Version No.			
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

ROLL NUMBER						
0	0	0	0	0	0	0
1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9

Answer Sheet No.____________

Sign. of Candidate___________

Sign. of Invigilator___________

MATHEMATICS HSSC–I

SECTION – A (Marks 20)

Time allowed: 25 Minutes

Section – A is compulsory. All parts of this section are to be answered on this page and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. **Do not use lead pencil.**

Q.1 Fill the relevant bubble for each part. All parts carry one mark.

1. In complex numbers, what is the multiplicative inverse of $2i$?
   - A. $\frac{-i}{2}$ ◯ B. $\frac{1}{2}$ ◯
   - C. $\frac{-1}{2}$ ◯ D. $\frac{i}{2}$ ◯

2. Which one of the following represents the negation of the statement $\sim p \to q$?
   - A. $\sim q \to p$ ◯ B. $\sim q \to \sim p$ ◯
   - C. $q \to \sim p$ ◯ D. $p \to \sim q$ ◯

3. What is the value of α, if $\begin{vmatrix} 2 & 3 & 0 \\ 3 & 9 & 6 \\ 2 & 15 & 1 \end{vmatrix} = \alpha \begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ 2 & 5 & 1 \end{vmatrix}$?
   - A. 3 ◯ B. 6 ◯
   - C. 9 ◯ D 15 ◯

4. What is the solution set of an equation $x^{-1} + \frac{1}{x^{-1}} = 2$?
   - A. {1} ◯ B. {–1, 1} ◯
   - C. {0, –1} ◯ D. {1, 1} ◯

5. If -1 is a root of $x^3 + kx^2 - x + 2 = 0$, then the value of k is:
   - A. -2 ◯ B. 2 ◯
   - C. -4 ◯ D. 0 ◯

6. The partial fractions of $\frac{7x+25}{(x+3)(x+4)}$ are:
   - A. $\frac{3}{x+4} + \frac{3}{x+3}$ ◯ B. $\frac{3}{x+4} + \frac{4}{x+3}$ ◯
   - C. $\frac{4}{x+4} - \frac{3}{x+3}$ ◯ D. $\frac{-4}{x+4} + \frac{3}{x+3}$ ◯

7. For an A.P: $2 + \frac{7}{2} + 5 + \frac{13}{2} + \cdots + a_{10}$, the value of S_{10} is:
   - A. $\frac{265}{2}$ ◯ B. $\frac{175}{2}$ ◯
   - C. $-\frac{95}{2}$ ◯ D. 195 ◯

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8. What should be the geometric mean between $\sqrt{2}$ and $3\sqrt{2}$?

A. $\frac{6}{\sqrt{2}}$ ◯ B. $6\sqrt{2}$ ◯

C. $2\sqrt{2}$ ◯ D. $\sqrt{6}$ ◯

9. For what value of n, ${}^{n}P_2 = 12$?

A. 4 ◯ B. 3 ◯

C. 12 ◯ D. 6 ◯

10. On tossing a pair of fair coins, what is the probability of appearing both tails?

A. $\frac{3}{4}$ ◯ B. $\frac{2}{4}$ ◯

C. $\frac{1}{4}$ ◯ D. $\frac{2}{3}$ ◯

11. Which one of the following is an expansion of $(1+x)^{-1}$?

A. $1 - x + x^2 - x^3 + \ldots$ ◯

B. $1 + x - x^2 + x^3 + \ldots$ ◯

C. $1 + x + x^2 + x^3 + \ldots$ ◯

D. $1 - x - x^2 - x^3 + \ldots$ ◯

12. How many terms are there in the expansion of $\left[\left(x - \frac{1}{x}\right)^{\frac{1}{2}}\right]^{60}$?

A. 30 ◯ B. 31 ◯

C. 15 ◯ D. 16 ◯

13. Which one of the following is the simplified form of $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$?

A. $\sec\theta$ ◯ B. $\sec^2\theta$ ◯

C. $2\sec^2\theta$ ◯ D. $2\sec\theta$ ◯

14. What will be the result if simplify $\cos x - \left[\cos\left(x + \frac{\pi}{2}\right) + \cos\left(x - \frac{\pi}{2}\right)\right]$?

