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# MATHEMATICS HSSC-II

## SECTION – A (Marks 20)

Time allowed: 25 Minutes

**Section – A is compulsory. All parts of this section are to be answered on this page and handed over to the Centre Superintendent. Deleting/overwriting is not allowed.**

*Do not use lead pencil.*

حصہ اول لازمی ہے۔ اس کے جوابات اسی صفحہ پر دے کر ناظم مرکز کے حوالے کریں۔ کاٹ کر دوبارہ لکھنے کی اجازت نہیں ہے۔ لیڈ پنسل کا استعمال ممنوع ہے۔

**Answer Sheet No.** _____

ہر سوال کے سامنے دیے گئے، کریکولم کے مطابق درست دائرہ کو پر کریں۔ **Invigilator Sign.** _____

**Fill the relevant bubble against each question according to curriculum:** **Candidate Sign.** _____

| | Question | A | B | C | D | A | B | C | D |
|---|---|---|---|---|---|---|---|---|---|
| 1. | For what value of $a$ the vectors $2\underline{i}+4\underline{j}-5\underline{k}$ and $-4\underline{i}-8\underline{j}+a\underline{k}$ are parallel? | $-10$ | $8$ | $-8$ | $10$ | ○ | ○ | ○ | ○ |
| 2. | What is the value of $f(-1)$ if $f^{-1}(x)=\frac{8-x}{2}$? | $\frac{7}{2}$ | $10$ | $6$ | $\frac{9}{2}$ | ○ | ○ | ○ | ○ |
| 3. | What is the value of $\lim_{x\to 0}\frac{1-\cos x}{x^2}$ ? | $2$ | $\frac{1}{4}$ | $\frac{1}{2}$ | $1$ | ○ | ○ | ○ | ○ |
| 4. | Which of the following is equal to $y_2$ if $xy=2$ ? | $\frac{-4}{x^3}$ | $\frac{2}{x^3}$ | $\frac{-2}{x^3}$ | $\frac{4}{x^3}$ | ○ | ○ | ○ | ○ |
| 5. | What is the value of $\frac{d}{dx}[fog(x)]$ if $f(x)=e^x$ and $g(x)=\sin x$ ? | $\cos x\, e^{\sin x}$ | $\sin(e^x)$ | $\cos(e^x)$ | $e^{\sin x}$ | ○ | ○ | ○ | ○ |
| 6. | Which of the following is the derivative of $x^x$ w.r.t.x ? | $\frac{x^x}{1+\ln x}$ | $x^x \ln x$ | $x^x(1+\ln x)$ | $\frac{x^x}{\ln x}$ | ○ | ○ | ○ | ○ |
| 7. | The derivative of odd function is always? | Both even and odd function | An even function | An odd function | Neither even nor odd function | ○ | ○ | ○ | ○ |
| 8. | Which of the following is equal to $\int(\text{Cosec}^2 x-1)dx$ ? | $-\cot x+x+C$ | $\cot x-x+C$ | $-\cot x-x+C$ | $\cot x+x+C$ | ○ | ○ | ○ | ○ |
| 9. | For what value of $x$ the function $f(x)=x^2-3x+7$ has a critical point? | $\frac{3}{2}$ | $\frac{-2}{3}$ | $\frac{-3}{2}$ | $\frac{2}{3}$ | ○ | ○ | ○ | ○ |
| 10. | If $\int_a^{\frac{\pi}{4}}\text{Cos}\,2x\,dx=\frac{1}{2}$ then what should be the value of $a$ ? | $\frac{\pi}{3}$ | $0$ | $1$ | $\frac{-\pi}{4}$ | ○ | ○ | ○ | ○ |

