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By Muhammad Tahir Aziz Takleed

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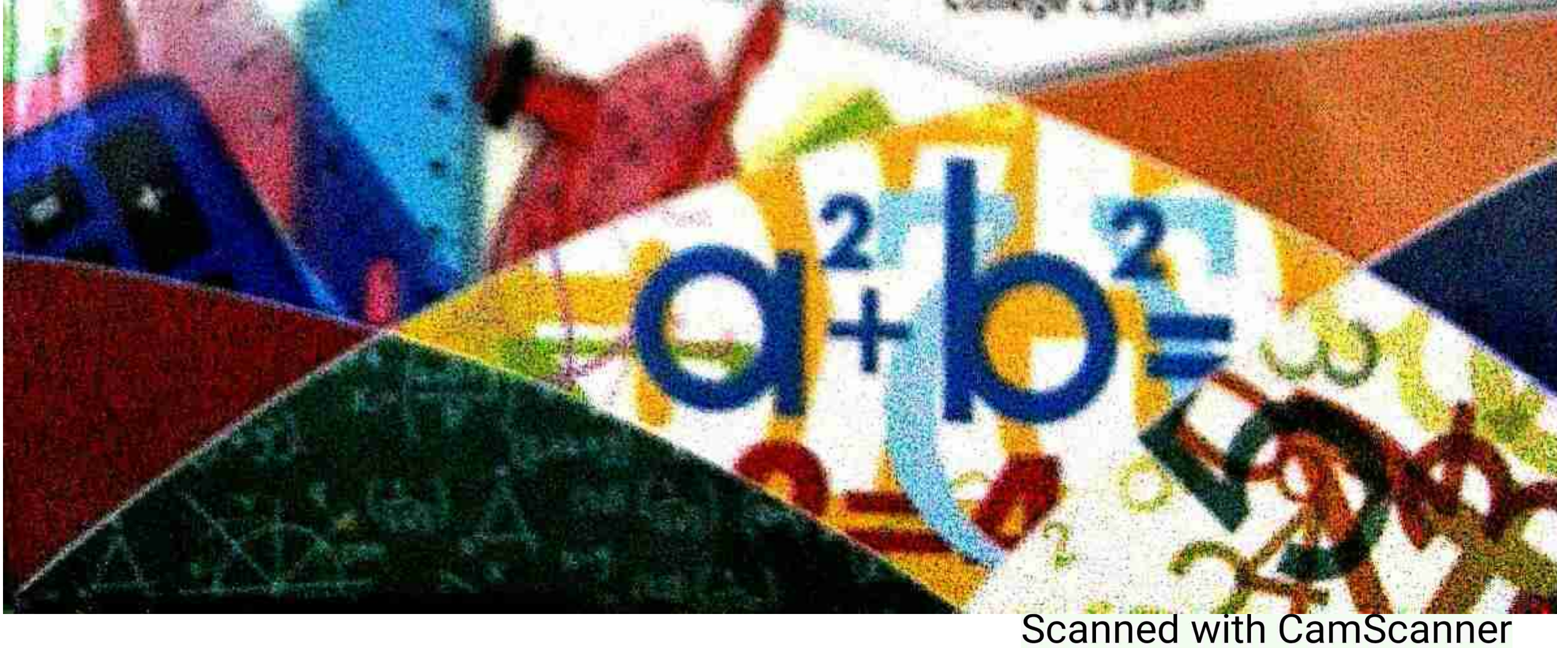

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Title

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Lecturer Math Past Paper PPSC 2011

Q.1 A ring R is called boolean Ring if, for all $x \in R$

(A) $x^2 = x$ (B) $x^2 = -x$

(C) $x^2 = 0$ (D) $x^2 = 1$

Solution:

Definition: A ring R with identity is called a Boolean if $x^2 = x, \forall x \in R$

Definition: A ring in which $x^2 = 0$ is called nilpotent ring

Theorem: Every Boolean ring is commutative ring

Q.2 The group of Quaternion is a non-Abelian group of order_____________.

(A) 6 (B) 8

(C) 10 (D) 4

Solution:

Group of Quaternion

$$Q_8 = \{\pm 1, \quad \pm i, \pm j, \pm k\}$$

With $(-1)^2 = 1$, $(i)^2 = -1$,

$(j)^2 = -1$, $(k)^2 = -1$,

$ij = k$, $jk = i$, $ki = j$, $ji = -k$,

$kj = -i$, $ik = -j$

The order of Quaternion is 8.

D_4 is other non-Abelian group of order 8

Q.3 Every group of prime order is _____________.

(A) An Abelian but not cyclic

(B) An Abelian

(C) A non Abelian group

(D) A cyclic group

Solution:

Theorem: Every group of prime order is cyclic.

Theorem: Every cyclic group is Abelian but converse is not true.

Theorem: A simple Abelian group $\Leftrightarrow$ the order is a prime number.

Q.4 Any two conjugate subgroups of a group are______.

(A) Equivalent (B) Similar

(C) Isomorphic (D) None of these

Solution:

Definition: Let G be a group. K and H be a subgroup of G, K and H are said conjugate subsets if there $\exists\, g \in G$ s.t $K = gHg^{-1}$

The mapping defined as $\varphi(h) = ghg^{-1}$

- If g and g′ are conjugate in G, Then they have same order
- If H and H′ are conjugate subgroup of G, then they have same order
- If H and H′ are conjugate subgroup of G, then they are isomorphic groups.

