

**Z.R. Bhatti**

# LAPLACE, FOURIER AND Z-TRANSFORMS

**with Applications**

**Second  
Edition**

BS 4-Years, M.Sc. Mathematics,  
M.Sc. Physics & B.Sc. Engineering

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$$

$$\mathcal{F}\{f(x)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} f(x) dx$$

$$\mathcal{Z}\{f(n)\} = \sum_{n=0}^{\infty} f(n) z^{-n}$$

According to New Syllabus  
Approved by the  
University of Punjab,  
Govt. College University, Lahore,  
Lahore College University for Women,  
Kinnaird College, F C College,  
University of Lahore, UCP, Comsat,  
FAST, GCU, Faisalabad, Global Institute Lahore,  
UMT, University of Sargodha,  
University of Gujrat,  
International Islamic University,  
Islamabad, Peshawar University,  
University of AJK,  
Bahauddin Zakariya University, Multan,  
Quaid-i-Azam University, Islamabad,  
University of Education, Lahore,  
Islamia University, Bahawalpur, UET, Lahore.



**ILMI KITAB KHANA**

**Kabir Street, Urdu Bazar, Lahore.**

# LAPLACE, FOURIER AND Z-TRANSFORMS

*with Applications*

Second Edition

For

BS 4-Years  
M.Sc., Mathematics  
M.Sc. Physics &  
B.Sc. Engineering

**M. ZAHID RAFIQ BHATTI**

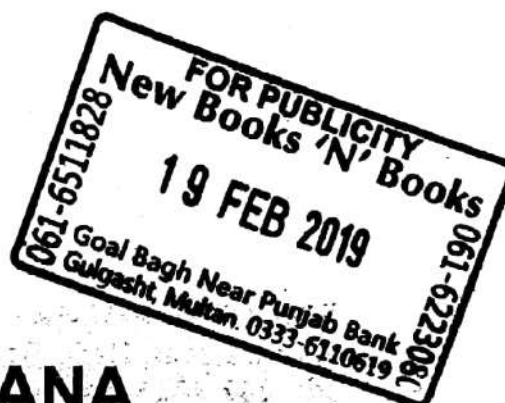
M.Sc.(PU), B.Ed.(PU), M.Phil.(QAU)

*Associate Professor*

Department of Mathematics

Govt. College of Science

Wahdat Road, Lahore.



**ILMI KITAB KHANA**

Kabir Street, Urdu Bazar, Lahore. Ph. 042-37353510, 042-37248129

Scanned by Asad Ali

Copyright ©2001, 2013

by

ILMI KITAB KHANA

Kabir Street, Urdu Bazar, Lahore.

All rights reserved. No part of this book may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, recording or by any information storage and retrieval system, without permission in writing from the Author and the Publisher.

**Title:** Laplace, Fourier and Z-transforms  
*with Applications*

**Author:** Z.R. Bhatti

**First Edition:** January, 2001  
**Second Edition:** November, 2013

**Publisher:** Ilmi Kitab Khana, Kabir Street, Urdu Bazar, Lahore  
**Printers:** Al-Hijaz Printers, 18-A Darbar Market, Lahore. Ph. 042-37238

**Price: Rs. 250/-**

ILMI KITAB KHANA  
Kabir Street, Urdu Bazar, Lahore.  
Ph. 042-37353510, 042-37248129  
Website: [ilmikitabkhana.com](http://ilmikitabkhana.com)



# DEDICATION

*I dedicate this humble work  
to my esteemed teacher*

**Prof. Muhammad Farooq**

Head (retired)

Department of Mathematics,  
Govt. College of Science,  
Wahdat Road, Lahore.

Scanned by Asad Ali





# **Preface**

(Second Edition)

This book is intended to serve as a text for the course in the Laplace, Fourier and Z-Transforms that is taken by the M.Sc. Mathematics, M.Sc. Physics, B.Sc. Engineering and BS 4-Years students. At the same time the book is also very useful for the candidates of competitive examinations. The language of the book is very simple, and I tried my best to explain articles with a number of suitable examples.

The book consists of four chapters. In the first chapter, the Laplace transform with its applications is discussed. Second chapter is concerned with the Fourier series. Fourier and Z-transforms are given in third and fourth chapters respectively.

The book is thoroughly revised and is self sufficient to explain the topics. Keeping in mind the problems of the students, I tried my best to explain each and every thing with details.

Finally I would like to thankful to all my colleagues and students whose kind corporation made it possible to write this book.

I hope that the readers of this book will continue to give me their valuable opinions and suggestions.

**Lahore**

**November, 2013**

**Z. R. BHATTI**

*Associate Professor*  
Department of Mathematics  
Govt. College of Science  
Wahdat Road, Lahore.

# Contents

## Preface

### **Chapter-1** **LAPLACE TRANSFORM**

1-1	Definition and Existence Theorem	1
	Exercise 1-1	13
1-2	Basic Properties of Laplace Transform	17
	Exercise 1-2	35
1-3	Inverse Laplace Transform	41
	Exercise 1-3	62
1-4	Convolution Theorem	69
	Exercise 1-4	81
1-5	Applications of Laplace Transform	88
	Exercise 1-5	122

### **Chapter-2** **FOURIER SERIES**

2-1	Fourier Series	126
	Exercise 2-1	137
2-2	Fourier Series with Arbitrary Periods	142
	Exercise 2-2	148
2-3	Fourier Cosine and Sine Series	151
	Exercise 2-3	154

### **Chapter-3** **FOURIER TRANSFORM**

3-1	Definition and Examples	157
3-2	Basic Properties of Fourier Transform	163
3-3	Some Useful Theorems	169
3-4	Fourier Cosine and Sine Theorems	175
3-5	Applications to Differential Equations	189

**Chapter-4**  
**Z-TRANSFORM**

4-1	Definition and Examples	209
4-2	Properties of Z-transforms	218
	Exercise 4-1	233
4-3	Z-transforms of Standard Discrete Functions	235
4-4	Convolution Theorem	247
4-5	Inverse Z-transforms	256
	Exercise 4-2	269
Answers		271
Glossary		287
Index		292

Scanned by Asad Ali

## Chapter

# 1

# LAPLACE TRANSFORM

Oliver Heaviside, in an effort to solve ordinary linear differential equations with facility, devised a method which led to the *Laplace transform*. The methods of Laplace transforms are very simple, and they give solutions of differential equations satisfying given boundary conditions directly without the use of the general solution. Since these particular solutions are the ones usually required in engineering, physics, mechanics, chemistry, and various fields of practical research, transforms are highly important.

### 1-1 Definition and Existence Theorem

The Laplace transform serves as a device for simplifying the solution of ordinary and partial differential equations. It associates a function  $f(t)$  with a function of another variable,  $F(s)$ , from which the original function can be recovered. Before defining the Laplace transform, it is essential to throw light on some basic and important definitions.

#### Piecewise Continuous Function

A function  $f$  defined in a closed interval  $a \leq x \leq b$  is called *piecewise continuous* or *sectionally continuous* if the interval can be split into a finite number of subintervals such that in each subinterval

- (i)  $f(x)$  is continuous,
- (ii)  $f(x)$  possesses (finite) limits at the left and right ends of each subinterval.

Condition (ii) means that  $f(x_i - 0)$  and  $f(x_i + 0)$  exist for all  $i = 1, 2, \dots, n$ , and also that  $f(a + 0)$  and  $f(b - 0)$  must exist.

For example, the function  $f$  defined by

$$f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ -1, & 1 \leq x \leq 2 \end{cases}$$