A Problem book in

Astronomy and Astrophysics

Compilation of problems from

International Olympiad in Astronomy and Astrophysics (2007-2012)

Edited by: Aniket Sule

on behalf of IOAA international board

July 23, 2013

Original Problems by:
academic committees of IOAAs held at:

- Thailand (2007)
- Indonesia (2008)
- Iran (2009)
- China (2010)
- Poland (2011)
- Brazil (2012)

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Thank you!

Editor would like to thank International Board of International Olympiad on Astronomy and Astrophysics (IOAA) and particularly, president of the board, Prof. Chatief Kunjaya, and general secretary, Prof. Gregorz Stachowaski, for entrusting this task to him. We acknowledge the hard work done by respective year's problem setters drawn from the host countries in designing the problems and fact that all the host countries of IOAA graciously agreed to permit use of the problems, for this book. All the members of the international board IOAA are thanked for their support and suggestions. All IOAA participants are thanked for their ingenious solutions for the problems some of which you will see in this book.

A Note about Problems

You will find a code in bracket after each problem e.g. (I07 - T20 - C). The first number simply gives the year in which this problem was posed I07 means IOAA2007. Second one gives information about the test and question number. T stands for theory, D for data analysis, O for observation and G for group tasks. In theory numbers greater than 15 denote long questions. The last letter denotes difficulty level of the question as perceived by the editor. A being the simplest and D the most difficult.
The prefix "*EA*" before a line in a problem indicates that this was not part of the original question but its an addition by editor.

General Marking Principle

1.	Giving correct answer without detailed calculation	Deduct 50% of the marks for that part
2.	Minor mistakes in the calculations e.g. wrong signs, symbols, substitutions	Deduct 20% of the marks for that part
3.	Units missing from final answers	Deduct 10% of the marks for that part
4.	Too few or too many significant digits in the final answer	Deduct 10% of the marks for that part
5.	Using incorrect physical concept (despite correct answers)	**No points given**
6.	Error propagated from earlier parts: minor errors	Full points (i.e. no deductions)
7.	Error propagated from earlier parts: major errors	Deduct 20% of the marks for the final answer

Preface

International Olympiads are high school competitions where each country sends a team of students accompanied by mentors and the students compete individually in a set of tests and awarded medals based on their performance. Currently, such Olympiads exist for Physics, Chemistry, Biology, Mathematics, Astronomy and Astrophysics, Informatics, Earth Sciences, Linguistics etc. There are also special Olympiads like International Junior Science Olympiads aimed at middle school and early high school students. These International Olympiads have also given birth to smaller, regional events like Asian Physics Olympiad, Latin American Astronomy and Astrophysics Olympiad and European Girls' Mathematics Olympiad to name a few.

International Olympiad on Astronomy and Astrophysics (IOAA) started in 2007. We have had 6 Olympiads so far. They were hosted by Thailand (2007), Indonesia (2008), Iran (2009), China (2010), Poland (2011) and Brazil (2012). The next two Olympiads will be hosted by Greece (2013) and Romania (2014). Currently roughly 30 teams participate in IOAA. Each team consists of upto five students chosen through national level selection process in the respective country and it is accompanied by two mentors. There are three rounds of tests. Theoretical problems, data analysis problems and night sky observation test. Theoretical round typically has 15 short questions and 2-3 long questions to be solved in five hours. Data analysis usually has two problems to be solved in four hours and night sky observation round has 4-5 tasks with typical time upto 5 minutes for each task.

The scores for three rounds for each participant are totalled with 50% weightage to theory and 25% each for the other two. Post totalling, the scores are renormalised by the "average score of the top three participants". As per these renormalised scores, participants scoring more than 90%, 78%, 65% and 50% qualify for Gold medals, Silver Medals, Bronze Medals and Honourable Mention certificates respectively. Additional special certificates are awarded for outstanding performances in one or more tests. From 2011, there is an additional team competition, where all members of the team work together on one or more problems.

This compilation includes problems from the first six IOAAs. The problems in the respective years were originally designed by the respective host countries. A sketch of possible solution was also provided. Post-IOAA, these problems were also made available on the respective IOAA websites. This compilation, however, aspires to present the problem set in a pedagogically more useful way. The problems are divided as per concepts involved and also graded as per difficulty level. At some places, additional sub-questions are added to stimulate thought process. Additional notes are added in solutions to make the solutions self explanatory. IOAA statutes demand that original solutions should not involve calculus. However, some times same problem can be solved more elegantly using calculus. Sometimes, a problem can be solved in more than one elegant ways. In such cases, both solutions are included. In some cases numerical values from original problems are tweaked slightly to make the problems more realistic. If a problem involves multiple concepts, it is included in the chapter where last of the concepts involved is covered. Numerical values use SI system of units wherever applicable. It is implicitly expected that students use proper number of significant digits and at least have a rough estimate of amount of error in their answer. The problems and solutions remain property of IOAA and any reproduction in any language should carry proper acknowledgements.

The current syllabus of IOAA is appended at the start of this book. As one can see, except the "non-calculus based solutions" restriction, the syllabus more or less covers same range of topics as covered in any typical undergraduate astrophysics course in any university. Thus, we hope that this book can serve as a useful problem set for those courses too. Needless to say, potential Astronomy and Astrophysics Olympiad participants would find this compilation extremely handy for their preparation.

Happy solving!

Aniket Sule

Table of Constants

The Sun			
Mass	$M_\odot$	=	1.9891×10^{30} Kg
Radius	$R_\odot$	=	6.955×10^{8} m
Luminosity	$L_\odot$	=	3.826×10^{26} W
Apperant magnitude at mid-day	$m_\odot$	=	-26.72
Absolute V-band magnitude	$M_{V_\odot}$	=	4.82
Absolute bolometric magnitude	M_{bol}	=	4.72
Apperant angular diameter	$\theta_\odot$	=	30'
Temperature on the surface	$T_\odot$	=	5778 K
Solar Constant (at Earth)	S	=	1366 W/m^2
The Earth			
Mass	$M_\oplus$	=	5.9736×10^{24} Kg
Radius	$R_\oplus$	=	6.3708×10^{6} m
Mean density	$\rho_\oplus$	=	5515 Kg/m^3
Gravitational acceleration on the surface	g	=	9.81 m/s^2
Inclination of the axis	ϵ	=	$23°26'$
Albedo	$\alpha_\oplus$	=	0.39
The Moon			
Mass	M_M	=	7.4377×10^{22} Kg
Radius	R_M	=	1.7374×10^{6} m
Mean distance from Earth	d_M	=	3.78×10^{8} m
Synodic period	P_{sy}	=	29.5306 days
Apperant magnitude (full moon)	m_{moon}	=	-12.74
Albedo	α	=	0.14
Inclination of the lunar orbit w.r.t. the ecliptic		=	5.14°
The Venus			
Radius	R_V	=	0.949 $R_\oplus$
Orbital semi-major axis	a_{Venus}	=	0.732 A.U.
Orbital period	T_{Venus}	=	224.70 days

Albedo	$\alpha_\oplus$	=	0.87
The Mars			
Mass	M_{Mars}	=	$6.421 \times 10^{23} kg$
Radius	R_{Mars}	=	3393 km
Orbital semi-major axis	a_{Mars}	=	1.52 A.U.
Rotational period	T_{Mars}	=	24.623 h
Orbital radius of Phobos	a_{Ph}	=	9380 km
The Jupiter			
Mass	M_J	=	1.898×10^{27} Kg
Orbital semi-major axis	a_{Jup}	=	5.204 A.U.
Number of arcseconds in a rad.		=	206265"
1 sidereal day		=	$23^h\ 56^m\ 4^s.1$
1 tropical year		=	365.2564 solar days
		=	3.1557×10^7 s
1 sidereal year		=	365.2422 solar days
1 Astronomical Unit (A.U.)	$a_{\oplus-\odot}$	=	1.4960×10^{11} m
1 lightyear (ly)		=	9.46×10^{15} m
		=	6.324×10^4 A. U.
1 parsec (pc)		=	3.0856×10^{16} m
		=	3.262 ly
Distance to the galactic centre	d_{GC}	=	8.3 ± 0.3 kpc
Speed of Light	c	=	2.99792458×10^8 m/s
Universal Gravitational Constant	G	=	$6.6726 \times 10^{-11}\ Nm^2kg^{-2}$
Planck Constant	h	=	$6.62 \times 10^{-34}\ J \cdot s$
Hubble Constant	H_0	=	(67.80 ± 0.77) Km/s/Mpc
Age of the Universe	t_0	=	13.77×10^9 years
Stephan's Constant	σ	=	$5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$
Boltzmann Constant	κ_B	=	$1.38 \times 10^{-23}\ JK^{-2}$
Wien's displacement law	$\lambda_m T$	=	2.898×10^{-3} m·K
Charge of a electron	q_e	=	1.602×10^{-19} C
Mass of a electron	m_e	=	9.1×10^{-31} kg
Mass of a proton	m_p	=	938.27 MeV/c^2
Mass of a neutron	m_n	=	939.56 MeV/c^2
Mass of a Deuterium atom	m_D	=	1875.60 MeV/c^2
Mass of a Helium-3 atom	m_{He3}	=	2808.30 MeV/c^2
Mass of a Helium-4 atom	m_{He}	=	4.002603 a.m.u.
		=	3727.40 MeV/c^2
Mass of a Carbon atom	m_C	=	12.000000 a.m.u.
Rest frequency of spin-flip transition of hydrogen	ν_{21}	=	1420.41 MHz

Distance of Barnard's star	$d_{Barnarad}$	=	1.83 pc
Coordinates of northern ecliptic pole (J2000.0)	(α_E, δ_E)	=	$18^h\ 00^m\ 00^s$, +66° 33.6'
Coordinates of northern galactic pole (J2000.0)	(α_G, δ_G)	=	$12^h\ 51^m$, +27° 08'
Diameter of human pupil		=	6 mm
Height of Christ the Redeemer		=	39.60 m
Height of Eiffel Tower		=	324 m

Basic Equations of spherical trigonometry (for a spherical triangle ABC)

$$\sin a \sin B = \sin b \sin A \tag{1}$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \tag{2}$$

$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A \tag{3}$$

Contents

Chapter 1

Celestial Mechanics

1.1 Theory

1. A Sun-orbiting periodic comet is the farthest at 31.5 A.U. and the closest at 0.5 A.U. What is the orbital period of this comet? (I07 - T06 - A)
2. For the comet in question 1 above, what is the area (in square A.U. per year) swept by the line joining the comet and the Sun? (I07 - T07 - A)
3. Most single-appearance comets enter the inner Solar System directly from the Oort Cloud. Estimate how long it takes a comet to make this journey. Assume that in the Oort Cloud, 35 000 AU from the Sun, the comet was at aphelion. (I11 - T01 - A)
4. Estimate the radius of a planet that a man can escape its gravitation by jumping vertically. Assume density of the planet and the Earth are the same. (I09 - T03 - A)
5. A spacecraft landed on the surface of a spherical asteroid with negligible rotation, whose diameter is 2.2 km, and its average density is 2.2 g/cm^3. Can the astronaut complete a circle along the equator of the asteroid on foot within 2.2 hours? (I10 - T06 - A)
6. Estimate the mass of a globular cluster with the radius $R = 20$ pc and root mean square velocity of stars in the cluster is 3 km/s. (I09 - T08 - B)
7. Estimate the number of stars in a globular cluster of diameter 40 pc, if the escape velocity at the edge of the cluster is 6 km/s and most of the stars are similar to the Sun. (I11 - T02 - B)
8. On 9 March 2011 the Voyager probe was 116.406 AU from the Sun and moving at 17.062 km/s. Determine the type of orbit the probe is on:

(a) elliptical, (b) parabolic, or (c) hyperbolic.
What is the apparent magnitude of the Sun as seen from Voyager? (I11 - T03 - B)

9. Calculate the ratio between the average densities of the Earth and the Sun, using ONLY the dataset below:
   – the angular diameter of the Sun, as seen from Earth.
   – the gravitational acceleration on Earth's surface.
   – the length of the year.
   – the fact that one degree in latitude at Earth's surface corresponds to 111 km. (I12 - T05 - B)
10. Assuming that dust grains are black bodies, determine the diameter of a spherical dust grain which can remain at 1 AU from the Sun in equilibrium between the radiation pressure and gravitational attraction of the Sun. Take the density of the dust grain to be $\rho = 10^3$ kg m^{-3}. (I11 - T08 - B)
11. Tidal forces result in a torque on the Earth. Assuming that, during the last several hundred million years,both this torque and the length of the sidereal year were constant and had values of6.0×10^{16} Nm and 3.15×10^7 s respectively, calculate how many days there were in a year 6.0×10^8 years ago. (I11 - T06 - B)
   Note: Moment of inertia of a homogeneous filled sphere of radius R and mass m is
   $$I = \frac{2}{5} m R^2 \tag{1.1}$$
12. A spacecraft is due to make a close pass of a space object and Scientists would like to investigate the object more carefully using a telescope on-board the spacecraft. For simplicity, let us reduce the problem to two dimensions and assume that the position of the space craft is stationary in (0,0) and the shape of the object is a disk and its boundary has the equation
   $$x^2 + y^2 - 10x - 8y + 40 = 0 \tag{1.2}$$
   Find the exact values of maximum and minimum of $\tan \phi$ where ϕ is the elevation angle of the telescope with respect to the "horizontal" direction (x-axis) during investigation from one edge to the other edge. (I08 - T13 - C)
13. Consider a Potential Hazardous Object (PHO) moving in a closed orbit under the influence of the Earth's gravitational force. Let u be the inverse of the distance of the object from the Earth and p be the magnitude of its linear momentum. As the object travels through points

A and B, values of u and p are noted as shown in the following table. Find the mass and the total energy of the object, and sketch the shape of u curve as a function of p from A to B. (I08 - T14 - C)

	p ($\times 10^9 kgms^{-1}$)	u ($\times 10^{-8} m^{-1}$)
A	0.052	5.15
B	1.94	194.17

14. Derive a relation for the escape velocity of an object, launched from the center of a proto-star cloud. The cloud has uniform density with the mass M and radius R. Ignore collisions between the particles of the cloud and the launched object. You are given the fact that if the object were allowed to fall freely from the surface, it would reach the center with a velocity equal to $\sqrt{\frac{GM}{R}}$. (I09 - T06 - C)
15. Gravitational forces of the Sun and the Moon lead to raising and lowering of sea water surfaces. Let ϕ be the difference in longitude between points A and B, where both points are at the equator and A is on the sea surface. Derive the horizontal acceleration of sea water at position A due to Moon's gravitational force at the time when the Moon is above point B according to observers on the Earth (express it in ϕ, the radius R of Earth, and the Earth-Moon distance r). (I08 - T08 - C)
16. **High Altitude Projectile:** A projectile which starts from the surface of the Earth at the sea level is launched with the initial speed of $v_0 = \sqrt{\frac{GM_\oplus}{R_\oplus}}$ and with the projecting angle (with respect to the local horizon) of $\theta = \frac{\pi}{6}$. Ignore the air resistance and rotation of the Earth.
    (a) Show that the orbit of the projectile is an ellipse with a semi-major axis of $a = R_\oplus$.
    (b) Calculate the highest altitude of the projectile with respect to the Earth surface (in the unit of $R_\oplus$).
    (c) What is the range of the projectile (surface distance between launching point and falling point) in the units of $R_\oplus$?
    (d) What is eccentricity (e) of this elliptical orbit?
    (e) Find the time of flight for the projectile. (I09 - T16 - C)
17. A spacecraft is launched from the Earth and it is quickly accelerated to its maximum velocity in the direction of the heliocentric orbit of the Earth, such that its orbit is a parabola with the Sun at its focus point, and grazes the Earth orbit. Take the orbit of the Earth and Mars as

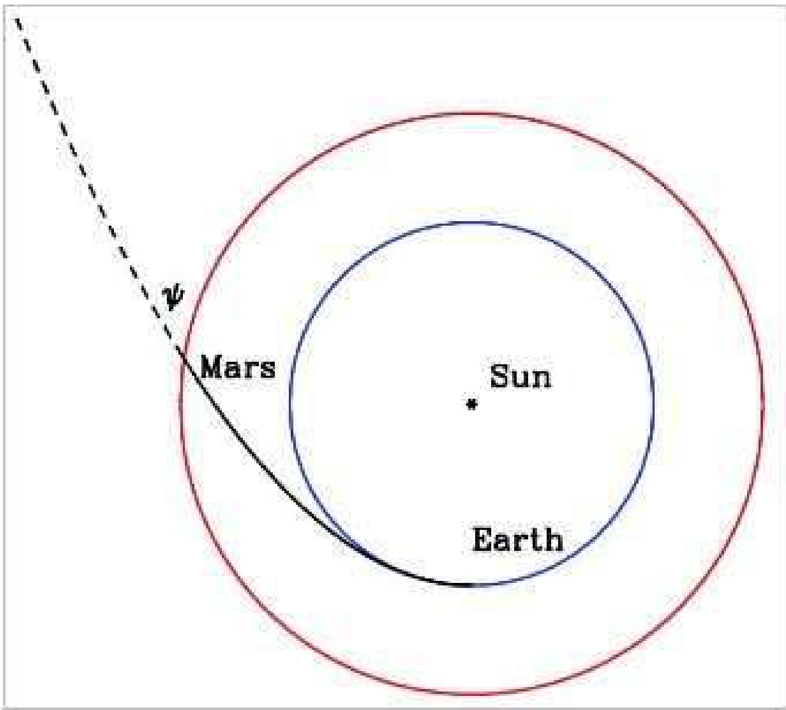


Figure 1.1 – The trajectory of the spacecraft (not in scale, seen from the north ecliptic pole). The inner circle is the orbit of the Earth, the outer circle is the orbit of Mars.

circles on the same plane. Make the following approximation: during most of the flight only the gravity from the Sun needs to be considered, but during the brief encounter with a planet, only the gravity of the planet needs to be considered. (I10 - T16 - D)

(a) What is the angle (ψ) between the path of the spacecraft and the orbit of the Mars (see Figure 1.1) as it crosses the orbit of the Mars, without considering the gravity effect of the Mars?

(b) Suppose the Mars happens to be very close to the crossing point at the time of the crossing, from the point of view of an observer on Mars, what is the approaching velocity and direction of approach (with respect to the Sun) of the spacecraft before it is significantly affected by the gravity of the Mars?

18. A satellite orbits the Earth on a circular orbit. The initial momentum of the satellite is given by the vector $\overrightarrow{p}$. At a certain time, an explosive charge is set off which gives the satellite an additional impulse $\Delta\overrightarrow{p}$, equal in magnitude to $|\overrightarrow{p}|$.

Let α be the angle between the vectors $\overrightarrow{p}$ and $\Delta\overrightarrow{p}$,and β between the radius vector of the satellite and the vector $\Delta\overrightarrow{p}$.
By thinking about the direction of the additional impulse $\Delta\overrightarrow{p}$, consider if it is possible to change the orbit to each of the cases given below. If it is possible and give values of α and β for which it is possible. If the orbit is not possible, mark NO.

(a) a hyperbola with perigee at the location of the explosion.

(b) a parabola with perigee at the location of the explosion.

(c) an ellipse with perigee at the location of the explosion.

(d) a circle.

(e) an ellipse with apogee at the location of the explosion.

Note that for $\alpha = 180°$ and $\beta = 90°$ the new orbit will be a line along which the satellite will free fall vertically towards the centre of the Earth. (I11 - T07 - D)

1.2 Data Analysis

1. **Galilean moons:** Computer simulation of the planet Jupiter and its 4 Galilean moons is shown on the screen similar to the view you may see through a small telescope. After observing the movement of the moons, please identify the names of the moons that appear at the end of the simulation. (Simulation was played on screen during the first fifteen minutes and the last fifteen minutes of the exam). (I07 - D01 - A)

Chapter 2

Celestial Coordinate Systems

2.1 Theory

1. Two persons, on the equator of the Earth separated by 180° in longitude, observe the Moon's position with respect to the background star field at the same time. If the declination of the Moon is zero, sketch the situation and calculate the difference in apparent right ascension seen by those two persons. (I08 - T01 - A)
2. The time interval between noon of 1 July and noon of 31 December is 183 solar days. What is this interval in sidereal days? (I07 - T03 - A)
3. In a typical Persian architecture, on top of south side windows there is a structure called "Tabeshband" (shader), which controls sunlight in summer and winter. In summer when the Sun is high, Tabeshband prevents sunlight to enter rooms and keeps inside cooler. In the modern architecture it is verified that the Tabeshband saves about 20% of energy cost. Figure 2.1 shows a vertical section of this design at latitude of 36°.0 N with window and Tabeshband. Using the parameters given in the figure, calculate the maximum width of the Tabeshband (x), and maximum height of the window (h) in such a way that:
   (a) No direct sunlight can enter to the room in the summer solstice at noon.
   (b) The direct sunlight reaches the end of the room (indicated by the point A in the figure) in the winter solstice at noon. (I09 - T04 - A)
4. The Damavand Mountain is located at the North part of Iran, in south coast of Caspian Sea. Consider an observer standing on the Damavand mountaintop (latitude = $35°57'$ N; longitude = $52°6'$ E; altitude

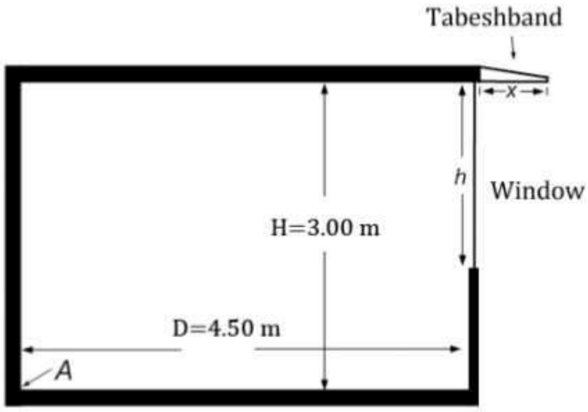


Figure 2.1 – Tabeshband

5.6×10^3 m from the mean sea level) and looking at the sky over the Caspian Sea. What is the minimum declination for a star, to be seen marginally circumpolar for this observer. Surface level of the Caspian Sea is approximately equal to the mean sea level. (I09 - T05 - A)

5. For an observer at latitude 42.5°N and longitude 71°W, estimate the time of sun rise on 21 December if the observer's civil time is -5 hours from GMT. Ignore refraction of the atmosphere, the size of the solar disc. (I07 - T01 - B)
   EA: How much is the error (in minutes) introduced by the atmospheric refraction, size of the solar disc as well as equation of time?

6. At Brazil's National Observatory, located at the city of Rio de Janeiro (22°54′ S, 43°12′ W), there is a sundial above the door of the dome of the 32cm telescope, facing to the north. The dial lies on the plane East-Zenith-West and the rod is parallel to the Earth's axis. For which declinations of the sun and during what period of the year (months and seasons) the clock (i) does not work during, at least, some fraction of the day? and (ii) does not work at all during the day? (I12 - T01 - B)

7. A full moon occurred on June 19, 2008 at 00^h30^m West Indonesian Time (local civil time for western part of Indonesia with reference geographic longitude as 105° E). Calculate the extreme possible durations of the Moon above the horizon for observers at Bosscha Observatory (longitude: 107° 35' 00".0 E, latitude: 6° 49' 00".0 S, Elevation: 1300.0 m, Time zone = UT $+7^h30^m$). (I08 - T03 - B)

8. A student tries to measure field of view (FOV) of the eyepiece of his/her telescope, using rotation of the Earth. To do this job, the observer

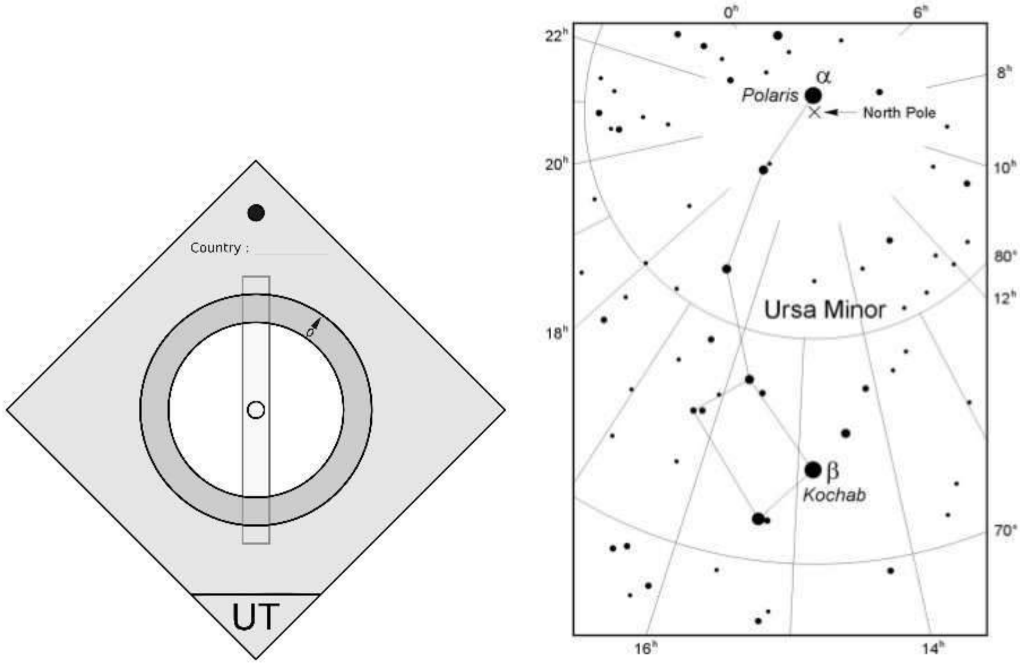


Figure 2.2 – Sky Map of Ursa Minor and Clock Card

points the telescope towards Vega (α Lyr., RA: 18.5^h , Dec: $+39°$), turns off its "clock drive" and measures trace out time to be $t = 5.3$ minutes. Trace out time is time taken by a star to cross the full diameter of the FOV. What is the FOV of this telescope in arc-minutes? (I09 - T07 - B)

9. **Nocturnal:** Circumpolar stars describe a full circle around the Celestial Pole over 24 hours. This can be used to make a simple clock. You are given a blank card with a movable ring, along with a clear strip with a central circle.

   If the card has a suitable scale, the clear strip is attached as in the diagram 2.2 (left) and the Pole Star is visible through the central circle, then the position of the star Kochab (β UMi) on the inner edge of the ring will give the current time.

   Design and mark on the card and ring suitable inner and outer scales (as required) such that, in Katowice, Poland for any night of the year, the side of the clock marked "UT" can be used to show current Universal Time, and the other side (marked "ST") can be used independently to show current Local Sidereal Time. For 27 August in Katowice, the lower culmination of Kochab is at 05:15 Central European Summer Time (UT+2). The coordinates of Kochab (β UMi) are : α : 14^h51^m,

$\delta : +74.2°$. ((I11 - T05 - C))

Notes:

- The blank card is marked with a line which should be held horizontally when the device is used.
- The clear strip will be attached later, after marking of the scale is finished.

10. The equation of the ecliptic in equatorial coordinates (α, δ) has the form:

$$\delta = arctan(\sin \alpha \tan \epsilon) \quad (2.1)$$

where ϵ is the angle of the celestial equator to the ecliptic plane.
Find an analogous relation h = $f(A)$ for the galactic equator in horizontal coordinates (A,h) for an observer at latitude $\varphi = 49°34'$ at local sidereal time $\theta = 0^h51^m$. (I11 - T13 - C)

11. Due to the precession of the Earth's axis, the region of sky visible from a location with fixed geographical coordinates changes with time. Is it possible that, at some point in time, Sirius will not rise as seen from Krakow, while Canopus will rise and set? (I11 - T12 - C)
Assume that the Earth's axis traces out a cone of angle 47°. Krakow is at latitude 50.1° N; the current equatorial coordinates (right ascension and declination) of these stars are:

Sirius	(α CMa)	:	6^h45^m,	$-16°43'$
Canopus	(α Car)	:	6^h24^m,	$-52°42'$

12. Christ, the Redeemer is the most famous Brazilian monument. But there are many similar statues in other Brazilian cities and across the world. Imagine that an exact copy of the monument was built on Borradaile Island, at latitude $\varphi = -66.55°$, the first place south of the Antarctic Circle reached by man.
Assume the island is exactly on the Antarctic Circle, and define a Cartesian coordinate system (O_{xy}) on the horizontal plane, with the origin O being at the base of the Christ, the O_x axis in the East-West direction and the O_y axis in the North-South direction. Determine the equation of the curve described by the tip of the Christ's head shadow on the horizontal plane, on a sunny solstice day and the minimum length of the shadow during that day (neglect the motion of the sun in declination during the day). Neglect the atmospheric effects. (I12 - T15 - C)

13. An observer in Salonika ($\varphi = +40.65°$), Greece, quietly contemplates the starry sky when he realizes that a very bright object ($\alpha = 5^h55^{min}, \delta =$

+7.41°, m = 0.45), when reaching its upper culmination, mysteriously detaches from the celestial sphere and continues moving at the same tangential speed, remaining in this movement for all eternity, Assume that the Earth stands still and the celestial sphere rotates. Then, determine the final alt-azimuthal coordinates of the object. How long will it take for its apparent magnitude to change to 6.00? (I12 - T14 - D)

14. An astronomer in the southern hemisphere contemplates the rise of the south ecliptic pole and wonders how fun it would be if the sky started spinning around the ecliptic pole, instead of the usual celestial pole.

    Sketch the displacement of this observer over the Earth's surface, to observe the stars revolving around south ecliptic pole in the same direction and with the same period that they usually revolve around the south celestial pole. Sketch the observer's trajectory for one entire day. Determine its velocity (direction and speed) when crossing the Equator for the first time. (I12 - T13 - D)

2.2 Data Analysis

1. **Solar System objects:** A set of data containing the apparent positions of 4 Solar System objects over a period of 1 calendar year is given in Table 2.1. Latitude of observing location was N18°47′00.0″ and longitude was E98°59′00.0″.
   (a) Identify objects A, B, C and D.
   (b) During the period of observation, which object could be observed for the longest duration at night time?
   (c) What was the date corresponding to the situation in 1b?
   (d) Assuming the orbits are coplanar (lie on the same plane) and circular, indicate the positions of the four objects and the Earth on the date in 1c, in the orbit diagram provided. The answer must show one of the objects as the Sun at the centre of the Solar System. Other objects including the Earth must be specified together with the correct values of elongation on that date. (I07 - D03 - D)

Date		Object A							Object B						
		R.A.			Dec.				R.A.			Dec.			
		h	m	s		°	′	″	h	m	s		°	′	″
Jan	1	18	44	7.11	-	23	3	11.92	9	18	19.95	+	16	30	33.13
	2	18	48	32.14	-	22	58	18.94	9	17	59.44	+	16	32	19.98
	3	18	52	56.86	-	22	52	58.47	9	17	38.28	+	16	34	9.51
	4	18	57	21.21	-	22	47	10.65	9	17	16.5	+	16	36	1.63
	5	19	1	45.16	-	22	40	55.67	9	16	54.11	+	16	37	56.27
	6	19	6	8.68	-	22	34	13.69	9	16	31.11	+	16	39	53.34
	7	19	10	31.74	-	22	27	4.91	9	16	7.53	+	16	41	52.72
	8	19	14	54.3	-	22	19	29.54	9	15	43.38	+	16	43	54.33
	9	19	19	16.35	-	22	11	27.82	9	15	18.68	+	16	45	58.07
	10	19	23	37.84	-	22	2	59.97	9	14	53.45	+	16	48	3.82
	11	19	27	58.77	-	21	54	6.25	9	14	27.71	+	16	50	11.49
	12	19	32	19.11	-	21	44	46.94	9	14	1.47	+	16	52	20.98
	13	19	36	38.84	-	21	35	2.3	9	13	34.75	+	16	54	32.19
	14	19	40	57.93	-	21	24	52.64	9	13	7.57	+	16	56	45.02
	15	19	45	16.37	-	21	14	18.25	9	12	39.96	+	16	58	59.37
	16	19	49	34.14	-	21	3	19.44	9	12	11.92	+	17	1	15.15
	17	19	53	51.23	-	20	51	56.52	9	11	43.49	+	17	3	32.27
	18	19	58	7.61	-	20	40	9.83	9	11	14.66	+	17	5	50.62
	19	20	2	23.28	-	20	27	59.67	9	10	45.47	+	17	8	10.11
	20	20	6	38.23	-	20	15	26.38	9	10	15.93	+	17	10	30.64
	21	20	10	52.44	-	20	2	30.27	9	9	46.06	+	17	12	52.1
	22	20	15	5.91	-	19	49	11.67	9	9	15.88	+	17	15	14.38
	23	20	19	18.63	-	19	35	30.9	9	8	45.41	+	17	17	37.36
	24	20	23	30.6	-	19	21	28.3	9	8	14.68	+	17	20	0.93
	25	20	27	41.8	-	19	7	4.21	9	7	43.7	+	17	22	24.96
	26	20	31	52.24	-	18	52	18.99	9	7	12.51	+	17	24	49.36
	27	20	36	1.91	-	18	37	13.01	9	6	41.12	+	17	27	14.01
	28	20	40	10.79	-	18	21	46.66	9	6	9.57	+	17	29	38.82
	29	20	44	18.89	-	18	6	0.34	9	5	37.86	+	17	32	3.69
	30	20	48	26.19	-	17	49	54.45	9	5	6.03	+	17	34	28.52
	31	20	52	32.68	-	17	33	29.39	9	4	34.1	+	17	36	53.21
Feb	1	20	56	38.36	-	17	16	45.58	9	4	2.1	+	17	39	17.64
	2	21	0	43.22	-	16	59	43.43	9	3	30.03	+	17	41	41.7
	3	21	4	47.25	-	16	42	23.36	9	2	57.94	+	17	44	5.28
	4	21	8	50.46	-	16	24	45.77	9	2	25.85	+	17	46	28.25
	5	21	12	52.84	-	16	6	51.09	9	1	53.78	+	17	48	50.51
	6	21	16	54.4	-	15	48	39.75	9	1	21.76	+	17	51	11.93
	7	21	20	55.14	-	15	30	12.16	9	0	49.82	+	17	53	32.4
	8	21	24	55.06	-	15	11	28.75	9	0	17.97	+	17	55	51.84
	9	21	28	54.18	-	14	52	29.96	8	59	46.26	+	17	58	10.13
	10	21	32	52.49	-	14	33	16.2	8	59	14.69	+	18	0	27.18
	11	21	36	50	-	14	13	47.91	8	58	43.29	+	18	2	42.91
	12	21	40	46.73	-	13	54	5.52	8	58	12.1	+	18	4	57.23
	13	21	44	42.68	-	13	34	9.45	8	57	41.12	+	18	7	10.05
	14	21	48	37.86	-	13	14	0.13	8	57	10.38	+	18	9	21.31
	15	21	52	32.29	-	12	53	37.99	8	56	39.9	+	18	11	30.93
	16	21	56	25.98	-	12	33	3.42	8	56	9.7	+	18	13	38.84
	17	22	0	18.94	-	12	12	16.82	8	55	39.79	+	18	15	44.94
	18	22	4	11.2	-	11	51	18.6	8	55	10.21	+	18	17	49.18
	19	22	8	2.76	-	11	30	9.12	8	54	40.97	+	18	19	51.45
	20	22	11	53.65	-	11	8	48.75	8	54	12.08	+	18	21	51.69
	21	22	15	43.89	-	10	47	17.87	8	53	43.58	+	18	23	49.81
	22	22	19	33.5	-	10	25	36.85	8	53	15.48	+	18	25	45.76
	23	22	23	22.49	-	10	3	46.09	8	52	47.81	+	18	27	39.46
	24	22	27	10.89	-	9	41	45.98	8	52	20.58	+	18	29	30.88
	25	22	30	58.7	-	9	19	36.93	8	51	53.82	+	18	31	19.96
	26	22	34	45.95	-	8	57	19.35	8	51	27.53	+	18	33	6.66
	27	22	38	32.64	-	8	34	53.67	8	51	1.75	+	18	34	50.93
	28	22	42	18.78	-	8	12	20.29	8	50	36.47	+	18	36	32.71
Mar	1	22	46	4.4	-	7	49	39.63	8	50	11.73	+	18	38	11.96
	2	22	49	49.5	-	7	26	52.12	8	49	47.54	+	18	39	48.61
	3	22	53	34.1	-	7	3	58.15	8	49	23.91	+	18	41	22.6
	4	22	57	18.21	-	6	40	58.15	8	49	0.86	+	18	42	53.9
	5	23	1	1.85	-	6	17	52.52	8	48	38.41	+	18	44	22.44
	6	23	4	45.05	-	5	54	41.66	8	48	16.58	+	18	45	48.18
	7	23	8	27.8	-	5	31	26	8	47	55.38	+	18	47	11.08
	8	23	12	10.14	-	5	8	5.92	8	47	34.81	+	18	48	31.12
	9	23	15	52.09	-	4	44	41.83	8	47	14.91	+	18	49	48.26
	10	23	19	33.65	-	4	21	14.14	8	46	55.67	+	18	51	2.48
	11	23	23	14.85	-	3	57	43.23	8	46	37.11	+	18	52	13.76
	12	23	26	55.71	-	3	34	9.51	8	46	19.24	+	18	53	22.09
	13	23	30	36.25	-	3	10	33.37	8	46	2.06	+	18	54	27.45
	14	23	34	16.48	-	2	46	55.19	8	45	45.59	+	18	55	29.83
	15	23	37	56.43	-	2	23	15.36	8	45	29.83	+	18	56	29.24
	16	23	41	36.11	-	1	59	34.24	8	45	14.78	+	18	57	25.64
	17	23	45	15.56	-	1	35	52.2	8	45	0.46	+	18	58	19.04
	18	23	48	54.79	-	1	12	9.57	8	44	46.86	+	18	59	9.41
	19	23	52	33.84	-	0	48	26.7	8	44	33.99	+	18	59	56.73
	20	23	56	12.72	-	0	24	43.88	8	44	21.87	+	19	0	40.98
	21	23	59	51.47	-	0	1	1.44	8	44	10.49	+	19	1	22.16
	22	0	3	30.11	+	0	22	40.3	8	43	59.87	+	19	2	0.26
	23	0	7	8.67	+	0	46	21.01	8	43	50.01	+	19	2	35.3
	24	0	10	47.16	+	1	10	0.35	8	43	40.91	+	19	3	7.28
	25	0	14	25.62	+	1	33	37.96	8	43	32.58	+	19	3	36.21
	26	0	18	4.04	+	1	57	13.49	8	43	25.02	+	19	4	2.11
	27	0	21	42.46	+	2	20	46.55	8	43	18.22	+	19	4	24.97
	28	0	25	20.89	+	2	44	16.79	8	43	12.2	+	19	4	44.79
	29	0	28	59.35	+	3	7	43.84	8	43	6.96	+	19	5	1.56
	30	0	32	37.85	+	3	31	7.33	8	43	2.49	+	19	5	15.27
	31	0	36	16.42	+	3	54	26.88	8	42	58.8	+	19	5	25.92
Apr	1	0	39	55.06	+	4	17	42.14	8	42	55.89	+	19	5	33.51
	2	0	43	33.8	+	4	40	52.73	8	42	53.76	+	19	5	38.03
	3	0	47	12.66	+	5	3	58.29	8	42	52.42	+	19	5	39.49
	4	0	50	51.65	+	5	26	58.45	8	42	51.87	+	19	5	37.89