A. $\cos x$ ◯ B. $2\cos x$ ◯

C. 0 ◯ D. $(1-\sqrt{3})\cos x$ ◯

15. If $\theta = \frac{\pi}{2}$, then which one of the following options is true?

A. $\cos 2\theta = \cos^2\theta + \sin^2\theta$ ◯

B. $\sin 2\theta = 2\sin\theta\cos\theta$ ◯

C. $\sin\theta = -2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$ ◯

D. $\cos\theta = \cos^2\left(\frac{\theta}{2}\right) + \sin^2\left(\frac{\theta}{2}\right)$ ◯

16. What is the period of a trigonometric function $\sin\left(\frac{\pi x}{2}\right)$?

A. 2 ◯ B. 4 ◯

C. $\frac{1}{2}$ ◯ D. $\frac{1}{4}$ ◯

17. In a triangle ABC, what will be e-radius opposite to vertex A?

A. $\frac{\Delta}{s-a}$ ◯ B. $\frac{\Delta}{s-b}$ ◯

C. $\frac{\Delta}{s-c}$ ◯ D. $\frac{s-a}{\Delta}$ ◯

18. The value of $\sec\left[\sin^{-1}\left(-\frac{1}{2}\right)\right]$ is:

A. $\frac{2}{\sqrt{3}}$ ◯ B. $\frac{-2}{\sqrt{3}}$ ◯

C. $\frac{1}{2}$ ◯ D. $-\frac{1}{2}$ ◯

19. If $\sin^{-1}x + (\sin^{-1}x + \cos^{-1}x) = \pi$, then value of x is:

A. $\frac{1}{2}$ ◯ B. $\frac{1}{\sqrt{2}}$ ◯

C. 0 ◯ D. -1 ◯

20. Which one of the following is a solution set of $\sin x = \frac{1}{2}$, where $x \in [0, 2\pi]$?

A. $\left\{\frac{\pi}{6}, \frac{\pi}{2}\right\}$ ◯ B. $\left\{\frac{5\pi}{6}, \frac{3\pi}{2}\right\}$ ◯

C. $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$ ◯ D. $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$ ◯

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Section – B (Marks 48)

Q2(i):

Solution: $z_1 = 1 - 2i,\ z_2 = 2 + 3i,\ z_3 = 4 - 3i$

(a) $\frac{\overline{z_2}}{z_3} = \frac{\overline{2+3i}}{4-3i} = \frac{2-3i}{4-3i}$

$\Rightarrow \frac{2-3i}{4-3i} \times \frac{4+3i}{4+3i} = \frac{(2-3i)(4+3i)}{(4-3i)(4-3i)} = \frac{8+6i-12i-9i^2}{16-9i^2}$

Since , $i^2 = -1$

So, $\frac{\overline{z_2}}{z_3} = \frac{8-6i+9}{16+9} = \frac{17}{25} - \frac{6}{25}i$

(b) $\overline{z_1} \cdot \overline{z_3} = (\overline{1 - 2i}) \cdot (\overline{4 - 3i})$

$\Rightarrow \overline{z_1} \cdot \overline{z_3} = (1 + 2i)(4 + 3i) = 4 + 3i + 8i + 6i^2 = 4 + 11i - 6 \qquad \because i^2 = -1$

$\Rightarrow \overline{z_1} \cdot \overline{z_3} = -2 + 11i$

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Q2(ii):

Solution: $A \cap B = B \cap A$

We first convert it into the logical form as: $p \wedge q = q \wedge p$

Now, we construct a truth table to prove this equality.

		L.H.S.	**R.H.S.**
p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

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Q2(iii):

Solution: $\begin{vmatrix} x & 1 & x+1 \\ 2 & x & 3 \\ x+1 & 4 & x \end{vmatrix} = 11 - 2x^2$

To find the value of x, we expand L.H.S for first row

$\Rightarrow \quad x\begin{vmatrix} x & 3 \\ 4 & x \end{vmatrix} - 1\begin{vmatrix} 2 & 3 \\ x+1 & x \end{vmatrix} + (x+1)\begin{vmatrix} 2 & x \\ x+1 & 4 \end{vmatrix} = 11 - 2x^2$

$\Rightarrow \quad x(x^2 - 12) - (2x - 3x - 3) + (x+1)(8 - x^2 - x) = 11 - 2x^2$

$\Rightarrow \quad \cancel{x^3} - 12x + \cancel{x} + 3 + 8x - \cancel{x^3} - x^2 + 8 - x^2 - \cancel{x} = 11 - 2x^2$

$\Rightarrow \quad -\cancel{2x^2} - 12x + \cancel{11} + \cancel{2x^2} - \cancel{11} = 0$

$\Rightarrow \quad -12x = 0$

$\Rightarrow \quad x = 0$

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Q2(iv):

