MATHEMATICAL METHODS

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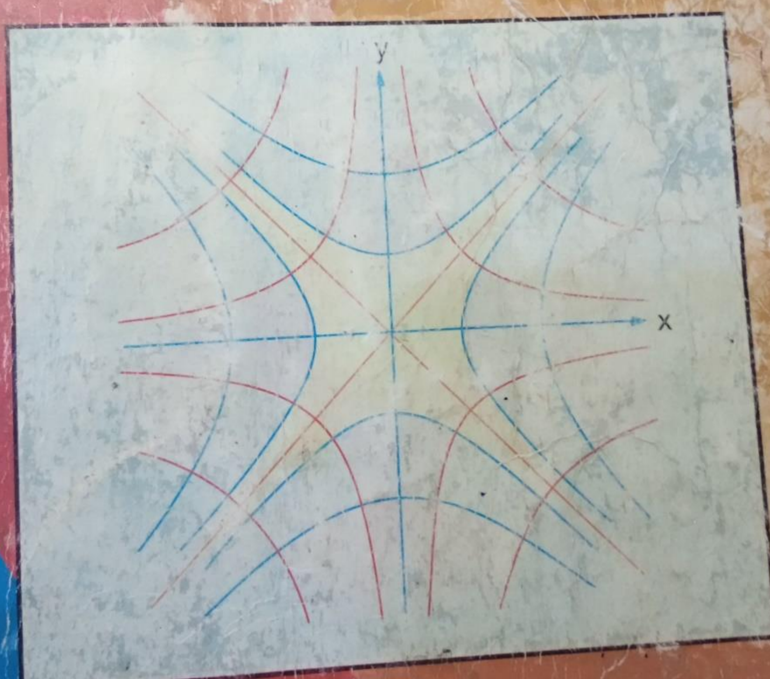

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CONTENTS

Section I
chp#1, ch# 8
& Vector

Section II

CHAPTER 6 VECTOR SPACES 223

CHAPTER 7 INNER PRODUCT SPACES 281

CHAPTER 8 INFINITE SERIES 325

CHAPTER 9 FIRST-ORDER DIFFERENTIAL EQUATIONS 377

Chapter 1

TRIGONOMETRY

COMPLEX NUMBERS

The reader is already familiar with complex numbers and a few of their properties. We now define a complex number in a formal manner.

(1.1) Definition. A **complex number** is an element (x, y) of the set

$$R^2 = \{(x, y) : x, y \in R)$$

obeying the following rules of addition and multiplication.

For $z_1 = (x_1, y_1),\ z_2 = (x_2, y_2) \in R^2$, we put

A1: $z_2 + z_2 = (x_1 + x_2, y_1 + y_2)$ **(Addition)**

M1: $z_1 z_2 = (x_1x_2 - y_1y_2, x_1y_2 + y_1x_2)$ **(Multiplication)**

Equality of Complex Numbers. Two complex numbers $z_1 = (x_1, y_1),\ z_2 = (x_2, y_2)$ are equal if and only if $x_1 = x_2,\ \ y_1 = y_2$.

We then write $z_1 = z_2$.

(1.2) Properties of Complex Numbers. Some important consequences of the definitions of addition (A1) and multiplication (M1) of complex numbers are as follows:

Properties of Addition Defined by (AI):

A2: For all $z_1 = (x_1, y_1),\ z_2 = (x_2, y_2),\ z_3 = (x_3, y_3) \in R^2$

$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$ **(Associative Law of Addition)**

Here, using **(A1)**, we have

$$\begin{aligned}(z_1 + z_2) + z_3 &= (x_1 + x_2, y_1 + y_2) + (x_3, y_3)\\ &= ((x_1 + x_2) + x_3, (y_1 + y_2) + y_3)\\ &= (x_1 + (x_2 + x_3), y_1 + (y_2 + y_3)). \quad \textbf{(Associative Law in } R\textbf{)}\\ &= (x_1, y_1) + (x_2 + x_3, y_2 + y_3)\end{aligned}$$

A3: There is a complex number $\mathit{0} = (0, 0) \in R^2$, called the **additive identity**, such that for all $z = (x, y) \in R^2$, **(Additive Identity)**

$$\mathit{0} + z = z + \mathit{0} = z$$

A4: For each $z = (x, y) \in R^2$, there is a $-z = (-x, -y) \in R^2$, called the **additive inverse of** z, such that **(Additive Inverse)**

$$z + (-z) = (0, 0) = \mathit{0}$$

A5: For all $z_1 = (x_1, y_1), z_2 = (x_2, y_2) \in R^2$

$$z_1 + z_2 = z_2 + z_1 \quad \textbf{(Commutative Law of Addition)}$$

Here

$$\begin{aligned}z_1 + z_2 &= (x_1 + x_2, y_1 + y_2)\\ &= (x_2 + x_1, y_2 + y_1) \quad \textbf{(Commutative Law in } R\textbf{)}\\ &= z_2 + z_1\end{aligned}$$

The properties **(A1 – A5)** in R^2 show that R^2, under addition, is an abelian group.

Properties of Multiplication Defined by (M1):

M2: For all $z_1 = (x_1, y_1), z_2 = (x_2, y_2), z_3 = (x_3, y_3) \in R^2$

$$(z_1 \cdot z_2) \cdot z_3 = z_1 \cdot (z_2 \cdot z_3) \quad \textbf{(Associative Law of Multiplication)}$$

Here

$$\begin{aligned}(z_1 \cdot z_2) \cdot z_3 &= (x_1x_2 - y_1y_2, x_1y_2 + y_1x_2)(x_3, y_3)\\ &= ((x_1x_2 - y_1y_2)x_3 - (x_1y_2 + y_1x_2)y_3,\\ &\qquad (x_1x_2 - y_1y_2)y_3 + (x_1y_2 + y_1x_2)x_3)\\ &= (x_1(x_2x_3 - y_2y_3) - y_1(x_2y_3 + y_2y_3),\\ &\qquad x_1(x_2y_3 + y_2y_3) + y_1(x_2y_3 - y_2x_3))\\ &= (x_1, y_1)(x_2x_3 - y_2y_3, x_2y_3 + y_2x_3)\\ &= z_1 \cdot (z_2 \cdot z_3)\end{aligned}$$

M3: The complex number $\underline{1} = (1, 0) \in R^2$ and it satisfies the condition

$$\underline{1} \cdot z = z \cdot \underline{1} = z \quad \textbf{(Multiplicative Identity)}$$

for all $z = (x, y) \in R^2$