Date		Object C							Object D						
		R.A.			Dec.				R.A.			Dec.			
		h	m	s		°	′	″	h	m	s		°	′	″
Jan	1	15	28	49.55	-	15	14	51.76	15	9	15.05	-	16	57	57.2
	2	15	32	37.4	-	15	27	53.69	15	11	50.45	-	17	8	47.63
	3	15	36	27.82	-	15	40	56.32	15	14	26.14	-	17	19	30.7
	4	15	40	20.75	-	15	53	58.28	15	17	2.13	-	17	30	6.3
	5	15	44	16.12	-	16	6	58.27	15	19	38.41	-	17	40	34.32
	6	15	48	13.9	-	16	19	55	15	22	14.98	-	17	50	54.66
	7	15	52	14.02	-	16	32	47.23	15	24	51.85	-	18	1	7.23
	8	15	56	16.44	-	16	45	33.75	15	27	29.01	-	18	11	11.93
	9	16	0	21.12	-	16	58	13.38	15	30	6.46	-	18	21	8.68
	10	16	4	28.02	-	17	10	44.96	15	32	44.21	-	18	30	57.4
	11	16	8	37.09	-	17	23	7.37	15	35	22.25	-	18	40	38.02
	12	16	12	48.29	-	17	35	19.51	15	38	0.58	-	18	50	10.46
	13	16	17	1.59	-	17	47	20.28	15	40	39.21	-	18	59	34.64
	14	16	21	16.94	-	17	59	8.65	15	43	18.13	-	19	8	50.51
	15	16	25	34.31	-	18	10	43.57	15	45	57.34	-	19	17	57.99
	16	16	29	53.66	-	18	22	4.02	15	48	36.85	-	19	26	57.02
	17	16	34	14.95	-	18	33	9.01	15	51	16.65	-	19	35	47.51
	18	16	38	38.13	-	18	43	57.56	15	53	56.73	-	19	44	29.41
	19	16	43	3.17	-	18	54	28.7	15	56	37.11	-	19	53	2.63
	20	16	47	30.03	-	19	4	41.47	15	59	17.76	-	20	1	27.09
	21	16	51	58.65	-	19	14	34.95	16	1	58.7	-	20	9	42.71
	22	16	56	29.01	-	19	24	8.21	16	4	39.91	-	20	17	49.4
	23	17	1	1.06	-	19	33	20.35	16	7	21.4	-	20	25	47.08
	24	17	5	34.74	-	19	42	10.51	16	10	3.16	-	20	33	35.67
	25	17	10	10.02	-	19	50	37.83	16	12	45.19	-	20	41	15.1
	26	17	14	46.85	-	19	58	41.5	16	15	27.49	-	20	48	45.31
	27	17	19	25.16	-	20	6	20.74	16	18	10.04	-	20	56	6.22
	28	17	24	4.91	-	20	13	34.8	16	20	52.84	-	21	3	17.77
	29	17	28	46.04	-	20	20	22.93	16	23	35.88	-	21	10	19.89
	30	17	33	28.49	-	20	26	44.44	16	26	19.15	-	21	17	12.5
	31	17	38	12.21	-	20	32	38.66	16	29	2.65	-	21	23	55.54
Feb	1	17	42	57.13	-	20	38	4.93	16	31	46.35	-	21	30	28.92
	2	17	47	43.19	-	20	43	2.65	16	34	30.26	-	21	36	52.57
	3	17	52	30.35	-	20	47	31.23	16	37	14.36	-	21	43	6.44
	4	17	57	18.54	-	20	51	30.12	16	39	58.66	-	21	49	10.46
	5	18	2	7.72	-	20	54	58.81	16	42	43.13	-	21	55	4.57
	6	18	6	57.82	-	20	57	56.81	16	45	27.79	-	22	0	48.74
	7	18	11	48.8	-	21	0	23.68	16	48	12.63	-	22	6	22.92
	8	18	16	40.61	-	21	2	19	16	50	57.63	-	22	11	47.07
	9	18	21	33.18	-	21	3	42.39	16	53	42.8	-	22	17	1.17
	10	18	26	26.46	-	21	4	33.49	16	56	28.13	-	22	22	5.19
	11	18	31	20.41	-	21	4	51.99	16	59	13.62	-	22	26	59.09
	12	18	36	14.96	-	21	4	37.58	17	1	59.26	-	22	31	42.86
	13	18	41	10.07	-	21	3	49.99	17	4	45.04	-	22	36	16.48
	14	18	46	5.67	-	21	2	29.01	17	7	30.95	-	22	40	39.92
	15	18	51	1.73	-	21	0	34.4	17	10	17	-	22	44	53.15
	16	18	55	58.17	-	20	58	5.99	17	13	3.17	-	22	48	56.15
	17	19	0	54.95	-	20	55	3.6	17	15	49.46	-	22	52	48.88
	18	19	5	52.02	-	20	51	27.1	17	18	35.86	-	22	56	31.32
	19	19	10	49.33	-	20	47	16.36	17	21	22.36	-	23	0	3.42
	20	19	15	46.83	-	20	42	31.31	17	24	8.96	-	23	3	25.17
	21	19	20	44.46	-	20	37	11.89	17	26	55.66	-	23	6	36.54
	22	19	25	42.17	-	20	31	18.08	17	29	42.44	-	23	9	37.52
	23	19	30	39.93	-	20	24	49.93	17	32	29.29	-	23	12	28.11
	24	19	35	37.65	-	20	17	47.49	17	35	16.21	-	23	15	8.29
	25	19	40	35.31	-	20	10	10.87	17	38	3.17	-	23	17	38.08
	26	19	45	32.84	-	20	2	0.17	17	40	50.17	-	23	19	57.46
	27	19	50	30.18	-	19	53	15.57	17	43	37.18	-	23	22	6.42
	28	19	55	27.29	-	19	43	57.23	17	46	24.21	-	23	24	4.97
Mar	1	20	0	24.12	-	19	34	5.36	17	49	11.22	-	23	25	53.1
	2	20	5	20.63	-	19	23	40.19	17	51	58.21	-	23	27	30.81
	3	20	10	16.76	-	19	12	41.98	17	54	45.18	-	23	28	58.1
	4	20	15	12.49	-	19	1	10.99	17	57	32.1	-	23	30	14.99
	5	20	20	7.76	-	18	49	7.55	18	0	18.98	-	23	31	21.49
	6	20	25	2.56	-	18	36	31.97	18	3	5.79	-	23	32	17.62
	7	20	29	56.84	-	18	23	24.62	18	5	52.54	-	23	33	3.42
	8	20	34	50.58	-	18	9	45.85	18	8	39.21	-	23	33	38.92
	9	20	39	43.75	-	17	55	36.07	18	11	25.8	-	23	34	4.16
	10	20	44	36.31	-	17	40	55.69	18	14	12.3	-	23	34	19.18
	11	20	49	28.26	-	17	25	45.14	18	16	58.7	-	23	34	24.04
	12	20	54	19.56	-	17	10	4.87	18	19	44.98	-	23	34	18.77
	13	20	59	10.2	-	16	53	55.33	18	22	31.16	-	23	34	3.43
	14	21	4	0.16	-	16	37	17.01	18	25	17.21	-	23	33	38.06
	15	21	8	49.43	-	16	20	10.37	18	28	3.12	-	23	33	2.71
	16	21	13	37.99	-	16	2	35.92	18	30	48.89	-	23	32	17.43
	17	21	18	25.83	-	15	44	34.15	18	33	34.52	-	23	31	22.25
	18	21	23	12.94	-	15	26	5.54	18	36	19.99	-	23	30	17.22
	19	21	27	59.33	-	15	7	10.62	18	39	5.3	-	23	29	2.37
	20	21	32	44.99	-	14	47	49.88	18	41	50.44	-	23	27	37.75
	21	21	37	29.92	-	14	28	3.87	18	44	35.4	-	23	26	3.42
	22	21	42	14.12	-	14	7	53.13	18	47	20.18	-	23	24	19.45
	23	21	46	57.6	-	13	47	18.24	18	50	4.77	-	23	22	25.91
	24	21	51	40.34	-	13	26	19.81	18	52	49.14	-	23	20	22.9
	25	21	56	22.35	-	13	4	58.42	18	55	33.28	-	23	18	10.49
	26	22	1	3.63	-	12	43	14.71	18	58	17.19	-	23	15	48.76
	27	22	5	44.17	-	12	21	9.3	19	1	0.83	-	23	13	17.8
	28	22	10	24	-	11	58	42.81	19	3	44.19	-	23	10	37.68
	29	22	15	3.1	-	11	35	55.88	19	6	27.27	-	23	7	48.5
	30	22	19	41.49	-	11	12	49.15	19	9	10.05	-	23	4	50.33
	31	22	24	19.18	-	10	49	23.26	19	11	52.52	-	23	1	43.26
Apr	1	22	28	56.18	-	10	25	38.84	19	14	34.66	-	22	58	27.4
	2	22	33	32.51	-	10	1	36.56	19	17	16.48	-	22	55	2.85
	3	22	38	8.18	-	9	37	17.06	19	19	57.94	-	22	51	29.71
	4	22	42	43.21	-	9	12	41	19	22	39.06	-	22	47	48.1

Date		Object A							Object B							Object C							Object D						
		R.A.			Dec.				R.A.			Dec.				R.A.			Dec.				R.A.			Dec.			
		h	m	s		°	'	"	h	m	s		°	'	"	h	m	s		°	'	"	h	m	s		°	'	"
	5	0	54	30.79	+	5	49	52.85	8	42	52.09	+	19	5	33.23	22	47	17.62	-	8	47	49.03	19	25	19.82	-	22	43	58.13
	6	0	58	10.09	+	6	12	41.12	8	42	53.1	+	19	5	25.55	22	51	51.42	-	8	22	41.81	19	28	0.21	-	22	39	59.92
	7	1	1	49.58	+	6	35	22.91	8	42	54.89	+	19	5	14.84	22	56	24.65	-	7	57	20.01	19	30	40.22	-	22	35	53.61
	8	1	5	29.26	+	6	57	57.84	8	42	57.46	+	19	5	1.14	23	0	57.33	-	7	31	44.3	19	33	19.85	-	22	31	39.3
	9	1	9	9.15	+	7	20	25.56	8	43	0.8	+	19	4	44.45	23	5	29.47	-	7	5	55.32	19	35	59.08	-	22	27	17.12
	10	1	12	49.27	+	7	42	45.71	8	43	4.91	+	19	4	24.8	23	10	1.1	-	6	39	53.75	19	38	37.92	-	22	22	47.21
	11	1	16	29.64	+	8	4	57.93	8	43	9.78	+	19	4	2.21	23	14	32.25	-	6	13	40.24	19	41	16.34	-	22	18	9.68
	12	1	20	10.26	+	8	27	1.87	8	43	15.41	+	19	3	36.69	23	19	2.94	-	5	47	15.46	19	43	54.35	-	22	13	24.66
	13	1	23	51.15	+	8	48	57.18	8	43	21.79	+	19	3	8.26	23	23	33.21	-	5	20	40.04	19	46	31.94	-	22	8	32.25
	14	1	27	32.34	+	9	10	43.54	8	43	28.92	+	19	2	36.92	23	28	3.08	-	4	53	54.64	19	49	9.1	-	22	3	32.58
	15	1	31	13.84	+	9	32	20.62	8	43	36.79	+	19	2	2.68	23	32	32.59	-	4	26	59.89	19	51	45.83	-	21	58	25.75
	16	1	34	55.67	+	9	53	48.12	8	43	45.39	+	19	1	25.55	23	37	1.77	-	3	59	56.4	19	54	22.12	-	21	53	11.88
	17	1	38	37.86	+	10	15	5.74	8	43	54.74	+	19	0	45.53	23	41	30.67	-	3	32	44.81	19	56	57.97	-	21	47	51.08
	18	1	42	20.43	+	10	36	13.21	8	44	4.81	+	19	0	2.63	23	45	59.3	-	3	5	25.74	19	59	33.37	-	21	42	23.48
	19	1	46	3.4	+	10	57	10.23	8	44	15.61	+	18	59	16.88	23	50	27.72	-	2	37	59.82	20	2	8.32	-	21	36	49.23
	20	1	49	46.78	+	11	17	56.52	8	44	27.13	+	18	58	28.31	23	54	55.96	-	2	10	27.7	20	4	42.8	-	21	31	8.5
	21	1	53	30.59	+	11	38	31.75	8	44	39.37	+	18	57	36.95	23	59	24.03	-	1	42	50.04	20	7	16.8	-	21	25	21.46
	22	1	57	14.85	+	11	58	55.62	8	44	52.32	+	18	56	42.83	0	3	51.99	-	1	15	7.53	20	9	50.29	-	21	19	28.26
	23	2	0	59.56	+	12	19	7.78	8	45	5.97	+	18	55	45.95	0	8	19.85	-	0	47	20.83	20	12	23.27	-	21	13	29.09
	24	2	4	44.74	+	12	39	7.91	8	45	20.32	+	18	54	46.34	0	12	47.66	-	0	19	30.64	20	14	55.72	-	21	7	24.09
	25	2	8	30.4	+	12	58	55.66	8	45	35.36	+	18	53	43.99	0	17	15.43	+	0	8	22.38	20	17	27.62	-	21	1	13.45
	26	2	12	16.55	+	13	18	30.69	8	45	51.08	+	18	52	38.91	0	21	43.22	+	0	36	17.55	20	19	58.95	-	20	54	57.33
	27	2	16	3.2	+	13	37	52.67	8	46	7.49	+	18	51	31.11	0	26	11.04	+	1	4	14.18	20	22	29.72	-	20	48	35.9
	28	2	19	50.36	+	13	57	1.25	8	46	24.57	+	18	50	20.59	0	30	38.93	+	1	32	11.59	20	24	59.89	-	20	42	9.34
	29	2	23	38.04	+	14	15	56.1	8	46	42.32	+	18	49	7.35	0	35	6.93	+	2	0	9.11	20	27	29.47	-	20	35	37.83
	30	2	27	26.24	+	14	34	36.89	8	47	0.73	+	18	47	51.41	0	39	35.07	+	2	28	6.06	20	29	58.44	-	20	29	1.56
May	1	2	31	14.97	+	14	53	3.27	8	47	19.81	+	18	46	32.78	0	44	3.39	+	2	56	1.76	20	32	26.79	-	20	22	20.7
	2	2	35	4.24	+	15	11	14.92	8	47	39.54	+	18	45	11.47	0	48	31.92	+	3	23	55.52	20	34	54.51	-	20	15	35.46
	3	2	38	54.04	+	15	29	11.5	8	47	59.91	+	18	43	47.5	0	53	0.7	+	3	51	46.66	20	37	21.59	-	20	8	46.02
	4	2	42	44.4	+	15	46	52.68	8	48	20.93	+	18	42	20.9	0	57	29.76	+	4	19	34.49	20	39	48.02	-	20	1	52.59
	5	2	46	35.29	+	16	4	18.13	8	48	42.57	+	18	40	51.68	1	1	59.13	+	4	47	18.32	20	42	13.78	-	19	54	55.36
	6	2	50	26.74	+	16	21	27.53	8	49	4.85	+	18	39	19.87	1	6	28.84	+	5	14	57.48	20	44	38.88	-	19	47	54.52
	7	2	54	18.73	+	16	38	20.54	8	49	27.73	+	18	37	45.49	1	10	58.94	+	5	42	31.26	20	47	3.3	-	19	40	50.29
	8	2	58	11.28	+	16	54	56.85	8	49	51.22	+	18	36	8.56	1	15	29.46	+	6	9	58.98	20	49	27.03	-	19	33	42.84
	9	3	2	4.37	+	17	11	16.13	8	50	15.31	+	18	34	29.11	1	20	0.42	+	6	37	19.94	20	51	50.07	-	19	26	32.37
	10	3	5	58.01	+	17	27	18.08	8	50	39.98	+	18	32	47.15	1	24	31.86	+	7	4	33.46	20	54	12.4	-	19	19	19.06
	11	3	9	52.2	+	17	43	2.38	8	51	5.23	+	18	31	2.69	1	29	3.82	+	7	31	38.86	20	56	34.02	-	19	12	3.1
	12	3	13	46.94	+	17	58	28.74	8	51	31.05	+	18	29	15.74	1	33	36.34	+	7	58	35.45	20	58	54.92	-	19	4	44.67
	13	3	17	42.23	+	18	13	36.88	8	51	57.43	+	18	27	26.31	1	38	9.44	+	8	25	22.56	21	1	15.09	-	18	57	23.93
	14	3	21	38.08	+	18	28	26.53	8	52	24.37	+	18	25	34.4	1	42	43.17	+	8	51	59.54	21	3	34.54	-	18	50	1.05
	15	3	25	34.5	+	18	42	57.44	8	52	51.86	+	18	23	40.03	1	47	17.56	+	9	18	25.71	21	5	53.25	-	18	42	36.24
	16	3	29	31.48	+	18	57	9.38	8	53	19.9	+	18	21	43.22	1	51	52.65	+	9	44	40.42	21	8	11.21	-	18	35	9.67
	17	3	33	29.03	+	19	11	2.1	8	53	48.47	+	18	19	43.99	1	56	28.48	+	10	10	43.01	21	10	28.42	-	18	27	41.59
	18	3	37	27.14	+	19	24	35.38	8	54	17.57	+	18	17	42.38	2	1	5.07	+	10	36	32.8	21	12	44.86	-	18	20	12.21
	19	3	41	25.83	+	19	37	48.97	8	54	47.19	+	18	15	38.42	2	5	42.45	+	11	2	9.08	21	15	0.5	-	18	12	41.8
	20	3	45	25.08	+	19	50	42.62	8	55	17.32	+	18	13	32.12	2	10	20.66	+	11	27	31.17	21	17	15.33	-	18	5	10.59
	21	3	49	24.89	+	20	3	16.07	8	55	47.96	+	18	11	23.5	2	14	59.71	+	11	52	38.33	21	19	29.32	-	17	57	38.86
	22	3	53	25.25	+	20	15	29.06	8	56	19.09	+	18	9	12.58	2	19	39.64	+	12	17	29.86	21	21	42.45	-	17	50	6.84
	23	3	57	26.15	+	20	27	21.33	8	56	50.72	+	18	6	59.34	2	24	20.47	+	12	42	5.04	21	23	54.7	-	17	42	34.8
	24	4	1	27.59	+	20	38	52.62	8	57	22.82	+	18	4	43.81	2	29	2.22	+	13	6	23.14	21	26	6.05	-	17	35	2.99
	25	4	5	29.56	+	20	50	2.7	8	57	55.4	+	18	2	25.97	2	33	44.92	+	13	30	23.45	21	28	16.48	-	17	27	31.68
	26	4	9	32.05	+	21	0	51.32	8	58	28.46	+	18	0	5.83	2	38	28.59	+	13	54	5.26	21	30	25.98	-	17	20	1.12
	27	4	13	35.04	+	21	11	18.24	8	59	1.98	+	17	57	43.41	2	43	13.25	+	14	17	27.84	21	32	34.51	-	17	12	31.58
	28	4	17	38.53	+	21	21	23.24	8	59	35.96	+	17	55	18.72	2	47	58.92	+	14	40	30.47	21	34	42.07	-	17	5	3.34
	29	4	21	42.5	+	21	31	6.09	9	0	10.39	+	17	52	51.77	2	52	45.61	+	15	3	12.46	21	36	48.63	-	16	57	36.65
	30	4	25	46.93	+	21	40	26.59	9	0	45.28	+	17	50	22.57	2	57	33.34	+	15	25	33.06	21	38	54.17	-	16	50	11.81
	31	4	29	51.8	+	21	49	24.51	9	1	20.6	+	17	47	51.15	3	2	22.13	+	15	47	31.59	21	40	58.68	-	16	42	49.07
Jun	1	4	33	57.1	+	21	57	59.68	9	1	56.35	+	17	45	17.53	3	7	11.98	+	16	9	7.31	21	43	2.13	-	16	35	28.73
	2	4	38	2.81	+	22	6	11.87	9	2	32.52	+	17	42	41.73	3	12	2.9	+	16	30	19.52	21	45	4.51	-	16	28	11.04
	3	4	42	8.91	+	22	14	0.93	9	3	9.12	+	17	40	3.77	3	16	54.91	+	16	51	7.5	21	47	5.79	-	16	20	56.3
	4	4	46	15.36	+	22	21	26.65	9	3	46.11	+	17	37	23.68	3	21	48	+	17	11	30.56	21	49	5.95	-	16	13	44.76
	5	4	50	22.16	+	22	28	28.87	9	4	23.51	+	17	34	41.47	3	26	42.19	+	17	31	27.98	21	51	4.99	-	16	6	36.69
	6	4	54	29.28	+	22	35	7.43	9	5	1.29	+	17	31	57.18	3	31	37.47	+	17	50	59.07	21	53	2.87	-	15	59	32.35
	7	4	58	36.7	+	22	41	22.17	9	5	39.45	+	17	29	10.8	3	36	33.84	+	18	10	3.13	21	54	59.58	-	15	52	31.99
	8	5	2	44.39	+	22	47	12.95	9	6	17.99	+	17	26	22.36	3	41	31.31	+	18	28	39.49	21	56	55.1	-	15	45	35.85
	9	5	6	52.34	+	22	52	39.64	9	6	56.89	+	17	23	31.85	3	46	29.88	+	18	46	47.46	21	58	49.43	-	15	38	44.17
	10	5	11	0.53	+	22	57	42.13	9	7	36.15	+	17	20	39.29	3	51	29.53	+	19	4	26.41	22	0	42.53	-	15	31	57.19
	11	5	15	8.94	+	23	2	20.32	9	8	15.76	+	17	17	44.69	3	56	30.29	+	19	21	35.68	22	2	34.39	-	15	25	15.14
	12	5	19	17.55	+	23	6	34.15	9	8	55.72	+	17	14	48.06	4	1	32.13	+	19	38	14.65	22	4	25	-	15	18	38.29
	13	5	23	26.35	+	23	10	23.54	9	9	36.03	+	17	11	49.44	4	6	35.05	+	19	54	22.73	22	6	14.33	-	15	12	6.89
	14	5	27	35.32	+	23	13	48.45	9	10	16.67	+	17	8	48.83	4	11	39.04	+	20	9	59.29	22	8	2.35	-	15	5	41.24
	15	5	31	44.44	+	23	16	48.84	9	10	57.64	+	17	5	46.29	4	16	44.09	+	20	25	3.75	22	9	49.04	-	14	59	21.66
	16	5	35	53.69	+	23	19	24.66	9	11	38.93	+	17	2	41.84	4	21	50.19	+	20	39	35.51	22	11	34.35	-	14	53	8.46
	17	5	40	3.05	+	23	21	35.85	9	12	20.54	+	16	59	35.5	4	26	57.3	+	20	53	33.97	22	13	18.26	-	14	47	1.99
	18	5	44	12.5	+	23	23	22.38	9	13	2.45	+	16	56	27.27	4	32	5.42	+	21	6	58.55	22	15	0.71	-	14	41	2.58
	19	5	48	22.02	+	23	24	44.18	9	13	44.66	+	16	53	17.18	4	37	14.51	+	21	19	48.68	22	16	41.68	-	14	35	10.59
	20	5	52	31.58	+	23	25	41.23	9	14	27.17	+	16	50	5.22	4	42	24.56	+	21	32	3.79	22	18	21.11	-	14	29	26.35
	21	5	56	41.18	+	23	26	13.51	9	15	9.96	+	16	46	51.39	4	47	35.53	+	21	43	43.34	22	19	58.98	-	14	23	50.21
	22	6	0	50.78	+	23	26	20.99	9	15	53.04	+	16	43	35.7	4	52	47.4	+	21	54	46.83	22	21	35.25	-	14	18	22.51
	23	6	5	0.37	+	23	26	3.67	9	16	36.41	+	16	40	18.16	4	58	0.13	+	22	5	13.74	22	23	9.86	-	14	13	3.58
	24	6	9	9.91	+	23	25	21.57	9	17	20.04	+	16	36	58.79	5	3	13.69	+	22	15	3.59	22	24	42.79	-	14	7	53.78
	25	6	13	19.39	+	23	24	14.7	9	18	3.95	+	16	33	37.59	5	8	28.04	+	22	24	15.94	22	26	13.98	-	14	2	53.44
	26	6	17	28.78	+	23	22	43.09	9	18	48.13	+	16	30	14.59	5	13	43.14	+	22	32	50.35	22	27	43.41	-	13	58	2.89
	27	6	21	38.05	+	23	20	46.79	9	19	32.57	+	16	26	49.8	5	18	58.94	+	22	40	46.4	22	29	11.03	-	13	53	22.48
	28	6	25	47.19	+	23	18	25.86	9	20	17.26	+	16	23	23.25	5	24	15.41	+	22	48	3.72	22	30	36.8	-	13	48	52.53
	29	6	29	56.15	+	23	15	40.35	9	21	2.2	+	16	19	54.96	5	29	32.5	+	22	54	41.93	22	32	0.68	-	13	44	33.36
	30	6	34	4.92	+	23	12	30.34	9	21	47.38	+	16	16	24.96	5	34	50.15	+	23	0	40.69	22	33	22.62	-	13	40	25.3
Jul	1	6	38	13.47	+	23	8	55.91	9	22	32.79	+	16	12	53.27	5	40	8.32	+	23	5	59.7	22	34	42.59	-	13	36	28.63
	2	6	42	21.77	+	23	4	57.16	9	23	18.42	+	16	9	19.92	5	45	26.95	+	23	10	38.66	22	36	0.54	-	13	32	43.65
	3	6	46	29.79	+	23	0	34.18	9	24	4.28	+	16	5	44.94	5	50	45.98	+	23	14	37.31	22	37	16.44	-	13	29	10.63
	4	6	50	37.51	+	22	55	47.09	9	24	50.34	+	16	2	8.33	5	56	5.37	+	23	17	55.4	22	38	30.24	-	13	25	49.82
	5	6	54	44.9	+	22	50	36	9	25	36.61	+	15	58	30.12	6	1	25.06	+	23	20	32.74	22	39	41.92	-	13	22	41.46
	6	6	58	51.94	+	22	45	1.04	9	26	23.08	+	15	54	50.31	6	6	44.99	+	23	22	29.14	22	40	51.43	-	13	19	45.76
	7	7	2	58.61	+	22	39	2.35	9	27	9.74	+	15	51	8.92	6	12	5.12	+	23	23	44.46	22	41	58.74	-	13	17	2.93