| | Question | A | B | C | D | A | B | C | D |
|---|---|---|---|---|---|---|---|---|---|
| 11. | Which of the following is the evaluation of $\int \frac{e^x}{e^x-3}dx$ ? | $(e^x-3)^{-2}+C$ | $\ln(e^x)+C$ | $\ln(e^x-3)+C$ | $e^x\ln(e^x-3)+C$ | ○ | ○ | ○ | ○ |
| 12. | The slope and $y-$ intercept of the line $2x+y-3=0$ are: | Slope = -2<br>y- intercept= -3 | Slope =2<br>y- intercept=3 | Slope =2<br>y-intercept=-3 | Slope = -2<br>y- intercept=3 | ○ | ○ | ○ | ○ |
| 13. | Perpendicular distance of the line $x+2y+4=0$ from the point $(1,0)$ is: | 1 | 2 | 5 | $\sqrt{5}$ | ○ | ○ | ○ | ○ |
| 14. | If line $l_1$ is making angle $45°$ with x-axis and line $l_2$ is perpendicular to y-axis then the angle between $l_1$ and $l_2$ is: | $\frac{\pi}{2}$ | 0 | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | ○ | ○ | ○ | ○ |
| 15. | Which one is the solution of inequality $-20>-2x$ ? | $10>x>2$ | $10<x<+\infty$ | $10>x>-\infty$ | $-10>x>-\infty$ | ○ | ○ | ○ | ○ |
| 16. | If $y^2-y=\text{Sin}\,x$ then which one is equal to $\frac{dy}{dx}$ ? | $\frac{\sin x}{1-2y}$ | $\frac{\cos x}{2y-1}$ | $\frac{\cos x}{1-2y}$ | $\frac{\sin x}{2y-1}$ | ○ | ○ | ○ | ○ |
| 17. | What is the length of latus rectum of parabola $(x+1)^2=8y+8$ ? | 4 | $\frac{1}{8}$ | 8 | $\frac{1}{4}$ | ○ | ○ | ○ | ○ |
| 18. | What is the centre of ellipse $\frac{(2x+1)}{9}+\frac{(y-1)}{5}=1$ ? | $\left(\frac{1}{2},-1\right)$ | $(1,-1)$ | $(-1,1)$ | $\left(-\frac{1}{2},1\right)$ | ○ | ○ | ○ | ○ |
| 19. | If $\theta$ is the angle between two vectors $\underline{a}$ and $\underline{b}$, and $\sin\theta=\frac{1}{2}$, $\lvert\underline{a}\rvert=\lvert\underline{b}\rvert=\sqrt{2}$ then what is the value of $\lvert\underline{a}\times\underline{b}\rvert$ ? | $\frac{3}{2}$ | 1 | $\frac{1}{2}$ | 2 | ○ | ○ | ○ | ○ |
| 20. | When a force $\underline{F}=3\underline{i}+2\underline{j}-4\underline{k}$ is applied to an object a displacement $\underline{d}=4\underline{i}+\alpha\underline{j}+4\underline{k}$ occurred. For what value of $\alpha$ work done would be 6 units? | 9 | 3 | 5 | 7 | ○ | ○ | ○ | ○ |



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# MATHEMATICS HSSC–II

**Time allowed: 2:35 Hours** **Total Marks Sections B and C: 80**

**NOTE: Attempt any twelve parts from Section 'B' and any four questions from Section 'C'. Write your answers neatly and legibly.**

## *SECTION – B (Marks 48)*

**Q. 2 Attempt any TWELVE parts. All parts carry equal marks.** **( 12 x 4 = 48 )**

(i) Let $f(x)=x^2+2$ and $g(x)=x+1$ then

**(a)** Find $gof(x)$ **(b)** Find $x$ for which $fog(x)=11$

(ii) For the function $f(x)=\begin{cases}3x+4, & 0\le x<3\\ 16-x, & 3\le x<12\\ x, & 12\le x<14\end{cases}$

**(a)** Explain whether $\lim_{x\to 3} f(x)$ exists **(b)** Discuss the continuity of the function $f(x)$ at $x=12$

(iii) If $x=a\cos^3\theta,\ y=b\sin^3\theta$ show that $a\frac{dy}{dx}+b\tan\theta=0$ (Chain Rule)

(iv) If $e^x+e^y=e^{x+y}$ then find the value of $\frac{dy}{dx}$ at $(1,1)$

(v) Show that $\cos(x+h)=\cos x-h\sin x-\frac{h^2}{2!}\cos x+\frac{h^3}{3!}\sin x+.....\infty$

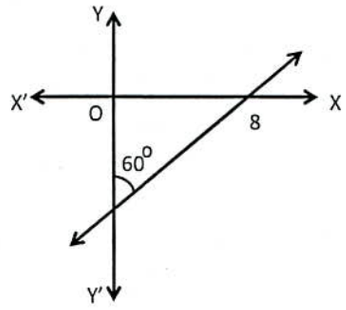
(vi) Find two positive integers whose sum is $9$ and product of one with square of the other is maximum.

