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Answer Sheet No._____

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# MATHEMATICS HSSC–II

## SECTION – A (Marks 20)

**Time allowed: 25 Minutes**

Section – A is compulsory. All parts of this section are to be answered on this page and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. **Do not use lead pencil.**

حصہ اوّل لازمی ہے۔ اس کے جوابات اسی صفحہ پر دے کر ناظم مرکز کے حوالے کریں۔ کاٹ کر دوبارہ لکھنے کی اجازت نہیں ہے۔ لیڈ پنسل کا استعمال ممنوع ہے۔

**Fill the relevant bubble against each question:**

ہر سوال کے سامنے دیے گئے درست دائرہ کو پر کریں۔

1. A function $f: x \to y$ defined by $f(x) = a,\ \forall x \in X,\ a \in y$ is called: ◯ Linear function ◯ Constant function ◯ Identity function ◯ Implicit function

2. If $f(x) = \sqrt{x^2 - 1}$ then Domain of $f$ is: ◯ $(-\infty, \infty)$ ◯ $[1, \infty)$ ◯ $[0, \infty)$ ◯ $(-\infty, -1] \cup [1, \infty)$

3. What result occurs in Evaluating $\lim_{x \to 3} \frac{x-3}{\sqrt{3} - \sqrt{x}}$ ◯ $-2\sqrt{3}$ ◯ $2\sqrt{3}$ ◯ $3\sqrt{2}$ ◯ $3\sqrt{3}$

4. If $f(x) = \cos x$, then what is the value of $f'(\sin^{-1} 3x)$ is: ◯ $-\sin 3x$ ◯ $-3x$ ◯ $\frac{-3}{\sqrt{1-9x^2}}$ ◯ $\frac{3}{\sqrt{1-9x^2}}$

5. If $f(x) = \ln x^2$ then what is the value of $f''(\sqrt{5})$ is: ◯ $\frac{-1}{5}$ ◯ $\frac{1}{5}$ ◯ $\frac{-2}{5}$ ◯ $\frac{2}{5}$

6. $(1 + x^2)\frac{d}{dx}(\tan^{-1} x + \cot^{-1} x) =$ ◯ 2 ◯ $\frac{2}{1+x^2}$ ◯ 0 ◯ $\frac{-2}{1+x^2}$

7. The integral $\int \frac{dx}{x \ln x}$ is equal to: ◯ $\ln x + c$ ◯ $\frac{1}{x} + c$ ◯ $\ln(\ln x) + c$ ◯ $\frac{(\ln x)^2}{2} + c$

8. What is the value of $k$ if $\int_0^1 (3x + k)dx = 2$ ◯ $\frac{1}{2}$ ◯ $\frac{-3}{2}$ ◯ $\frac{-1}{2}$ ◯ $\frac{2}{3}$

9. What is the area between the x-axis and curve $y = \cos\frac{x}{2}$, from $-\pi$ to $\pi$? ◯ 1 ◯ 2 ◯ 4 ◯ 8

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| 10. | The equation of a line $\frac{x}{P\sec\alpha}+\frac{y}{P\operatorname{cosec}\alpha}=1$ is called: | ◯ Symmetric equation of a line | ◯ Two intercept form of a line | ◯ Slope intercept form of a line | ◯ Normal form of a line |
| 11. | For what value of $k$ the lines $kx-2y+5=0$ and $x-2ky+3=0$ are parallel line? | ◯ 3 | ◯ $\pm1$ | ◯ 2 | ◯ $\pm2$ |
| 12. | The equation of the vertical line through $(-6,5)$ | ◯ $-5$ | ◯ $-6$ | ◯ 5 | ◯ 6 |
| 13. | Which one satisfies the inequality $x+2y<6$ | ◯ $(4,1)$ | ◯ $(1,3)$ | ◯ $(1,4)$ | ◯ $(3,1)$ |
| 14. | What is the length of tangent from $(1,1)$ to the circle $x^2+y^2-2x+3y+6=0$ ? | ◯ 1 | ◯ 2 | ◯ 3 | ◯ 4 |
| 15. | What is the eccentricity of an ellipse $\frac{x^2}{16}+\frac{y^2}{4}=1$ | ◯ $\frac{1}{\sqrt{3}}$ | ◯ $\frac{2}{\sqrt{3}}$ | ◯ $\frac{\sqrt{3}}{2}$ | ◯ $\sqrt{3}$ |
| 16. | What is the length of latus rectum of the hyperbola whose equation is $\frac{x^2}{16}-\frac{y^2}{9}=1$ ? | ◯ $\frac{2}{9}$ | ◯ $\frac{9}{2}$ | ◯ $\frac{4}{9}$ | ◯ $\frac{9}{4}$ |
| 17. | What is the Directrix of Parabola with vertex at origin and focus at $(8,0)$ ? | ◯ $x+8=0$ | ◯ $x-8=0$ | ◯ $x+4=0$ | ◯ $x+2=0$ |
| 18. | What is the projection of vector $-2\hat{i}+3\hat{j}+7\hat{k}$ on $2\hat{j}+\hat{k}$ ? | ◯ $\frac{\sqrt{13}}{5}$ | ◯ $\frac{13}{\sqrt{5}}$ | ◯ $\frac{5}{\sqrt{13}}$ | ◯ $\frac{\sqrt{5}}{13}$ |
| 19. | What is the angle between the vectors $2\overline{i}+\overline{j}+\overline{k}$ , $-\overline{i}+2\overline{j}$ are? | ◯ $\frac{\pi}{3}$ | ◯ $\frac{2\pi}{3}$ | ◯ $\frac{\pi}{6}$ | ◯ $\frac{\pi}{2}$ |
| 20. | For what value of $\alpha$ the vectors $2\overline{i}$ , $\overline{j}+\overline{k}$ and $\overline{i}+\alpha\overline{j}+2\overline{k}$ are coplanar? | ◯ $-2$ | ◯ 2 | ◯ 3 | ◯ $-3$ |