Date		Object A							Object B							Object C							Object D						
		R.A.			Dec.				R.A.			Dec.				R.A.			Dec.				R.A.			Dec.			
		h	m	s		°	'	"	h	m	s		°	'	"	h	m	s		°	'	"	h	m	s		°	'	"
	8	7	7	4.9	+	22	32	40.08	9	27	56.59	+	15	47	25.97	6	17	25.38	+	23	24	18.59	22	43	3.82	-	13	14	33.16
	9	7	11	10.78	+	22	25	54.41	9	28	43.62	+	15	43	41.46	6	22	45.73	+	23	24	11.45	22	44	6.62	-	13	12	16.65
	10	7	15	16.25	+	22	18	45.52	9	29	30.84	+	15	39	55.43	6	28	6.12	+	23	23	23	22	45	7.12	-	13	10	13.6
	11	7	19	21.28	+	22	11	13.58	9	30	18.23	+	15	36	7.9	6	33	26.48	+	23	21	53.24	22	46	5.27	-	13	8	24.21
	12	7	23	25.86	+	22	3	18.81	9	31	5.8	+	15	32	18.91	6	38	46.77	+	23	19	42.18	22	47	1.02	-	13	6	48.71
	13	7	27	29.99	+	21	55	1.4	9	31	53.52	+	15	28	28.48	6	44	6.94	+	23	16	49.88	22	47	54.32	-	13	5	27.32
	14	7	31	33.64	+	21	46	21.54	9	32	41.41	+	15	24	36.66	6	49	26.92	+	23	13	16.39	22	48	45.13	-	13	4	20.27
	15	7	35	36.82	+	21	37	19.43	9	33	29.44	+	15	20	43.45	6	54	46.66	+	23	9	1.81	22	49	33.38	-	13	3	27.81
	16	7	39	39.5	+	21	27	55.24	9	34	17.62	+	15	16	48.88	7	0	6.11	+	23	4	6.24	22	50	19.03	-	13	2	50.15
	17	7	43	41.67	+	21	18	9.18	9	35	5.94	+	15	12	52.95	7	5	25.23	+	22	58	29.81	22	51	2.01	-	13	2	27.5
	18	7	47	43.34	+	21	8	1.43	9	35	54.39	+	15	8	55.67	7	10	43.96	+	22	52	12.7	22	51	42.27	-	13	2	20.04
	19	7	51	44.49	+	20	57	32.21	9	36	42.98	+	15	4	57.05	7	16	2.25	+	22	45	15.08	22	52	19.77	-	13	2	27.93
	20	7	55	45.12	+	20	46	41.72	9	37	31.7	+	15	0	57.1	7	21	20.06	+	22	37	37.2	22	52	54.45	-	13	2	51.3
	21	7	59	45.21	+	20	35	30.18	9	38	20.55	+	14	56	55.83	7	26	37.34	+	22	29	19.3	22	53	26.26	-	13	3	30.26
	22	8	3	44.77	+	20	23	57.84	9	39	9.52	+	14	52	53.26	7	31	54.04	+	22	20	21.68	22	53	55.16	-	13	4	24.88
	23	8	7	43.77	+	20	12	4.93	9	39	58.61	+	14	48	49.41	7	37	10.12	+	22	10	44.64	22	54	21.11	-	13	5	35.2
	24	8	11	42.22	+	19	59	51.7	9	40	47.82	+	14	44	44.3	7	42	25.54	+	22	0	28.53	22	54	44.07	-	13	7	1.22
	25	8	15	40.1	+	19	47	18.4	9	41	37.14	+	14	40	37.96	7	47	40.25	+	21	49	33.7	22	55	4	-	13	8	42.9
	26	8	19	37.41	+	19	34	25.29	9	42	26.56	+	14	36	30.42	7	52	54.21	+	21	38	0.57	22	55	20.87	-	13	10	40.18
	27	8	23	34.14	+	19	21	12.66	9	43	16.08	+	14	32	21.71	7	58	7.38	+	21	25	49.54	22	55	34.65	-	13	12	52.91
	28	8	27	30.27	+	19	7	40.77	9	44	5.69	+	14	28	11.86	8	3	19.73	+	21	13	1.05	22	55	45.32	-	13	15	20.93
	29	8	31	25.81	+	18	53	49.9	9	44	55.4	+	14	24	0.89	8	8	31.22	+	20	59	35.56	22	55	52.86	-	13	18	4
	30	8	35	20.74	+	18	39	40.35	9	45	45.18	+	14	19	48.84	8	13	41.81	+	20	45	33.56	22	55	57.26	-	13	21	1.84
	31	8	39	15.06	+	18	25	12.39	9	46	35.03	+	14	15	35.74	8	18	51.48	+	20	30	55.54	22	55	58.52	-	13	24	14.08
Aug	1	8	43	8.76	+	18	10	26.33	9	47	24.95	+	14	11	21.59	8	24	0.19	+	20	15	42.02	22	55	56.64	-	13	27	40.32
	2	8	47	1.84	+	17	55	22.45	9	48	14.94	+	14	7	6.44	8	29	7.91	+	19	59	53.53	22	55	51.64	-	13	31	20.07
	3	8	50	54.29	+	17	40	1.05	9	49	4.99	+	14	2	50.28	8	34	14.63	+	19	43	30.62	22	55	43.53	-	13	35	12.8
	4	8	54	46.12	+	17	24	22.44	9	49	55.09	+	13	58	33.13	8	39	20.33	+	19	26	33.86	22	55	32.34	-	13	39	17.94
	5	8	58	37.32	+	17	8	26.93	9	50	45.24	+	13	54	15.03	8	44	25	+	19	9	3.84	22	55	18.1	-	13	43	34.86
	6	9	2	27.91	+	16	52	14.82	9	51	35.45	+	13	49	55.99	8	49	28.61	+	18	51	1.16	22	55	0.85	-	13	48	2.92
	7	9	6	17.88	+	16	35	46.45	9	52	25.69	+	13	45	36.05	8	54	31.15	+	18	32	26.44	22	54	40.61	-	13	52	41.44
	8	9	10	7.24	+	16	19	2.11	9	53	15.98	+	13	41	15.25	8	59	32.63	+	18	13	20.32	22	54	17.45	-	13	57	29.74
	9	9	13	55.99	+	16	2	2.14	9	54	6.31	+	13	36	53.62	9	4	33.03	+	17	53	43.44	22	53	51.39	-	14	2	27.11
	10	9	17	44.15	+	15	44	46.84	9	54	56.66	+	13	32	31.19	9	9	32.34	+	17	33	36.45	22	53	22.49	-	14	7	32.82
	11	9	21	31.73	+	15	27	16.5	9	55	47.03	+	13	28	8.01	9	14	30.57	+	17	13	0	22	52	50.79	-	14	12	46.1
	12	9	25	18.72	+	15	9	31.42	9	56	37.42	+	13	23	44.09	9	19	27.72	+	16	51	54.74	22	52	16.36	-	14	18	6.17
	13	9	29	5.14	+	14	51	31.88	9	57	27.83	+	13	19	19.45	9	24	23.78	+	16	30	21.33	22	51	39.27	-	14	23	32.2
	14	9	32	51.01	+	14	33	18.17	9	58	18.25	+	13	14	54.1	9	29	18.76	+	16	8	20.42	22	50	59.57	-	14	29	3.32
	15	9	36	36.34	+	14	14	50.55	9	59	8.67	+	13	10	28.06	9	34	12.68	+	15	45	52.67	22	50	17.36	-	14	34	38.62
	16	9	40	21.14	+	13	56	9.33	9	59	59.1	+	13	6	1.35	9	39	5.55	+	15	22	58.76	22	49	32.73	-	14	40	17.15
	17	9	44	5.43	+	13	37	14.77	10	0	49.53	+	13	1	33.97	9	43	57.37	+	14	59	39.37	22	48	45.78	-	14	45	57.91
	18	9	47	49.22	+	13	18	7.19	10	1	39.97	+	12	57	5.95	9	48	48.16	+	14	35	55.16	22	47	56.61	-	14	51	39.88
	19	9	51	32.51	+	12	58	46.88	10	2	30.4	+	12	52	37.33	9	53	37.94	+	14	11	46.85	22	47	5.36	-	14	57	22.01
	20	9	55	15.33	+	12	39	14.15	10	3	20.82	+	12	48	8.12	9	58	26.72	+	13	47	15.13	22	46	12.15	-	15	3	3.2
	21	9	58	57.69	+	12	19	29.3	10	4	11.24	+	12	43	38.42	10	3	14.51	+	13	22	20.73	22	45	17.11	-	15	8	42.35
	22	10	2	39.59	+	11	59	32.65	10	5	1.63	+	12	39	8.36	10	8	1.34	+	12	57	4.36	22	44	20.41	-	15	14	18.33
	23	10	6	21.05	+	11	39	24.52	10	5	51.98	+	12	34	37.64	10	12	47.23	+	12	31	26.75	22	43	22.19	-	15	19	50
	24	10	10	2.08	+	11	19	5.24	10	6	42.33	+	12	30	6.2	10	17	32.2	+	12	5	28.62	22	42	22.61	-	15	25	16.22
	25	10	13	42.68	+	10	58	35.13	10	7	32.64	+	12	25	34.43	10	22	16.26	+	11	39	10.72	22	41	21.86	-	15	30	35.83
	26	10	17	22.88	+	10	37	54.52	10	8	22.92	+	12	21	2.34	10	26	59.44	+	11	12	33.76	22	40	20.12	-	15	35	47.67
	27	10	21	2.68	+	10	17	3.75	10	9	13.15	+	12	16	29.92	10	31	41.76	+	10	45	38.48	22	39	17.56	-	15	40	50.6
	28	10	24	42.09	+	9	56	3.16	10	10	3.34	+	12	11	57.19	10	36	23.25	+	10	18	25.63	22	38	14.39	-	15	45	43.48
	29	10	28	21.11	+	9	34	53.09	10	10	53.46	+	12	7	24.18	10	41	3.94	+	9	50	55.93	22	37	10.8	-	15	50	25.18
	30	10	31	59.78	+	9	13	33.88	10	11	43.53	+	12	2	50.9	10	45	43.85	+	9	23	10.12	22	36	7.01	-	15	54	54.62
	31	10	35	38.08	+	8	52	5.86	10	12	33.54	+	11	58	17.38	10	50	23.01	+	8	55	8.93	22	35	3.2	-	15	59	10.76
Sep	1	10	39	16.06	+	8	30	29.38	10	13	23.48	+	11	53	43.64	10	55	1.46	+	8	26	53.1	22	33	59.6	-	16	3	12.63
	2	10	42	53.71	+	8	8	44.78	10	14	13.36	+	11	49	9.72	10	59	39.22	+	7	58	23.36	22	32	56.38	-	16	6	59.33
	3	10	46	31.05	+	7	46	52.42	10	15	3.16	+	11	44	35.65	11	4	16.34	+	7	29	40.47	22	31	53.75	-	16	10	30.06
	4	10	50	8.1	+	7	24	52.63	10	15	52.88	+	11	40	1.48	11	8	52.84	+	7	0	45.16	22	30	51.89	-	16	13	44.09
	5	10	53	44.88	+	7	2	45.76	10	16	42.52	+	11	35	27.25	11	13	28.77	+	6	31	38.18	22	29	50.98	-	16	16	40.8
	6	10	57	21.4	+	6	40	32.14	10	17	32.08	+	11	30	53	11	18	4.15	+	6	2	20.27	22	28	51.17	-	16	19	19.65
	7	11	0	57.68	+	6	18	12.1	10	18	21.53	+	11	26	18.78	11	22	39.02	+	5	32	52.15	22	27	52.64	-	16	21	40.15
	8	11	4	33.74	+	5	55	45.96	10	19	10.89	+	11	21	44.6	11	27	13.42	+	5	3	14.56	22	26	55.53	-	16	23	41.9
	9	11	8	9.6	+	5	33	14.02	10	20	0.15	+	11	17	10.51	11	31	47.38	+	4	33	28.22	22	25	59.98	-	16	25	24.57
	10	11	11	45.3	+	5	10	36.58	10	20	49.3	+	11	12	36.53	11	36	20.96	+	4	3	33.84	22	25	6.14	-	16	26	47.85
	11	11	15	20.83	+	4	47	53.94	10	21	38.34	+	11	8	2.67	11	40	54.19	+	3	33	32.12	22	24	14.14	-	16	27	51.52
	12	11	18	56.25	+	4	25	6.38	10	22	27.26	+	11	3	28.95	11	45	27.12	+	3	3	23.77	22	23	24.1	-	16	28	35.38
	13	11	22	31.55	+	4	2	14.22	10	23	16.08	+	10	58	55.41	11	49	59.79	+	2	33	9.5	22	22	36.15	-	16	28	59.3
	14	11	26	6.78	+	3	39	17.74	10	24	4.77	+	10	54	22.06	11	54	32.24	+	2	2	50.01	22	21	50.4	-	16	29	3.17
	15	11	29	41.95	+	3	16	17.25	10	24	53.34	+	10	49	48.93	11	59	4.52	+	1	32	26.03	22	21	6.97	-	16	28	46.94
	16	11	33	17.08	+	2	53	13.05	10	25	41.79	+	10	45	16.05	12	3	36.66	+	1	1	58.26	22	20	25.94	-	16	28	10.59
	17	11	36	52.2	+	2	30	5.47	10	26	30.11	+	10	40	43.47	12	8	8.72	+	0	31	27.42	22	19	47.43	-	16	27	14.13
	18	11	40	27.33	+	2	6	54.83	10	27	18.29	+	10	36	11.22	12	12	40.73	+	0	0	54.24	22	19	11.51	-	16	25	57.62
	19	11	44	2.48	+	1	43	41.44	10	28	6.34	+	10	31	39.34	12	17	12.75	-	0	29	40.56	22	18	38.27	-	16	24	21.13
	20	11	47	37.68	+	1	20	25.64	10	28	54.24	+	10	27	7.87	12	21	44.8	-	1	0	16.24	22	18	7.79	-	16	22	24.79
	21	11	51	12.95	+	0	57	7.78	10	29	41.99	+	10	22	36.85	12	26	16.93	-	1	30	52.06	22	17	40.13	-	16	20	8.7
	22	11	54	48.3	+	0	33	48.18	10	30	29.58	+	10	18	6.34	12	30	49.19	-	2	1	27.3	22	17	15.35	-	16	17	33.04
	23	11	58	23.75	+	0	10	27.2	10	31	17.01	+	10	13	36.36	12	35	21.61	-	2	32	1.2	22	16	53.51	-	16	14	37.97
	24	12	1	59.32	-	0	12	54.79	10	32	4.28	+	10	9	6.98	12	39	54.23	-	3	2	33.01	22	16	34.65	-	16	11	23.65
	25	12	5	35.02	-	0	36	17.46	10	32	51.36	+	10	4	38.21	12	44	27.1	-	3	33	1.98	22	16	18.83	-	16	7	50.29
	26	12	9	10.88	-	0	59	40.42	10	33	38.27	+	10	0	10.09	12	49	0.26	-	4	3	27.36	22	16	6.08	-	16	3	58.07
	27	12	12	46.92	-	1	23	3.31	10	34	25	+	9	55	42.66	12	53	33.74	-	4	33	48.4	22	15	56.43	-	15	59	47.22
	28	12	16	23.14	-	1	46	25.77	10	35	11.54	+	9	51	15.95	12	58	7.6	-	5	4	4.34	22	15	49.9	-	15	55	17.96
	29	12	19	59.57	-	2	9	47.42	10	35	57.88	+	9	46	49.99	13	2	41.87	-	5	34	14.42	22	15	46.52	-	15	50	30.56
	30	12	23	36.23	-	2	33	7.88	10	36	44.04	+	9	42	24.82	13	7	16.59	-	6	4	17.88	22	15	46.27	-	15	45	25.33
Oct	1	12	27	13.13	-	2	56	26.78	10	37	29.99	+	9	38	0.5	13	11	51.8	-	6	34	13.96	22	15	49.15	-	15	40	2.6
	2	12	30	50.29	-	3	19	43.73	10	38	15.74	+	9	33	37.08	13	16	27.53	-	7	4	1.86	22	15	55.15	-	15	34	22.76
	3	12	34	27.72	-	3	42	58.36	10	39	1.28	+	9	29	14.61	13	21	3.83	-	7	33	40.82	22	16	4.25	-	15	28	26.19
	4	12	38	5.45	-	4	6	10.29	10	39	46.59	+	9	24	53.14	13	25	40.73	-	8	3	10.07	22	16	16.39	-	15	22	13.27
	5	12	41	43.5	-	4	29	19.18	10	40	31.68	+	9	20	32.71	13	30	18.27	-	8	32	28.82	22	16	31.56	-	15	15	44.42
	6	12	45	21.88	-	4	52	24.66	10	41	16.54	+	9	16	13.35	13	34	56.49	-	9	1	36.31	22	16	49.71	-	15	9	0.02
	7	12	49	0.63	-	5	15	26.39	10	42	1.17	+	9	11	55.11	13	39	35.43	-	9	30	31.77	22	17	10.79	-	15	2	0.46
	8	12	52	39.75	-	5	38	24.03	10	42	45.55	+	9	7	38.01	13	44	15.12	-	9	59	14.45	22	17	34.76	-	14	54	46.12
	9	12	56	19.29	-	6	1	17.24	10	43	29.7	+	9	3	22.08	13	48	55.6	-	10	27	43.57	22	18	1.57	-	14	47	17.37

Date		Object A							Object B							Object C							Object D						
		R.A.			Dec.				R.A.			Dec.				R.A.			Dec.				R.A.			Dec.			
		h	m	s		°	'	"	h	m	s		°	'	"	h	m	s		°	'	"	h	m	s		°	'	"
	10	12	59	59.25	-	6	24	5.7	10	44	13.6	+	8	59	7.34	13	53	36.92	-	10	55	58.38	22	18	31.19	-	14	39	34.56
	11	13	3	39.67	-	6	46	49.06	10	44	57.25	+	8	54	53.83	13	58	19.1	-	11	23	58.12	22	19	3.55	-	14	31	38.04
	12	13	7	20.56	-	7	9	26.98	10	45	40.65	+	8	50	41.58	14	3	2.19	-	11	51	42.03	22	19	38.62	-	14	23	28.15
	13	13	11	1.96	-	7	31	59.12	10	46	23.79	+	8	46	30.64	14	7	46.23	-	12	19	9.32	22	20	16.34	-	14	15	5.2
	14	13	14	43.87	-	7	54	25.14	10	47	6.67	+	8	42	21.03	14	12	31.23	-	12	46	19.25	22	20	56.66	-	14	6	29.51
	15	13	18	26.33	-	8	16	44.66	10	47	49.29	+	8	38	12.81	14	17	17.24	-	13	13	11.01	22	21	39.55	-	13	57	41.38
	16	13	22	9.35	-	8	38	57.34	10	48	31.63	+	8	34	6.03	14	22	4.28	-	13	39	43.84	22	22	24.94	-	13	48	41.1
	17	13	25	52.95	-	9	1	2.8	10	49	13.69	+	8	30	0.73	14	26	52.39	-	14	5	56.95	22	23	12.8	-	13	39	28.93
	18	13	29	37.14	-	9	23	0.67	10	49	55.47	+	8	25	56.96	14	31	41.58	-	14	31	49.54	22	24	3.07	-	13	30	5.14
	19	13	33	21.95	-	9	44	50.55	10	50	36.96	+	8	21	54.78	14	36	31.88	-	14	57	20.82	22	24	55.7	-	13	20	29.98
	20	13	37	7.39	-	10	6	32.08	10	51	18.14	+	8	17	54.23	14	41	23.32	-	15	22	29.99	22	25	50.66	-	13	10	43.67
	21	13	40	53.48	-	10	28	4.84	10	51	59.02	+	8	13	55.36	14	46	15.91	-	15	47	16.25	22	26	47.89	-	13	0	46.43
	22	13	44	40.23	-	10	49	28.44	10	52	39.58	+	8	9	58.22	14	51	9.67	-	16	11	38.79	22	27	47.35	-	12	50	38.47
	23	13	48	27.66	-	11	10	42.47	10	53	19.83	+	8	6	2.86	14	56	4.63	-	16	35	36.81	22	28	49	-	12	40	19.96
	24	13	52	15.78	-	11	31	46.51	10	53	59.75	+	8	2	9.31	15	1	0.79	-	16	59	9.5	22	29	52.79	-	12	29	51.08
	25	13	56	4.6	-	11	52	40.15	10	54	39.34	+	7	58	17.62	15	5	58.17	-	17	22	16.07	22	30	58.69	-	12	19	11.97
	26	13	59	54.14	-	12	13	22.98	10	55	18.6	+	7	54	27.82	15	10	56.78	-	17	44	55.72	22	32	6.65	-	12	8	22.82
	27	14	3	44.4	-	12	33	54.56	10	55	57.52	+	7	50	39.96	15	15	56.63	-	18	7	7.65	22	33	16.64	-	11	57	23.8
	28	14	7	35.39	-	12	54	14.46	10	56	36.09	+	7	46	54.09	15	20	57.72	-	18	28	51.09	22	34	28.6	-	11	46	15.1
	29	14	11	27.13	-	13	14	22.27	10	57	14.32	+	7	43	10.28	15	26	0.05	-	18	50	5.24	22	35	42.48	-	11	34	56.94
	30	14	15	19.6	-	13	34	17.54	10	57	52.18	+	7	39	28.58	15	31	3.62	-	19	10	49.32	22	36	58.23	-	11	23	29.57
	31	14	19	12.84	-	13	53	59.84	10	58	29.67	+	7	35	49.06	15	36	8.41	-	19	31	2.54	22	38	15.8	-	11	11	53.21
Nov	1	14	23	6.83	-	14	13	28.75	10	59	6.79	+	7	32	11.76	15	41	14.43	-	19	50	44.14	22	39	35.12	-	11	0	8.11
	2	14	27	1.59	-	14	32	43.86	10	59	43.53	+	7	28	36.74	15	46	21.66	-	20	9	53.35	22	40	56.14	-	10	48	14.5
	3	14	30	57.13	-	14	51	44.76	11	0	19.88	+	7	25	4.03	15	51	30.09	-	20	28	29.42	22	42	18.82	-	10	36	12.61
	4	14	34	53.46	-	15	10	31.06	11	0	55.84	+	7	21	33.66	15	56	39.72	-	20	46	31.65	22	43	43.1	-	10	24	2.66
	5	14	38	50.59	-	15	29	2.37	11	1	31.4	+	7	18	5.68	16	1	50.53	-	21	3	59.31	22	45	8.93	-	10	11	44.87
	6	14	42	48.54	-	15	47	18.31	11	2	6.56	+	7	14	40.11	16	7	2.5	-	21	20	51.71	22	46	36.27	-	9	59	19.43
	7	14	46	47.31	-	16	5	18.49	11	2	41.32	+	7	11	16.99	16	12	15.62	-	21	37	8.17	22	48	5.08	-	9	46	46.56
	8	14	50	46.9	-	16	23	2.55	11	3	15.67	+	7	7	56.36	16	17	29.86	-	21	52	48.05	22	49	35.3	-	9	34	6.44
	9	14	54	47.34	-	16	40	30.1	11	3	49.6	+	7	4	38.26	16	22	45.2	-	22	7	50.7	22	51	6.9	-	9	21	19.28
	10	14	58	48.62	-	16	57	40.76	11	4	23.11	+	7	1	22.73	16	28	1.62	-	22	22	15.5	22	52	39.85	-	9	8	25.24
	11	15	2	50.76	-	17	14	34.17	11	4	56.2	+	6	58	9.83	16	33	19.09	-	22	36	1.84	22	54	14.09	-	8	55	24.52
	12	15	6	53.75	-	17	31	9.93	11	5	28.85	+	6	54	59.6	16	38	37.56	-	22	49	9.13	22	55	49.6	-	8	42	17.29
	13	15	10	57.61	-	17	47	27.67	11	6	1.06	+	6	51	52.1	16	43	57.01	-	23	1	36.81	22	57	26.35	-	8	29	3.71
	14	15	15	2.32	-	18	3	27	11	6	32.83	+	6	48	47.37	16	49	17.39	-	23	13	24.33	22	59	4.29	-	8	15	43.93
	15	15	19	7.89	-	18	19	7.54	11	7	4.14	+	6	45	45.48	16	54	38.66	-	23	24	31.15	23	0	43.39	-	8	2	18.12
	16	15	23	14.33	-	18	34	28.9	11	7	34.98	+	6	42	46.48	17	0	0.79	-	23	34	56.77	23	2	23.63	-	7	48	46.41
	17	15	27	21.62	-	18	49	30.68	11	8	5.35	+	6	39	50.42	17	5	23.71	-	23	44	40.7	23	4	4.98	-	7	35	8.94
	18	15	31	29.77	-	19	4	12.5	11	8	35.25	+	6	36	57.36	17	10	47.38	-	23	53	42.48	23	5	47.42	-	7	21	25.83
	19	15	35	38.76	-	19	18	33.97	11	9	4.66	+	6	34	7.33	17	16	11.76	-	24	2	1.67	23	7	30.91	-	7	7	37.19
	20	15	39	48.61	-	19	32	34.7	11	9	33.58	+	6	31	20.4	17	21	36.78	-	24	9	37.85	23	9	15.43	-	6	53	43.13
	21	15	43	59.28	-	19	46	14.29	11	10	1.99	+	6	28	36.59	17	27	2.39	-	24	16	30.66	23	11	0.97	-	6	39	43.74
	22	15	48	10.79	-	19	59	32.38	11	10	29.91	+	6	25	55.96	17	32	28.54	-	24	22	39.75	23	12	47.51	-	6	25	39.11
	23	15	52	23.11	-	20	12	28.59	11	10	57.32	+	6	23	18.54	17	37	55.16	-	24	28	4.81	23	14	35.02	-	6	11	29.34
	24	15	56	36.24	-	20	25	2.55	11	11	24.21	+	6	20	44.39	17	43	22.19	-	24	32	45.57	23	16	23.5	-	5	57	14.54
	25	16	0	50.15	-	20	37	13.89	11	11	50.57	+	6	18	13.56	17	48	49.56	-	24	36	41.8	23	18	12.9	-	5	42	54.82
	26	16	5	4.81	-	20	49	2.26	11	12	16.41	+	6	15	46.11	17	54	17.21	-	24	39	53.3	23	20	3.22	-	5	28	30.34
	27	16	9	20.22	-	21	0	27.29	11	12	41.71	+	6	13	22.12	17	59	45.05	-	24	42	19.89	23	21	54.41	-	5	14	1.28
	28	16	13	36.34	-	21	11	28.64	11	13	6.46	+	6	11	1.63	18	5	13.01	-	24	44	1.43	23	23	46.46	-	4	59	27.79
	29	16	17	53.16	-	21	22	5.95	11	13	30.65	+	6	8	44.7	18	10	41.03	-	24	44	57.82	23	25	39.33	-	4	44	50.06
	30	16	22	10.65	-	21	32	18.92	11	13	54.28	+	6	6	31.36	18	16	9.02	-	24	45	8.97	23	27	33	-	4	30	8.26
Dec	1	16	26	28.79	-	21	42	7.23	11	14	17.35	+	6	4	21.64	18	21	36.92	-	24	44	34.86	23	29	27.45	-	4	15	22.56
	2	16	30	47.58	-	21	51	30.61	11	14	39.85	+	6	2	15.58	18	27	4.67	-	24	43	15.49	23	31	22.65	-	4	0	33.13
	3	16	35	6.98	-	22	0	28.77	11	15	1.77	+	6	0	13.2	18	32	32.18	-	24	41	10.91	23	33	18.57	-	3	45	40.13
	4	16	39	26.99	-	22	9	1.47	11	15	23.11	+	5	58	14.54	18	37	59.4	-	24	38	21.21	23	35	15.21	-	3	30	43.72
	5	16	43	47.57	-	22	17	8.46	11	15	43.86	+	5	56	19.62	18	43	26.27	-	24	34	46.51	23	37	12.54	-	3	15	44.07
	6	16	48	8.71	-	22	24	49.5	11	16	4.03	+	5	54	28.48	18	48	52.7	-	24	30	26.97	23	39	10.54	-	3	0	41.33
	7	16	52	30.39	-	22	32	4.38	11	16	23.61	+	5	52	41.16	18	54	18.65	-	24	25	22.78	23	41	9.19	-	2	45	35.66
	8	16	56	52.59	-	22	38	52.88	11	16	42.58	+	5	50	57.69	18	59	44.04	-	24	19	34.19	23	43	8.48	-	2	30	27.21
	9	17	1	15.27	-	22	45	14.81	11	17	0.95	+	5	49	18.12	19	5	8.81	-	24	13	1.45	23	45	8.39	-	2	15	16.14
	10	17	5	38.42	-	22	51	9.97	11	17	18.71	+	5	47	42.5	19	10	32.91	-	24	5	44.85	23	47	8.91	-	2	0	2.59
	11	17	10	2	-	22	56	38.18	11	17	35.85	+	5	46	10.85	19	15	56.27	-	23	57	44.72	23	49	10.02	-	1	44	46.71
	12	17	14	26	-	23	1	39.27	11	17	52.36	+	5	44	43.24	19	21	18.84	-	23	49	1.41	23	51	11.71	-	1	29	28.63
	13	17	18	50.38	-	23	6	13.07	11	18	8.24	+	5	43	19.7	19	26	40.55	-	23	39	35.29	23	53	13.97	-	1	14	8.49
	14	17	23	15.11	-	23	10	19.42	11	18	23.48	+	5	42	0.27	19	32	1.37	-	23	29	26.78	23	55	16.79	-	0	58	46.43
	15	17	27	40.18	-	23	13	58.18	11	18	38.07	+	5	40	45	19	37	21.23	-	23	18	36.28	23	57	20.15	-	0	43	22.54
	16	17	32	5.53	-	23	17	9.21	11	18	52.02	+	5	39	33.91	19	42	40.09	-	23	7	4.27	23	59	24.05	-	0	27	56.96
	17	17	36	31.16	-	23	19	52.38	11	19	5.3	+	5	38	27.05	19	47	57.91	-	22	54	51.2	0	1	28.49	-	0	12	29.78
	18	17	40	57.02	-	23	22	7.57	11	19	17.92	+	5	37	24.44	19	53	14.64	-	22	41	57.58	0	3	33.46	+	0	2	58.9
	19	17	45	23.09	-	23	23	54.7	11	19	29.88	+	5	36	26.11	19	58	30.25	-	22	28	23.93	0	5	38.95	+	0	18	29
	20	17	49	49.32	-	23	25	13.69	11	19	41.17	+	5	35	32.07	20	3	44.69	-	22	14	10.8	0	7	44.97	+	0	34	0.43
	21	17	54	15.7	-	23	26	4.47	11	19	51.78	+	5	34	42.37	20	8	57.94	-	21	59	18.78	0	9	51.51	+	0	49	33.09
	22	17	58	42.17	-	23	26	27	11	20	1.72	+	5	33	57.03	20	14	9.95	-	21	43	48.47	0	11	58.56	+	1	5	6.89
	23	18	3	8.7	-	23	26	21.28	11	20	10.97	+	5	33	16.09	20	19	20.7	-	21	27	40.51	0	14	6.12	+	1	20	41.71
	24	18	7	35.25	-	23	25	47.28	11	20	19.53	+	5	32	39.59	20	24	30.15	-	21	10	55.58	0	16	14.18	+	1	36	17.42
	25	18	12	1.77	-	23	24	45.01	11	20	27.4	+	5	32	7.57	20	29	38.26	-	20	53	34.33	0	18	22.73	+	1	51	53.87
	26	18	16	28.22	-	23	23	14.49	11	20	34.56	+	5	31	40.06	20	34	45.02	-	20	35	37.47	0	20	31.76	+	2	7	30.91
	27	18	20	54.55	-	23	21	15.73	11	20	41.02	+	5	31	17.07	20	39	50.38	-	20	17	5.68	0	22	41.26	+	2	23	8.38
	28	18	25	20.74	-	23	18	48.79	11	20	46.77	+	5	30	58.62	20	44	54.33	-	19	57	59.68	0	24	51.21	+	2	38	46.12
	29	18	29	46.75	-	23	15	53.73	11	20	51.82	+	5	30	44.69	20	49	56.86	-	19	38	20.18	0	27	1.61	+	2	54	23.98
	30	18	34	12.54	-	23	12	30.64	11	20	56.16	+	5	30	35.28	20	54	57.95	-	19	18	7.93	0	29	12.46	+	3	10	1.81
	31	18	38	38.09	-	23	8	39.63	11	20	59.79	+	5	30	30.4	20	59	57.59	-	18	57	23.68	0	31	23.73	+	3	25	39.47

Chapter 3

Geometric Astronomy and Time

3.1 Theory

1. The largest angular separation between Venus and the Sun, when viewed from the Earth, is 46°. Calculate the radius of Venus's circular orbit in A.U. (I07 - T02 - A)
2. One night during a full Moon, the Moon subtends an angle of 0.46° to an observer. What is the observer's distance to the Moon on that night? (I07 - T04 - A)
3. An observer was able to measure the minute differences in the positions of the stars, due to the Earth's motion around the Sun, to a star as distant as 100 parsecs away. What was the minimum angular difference in arc seconds this observer could measure? (I07 - T05 - A)
4. What is the time interval between two consecutive oppositions of Mars? Assume the orbit is circular. (I12 - T03 - A)
5. Mars arrived at its great opposition at UT 17^h56^m Aug.28, 2003. The next great opposition of Mars will be in 2018, estimate the date of that opposition. The semi-major axis of the orbit of Mars is 1.524 AU. (I10 - T11 - A)
6. Calculate the length of the sidereal day on Earth. What would be the length of the solar and sidereal days, in the current time measures (our solar hours, minutes and seconds), if the Earth would rotate in the opposite direction, but with the same rotation speed? (I12 - T02 - A)
7. A satellite is orbiting around the Earth in a circular orbit along the celestial equator. An observer in Tehran at the latitude of $\varphi = 35.6°$

observes that the satellite has a zenith angle of $z = 46.0°$, when it transits the local meridian. Calculate the distance of the satellite from the center of the Earth (in the Earth radius unit). (I09 - T14 - B)

8. Assuming that Phobos moves around Mars on a perfectly circular orbit in the equatorial plane of the planet, give the length of time Phobos is above the horizon for a point on the Martian equator. (I11 - T04 - B)
9. Assume that you are living in the time of Copernicus and do not know anything about Kepler's laws. You might calculate Mars-Sun distance in the same way as he did. After accepting the revolutionary belief that all the planets are orbiting around the Sun, not around the Earth, you measure that the orbital period of Mars is 687 days, then you observe that 106 days after opposition of Mars, the planet appears in quadrature. Calculate Mars-Sun distance in astronomical unit (AU). (I09 - T13 - C)
10. **The planet Taris:** The planet Taris is home of the Korribian civilisation The Korribian species is a highly intelligent life form. They speak of Korribianese language. The Korribianese-English dictionary is shown in table 3.1. Read it carefully. Korribian astronomers have been studying the heavens for thousands of years. Their knowledge can be summarized as follows:
    - Taris orbits its host star Sola in a circular orbit, at a distance of 1 Tarislength.
    - Taris orbits Sola in 1 Tarisyear.
    - The Obliquity of Taris is 3^o with respect **to the normal** to its orbit.
    - There are exactly 10 Tarisdays in 1 Tarisyear.
    - Taris has two moons, named Endor and Extor. Both have circular orbits.
    - The sidereal orbital period of Endor (around Taris) is exactly 0.2 Tarisdays.
    - The sidereal orbital period of Extor (around Taris) is exactly 1.6 Tarisdays.
    - The distance between Taris and Endor is 1 Endorlength.
    - Corulus, another planet, also orbits Sola in a circular orbit. Corulus has one moon.
    - The distance between Sola and Corulus is 9 Tarislengths.
    - The tarisyear begins when Solaptic longitude of the Sola is zero.

    (a) Draw the Solar-system, and indicate all planets and moons.

    (b) How often does Taris rotate around its axis during one Tarisyear?

    (c) What is the distance between Taris and Extor, in Endorlengths?

Korribianese	**English Translation**
Corulus	A planet orbiting Sola
Endor	(i) Goddess of the night; (ii) a moon of Taris
Endorlength	The distance between Taris and Endor
Extor	(i) God of peace; (ii) a moon of Taris
Sola	(i) God of life; (ii) the star which Taris and Corulus orbit
Solaptic	Apparent path of Sola and Corulus as viewed from Taris
Taris	A planet orbiting the star Sola, home of the Korribians
Tarisday	The time between successive midnights on the planet Taris
Tarislength	The distance between Sola and Taris
Tarisyear	Time taken by Taris to make one revolution around Sola

Table 3.1 – Korribianese - English Dictionary

(d) What is the orbital period of Corulus, in Tarisyears?

(e) What is the distance between Taris and Corulus when Corulus is in opposition?

(f) If at the beginning of a particular tarisyear, Corulus and taris were in opposition, what would be Solaptic longitude (as observed from Taris) of Corulus 'n' tarisdays from the start of that year?

(g) What would be the area of the triangle formed by Sola, Taris and Corulus exactly one tarisday after the opposition? (I10 - T17 - C)

11. An observer observed a transit of Venus near the North Pole of the Earth. The transit path of Venus is shown in the figure 3.1. A, B, C, D are all on the path of transit and marking the center of the Venus disk. At A and B, the center of Venus is superposed on the limb of the Sun disk; C corresponds to the first contact while D to the fourth contact, $\triangle AOB = 90°$, MN is parallel to AB. The first contact occured at 9:00 UT. Calculate the time of the fourth contact. (I10 - T14 - D)

12. On average, the visual diameter of the Moon is slightly less than that of the Sun, so the frequency of annular solar eclipses is slightly higher than total solar eclipses. For an observer on the Earth, the longest total solar eclipse duration is about 7.5 minutes, and the longest annular eclipse duration is about 12.5 minutes. Here, the longest duration is the time interval from the second contact to the third contact. Suppose we count the occurrences of both types of solar eclipses for a very long time, estimate the ratio of the occurrences of annular solar eclipses and total solar eclipses. Assume the orbit of the Earth to be circular and the eccentricity of the Moon's orbit is 0.0549. Count all hybrid eclipses as annular eclipses. (I10 - T15 - D)

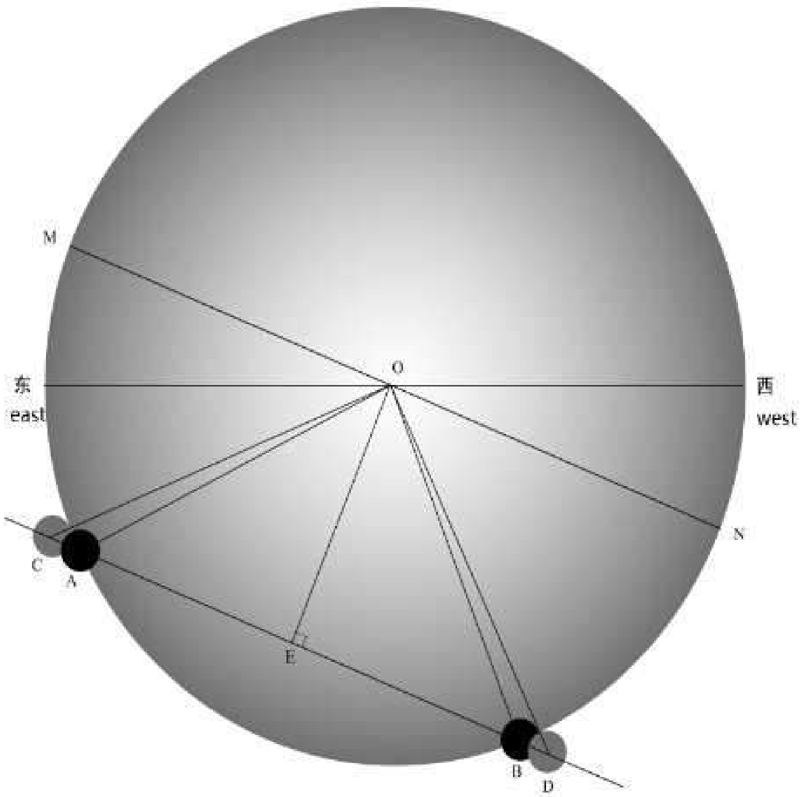


Figure 3.1 – Transit of Venus

3.2 Data Analysis

1. **The Moon's age:** The 60th anniversary celebrations of King Bhumibol Adulyadej's accession to the throne of Thailand (GMT +07) were held from 8th to 13th June 2006. Photographs of the Moon taken at the same hour each night are shown below:

8th June, 2006 9th June, 2006 10th June, 2006

11th June, 2006 12th June, 2006 13th June, 2006

Assuming that Albert Einstein's birth was at noon on 14th March, 1879, use the data provided above to find the Moon's age (number of days since the new moon) on his birth date in Germany (GMT + 01). Estimate the errors in your calculation. (I07 - D02 - B)

2. **Venus:** An observer in Deh-Namak, Iran (location of IOAA 2009 observational round) has observed the Venus for seven months, started from September 2008 and continued until March 2009. During these observations, a good quality CCD camera and an image processing software were used to take high resolution images and to extract high precision data. Table 3.2 shows the collected data during the observation. In this table,

Column 1	Date of observation.
Column 2	Earth-Sun distance in astronomical unit (AU) for observation date and time. This value is taken from high precision tables.
Column 3	Observed phase of Venus, Percent of Venus disk illuminated by the Sun as observed from the Earth.
Column 4	Observed elongation of Venus, the angular distance between center of the Sun and center of Venus in degrees.

Table 3.2 – Phase data for Venus

Date	**Earth - Sun Distance (AU)**	**Phase (%)**	**Elongation (SEV) (°)**
20/9/2008	1.0043	88.4	27.56
10/10/2008	0.9986	84.0	32.29
20/10/2008	0.9957	81.6	34.53
30/10/2008	0.9931	79.0	36.69
9/11/2008	0.9905	76.3	38.71
19/11/2008	0.9883	73.4	40.62
29/11/2008	0.9864	70.2	42.38
19/12/2008	0.9839	63.1	45.29
29/12/2008	0.9834	59.0	46.32
18/1/2009	0.9838	49.5	47.09
7/2/2009	0.9863	37.2	44.79
17/2/2009	0.9881	29.6	41.59
27/2/2009	0.9904	20.9	36.16
19/3/2009	0.9956	3.8	16.08

(a) Using given data in table 3.2, calculate the Sun-Venus-Earth angle ($\angle SVE$) for all observing dates.This is angular separation between the Sun and the Earth as seen from the Venus.
Note: Remember that the terminator line (line separating bright and dark regions of the Venus surface) is always an arc of an ellipse.

(b) Calculate Sun - Venus distance in AU for all observation dates.

(c) Plot Sun - Venus distance versus observing date.

(d) Find perihelion ($r_{v,min}$) and aphelion ($r_{v,max}$) distances of Venus from the Sun.

(e) Calculate semi-major axis (a) of the Venus orbit.

