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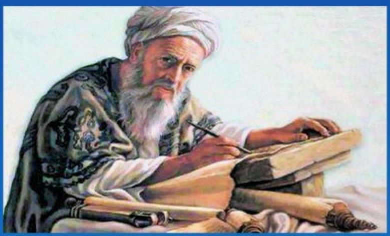
SEVEN MUSLIMS NOTES

PHYSICS 11



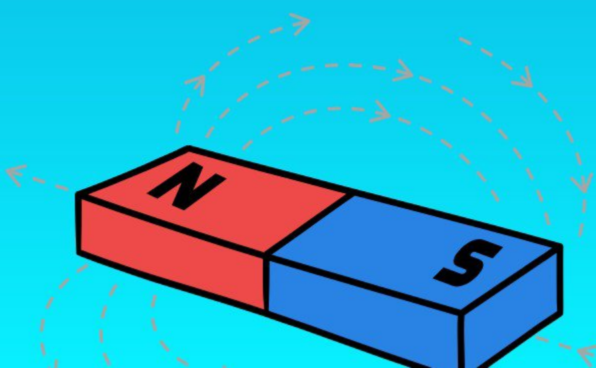
Al-Biruni (973–1048) calculated the Earth's radius and worked on the physics of planetary motion.

Best Regards to

Sir Muhammad Ali

(Physics Lecturer KIPS College)

$E=mc^2$



CHAPTER 7

OSCILLATIONS

Q.1: Define oscillatory motion.

OSCILLATORY MOTION

"To and fro motion of an object about its mean position is called vibratory or oscillatory/vibratory motion."

Examples:

1.Motion of simple pendulum

2.Mass spring system

3.Ball and bowl system

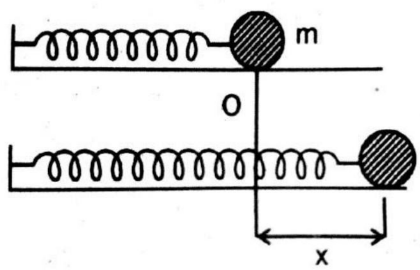
Q.2: Explain simple harmonic motion. (Or) Drive an expression for the acceleration of a body vibrating under elastic restoring force.

SIMPLE HARMONIC MOTION

"To and fro motion of an object about its mean position in which acceleration is directly proportional to displacement and always directed towards mean position is called simple harmonic motion."

Explanation:

Consider a mass "m" attached to a spring capable of moving on a frictionless horizontal surface. Let the mass is displaced from mean position to extreme position "A" by the action of force "F" according to Hook's law



$$F \propto x$$

$$F = kx$$

$$K = \text{spring constant}$$

Due to the elasticity of spring mass moves back towards the mean position by a restoring force

Fr Given as

$$F_r = -kx \quad \ldots\ldots\ldots\ldots\ldots(1)$$

Negative shows that direction of restoring force is opposite to x.

By newton's second law

$$F = ma \quad \ldots\ldots\ldots\ldots\ldots..(2)$$

Comparing equation 1 and 2

$$-Kx = ma$$

$$a = (-\frac{k}{m})x$$

$$= -\text{ constant } x$$

$$a \propto -x$$

Q.3: Define the following terms

1. **Instantaneous displacement.**
2. **Amplitude**
3. **Vibration**
4. **Time period**
5. **Time period**
6. **Frequency**
7. **Angular frequency**

1. **INSTANTANEOUS DISPLACEMENT**

   "The value of displacement of a body executing SHM at any instant of time is called instantaneous displacement."

2. **AMPLITUDE**

   "The maximum displacement of a body executing SHM from its mean position is called its amplitude."

3. **VIBRATION**

   "One complete round trip of a body in SHM is called one vibration."

4. **TIME PERIOD**

   "Time to complete one vibration is called time period."

5. **FREQUENCY**

   "Number of vibrations per second is called frequency."