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# MATHEMATICS HSSC–II

**Time allowed: 2:35 Hours** **Total Marks Sections B and C: 80**

**NOTE:** **Attempt any twelve parts from Section 'B' and any four questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet–B if required. Write your answers neatly and legibly. Graph paper will be provided on Demand.**

## SECTION – B (Marks 48)

**Q. 2** **Attempt any TWELVE parts. All parts carry equal marks.** **( 12 x 4 = 48 )**

(i) For the real valued function, $f(x)$ is defined by $f(x)=\sqrt{x^3+4}$ find $f^{-1}(x)$. Also verify $f(f^{-1}(x))=x$

(ii) Evaluate $\lim_{x\to 0}\dfrac{\operatorname{cosec} x-\cot x}{x}$

(iii) If $y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+....\infty}}}$ prove that $(2y-1)\dfrac{dy}{dx}=\cos x$

(iv) Show that $\sin(x+h)=\sin x+h\cos x-\dfrac{h^2}{2!}\sin x-\dfrac{h^3}{3!}\cos x+.....$ (by Taylor's Series)

(v) If $y=\sin^{-1}\dfrac{x}{a}$ then show that $y_2=x(a^2-x^2)^{-\frac{3}{2}}$

(vi) Evaluate $\displaystyle\int\frac{dx}{3x(\ln 3x)^4}$

(vii) Evaluate $\displaystyle\int_0^3\frac{x^3+9x+3}{x^2+9}\,dx$

(viii) Solve the differential equation $\dfrac{dy}{dx}+\dfrac{4xy}{4y+2}=x$

(ix) Find an equation of the perpendicular bisector of a line joining the points $A(5,6)$ and $B(8,4)$.

(x) Find the value of $k$ such that the lines $2x-2y+2=0$, $3x-5y-1=0$ and $2x+ky+8=0$ meet at a point.

(xi) Graph the feasible region of the system of linear inequalities by shading
$5x+7y\le 35$ , $-x+3y\le 3$ , $x\ge 0$ , $y\ge 0$

(xii) Find the equation of a circle passing through the points $A(2,3)$, $B(0,2)$ having centre at $3x+2y-3=0$

(xiii) Find the equation of Parabola with focus $(3,2)$ and directrix $2x-y+5=0$.

(xiv) Find the equation of tangent to hyperbola $9x^2-4y^2=36$ parallel to the line $3x+2y+7=0$

(xv) Find the scalar '$\alpha$' so that vectors $3\bar{i}+\alpha\bar{j}+4\bar{k}$ and $4\bar{i}+5\bar{j}+\alpha\bar{k}$ are perpendicular to each other.