Solution: $\frac{18}{x^4} + \frac{1}{x^2} = 4$

$\Rightarrow \frac{18+x^2}{x^4} = 4$

$\Rightarrow \quad 18 + x^2 = 4x^4$

OR

$\Rightarrow \quad 4x^4 - x^2 - 18 = 0$

$\Rightarrow \quad 4x^4 - 9x^2 + 8x^2 - 18 = 0$

$\Rightarrow \quad x^2(4x^2 - 9) + 2(4x^2 - 9) = 0$

$\Rightarrow \quad (4x^2 - 9)(x^2 + 2) = 0$

Here, $4x^2 - 9 = 0$ and $x^2 + 2 = 0$

$\Rightarrow \quad x^2 = \frac{9}{4}$ and $x^2 = -2$

Taking square-root on bothsides

$\Rightarrow \quad x = \pm\frac{3}{2}$ and $x = \pm\sqrt{2}i$ (Complex)

Solution Set $= \{\pm\frac{3}{2}\}$

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Q2(v):

Solution: Resolve $\frac{3x^2+7x+28}{x(x^2+x+7)}$ into Partial Fraction

$\Rightarrow \frac{3x^2+7x+28}{x(x^2+x+7)} = \frac{A}{x} + \frac{Bx+C}{x^2+x+7}$ (i)

Multiplying by $x(x^2+x+7)$

$\Rightarrow 3x^2+7x+28 = A(x^2+x+7)+(Bx+C)(x)$ (ii)

For $x = 0$, Eq. (ii) gives

$\Rightarrow 3(0)^2+7(0)+28 = A(0^2+0+7)+(B(0)+C)(0)$

$\Rightarrow 28 = 7A, \quad \Rightarrow A = 4$

Expanding Eq. (ii), we get

$\Rightarrow 3x^2+7x+28 = Ax^2+Ax+7A+Bx^2+Cx$

Equating the coefficients of;

$x^2; \quad 3 = A+B, \Rightarrow 4+B = 3, \quad \Rightarrow B = -1$

$x; \quad 7 = A+C, \Rightarrow 4+C = 7, \quad \Rightarrow C = 3$

Substituting the values of $A, B, and\ C$ in Eq. (i)

$\Rightarrow \frac{3x^2+7x+28}{x(x^2+x+7)} = \frac{4}{x} + \frac{(-1)x+3}{x^2+x+7}$

$\Rightarrow \frac{3x^2+7x+28}{x(x^2+x+7)} = \frac{4}{x} - \frac{x-3}{x^2+x+7}$

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Q2(vi):

Solution: Since, $f(x+p) = f(x)$

Where, p is called the period of the function.

For Cosine function, we check the periodic function for $p = 2\pi$,

(i.e)

$\cos(x+2\pi) = cosx\cos(2\pi) - sinx\sin(2\pi)$ (By Fundamental law of trigonometry)

Here, $\cos(2\pi) = 1$, and $\sin(2\pi) = 0$

$\Rightarrow \cos(x+2\pi) = cosx\ (1) - sinx\ (0)$

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$\Rightarrow \quad \cos(x+2\pi) = cosx$

Hence, the period of Cosine is 2π.

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Q2(vii):

Solution: Sum of an A.P. is;

$S_n = \frac{n}{2}[2a_1 + (n-1)d]$

Sum of first 30 $-$terms of an A.P. is;

$\Rightarrow \quad S_{30} = \frac{30}{2}[2a + (30-1)d]$

$\Rightarrow \quad S_{30} = 15[2a + 29d]$

Similarly, the sum of first 6 $-$terms is;

$\Rightarrow \quad S_6 = 3[2a + 5d]$

The square of sum of first 6 $-$terms will be;

$\Rightarrow \quad S_6^2 = 9(2a+5d)^2 = 9(4a^2 + 20ad + 25d^2)$

Since, $S_{30} = S_6^2$

$\Rightarrow \quad 15(2a + 29d) = 9(4a^2 + 20ad + 25d^2)$

$\Rightarrow \quad 5(2a + 29d) = 3(4a^2 + 20ad + 25d^2)$

$\Rightarrow \quad 10a + 145d = 12a^2 + 60ad + 75d^2$ (Proved)

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Q2(viii):

Solution:

Number of boys = 8

Number of girls = 6

Person chosen = 4

The cases for atleast one girl are;

Case – 1: 1 girl + 3 boys $= \binom{6}{1} \times \binom{8}{3}$

Case – 2: 2 girls + 2 boys $= \binom{6}{2} \times \binom{8}{2}$

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Case – 3: 3 girls + 1 boy $= \binom{6}{3} \times \binom{8}{1}$

Case – 4: 4 girls + 0 boy $= \binom{6}{4} \times \binom{8}{0}$

Probability of more girls than boys is;

$$= \frac{\binom{6}{3}\times\binom{8}{1}}{\binom{14}{4}} + \frac{\binom{6}{4}\times\binom{8}{0}}{\binom{14}{4}} = \frac{160}{1001} + \frac{15}{1001} = \frac{175}{1001} = \frac{25}{143}$$

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Q2(ix):

Solution: There are 7 places to fill;