(f) Calculate eccentricity (e) of the Venus orbit. ((I09 - D02 - C))

Chapter 4

Optics and Detectors

4.1 Theory

1. A crater on the surface of the Moon has a diameter of 80 km. Is it possible to resolve this crater with naked eyes, assuming the eye pupil aperture is 5 mm? (I07 - T13 - A)
2. What would be the diameter of a radio telescope, working at a wavelength of $\lambda = 1$ cm, with the same resolution as an optical telescope of diameter D = 10 cm? (I11 - T05 - A)
3. The Galactic Center is believed to contain a super-massive black hole with a mass $M = 4 \times 10^6 M_\odot$. The astronomy community is trying to resolve its event horizon, which is a challenging task. For a non-rotating black hole, this is the Schwarzschild radius, $R_\odot = 3(M/M_\odot)$ km. Assume that we have an Earth-sized telescope (using Very Long Baseline Interferometry). What wavelengths should we adopt in order to resolve the event horizon of the black hole? (I10 - T08 - B)
4. What is the minimum diameter of a telescope, observing in the visible and near ultraviolet bands and located at one of the Lagrangian points L_4 or L_5s of the Sun-Earth system, in order to be able to detect the Earth's wobbling relative to the ecliptic plane caused by the gravitational action of the Moon? (I12 - T12 - B)
5. The coordinates of the components of Visual Binary star μ Sco as on the August 28, 2008 are given in the table below.

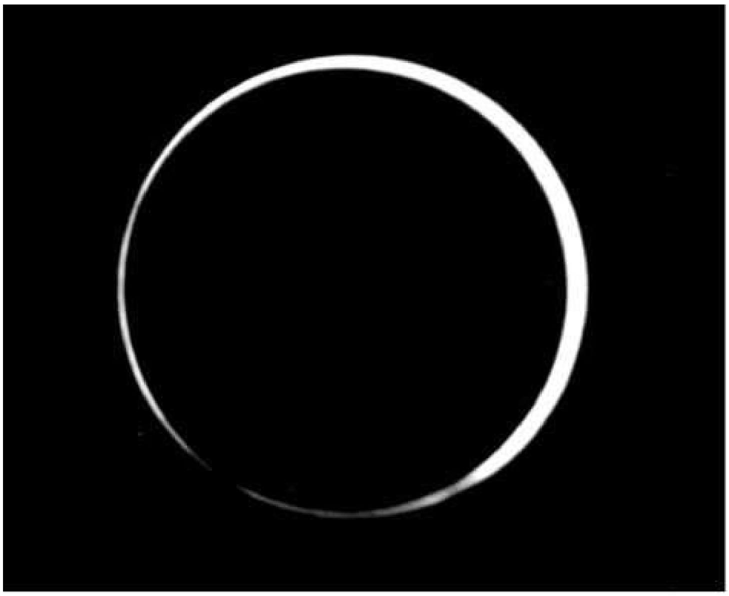
Star	α (RA)	δ (Dec)
μ Sco (primary)	$20^h17^m38.90^s$	$-12°30'30''$
μ Sco (secondary)	$20^h18^m03.30^s$	$-12°32'41''$

The stars are observed using Zeiss refractor telescope at the Bosscha Observatory whose aperture and focal length are 600 mm and 10,780 mm respectively. The telescope is equipped with 765×510 pixels CCD camera. The pixel size of the chip is $9\mu m \times 9\mu m$. (I08 - T10 - B)

(a) Can both components of the binary be inside the single frame of the camera?

(b) What is the position angle of the secondary star, with respect to the North?

6. Below is a picture on a 35 mm film of an annular eclipse, which was observed from Dumai, North Sumatra on August 22, 1998. This picture was taken with a telescope having effective diameter of 10 cm and f-ratio of 15. The diameter of the Sun's disk in original picture on the film is 13.817 mm and the diameter of the Moon's disk is 13.235 mm. Estimate the distances of the Sun and the Moon (expressed in km) from the Earth and the percentage of the solar disk covered by the Moon during the annular eclipse. (I08 - T11 - C)



7. On April 2, 2008 a telescope (10 cm diameter, f /10) at the Bosscha Observatory was used to observe the Sun and found an active region 0987 (based on the NOAA number) at 8° South and 40° West from the center of the solar disk. The region was recorded with a SBIG ST-8 CCD camera ($1600 \times 1200 pixels$, (9 μm $\times 9\mu$m)/pixel) and size of the spot was 5×4 pixels. According to the Astronomical Almanac, the solar diameter on that day is 32'. How large is the corrected area of the

active region in unit of millionth of solar hemisphere (msh)? (I08 - T02 - C)

4.2 Data Analysis

1. **CCD image:**

   Picture 4.1 presents a negative image of sky taken by a CCD camera attached to a telescope whose parameters are presented in the accompanying table (which is part of the FITS datafile header).

   Picture 4.2 consists of two images: the one on the right (picture 4.2B) is an enlarged view of part of Picture 4.1 and the one on the left (picture 4.2A) is an enlarged image of the same part of the sky taken some time earlier.

   Picture 4.3 presents a sky map which includes the region shown in the CCD images.

   The stars in the images are far away and should ideally be seen as point sources. However, diffraction on the telescope aperture and the effects of atmospheric turbulence (known as 'seeing') blur the light from the stars. The brighter the star, the more of the spread-out light is visible above the level of the background sky. ((I10 - D01 - B))

   Questions:

   (a) Identify any 5 bright stars (mark them by Roman numerals) from the image and mark them on both the image and map.

   (b) Mark the field of view of the camera on the map.

   (c) Use this information to obtain the physical dimensions of the CCD chip in mm.

   (d) Estimate the size of the blurring effect in arcseconds by examining the image of the star in Picture 4.2. (Note that due to changes in contrast necessary for printing, the diameter of the image appears to be about 3.5 times the full width at half maximum (FWHM) of the profile of the star.)

   (e) Compare the result with theoretical size of the diffraction disc of the telescope.

   (f) Seeing of 1" often considered to indicate good conditions. Calculate the size of the star image in pixels if the atmospheric seeing was 1" and compare it with the result from question 1d.

   (g) Two objects observed moving relative to the background stars have been marked on Picture 4.1. The motion of one ("Object 1") was

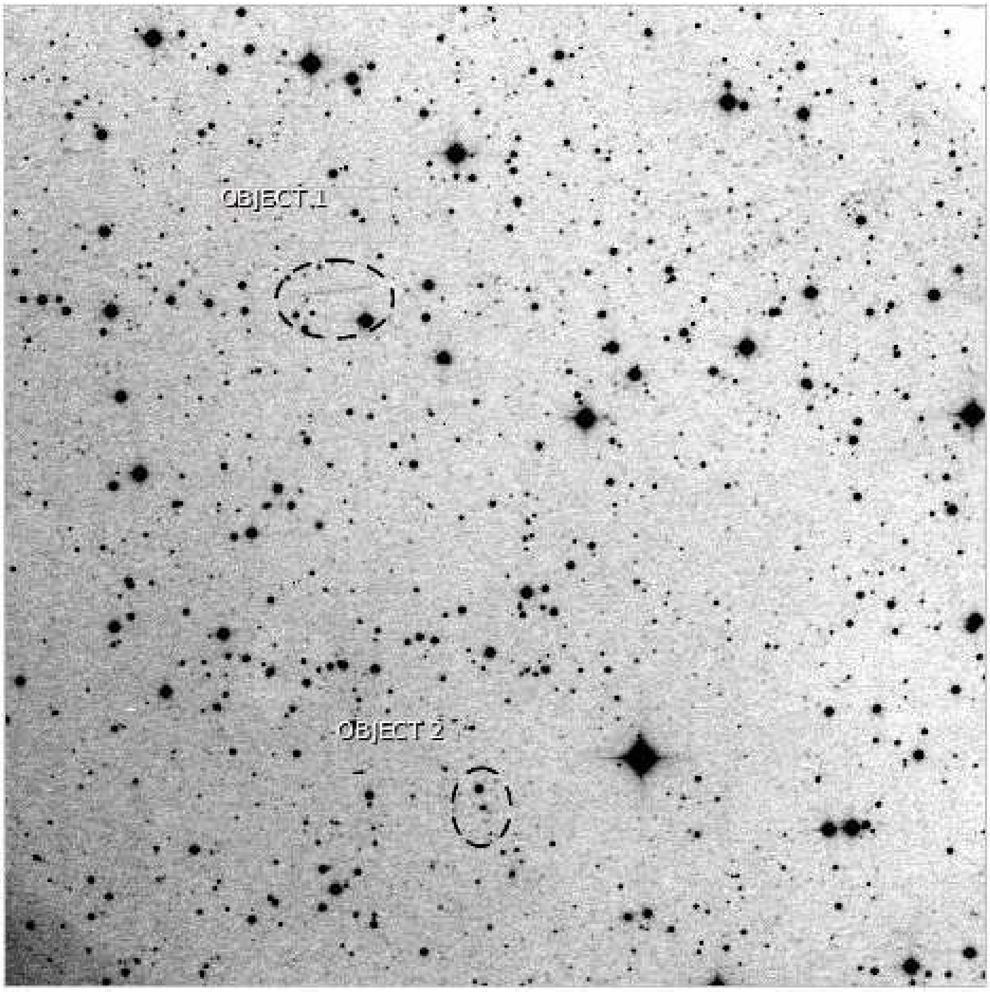


Figure 4.1 – CCD photograph showing part of the sky

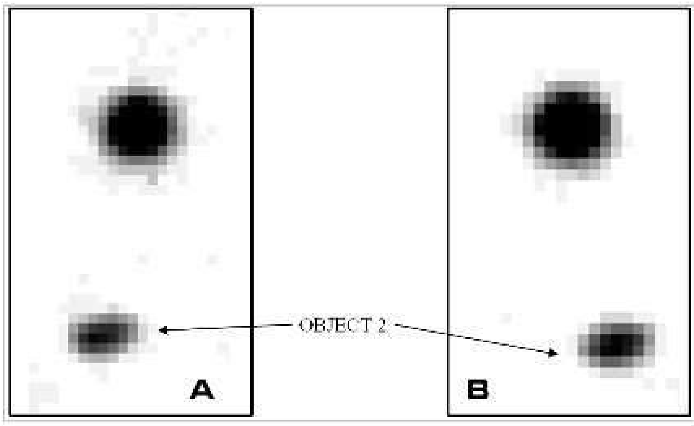


Figure 4.2 – In this image, B is simply enlargement of picture 4.1 around Object 2. On the other hand A shows the same area observed some time earlier.

fast enough that it left a clear trail on the image. The motion of the other ("Object 2") is more easily seen on the enlarged image (Picture 4.2B) and another image taken some time earlier (Picture 4.2A). Using the above results, determine the angular velocity on the sky of both objects. Choose which of the statements in the list below are correct, assuming that the objects are moving on circular orbits.

The probable causes of the different angular velocities are:
– different masses of the objects,
– different distances of the objects from Earth,
– different orbital velocities of the objects,
– different projections of the objects' velocities,

For Picture 4.1, the data header file is as fllows:

```
BITPIX  =    16                     / Number of bits per pixel
NAXIS   =     2                     / Number of axes
NAXIS1  =  1024                     / Width of image  (in pixels)
NAXIS2  =  1024                     / Height of image (in pixels)
DATE-OBS=  2010-09-07  05:00:40.4   / Middle of exposure
TIMESYS =   UT                      / Time Scale
EXPTIME =   300.00                  / Exposure time (seconds)
OBJCTRA =  22 29 20.031             / RA of center of the image
OBJCTDEC= +07 20 00.793             / DEC of center of the image
```

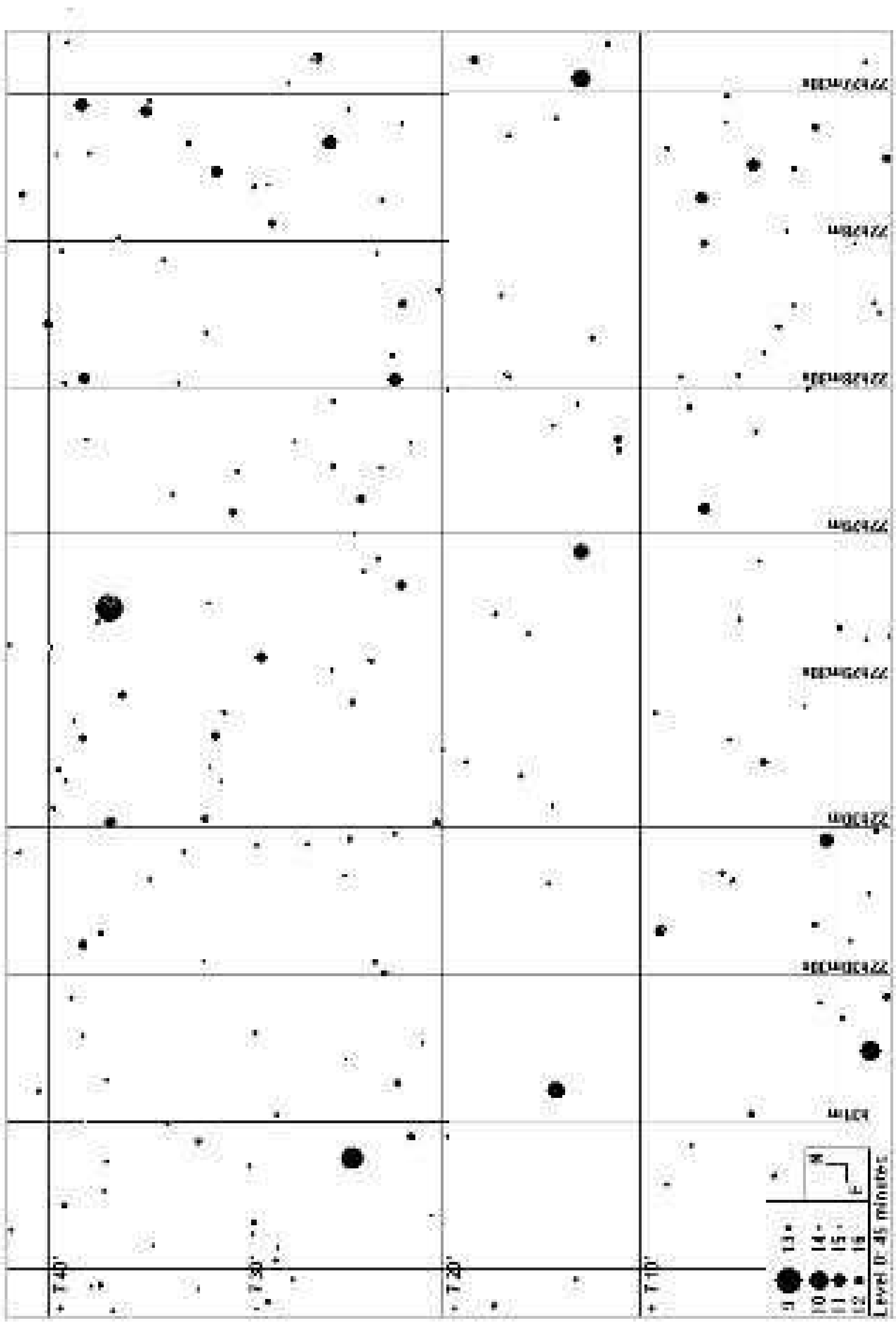


Figure 4.3 – Sky Map which contains corresponding region

Telescope focal length	1.20 m
CCD pixel size	$25 \times 25 \mu m$
Exposure time	450 s
Telescope zenith angle	25°
Average extinction coefficient in band	0.3 mag/airmass

Table 4.1 – Telescope Data

Table 4.2 – Stellar Magnitudes

star	m_t
1	9.03
3	6.22
4	8.02

```
FOCALLEN=    3.180m                    / Focal length of the telescope
TELESCOP=    0.61m                     / Telescope aperture
```

In picture 4.2A, all the data above remains the same except

```
DATE-OBS= 2010-09-07  04:42:33.3    / Middle of exposure
```

2. **CCD Image Processing:** As an exercise of image processing, this problem involves use of a simple calculator and tabular data (table 4.3) which contains the pixel values of an image during the given exposure time. This image, which is a part of a larger CCD image, was taken by a small CCD camera, installed on an amateur telescope and using a V band filter. Figure 4.4 shows this 50×50 pixels image that contains 5 stars.

   In table 4.3 the first row and column indicates the pixels' x and y coordinates. Table 4.1 gives the telescope and the image specifications. *EA:* In the IOAA 2009, this table was printed on a single A3 sheet as 50×50 grid. Here this table is divided in two parts to fit to the size of the book. Part II of the table should be put to the **right** of part I to complete the grid.

   The observer identified stars 1, 3 and 4 by comparing this image with standard star catalogues. Table 4.2 shows stars true magnitudes (m_t) as given in the catalogue.

   (a) Using the available data, determine the instrumental magnitudes of the stars in the image. Assume the dark current is negligible and the image is flat fielded. For simplicity you can use a square aperture.
   Note: The instrumental magnitude is calculated using the differ-

2	4	4	5	6	6	0	9	0	5	3	1	1	4	5	2	5	5	3	8	5	6	3	1	9
10	1	6	8	10	0	6	4	7	1	9	2	3	8	3	4	8	8	2	9	5	9	7	0	9
7	6	6	4	6	5	2	10	4	3	2	2	7	1	6	4	9	3	6	3	7	3	1	6	4
5	8	2	8	1	3	4	4	4	4	3	5	6	8	2	8	9	2	6	1	9	3	8	6	1
7	1	0	9	2	5	8	5	2	5	8	3	1	6	2	9	4	3	8	4	0	6	7	6	8
2	8	7	5	1	2	8	2	8	7	4	10	4	4	8	8	9	1	2	7	2	8	4	3	2
10	2	6	4	0	9	10	8	7	0	9	0	9	1	1	0	7	3	2	4	9	7	6	8	1
3	7	1	4	5	2	5	8	1	7	6	8	8	10	5	9	2	5	7	8	1	2	4	4	8
10	6	10	4	9	7	3	1	9	7	5	7	3	1	3	6	8	1	3	6	4	9	4	9	1
8	2	8	7	8	2	0	9	4	2	7	7	4	8	0	9	2	6	7	0	3	7	7	0	2
6	5	1	10	4	7	5	8	0	9	3	4	7	2	9	9	8	5	1	1	2	1	9	8	1
8	5	10	8	1	3	2	6	4	1	10	5	0	10	1	8	4	0	5	6	4	9	7	3	6
3	6	7	6	4	4	4	1	12	13	7	4	4	8	5	2	3	7	4	9	3	6	6	3	9
2	2	2	10	4	10	0	9	202	798	206	5	1	1	7	5	1	7	2	1	1	5	6	3	3
1	8	5	6	6	1	6	14	803	3239	804	18	5	9	4	6	3	6	3	6	1	3	3	2	7
6	3	10	8	6	8	3	5	204	798	206	6	4	9	9	5	8	8	0	4	5	7	2	8	2
7	1	10	3	2	10	9	4	7	13	8	3	10	5	6	5	3	0	3	8	5	3	6	3	0
3	6	8	9	6	1	9	2	3	4	1	6	7	0	10	7	2	3	2	6	3	2	8	4	6
4	3	0	6	3	7	4	8	10	8	6	7	6	1	6	0	10	4	4	10	6	2	2	7	5
6	7	5	5	9	6	2	8	9	10	6	5	8	10	2	1	7	9	3	7	3	1	7	9	10
1	9	6	4	8	3	5	5	1	8	7	1	8	5	4	0	1	6	5	2	8	2	5	10	4
4	7	7	2	8	6	7	9	9	1	8	6	9	9	2	8	5	4	9	1	9	8	10	5	10
4	5	8	9	5	6	7	8	6	6	8	5	3	1	1	6	1	4	3	1	3	43	165	40	11
4	9	7	1	5	2	4	5	9	2	2	10	2	3	8	4	4	7	4	10	40	2665	10786	2667	42
5	7	4	8	1	7	7	2	4	6	6	3	9	9	1	9	6	1	4	10	165	10785	43705	10780	170
1	5	4	0	3	3	2	6	4	9	1	2	3	0	1	0	9	6	3	9	45	2663	10783	2658	47
4	6	1	1	7	9	3	1	9	4	8	1	2	10	2	2	8	6	6	8	4	44	164	42	3
1	9	4	1	3	7	6	10	9	9	3	0	4	5	5	4	6	7	5	6	7	2	10	1	6
4	0	2	9	1	5	7	8	5	7	8	2	1	9	9	2	7	10	5	1	3	4	9	6	0
2	1	8	8	6	4	5	4	3	1	9	2	0	6	3	7	10	5	2	8	2	10	2	5	7
3	1	2	5	5	8	4	5	3	1	10	4	0	7	4	3	2	3	9	5	8	3	1	9	5
3	5	4	8	1	3	9	3	5	9	1	9	6	9	8	5	3	6	8	4	4	1	2	2	7
3	8	3	6	7	9	3	0	1	3	8	2	3	8	5	1	1	7	9	9	7	5	5	7	5
5	7	0	9	3	5	1	8	0	8	7	6	5	6	1	8	1	4	5	3	5	7	5	10	9
7	6	6	8	9	7	2	8	9	9	5	6	6	8	3	9	6	9	6	2	1	3	3	8	3
1	10	2	8	8	3	0	7	1	5	3	4	2	7	1	0	5	1	5	0	6	1	9	6	0
7	9	7	2	5	4	8	0	2	5	3	1	1	3	3	4	9	0	7	2	5	9	3	6	2
6	1	0	6	4	5	7	6	7	7	9	9	9	5	7	6	2	3	3	8	7	7	5	5	9
3	2	2	1	8	2	1	6	3	3	10	7	7	7	6	3	1	3	1	1	3	1	1	7	9
1	8	5	9	2	5	1	0	9	7	10	7	7	5	1	9	3	7	9	9	7	6	4	4	8
6	0	6	7	7	4	8	10	7	9	6	3	6	7	0	1	2	9	2	4	9	2	7	3	3
1	7	5	2	3	7	4	9	1	8	2	9	5	7	2	9	2	7	8	2	0	9	1	5	2
1	7	1	6	2	10	4	8	7	7	2	7	4	8	5	6	0	5	4	3	7	10	6	5	3
8	3	1	0	3	4	8	7	7	7	9	2	1	6	2	7	10	10	4	1	7	5	5	10	4
5	10	8	8	4	10	8	5	9	6	3	2	4	5	8	2	2	2	9	2	9	3	0	1	6
1	6	3	0	9	5	4	0	9	6	2	7	8	6	7	7	4	6	3	3	2	4	8	2	4
7	4	2	8	8	2	4	8	3	4	8	1	2	8	0	10	1	6	8	6	8	0	6	2	10
1	8	2	2	5	5	2	2	5	6	4	1	5	6	5	8	4	7	8	6	8	4	0	8	3
8	4	2	7	1	0	3	5	1	1	8	9	8	1	1	5	7	6	5	3	8	6	3	3	6
6	2	8	4	6	7	5	3	1	2	2	3	5	3	2	5	1	3	3	1	1	9	6	6	2

Table 4.3 – Part I of the CCD readout table: Columns 1 to 25

ence between the measured *flux* from the star in the aperture and the *flux* from an equivalent area of dark sky.

(b) The instrumental magnitude of a star in a CCD image is related to true magnitude as

$$m_I = m_t + KX - Z_{mag} \tag{4.1}$$

where K is extinction, X is airmass, m_I and m_t are respectively instrumental and true magnitude of star and Z_{mag} is zero point constant. Calculate the zero point constant(Z_{mag}) for identified stars and hence, calculate average zero point constant (Z_{mag}). *Note:* Zero point constant is the constant difference between extinction-free instrumental magnitudes and the true magnitude.

(c) Calculate true magnitudes of stars 2 and 5.

(d) Calculate CCD pixel scale for the CCD camera in units of arcsec.

(e) Calculate average brightness of dark sky in magnitude per square arcsec (m_{sky}).

(f) Use a suitable plot to estimate astronomical seeing in arcsec. (I09 - D01 - C)

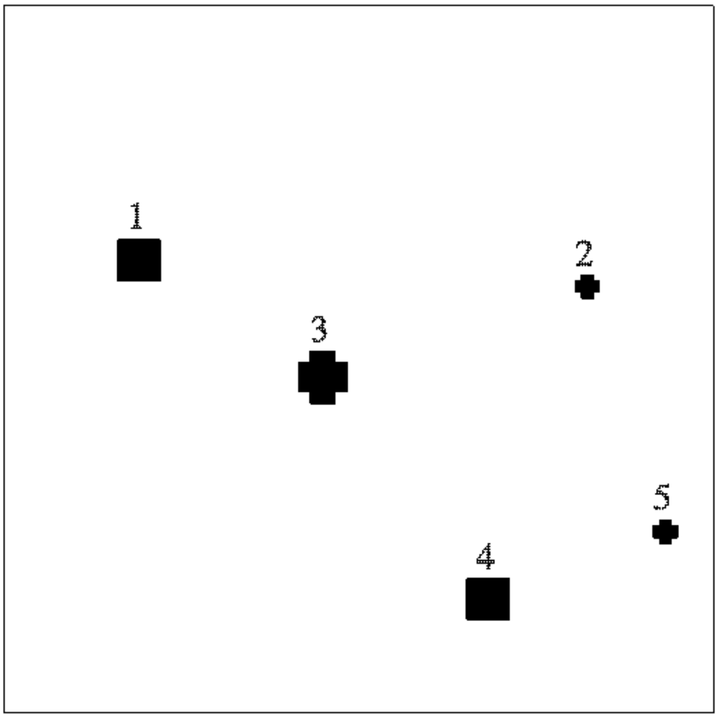


Figure 4.4 – CCD image

6	7	3	1	7	8	3	2	6	6	6	2	10	4	10	2	9	3	2	9	2	2	2	5	1
6	1	3	10	2	6	8	8	5	3	0	4	1	3	7	5	5	1	1	1	1	3	4	0	6
3	4	3	3	9	1	0	3	5	9	10	7	2	9	10	7	7	3	9	3	9	2	3	9	8
0	0	6	8	8	5	1	8	4	8	9	7	0	9	1	4	9	9	8	9	3	8	5	7	9
7	2	4	9	3	5	8	9	5	0	5	1	7	1	0	9	9	9	1	3	9	1	4	8	8
4	8	5	5	9	5	1	2	5	8	6	7	1	4	7	5	4	1	10	6	2	2	2	2	3
4	9	9	7	8	4	3	8	7	6	3	9	2	9	2	4	6	3	8	5	5	8	1	5	7
1	3	0	10	2	8	3	9	10	1	8	6	3	1	4	5	6	9	1	5	2	2	9	4	1
9	1	4	4	8	3	2	7	7	7	3	1	2	9	3	5	3	5	7	3	5	0	5	2	7
3	6	1	8	5	5	3	4	7	5	2	3	6	9	4	6	4	2	8	0	3	2	3	1	3
0	2	2	7	4	3	0	5	6	5	4	5	1	3	8	5	3	8	3	6	4	4	4	6	10
9	5	2	2	1	10	2	6	4	2	1	8	3	0	2	5	5	3	6	10	4	7	3	6	4
7	8	5	0	2	5	10	8	4	8	6	10	5	10	1	6	3	9	1	6	7	4	5	7	6
4	0	3	1	0	5	1	9	5	8	3	5	0	6	8	7	8	5	1	5	3	3	4	8	7
7	10	8	5	5	1	2	3	4	3	1	5	5	4	12	6	1	5	2	5	1	5	6	5	3
1	4	8	4	9	4	2	4	6	3	7	6	5	92	353	94	5	1	2	2	4	0	3	4	1
2	5	6	2	5	2	7	8	9	8	9	8	11	356	1426	355	9	1	9	6	3	1	8	3	9
8	7	3	8	0	0	9	6	7	6	9	8	8	89	351	96	6	8	4	7	4	4	0	9	3
2	3	0	6	6	2	9	0	7	4	3	2	1	6	12	8	2	9	7	3	2	8	7	7	8
8	4	8	10	7	4	1	3	6	9	9	2	8	9	10	5	8	5	7	7	8	6	9	3	5
2	8	7	5	6	2	1	5	1	10	4	6	1	10	7	8	9	5	3	6	1	6	10	9	2
3	7	0	4	5	7	0	9	5	7	6	5	1	9	10	4	9	1	8	5	3	7	8	7	4
10	7	3	8	1	2	4	7	4	5	2	8	10	1	8	0	5	1	9	1	3	5	7	9	1
4	6	8	10	1	9	7	10	3	1	10	6	6	2	2	9	10	2	3	10	9	6	5	6	1
10	6	5	3	9	8	10	4	9	10	10	3	3	10	0	9	2	4	4	1	5	10	3	3	6
10	2	0	4	5	5	4	6	8	5	2	5	10	4	5	9	6	9	8	1	1	1	1	5	0
3	9	3	9	6	4	7	3	1	7	8	3	4	3	10	6	5	6	0	0	8	8	6	9	1
9	1	5	8	5	9	5	2	2	0	0	1	3	6	6	2	2	1	1	10	7	7	9	0	2
8	3	6	0	3	3	5	7	0	6	6	0	9	9	3	3	3	3	3	5	1	1	4	6	6
7	9	5	0	6	5	1	5	9	7	7	7	6	4	6	1	3	8	9	6	3	8	3	8	5
7	8	7	5	0	8	6	0	1	5	5	6	0	5	9	1	6	2	6	0	5	3	2	2	4
2	2	6	5	1	2	0	9	6	8	5	5	9	0	6	7	5	10	2	5	7	7	1	3	4
1	9	7	2	9	4	9	2	6	4	2	2	0	10	3	2	4	4	1	6	9	4	9	2	3
10	8	10	4	4	1	1	3	8	6	4	5	8	9	4	8	9	10	27	94	24	2	2	5	10
8	2	9	7	1	9	2	8	9	5	2	2	0	5	7	3	9	11	95	355	96	7	7	8	8
3	5	4	1	8	1	2	1	5	8	6	5	5	0	2	8	10	10	23	94	25	8	7	0	8
5	3	3	5	8	5	1	2	6	7	10	7	4	6	2	9	6	6	9	11	1	6	2	10	2
1	7	3	1	2	5	2	4	1	8	1	0	3	4	2	6	4	2	4	9	6	6	7	3	0
4	2	8	0	5	5	4	0	1	8	1	6	5	7	7	9	0	2	1	3	8	2	8	6	2
3	7	4	2	1	5	2	6	5	0	14	38	12	7	7	6	0	2	3	5	4	5	4	3	4
3	7	9	6	0	6	0	0	7	15	505	2041	510	9	0	1	6	7	9	9	8	10	2	1	1
9	1	9	6	4	2	2	6	7	37	2040	8236	2031	35	7	7	1	4	1	7	2	5	2	0	8
3	6	2	6	2	4	1	3	3	16	508	2040	504	8	0	8	9	2	7	7	3	6	2	2	7
8	2	3	0	8	2	8	10	1	2	8	40	10	2	8	3	3	7	4	9	9	2	7	3	1
6	1	2	6	0	2	2	7	4	10	2	3	2	8	6	4	8	4	3	7	6	3	8	1	1
3	2	10	6	8	7	8	2	5	8	8	10	2	1	9	9	0	6	3	5	8	9	8	7	0
2	10	8	5	8	6	2	3	1	6	9	5	1	4	2	4	9	8	10	3	8	1	2	8	5
4	0	6	8	7	0	3	4	6	7	3	2	3	4	5	1	9	9	3	9	0	2	3	5	1
10	3	4	8	8	10	8	8	4	2	8	2	6	3	4	8	2	9	3	3	6	2	0	9	8
7	7	2	9	8	2	2	3	3	7	9	5	2	4	4	2	5	3	4	4	7	7	0	1	6

Table 4.4 – Part II of the table: Columns 26 to 50

Chapter 5

Physics of Stars and Planets

5.1 Theory

1. At what wavelength does a star with the surface temperature of 4000 °K emit most intensely? (I07 - T08 - A)
2. Estimate the effective temperature of the photosphere of the Sun using the naked eye colour of the Sun. (I10 - T13 - A)
3. The difference in brightness between two main sequence stars in an open cluster is 2 magnitudes. Their effective temperatures are 6000K and 5000K respectively. Estimate the ratio of their radii. (I10 - T12 - A)
4. Luminous Blue Variable (LBV) stars greatly vary in visual brightness; however, their bolometric magnitude remains constant. Imagine a LBV star with a black body temperature of 5000 K at its maximum visual brightness, and 30,000 K at its minimum visual brightness. Calculate the ratio of the star radius between both situations above. (I12 - T07 - A)
5. Calculate the total luminosity of a star whose surface temperature is 7500 K, and whose radius is 2.5 times that of our Sun. Give your answer in units of the solar luminosity, assuming the surface temperature of the Sun to be 5800 K. (I07 - T09 - A)
6. If the escape velocity from a solar mass objects's surfce exceeds the speed of light, what would be its radius? (I10 - T02 - A)
7. If the Sun were to collapse gravitationally to form a non-rotating black hole, what would be the radius of its event horizon (its Schwarzschild radius)? (I07 - T14 - A)

8. Calculate the mean mass density for a super massive black hole with total mass $1 \times 10^8 M_\odot$ inside the Schwarzschild radius. (I09 - T01 - A)
9. The radiation incoming to the Earth from the Sun must penetrate the Earth's atmosphere before reaching the earth surface. The Earth also releases radiation to its environment and this radiation must penetrate the Earth's atmosphere before going out to the outer space. In general, the transmittance (t_1) of the Sun radiation during its penetration to the Earth's atmosphere (t_1) is higher than that of the radiation from the Earth (t_2). Let $T_\odot$ be the effective temperature of the Sun, $R_\odot$ the radius of the Sun, $R_\oplus$ the radius of the Earth, and $d_{\oplus-\odot}$ the distance between the Sun and the Earth. Derive the temperature of the Earth's surface as a function of the aforementioned parameters. (I08 - T09 - B)
10. Most of the energy emitted by the Sun is generated in its core via the so-called proton-proton (p-p) nuclear chain reaction, which has three different branches. The most energetic branch transforms $2He^3$ into $He^4 + 2H^1$. Calculate the energy released (in MeV) and the fractional reduction of the mass of the particles involved in this reaction. (I12 - T06 - A)
11. Suppose a star has a mass of $20M_\odot$. If 20% of the star's mass is now in the form of helium, calculate the helium-burning lifetime of this star. Assume that the luminosity of the star is $100L_\odot$, in which 30% is contributed by helium burning.Assume no additional helium is being produced in this time and all helium is available for burning.is 12.000000 amu. The fusion of helium into carbon by triple-α process is given by $3\ ^4He \rightarrow ^{12} C + \gamma$ (I08 - T04 - B)
12. A main sequence star at a distance of 20 pc is barely visible through a certain space-based telescope which can record all wavelengths. The star will eventually move up along the giant branch, during which time its temperature drops by a factor of 3 and its radius increases 100-fold. What is the new maximum distance at which the star can still be (barely) visible using the same telescope? (I08 - T07 - B)
13. Estimate the number of solar neutrinos which should pass through a $1m^2$ area of the Earth's surface perpendicular to the Sun every second. Use the fact that each fusion reaction in the Sun produces 26.8 MeV of energy and 2 neutrinos. (I11 - T14 - B)
14. **A Planet & Its Surface Temperature:** A fast rotating planet of radius R with surface albedo α is orbiting a star of luminosity L. The orbital radius is d. It is assumed here that, at equilibrium, all of the

energy absorbed by the planet is re-emitted as a blackbody radiation. (I07 - T16 - B)

(a) What is the radiation flux from the star at the planet's surface?

(b) What is the total rate of energy absorbed by the planet?

(c) What is the reflected luminosity of the planet?

(d) What is the average blackbody temperature of the planet's surface?

(e) If we were to assume that one side of the planet is always facing the star, what would be the average surface temperature of that side?

(f) For the planet in problem 14d, if $\alpha = 0.25$, $d = 1.523$ A.U. and $L = L_\odot$, calculate surface temperature of the planet in Kelvins.