(xvi) Find the volume of the tetrahedron whose vertices are $A(-2,1,4)$ , $B(3,2,5)$ , $C(-3,-5,0)$ , $D(5,8,9)$

## SECTION – C (Marks 32)

**Note:** **Attempt any FOUR questions. All questions carry equal marks.** **( 4 x 8 = 32 )**

**Q. 3** Let $f(x)=\begin{cases} mx+3 & \text{if } x<3 \\ m+n & \text{if } x=3 \\ -x+9 & \text{if } x>3 \end{cases}$

a. Find $\lim_{x\to 3^-} f(x)$ and $\lim_{x\to 3^+} f(x)$

b. Find the $\lim_{x\to 3} f(x) = f(3)$

c. Find the value of $m$ and $n$ such that $f(x)$ is continuous at $x=3$

d. After finding the values of $m$ and $n$, sketch the graph of the function

**Q. 4** The perimeter of a triangle is 18 centimetres. If one side is of length 8 cm. What are lengths of the other sides for maximum area of a triangle?

a. Find function $f(x)$

b. Find $f'(x)$ and $f''(x)$

c. Find the values of $f(x)$ for which has maximum or minimum values?

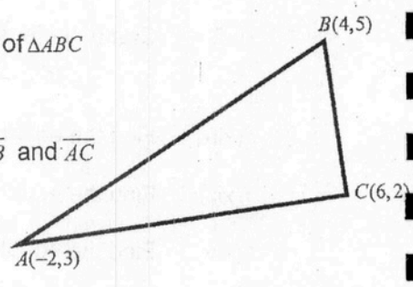
d. Find the sides of triangle ABC

**Q. 5** Evaluate the integral $\int \frac{2x^2+5x+3}{(x-2)^2(x^2+x+1)}dx$

a. Resolve $\frac{2x^2+5x+3}{(x-2)^2(x^2+x+1)}$ into Partial fraction

b. After Partial Fraction Integrate the result $\int \frac{2x^2+5x+3}{(x-2)^2(x^2+x+1)}dx$

**Q. 6** The diagram shows a triangle ABC where $A(-2,3)$, $B(4,5)$, $C(6,2)$ are vertices of $\Delta ABC$




a. Find the slopes of side $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$

b. Find the angle between the sides $\overline{AB}$ and $\overline{BC}$ and angle between $\overline{AB}$ and $\overline{AC}$

c. Find the equations of sides $\overline{AB}$ and $\overline{BC}$

d. Find the area of triangle ABC check these three points are collinear

**Q. 7** An agent wishes to purchase a number of chairs and tables. He has only Rs. 12000 to invest and has space at most for 28 items. A chair costs him Rs. 480 and a table costs Rs. 300. His expectation is that he can sell a chair at a profit of Rs. 200 and table at a profit of Rs. 150. Assuming that he can sell all the items that he can buy. How should he invest his money in order to maximize his profit?

**Q. 8** Find the Centre, Foci, Eccentricity, Vertices and Equation of directrices of the conic $25x^2+4y^2-250x-16y+541=0$

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Answer Sheet No._____

Sign. of Candidate _____

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Section – A is compulsory. All parts of this section are to be answered on this page and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use **lead pencil.**

# MATHEMATICS HSSC–II
## SECTION – A (Marks 20)
## Time allowed: 25 Minutes

حصہ اوّل لازمی ہے۔ اس کے جوابات اسی صفحہ پر دے کر ناظم مرکز کے حوالے کریں۔ کاٹ کر دوبارہ لکھنے کی اجازت نہیں ہے۔ لیڈ پنسل کا استعمال ممنوع ہے۔

ہر سوال کے سامنے دیے گئے درست دائرہ کو پر کریں۔

**Fill the relevant bubble against each question:**

1. $x = a\cos\theta$, $y = b\sin\theta$ are parametric equations of: ◯ Circle ◯ Parabola ◯ Ellipse ◯ Hyperbola

2. Which of the following represents $f^{-1}(5)$ if $f(x) = x^{\frac{1}{3}} + 2$ ◯ 1 ◯ 3 ◯ 9 ◯ 27

3. In which of the following intervals, $f(x) = 4x - 2x^2$ is increasing? ◯ $0 \le x \le 1$ ◯ $0 < x < 1$ ◯ $1 \le x \le 2$ ◯ $0 < x < 2$

4. What result will occur, in evaluating $\lim_{x\to\infty}\left(1+\frac{2}{n}\right)^{3n}$ ◯ $e$ ◯ $e^2$ ◯ $e^3$ ◯ $e^6$

5. For a function $f(x) = a\sin 3x$ and $f'(\frac{\pi}{3}) = 6$ then what is the value of $a$? ◯ $-2$ ◯ 2 ◯ 3 ◯ 6

6. $\frac{d}{dx}(\sec^{-1}x + \operatorname{cosec}^{-1}x) =$ ◯ $-1$ ◯ 0 ◯ 1 ◯ 2

7. $\int \frac{1}{\sqrt{x}}\frac{dx}{(\sqrt{x}+1)} =$ ◯ $-\ln(\sqrt{x}+1)+c$ ◯ $\ln(\sqrt{x}+1)+c$ ◯ $2\ln(\sqrt{x}+1)+c$ ◯ $-2\ln(\sqrt{x}+1)+c$