3 places of alphabet (out of 26) can be filled in $= 26 \times 25 \times 24 = 15600$

4 places of digits (out of 10 – (0 to 9)) can be filled in $= 10 \times 9 \times 8 \times 7 = 5040$

Number of different plates $= 15600 \times 5040 = 78624000$

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Q2(x):

Solution: $\frac{1}{\sqrt{9+x}} = (9+x)^{-\frac{1}{2}} = 9^{-\frac{1}{2}}\left(1+\frac{x}{9}\right)^{-\frac{1}{2}}$

$$\Rightarrow \quad 3^{-1}\left(1+\frac{x}{9}\right)^{-\frac{1}{2}} = \frac{1}{3}\left(1+\frac{x}{9}\right)^{-\frac{1}{2}}$$

Expanding by Binomial Series

$$\Rightarrow \frac{1}{3}\left\{1 + \left(-\frac{1}{2}\right)\left(\frac{x}{9}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}\left(\frac{x}{9}\right)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{3!}\left(\frac{x}{9}\right)^3 + \cdots\right\}$$

$$\Rightarrow \frac{1}{3}\left\{1 - \frac{x}{18} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2\cdot1}\cdot\frac{x^2}{81} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3\cdot2\cdot1}\cdot\frac{x^3}{729} + \cdots\right\}$$

$$\Rightarrow \frac{1}{3}\left\{1 - \frac{x}{18} + \frac{1}{8}\cdot\frac{x^2}{27} - \frac{5}{16}\cdot\frac{x^3}{729} + \cdots\right\}$$

$$\Rightarrow \frac{1}{3}\left\{1 - \frac{x}{18} + \frac{x^2}{216} - \frac{5x^3}{11664} + \cdots\right\} = \frac{1}{3} - \frac{x}{54} + \frac{x^2}{648} - \frac{5x^3}{34992} + \cdots$$

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Q2(xi):

Solution: Radius $= 15cm$

Interior angle $= \frac{360^o}{5} = 72^o$

Using law of cosine, we obtain

$\Rightarrow \quad c^2 = a^2 + b^2 - 2abcos\gamma$

$\Rightarrow \quad c^2 = (15)^2 + (15)^2 - 2(15)(15)cos72^o$

$\Rightarrow \quad c^2 = 225 + 225 - 450(0.309)$

$\Rightarrow \quad c^2 = 310.95$

Taking square-root

$\Rightarrow \quad c = 17.63cm$

Perimeter of Pentagon $= 5(17.63) = 88.17cm$

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Q2(xii):

Solution: $\frac{sin3\theta+sin5\theta+sin7\theta}{cos3\theta+cos5\theta+cos7\theta} = tan5\theta$

Taking, $L.H.S. = \frac{sin3\theta+sin7\theta+sin5\theta}{cos3\theta+cos7\theta+cos5\theta}$

$\Rightarrow \quad L.H.S. = \frac{2\sin\left(\frac{10\theta}{2}\right)\cos\left(\frac{4\theta}{2}\right)+sin5\theta}{2\cos\left(\frac{10\theta}{2}\right)\cos\left(\frac{4\theta}{2}\right)+cos5\theta}$

$\Rightarrow \quad L.H.S. = \frac{2\sin 5\theta\cos 2\theta+sin5\theta}{2\cos 5\theta\cos 2\theta+cos5\theta} = \frac{sin5\theta(2cos2\theta+1)}{cos5\theta(2\cos 2\theta+1)}$

$\Rightarrow \quad L.H.S. = \frac{sin5\theta}{cos5\theta} = tan5\theta = R.H.S.$ (Proved)

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Q2(xiii):

Solution: $y = sec2x$; $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

x	$-\pi/2$	$-\pi/3$	$-\pi/6$	0	$\pi/6$	$\pi/3$	$\pi/2$
$y = sec2x$	-1	-2	2	1	2	-2	-1
$P(x,y)$	$\left(-\frac{\pi}{2},-1\right)$	$\left(-\frac{\pi}{3},-2\right)$	$\left(-\frac{\pi}{6},2\right)$	$(0,1)$	$\left(\frac{\pi}{6},2\right)$	$\left(\frac{\pi}{3},-2\right)$	$\left(\frac{\pi}{2},-1\right)$

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Plot the graph on graph paper using the values calculated in the above table.