15. Estimate the minimum energy a proton would need to penetrate the Earth's magnetosphere. Assume that the initial penetration is perpendicular to a belt of constant magnetic field 30 μ T and thickness 1.0×10^4 km. Prepare the sketch of the particle trajectory. (Note that at such high energies the momentum can be replaced by the expression E/c. Ignore any radiative effects). (I11 - T10 - B)

16. A main sequence star with the radius and mass of $R = 4R_\odot$, $M = 6M_\odot$ has an average magnetic field of 1×10^{-4}T. Calculate the strength of the magnetic field of the star when it evolves to a neutron star with the radius of 20 km. (I09 - T10 - B)

17. If 0.8% of the initial total solar mass could be transformed into energy during the whole life of the Sun, estimate the maximum possible life time for the Sun. Assume that the solar luminosity remains constant. (I10 - T05 - B)

18. Calculate how much the radius of the Earth's orbit increases as a result of the Sun losing mass due to the thermo-nuclear reactions in its center in 100 years. Assume that the Earth's orbit remains circular during this period. (I09 - T12 - C)

19. A pulsar, located 1000 pc far from Earth and 10,000 times more luminous than our Sun, emits radiation only from its two opposite poles, creating an homogeneous emission beam shaped as double cone with opening angle $\alpha = 4°$. Assuming the angle between the rotation axis and the emission axis is 30°, and assuming a random orientation of the pulsar beams in relation to an observer on Earth, what is the probability of detecting the pulses? In case we can see it, what is the apparent bolometric magnitude of the pulsar? (I12 - T08 - C)

5.2 Data Analysis

1. A few facts about photometry of asteroids:

   – Asteroids are small, irregularly shaped objects of our Solar System that orbit the Sun in approximately elliptical orbits.
   – Their brightness, observed at a given instant from Earth, depends on the surface area illuminated by the Sun and the part of the asteroid which is visible to the observer. Both vary as the asteroid moves.
   – The way the sunlight is reflected by the surface of the asteroid depends on its texture and on the angle between the Sun, the asteroid and the observer (phase angle), which varies as the Earth and the asteroid move along their orbits. In particular, asteroids with surfaces covered by fine dust (called regoliths) exhibit a sharp increase in brightness at phase angles φ close to zero (i.e., when they are close to the opposition).
   – Since the observed flux of any source decreases with the square of the distance, the observed magnitude of an asteroid also depends on its distance from the Sun and from the observer at the time of the observation. Their apparent magnitude (m) outside the atmosphere is then

   $$M(t) = m_r(t) + 5\log(RD) \tag{5.1}$$

   where m_r is usually called reduced magnitude (meaning the magnitude the asteroid would have if its distances from the Sun and the Earth were reduced both to 1 AU) and depends only on the visible illuminated surface area and on phase angle effects. R and D are the heliocentric and geocentric distances respectively.

   Consider now the following scenario. Light curves of a given asteroid were obtained at three different nights at different points of its orbit, and at each time a photometric standard star was observed in the same frame of the asteroid. Table 4.1 shows the geometric configuration of the asteroid at each night (phase angle φ in degrees, R and D in AUs), and the catalogued calibrated magnitude of the same standard star. Consider this calibrated magnitude as the final apparent magnitude after correcting for all the extinction effects.

   Table 5.2 contains the time of each observation (in hours) as an offset from the first observation of that night, the air mass and the uncalibrated magnitude of the asteroid and the uncalibrated magnitude of the star for each night.

Air mass is the dimensionless thickness of the atmosphere along the line of sight, and is equal to 1 when looking to zenith.

(a) Plot the star's uncalibrated magnitude versus air mass for each set.

(b) Calculate the extinction coefficient for each night. Note that the magnitude outside the atmosphere is given by:

$$M = m - \beta X + A \tag{5.2}$$

where β is the extinction coefficient, A is the zero point coefficient for the night, X is the airmass of the observation and m is the magnitude measured from the ground.
You are also given that in the method of least squares estimation of angular coefficients,
For a straight lines described by,

$$y_i = \alpha + \beta x_i \tag{5.3}$$

$$\beta = \frac{\sum\limits_{i=1}^{n} x_i y_i - \frac{1}{n}\sum\limits_{i=1}^{n} x_i \sum\limits_{i=1}^{n} y_i}{\sum\limits_{i=1}^{n}(x_i)^2 - \frac{1}{n}(\sum\limits_{i=1}^{n} x_i)^2} \tag{5.4}$$

(c) Were the observations affected by clouds in one night?

(d) Plot the corrected magnitude of the asteroid against time for each set of observations. If the standard star is in the same frame as the object, the corrected magnitude of the object can be obtained by:

$$M_{ast} = m_{ast} - m_{star} + M_{star} \tag{5.5}$$

where capital letters denote corrected and small letters denote uncorrected magnitudes.

(e) Determine the rotation period of the asteroid for each night. Consider that the light curve for this asteroid has two minima and two maxima, and that the semi-period is an average of intervals between the two maxima and the two minima.

(f) Determine the amplitude (difference from maximum to minimum) of the light curve for each night.

(g) Plot the corrected reduced magnitude m_r (use the mean value of each light curve) versus phase angle φ.

Night	D	R	φ	M_{star}
A	0.36	1.35	0.0	8.2
B	1.15	2.13	8.6	8.0
C	2.70	1.89	15.6	8.1

Table 5.1 – Geometric configuration of the asteroid on different nights

(h) Calculate the angular coefficient (slope) of the phase curve (the plot in the previous part) considering only the points away from the opposition.

(i) Is there any reason to assume a surface covered by fine dust (regolith)? ((I12 - D01 - B))

2. **The age of Meteorite:** The basic equation of radioactive decay can be written as $N(t) = N_0 e^{-\lambda t}$, where $N(t)$ and N_0 are the number of remaining atoms of the radioactive isotope (or parent isotope) at time t and its initial number at $t = 0$, respectively, while λ is the decay constant. The decay of the parent produces daughter nuclides $D(t)$, or radiogenics, which is defined as $D(t) = N_0 - N(t)$.

   A group of astronomers investigated a number of meteorite samples to determine their ages. They took samples from two meteorites, one containing Allende chondrites (A) and the other one containing basaltic Achondrite (B). By examining these samples, they measured the abundance of ^{87}Rb and ^{87}Sr assuming that ^{87}Sr was entirely produced by the decay of ^{87}Rb. The value of λ is 1.42×10^{-11} per year for this isotopic decay of ^{87}Rb. In addition, quantities non-radiogenic element ^{86}Sr are also measured. The results of measurement are given in the table below, expressed in ppm (part per million). (I08 - D03 - C)

Sample	**Type**	^{86}Sr **(ppm)**	^{87}Rb **(ppm)**	^{87}Sr **(ppm)**
1	A	29.6	0.3	20.7
2	B	58.7	68.5	44.7
3	B	74.2	14.4	52.9
4	A	40.2	7.0	28.6
5	A	19.7	0.4	13.8
6	B	37.9	31.6	28.4
7	A	33.4	4.0	23.6
8	B	29.8	105.0	26.4
9	A	9.8	0.8	6.9
10	B	18.5	44.0	15.4

Δt	Air	Night A		Night B		Night C	
(hr)	mass	m_{ast}	m_{star}	m_{ast}	m_{star}	m_{ast}	m_{star}
0	1.28	7.44	8.67	11.64	8.58	13.24	8.38
0.44	1.18	7.38	8.62	11.53	8.54	13.21	8.36
0.89	1.11	7.34	8.59	11.56	8.6	13.13	8.34
1.33	1.06	7.28	8.58	11.49	8.52	13.11	8.33
1.77	1.02	7.32	8.58	11.58	8.48	13.11	8.32
2.21	1	7.33	8.56	11.79	8.63	13.15	8.32
2.66	1	7.33	8.56	11.67	8.53	13.17	8.32
3.1	1.01	7.3	8.56	11.53	8.46	13.17	8.32
3.54	1.03	7.27	8.58	11.47	8.48	13.13	8.33
3.99	1.07	7.27	8.58	11.63	8.67	13.15	8.34
4.43	1.13	7.31	8.61	11.51	8.51	13.14	8.34
4.87	1.21	7.37	8.63	11.65	8.55	13.14	8.37
5.31	1.32	7.42	8.67	11.77	8.61	13.21	8.38
5.76	1.48	7.49	8.73	11.88	8.75	13.3	8.43
6.2	1.71	7.59	8.81	11.86	8.78	13.34	8.47
6.64	2.06	7.69	8.92	12.03	9.03	13.39	8.54
7.09	2.62	7.87	9.14	12.14	9.19	13.44	8.65
7.53	3.67	8.21	9.49	12.63	9.65	13.67	8.87

Table 5.2 – Magnitudes of the asteroid and reference star for three nights (A, B and C) at different times

(a) Express the time t in term of $\frac{D(t)}{N(t)}$.

(b) Determine the half-life $T_{1/2}$, i.e. the time required to obtain a half number of parents after decay.

(c) Knowledge of the ratio between two isotopes is more valuable than just the absolute abundance of each isotop. It is quite likely that there was some initial strontium present. by taking $\frac{^{87}Rb}{^{86}Sr}$ as an independent variable and $\frac{^{87}Sr}{^{86}Sr}$ as dependent variable, estimate a simple linear regression model that describes the data.

(d) Plot $\frac{^{87}Rb}{^{86}Sr}$ versus $\frac{^{87}Sr}{^{86}Sr}$ and also the regression line for each type of the meteorites.

(e) Subsequently, help the astronomers to determine the age of each type of the meteorites and error in its estimation. Which type is older?

(f) Determine the initial value of $\left(\frac{^{87}Sr}{^{86}Sr}\right)_0$ for each type of meteorites along with errors.

notes:

– *EA:* Due to unique nature of this problem, you may have to use minimum 7 decimal digits for intermediate calculations. This requirement is not consistant with regular practice of deciding number of decimal digits as per significant figures. Nonetheless, it is essential.

– If we fit a straight line $Y = a + bX$, to a set of points (X_i, Y_i) via simple linear regression, then

$$b = \frac{SS_{xy}}{SS_{xx}} \tag{5.6}$$

$$a = \overline{y} - b\overline{x} \tag{5.7}$$

$$S_a = \sqrt{\frac{SS_{yy} - \dfrac{SS_{xy}^2}{SS_{xx}}}{(n-2)SS_{xx}} \times \sum X_i^2} \tag{5.8}$$

$$S_b = \sqrt{\frac{SS_{yy} - \dfrac{SS_{xy}^2}{SS_{xx}}}{(n-2)SS_{xx}}} \tag{5.9}$$

where S_a and S_b are std. deviations in the estimation of a and b,

$$SS_{xx} = \sum_{i=1}^{n} X_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right)^2 \tag{5.10}$$

$$SS_{yy} = \sum_{i=1}^{n} Y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} Y_i\right)^2 \tag{5.11}$$

$$SS_{xy} = \sum_{i=1}^{n} X_i Y_i - \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right)\left(\sum_{i=1}^{n} Y_i\right) \tag{5.12}$$

Chapter 6

Stellar Observations

6.1 Theory

1. In a binary system, the apperant magnitude of the primary star is 1.0 and that of the secondary star is 2.0 Find the maximum combined magnitude of this system. (I10 - T01 - A)
2. A supernova shines with a luminosity 10^{10} times that of the Sun. If such a supernova appears in our sky as bright as the Sun, how far away (in parsec) from us must it be located? (I07 - T11 - A)
3. The magnitude of the faintest star you can see with naked eyes is $m = 6$, whereas that of the brightest star in the sky is $m = -1.5$. What is the energy-flux ratio of the faintest to that of the brightest? (I07 - T15 - A)
4. What would be full Moon's visual magnitude if its albedo was equal to 1? (I12 - T04 - A)
5. A K-type star on the Main Sequence has a luminosity of $0.4L_{\odot}$. This star is observed to have a flux of $6.23 \times 10^{-14} W.m^{-2}$. What is the distance (in parsec) to this star? You may neglect the atmospheric effect. (I07 - T10 - A)
6. Estimate the number of photons per second that arrive on our eye at $\lambda = 550$ nm (V-band) from a G2 main sequence star with apparent magnitude of m = 6 (the threshold of naked eye visibility). Assume all the radition from this star is in $\lambda = 550$ nm. (I09 - T02 - A)
7. We are interested in finding habitable exoplanets. One way to achieve this is through the dimming of the star, when the exoplanet transits across the stellar disk and blocks a fraction of the light. Estimate the maximum luminosity ratio for an Earth-like planet orbiting a star sim-

ilar to the Sun. (I10 - T07 - B)

8. A star has a measured I-band magnitude of 22.0. How many photons per second are detected from this star by the Gemini Telescope(8m diameter)? Assume that the overall quantum efficiency is 40% and the filter passband is flat. (I10 - T09 - B)

Filter	λ_0 (nm)	$\Delta\lambda$ (nm)	F_{Vega} $(Wm^{-2}nm^{-1})$
I	8.00×10^2	24.0	8.30×10^{-12}

9. What is the angular amplitude of the oscillatory motion of the Sun, due to the existence of Jupiter, as measured by an observer located at Barnard's Star? What is the period of this oscillation? (I11 - T11 - B)

10. An old planetary nebula, with a white dwarf (WD) in its center, is located 50 pc away from Earth. Exactly in the same direction, but behind the nebula, lies another WD, identical to the first, but located at 150 pc from the Earth. Consider that the two WDs have absolute bolometric magnitude +14.2 and intrinsic color indexes B - V = 0.300 and U - V = 0.330. Extinction occurs in the interstellar medium and in the planetary nebula.

    When we measure the color indices for the closer WD (the one who lies at the center of the nebula), we find the values B - V = 0.327 and U - B = 0.038. In this part of the Galaxy, the interstellar extinction rates are 1.50, 1.23 and 1.00 magnitudes per kiloparsec for the filters U, B and V, respectively. Calculate the color indices as they would be measured for the second star. (I12 - T09 - C)

11. An astronomer on Earth observes a globular cluster, which has an angular diameter α and contains N stars, each one with the same absolute magnitude M_0, and is at a distance D from the Earth. A biologist is in the center of that cluster. (I12 - T16 - C)

    (a) What is the difference between the combined visual magnitudes of all stars observed by the astronomer and the biologist. Consider that the spatial distribution of stars in the cluster is perfectly homogeneous and the biologist is measuring the combined magnitude of the entire cluster.

    (b) What is the diameter of the astronomer's telescope, considering he wants to visualize the cluster with the same brightness that the biologist sees?

    (c) What would be the difference between the visual magnitudes observed by the two scientists, if the diameter of the field of view of the biologist is also α.

12. A UBV photometric (UBV Johnson's) observation of a star gives $U = 8.15$, $B = 8.50$, and $V = 8.14$. Based on the spectral class, one gets the intrinsic colour $(U - B)_{\circ} = -0.45$. If the star is known to have radius of $2.3R_{\odot}$, absolute bolometric magnitude of -0.25, and bolometric correction (BC) of -0.15, determine:
    (a) the intrinsic magnitudes U, B, and V of the star,
    (b) the effective temperature of the star,
    (c) the distance to the star.

    Note: Take, for the typical interstellar matters, the ratio of total to selective extinction $R_V = 3.2$ and the colour excess in $(B - V)$ to be about 72% of the colour excess in $(U - B)$. (I08 - T17 - D)

6.2 Data Analysis

1. **Light curves of stars:** A pulsating variable star KZ Hydrae was observed with a telescope equipped with a CCD camera. Figure 6.1 shows a CCD image of KZ Hyd marked together with the comparison star and the check star. Table 6.1 lists the observation time in Heliocentric Julian dates, the magnitude differences of KZ Hyd and the check star relative to the comparison star in V and R band.
    (a) Draw the light curves of KZ Hyd relative to the comparison star in V and R band, respectively.
    (b) What are the average magnitude differences of KZ Hyd relative to the comparison star in V and R, respectively?
    (c) What are the photometric precisions in V and R, respectively?
    (d) Estimate the pulsation periods of KZ Hyd in V and R.
    (e) Give the estimation of the peak-to-peak pulsation amplitudes of KZ Hyd in V and R
    (f) What is the phase delay between the V and R bands, in term of the pulsation period? ((I10 - D02 - A))

2. Determination of stellar masses in a visual binary system: The star α-Centauri (Rigel Kentaurus) is a triple star consists of two main-sequence stars α- Centauri A and α-Centauri B representing a visual binary system, and the third star, called Proxima Centauri, which is smaller, fainter and farther than the other two stars. The angular distance between α-Centauri A and α-Centauri B is 17.59". The binary system has an orbital period of 79.24 years. The visual magnitudes of

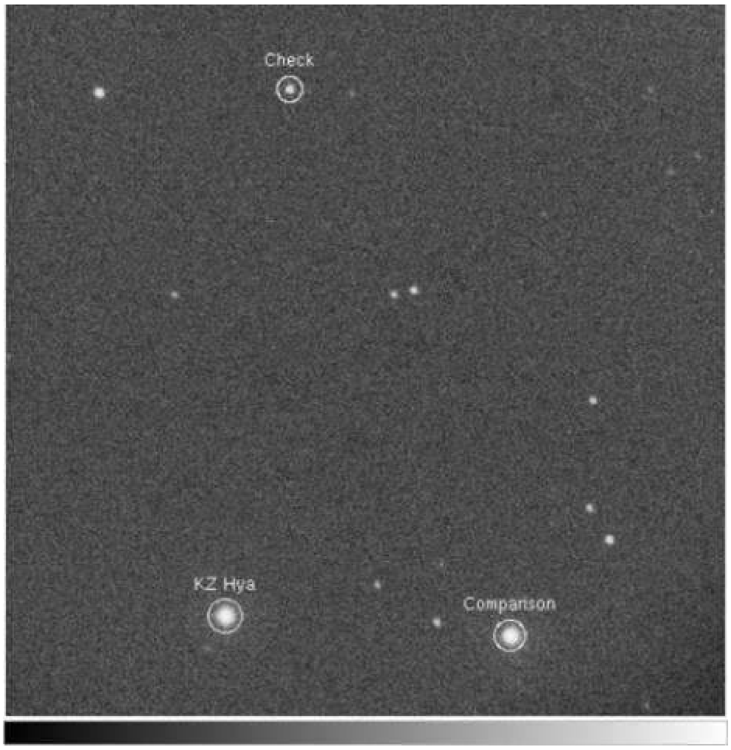


Figure 6.1 – Image showing star KZ Hydrae

$HJD*$	δV	δV_{chk}	$HJD*$	δR	δR_{chk}
3.162	0.068	4.434	3.1679	0.260	2.789
3.1643	0.029	4.445	3.1702	0.185	2.802
3.1667	-0.011	4.287	3.1725	-0.010	2.789
3.1691	-0.100	4.437	3.1749	-0.147	2.809
3.1714	-0.310	4.468	3.1772	-0.152	2.809
3.1737	-0.641	4.501	3.1796	-0.110	2.789
3.1761	-0.736	4.457	3.1820	-0.044	2.803
3.1784	-0.698	4.378	3.1866	0.075	2.805
3.1808	-0.588	4.462	3.1890	0.122	2.793
3.1831	-0.499	4.326	3.1914	0.151	2.793
3.1855	-0.390	4.431	3.1938	0.177	2.782
3.1878	-0.297	4.522	3.1962	0.211	2.795
3.1902	-0.230	4.258	3.1986	0.235	2.796
3.1926	-0.177	4.389	3.2011	0.253	2.788
3.195	-0.129	4.449	3.2035	0.277	2.796
3.1974	-0.072	4.394	3.2059	0.288	2.783
3.1998	-0.036	4.362	3.2083	0.296	2.796
3.2023	-0.001	4.394	3.2108	0.302	2.791
3.2047	0.016	4.363	3.2132	0.292	2.806
3.2071	0.024	4.439	3.2157	0.285	2.779
3.2096	0.036	4.078	3.2181	0.298	2.779
3.2120	0.020	4.377	3.2206	0.312	2.787
3.2145	0.001	4.360	3.2231	0.313	2.804
3.2169	0.001	4.325	3.2255	0.281	2.796
3.2194	0.005	4.355	3.2280	0.239	2.795
3.2219	0.041	4.474	3.2306	0.115	2.792
3.2243	0.009	4.369	3.2330	-0.111	2.788
3.2267	-0.043	4.330	3.2354	-0.165	2.793
3.2293	-0.183	4.321	3.2378	-0.152	2.781
3.2318	-0.508	4.370	3.2403	-0.088	2.787
3.2342	-0.757	4.423	3.2428	-0.014	2.780
3.2366	-0.762	4.373	3.2452	0.044	2.766
3.2390	-0.691	4.427	3.2476	0.100	2.806
3.2415	-0.591	4.483	3.2500	0.119	2.791
3.2440	-0.445	4.452	3.2524	0.140	2.797
3.2463	-0.295	4.262	3.2548	0.190	2.825

Table 6.1 – Data for the light curves of KZ Hyd in V and R. δV and δR are KZ Hyd relative to the comparison star in V and R. $\delta V_{chk} and \delta R_{chk}$ are for the check star relative to the comparison in V and R. $HJD* = HJD - 2453800$

α-Centauri A and α-Centauri B are -0.01 and 1.34 respectively. Their color indices are 0.65 and 0.85 respectively. Use the data below to answer the following questions.

$(B-V)_0$	T_{eff}	BC
-0.25	24500	2.30
-0.23	21000	2.15
-0.20	17700	1.80
-0.15	14000	1.20
-0.10	11800	0.61
-0.05	10500	0.33
0.00	9480	0.15
0.10	8530	0.04
0.20	7910	0
0.30	7450	0
0.40	6800	0
0.50	6310	0.03
0.60	5910	0.07
0.70	5540	0.12
0.80	5330	0.19
0.90	5090	0.28
1.00	4840	0.40
1.20	4350	0.75

where 'BC' stands for the Bolometric correction and $(B-V)_0$ for the intrinsic Color of the star. (I08 - D02 - C)

(a) Plot the curve BC versus $(B-V)_0$.

(b) Determine the apparent bolometric magnitudes of α-Centauri A and α-Centauri B using the corresponding curve.

(c) Calculate the mass of each star.

Notes:

– Bolometric correction (BC) is a correction that must be made to the apparent magnitude of an object in order to convert an object's visible band magnitudes to its bolometric magnitudes, i.e.,

$$BC = M_v - M_{bol} = m_v - m_{bol} \tag{6.1}$$

– Mass-Luminosity relation:

$$M_{bol} = -10.2 \log\left(\frac{M}{M_\odot}\right) + 4.9 \tag{6.2}$$

where M and $M_\odot$ are masses of the star and the Sun respectively.

Chapter 7

Binaries and Variables

7.1 Theory

1. A binary system is 10 pc away, the largest angular separation between the components is 7.0", the smallest is 1.0". Assume that the orbital period is 100 years, and that the orbital plane is perpendicular to the line of sight. If the semi-major axis of the orbit of one component corresponds to 3.0", that is $a_1 = 3.0"$, estimate the mass of each component of the binary system, in terms of solar mass. (I10 - T04 - A)
2. An eclipsing close binary system consists of two giant stars with the same sizes. As a result of mutual gravitational force, stars are deformed from perfect sphere to the prolate spheroid with a = 2b, where a and b are semi-major and semi-minor axes (the major axes are always co-linear). The inclination of the orbital plane to the plane of sky is 90° . Calculate the amplitude of light variation in magnitude ((Δm)) as a result of the orbital motion of two stars. Ignore temperature variation due to tidal deformation and limb darkening on the surface of the stars. *Hint:* A prolate spheroid is a geometrical shape made by rotating of an ellipse around its major axis, like rugby ball or melon. (I09 - T15 - B)
3. **Orbital motion:** The scale diagram 7.1 represents the relative orbit of a physically double star. A star of mass m moves around a star of mass M in the indicated direction, where $m \ll M$. The major axis of the ellipse is aligned with the direction to the observer, and the motion of the star is in the plane of the diagram.
   (a) Find the part of the ellipse where the angular velocity ω of m is less than its mean angular velocity $\langle\omega\rangle$, and indicate this as accurately as possible on the scale diagram on the answer sheet.
   Note: The instantaneous angular velocity ω of m is equal to the

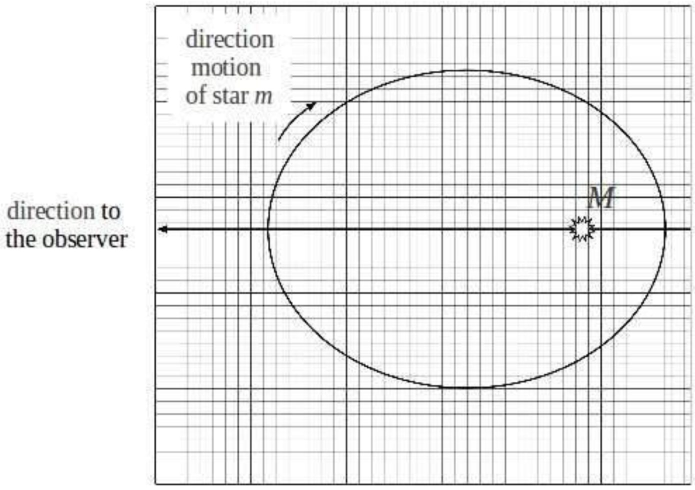


Figure 7.1 – Direction of Orbit of the double star

mean angular velocity $\langle\omega\rangle$ when the distance between stars $r = \sqrt{ab}$, where a and b are the semi-axes of the orbit.

Also mark those places on the ellipse for which the observer will see:

(b) extreme tangential (perpendicular to line of sight) velocity: $v_{t_{max}}$ and $v_{t_{min}}$,

(c) extreme radial (parallel to line of sight) velocity: $v_{r_{max}}$ and $v_{r_{min}}$. ((I11 - G02 - B))

4. **Minimum of an eclipsing binary:** The figure 7.2 shows the secondary (shallower) minimum of the bolometrically - corrected light curve of an eclipsing binary star. The difference between magnitudes $m_{1,Bol}$â$m_{0,Bol} = 0.33$ magnitude. We also know from simultaneous spectroscopy that the star with the smaller radius was totally eclipsed by the larger star during the secondary minimum (As only one spectrum was observable during the minimum).

   Determine the change of brightness of this binary during the primary minimum and draw the shape of the primary minimum using the same scale as the secondary minimum. Label the graph with all appropriate parameters. You may assume that the eclipses are central, that the stars are spheres of constant surface brightness, and that the distance between the stars does not change. ((I11 - G04 - B))

5. A binary star system consists of M_1 and M_2 separated by a distance D. M_1 and M_2 are revolving with an angular velocity ω in circular

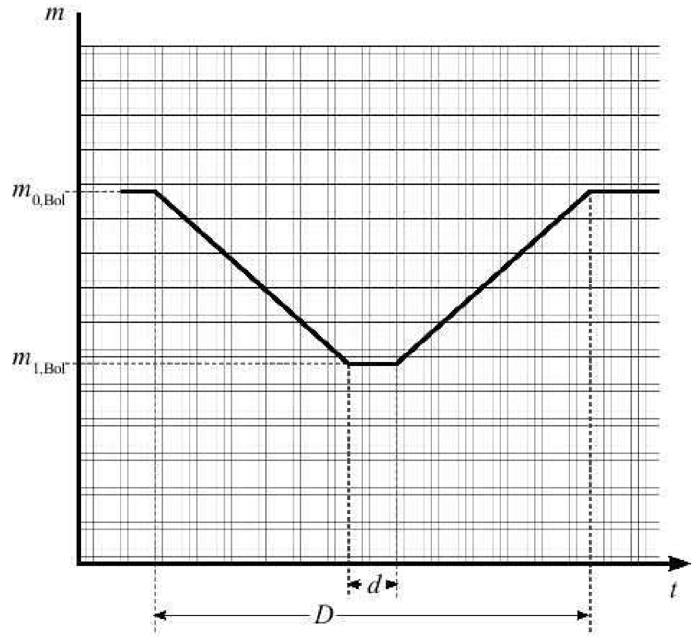


Figure 7.2 – Light curve of an eclipsing binary

orbits about their common centre of mass. Mass is continuously being transferred from one star to the other. This transfer of mass causes their orbital period and their separation to change slowly with time.

In order to simplify the analysis, we will assume that the stars are like point particles and that the effects of the rotation about their own axes are negligible. (I07 - T17 - B)

(a) What is the total angular momentum and kinetic energy of the system?

(b) Find the relation between the angular velocity ω and the distance D between the stars.

(c) In a time duration Δt , a mass transfer between the two stars results in a change of mass ΔM_1 in star M_1, find the quantity $\Delta\omega$ in terms of ω, M_1, M_2 and ΔM_1.

(d) In a certain binary system, $M_1 = 2.9M_\odot$, $M_2 = 1.4M_\odot$ and the orbital period, $T = 2.49$ days. After 100 years, the period T has increased by 20 s. Find the value of $\frac{\Delta M_1}{M_1 \Delta t}$ (in the unit "per year").

(e) In which direction is mass flowing, from M_1 to M_2, or M_2 to M_1?

(f) Find also the value of $\frac{\Delta D}{D \delta t}$ (in the unit "per year").
You may use these approximations:

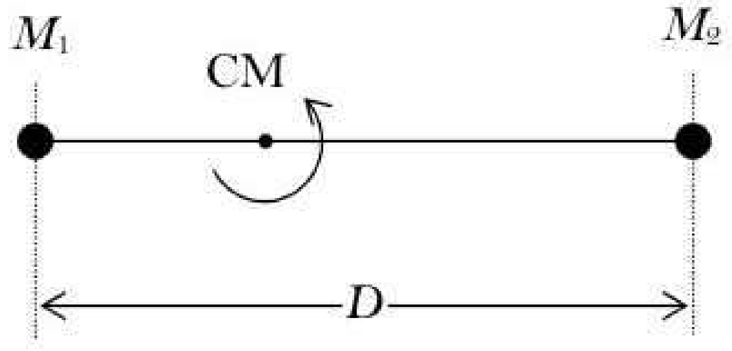


Figure 7.3 – Mass-Transfer Binaries

$(1+x)^n \sim 1+nx$, when $x \ll 1$;
$(1+x)(1+y) \sim 1+x+y$, when $x, y \ll 1$.

6. A transit of duration 180 minutes was observed for a planet which orbits the star HD209458 with a period of 84 hours. The Doppler shift of absorption lines arising in the planet's atmosphere was also measured, corresponding to a difference in radial velocity of 30 km/s (relative to Earth) between the beginning and the end of the transit. Assuming a circular orbit exactly edge-on to the observer, find the approximate radius and mass of the star and the radius of the orbit of the planet. (I11 - T16 - B)
7. The planetarium program 'Guide' gives the following data for two main sequence, one solar mass stars:

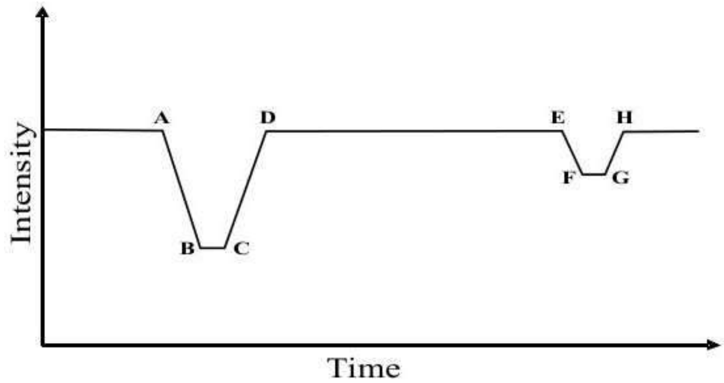
Star	1	2
Right Ascension	$14^h29^m44.95^s$	$14^h39^m39.39^s$
Declination	$-62°40'46.14$“	$-6050'22.10''$
Distance	1.2953 pc	1.3475 pc
Proper motion in R.A.	-3.776" / year	-3.600" / year
Proper motion in Dec.	0.95" / year	0.77" / year

Based on these data, determine whether these stars form a gravitationally bound system.

Note: the proper motion in R.A. has been corrected for the declination of the stars. (I11 - T18 - C)

8. An eclipsing binary star system has a period of 30 days. The light curve in the figure below shows that the secondary star eclipses the primary star (from point A to point D) in eight hours (measured from the time of first contact to final contact), whereas from point B to point C, the total eclipse period is one hour and eighteen minutes. The spectral

analysis yields the radial velocity of the primary star to be 30 km/s and of the secondary star to be 40 km/s. If we assume that the orbits are circular and has an inclination of $i = 90°$, determine the radii and the masses of both stars in unit of solar radius and solar mass. (I08 - T16 - D)



7.2 Data Analysis

1. Cepheids are very bright variable stars whose mean absolute magnitudes are functions of their pulsation periods. This allows astrophysicists to easily determine their intrinsic luminosities from the variation in their observed, apparent magnitudes. In table 7.1, P_0 is the pulsation period in days and $\langle M_V \rangle$ is the mean absolute visual magnitude.

   (a) Plot all Cepheids in a scatter diagram. $\log_{10}(P_0)$ should be the abscissa and $\langle M_V \rangle$ should be the ordinate.

   (b) Fit, using least squares method, a straight line to the $\langle M_V \rangle$ vs $\log_{10}(P_0)$ plot. This equation allows one to obtain the absolute magnitude from the pulsation period for any Cepheid.

   (c) Two panels in figure 7.4 show the light curves of two Cepheids. Use the available data to estimate the distances to each of these two Cepheids. Also estimate the uncertainty in the distance determination (only rough estimate is expected).

   (d) Comparing the difference between the distances of the two stars with the typical size of a galaxy, would it be likely for these two stars to be in the same galaxy? ((I12 - D02 - A))

2. **Analysis of times of minima:** Figure 7.5 shows the light curve of the eclipsing binary V1107 Cas, classified as a W Ursae Majoris type. Table 7.2 contains a list of observed minima of the light variation. The

Cepheid	P_0 (days)	$\langle M_V \rangle$
SU Cas	1.95	-1.99
V1726 Cyg	4.24	-3.04
SZ Tau	4.48	-3.09
CV Mon	5.38	-3.37
QZ Nor	5.46	-3.32
α UMi	5.75	-3.42
V367 Sct	6.30	-3.58
U Sgr	6.75	-3.64
DL Cas	8.00	-3.80
S Nor	9.75	-3.95
ζ Gem	10.14	-4.10
X Cyg	16.41	-4.69
WZ Sgr	21.83	-5.06
SW Vel	23.44	-5.09
SV Vul	44.98	-6.04

Table 7.1 – Cepheid Period-Magnitudes data

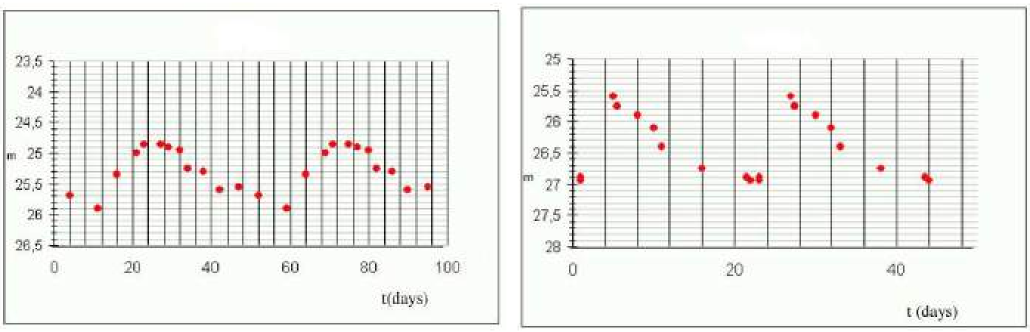


Figure 7.4 – Light Curves of two Cepheids

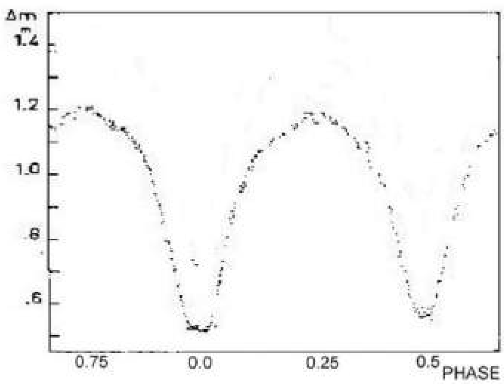


Figure 7.5 – Light Curve of V1107 Cas

columns contain: the serial number of the minimum, the date on which the minimum was observed, the heliocentric time of minimum expressed in Julian days and an error (in fractions of a day). Using these:

(a) Determine an initial period of V1107 Cas, assuming that the period of the star is constant during the interval of observations. Assume that observations during one night are continuous. Duration of the transit is negligible.

(b) Make what is known as an (O-C) diagram (for "observed - calculated") of the times of minima, as follows:
on the x-axis put the number of periods elapsed (the "epoch") since a chosen initial moment M_0; on the y-axis the difference between the observed moment of minimum M_{obs} and the moment of minimum calculated using the formula ("ephemeris"):

$$M_{calc} = M_0 + P \times E \tag{7.1}$$

where E, the epoch, is exactly an integer or half-integer, and P is the period in days.