(vii) The side of a cube is measured to be $20cm$ with a maximum error of $0.12cm$ in its measurement. Find the maximum error in the calculated volume of the cube.

(viii) Evaluate $\int\frac{\cos x}{\sin x.\ln\sin x}dx$

(ix) Find the equation of the curve for which $\frac{dy}{dx}+\frac{2xy}{2y+1}=x$. The curve passes through the point $(2,1)$

(x) Find the equation of a straight line whose $x-$ intercept is $8$ and making angle of $60°$ with the positive direction of $y-$ axis

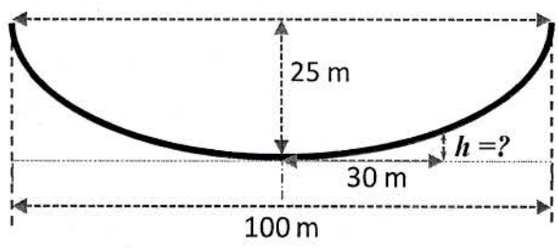



(xi) If $\int_{-3}^{3}\left(x^3+kx^2\right)dx=54$ find the value of $k$

(xii) Find the area of a triangle with one vertex as point of intersection of the lines $x+y-5=0$ and $2x-y+2=0$, and points $(2,-3)$ and $(3,4)$ are other two vertices.

(xiii) Find the equation of a circle concentric with the circle $x^2+y^2-8x+4=0$ and is tangent to the line $x+2y+6=0$

(xiv) A parabolic arch has a $100m$ base and height $25m$. Find the height of the arch at the point $30m$ from the centre of the base.




(xv) Verify that $\underline{a}$ and $\underline{b}$ are perpendicular to $\underline{b}\times\underline{a}$ where $\underline{a}=3\underline{i}-\underline{j}+5\underline{k}$ and $\underline{b}=4\underline{i}+3\underline{j}-2\underline{k}$

(xvi) Prove that in any triangle $\Delta ABC$ $\quad b=c\cos A+a\cos C$

## SECTION – C (Marks 32)

**Note:** **Attempt any FOUR questions. All questions carry equal marks.** **( 4 x 8 = 32 )**

**Q. 3** Let

$$f(x)=\begin{cases}2-x, & 0\le x<1\\ k, & x=1\\ 2x-1, & 1<x\le 2\end{cases}$$

then

a. What are domain and range of $f(x)$

b. Find the value of $k$ for which $f(x)$ is continuous at $x=1$

c. State value of $x$ where only left-hand limit and only right-hand limit exist.

d. Sketch the graph of $f(x)$ *(Use of Graph Paper is not necessary)*

**Q. 4** A box has square base and the sum of one side of the base and height of the box is $12$. If the length of one side of box is $x\,cm$

a. Express volume $V$ of the box in terms of $x$

b. Find $\frac{d}{dx}(V)$ and $\frac{d^2}{dx^2}(V)$

c. Find the dimensions of the box for which the volume of the box is maximum. Hence find maximum volume of the box.

**Q. 5** For a curve $f(x)=x^2-2x, x\in[-1,3]$

a. Find $\int_{-1}^{3} f(x)dx$

b. Sketch the graph of $f(x)$ and shade the area bounded by x-axis and the curve $f(x)$ (Use of Graph Paper is not necessary)

c. Find the area bounded by x-axis and the curve $f(x)$

**Q. 6** If the sides of a triangle $\Delta ABC$ are $7x-y-10=0$, $10x+y-41=0$, $3x+2y+3=0$

then

a. Find the vertices $A, B, C$ of the triangle

b. Find the equations of altitudes of $\Delta ABC$

c. Show that the altitudes of $\Delta ABC$ are concurrent

d. Find area of $\Delta ABC$

**Q. 7** A car detailing company performs two types of detailing: deluxe and ordinary. The deluxe detailing requires $1$ hour inspection and $3$ hours maintenance time. While an ordinary detailing requires $1$ hour inspection and $2$ hours maintenance time. There are $100$ hours of inspection and $240$ hours of maintenance time during coming two weeks. How many detailing(s) of each type should the company perform to get maximum profit if the company gets a profit of $Rs\,2000$ on deluxe detailing and $Rs\,1500$ in ordinary detailing.