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Q2(xiv):

Solution: $s^2 = \triangle \cot\left(\frac{\alpha}{2}\right)\cot\left(\frac{\beta}{2}\right)\cot\left(\frac{\gamma}{2}\right)$

Taking, $R.H.S. = \triangle \cot\left(\frac{\alpha}{2}\right)\cot\left(\frac{\beta}{2}\right)\cot\left(\frac{\gamma}{2}\right)$

Where, $\cot\left(\frac{\alpha}{2}\right) = \sqrt{\frac{s(s-a)}{(s-b)(s-c)}}$; $\cot\left(\frac{\beta}{2}\right) = \sqrt{\frac{s(s-b)}{(s-c)(s-a)}}$; $\cot\left(\frac{\gamma}{2}\right) = \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$

$\Rightarrow \quad R.H.S. = \triangle \sqrt{\frac{s(s-a)}{(s-b)(s-c)} \cdot \frac{s(s-b)}{(s-c)(s-a)} \cdot \frac{s(s-c)}{(s-a)(s-b)}}$

$\Rightarrow \quad R.H.S. = \triangle \sqrt{\frac{s^2 \cdot s(s-a)(s-b)(s-c)}{(s-a)^2(s-b)^2(s-c)^2}} = \triangle \sqrt{\frac{s^2 \cdot s^2}{s(s-a)(s-b)(s-c)}}$

$\Rightarrow \quad R.H.S. = \triangle\left(\frac{s^2}{\triangle}\right) = s^2 = L.H.S.$ (Proved)

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Q2(xv):

Solution: $\cot^{-1}\left(\frac{119}{120}\right) = 2\sin^{-1}\left(\frac{5}{13}\right)$

Let $y = \cot^{-1}\left(\frac{119}{120}\right)$

$\Rightarrow \quad coty = \frac{119}{120}$

Where, $cscy = \sqrt{1 + \cot^2 y} = \sqrt{1 + \frac{14161}{14400}} = \sqrt{\frac{28561}{14400}}$

$\Rightarrow \quad cscy = \frac{169}{120}$

$\Rightarrow \quad siny = \frac{120}{169}$

Here, $cosy = \sqrt{1 - \sin^2 y} = \sqrt{1 - \frac{14400}{28561}} = \frac{119}{169}$

Using Half angle identity

$\Rightarrow \quad \sin\left(\frac{y}{2}\right) = \sqrt{\frac{1-cosy}{2}} = \sqrt{\frac{1}{2}\left(1 - \frac{119}{169}\right)} = \sqrt{\frac{25}{169}}$

$\Rightarrow \quad \sin\left(\frac{y}{2}\right) = \frac{5}{13}$; $\Rightarrow \quad y = 2\sin^{-1}\left(\frac{5}{13}\right)$

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Hence, $\cot^{-1}\left(\frac{119}{120}\right) = 2\sin^{-1}\left(\frac{5}{13}\right)$ Proved

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Q2(xvi):

Solution: We shall show that $2\cos^2\theta + 3\cos\theta - 2 = 0$

Where, $2\tan^2\theta cos\theta = 3$

$\Rightarrow \quad 2\left(\frac{\sin^2\theta}{\cos^2\theta}\right) cos\theta = 3$

$\Rightarrow \quad 2(1 - \cos^2\theta) = 3cos\theta$

$\Rightarrow \quad 2 - 2\cos^2\theta = 3cos\theta$

$\Rightarrow \quad 2\cos^2\theta + 3cos\theta - 2 = 0$ (Proved)

Also, to find the solution, we take $2\cos^2\theta + 3cos\theta - 2 = 0$

$\Rightarrow \quad 2\cos^2\theta + 4cos\theta - cos\theta - 2 = 0$

$\Rightarrow \quad 2\,cos\,\theta(cos\theta + 2) - 1(cos\theta + 2) = 0$

$\Rightarrow \quad (cos\theta + 2)(2cos\theta - 1) = 0$

Here, $cos\theta + 2 = 0$ and $2cos\theta - 1 = 0$

$\Rightarrow \quad cos\theta = -2$ (Not possible) and $\Rightarrow \quad cos\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = \frac{\pi}{3}$ and $\Rightarrow \quad \theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

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Section – C (Marks 32)

Q3.

Solution: $x + 2y + 3z = 3 \;\; ; \;\; 2x + 3y + z = 1 \;\; ; \;\; 3x + y + 2z = 2$

Let A_b be the augmented matrix, then

$$\Rightarrow \quad A_b = \begin{bmatrix} 1 & 2 & 3 & 3 \\ 2 & 3 & 1 \vdots & 1 \\ 3 & 1 & 2 & 2 \end{bmatrix}$$

