Z.R. Bhatti

LAPLACE, FOURIER AND Z-TRANSFORMS

with Applications

Second Edition

BS 4-Years, M.Sc. Mathematics,
M.Sc. Physics & B.Sc. Engineering

$$\mathscr{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$$

$$\mathscr{F}\{f(x)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx} f(x)dx$$

$$Z\{f(n)\} = \sum_{n=0}^{\infty} f(n)z^{-n}$$

According to New Syllabus
Approved by the
University of Punjab,
Govt. College University, Lahore,
Lahore College University for Women,
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ILMI KITAB KHANA
Kabir Street, Urdu Bazar, Lahore.

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DEDICATION

I dedicate this humble work

to my esteemed teacher

Prof. Muhammad Farooq

Head (retired)

Department of Mathematics,

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Wahdat Road, Lahore.

Preface

(Second Edition)

This book is intended to serve as a text for the course in the Laplace, Fourier and *Z*-Transforms that is taken by the M.Sc. Mathematics, M.Sc. Physics, B.Sc. Engineering and BS 4-Years students. At the same time the book is also very useful for the candidates of competitive examinations. The language of the book is very simple, and I tried my best to explain articles with a number of suitable examples.

The book consists of four chapters. In the first chapter, the Laplace transform with its applications is discussed. Second chapter is concerned with the Fourier series. Fourier and *Z*-transforms are given in third and fourth chapters respectively.

The book is thoroughly revised and is self sufficient to explain the topics. Keeping in mind the problems of the students, I tried my best to explain each and every thing with details.

Finally I would like to thankful to all my colleagues and students whose kind corporation made it possible to write this book.

I hope that the readers of this book will continue to give me their valuable opinions and suggestions.

Lahore
November, 2013

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Chapter

1 LAPLACE TRANSFORM

Oliver Heaviside, in an effort to solve ordinary linear differential equations with facility, devised a method which led to the *Laplace transform*. The methods of Laplace transforms are very simple, and they give solutions of differential equations satisfying given boundary conditions directly without the use of the general solution. Since these particular solutions are the ones usually required in engineering, physics, mechanics, chemistry, and various fields of practical research, transforms are highly important.

1-1 Definition and Existence Theorem

The Laplace transform serves as a device for simplifying the solution of ordinary and partial differential equations. It associates a function $f(t)$ with a function of another variable, $F(s)$, from which the original function can be recovered. Before defining the Laplace transform, it is essential to throw light on some basic and important definitions.

Piecewise Continuous Function

A function f defined in a closed interval $a \le x \le b$ is called *piecewise continuous* or *sectionally continuous* if the interval can be split into a finite number of subintervals such that in each subinterval

(i) $f(x)$ is continuous,

(ii) $f(x)$ posses (finite) limits at the left and right ends of each subinterval.

Condition (ii) means that $f(x_i-0)$ and $f(x_i+0)$ exist for all $i=1,2,\ldots,n$, and also that $f(a+0)$ and $f(b-0)$ must exist.

For example, the function f defined by

$$f(x)=\begin{cases} 1, & 0\le x\le 1 \\ -1, & 1\le x\le 2 \end{cases}$$