographed by Sullivan at the Yerkes Observatory with the 40-inch telescope.*

303. Ring Nebulæ.—A few nebulæ have the form of almost perfect rings, the best example of which is the one between Beta Lyræ and Gamma Lyræ (Fig. 193). This nebula has a fifteenth-magnitude star near its center which has been suspected of being variable. It is probably associated with the nebula, though this is not certain. The spectrum of the ring nebula in Lyra has been examined and it has been found that hydrogen extends out considerably beyond the helium. The origin and development of these remarkable objects are quite beyond conjecture at present.

304. Planetary Nebulæ.—The planetary nebulæ are supposed to be next to the O-type stars in evolution, and the O-type stars are supposed to precede the B-type stars. They are in all cases apparently small in size, usually rather dense, particularly near their centers, and they have rather well-defined outlines. They were named by Herschel from their resemblance to faint planetary disks.

The spectra of about 75 planetary nebulæ have been examined. Perhaps the most important result of this examination is that their radial velocities (24 miles per second) are at least three times those of the stars of Type B. This is squarely opposed to the theory that they condense into stars of Types O and B. If this theory is maintained, an explanation of the greatly decreased velocities is demanded, and none is at hand. On the other hand, the novæ go first into planetary nebulæ and then into Wolf-Rayet stars.

The central parts of planetary nebulæ give the lines of nebulium and hydrogen; the outermost parts give the hydrogen lines alone. That is, hydrogen forms an atmosphere around the denser nebulium and hydrogen cores.

FIG. 194.—A planetary nebula. *Photographed with the Crossley reflector at the Lick Observatory.*

The problem of the rotation of planetary nebulæ is now being taken up at a number of observatories. By an adaptation of the spectroscope first employed by Keeler on the rings of Saturn, and used more recently by Slipher at the Lowell Observatory on planets and spiral nebulæ, Campbell and Moore have found that two of these remarkable objects are rotating around axes approximately at right angles to a plane passing through the earth and the longer axes of the nebulæ. On the basis of the observed relative velocities of 3.1 to 3.7 miles per second, and plausible assumptions regarding the distance of the nebulæ, they found that their masses are between 3 and 100 times that of the sun, with periods of rotation between 600 and 14,000 years. With such slow rates of rotation there is no possibility of these objects ever dividing into two parts and forming a binary star, in spite of the fact that their density probably does not exceed one millionth that of our atmosphere at sea level.

XXIV. QUESTIONS

1. If 500,000,000 stars were scattered uniformly over the celestial sphere, what would be the apparent angular distance between adjacent stars? If another star were placed at random on the sky, what would be the probability that it would be within $1''$ of one of these stars?

2. In the part of the sky covered by Aitken's survey of double stars (north of declination $-14°$) there are about 200,000 stars brighter than the tenth magnitude; what is the average distance between adjacent members of this list of stars? Aitken found 5400 pairs separated by less than $5''$; what is the probability that a particular one of these cases is accidental? What is the probability that they are all accidental? According to the laws of probability, how many of the 5400 stars, in a random arrangement, should be separated less than $5''$?

3. Suppose the apparent distance between two stars must be at least $0''.2$ in order that they may be seen as two distinct stars with the largest telescopes; suppose the distance of a double star is 500 parsecs; what must be the distance, in astronomical units, between the components in order that they may be seen as separate stars? If the mass of each star is equal to that of the sun, what will be their period of revolution (Art. 154)? If their dimensions and surface brilliancy are the same as those of the sun, what will be their magnitude taken together?

4. Suppose the relative velocity of the two components of a double star must be 5 miles per second in order that it may be possible to determine by the spectroscope that the star is a binary; how near must the components be to each other in order that it may be possible to find that the star is a binary if their combined mass is one tenth that of the sun? Equal to that of the sun? Ten times that of the sun?

5. Suppose the density of the components of a binary star is equal to that of the sun and that the two components (assumed spherical) are in contact; what is their period of revolution if their combined mass is one tenth that of the sun? Equal to that of the sun? Ten times that of the sun? What are their relative velocities in the respective cases? What are their temperatures in the respective cases [Art. 298 (*c*)]? What are their luminosities in the respective cases?

6. Suppose the two components of an eclipsing variable are equal in mass and that their density is that of the sun; what is the ratio of the time of eclipse to the period of revolution if their combined mass is one tenth that of the sun? Equal to that of the sun? Ten times that of the sun? Solve the problem if their density is one tenth that of the sun, and also if it is ten times that of the sun.

7. Which of the ten phenomena of Art. 296 fail to arrange the stars strictly in the order B, A, F, G, K, M? Which of the ten phenomena are

opposed to the hypothesis that the spectral type of a star depends on its mass? Which of the ten phenomena are opposed to the hypothesis that the arrangement of stars according to age is M, A, B, A, F, G, K, M (the hypothesis of Lockyer and Russell)?

8. The apparent areas of the sun and the denser part of the Orion nebula are about the same, and the sun is about 30 magnitudes brighter than the nebula. Suppose the amount of light they radiate is proportional to the fourth powers of their absolute temperatures. What is the temperature of the Orion nebula? If its diameter is 20,000,000 times that of the sun, what is its mass (computed from the relation connecting temperature, mass, and density of a gaseous body)? Under the same assumptions, what is its mean density? (The student will not fail to remember that some of the assumptions on which the computation rests are questionable.)

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