**Q. 8** Find the centre, foci, eccentricity, vertices and directrices of the given conic $9x^2-y^2-36x-6y+18=0$

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# MATHEMATICS HSSC-II

## SECTION – A (Marks 20)

Time allowed: 25 Minutes

**Section – A is compulsory. All parts of this section are to be answered on this page and handed over to the Centre Superintendent. Deleting/overwriting is not allowed.**

*Do not use lead pencil.*

حصہ اول لازمی ہے۔ اس کے جوابات اسی صفحہ پر دے کر نا ظم مرکز کے حوالے کریں۔ کاٹ کر دوبارہ لکھنے کی اجازت نہیں ہے۔ لیڈ پنسل کا استعمال ممنوع ہے۔

**Answer Sheet No.** _____

ہر سوال کے سامنے دیے گئے، کریکولم کے مطابق درست دائرہ کو پر کریں۔ **Invigilator Sign.** _____

**Fill the relevant bubble against each question according to curriculum:** **Candidate Sign.** _____

| | Question | A | B | C | D | A | B | C | D |
|---|---|---|---|---|---|---|---|---|---|
| 1. | What is the value of $f(-4)$ if $f^{-1}(x)=\frac{1-x}{2}$ ? | $\frac{5}{2}$ | $-\frac{3}{2}$ | 9 | $-7$ | ○ | ○ | ○ | ○ |
| 2. | If the distance $(x)$ travelled by a car in time $(t)$ is given by $x=2t^3-24\ln(t+1)$. What will be the velocity of the car for $t=5$? $\left(Velocity=\frac{dx}{dt}\right)$ | 128 | 134 | 140 | 146 | ○ | ○ | ○ | ○ |
| 3. | What is the value of $\lim_{x\to 0}(x\sin x)$ ? | $\infty$ | $-1$ | 1 | 0 | ○ | ○ | ○ | ○ |
| 4. | What is the value of $\frac{d}{dx}(gof(x))$ if $f(x)=\sin x$ and $g(x)=\ln x$ | $\sin(\ln x)$ | $\tan x$ | $\cot x$ | $\ln(\sin x)$ | ○ | ○ | ○ | ○ |
| 5. | If $y=\ln\sqrt{\tan x}$ then what is the value of $\frac{dy}{dx}$ at $x=\frac{\pi}{4}$ ? | $\infty$ | 1 | 0 | $\frac{1}{2}$ | ○ | ○ | ○ | ○ |
| 6. | If $y=\left(1-x^{\frac{1}{2}}\right)\left(1+x^{\frac{1}{2}}\right)$ then what is the value of $\frac{dy}{dx}$ ? | 1 | $-1$ | $x$ | $\sqrt{x}$ | ○ | ○ | ○ | ○ |
| 7. | Which of the following is the evaluation of $\int(\mathrm{Sec}^2 x-1)dx$ ? | $\tan x+x+C$ | $\tan x-x+C$ | $-\tan x+x+C$ | $-\tan x-x+C$ | ○ | ○ | ○ | ○ |
| 8. | For which of the following intervals functions $f(x)=2x^2-8x+4$ is increasing. | $(-\infty,2]$ | $[0,+\infty)$ | $[2,+\infty)$ | $[0,2]$ | ○ | ○ | ○ | ○ |
| 9. | If $\int_{-\pi/2}^{a}\mathrm{Cos}\,x dx=2$ then what should be the value of $a$ ? | $\frac{\pi}{2}$ | $\frac{\pi}{6}$ | $\frac{\pi}{3}$ | $-\frac{\pi}{3}$ | ○ | ○ | ○ | ○ |

