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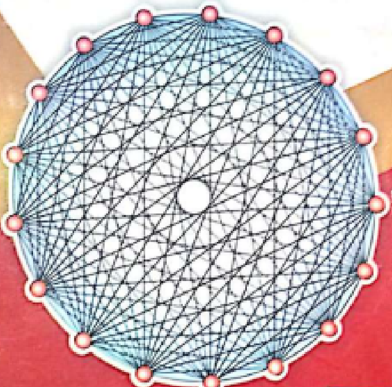
LECTURER'S
Mathematics
TEST GUIDE



Z.R. Bhatti

For

The Recruitment of Lecturers,
Assistant Professors, Subject Specialists,
Senior Subject Specialists and Other Related Exams

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Solution:

$\dfrac{x^2-2}{1-2x}>1$

$\Rightarrow \dfrac{x^2-2}{1-2x}-1>0$

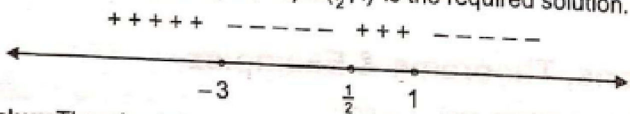
$\Rightarrow \dfrac{x^2+2x-3}{1-2x}>0$

$\Rightarrow \dfrac{(x-1)(x+3)}{1-2x}>0$

Interval	Test Point	Sign of $\dfrac{(x-1)(x+3)}{1-2x}$
$(-\infty,-3)$	-4	$\frac{+}{+}=+$
$(-3,\frac{1}{2})$	-2	$\frac{-}{+}=-$
$(\frac{1}{2},1)$	$\frac{3}{4}$	$\frac{-}{-}=+$
$(1,\infty)$	2	$\frac{+}{-}=-$

The values of x for which $x-1=0$, $x+3=0$ or $1-2x=0$ are $x=1, x=-3$ and $x=\frac{1}{2}$. These points divide the number line into four open intervals $(-\infty,-3),(-3,\frac{1}{2}),(\frac{1}{2},1)$, and $(1,\infty)$. We choose $-4,-2,\frac{3}{4}$ and 2 in the intervals $(-\infty,-3),(-3,\frac{1}{2}),(\frac{1}{2},1)$ and $(1,\infty)$ respectively and use these points as test points.

The pattern of signs in the intervals is shown on the number line in the following figure. We conclude that the set $(-\infty,-3)\cup(\frac{1}{2},1)$ is the required solution.



Absolute Value: The *absolute value* or *magnitude* of a real number x is denoted by $|x|$ and is defined as $|x|=\begin{cases} x & \text{if } x\ge 0 \\ -x & \text{if } x<0\end{cases}$

Theorem-2 (*Leibniz's Theorem*): *If u and v are functions of x and both possess nth order derivatives with respect to x, then*

$$(uv)_n={}^nC_0u_nv+{}^nC_1u_{n-1}v_1+{}^nC_2u_{n-2}v_2+\dots+{}^nC_{n-1}u_1v_{n-1}+{}^nC_nuv_n$$

Proof: We prove this theorem by Mathematical Induction. Using product theorem of differentiation, we have

$$(uv)_1=\frac{d}{dx}(uv)=\frac{du}{dx}v+u\frac{dv}{dx}=u_1v+uv_1={}^1C_0u_1v+{}^1C_1uv_1 \quad \because {}^1C_0=1={}^1C_1$$

This shows that the given theorem is true for $n=1$, so let it be true for $n=k\in N$, i.e.

$(uv)_k={}^kC_0u_kv+{}^kC_1u_{k-1}v_1+{}^kC_2u_{k-2}v_2+\dots+{}^kC_{k-1}u_1v_{k-1}+{}^kC_kuv_k$

$(uv)_{k+1}={}^kC_0\frac{d}{dx}(u_kv)+{}^kC_1\frac{d}{dx}(u_{k-1}v_1)+{}^kC_2\frac{d}{dx}(u_{k-2}v_2)+\dots+{}^kC_{k-1}\frac{d}{dx}(u_1v_{k-1})+{}^kC_k\frac{d}{dx}(uv_k)$

$(uv)_{k+1}={}^kC_0[u_{k+1}v+u_kv_1]+{}^kC_1[u_kv_1+u_{k-1}v_2]+{}^kC_2[u_{k-1}v_2+u_{k-2}v_3]+$

$\dots+{}^kC_{k-1}[u_2v_{k-1}+u_1v_k]+{}^kC_k[u_1v_k+uv_{k+1}]$

$(uv)_{k+1}={}^kC_0u_{k+1}v+[{}^kC_0+{}^kC_1]u_kv_1+[{}^kC_1+{}^kC_2]u_{k-1}v_2+\dots+[{}^kC_{k-1}+{}^kC_k]u_1v_k+{}^kC_kuv_{k+1}$

Since ${}^kC_{r-1}+{}^kC_r={}^{k+1}C_r$, ${}^kC_0=1={}^{k+1}C_0$, ${}^kC_k=1={}^{k+1}C_{k+1}$, so the last equation reduces to

$$(uv)_{k+1}={}^{k+1}C_0u_{k+1}v+{}^{k+1}C_1u_kv_1+{}^{k+1}C_2u_{k-1}v_2+\dots+{}^{k+1}C_ku_1v_k+{}^{k+1}C_{k+1}uv_{k+1}$$

This shows that theorem is true for $n=k+1$, so by principle of mathematical induction, we have $(uv)_n={}^nC_0u_nv+{}^nC_1u_{n-1}v_1+{}^nC_2u_{n-2}v_2+\dots+{}^nC_{n-1}u_1v_{n-1}+{}^nC_nuv_n$.