8. Which one of the following results occurs of the integral $\int_0^2 \frac{dx}{x^2+4}$ ◯ $\frac{\pi}{6}$ ◯ $\frac{\pi}{8}$ ◯ $\frac{\pi}{4}$ ◯ $\frac{\pi}{2}$

9. If $\int_0^2 f(x) = 3$ then what is the value of $k$ if $\int_0^2 (3f(x)+4)dx = k$ ◯ 11 ◯ 17 ◯ 20 ◯ 23

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| 10. | The points $A(2,5)$ and $B(3,-2)$ are the ends of a diameter of a circle, what is the radius of a circle? | ◯ $2\sqrt{5}$ | ◯ $5\sqrt{2}$ | ◯ $\frac{5}{\sqrt{2}}$ | ◯ $\frac{2}{\sqrt{5}}$ |
| 11. | A line cuts the x-axis at $(2,0)$ and y-axis at $(0,-4)$, then equation of a line is: | ◯ $2x-y-4=0$ | ◯ $2x-y+4=0$ | ◯ $x+2y-4=0$ | ◯ $x-2y+4=0$ |
| 12. | Pair of lines represented by Homogeneous equation $ax^2+2hxy+by^2=0$ through origin will be real and coincident if: | ◯ $h^2>ab$ | ◯ $h^2<ab$ | ◯ $h^2=ab$ | ◯ $a+b=0$ |
| 13. | The solution set of $2y+5>4y-3$ | ◯ $y>-4$ | ◯ $y>8$ | ◯ $y<-4$ | ◯ $y<4$ |
| 14. | The line $y=mx+c$ will be tangent to a circle $x^2+y^2=a^2$ if: | ◯ $c=\pm m\sqrt{1+a^2}$ | ◯ $c=\pm a\sqrt{1+m^2}$ | ◯ $c=\pm m\sqrt{1-a^2}$ | ◯ $c=\pm a\sqrt{1-m^2}$ |
| 15. | What is the Length of Latus Rectum of Parabola $x^2=5y$ | ◯ 5 | ◯ 20 | ◯ $\frac{5}{4}$ | ◯ 10 |
| 16. | Which one of the following represents the graph of $9x^2-18x+4y^2+8y-23=0$ ? | ◯ Circle | ◯ Parabola | ◯ Ellipse | ◯ Hyperbola |
| 17. | The co-vertices of hyperbola $\frac{x^2}{16}-\frac{y^2}{4}=1$ are: | ◯ $(0,\pm4)$ | ◯ $(\pm2,0)$ | ◯ $(\pm4,0)$ | ◯ $(0,\pm2)$ |
| 18. | The area of the triangle whose adjacent sides are $3\bar{i}+4\bar{j}$ and $12\bar{i}+9\bar{j}$ is: | ◯ $\frac{45}{2}$ | ◯ $\frac{21}{2}$ | ◯ $\frac{55}{2}$ | ◯ $\frac{25}{2}$ |
| 19. | If vectors $\bar{v}=\bar{i}-3\bar{j}+4\bar{k}$ and $\bar{w}=\lambda\bar{i}+9\bar{j}-12\bar{k}$ are parallel then what is the value of $\lambda$ ? | ◯ $-3$ | ◯ 3 | ◯ $-9$ | ◯ 9 |
| 20. | What is the volume of a parallelepiped if its edges are $2\underline{i}-4\underline{j}+5\underline{k}$, $2\underline{i}-3\underline{j}+6\underline{k}$, $-\underline{j}-\underline{k}$ ? | ◯ 0 | ◯ 3 | ◯ 15 | ◯ 24 |



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# MATHEMATICS HSSC–II

Time allowed: 2:35 Hours | Total Marks Sections B and C: 80

**NOTE:** Attempt any twelve parts from Section 'B' and any four questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet–B if required. Write your answers neatly and legibly. Graph paper will be provided on Demand.