| | Question | A | B | C | D | A | B | C | D |
|---|---|---|---|---|---|---|---|---|---|
| 10. | Which of the following is the evaluation of $\int \frac{e^x}{e^x+2}dx$ ? | $\ln(e^x+2)+C$ | $\ln(e^x-2)+C$ | $x-2e^{-x}+C$ | $x+2e^{-x}+C$ | ○ | ○ | ○ | ○ |
| 11. | The $x-$ intercept and $y-$ intercept of the line $3x-y+6=0$ are: | $x-$ intersecpt=2<br>$y-$ ntersecpt $=-6$ | $x-$ intersecpt$=-2$<br>$y-$ ntersecpt $=-6$ | $x-$ intersecpt$=-2$<br>$y-$ ntersecpt $=6$ | $x-$ intersecpt=2<br>$y-$ ntersecpt $=6$ | ○ | ○ | ○ | ○ |
| 12. | For what value of $k$ the distance from the point $(0,1)$ to line $3x-4y+k=0$ is $3$ ? | 15 | $-19$ | 19 | 11 | ○ | ○ | ○ | ○ |
| 13. | If the slope of line $l_1$ is $2$ and it makes angle $45°$ with line $l_2$ . What is the slope of line $l_2$ ? $\left(\text{Tan}\,\theta=\frac{m_2-m_1}{1+m_1m_2}\right)$ | 0 | 1 | $-\frac{1}{2}$ | $-3$ | ○ | ○ | ○ | ○ |
| 14. | For which of the following inequalities, $(3,3)$ is **NOT** the solution? | $x+y>2$ | $3x+5y<7$ | $3x+5y>7$ | $3x-7y<3$ | ○ | ○ | ○ | ○ |
| 15. | Which of the following is the slope of normal to curve $x^2-y^2-12=0$ at $P(4,2)$ ? | 2 | $-2$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | ○ | ○ | ○ | ○ |
| 16. | What are the vertices of an ellipse with equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ? | $(\pm a,0)$ | $(0,\pm a)$ | $(\pm b,0)$ | $(\pm c,0)$ | ○ | ○ | ○ | ○ |
| 17. | Eccentricity of an ellipse '$e$' is: | $e>1$ | $e<1$ | $e=0$ | $e=1$ | ○ | ○ | ○ | ○ |
| 18. | If $\lvert\underline{a}\rvert=\lvert\underline{b}\rvert=\sqrt{2}$ and $\lvert\underline{a}\times\underline{b}\rvert=2$ then what is the angle between $\underline{a}$ and $\underline{b}$ ? | $\frac{\pi}{3}$ | $\frac{\pi}{4}$ | $\frac{\pi}{6}$ | $\frac{\pi}{2}$ | ○ | ○ | ○ | ○ |
| 19. | A displacement $\underline{d}=-\underline{i}+3\underline{j}+8\underline{k}$ occurred when a constant force $\underline{F}=\alpha\underline{i}+3\underline{j}+5\underline{k}$ is applied to an object. If the work done is $45$ , then what would be the value of $\alpha$ ? | $-1$ | 4 | $-4$ | 5 | ○ | ○ | ○ | ○ |
| 20. | What is the volume of a tetrahedron determined by the vectors $\underline{i}$, $2\underline{j}$ and $3\underline{k}$ ? | 1 | $\frac{1}{6}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | ○ | ○ | ○ | ○ |



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# MATHEMATICS HSSC–II

**Time allowed: 2:35 Hours** **Total Marks Sections B and C: 80**

**NOTE: Attempt any twelve parts from Section 'B' and any four questions from Section 'C'. Write your answers neatly and legibly.**

## *SECTION – B (Marks 48)*

**Q. 2 Attempt any TWELVE parts. All parts carry equal marks.** **( 12 x 4 = 48 )**

(i) Let $f(x)=1+2x$ and $g(x)=\frac{x}{x-1}$ then

**(a)** Find $fog(x)$ **(b)** Find $x$ for which $gof(x)=7$

(ii) For $f(x)=\begin{cases}7-4x & x<1\\ x^2+2 & x\geq 1\end{cases}$

**(a)** Explain whether $\lim_{x\to 1} f(x)$ exists **(b)** Discuss the continuity at $x=1$

(iii) Differentiate w.r.t. $x$ $y=\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$

(iv) If $\sqrt{x}+\sqrt{y}=4$ then find value of $\frac{dy}{dx}$ at $x=1$

(v) Show that $e^{x+h}=e^x\left\{1+h+\frac{h^2}{2}+\frac{h^3}{3}+.....\infty\right\}$

(vi) Find the dimensions of a rectangular garden having perimeter $80$ meters, if its area is to be maximum.