To reduce A_b into reduced echelon form by elementary Row operations

$$\Rightarrow \quad R_2 - (2)R_1 \;, R_3 - (3)R_1 \;\sim^R \begin{bmatrix} 1 & 2 & 3 & 3 \\ 0 & -1 & -5 \vdots & -5 \\ 0 & -5 & -7 & -7 \end{bmatrix}$$

$$\Rightarrow \quad (-1)R_2 \;\sim^R \begin{bmatrix} 1 & 2 & 3 & 3 \\ 0 & 1 & 5 \vdots & 5 \\ 0 & -5 & -7 & -7 \end{bmatrix}$$

$$\Rightarrow \quad R_1 - (2)R_2 \;, R_3 + (5)R_2 \;\sim^R \begin{bmatrix} 1 & 0 & -7 & -7 \\ 0 & 1 & 5 \vdots & 5 \\ 0 & 0 & 18 & 18 \end{bmatrix}$$

$$\Rightarrow \quad \left(\frac{1}{18}\right) R_3 \;\sim^R \begin{bmatrix} 1 & 0 & -7 & -7 \\ 0 & 1 & 5 \vdots & 5 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \quad R_1 + (7)R_3 \;, R_2 - (5)R_3 \;\sim^R \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 \vdots & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$\therefore \quad x = 0 \;,\; y = 0 \;,\; z = 0$

Hence, the solution is trivial, (i.e.) {(0,0,0)}

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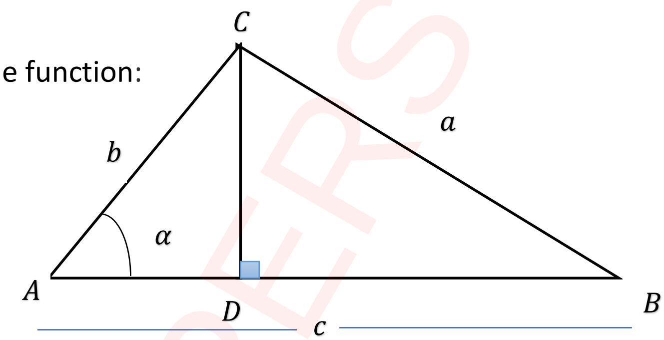
Q4.

Proof (a): $a^2 = b^2 + c^2 - 2bc\ cos\alpha$



In the right triangle ACD, by the definition of cosine function:

$cos\ \alpha = \frac{AD}{b} \Rightarrow AD = b\ cos\alpha$ (i)

$\Rightarrow DB = c - b\ cos\alpha$ (ii)

In the triangle ACD, according to Sine definition

$sin\ \alpha = \frac{CD}{b} \Rightarrow CD = b\ sin\alpha$ (iii)

In the triangle BDC, if we apply the Pythagorean Theorem, then

$a^2 = (BD)^2 + (CD)^2$ (iv)

Substituting for BD and CD from equations (ii) and (iii) in equation (iv)

$\Rightarrow a^2 = (c - b\ cos\alpha)^2 + (b\ sin\alpha)^2$

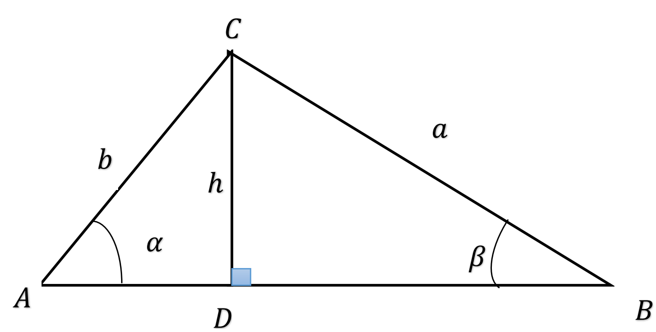
$\Rightarrow a^2 = c^2 - 2bc\ cos\alpha + b^2\cos^2\alpha + b^2\sin^2\alpha$

$\Rightarrow a^2 = b^2(\cos^2\alpha + \sin^2\alpha) + c^2 - 2bc\ cos\alpha$

Where, $\cos^2\alpha + \sin^2\alpha = 1$

$\Rightarrow a^2 = b^2 + c^2 - 2bc\ cos\alpha$ (Proved)

Proof (b): $\frac{a}{sin\alpha} = \frac{b}{sin\beta} = \frac{c}{sin\gamma}$



In right-angled triangle ADC

$sin\alpha = \frac{h}{b} \Rightarrow h = b\ sin\alpha$ (i)

In right-angled triangle BDC,

$sin\beta = \frac{h}{a} \Rightarrow h = a\ sin\beta$ (ii)

Comparing Equations (i) and (ii), we get

$a\ sin\beta = b\ sin\alpha$

$\Rightarrow \frac{a}{sin\alpha} = \frac{b}{sin\beta}$ (iii)

Similarly, it can also be proved that $\frac{b}{sin\beta} = \frac{c}{sin\gamma}$ or $\frac{c}{sin\gamma} = \frac{a}{sin\alpha}$

Thus, $\frac{a}{sin\alpha} = \frac{b}{sin\beta} = \frac{c}{sin\gamma}$ (Proved)

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Q5.