## SECTION – B (Marks 48)

**Q. 2** Attempt any TWELVE parts. All parts carry equal marks. **( 12 x 4 = 48 )**

(i) Let the real valued function, $f$ and $g$ defined by $f(x)=4x+1$ and $g(x)=2x^2+5x$ obtain the expression for:

a. $f((g))$ b. $g(f(x))$ c. $f(f(x))$ d. $g(f(x))$

(ii) Evaluate $\lim_{x\to 0}\dfrac{\sqrt{x+5}-\sqrt{5}}{x}$

(iii) Find $\dfrac{dy}{dx}$ if $x=\dfrac{3at}{1+t^3}$, $y=\dfrac{3at^2}{1+t^3}$

(iv) If $y=\tan(4\tan^{-1}\dfrac{x}{4})$ show that $\dfrac{dy}{dx}=\dfrac{16(1+y^2)}{16+x^2}$

(v) Use implicit rule to find the second derivative of the function $y=x+\tan^{-1}y$

(vi) If $x=\cos\theta$ ; $y=\cos n\theta$ show that $(1-x^2)y_2-xy_1+n^2y=0$

(vii) Find the area between the x-axis and the curve $f(x)=x^2-2x$ from $x=0$ to $x=3$

(viii) Evaluate $\int x^3\sqrt{1+x^2}\,dx$

(ix) Find the point two-fifth of the way along the line segment $A(-3,5)$ to $B(5,3)$.

(x) Find the angle $\theta$ form the lines $L_1$ and $L_2$: $L_1: 7x+3y-9=0$, $L_2: 5x-2y+2=0$

(xi) Graph the feasible solution region of the system of linear inequalities by shading, also find the corner points. $3x+7y\le 21$ , $x-y\le 3$, $x\ge 0$, $y\ge 0$

(xii) Find the equation of parabola with focus $(1,3)$ and vertex $(4,3)$.

(xiii) Find the equation of parabola, with Directrix, $y=3$ and vertex $(2,2)$.

(xiv) Write the equation of ellipse with vertices at $(-1,2)$ and $(7,2)$ and 2 is the length of semi minor axis whereas major axis is horizontal.

(xv) Prove that $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

(xvi) Find constant $\alpha$ so that vectors are coplaner $\bar{i}-\alpha\bar{j}-\bar{k}$, $\bar{i}+\bar{j}+2\bar{k}$ and $\alpha\bar{i}-\bar{j}+\bar{k}$

## SECTION – C (Marks 32)

**Note:** **Attempt any FOUR questions. All questions carry equal marks.** **( 4 x 8 = 32 )**

**Q. 3** If $\theta$ is measured in radian then prove that $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$

a. Draw the figure and give explanation.

b. Find area of triangles in figure.

c. From figure, see the inequalities of area and prove the theorem.

**Q. 4** Consider the function $f(x)=\sin x+\frac{1}{\sqrt{2}}\cos 2x$ where $x\in(0,2\pi)$

find the extreme values of the functions in the interval $x\in(0,2\pi)$

a. Find function $f'(x)$

b. Find $f''(x)$

c. Find the values of $x\in(0,2\pi)$ for which $f(x)$ has maximum or minimum values

d. Find possible extreme values of $f(x)$

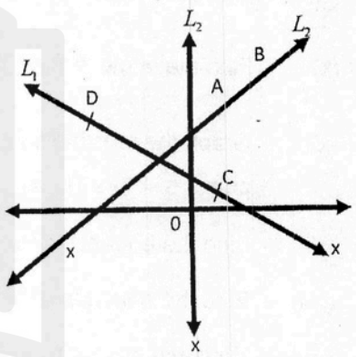
**Q. 5** Integrate $\int\frac{2x+5}{(x-3)^2(x^2-x+5)}dx$

a. Resolve $\frac{2x+5}{(x-3)^2(x^2-x+5)}$ into Partial fraction

b. After Partial Fraction Integrate the result $\int\frac{2x+5}{(x-3)^2(x^2-x+5)}dx$

**Q. 6** The diagram shows two Lines $L_1$ and $L_2$ passing through points:

$L_1$: joins $A(2,7)$ and $B(7,10)$ $L_2$: joins $C(1,1)$ and $D(-5,3)$




a. Find the slope of lines $L_1$ and $L_2$

b. Find the angle between the lines $L_1$ and $L_2$

c. Find the equations of line $L_1$ and $L_2$

d. Find the point of contact where line $L_1$ and $L_2$ intersect

**Q. 7** Find the maximum and minimum values of $f$ and $g$ defined as $f(x)=3x+5y$ and $g(x)=6x+8y$ under the constraints. $2x-3y\le 6$ , $2x+y\ge 2$ , $2x+3y\le 12$ , $x\ge 0, y\ge 0$

**Q. 8** Find the equations of tangent and normal lines at a point $\left(3,\frac{12}{5}\right)$ to ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ For what value of $C$ the line $x+y+c=0$ will touch the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$