   Formula:

   $$F = \frac{n}{t}$$

   For one vibration

   $$F = \frac{1}{t}$$

6. **ANGULAR FREQUENCY**

   The angular frequency of a body is given as

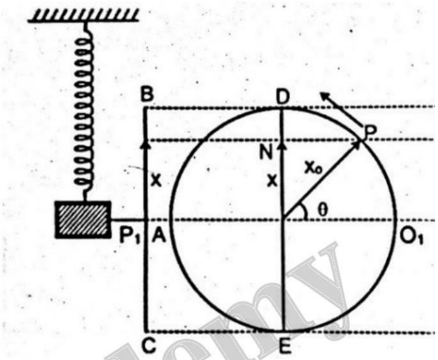
$$\omega = \frac{\theta}{t} = \frac{2\pi}{T}$$

$$\omega = 2\pi f \qquad \left(\because f = \frac{1}{T}\right)$$

Q.4: Discuss simple harmonic motion on the bases of uniform circular motion. Also calculate instantaneous displacement, instantaneous velocity and instantaneous acceleration.

UNIFORM CIRCULAR MOTION AND SHM

Vertically suspended mass spring system:

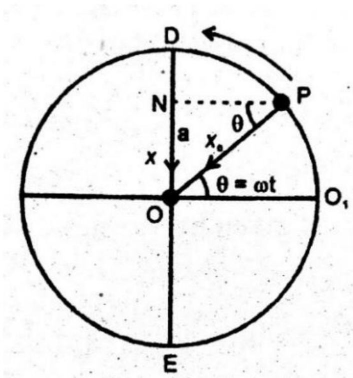


Consider a vertically suspended mass spring system that can vibrate simple harmonically between point B and C with amplitude x_o, frequency f and time period T. The position of the mass at any instant is represented by a pointer P_1. If at t = 0 pointer P_1 starts It's motion from mean position A then it will be at point B, A, C and back to A in time $\frac{T}{4}, \frac{T}{2}, \frac{3T}{4}$, T.

UNIFORM CIRCULAR MOTION

Let us discuss motion of particle P moving in a circle of radius x_o with constant speed the projection N of the particle vibrate along the direction DE of the circle as the particle moves in the circle. If at t = 0 particle P starts its motion from point O_1 then the projection N moves from point O to D, O, E and back to point O in time $\frac{T}{4}, \frac{T}{2}, \frac{3T}{4}$, T. Thus projection N and the mass m is performing SHM.

INSTANTANEOUS DISPLACEMENT



If particle P starts its motion from point O_1 at t= 0 and makes angle θ with the initial position then the displacement of Projection N at any instant is calculated as

$$\text{Sin}\angle p = \frac{P}{H} = \frac{x}{x_o}$$

$$x = x_o \sin\angle p$$

$$x = x_o \sin\theta$$

$$x = x_o \sin\omega t$$

or $$x = x_o \sin 2\pi f t \qquad \because \omega = \frac{2\pi}{T} = 2\pi f$$

At $\theta = 0$; $x = 0$

At $\theta = 90°$; $x = x_o$

At $\theta = 180°$; $x = 0$

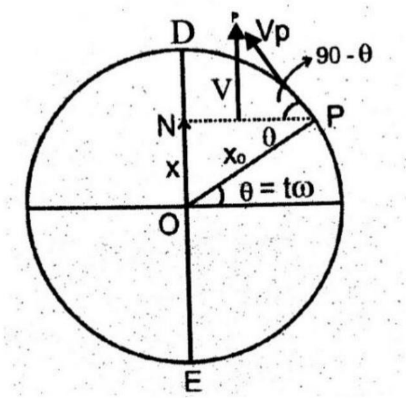
At $\theta = 270°$; $x = -x_o$

At $\theta = 360°$; $x = 0$

INSTANTANEOUS VELOCITY

The velocity of particle P moving in a circular path at any instant is given as

$$V_p = x_o\omega \ldots\ldots\ldots\ldots (1)$$



The velocity of Projection N at any instant is the vertical component of V_p and is given as

$$\sin(90-\theta) = \frac{V}{V_p}$$

$$V = V_p \text{Sin}(90-\theta)$$

Use in equation 1

$$V = x_o\omega\cos\theta \ldots\ldots (2)$$

In ΔONP

$$x_o^2 = x^2 + \overline{NP}^2$$

$$\overline{NP} = \sqrt{x_o^2 - x^2}$$

Also

$$\cos\theta = \frac{B}{H} = \frac{\overline{NP}}{\overline{OP}}$$

$$\cos\theta = \frac{\sqrt{x_o^2 - x^2}}{x_o}$$

Use in equation 2

$$V = \frac{x_o\omega\sqrt{x_o^2 - x^2}}{x_o}$$

$$V = \omega\sqrt{x_o^2 - x^2}$$

At mean position

$$x = 0$$

$$V_{max} = x_o\omega$$

At extreme position

$$x = x_o$$

$$V = 0$$

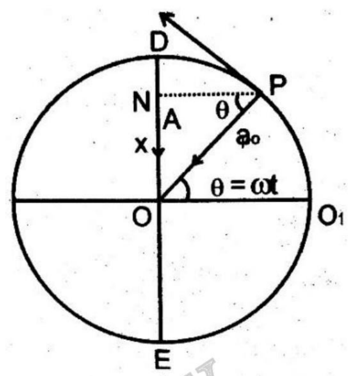
INSTANTANEOUS ACCELERATION

The centripetal acceleration of particle P moving in a circle of radius x_o is given as