Solution:

(a) nth term of the series is;

$$a_n = \frac{4}{5}\left(\frac{2}{3}\right)^{n-1}$$

For $n = 1$

$$\Rightarrow \quad a_1 = \frac{4}{5}\left(\frac{2}{3}\right)^{1-1} = \frac{4}{5}\left(\frac{2}{3}\right)^{0}$$

$$\Rightarrow \quad a_1 = \frac{4}{5}$$

For $n = 2$

$$\Rightarrow \quad a_2 = \frac{4}{5}\left(\frac{2}{3}\right)^{2-1} = \frac{4}{5}\left(\frac{2}{3}\right)^{1}$$

$$\Rightarrow \quad a_2 = \frac{8}{15}$$

For $n = 3$

$$\Rightarrow \quad a_3 = \frac{4}{5}\left(\frac{2}{3}\right)^{3-1} = \frac{4}{5}\left(\frac{2}{3}\right)^{2}$$

$$\Rightarrow \quad a_3 = \frac{16}{45}$$

$\therefore \quad \frac{4}{5} + \frac{8}{15} + \frac{16}{45} + \cdots$, the Geometric series with Common Ratio $= \frac{2}{3}$

(b) To find the sum of first ten terms of the geometric series, we have

$$S_n = \frac{a_1(1-r)^n}{1-r}$$

Here, $a_1 = \frac{4}{5}, \quad r = \frac{2}{3} < 1 \, , n = 10$

$$\Rightarrow \quad S_{10} = \frac{\frac{4}{5}\left(1-\frac{2}{3}\right)^{10}}{1-\frac{2}{3}} = \frac{4}{5}\left(\frac{1}{3}\right)^{10} \times 3$$

$\Rightarrow \quad S_{10} = \frac{4}{5\times 3^9} = \frac{4}{98415}$

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Q6.

Solution: We shall prove that $63y^2 + 84y + 19 = 0$

Where, $y = -\frac{1}{3} + \frac{1}{3^3} + \frac{1.3}{2!}.\frac{1}{3^5} + \frac{1.3.5}{3!}.\frac{1}{3^7} + \cdots$

$$\Rightarrow \quad y + \frac{1}{3} = \frac{1}{3}\left(\frac{1}{3^2} + \frac{1.3}{2!}.\frac{1}{3^4} + \frac{1.3.5}{3!}.\frac{1}{3^6} + \cdots\right)$$

$$\Rightarrow \quad 3y + 1 + 1 = 1 + \frac{1}{3^2} + \frac{1.3}{2!}.\frac{1}{3^4} + \frac{1.3.5}{3!}.\frac{1}{3^6} + \cdots$$

$\Rightarrow \quad 3y + 2 = 1 + \frac{1}{3^2} + \frac{1.3}{2!}.\frac{1}{3^4} + \frac{1.3.5}{3!}.\frac{1}{3^6} + \cdots$ (A)

Let the R.H.S. of the series be identical as;

$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$ (B)

Equating the second and third terms on R.H.S. of above equations, we get

$nx = \frac{1}{3^2} = \frac{1}{9}$ (i)

$\frac{n(n-1)}{2!}x^2 = \frac{1.3}{2!}.\frac{1}{3^4}$ (ii)

From Eq. (i); $x = \frac{1}{3^2 n}$ (iii)

Substituting Eq. (iii) in Eq. (ii)

$$\frac{n(n-1)}{2!}\left(\frac{1}{3^2 n}\right)^2 = \frac{1.3}{2!}.\frac{1}{3^4}$$

$$\Rightarrow \quad \frac{n(n-1)}{n^2} = \frac{1.3}{2!} \times \frac{1}{3^4} \times 2! \times 3^4$$

$$\Rightarrow \quad n - 1 = 3n$$

$$\Rightarrow \quad n = -\frac{1}{2}$$

Utilizing the value of n in Eq. (iii), it gives

$$x = -\frac{2}{9}$$

Now, substituting the values of x and n in the L.H.S. of Eqs. (A) and (B), we obtain

$$3y + 2 = \left(1 - \frac{2}{9}\right)^{-\frac{1}{2}}$$

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$$\Rightarrow \quad 3y + 2 = \left(\frac{9}{7}\right)^{\frac{1}{2}}$$

Squaring the bothsides

$$\Rightarrow \quad (3y+2)^2 = \frac{9}{7}$$

$$\Rightarrow \quad 7(9y^2 + 12y + 4) = 9$$

$$\Rightarrow \quad 63y^2 + 84y + 19 = 0$$

Hence, proved.

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Q7.