(c) Using this (O-C) diagram, improve the determination of the initial moment M_0 and the period P and estimate the errors in their values.

(d) Calculate the predicted times of minima of V1107 Cas, in heliocentric JD, occurring between 19^h, 1 September 2011 UT and 02^h, 2 September 2011 UT. ((I11 - D01 - C))

No.	Date of minimum (UT)	Time of minimum (Heliocentric JD)	Error
1	22 December 2006	2 454 092.4111	0.0004
2	23 December 2006	2 454 092.5478	0.0002
3	23 September 2007	2 454 367.3284	0.0005
4	23 September 2007	2 454 367.4656	0.0005
5	15 October 2007	2 454 388.5175	0.0009
6	15 October 2007	2 454 388.6539	0.0011
7	26 August 2008	2 454 704.8561	0.0002
8	5 November 2008	2 454 776.4901	0.0007
9	3 January 2009	2 454 835.2734	0.0007
10	15 January 2009	2 454 847.3039	0.0004
11	15 January 2009	2 454 847.4412	0.0001
12	16 January 2009	2 454 847.5771	0.0004

Table 7.2 – Observed times of minima for V1107 Cassiopeae

Chapter 8

Galactic Astrophysics

8.1 Theory

1. The "spin-flip" transition of atomic hydrogen at rest generates the electromagnetic wave of the frequency $\nu_0 = 1420.406$ MHz. Such an emission from a gas cloud near the galactic center is observed to have a frequency $\nu = 1421.65$ MHz. Calculate the velocity of the gas cloud. Is it moving towards or away from the Earth? (I07 - T12 - A)
2. Radio wavelength observations of gas cloud swirling around a black hole in the center of our galaxy show that radiation from the hydrogen spin-flip transition is detected at a frequency of 1421.23 MHz. If this gas cloud is located at a distance of 0.2 pc from the black hole and is orbiting in a circle, determine the speed of this cloud and whether it is moving toward or away from us and calculate the mass of the black hole. (I08 - T06 - A)
3. Assuming that the G-type main-sequence stars (such as the Sun) in the disc of the Milky Way obey a vertical exponential density profile with a scale height of 300pc, by what factor does the density of these stars change at 0.5 and 1.5kpc from the mid-plane relative to the density in the mid-plane? (I10 - T10 - B)
4. The Galactic longitude of a star is $I = 15°$. Its radial velocity with respect to the Sun is $V_r = 100$ km/s. Assume stars in the disk of the Galaxy are orbiting around the galactic center in circular orbits, in the same sense in the galactic plane as the Sun, and have constant velocity of $V_0 = 250$ km/s. Calculate distance of the star from the center of the Galaxy. (I09 - T09 - C)
5. **Apparent number density of stars in the Galaxy:** Let us model the number density of stars in the disk of Milky Way Galaxy with a

simple exponential function of

$$n(r) = n_0 e^{-\left(\frac{r-R_0}{R_d}\right)} \tag{8.1}$$

where r represents the distance from the centre of the Galaxy, R_0 is the distance of the Sun from the centre of the Galaxy, R_d is the typical size of disk and n_0 is the stellar density of disk at the position of the Sun. All distances are expressed in kpc. An astronomer observes the center of the Galaxy within a small field of view. We take a particular type of Red giant stars as the standard candles for the observation with approximately constant absolute magnitude of $M = -0.2$,

(a) A telescope has a limiting magnitude of $m = 18$. Calculate the maximum distance to which this telescope can detect these red giant stars. For simplicity we ignore the presence of interstellar medium so there is no extinction.

(b) Assume an extinction of 0.7 mag/kpc for the interstellar medium. Repeat the calculation as done in the part 5a and obtain a rough number for the maximum distance these red giant stars can be observed.

(c) Give an expression for the number of these red giant stars per magnitude within a solid angle of Ω that we can observe with apparent magnitude in the range of m and $m + \Delta m$, (i.e. $\frac{\Delta N}{\Delta m}$). Red giant stars contribute fraction f of overall stars. In this part assume no extinction in the interstellar medium as part 5a. Assume the size of the disk is infinite. (I09 - T17 - C)
Hint: The Tylor series expansion, for $x \ll 1$

$$\log_{10}(1+x) = \frac{1}{\ln 10}\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...\right) \tag{8.2}$$

6. Astronomers studied a spiral galaxy with an inclination angle of 90° from the plane of the sky ("edge-on") and apparent magnitude 8.5. They measured the rotational velocity and radial distance from the galactic center and plotted its rotation curve. (I12 - T17 - C)

(a) Approximate the rotation curve in figure 8.1 with a continuous function V(D) composed of two straight lines.

(b) Using the same observations, they estimated that the rotation period of the pressure wave in the galactic disk is half of the rotation period of the mass of the disk. Estimate the time it takes for a spiral arm to take another turn around the galactic center (use the function constructed in previous part).

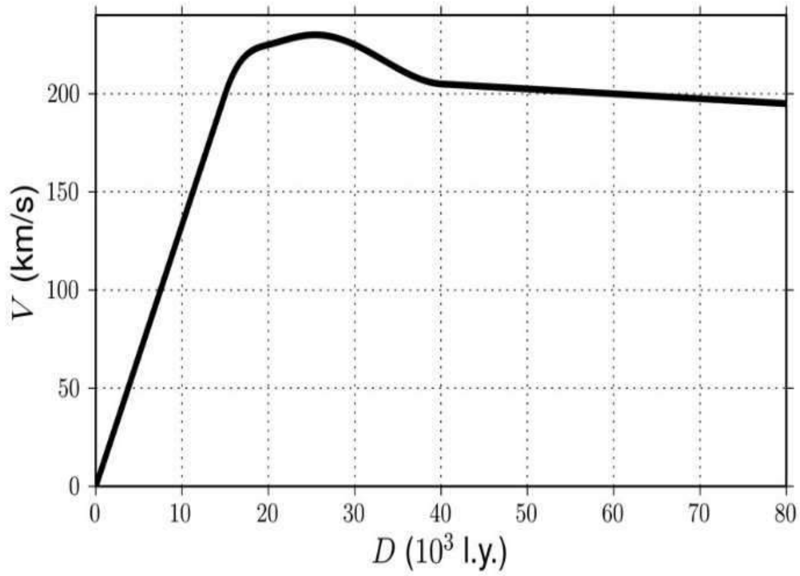


Figure 8.1 – Rotation Curve of the spiral galaxy

(c) Calculate the distance to the galaxy using the Tully-Fisher relation.

(d) Calculate the maximum and minimum values of the observed wavelengths of the hydrogen lines corresponding to 656.28 nm in the spectrum of this galaxy.
Hint: also take into account the cosmological expansion.

(e) Using figure 8.1, estimate the mass of the galaxy up to a radius of 3×10^4 light-years.

(f) Estimate the number of stars of the galaxy, assuming that: * the mean mass of the stars is equal to one solar mass and one third of the baryonic mass of the galaxy is in the form of stars, and; * the fraction of baryonic to dark matter in the galaxy is the same as the fraction for the whole Universe.

Notes:

- *Tully-Fisher Relation:* The luminosity of a galaxy is proportional to the 4^{th} power of the velocity variation ($\Delta V = V_2 - V_1$), which is the largest difference of rotational velocities in the galaxy. The proportionality constant of this relation is $\kappa = 0.317 L_\odot$ (km/s)$^{-4}$, where $L_\odot$ is the solar luminosity.
- *Approximate composition of the Universe:*
  - 4% of baryons

  - 22% of dark matter
  - 74% of dark energy.

7. Interstellar distances are large compared to the sizes of stars. Thus, stellar clusters and galaxies which do not contain diffuse matter essentially do not obscure objects behind them. Estimate what proportion of the sky is obscured by stars when we look in the direction of a galaxy of surface brightness $\mu = 18.0$ mag arcsec^{-2}. Assume that the galaxy consists of stars similar to the Sun. (I11 - T09 - D)

8.2 Data Analysis

1. **Weighing a galaxy:** The image 8.2 shows a photograph of the spiral galaxy NGC 7083, which lies at a distance of 40 Mpc, and image 8.3 a fragment of its spectrum. The slit of the spectrograph was aligned with the major axis of the image of the galaxy. The x-axis of the spectrum represents wavelength, and the y-axis represents the angular distance of the emitting region from the core of the galaxy, where 1 pixel corresponds to 0.82 arcsec. Two bright emission lines are visible, with rest wavelengths of $\lambda_1 = 656.4$ nm, $\lambda_2 = 658.4$ nm. Use the spectrum to plot the rotation curve of the galaxy and estimate the mass of the central bulge.
   Assumption: central bulge is spherical. The photograph of the galaxy has the correct proportions. ((I11 - D02 - B))

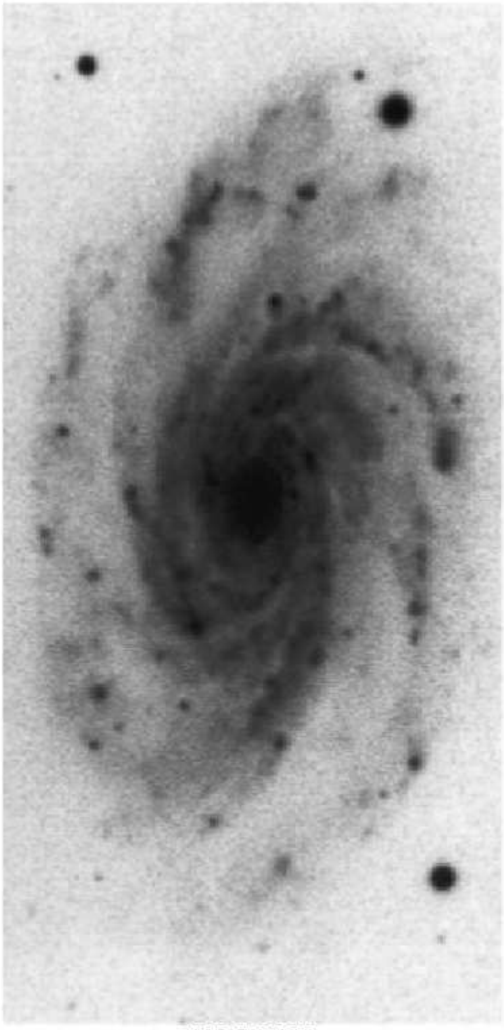


Figure 8.2 – Spiral Galaxy NGC 7083.

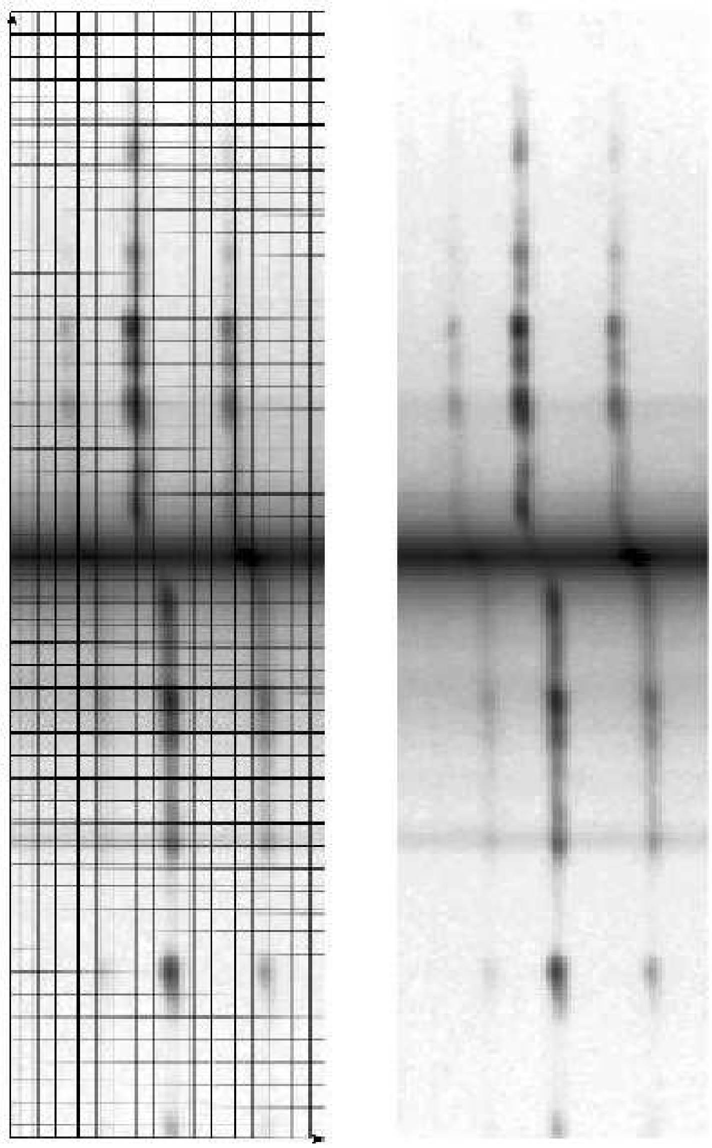


Figure 8.3 – Spectrum of spiral Galaxy NGC 7083. In the image on left side, grid marks pixels

Chapter 9

Extragalactic Astrophysics

9.1 Theory

1. Given that the cosmic background radiation has the spectrum of a black body throughout the evolution of the Universe, determine how its temperature changes with redshift z. In particular, give the temperature of the background radiation at the epoch $z \approx 10$ (that of the farthest currently observed objects). The current temperature of the cosmic background radiation is 2.73 K. (I11 - T15 - A)
2. The observed redshift of a QSO is z = 0.20, estimate its distance. The Hubble constant is $72 kms^{-}1Mpc^{-1}$. (I10 - T03 - A)
3. The average temperature of the Cosmic Microwave Background (CMB) is currently T = 2.73 K. It yields the origin of CMB to be at redshift $z_{CMB} = 1100$. The current densities of the Dark Energy, Dark Matter, and Normal Matter components of the Universe as a whole are, $\rho_{DE} = 7.1\times10^{-30} g/cm^3$, $\rho_{DM} = 2.4\times10^{-30} g/cm^3$ and $\rho_{NM} = 0.5\times10^{-30} g/cm^3$, respectively. What is the ratio between the density of Dark Matter to the density of Dark Energy at the time CMB was emitted, if we assume that the dark energy is the energy of empty space? (I08 - T05 - B)
4. Galaxy NGC 2639 is morphologically identified as an Sa galaxy with measured maximum rotational velocity $v_{max} = 324$ km/s. After corrections for any extinction, its apparent magnitude in B is $m_B = 12.22$. It is customary to measure a radius R_{25} (in units of kpc) at which the galaxy's surface brightness falls to 25 $mag_B/arcsec^2$.
   Spiral galaxies tend to follow a typical relation:

   $$logR_{25} = -0.249M_B - 4.00 \tag{9.1}$$

   where M_B is the absolute magnitude in B.

Apply the B-band Tully-Fisher relation for Sa spirals,

$$M_B = -9.95log(v_{max}) + 3.15, \qquad (9.2)$$

where v_{max} is in km/s, to calculate the mass of NGC 2639 out to R_{25}. If colour index of the sun is $(B_\odot - V_\odot) = 0.64$, write the mass in units of solar mass $M_\odot$ and its luminosity B-band in unit of $L_\odot$.
(I08 - T15 - B)

5. Consider a type Ia supernova in a distant galaxy which has a luminosity of $5.8\times10^9 L_\odot$ at its maximum light. Suppose you observe this supernova using your telescope and find that its brightness is 1.6×10^{-7} times the brightness of Vega. The redshift of its host galaxy is known to be $z = 0.05$. Calculate the distance of this galaxy (in pc) and also the Hubble time. (I08 - T12 - B)

6. Assume the mass of neutrinos is $m_v = 10^{-5} m_e$. Calculate the number density of neutrinos needed to compensate the dark matter of the universe. Assume the universe is flat and 25% of its mass is dark matter. *Hint:* Take the classical total energy equal to zero. (I09 - T11 - C)

7. Assume that the universe currently is well described by a density parameter $\Omega_0 = 1$, there is no dark energy, and the current temperature of the universe is 2.73 K. Knowing that the temperature of the universe is inversely proportional to its radius (the scale factor), compute how long, starting from the present time, it will take to the Universe to cool down by 0.1 K. (I12 - T10 - C)

8. Based on the spectrum of a galaxy with redshift $z = 6.03$ it was determined that the age of the stars in the galaxy is from 560 to 600 million years. At what z did the epoch of star formation occur in this galaxy?

   Assume that the rate of expansion of the Universe is given by a flat cosmological model with cosmological constant $\Lambda = 0$. (In such a model the scale factor $a \propto t^{\frac{2}{3}}$, where t is the time since the Big Bang.) (I11 - T11 - C)

9. The deflection of light by a gravitational field was first predicted by Einstein in 1912, a few years before the publication of the General Relativity in 1916. A massive object that causes a light deflection behaves like a classical lens. This prediction was confirmed by Sir Arthur Stanley Eddington in 1919. Consider a spherically symmetric object with a mass M. This object will act like a lens, with an impact parameter ξ measured from the centre of the object. The deflection equation in this case is given by:

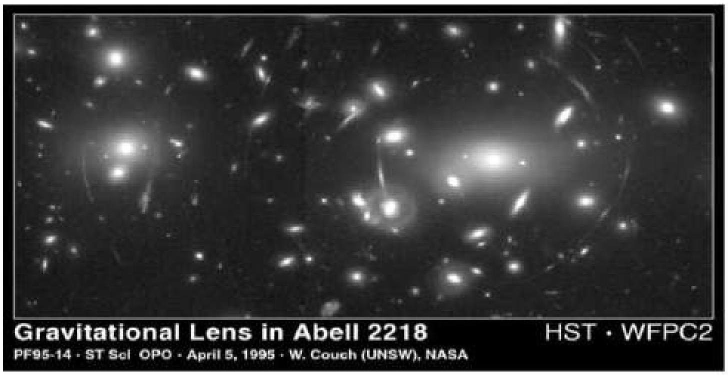


Figure 9.1 – Image of Abel Cluster

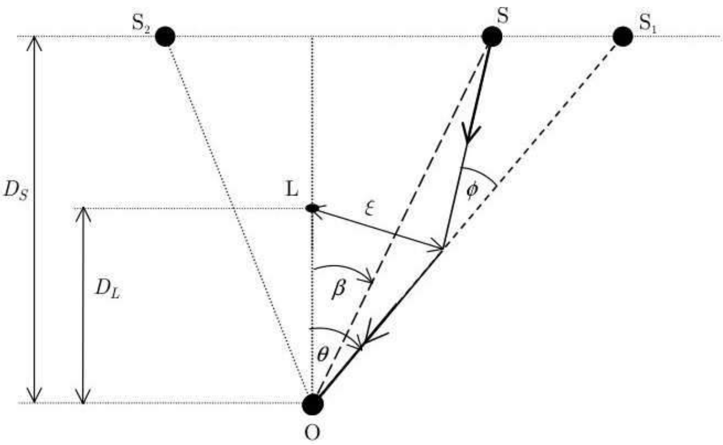


Figure 9.2 – Geometric Model of Gravitational Lensing

$$\phi = \frac{4GM}{\xi c^2} \tag{9.3}$$

In a simpified model, the impact parameter may be seen as the shortest separation between the centre of the lens and path of a particular light ray. In figure 9.2, the massive object which behaves like a lens is at L. Light rays emitted from the source S being deflected by the lens are observed by observer O as images S_1 and S_2. Here, ϕ, β, and θ are very very small angles. (I07 - T18 - C)

(a) For a special case in which the source is perfectly aligned with the lens such that $\beta = 0$, show that a ring-like image will occur with the angular radius,called Einstein radius θ_E, given by:

$$\theta_E = \sqrt{\left(\frac{4GM}{c^2}\right)\left(\frac{D_S - D_L}{D_L D_S}\right)} \tag{9.4}$$

(b) The distance (from Earth) of a source star is about 50 kpc. A solar-mass lens, along line-of-sight, is about 10 kpc from the star. Calculate the angular radius of the Einstein ring formed by this solar-mass lens with the perfect alignment.

(c) What is the resolution of the Hubble space telescope with 2.4 m diameter mirror? Can the Hubble telescope resolve the Einstein ring in 9b?

(d) In figure 9.2, for an isolated point source S, there will be two images (S_1 and S_2) formed by the gravitational lens. Find the positions (θ_1 and θ_2)of the two images. Answer in terms of β and θ_E.

(e) Find the ratio $\frac{\theta_{1,2}}{\beta}$ ($\frac{\theta_1}{\beta}$ or $\frac{\theta_2}{\beta}$) in terms of η. Here $\theta_{1,2}$ represents each of the image positions in 9d and η stands for the ratio $\frac{\beta}{\theta_E}$.

(f) Find also the values of magnifications $\frac{\Delta\theta}{\Delta\beta}$ in terms of η for $\theta = \theta_{1,2}$ ($\theta = \theta_1$ or $\theta = \theta_2$), when $\Delta\beta \ll \beta$, and $\Delta\theta \ll \theta$.

10. Within the field of a galaxy cluster at a redshift of $z = 0.500$, a galaxy which looks like a normal elliptical is observed, with an apparent magnitude in the B filter $m_B = 20.40$ mag. The luminosity distance corresponding to a redshift of $z = 0.500$ is $d_L = 2754$ Mpc. The spectral energy distribution (SED) of elliptical galaxies in the wavelength range 250 nm to 500 nm is adequately approximated by the formula:

$$L_\lambda(\lambda) \propto \lambda^4 \tag{9.5}$$

(i.e., the spectral density of the object's luminosity, known also as the monochromatic luminosity, is proportional to λ^4.)

(a) What is the absolute magnitude of this galaxy in the B filter ?

(b) Can it be a member of this cluster?

Hint: Try to establish a relation that describe the dependence of the spectral density of flux on distance for small wavelength interval. Normal elliptical galaxies have maximum absolute magnitude equal to -22 mag. For blue band, effective midpoint wavelength is 445nm and FWHM is 94nm. (I11 - T17 - D)

11. Measurement of the cosmic microwave background radiation (CMB) shows that its temperature is practically the same at every point in the sky to a very high degree of accuracy. Let us assume that light emitted at the moment of recombination ($T_r \approx 3000$ K, $t_r \approx 300000$ years) is only reaching us now ($T_0 \approx 3$ K, $t_0 \approx 1.5 \times 10^{10}$ years). Scale factor (a) is defined as such $a_0 = a(t = t_0) = 1$ and $a_t = a(t < t_0) < 1$. Note that the time from the end of inflation ($t = 10^{-32}$ seconds) to the time of recombination was the radiation dominated period and the matter dominated period started at the recombination. During the radiation dominated period a is proportional to $t^{1/2}$, while during the matter dominated period a is proportional to $t^{2/3}$. (I08 - T18 - D)

    (a) Estimate the horizon distances when recombination took place. Assume that temperature T is proportional to $1/a$.

    (b) Consider two points in CMB imprint, which are currently observed at a separation angle $\alpha = 5°$. At the recombination epoch, would it have been possible for observers at these two points to communicate with each other using photons?

    (c) Estimate the size of our Universe at the end of inflation period.

    Note: "Horizon distance" is the present day angular separation between two points, such that observer at one of the point would be just able to receive photon from the other point.

9.2 Data Analysis

1. **Virgo Cluster:** The Virgo Cluster of galaxies is the nearest large cluster which extends over nearly 10 degrees across the sky and contains a number of bright galaxies. It will be interesting to find the distance to Virgo and to deduce certain cosmological information from it. The table below provides the distance estimates using various distance indicators (listed in the left column). The right column lists the mean distance (d_i)$\pm$ the standard deviation (s_i). (I08 - D01 - B)

	Distance Indicator	**Virgo Distance (Mpc)**
1	Cepheids	14.9 ± 1.2
2	Novae	21.1 ± 3.9
3	Planetary Nebulae	15.2 ± 1.1
4	Globular Cluster	18.8 ± 3.8
5	Surface Brightness Fluctuations	15.9 ± 0.9
6	Tully-Fisher relation	15.8 ± 1.5
7	Faber-Jackson relation	16.8 ± 2.4
8	Type Ia Supernovae	19.4 ± 5.0

(a) As different methods give slightly different distances, we need to compute the average distance. This average, calculated by applying the weighted mean, can be taken as an estimate to the distance to Virgo Cluster.

$$d_{avg} = \frac{\sum_{i=1}^{n} \frac{d_i}{s_i^2}}{\sum_{i=1}^{n} s_i^2} \tag{9.6}$$

EA: Here weights are simply inverse of variance. Lower the standard deviation, higher the confidence hence higher the weight.

(b) What is the rms uncertainity (in units of Mpc) in that estimate?

(c) Spectra of the galaxies in Virgo indicate an average recession velocity of 1136 km/sec for the cluster. Can you estimate the Hubble constant H_0 and error in its estimation?

(d) What is Hubble Time (age of the Universe) using the value of Hubble constant you found and the rms uncertainity in its value?

Chapter 10

Night Sky Observation

10.1 General Night Sky

1. Move the pointer along the celestial equator. (I07 - O01 - A)
2. Aim the pointer at the vernal equinox. (I07 - O02 - A)
3. Aim the pointer at the star named alpha-Arietis (α-Ari). (I07 - O04 - A)
4. In the constellation of Pegasus and its vicinity there is an obvious square of bright stars (Great Square of Pegasus), aim the pointer at the brightest star of the square. (I07 - O03 - A)
5. Use your green laser pointer to aim at the stars Antares (α - Sco), Vega (α - Lyr), Altair (α- Aql) and Peacock (α- Pav). Also point to the constellation Corona Australis. (I12 - O01 - A)
6. Use your laser pointer to aim at 3 zodiacal constellations of your choice. (I12 - O11 - A)
7. Planetarium is projecting the sky of Bejing, at 21:00 local time in mid-August. The staff will point 5 constellations in the sky with laser pointer, one minute for each. Identify the constellations (IAU names) in the order in which the 5 constellations have been shown. (I10 - O01 - B)
8. Planetarium is projecting the sky of Bejing, at 21:00 local time in mid-August. Identify any 5 constellations (IAU names) you can see that are crossed by the celestial equator. (I10 - O02 - B)
9. Figure 10.1 (frame size $\cong 100° \times 70°$) shows a part of the sky, for 22 October 2009 at 21:00 local time. Four bright stars in Perseus and Andromeda constellations are missing in this chart. Find these missing stars by looking at the sky. Then, draw a cross on the location of

Figure 10.1 – Photograph of Northern Sky

each missing bright star in these two constellations on the chart. Use numbers in table 9 to indicate these crosses.
Note : Polaris is indicted by "N" symbol in the figure. (I09 - O01 - C)

10. Figure 10.2 shows a part of the southern sky chart for August 21, 2008 at 07.00 p.m. local time. Unfortunately, a number of bright stars in Capricorn and Scorpio constellations are missing. Now, you have to find those missing bright stars in both constellations by looking at the sky directly. (I08 - O01 - D)
    (a) Draw small circles on the locations of the missing bright stars in the Capricorn and Scorpio constellations.

Table 10.1 – The Bayer and common name of some stars around Perseus

Number	**Bayer Name**	**Number**	**Bayer Name**
1	α-Per (Mirfak)	7	γ-And (Almach)
2	α-And (Alpheratz)	8	δ-And
3	ϵ-Per	9	51-And
4	ξ-Per (Menkib)	10	β-And (Mirach)
5	γ-Per	11	ζ-Per (Atik)
6	β-Per (Algol)		

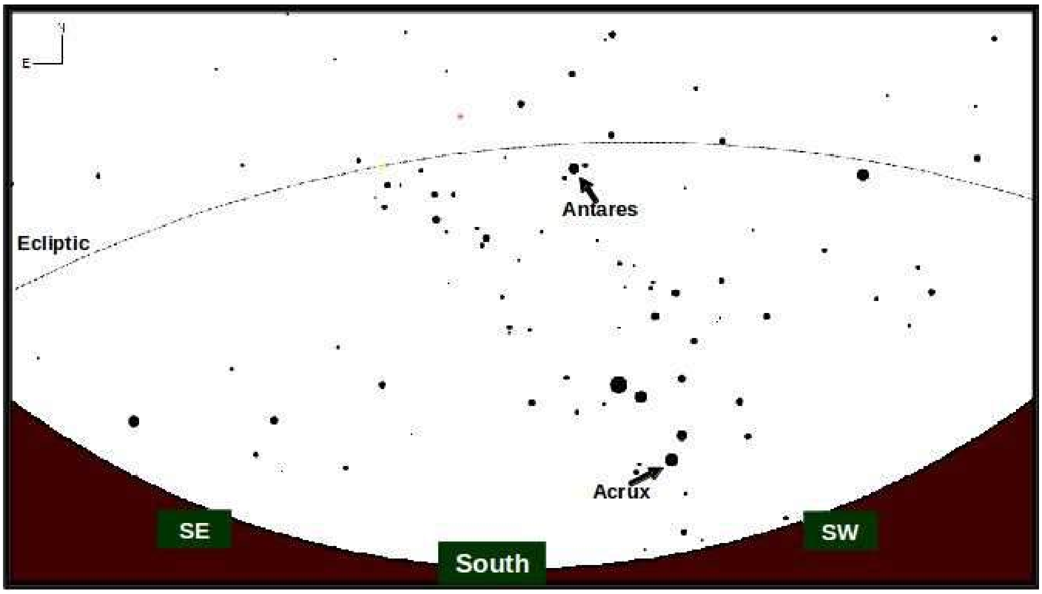


Figure 10.2 – Sky Map for Scorpio and surrounding regions

(b) Identify them by putting the numbers on the sky chart, as many as possible based on the Table 10.

(c) Afterwards, draw on the sky chart, the borders of Scorpio and Capricorn constellations.

11. **Constellations:** Jan Hevelius (1611-1687) introduced 11 new constellations onto the sky. The International Astronomical Union confirmed 7 of those in 1928:

No.	IAU Abbr.	Latin name	Translation	Coordinates of the centre R. A. α	Dec. δ
1	CVn	Canes Venatici	Hunting dogs	13h 00m	+40°
2	Lac	Lacerta	Lizard	22h 30m	+46°
3	LMi	Leo Minor	Smaller Lion	10h 10m	+32°
4	Lyn	Lynx	Lynx	8h 00m	+48°
5	Sct	Scutum	Shield	18h 40m	-10°
6	Sex	Sextans	Sextant	10h 15m	-3°
7	Vul	Vulpecula	(Little) Fox	20h 15m	+24°

(a) Mark each of the above constellations on the map 10.3 by a point lying anywhere within the constellation. Label the marks using the appropriate serial number or IAU name.

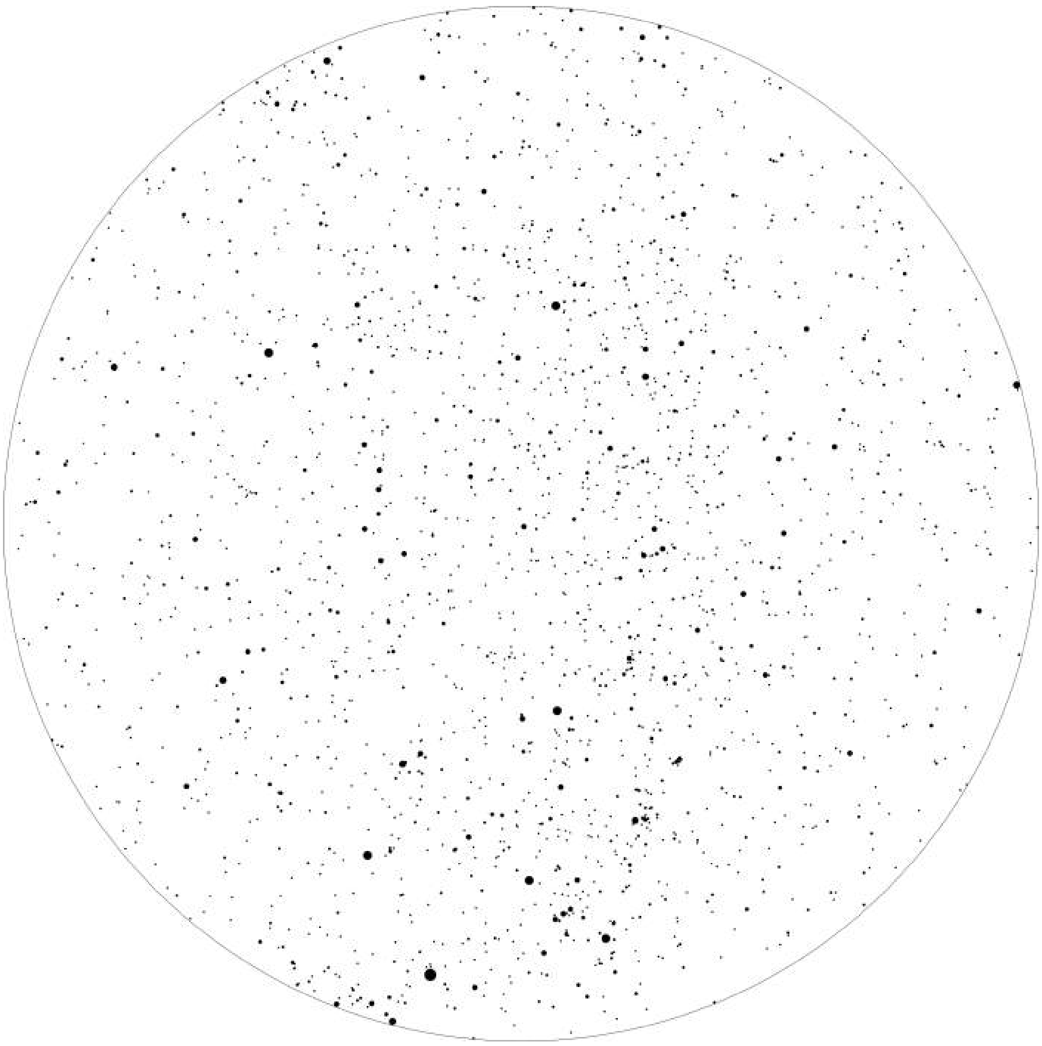


Figure 10.3 – Map of Sky used in IOAA 2011

Table 10.2 – The Bayer and common name of some stars around Scorpio

Number	Common Names	Number	Common Names
1	Rukbat (α Sgr)	18	Albali (ϵAqr)
2	Graffias (β Sco)	19	Altair (α Aql)
3	Nunki (σ Sgr)	20	Shaula (λ Sco)
4	Deneb (α Cyg)	21	Vrischika (π Sco)
5	Zaniah (η Vir)	22	Arich (γ Vir)
6	Tarazed (γ Aql)	23	Deneb Algedi (δ Cap)
7	Dabih (β Cap)	24	Heze (ζ Vir)
8	Girtab (κ Sco)	25	Nusakan (β CrB)
9	Spica (α Vir)	26	Wei (ϵ Sco)
10	Sabik (η Oph)	27	Syrma (ι Vir)
11	Dschubba (δ Sco)	28	Nashira (γ Cap)
12	Kaus Australis (ϵ Sgr)	29	Lesath (ν Sco)
13	Algiedi (α Cap)	30	Zavijava (β Vir)
14	Sadr (γ Cyg)	31	Arcturus (α Boo)
15	Vindemiatrix (ϵ Vir)	32	Megrez (δ UMa)
16	Antares (α Sco)	33	Chara (β CVn)
17	Yen (ζ Cap)	34	Sargas (θ Sco)

(b) Mark on the same map, positions of any 13 objects from the Messier Catalogue (not necessarily from the constellations above), giving the Messier number (“M xx”) for each.