$$a_p = \frac{V^2}{x_o} = \frac{x_o^2\omega^2}{x_o}$$

$$a_p = x_o\omega^2 \quad \ldots\ldots\ldots..(1)$$

The instantaneous acceleration a of Projection N is the vertical component of a_p Thus from ΔONP



$$\text{Sin}\angle p = \frac{a}{a_p}$$

$$a = a_p \sin\angle p\ldots\ldots\ldots(2)$$

In terms of displacement

$$\text{Sin}\angle p = \frac{x}{x_o}\ldots\ldots\ldots\ldots\ldots(3)$$

Use equation no 1 and 3 in 2.

$$a = x_o\,\omega^2\left(\frac{x}{x_o}\right)$$

$$a = -x\omega^2$$

The negative sign shows that direction of a is always towards the mean position.

Q.5: Define Phase angle.

PHASE

"The angle that specifies position and direction of motion of a particle moving in a circular path is called phase."

Q.6: Discuss the motion of mass attach with one end of spring placed on horizontal surface.

HORIZONTAL MASS SPRING SYSTEM

The acceleration of a particle in simple harmonic motion at any instant is given as:

$$a = \left(-\frac{k}{m}\right)x\ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

Also

$$a = -\omega^2 x\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(2)$$

Comparing equation 1 and 2

$$\omega^2 = \frac{k}{m}$$

$$\omega = \sqrt{\frac{k}{m}}$$

Time period of mass m is

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Instantaneous displacement x of the mass is given by

$$x = x_o \sin\omega t = x_o \sin\sqrt{\frac{k}{m}}t$$

The instantaneous velocity of mass is given by

$$v = \omega\sqrt{x_o^2 - x^2}$$

$$v = \sqrt{\frac{k}{m}(x_o^2 - x^2)}$$

$$v = \sqrt{\frac{k}{m}x_o^2(1 - \frac{x^2}{x_o^2})}$$

$$V = x_o\sqrt{\frac{k}{m}(1 - \frac{x^2}{x_o^2})}$$

The maximum velocity of mass m at mean position x=0 is

$$v_{max} = x_o\sqrt{\frac{k}{m}}$$

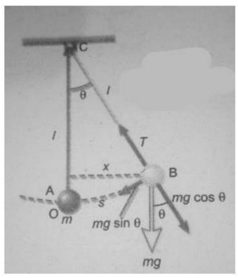
Q.7: What is simple pendulum. Show that its motion is S.H.M. Also derive the expression for time period and frequency.

SIMPLE PENDULUM

"A small heavy mass suspended by an inextensible string is called simple pendulum."

Explanation:

Consider a simple pendulum of mass m and length *l*. When such a pendulum is displaced from its mean position to extreme position A and released, it starts oscillation to and fro about mean position O. The restoring force acting on mass m at point A is



$$F = -mg\sin\theta$$

For small angles

$$\sin\theta = \theta$$

So

$$F = -mg\theta$$

By newton's second law

$$ma = -mg\theta$$

$$a = -g\theta \quad \ldots\ldots\ldots..(1)$$

Also

$$\theta = \frac{\text{Arc length}}{\text{radius}}$$

$$\theta = \frac{x}{l}$$

Use in 1

$$a = -g\frac{x}{l} \quad \ldots\ldots\ldots\ldots\ldots(2)$$

$$a = -\text{constant } x$$

$$a \propto -x$$

$$a = -\omega^2 x$$

Comparing equation 1 and 2

$$\omega^2 = \frac{g}{l}$$

$$\omega = \sqrt{\frac{g}{l}}$$

Then

$$T = \frac{2\pi}{\omega}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

Dependence of time period:

The time period of simple pendulum depends upon:

1. Length of the pendulum
2. Gravitational acceleration

It is Independent of gravitational acceleration.