Solution: $\cos\frac{\pi}{18} \,.\cos\frac{\pi}{6}.\cos\frac{5\pi}{18}.\cos\frac{7\pi}{18} = \frac{3}{16}$

We take;

$$L.H.S. = \cos\frac{\pi}{18}\,.\cos\frac{\pi}{6}.\cos\frac{5\pi}{18}.\cos\frac{7\pi}{18}$$

$$L.H.S. = \cos\frac{\pi}{6}.\frac{1}{2}\left(2\cos\frac{5\pi}{18}\,.\cos\frac{\pi}{18}\right).\cos\frac{7\pi}{18}$$

Where, $2cos\alpha\ cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$

$\therefore$

$$L.H.S. = \frac{\sqrt{3}}{4}\left(\cos\left(\frac{6\pi}{18}\right) + \cos\left(\frac{4\pi}{18}\right)\right).\cos\frac{7\pi}{18}$$

$$L.H.S. = \left(\frac{\sqrt{3}}{8} + \frac{\sqrt{3}}{4}\cos\left(\frac{2\pi}{9}\right)\right).\cos\frac{7\pi}{18}$$

$$L.H.S. = \frac{\sqrt{3}}{8}.\cos\frac{7\pi}{18} + \frac{\sqrt{3}}{8}\left(2\cos\left(\frac{2\pi}{9}\right).\cos\left(\frac{7\pi}{18}\right)\right)$$

$$L.H.S. = \frac{\sqrt{3}}{8}.\cos\frac{7\pi}{18} + \frac{\sqrt{3}}{8}\left(\cos\left(\frac{11\pi}{18}\right) + \cos\left(\frac{3\pi}{18}\right)\right)$$

$$L.H.S. = \frac{\sqrt{3}}{8}.\cos\frac{7\pi}{18} + \frac{\sqrt{3}}{8}.\cos\frac{11\pi}{18} + \frac{\sqrt{3}}{8}.\frac{\sqrt{3}}{2}$$

$$L.H.S. = \frac{\sqrt{3}}{8}\{\left(cos\frac{7\pi}{18}\right) + \left(cos\frac{11\pi}{18}\right)\} + \frac{3}{16}$$

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$$L.H.S. = \frac{\sqrt{3}}{8}\left(2\cos\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{9}\right)\right) + \frac{3}{16}$$

$$L.H.S. = \frac{3}{16} = R.H.S.$$

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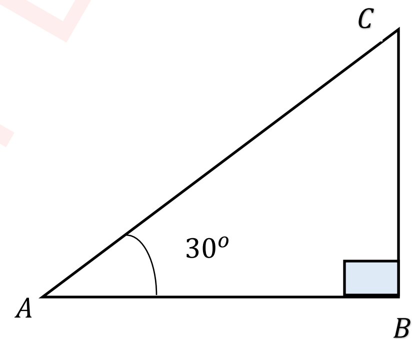
Q8.

Solution: ABC is a right-angled triangle with $m\angle B = 90^o$, $m\angle A = 30^o$ and $m\overline{AB} = 3cm$



(a) First, we calculate $m\angle C$ by

$$m\angle A + m\angle B + m\angle C = 180^o$$

$$\Rightarrow \quad 30^o + 90^o + m\angle C = 180^o$$

$$\Rightarrow \quad m\angle C = 180^o - 120^o$$

$$\Rightarrow \quad m\angle C = 60^o$$

Second, $cos30^o = \frac{3}{m\overline{AC}} \quad \Rightarrow \quad m\overline{AC} = 2\sqrt{3}cm$

Third, $sin30^o = \frac{m\overline{BC}}{2\sqrt{3}} \quad \Rightarrow \quad m\overline{BC} = \sqrt{3}cm$

(b) Area of triangle $(\Delta ABC) = \frac{1}{2}(m\overline{AB})(m\overline{BC})$

$$\Rightarrow \quad \Delta = \frac{1}{2}(3)(\sqrt{3})$$

$$\Rightarrow \quad \Delta = \frac{3\sqrt{3}}{2}\ cm^2$$

(c) Radius of circum-circle $(R) = \frac{abc}{4\Delta}$

$$\Rightarrow \quad R = \frac{(3)(2\sqrt{3})(\sqrt{3})}{4\left(\frac{3\sqrt{3}}{2}\right)} = \sqrt{3}cm$$

(d) Radius of in-circle $(r) = \frac{\Delta}{s}$

Where, $s = \frac{a+b+c}{2} = \frac{3+2\sqrt{3}+\sqrt{3}}{2} = \frac{3(1+\sqrt{3})}{2}$

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$$\Rightarrow \quad r = \frac{\left(\frac{3\sqrt{3}}{2}\right)}{\left(\frac{3(1+\sqrt{3})}{2}\right)}$$

$$\Rightarrow \quad r = \frac{\sqrt{3}}{1+\sqrt{3}}$$

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