The map is prepared for epoch J 2000.0 and uses a polar projection with a linear scale in declination. It includes stars brighter than about 5^{th} magnitude. (I11 - G01 - D)

10.2 Magnitudes and Angular Sizes

1. Start from the star named Aldebaran (α-Tau) in the constellation Taurus, turn the pointer 35 degrees northward followed by 6 degrees westward (in equatorial coordinate). Then, aim the pointer at the brightest star in the field of view. (I07 - O05 - A)
2. The open star cluster “Hyades” in constellation Taurus is one of the nearest clusters to us, being only 151 light years away. Figure 10.4 lists brightness of some stars indicated by the apparent magnitude in parentheses. Please estimate the apparent magnitude of the star Gamma-Tauri (γ-Tau) to the nearest first decimal digit. (I07 - O06 - B)

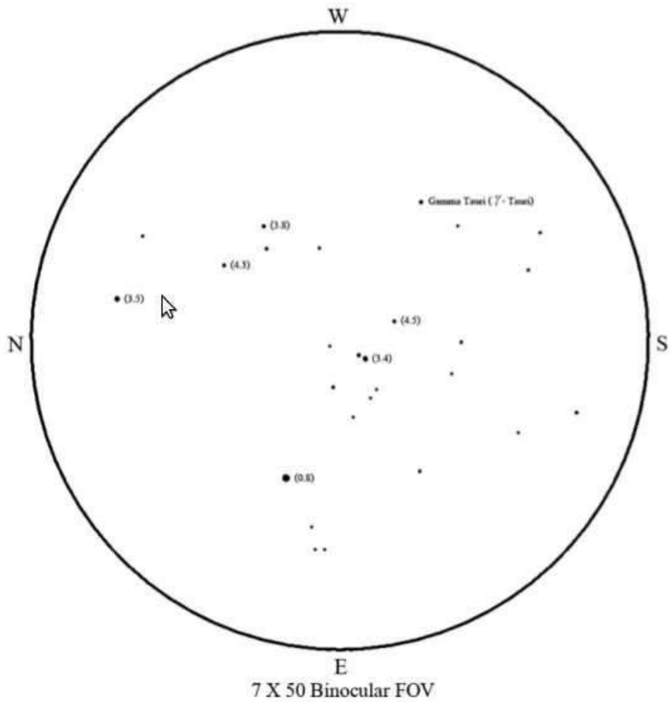


Figure 10.4 – Sky Map for Taurus region

Table 10.3 – Angular Distance

Pairs of stars	**Angular Distance (degrees)**
1 (Errai) and 2 (Alfirk)	
1 (Errai) and 3 (Alderamin)	

3. (a) Figure 10.5 shows a part of the sky which contains Cepheus constellation, for 22 October 2009 at 22:00 local time. Five bright stars in Cepheus constellation are identified by numbers (1, 2, 3, 4, 5) and common names. Estimate the angular distances (in units of degrees) between two pairs of stars shown in table 10.3. (I09 - O02 - B)

   (b) Use table 10.4 and figure 10.5, then Estimate the "apparent visual magnitude" of stars 2 (Alfirak) and 3 (Alderamin) in table 10.4.

10.3 Instrument Aided Observations

1. Identifying telescope components: (I11 - G03 - A)

   (a) Look at the pictures of the telescope in figure 10.6 and match the

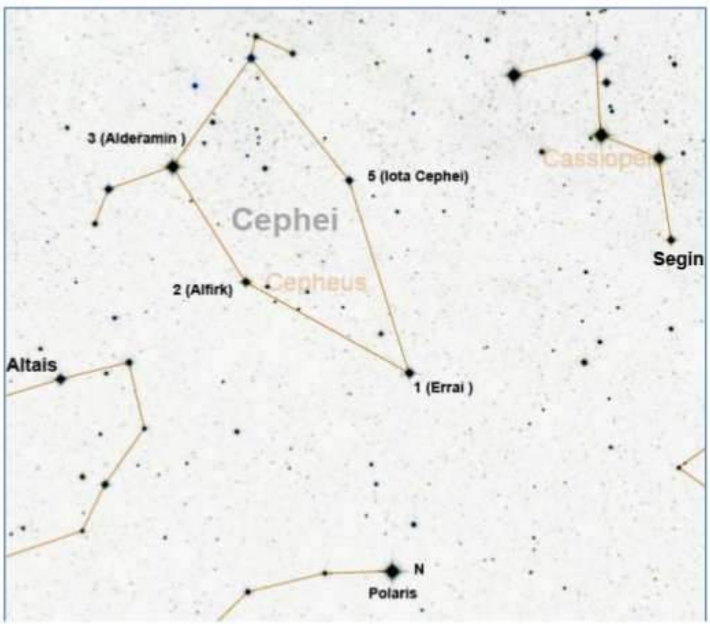


Figure 10.5 – Sky Map for Cepheus region

Table 10.4 – Star Magnitudes

Star Name	m_v
Polaris	1.95
Altais	3.05
Segin	3.34
2 (Alfirk)	
3 (Alderamin)	

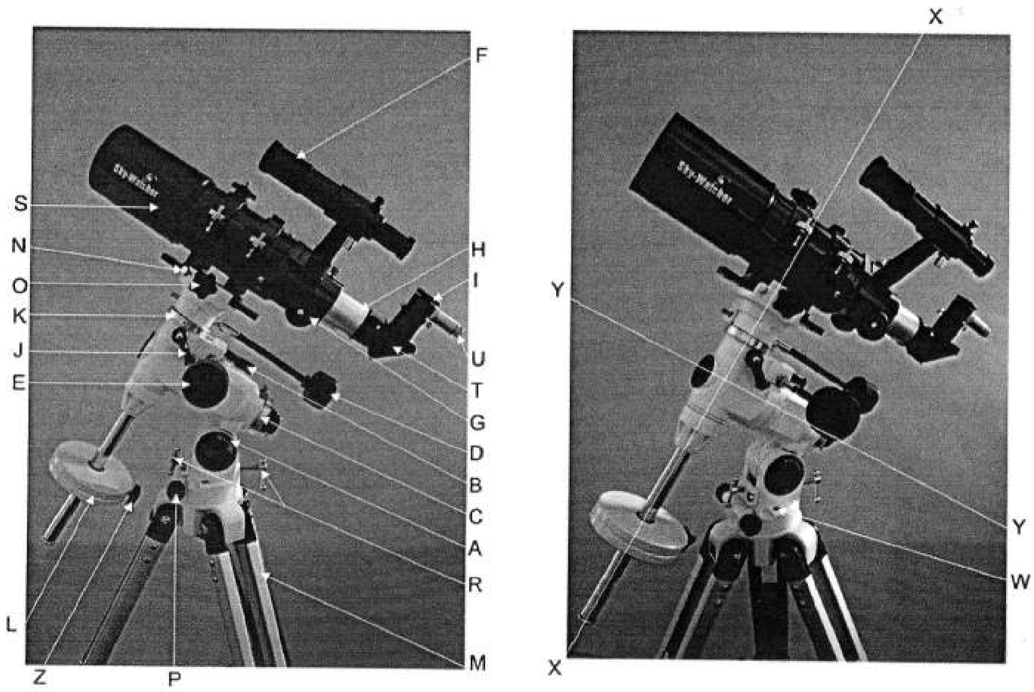


Figure 10.6 – Parts of a telescope

names of the items with the corresponding letters in table 10.5.

(b) Select and circle the correct answer for each of the questions below:

21. Mount design :

a. Fork b. Transit c. Dobsonian Alt-Azimuth d. German Equatorial

22. Optical type :

a. Newtonian b. Cassegrain c. Keplerian d. Galilean

23. Objective aperture :

a. 60 mm b. 80 mm c. 90 mm d. 100 mm

and objective lens focal length :

a. 400 mm b. 500 mm c. 600 mm d. 800 mm

24. Eyepiece focal length :

a. 4 mm b. 6 mm c. 12.5 mm d. 25 mm

25. Used for visual observations of the sky, the finder scope gives a picture which is :

a. normal b. rotated by 180° c. reflected in one axis d. rotated by 90°

26. Used for visual observations with the diagonal mirror, the instrument gives a picture which is :

Table 10.5 – Parts of a telescope

	Item name	Letter
	(example) Tripod	**M**
1.	Counterweight	
2.	Right Ascension Setting Circle (R.A. Scale)	
3.	Declination Setting Circle (Declination Scale)	
4.	Right Ascension locking knob	
5.	Declination locking knob	
6.	Geographical latitude scale	
7.	Finder scope	
8.	Focuser tube	
9.	Focuser knob	
10.	Eyepiece	
11.	Declination Axis	
12.	Right Ascension Axis (Polar Axis)	
13.	Right Ascension slow motion adjustment	
14.	Declination flexible slow motion adjustment	
15.	90° diagonal mirror	
16.	Azimuth adjustment knobs	
17.	Altitude adjustment screws	
18.	Lock screw	
19.	Spirit level bubble	
20.	Eyepiece reticle light switch and brightness control	

a. normal b. rotated by 180° c. reflected in one axis d. rotated by 90°

(c) Determine the following theoretical instrument parameters:

27. Magnification :

28. Focal ratio :

29. Resolution (in arcseconds):

30. Limiting magnitude: (I11 - G03 - A)

2. Assemble a telescope indoor with an equatorial mount. Your team will be given a maximum of 10 minutes to assemble it. The direction of the North will be indicated to you. After the end of the round, the students should take apart the telescope again.
   The co ordinates of Beijing: Longitude: 116° 48'E, Latitude: 40° 32'N. (I10 - G01 - A)

3. Point your telescope to the binary star ϵ - Trianguli Australis using figure 10.7 as a guide. If you are not able to point at the star, the instructor will do it for you, but you will lose 50% of the points. The components of this pair have magnitudes 4.1 and 9.3 and are separated by 82". Indicate color of each star: (I12 - O04 - A)

   Brighter: White / blue ( ) Yellow ( ) Red ( )
   Dimmer: White / blue ( ) Yellow ( ) Red ( )

4. Point your telescope to the binary star Albireo (β- Cygni) using figure 10.8 as guide. If you are not able to point at the star, the examiner will do it for you but you lose 50% of the points. The components of that pair are of magnitude 3.2 and 4.7, separated by 34.8". Indicate color of each star: (I12 - O14 - A)

   Brighter: White ( ) Blue ( ) Yellow ( ) Red ( )
   Dimmer: White ( ) Blue ( ) Yellow ( ) Red ( )

5. For this task, 4 telescopes have been pointed to four specific objects. Identify which one correspond to: Open Cluster (OC), Globular Cluster (GC), Emission Nebulae (EN) and Planetary Nebulae (PN). (I12 - O05 - A)

   Object 1 ( ) Object 2 ( ) Object 3 ( ) Object 4 ( )

6. Use figure 10.9 to point to ν - Scorpii. If you are not able to point at the star, you can ask the examiner to do it for you but you will not get marks for pointing. Note: Use 2x barlow + 10mm eyepiece to draw what you see through the eyepiece. It is not necessary to mark the North. (I12 - O12 - B)

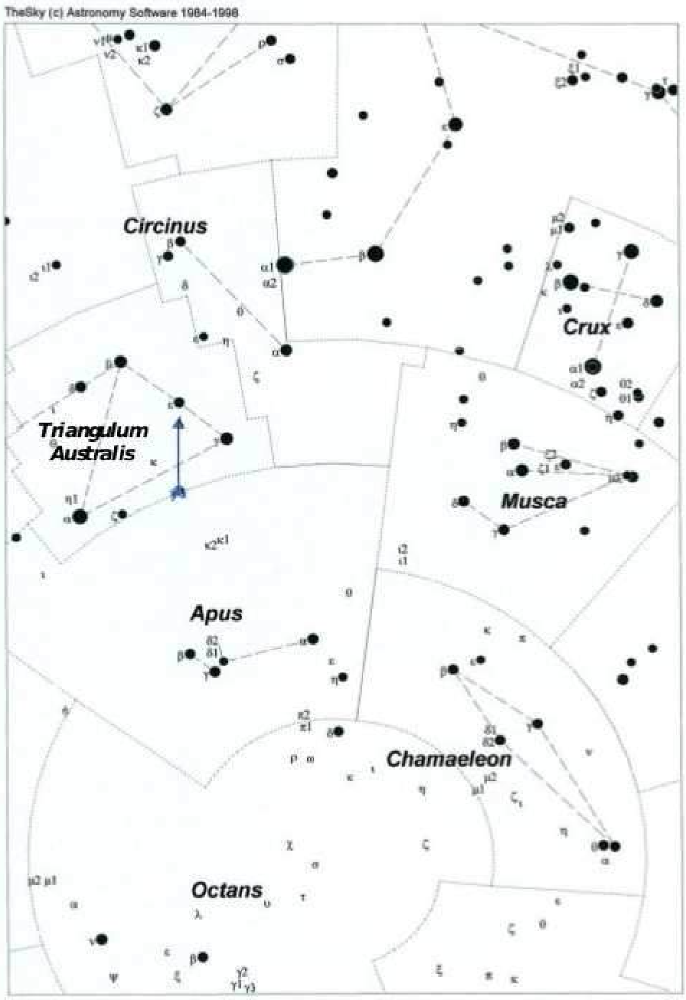


Figure 10.7 – Sky map of Southern Pole region

CHART 6 – Cygnus

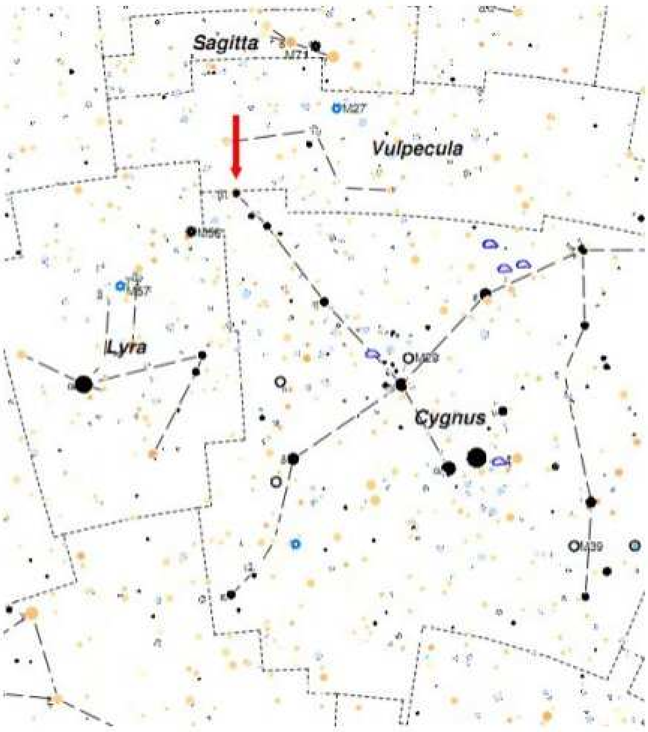


Figure 10.8 – Sky map of Cygnus region

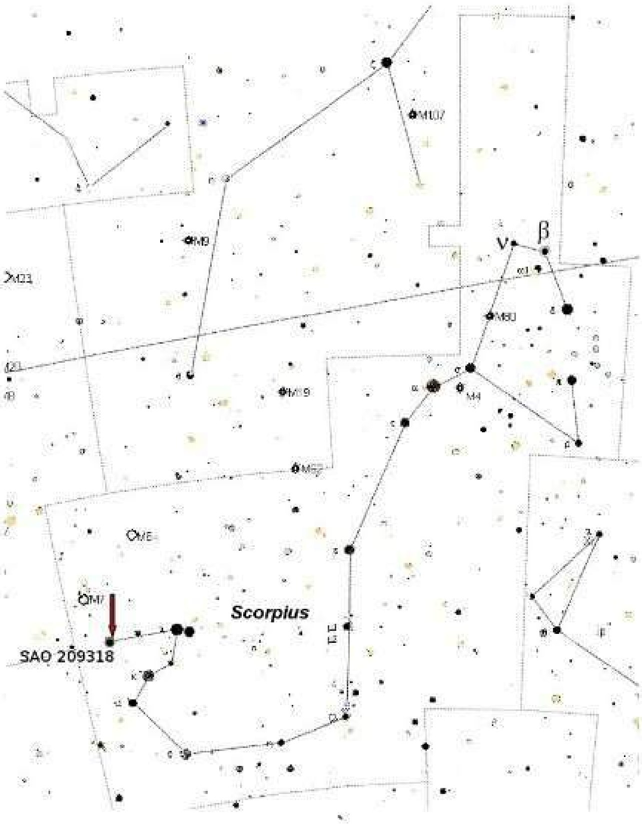


Figure 10.9 – Map of Scorpio Region marked with SAO 209318 and ν-Sco

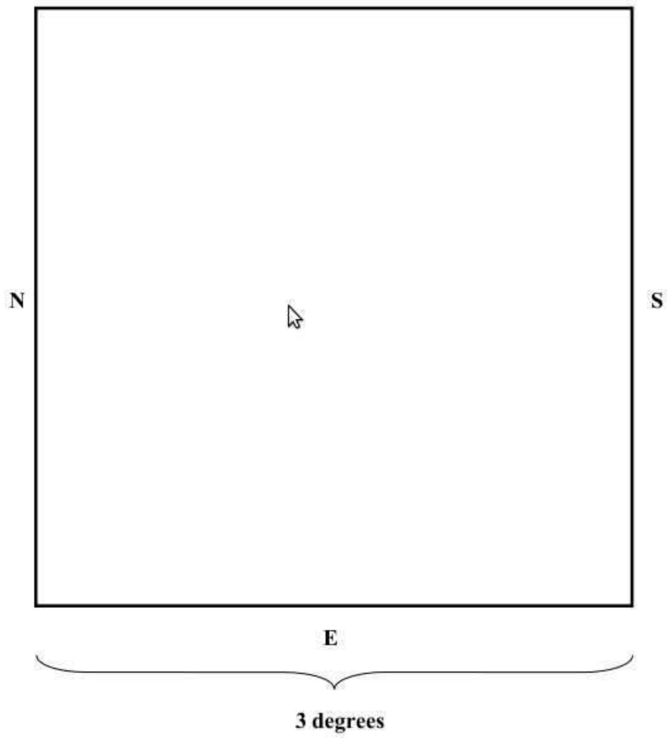


Figure 10.10 – Field for drawing of Andromeda Galaxy

7. Point your telescope to the star SAO 209318 using figure 10.9. Notice a small nebulous patch close to that star. Use your 10mm or 10mm + 2x Barlow to estimate the distance between the star and the nebulous patch, in arc minutes. (coordinates to SAO 209318 are α: 17h 50m 51s and δ: $-37°$ 02'). Express your answer using 0.5' precision. Note that the field of view of the 10mm eyepiece on this telescope is 24 arcmin or 0.4°. If you are not able to point at the star, the examiner will do it for you but you lose 50% of the points. (I12 - O13 - B)
8. Observe the Andromeda Galaxy (M31) then draw the approximate shape and size of the galaxy that you see through the binoculars in the frame 10.10 with correct orientation (in equatorial coordinates). The field of view of the binoculars is 6.8 degrees. (I07 - O07 - A)
9. The Telescope is pointing to Caph in Cassiopeia constellation (RA: 0h 9.7m; Dec: 59° 12'). Using the clock beside the telescope, write down the local time (with the format of HH:MM:SS). Note down readings on dials on the telescope mount. (I09 - O03 - A)
   (a) Point your telescope to any one of the following stars. Write down

the name of the selected star notify the examiner to check it.
Deneb (Alpha Cygni) / Alfirk (Beta Cephei) / Algol (Beta Persei) / Capella (Alpha Aurigae)

(b) Note down readings on dials on the telescope mount again. Estimate the "declination" and the "hour angle" of the star you have chosen.

10. The telescope for this task is aimed at open cluster NGC 6231. Ensure that 10mm Plössl eyepiece is put on the telescope. Estimate the diameter of the field of view of this telescope using the two angular distances shown on figure 10.11 as reference. Express your answer in arc minutes. (I12 - O02 - C)

11. Use figure 10.12 to estimate the magnitude of the missing star, shown as a cross, inside NGC 6231. Use magnitude of other stars as reference. (I12 - O03 - C)
Note: To avoid confusion between decimal point and real stars, decimal points are not shown. So, magnitude 60 correspond to magnitude 6.0. Give your answer with a precision of 0.1 magnitude. Also note that 8.0 star in the chart is a variable.

12. You have to identify as many stars as possible in the field of a celestial photograph using the telescope and CCD provided. You can choose one out of the five recommended regions in the sky listed below. Then, point the telescope to the direction of the selected sky region. Take three photographs with different exposure times and record the images of the sky by the CCD camera. Save the observational data. Transfer the data to the printing facilities to print out the result. Ask the technical assistant for a help. Choose the best prints-out and use the image to identify the stars in the field of observations. Step by step process for this is given below. (I08 - O02 - D)

(a) Choose only one out of the following five recommended regions to the directions of (marked by the following bright clusters):
– M7: $(\alpha = 17^h 53^m.3, \delta = -34°46'.2)$
– M8: $(\alpha = 18^h 04^m.2, \delta = -24°22'.0)$
– M20: $(\alpha = 18^h 02^m.4, \delta = -22°58'.7)$
– M21: $(\alpha = 18^h 04^m.2, \delta = -22°29'.5)$
– M23: $(\alpha = 17^h 57^m.0, \delta = -18°59'.4)$
You may not change your choice.

(b) Point the telescope to the chosen cluster using telescope controller. If necessary you may move the telescope slightly to get the best position in the frame of the CCD by checking the display of "CC-Dops" software.

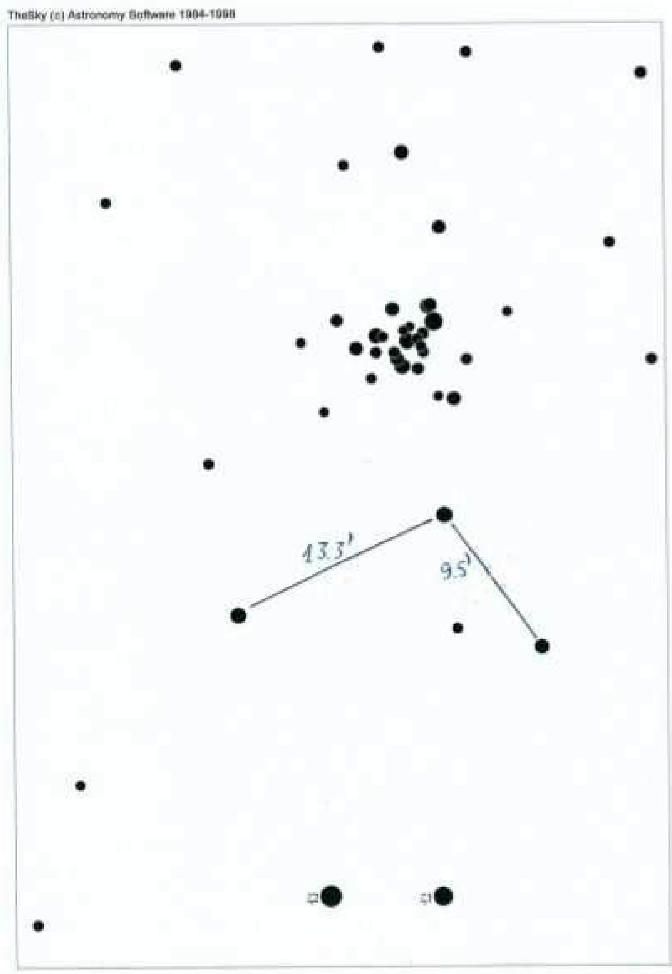


Figure 10.11 – Image of NGC 6231 with angular separation

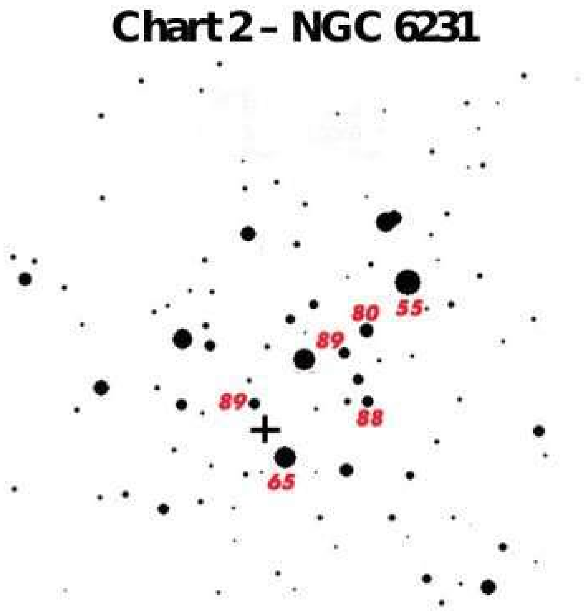


Figure 10.12 – Image of NGC 6231 with magnitudes

(c) Display the region in The Sky map software provided in the computer, to confirm that the telescope is pointed to the selected object in the sky. You may change field of view of the sky chart.

(d) You may invert the background images into white color, as in the chart mode. Copy and paste the sky chart from The Sky into the answer page. Use "Ctrl-c" and "Ctrl-v" buttons from the keyboard, respectively.

(e) Type the equatorial coordinates of the centre of that object in the answer page as indicated in The Sky.

(f) Take three photographs of the chosen object by using the attached CCD camera and CCDops software, with various exposure times. Choose exposure time in the range between 1 and 120 seconds. Image is automatically subtracted with dark frame of the same exposure time.

(g) You must invert the background images into white color. Copy and paste images from CCDops into the answer page. Use "Ctrl-c" and "Ctrl-v" buttons from the keyboard, respectively.

(h) Save your answer page into hard disk in a Microsoft Word file format.

(i) Print your answer page which consists of the photographs and the corresponding sky chart.

(j) Go to the identification room and bring with you the prints-out of the sky chart and the photographs. Ask some help from technical assistant, if necessary.

(k) Choose the best out of the three printed images and identify as many objects as possible on it.

(l) Use the assigned computer and The Sky software to identify objects. Type your identification to your answer page.

(m) Type on the answer page the names (or catalogue number), RA, Dec, and magnitude of each identified star and put the sequential number on the photograph. Make sure that you list the stars on the answer page following the same order and number as on the photograph.

(n) Estimate the limiting magnitude of the photograph you choose, empirically.

10.4 Planetarium Based Questions

1. Planetarium is projecting the sky of Bejing, on some day, just about 1 hour after the sunset. Estimate which month (use 1-12) it should be according to the displayed night sky. What is the age[1] of the moon (in 1-30 days)? (I10 - O03 - B)

   The co ordinates of Beijing: Longitude: 116° 48'E, Latitude: 40° 32'N.

2. Simulation of the Earth Sky (I11 - O01 - C)

   (a) In the sky projected, one can notice cresent moon, a nova and a comet. On the map of the sky (use figure 10.3), mark nova with a cross and label it as "N". Mark the Moon with a Moon symbol and draw the shape and position of the comet.

   (b) In the table below, circle only those objects which are above the astronomical horizon.

M20 - Triffid Nebula	o Cet - Mira	δ CMa - Wezen
α Cyg - Deneb	M57 - Ring Nebula	β Per - Algol
δ Cep - Alrediph	α Boo - Arcturus	M44 - Praesepe (Beehive Cluster)

1. Age of the moon is number of days since last new moon.

(c) Coordinate grid will now be switched on. When it is visible, mark on the map the northern part of the local meridian (from the zenith to the horizon) and the ecliptic north pole (with a cross and marked "P").

(d) For the displayed sky, note down:
geographical latitude of the observer (ϕ):
Local Sidereal Time (θ):
approximate time of year (by circling the calendar month):
Jan, Feb, Mar, Apr, May, Jun, Jul, Aug, Sep, Oct, Nov, Dec.

(e) Give the names of the objects, whose approximate horizontal coordinates are :
azimuth $A_1 = 45°$ and altitude $h_1 = 58°$:
azimuth $A_2 = 278°$ and altitude $h_2 = 20°$:
(If you can, use Bayer designations, IAU abbreviations and Messier numbers or English or Latin names.)

(f) Give the horizontal coordinates (azimuth, altitude) of:
Sirius (α CMa): $A_3 =$; $h_3 =$
The Andromeda Galaxy (M31): $A_4 =$; $h_4 =$

(g) Give the equatorial coordinates of the star marked on the sky with a red arrow: $\alpha =$; $\delta =$

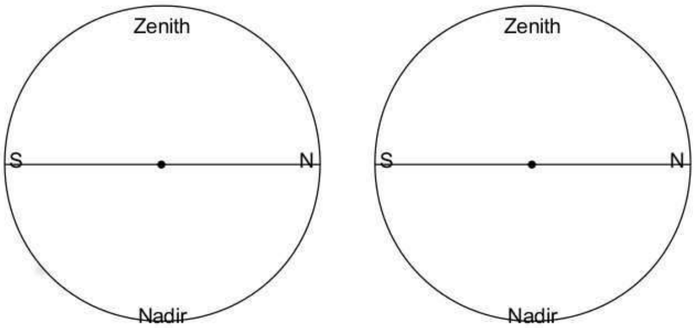
3. Simulation of Martian sky: The sky projected now is as it appears for an observer standing at some distance from a martian base. The martian base is visible on the horizon. (I11 - O02 - C)

(a) Give the areographic (Martian) latitude of the observer : $\varphi =$

(b) Give the altitudes of upper (h_u) and lower (h_l) culmination of:
Pollux (β Gem) : $h_u =$; $h_l =$
Deneb (α Cyg) : $h_u =$; $h_l =$

(c) Give the areocentric (Martian) declination of:
Regulus (α Leo) $\delta=$
Toliman (α Cen) $\delta=$

(d) Sketch diagrams to illustrate your working in the two sub-questions 3b and 3c above:

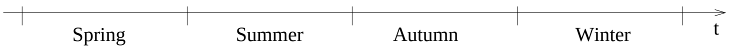


(e) On the map of the sky, mark the Martian celestial North Pole with a cross and label it as “M”.

(f) Estimate the azimuth (measured from the South) of the observer as seen from the Martian base visible on the horizon.

(g) Estimate the location of the base on Mars and circle the appropriate description:
a. near the northern Tropic circle — b. near the Equator
c. near the northern Arctic circle — d. near the North Pole

(h) The time axis below shows the Martian year and the seasons in the northern hemisphere. Mark the date corresponding to the sky on the axis.



Chapter 11

Solutions: Celestial Mechanics

11.1 Theory

1. (perihelion distance) + (aphelion distance) = (major axis of the orbit).
   Thus, semi-major axis, $a = \frac{31.5 + 0.5}{2} = 16$ A. U.
   According to Kepler's third law, we have

$$T^2 \propto a^3 \tag{11.1}$$
$$T^2 = (\text{constant})a^3$$

   This constant is $1 \frac{(year)^2}{(A.U.)^3}$, for our Solar system, when T is measured in years and a in A.U.s. This is a useful trick to remember for comparision of periods of the solar system object.
   For this comet we have,

$$T^2 = 16^3 = (4)^6 = \left(4^3\right)^2$$
$$T = 64.0 \text{ years} \tag{11.2}$$

2. This comet has period of 64 years and semi-major axis measuring 16 A. U.
   By Kepler's second law, the comet will sweep equal area in equal time.

$$\therefore \text{Area per year} = \frac{\text{Area of elliptical orbit}}{T} = \frac{\pi ab}{T} \tag{11.3}$$

   The perihelion distance is given by $a(1 - e) = 0.5$ A. U., where e is the eccentricity.

$$
\begin{aligned}
b^2 &= a^2(1 - e^2) && (11.4)\\
&= a^2\left(1 - \left(1 - \frac{0.5}{a}\right)^2\right)\\
&= a^2 - (a - 0.5)^2\\
&= 16^2 - 15.5^2\\
b &= 3.97 A.U. && (11.5)\\
\therefore \text{Area per year} &= \frac{\pi \times 16 \times 3.97}{64}\\
&= 3.1\ (\text{A.U.})^2/\text{year} && (11.6)
\end{aligned}
$$

3. As the comet is entering inner solar system, its perihelion distance will be of the order of 10 A.U. or less, which is very small as compared to aphelion distance.

$$
\begin{aligned}
a &= \frac{d_{peri} + d_{ap}}{2} && (11.7)\\
&\approx \frac{d_{ap}}{2} = \frac{35000}{2}\\
&= 17500 AU && (11.8)\\
T &= a^{1.5} && (11.9)\\
&= 2.3 \times 10^6\ years && (11.10)\\
t_{journey} &= \frac{T}{2} && (11.11)\\
&= 1.2 \times 10^6 \text{ years.} && (11.12)
\end{aligned}
$$

4. Let us assume that A normal man can jump vertically up to 50 cm on the Earth. This number is purely a rough estimate and one can choose

any number between 10 cm to 50 cm.

$$v_{jump} = \sqrt{2gh} \tag{11.13}$$

$$v_{escape} = \sqrt{\frac{2GM}{R}} \tag{11.14}$$

$$\therefore 2gh = \frac{2GM}{R} = \frac{2G}{R}\frac{4\pi\rho R^3}{3}$$

$$R = \sqrt{\frac{3gh}{4\pi\rho G}} \tag{11.15}$$

$$= \sqrt{\frac{3 \times 9.81 \times 0.5}{4\pi \times 5515 \times 6.67 \times 10^{-11}}}$$

$$= 1.78 \times 10^3 m \tag{11.16}$$

$$R \approx 2km \tag{11.17}$$

5. The mass of the asteroid is

$$m = \frac{4}{3}\pi r^3 \rho \tag{11.18}$$

$$= \frac{4}{3}\pi (1.1 \times 10^3)^3 \times 2.2 \times 10^3$$

$$= 1.23 \times 10^{13} Kg \tag{11.19}$$

Since,$m_{astronaut} \ll m$ it can be safely ignored. Then, critical velocity is,

$$v_{crit} = \sqrt{\frac{Gm}{r}} \tag{11.20}$$

$$= \sqrt{\frac{6.6726 \times 10^{-11} \times 1.23 \times 10^{13}}{1100}}$$

$$= 0.863\ Km/s \tag{11.21}$$

If the velocity of the astronaut is greater then v_{crit}, he will start orbiting the satellite. That means he will not be able to "walk" on the satellite. If the astronaut wants to complete a circle along the equator of the asteroid on foot within 2.2 hours,

$$v_2 = \frac{2\pi \times 1100}{2.2 \times 3600} \tag{11.22}$$

$$= 0.873 Km/s \tag{11.23}$$

As $v_2 > v_{crit}$, the astronaut will NOT be able to complete the circle on foot within stipulated time.