It is Independent of mass is the pendulum.

Q.8: Explain the energy conservation in simple harmonic motion.

ENERGY CONSERVATION IN SHM

Consider a horizontal mass spring system in which mass m is displaced from its mean position to a displacement x_o against the elastic restoring force such that a = 0

According to Hook's law:

$$F = kx_o$$

Let

$$\text{When } F = 0 \quad ; \text{displacement} = 0$$

$$\text{When } F = kx_o \text{ ; displacement} = x_o$$

$$F_{avg} = \frac{0 + kx_o}{2} = \frac{1}{2}kx_o$$

Thus

$$w = Fd = \left(\frac{1}{2}kx_o\right)x_o$$

$$w = \frac{1}{2}kx_o^2$$

This work appears as elastic potential energy in the spring

$$P.E_{max} = \frac{1}{2}kx_o^2$$

At any instant

$$P.E = \frac{1}{2}kx^2$$

The velocity of mass m in SHM at any instant

$$v = x_o\sqrt{\frac{k}{m}(1 - \frac{x^2}{x_o^2})}$$

Then

$$k.E = \frac{1}{2}mv^2 = \frac{1}{2}m\left(x_o^2\frac{k}{m}\left(1 - \frac{x^2}{x_o^2}\right)\right)$$

$$k.E = \frac{1}{2}kx_o^2\left(1 - \frac{x^2}{x_o^2}\right)$$

At mean position x=0

$$k.E_{max} = \frac{1}{2}kx_o^2$$

The total energy of mass in SHM at any instant is

$$\text{Total energy} = E = P.E + K.E$$

$$E = \frac{1}{2}kx^2 + \frac{1}{2}kx_o^2\left(1 - \frac{x^2}{x_o^2}\right)$$

$$E = \frac{1}{2}kx^2 + \frac{1}{2}kx_o^2 - \frac{1}{2}kx^2$$

$$E = \frac{1}{2}kx_o^2$$

Conclusion:

During the whole motion of mass m is SHM the maximum potential energy at extreme position is changed completely into kinetic energy at mean position but the total energy at every point remains

Q.9: Define free and forced oscillations.

same.

FREE OSCILLATION

"A body is said to be expecting free oscillations when it oscillates without the inference of an external force."

Example:

Oscillation of simple pendulum which is slightly displaced from its mean position.

NATURAL FREQUENCY

"The frequency of free oscillations of a body is called its natural frequency."

FORCED OSCILLATION

"If a freely oscillating system is subjected to an external force then force oscillations will take place."

Example:

Oscillations of vibrating simple pendulum which is struck repeatedly.

DRIVEN HARMONIC OSCILLATOR

"A physical system under going forced oscillations is called driven harmonic oscillator."

Q.10: Define and explain the phenomenon of resonance with an

RESONANCE

" Resonance occurs when the frequency of applied force is equal to the natural frequency of driven harmonic oscillator. "

Properties of Resonance:

- At resonance amplitude of vibration increases.
- Energy absorption by the system is maximum.

Examples:

We come across many examples of resonance in every day life. A swing is a good example of mechanical resonance. It is like a pendulum with a single natural frequency depending on its length. If a series of regular pushes are given to the swing, its motion can be built up enormously. If pushes are given irregularly, the swing will hardly vibrate.

The column of soldiers, while marching on a bridge of long span are advised to break their steps. Their rhythmic march might set up oscillations of dangerously large amplitude in the bridge structure.

Tuning a radio is the best example of electrical resonance. When we turn the knob of a radio, to tune a station, we are changing the natural frequency of the electrical circuit of the receiver, to make it equal to the transmission frequency of the radio station. When the two frequencies match, energy absorption is maximum and this is the only station we hear.

Another good example of resonance is the heating and cooling of food very efficiently and evenly by microwave oven. The waves produced in this type of oven have a wavelength of 12cm at a frequency of 2450 MHz. At this frequency the waves are absorbed due to resonance by water and fat molecules in the food, heating them up and so cooking the food.

Q.11: Define damped oscillations and damping. What are its applications

DAMPED OSCILLATIONS

" Such oscillations in which the amplitude decreases steadily with time are called damped oscillations."

DAMPING

"Damping is a process whereby energy is dissipated from an oscillating system."

APPLICATIONS

An application of damped oscillations is the shock absorber of a car which provides a damping force to prevent excessive oscillations.