Q. 5 If H is a subgroup of index____________, then H is a normal subgroup of G.

(A) 2 (B) 4

(C) Prime number (D) None of these

Solution:

Index of Group: Let H be a subgroup of G. Then $[G:H]$ = number of distinct left or right cosets of H in G.

Theorem: Any subgroup of index 2 is a normal subgroup

Q.6 nZ is a maximal ideal of a ring Z if and only if n is________.

(A) Prime Number

(B) Composite Number

(C) Natural Number

(D) None of these

Solution:

- The proper (Maximal) ideal of Z exectly pZ where p is prime
- By canonical corresponding $Z \to \frac{Z}{nZ}$, the maximal ideals of $\frac{Z}{nZ}$ corresponding exactly to the maximal ideals of Z that contain nZ.
- The positive divisor of 17 are 1 and 17

$$\langle 1 \rangle = Z_{17}$$
$$\langle 17 \rangle = \{0\}$$

- $\langle 1 \rangle$ and $\langle 17 \rangle$ are improper ideals there is no prime divisor so there is no proper ideals of Z_{17}
- Let Z_8 the positive divisor of 8 are 1,2,4,8. The ideals in Z_8 are

$$(1) = Z_8$$
$$(2) = \{0,2,4,6\}$$
$$(4) = \{0,4\}$$
$$(8) = \{0\}$$

From above (2) is maximal (2 is prime), (1) and (8) are improper so, neither prime nor maximal. (4)⊊(2) so 4 not is maximal.

Q.7 Let G be a cyclic group of order 24 generated by a then order of a^{10}

(A) 2 (B) 12

(C) 10 (D) None of these

Solution:

$$O(G) = 24$$

Let $O(a^{10}) = m$

Then $(a^{10})^m = e \Rightarrow O(a) = 10m$

As we know order of element divide the order of group

So $24 | 10m$

$\Rightarrow 10m = 24k \quad k \in Z$

$$m = \frac{24}{2} = 12$$

Other method

$$|a^k| = \frac{|G|}{gcd(|G|,k)}$$
$$gcd(|G|,k) = gcd(24,10) = 2$$
$$|a^k| = \frac{24}{2} = 12$$
$$m = \frac{24}{10}k$$
$$m = \frac{12}{5}k$$

For least positive number

$$k = 5 \Rightarrow m = 12$$
$$O(a^{10}) = m = 12$$

Q.8 If a vector space V has a basis of n vectors, every basis of V must consist of exactly _____ vectors.

(A) $n+1$ (B) n

(C) $n+1$ (D) None of these

Solution:

Theorem: If a vector space V has a basis of n vectors, every basis of V must consist of exactly n vectors.

Q.9 An indexed set of vectors $\{v_1, v_2, \dots, v_p\}$ in R^n is said to be ________ if the vector equation $x_1v_1 + x_2v_2 + \cdots + x_pv_p = 0$ has only the trivial solution.

(A) Linearly Independent

(B) Basis

(C) Linear dependent

(D) None of these

Solution:

Definition: A **linear combination of vectors $v_1, v_2, \dots, v_n$** with coefficients $x_1, x_2, \dots, x_n$ is a vector $x_1v_1 + x_2v_2 + \cdots + x_nv_n$

Definition: A linear combination $x_1v_1 + x_2v_2 + \cdots + x_nv_n$ are called **trivial** if all the coefficients $x_1, x_2, \dots, x_n$ are zero.

Definition: A linear combination $x_1v_1 + x_2v_2 + \cdots + x_nv_n$ are called **non trivial** if at least one of them coefficients $x_1, x_2, \dots, x_n$ is not zero.

Definition: The vectors $v_1, v_2, \dots, v_n$ are called linearly independent if there are trivial combination of these vectors equal to the zero vector

Definition: The vectors $v_1, v_2, \dots, v_n$ are called linearly dependent if there are non-trivial combination of these vectors equal to the zero vector

Definition: If a vector V has a basis n, then any set in V containing more then n vectors must be linearly dependent.

Q.10 The set C_n of all root of unity for a fixed positive integer n is a group under________.

(A) Addition

(B) Addition modulo n

(C) Multiplication

(D) Multiplication modulo n

Solution:

These are the sets of roots of unity

$$C_2 = \{1. -1\}$$

$$C_3 = \{1. \omega, \omega^2\}$$

$$C_4 = \{1. -1, i, -i\}$$

Clearly all these are group under multiplication

Q.11 Intersection of any collection of normal subgroup G________________.

(A) Is normal subgroup

(B) May not be normal subgroup

(C) Is cyclic subgroup

(D) Is Abelian subgroup

Solution:

Theorem: The intersection of two normal subgroups is normal subgroup. If $X \le G, Y \le G$ then $(X \cap Y) \le G$

Q.12 $\frac{Z}{2Z}$ is a quotient group of order ________.

(A) 1 (B) 2

(C) Infinite (D) None of these

Solution:

- The order of quotient group G/H is given by lagrange Theorem