

Introduction to

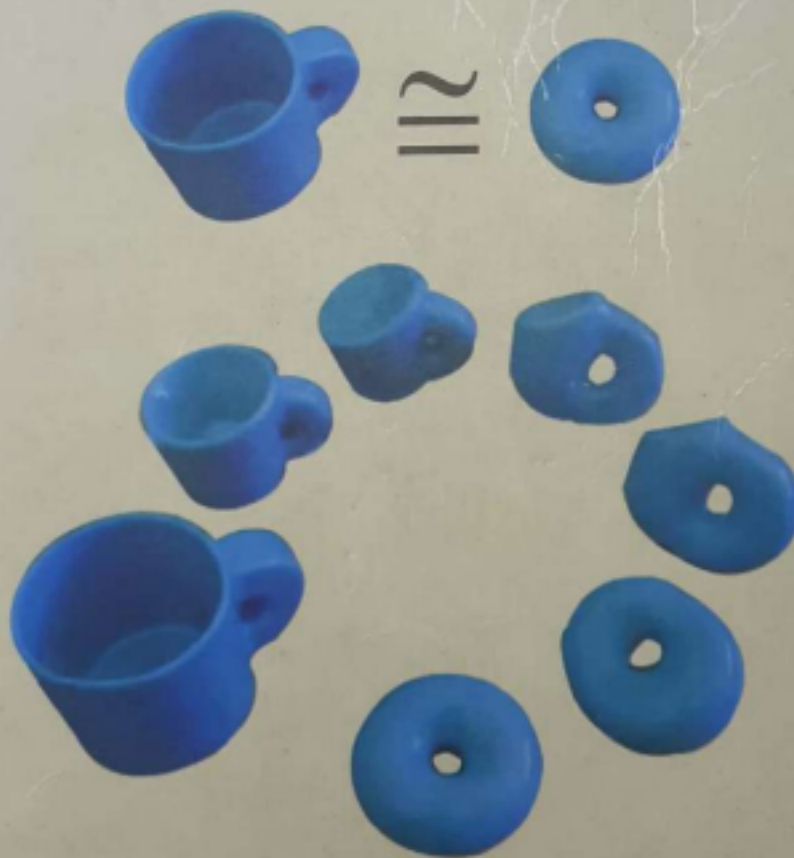
TOPOLOGY

3rd Edition

For

BS 4-Years

M.Sc. Mathematics



Z.R. Bhatti

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TOPOLOGY

3rd Edition

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**BS 4-Years
M.Sc. Mathematics**

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Preface

(Third Edition)

The book is revised thoroughly keeping in mind the paper patterns of BS Mathematics and M.Sc. Mathematics of different universities of Pakistan. Each exercise is divided into three parts. First part of each exercise consists of MCQs, second part consists of Short Questions and third part consists of Long Questions.

The essential topics covering the syllabi of BS Mathematics and M.Sc. Mathematics are given in 5 chapters.

The first chapter is devoted for the basic definitions and theorems of topological spaces. Continuity and countability are discussed in the second chapter. Separation, Connectedness and Compactness are discussed in the third, fourth and fifth chapters respectively.

I wish to thank to my friends and colleagues for their appreciation and valuable feedback.

Lahore
January, 2017

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Preface

(First Edition)

This book is intended to serve as a text for the course in the Topology that is taken by the M.Sc. Mathematics, B.Sc. Hons. and M.Sc. Hons. students. The aim of writing this book is to facilitate our students who do not often understand the tough foreign language. I tried my best to explain the articles and examples with detail in a simple and lucid manner. The second purpose of writing this book is to avoid the presentation of the unnecessary material which loses the interest and concentration of our students.

Hopefully the book will be very beneficiary for the readers who take the course of Topology at the beginning level.

I would like to thank to all my colleagues who always appreciate and encourage me in serving the Nation by writing books.

Lahore
January, 2005

Muhammad Zahid Rafiq Bhatti

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TOPOLOGICAL SPACES

- 1-1 Introduction
- 1-2 Definitions and Examples
- 1-3 Subspaces
- 1-4 Elementary Concepts
- 1-5 Interior, Exterior and Boundary of a Set

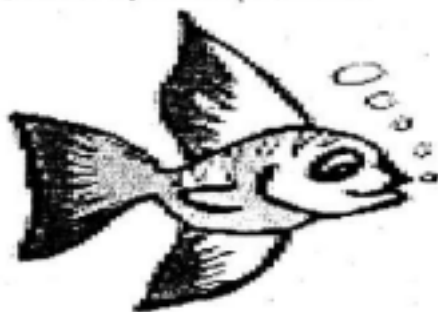
1-1 Introduction

In mathematics, *topology* (from the Greek τόπος, *place*, and λόγος, *study*) is concerned with the properties of space that are preserved under continuous deformations, such as stretching and bending, but not tearing or gluing. This can be studied by considering a collection of subsets, called *open sets*, which satisfy certain properties, turning the given set into what is known as a *topological space*.

Topology is often described as ***rubber sheet geometry***. Topologists study those properties of shapes that remain the same when the shapes are bent, stretched or twisted. What if, instead of paper sheets, we used rubber sheets to write and draw on? With advances in computer technologies, we really do not need to imagine this thought at all. For example, inspect the Pictures-1, 2 and 3 below.



Picture-1



Picture-2



Picture-3

If someone were to grab the right and left hand sides of the Picture-1 and pull, notice in Picture-2 how the image distorts from its original state. Now, let's squash the poor fish and notice in Picture-3 how the image distorts.