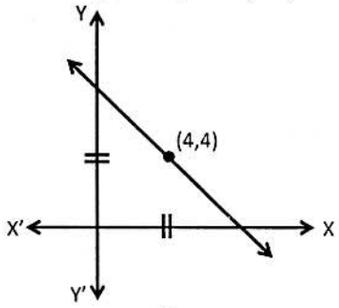
(vii) Find the approximate increase in the volume of a cube if the length of each edge changes from $5$ to $5.02\,cm$

(viii) Evaluate $\int\frac{\sin x-\cos x}{\sin x+\cos x}dx$

(ix) Evaluate $\int_{\frac{\pi}{2}}^{\pi} \text{Cos}\left(\frac{x}{2}\right)dx$

(x) Find the equation of the curve for which $x^2(2y+1)\frac{dy}{dx}=1$ . The curve passes through the point $(2,1)$

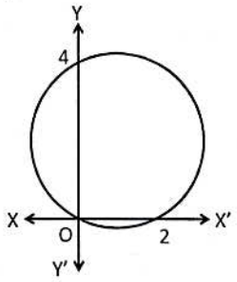
(xi) Find the equation of a straight line making equal intercepts on the axes and passes through the point $(4,4)$



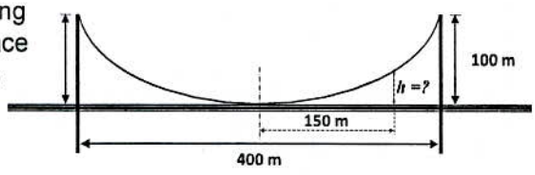

(xii) For what value of $k$ the following lines are concurrent

$$x+ky-2=0$$
$$2x-y+4=0$$
$$x-11y+14=0$$

(xiii) Find the equation of circle which passes through origin and whose intercepts on coordinate axes are $2$ and $4$




(xiv) A suspension bridge with weight uniformly distributed along the length has two towers $100m$ high above the road surface and $400m$ apart. The cables are parabolic in shape and are tangent to road surface at the centre of the bridge. Find the height of the cable at a point $150m$ from the centre.




(xv) Find a unit vector perpendicular to a plane containing $\underline{a}=2\underline{i}-6\underline{j}-3\underline{k}$ and $\underline{b}=4\underline{i}+3\underline{j}-\underline{k}$ . Also verify.

(xvi) Prove that in any triangle $\Delta ABC$, $c^2=a^2+b^2-2ab\ \text{Cos}C$

## SECTION – C (Marks 32)

**Note:** **Attempt any FOUR questions. All questions carry equal marks.** **( 4 x 8 = 32 )**

**Q. 3** Let

$$f(x)=\begin{cases}3ax+b, & x>1\\ 11, & x=1\\ 5ax-2b, & x<1\end{cases}$$

then

a. Find $\lim_{x\to 1^-} f(x)$ and $\lim_{x\to 1^+} f(x)$

b. Find the values of $a$ and $b$ for which $f(x)$ is continuous at $x=1$

c. Sketch the graph of $f(x)$ after finding values of $a$ and $b$ (*Use of Graph Paper is not necessary*)

**Q. 4** A solid rectangular block has a base of length $2x\,cm$ and width $x\,cm$. Height of the block is $y\,cm$ and volume of the block is $72cm^3$

a. Express total surface area $(A)$ of the block as a function of $x$

b. Find $\frac{d}{dx}(A)$ and $\frac{d^2}{dx^2}(A)$

c. Find the dimensions of the block for which its surface area is minimum.

**Q. 5** For a curve $f(x)=x^2-4,\ x\in[0,4]$

a. Find $\int_0^4 f(x)dx$

b. Sketch the graph of the curve and shade the region bounded by x-axis and the curve $f(x)$ (*Use of Graph Paper is not necessary*)

c. Find the shaded area bounded by the curve $f(x)$ and x-axis

**Q. 6** If $\Delta ABC$ is a triangle with vertices $A(0,0),\ B(8,6),\ C(12,0)$ then

a. Find mid-points and slopes of three sides of the triangle

b. Find the equations of right bisectors of the sides of $\Delta ABC$

c. Show that the right bisectors of $\Delta ABC$ are concurrent

d. Find area of $\Delta ABC$

**Q. 7** A farmer plans to mix two types of food to make a low cost feed for the animals in his farm. A bag of food P costs Rs. 40 and contains 5 units of protiens and 4 units of vitamins. A bag of food Q costs Rs. 50 and contains 4 units of proteins and 8 units of vitamins. How many bags of food P and Q should be consumed by the animals each day in order to meet the minimum daily requirement of 120 units of protiens and 144 units of vitamins at a minimum cost?

**Q. 8** Find the centre, foci, eccentricity, vertices and directrices of the given conic $x^2+16x+4y^2-16y+76=0$