6. As the stars are all part of the cluster, the r.m.s. velocity must be smaller than the escape velocity. As an rough estimate, one can take

$$v_{esc} = \sqrt{2} v_{rms} \tag{11.24}$$

However, this is just a convenient assumption and one is free to make similar other assumptions.

$$\begin{aligned}
\frac{1}{2} M v_{esc}^2 &= \frac{1}{2} M \frac{GM}{R} && (11.25)\\
v_{esc}^2 &= \frac{2GM}{R} \\
\therefore v_{rms}^2 &= \frac{GM}{R} \\
\therefore M &= \frac{R v_{rms}^2}{G} && (11.26)\\
&= \frac{20 \times 3.0856 \times 10^{16} \times 9 \times 10^6}{6.672 \times 10^{-11}} \\
&= 8.3 \times 10^{34} \text{ Kg} && (11.27)\\
M &= 4.2 \times 10^4 M_\odot && (11.28)
\end{aligned}$$

7. The escape velocity of object on the edge of the cluster is given by,

$$\begin{aligned}
v_e &= \sqrt{\frac{2GM_{cl}}{R_{cl}}} && (11.29)\\
\therefore M_{cl} &= N M_\odot = \frac{R_{cl} v_e^2}{2G} \\
\therefore N &= \frac{R_{cl} v_e^2}{2GM_\odot} && (11.30)\\
&= \frac{20 \times 3.0856 \times 10^{16} (6 \times 10^3)^2}{2 \times 6.6726 \times 10^{-11} \times 1.9891 \times 10^{30}} \\
N &\approx 8.4 \times 10^4 && (11.31)
\end{aligned}$$

8. To determine type of orbit, we find total energy of the spacecraft.

$$\begin{aligned} E_{Tot} &= KE + PE \\ &= \frac{1}{2}mv^2 - \frac{GMm}{R} \qquad (11.32) \\ &= m\left(\frac{(17062)^2}{2} - \frac{6.6726 \times 10^{-11} \times 1.9891 \times 10^{30}}{116.406 \times 1.4960 \times 10^{11}}\right) \\ &= m\left(145.556 \times 10^6 - 7.62158 \times 10^6\right) \\ E_{Tot} &= 1.3793 \times 10^8 m \qquad (11.33) \\ E_{Tot} &> 0 \qquad (11.34) \end{aligned}$$

Hence the orbit is **Hyperbolic**. The magnitude of the Sun will be given by,

$$m_1 - m_2 = -2.5 \log\left(\frac{f_1}{f_2}\right) \qquad (11.35)$$

$$m_1 - m_2 = -2.5 \log\left(\frac{d_2^2}{d_1^2}\right)$$

$$m_1 = m_2 + 5 \log\left(\frac{d_1}{d_2}\right) \qquad (11.36)$$

$$= -26.72 + 5 \log\left(\frac{116.406}{1}\right)$$

$$m_1 = -16.39 \qquad (11.37)$$

9. Ratio of average densities of the Sun and the Earth will be given by

$$\frac{\rho_\oplus}{\rho_\odot} = \frac{M_\oplus}{(R_\oplus)^3}\frac{(R_\odot)^3}{M_\odot} \qquad (11.38)$$

$$T_\oplus = 2\pi\sqrt{\frac{a^3}{GM_S}} \qquad (11.39)$$

Let $\theta_\odot$ be the angular diameter of Sun in radians.

$$\theta_\odot = \frac{2R_\odot}{a} \qquad (11.40)$$

$$\Rightarrow a = \frac{2R_\odot}{\theta_\odot}$$

$$T^2 = 4\pi^2 \times \frac{8{R_\odot}^3}{GM_\odot\theta_\odot^3}$$

$$\Rightarrow \frac{R_\odot^3}{M_\odot} = \frac{GT^2\theta_\odot^3}{32\pi^2} \qquad (11.41)$$

Using expression for arc legnth of a circle and definition of g,

$$g = \frac{GM_\oplus}{(R_\oplus)^2}$$
$$\Rightarrow \frac{M_\oplus}{(R_\oplus)^2} = \frac{g}{G} \tag{11.42}$$
$$R_\oplus \varphi = S$$
$$\frac{1}{R_\oplus} = \frac{\varphi}{S} \tag{11.43}$$

Combining equations 11.41, 11.42 and 11.43, we get

$$\frac{\rho_\oplus}{\rho_\odot} = \frac{M_\oplus}{(R_\oplus)^2}\frac{1}{R_\oplus}\frac{(R_\odot)^3}{M_\odot}$$
$$= \frac{g}{G}\frac{\varphi}{S}\frac{GT^2\theta_\odot^3}{32\pi^2}$$
$$= \frac{g\varphi\theta_\odot^3 T^2}{32\pi^2 S} \tag{11.44}$$
$$= g \times \frac{\pi}{180} \times \left(\frac{\pi}{360}\right)^3 \times \frac{T^2}{32\pi^2 S}$$
$$= \frac{gT^2\pi^2}{32 \times 180 \times (360)^3 S} \tag{11.45}$$
$$= \frac{9.81 \times (3.1557 \times 10^7)^2 \pi^2}{32 \times 180 \times (360)^3}$$
$$\frac{\rho_\oplus}{\rho_\odot} = 3.23 \tag{11.46}$$

10. Here we have to balance the gravitational attraction of the Sun with the radiation pressure. The force of radiation pressure is simply solar radiation flux received by an object divided by the speed of light. Assuming that the dust grains completely abosorb all the radiation incident on them, Force due to gravitational attraction $= \frac{GMm}{r^2}$ Force due to radiation process $= \frac{L}{4\pi r^2}\frac{\pi D^2}{4c}$ [Dust grains completely absorbs

all radiation] $\frac{LD^2}{16r^2c}$ For equilibrium,

$$F_{RP} = \frac{L_\odot}{4\pi a_\oplus^2}\frac{\pi r_{dust}^2}{c} \tag{11.47}$$
$$F_{grav} = F_{RP} \tag{11.48}$$
$$\therefore \frac{GM_\odot m}{a_\oplus^2} = \frac{L_\odot r_{dust}^2}{4a_\oplus^2 c}$$
$$m = \frac{L_\odot r_{dust}^2}{4GM_\odot c}$$
$$\frac{4\pi}{3}r_{dust}^3\rho = \frac{L_\odot r_{dust}^2}{4GM_\odot c}$$
$$\therefore r_{dust} = \frac{3L_\odot}{16\pi\rho GM_\odot c} \tag{11.49}$$
$$= \frac{3 \times 3.826 \times 10^{26}}{16\pi \times 10^3 \times 6.6726 \times 10^{-11} \times 1.9891 \times 10^{30} 3 \times 10^8}$$
$$r_{dust} \approx 0.6\mu m \tag{11.50}$$

11.

$$\tau = I\alpha \tag{11.51}$$
$$= \frac{2}{5}M_\oplus R_\oplus^2\alpha$$
$$\omega_i = \omega_f + \alpha t \tag{11.52}$$
$$= \frac{2\pi}{day_{sidereal}} + \frac{\tau}{\frac{2}{5}M_\oplus R_\oplus^2} \times 6 \times 10^8 \times (1\ year)$$
$$= \frac{2\pi}{23^h 56^m 4^s.1} + \frac{5 \times 6 \times 10^{16} \times 6 \times 10^8 \times 3.1557 \times 10^7}{2 \times 5.9736 \times 10^{24} \times (6.3708 \times 10^6)^2}$$
$$= 7.292 \times 10^{-5} + 1.171 \times 10^{-5}$$
$$\omega_i = 8.463 \times 10^{-5}\ rad/s \tag{11.53}$$
$$days_i = \frac{1\ sidereal\ year}{T_i} = \frac{1\ sidereal\ year \times \omega_i}{2\pi} \tag{11.54}$$
$$= \frac{3.15 \times 10^7 \times 8.463 \times 10^{-5}}{2\pi}$$
$$= 424.3\ days$$
$$days_i \approx 420\ days \tag{11.55}$$

12. The mechanical energy is conserved and has the form

$$E = \frac{p^2}{2m} - \frac{GMm}{r} = \frac{p^2}{2m} - GMmu \tag{11.56}$$

$$E_A = E_B \tag{11.57}$$

$$\frac{p_A^2}{2m} - GMmu_A = \frac{p_B^2}{2m} - GMmu_B \tag{11.58}$$

$$\frac{p_A^2}{2m} - \frac{p_B^2}{2m} = GMmu_A - GMmu_B \tag{11.59}$$

$$\frac{(p_A^2 - p_B^2)}{2m} = GMm(u_A - u_B) \tag{11.60}$$

$$\therefore m = \sqrt{\frac{(p_A^2 - p_B^2)}{2GM(u_A - u_B)}} \tag{11.61}$$

$$m = \sqrt{\frac{(5.2 \times 10^7)^2 - (1.94 \times 10^9)^2}{2 \times 6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times (5.15 - 194.17) \times 10^{-8}}} \tag{11.62}$$

$$= 5.0 \times 10^4 kg \tag{11.63}$$

$$E = E_A = \frac{p_A^2}{2m} - GMmu_A \tag{11.64}$$

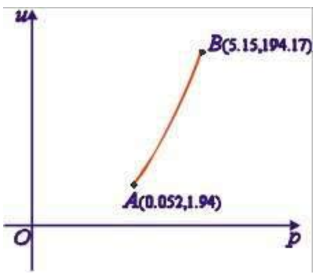
$$E = \frac{(5.2 \times 10^7)^2}{2 \times 50000} - 6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 5 \times 10^4 \times 5.15 \times 10^{-8} \tag{11.65}$$

$$= -1.0 \times 10^{12} J \tag{11.66}$$

To sketch the curve, we use the equation

$$E = \frac{p^2}{2m} - GMmu \tag{11.67}$$

$$\rightarrow u = \frac{p^2}{2GMm^2} - \frac{E}{GMm} \tag{11.68}$$



This is a parabolic curve.

13. We can solve the question in two ways:

(I) Algebraic Solution
As the spacecraft is stationary at (0,0), let $y = mx$ be the line of sight of the telescope.
Then, the intersection of the line of sight and the circle is

$$x^2 + (mx)^2 - 10x - 8(mx) + 40 = 0 \tag{11.69}$$
$$(1 + m^2)x^2 - (10 + 8m)x + 40 = 0 \tag{11.70}$$

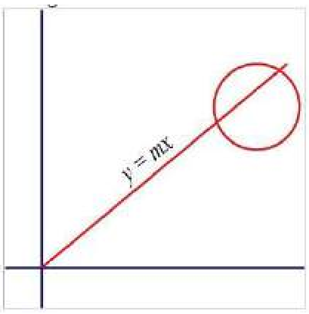
The equation will have solution if its discriminant ($D = b^2 - 4ac$) is greater than or equal to zero.

$$D = b^2 - 4ac \geq 0 \tag{11.71}$$
$$(-10 - 8m)^2 - 4(1 + m^2)40 \geq 0 \tag{11.72}$$
$$64m^2 + 160m + 100 - 160m^2 - 160 \geq 0 \tag{11.73}$$
$$-96m^2 + 160m - 60 \geq 0 \tag{11.74}$$
$$-24m^2 + 40m - 15 \geq 0 \tag{11.75}$$

The solution of the inequality is

$$\frac{(10 - \sqrt{10})}{12} \leq m \leq \frac{(10 + \sqrt{10})}{12} \tag{11.76}$$

Therefore these extreme values will be values of $\tan\phi$.



(II) Geometric Solution

$$x^2 + y^2 - 10x - 8y + 40 = 0 \tag{11.77}$$
$$x^2 - 10x + 25 + y^2 - 8y + 16 - 1 = 0 \tag{11.78}$$
$$(x - 5)^2 + (y - 4)^2 = 1 \tag{11.79}$$

The centre of the circle is $(5, 4)$ and the radius is 1 unit.
Now, we should find tangents from origin to this circle to give

minimum and maximum angles of elevation. For this purpose, we define angle α as the angle made by the center of the disk with x-axis and angle β as angle subtended by radius of the disk at the origin.

$$\tan\alpha = \frac{4}{5} \tag{11.80}$$

$$\tan\beta = \frac{1}{\sqrt{5^2+4^2-1}} = \frac{1}{\sqrt{40}} = \frac{1}{2\sqrt{10}} \tag{11.81}$$

$$\tan\phi_{min} = \tan(\alpha-\beta) \tag{11.82}$$

$$= \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} \tag{11.83}$$

$$= \frac{\frac{4}{5} - \frac{1}{2\sqrt{10}}}{1+\frac{4}{10\sqrt{10}}} \tag{11.84}$$

$$= \frac{(8\sqrt{10}-5)}{(10\sqrt{10}+4)} \tag{11.85}$$

$$= \frac{(8\sqrt{10}-5)(10\sqrt{10}-4)}{(10\sqrt{10}+4)(10\sqrt{10}-4)} \tag{11.86}$$

$$= \frac{(800-50\sqrt{10}-32\sqrt{10}+20)}{(1000-16)} \tag{11.87}$$

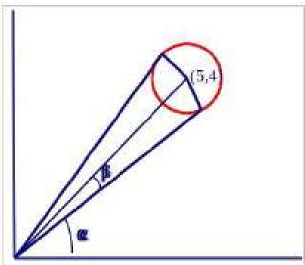
$$= \frac{82(10-\sqrt{10})}{82\times 12} \tag{11.88}$$

$$= \frac{(10-\sqrt{10})}{12} \tag{11.89}$$

$$\tan\phi_{max} = \tan(\alpha+\beta) \tag{11.90}$$

$$= \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} \tag{11.91}$$

$$= \frac{(10+\sqrt{10})}{12} \tag{11.92}$$



14. For the cloud, let O be the centre and A be some point on the surface.

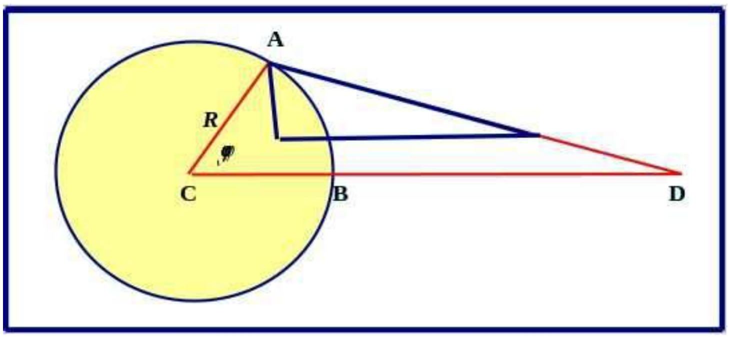
Thus, the object is traveling from O to A. Let V_i be gravitational potential at point i.
If the particle was free-falling, then its velocity at the surface will be be zero.

$$KE_A + PE_A = KE_O + PE_O \tag{11.93}$$
$$0 + V_A = \frac{GMm}{2R} + V_O \tag{11.94}$$
$$\therefore V_O = V_A - \frac{GMm}{2R}$$
$$= -\frac{GMm}{R} - \frac{GMm}{2R}$$
$$V_O = -\frac{3GMm}{2R} \tag{11.95}$$

To escape, total energy of the particle should be zero. If $V_O = V_{esc}$

$$KE_O + PE_O = 0 \tag{11.96}$$
$$\frac{1}{2}V_{esc}^2 + V_O = 0 \tag{11.97}$$
$$\frac{1}{2}V_{esc}^2 = \frac{3GM}{2R}$$
$$\therefore V_{esc} = \sqrt{\frac{3GM}{R}} \tag{11.98}$$

15. In this figure, D is the centre of the Moon. The moon's gravitational acceleration at A and the centre of the Earth C are,



$$\overrightarrow{g_A} = -\frac{GM_m}{r_A^3}\overrightarrow{r_A} \tag{11.99}$$
$$\overrightarrow{g_C} = -\frac{GM_m}{r_C^3}\overrightarrow{r_C} = -\frac{GM_m}{r^3}\overrightarrow{r_C} \tag{11.100}$$

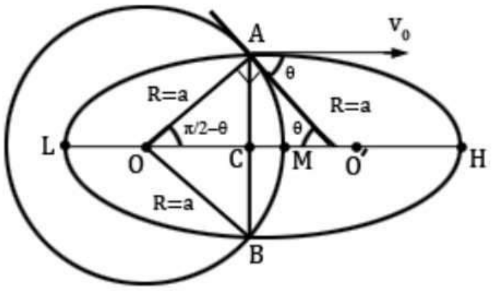


Figure 11.1 – High altitude projectile

Both these vectors point towards the Moon. From the figure, $r_A = AD$ and $r_C = CD$.
The Moon's net gravitational acceleration felt by test mass at A is,

$$\overrightarrow{g'_A} = \overrightarrow{g_A} - \overrightarrow{g_C} = -\left(\frac{GM_m}{r_A^3}\overrightarrow{r_A} - \frac{GM_m}{r^3}\overrightarrow{r_C}\right) \tag{11.101}$$

From the figure, $\overrightarrow{r_A} = \overrightarrow{r_C} + \overrightarrow{R}$. Thus,

$$\overrightarrow{g'_A} = -\frac{GM_m}{r_A^3}\overrightarrow{R} - \left(\frac{GM_m}{r_A^3} - \frac{GM_m}{r^3}\right)\overrightarrow{r_C} \tag{11.102}$$

Now, first term of the R.H.S. is purely radial. Thus, for horizontal acceleration of the sea water at A, we have to only take horizontal component of the second term. Distance r_A can be found by cosine rule.

$$\overrightarrow{g'_{A_{(hori)}}} = \left(\frac{GM_m}{r^3} - \frac{GM_m}{r_A^3}\right) r\sin\phi \tag{11.103}$$

$$= \left(\frac{1}{r^3} - \frac{1}{(r^2 + R^2 - 2Rr\cos\phi)^{3/2}}\right) GM_m r\sin\phi \tag{11.104}$$

16. In the figure 11.1, the projectile is launched from point A on the surface of the earth, it reaches highest point at H and then hits the Earth surface back at B. The principal focus of the orbit is at the centre of the earth (O). Point M is on the surface of the earth along the major axis (LOH). Line AB cuts the major axis at C and O' is the other focus of the ellipse. Tangent at point A meets the major axis between M and O'.

    One should remember that when the projectile goes from the surface to high altitude, the gravitational acceleration will be variable along the path. Hence approximated relations, which assume constant g, cannot be used.

(a) Total energy of the projectile is

$$E = \frac{1}{2}mv_0^2 - \frac{GM_\oplus m}{R_\oplus} \tag{11.105}$$
$$= \frac{GM_\oplus m}{2R_\oplus} - \frac{GM_\oplus m}{R_\oplus} \tag{11.106}$$
$$= -\frac{GM_\oplus m}{2R_\oplus} < 0 \tag{11.107}$$

As the total energy is less than 0, it means that orbit might be ellipse or circle. As $\theta > 0$, the orbit is an ellipse. Total energy for an ellipse is

$$E = -\frac{GMm}{2a} \tag{11.108}$$

Comparing equation 11.107 and 11.108, we conclude

$$a = R \tag{11.109}$$

(b) Let b be the semi-minor axis of the orbit.

$$b = a\sqrt{1-e^2} \tag{11.110}$$
$$= R_\oplus\sqrt{1-e^2} \tag{11.111}$$
$$b = R_\oplus \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}R_\oplus \tag{11.112}$$
$$\therefore \sqrt{1-e^2} = \frac{\sqrt{3}}{2}$$
$$1 - e^2 = \frac{3}{4}$$
$$e^2 = \frac{1}{4}$$
$$\therefore e = 0.5 \tag{11.113}$$
$$r_{max} = R_\oplus(1+e) \tag{11.114}$$
$$= 1.5R_\oplus \tag{11.115}$$
$$\therefore h_{max} = HM = HO - OM \tag{11.116}$$
$$= r_{max} - R_\oplus$$
$$h = 0.5R_\oplus \tag{11.117}$$

(c) As there is no air drag or other dissipitive force, paths AHB will be symmetric about the major axis. As $\measuredangle COA = 60°$, we can conclude, $\measuredangle BOA = 120°$. Thus range from A to B along the surface will be,

$$R = \widehat{AB} = \frac{120}{360} \times 2\pi R_{\oplus} \tag{11.118}$$
$$R = \frac{2\pi}{3} R_{\oplus} \tag{11.119}$$

(d) As already shown in equation 11.113, $e = 0.5$

(e) We can show that C is actually the centre of the ellipse by showing that $OC = ae$.

$$l(\overline{OC}) = l(\overline{OC}) \sin(\measuredangle OAC) \tag{11.120}$$
$$= a \sin 30°$$
$$l(\overline{OC}) = 0.5a = ae \tag{11.121}$$
$$\therefore b = a\sqrt{1 - e^2} \tag{11.122}$$
$$= a\sqrt{1 - (0.5)^2}$$
$$= a\sqrt{\left(1 - \frac{1}{4}\right)} = a\sqrt{\frac{3}{4}}$$
$$b = \frac{\sqrt{3}a}{2} \tag{11.123}$$

Now by Kepler's second law, time spent to cover particular part of the orbit will be proportional to the area swept by the radius vector. Thus, we want to find $Area(OAHBO)$.

$$\begin{aligned}
Area(OAHBO) &= Area(CAHBC) + Area(\triangle AOB) && (11.124)\\
&= \frac{\pi ab}{2} + \frac{1}{2} \times l(\overline{OC}) \times l(\overline{AB}) && (11.125)\\
&= \frac{\pi ab}{2} + \frac{1}{2} \times 0.5a \times 2b\\
&= \frac{\pi ab}{2} + \frac{ab}{2}\\
Area(OAHBO) &= \frac{(\pi+1)ab}{2} && (11.126)\\
T &= \sqrt{\frac{4\pi^2 a^3}{GM_\oplus}} && (11.127)\\
\therefore \frac{t}{T} &= \frac{Area(OAHBO)}{Area(LAHBL)} && (11.128)\\
\therefore t &= \frac{(\pi+1)ab}{2\pi ab} \times 2\pi\sqrt{\frac{a^3}{GM_\oplus}} && (11.129)\\
&= (\pi+1)\sqrt{\frac{a^3}{GM_\oplus}} && (11.130)\\
&= (\pi+1)\sqrt{\frac{(6.3708 \times 10^6)^3}{6.672 \times 10^{-11} \times 5.9736 \times 10^{24}}}\\
&= 3336\ sec && (11.131)\\
t &= 55^m 36^s && (11.132)
\end{aligned}$$

17. As the orbit is parabolic,

$$\begin{aligned}
E_{sat} &= \left(\frac{-GM_\odot}{r_{sat}} + \frac{1}{2}v_{sat}^2\right) m_{sat} = 0 && (11.133)\\
\therefore v_{sat} &= \sqrt{\frac{2GM_\odot}{r_{sat}}} && (11.134)
\end{aligned}$$

(a) Angular momentum of the satellite at the launch is,

$$\begin{aligned}
L &= m_s a t v_{sat,\oplus} a_\oplus\\
&= m_{sat}\sqrt{2GM_\odot a_\oplus} && (11.135)
\end{aligned}$$

Let v_T is the tangential velocity of the satellite at the Mars' orbit.

Conserving the angular momentum,

$$m_{sat}\sqrt{2GM_{\odot}a_{\oplus}} = m_{sat}v_T a_m$$

$$\therefore v_T = \frac{\sqrt{2GM_{\odot}a_{\oplus}}}{a_m} \tag{11.136}$$

$$\cos\psi = \frac{v_T}{v} = \frac{\sqrt{2GM_{\odot}a_{\oplus}}}{a_m}\sqrt{\frac{a_m}{2GM_{\odot}}} \tag{11.137}$$

$$= \sqrt{\frac{a_{\oplus}}{a_m}} = \sqrt{\frac{1}{1.52}}$$

$$= 0.8111 \tag{11.138}$$

$$\therefore \psi = 35.80° \tag{11.139}$$

(b) If the Mars is very close to the satellite, then orbital velocity of the Mars will be parallel to the tangential velocity of the satellite.

$$\overrightarrow{v_{rel}} = \overrightarrow{v_{rad}} + \overrightarrow{v_T} - \overrightarrow{v_m} \tag{11.140}$$

$$= v_{rad}\hat{r} + (v_T - v_m)\hat{\theta} \tag{11.141}$$

$$v_m = \sqrt{\frac{GM_{\odot}}{a_m}} \tag{11.142}$$

$$= \sqrt{\frac{6.6726 \times 10^{-11} \times 1.9891 \times 10^{30}}{1.52 \times 1.496 \times 10^{11}}}$$

$$v_m = 24.16\ km/s \tag{11.143}$$

$$v = \sqrt{\frac{2GM_{\odot}}{r_m}} = \sqrt{2}v_m \tag{11.144}$$

$$= 34.17\ km/s \tag{11.145}$$

$$v_T = v\cos\psi = 34.17 \times 0.8111 \tag{11.146}$$

$$= 27.71\ km/s \tag{11.147}$$

$$v_{rad} = v\sin\psi \tag{11.148}$$

$$= 19.99\ km/s \tag{11.149}$$

$$\therefore \overrightarrow{v_{rel}} = 19.99\hat{r} + 3.55\hat{\theta} \tag{11.150}$$

$$|\overrightarrow{v_{rel}}| = 20.30\ km/s \tag{11.151}$$

$$\theta = \tan^{-1}\left(\frac{3.55}{19.99}\right) \tag{11.152}$$

$$= 10.07° \tag{11.153}$$

Thus the angle is about 10° eastwards. As the satellite has been launched in the direction of the Earth's motion and given the fact that the Mars and the Earth have same sense of revolution around the Sun, $\overrightarrow{v_T}$ and $\overrightarrow{v_m}$ are in the same direction and hence the magnitudes have to be subtracted.

18. In this question the trick is to realise that the two angles α and β are not in the same plane. In other words, the impulse given $(\Delta\overrightarrow{p})$ is not in the plane of the orbit of the satellite. If the orbit is in x-y plane, we can say,

$$\overrightarrow{p'} = \overrightarrow{p} + \Delta\overrightarrow{p} \tag{11.154}$$

$$p'_{tangential} = p + p\cos\alpha\sin\beta \tag{11.155}$$

$$p'_{radial} = p\cos\beta \tag{11.156}$$

$$p'_z = p\sin\alpha\sin\beta \tag{11.157}$$

We note that if $\cos\beta \neq 0$ then, the point of explosion will no longer remain either perigee or apogee point. Thus,

$$\beta = 90^\circ \ or \ 270^\circ \tag{11.158}$$

Thus, we get

$$\begin{aligned} p' &= \sqrt{p^2(1+\cos\alpha)^2 + p^2\sin^2\alpha} && (11.159) \\ &= p\sqrt{1+\cos^2\alpha + 2\cos\alpha + \sin^2\alpha} \\ p' &= p\sqrt{2(1+\cos\alpha)} \\ &= p\sqrt{4\cos^2(\alpha/2)} \\ p' &= 2p\cos(\alpha/2) && (11.160) \end{aligned}$$

Now, for orbit to remain circular, we need $p' = p$.
For parabola we need, $p' = \sqrt{2}p$.
For ellipse, we need p' to be smaller than the value for parabola and for hyperbola we need values greater than that of parabola. Thus,

Type	α		
hyperbola perigee = initial point	$[0^\circ : 90^\circ)$	or	$(270^\circ : 360^\circ]$
parabola perigee = initial point	90°	or	270°
ellipse perigee = initial point	$(90^\circ : 120^\circ)$	or	$(240^\circ : 270^\circ)$
circle	120°	or	240°
ellipse apogee = initial point	$[120^\circ : 180^\circ)$	or	$(180^\circ : 240^\circ]$

At exactly $\alpha = 180^\circ$, we get $p' = 0$ and the satellite will fall on the earth. This also assumes the Earth to be a point object. However, in reality, for angles close to $\alpha = 180^\circ$, the orbit will pass so close to the centre of the Earth, the satellite will hit the Earth surface.

11.2 Data Analysis

1. It was expected that students should pay attention the motion of the moons. By Kepler's third law, shorter the orbital radius, shorter the period. Students were expected to know the sequence of moons (from closer to Jupiter to farther) as Io, Europa, Ganymede and Callisto.

Chapter 12

Solutions: Celestial Coordinate Systems

12.1 Theory

1.

$$\alpha = \frac{\text{Diameter of Earth}}{\text{Distance to Moon}} \tag{12.1}$$

$$= \frac{D_\oplus}{d_{e-m}} \text{ radians } = \frac{D_\oplus}{d_{e-m}} \times \frac{180 \times 60}{\pi} \text{ arcminutes} \tag{12.2}$$

$$= \frac{12742}{378000} \times \frac{180 \times 60}{\pi} \text{ arcminutes} \tag{12.3}$$

$$= 116' = \frac{116}{15} \text{ minutes of RA} \tag{12.4}$$

Thus, the difference in apparent right ascension is 7.73 minutes.

2. As the solar day is measured from noon to noon, it is slightly longer than the rotation period of the Earth (sidereal day). Over the course of one solar year, this small time is accumulated to one additional sidereal day. Thus, in exactly half a year, it would have accumulated to 0.5 extra days.

   Thus, there will be 183.5 sidereal days from the noon of 1 July to the noon of 31 December.

   Mathematically, the sidereal day is $3^m\ 56^s$ shorter than the solar day.

$$\therefore 183 \ solar \ days = 183^d + 183 \times (3^m 56^s)$$
$$= 183^d 11^h 50^m 48^s$$
$$\approx 183.5 \text{ days} \tag{12.5}$$

3. For condition (a), we have to ensure that the ray of light, grazing the outer edge of the Tabeshband, should hit lower edge of the window. For condition (b), we have to ensure that the ray of light, grazing the outer edge of the Tabeshband, should reach point A as in figure 2.1. The zenith angle of the Sun at the summer and winter solstice will be given by,

$$z_s = \phi - 23.5^\circ = 12.5^\circ \tag{12.6}$$
$$z_w = \phi + 23.5^\circ = 59.5^\circ \tag{12.7}$$

From the figure, (right) at the summer solstice we have

$$\tan(z_s) = \frac{x}{h} \tag{12.8}$$
$$= \tan 12.5^\circ = 0.222 \tag{12.9}$$

And in the winter solstice,

$$\tan(z_w) = \frac{D + x}{H} \tag{12.10}$$
$$= \tan 59.5^\circ = 1.70 \tag{12.11}$$
$$\therefore x = 1.70H - D$$
$$x = 0.593m \tag{12.12}$$
$$h = x/0.222 \tag{12.13}$$
$$h = 2.67m \tag{12.14}$$

4. To find the minimum declination, we have to consider various effects. Firstly, we have to note the latitude of the observer. Then we note that as the observer is at a high altitude, he/she will be able to see some part of the sky below the standard horizon. This is known as horizon depression (θ). Thirdly, atmospheric refraction at the horizon (α) is about 34'.

$$\cos\theta = \frac{R}{R + h} = \frac{6370.8}{6370.8 + 5.6} \tag{12.15}$$
$$\theta = 2.40^\circ = 2^\circ 24' \tag{12.16}$$
$$\therefore \delta = 90^\circ - \phi - \theta - \alpha \tag{12.17}$$
$$= 90^\circ - (35^\circ 57' + 2^\circ 24' + 34')$$
$$= 90^\circ - 38^\circ 55'$$
$$\therefore \delta = 51^\circ 5' N \tag{12.18}$$

5. Recall coordinate transformation relations between equatorial and horizontal coordinates.
For the special case of rising or setting of an astronomical object, the relation is reduced to a simplified form

$$\cos H = -\tan\delta \tan\phi \tag{12.19}$$

For 21st December, $\delta_\odot = -23.5°$ and question states $\phi_{Observer} = 42.5°$. Thus,

$$\begin{aligned} H &= \cos^{-1}(-\tan(-23.5)\tan 42.5) \\ &= \cos^{-1}(0.4348 \times 0.9163) \\ &= \cos^{-1}(0.3984) \\ H &\approx 4^h 26^m 5^s \end{aligned} \tag{12.20}$$

Thus, equation 12.20 shows that the Sun rises 4 hours 26 minutes before local noon or at 7:34am local time.
Next we notice that the longitude of observer is $\lambda_{Observer} = 71°$ E.
As local time changes by 4 minute for every degree of longitude, his local time must be 4 hours and 44 minutes ahead of GMT. But the problem states that his civil time is 5 hours after GMT.
This means his civil time is 16 minutes behind of his local time.
Applying this correction, the Sunrise time as per the observer's watch will be,

$$t = 7:34 - 0:16 = 7:18am \tag{12.21}$$

EA: Correction for atmospheric refraction is roughly 34' and angular radius of solar disk can be upto 16'. As a combined effect of these two factors, the top edge of the Sun becomes visible when the centre of the Sun is about 50' below horizon.
In the figure, $\overline{ABC}$ is the horizon line. A is the position where the centre of the Sun will rise. B is the place where top edge of the Sun becomes visible when the centre of the Sun is at S. Thus, $l(\overline{BS}) = 50'$.
It can be seen that

$$\measuredangle BCS = \measuredangle BSA = \phi \tag{12.22}$$

$$\therefore AS = \frac{l(\overline{BS})}{\cos\phi} = \frac{50'}{\cos 42.5°} \approx 67'.8 \tag{12.23}$$

As this is a small length, we approximate the Sun's speed as 4 minutes per degree. Thus, the Sun will rise about 4.5 minutes earlier due to these two factors. The equation of time shows that the Sun will be about 2 minutes (1.7 minutes to be precise) ahead on 21st December.

Thus total error introduced by all these effects is about 6 minutes.

6. When ever the Sun is towards the South of prime vertical (i.e. the great circle through East, West and Zenith), the sundial will not work.

   (i) When the Sun is south of equator i.e. $-23°26' < \delta < 0$, it will set south of west and thus across the prime vertical. Thus for this period from September 23 to March 21, there will be at least some part of the day for which the Sun will not cast any shadow on the sundial. This corresponds to the spring and summer season of southern hemisphere.

   (ii) It will not work the whole day, if the Sun is south of prime vertical for the whole day i.e. it should cross the meridian on the south of zenith. This will happen when declination of the Sun is less than latitude of the place.

$$-\epsilon < \delta < \phi \tag{12.24}$$
$$-23°27' < \delta < -22°54' \tag{12.25}$$
$$\lambda = \sin^{-1}\left(\frac{\sin\delta}{\sin\epsilon}\right) \tag{12.26}$$
$$= \sin^{-1}(0.9778)$$
$$77.91° < \lambda < 102.09° \tag{12.27}$$

   This corresponds to 11.9 days on the either side of winter solstice (22nd Dec.

7. Note that the date of observation is (June 19) very close to summer solstice. Thus,
   $\delta \approx +23.°.5,\ \&\ \alpha \approx 6^h$
   At full moon phase, the Moon is nearly diametrically opposite to the Sun along the ecliptic. The diametrically opposite position will be,
   $\delta \approx -23.°.5,\ \&\ \alpha \approx 18^h$
   We further note that the Moon can be upto 5°.25 above or below the ecliptic. Thus, δ can be between $-18°.25$ and $-28°.75$ depending on

the position of node.

$$\cos(H) = -\tan\delta\tan\phi \tag{12.28}$$
$$\therefore H_1 = \cos^{-1}(-\tan\delta\tan\phi) \tag{12.29}$$
$$= \cos^{-1}(-\tan(-18.5)\tan(-6°49')) \tag{12.30}$$
$$= 92°.3 = 6^h9^m \tag{12.31}$$
$$\therefore H_1 = \cos^{-1}(-\tan\delta\tan\phi) \tag{12.32}$$
$$= \cos^{-1}(-\tan(-28.75)\tan(-6°49')) \tag{12.33}$$
$$= 93°.8 = 6^h15^m \tag{12.34}$$

As a first approximation, the duration will be given by $2H$ i.e. Its extreme values can be from 12 hours 18 minutes and 12 hours 30 minutes.

However, we have to include two important corrections to this value. Firstly, due to the lunar orbital motion, the Moon is moving backwards by slightly less than 53 minutes per day. Thus, in 12.5 hours, it will move backwards by nearly 27 minutes and will be visible for 27 more minutes. Secondly, as the refraction near horizon is nearly half a degree, the moon will be seen longer by 2 minutes each at rising and setting. Thus, total correction will be for 31 minutes.

Hence, extreme possible values are 12 hours 49 minutes and 13 hours 1 minute.

One may note that some of the data given in this problem like longitude, civil time and elevation is not used in the solution. It is expected that student solving such problems chooses data judiciously and not get confused by excess data.

8. Since FOV is very small, motion of Vega across the FOV can be approximated by a straight line. We will calculate angle of rotation of the sky near Vega's location. We must remember to include the declination effect.

$$\therefore \theta = 360° \times \frac{t}{T_{sidereal}} \times \cos\delta \tag{12.35}$$
$$= 360° \times \frac{5.3}{(23 \times 60 + 56)} \times \cos 39$$
$$\theta \approx 62' \tag{12.36}$$

9. Recall that hour angle of a star, its R.A. and local sidereal time are related by

$$\alpha = t - H \tag{12.37}$$

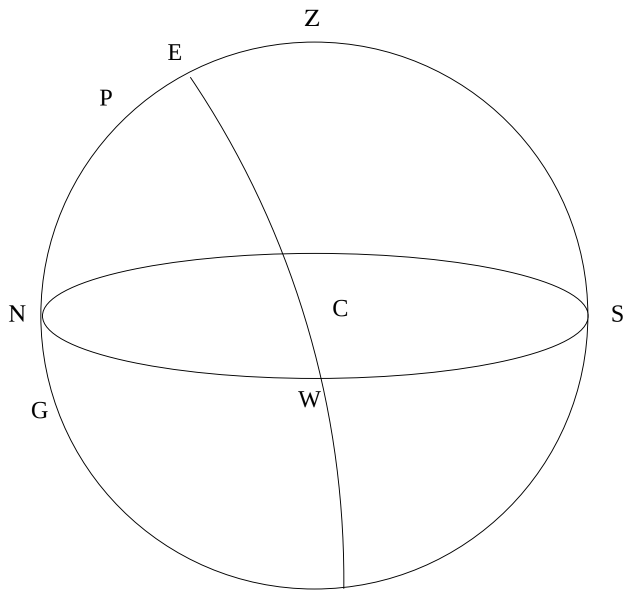


Figure 12.1 – Galactic plane in horizontal coordinates

At lower culmination, hour angle is 12 hours. Hence, the sidereal time at that moment will be

$$\begin{aligned} t &= \alpha + H \\ &= 14^h 51^m + 12^h 00^m \\ t &= 2^h 51^m \end{aligned} \tag{12.38}$$

This will be your lowermost marker on the dial. Next we should ensure that scale is marked "anti-clockwise" as that will be direction of motion of the stars. On the UT side of the card, we again note that lowermost point will correspond to 03:15 for the given date and scale will again be anti-clockwise. Now as sidereal day is shorter than the solar day by 4 minutes, the clock will need second dial to calibrate for this effect. This can be done easily by calibrating inner dial with dates.

10. As the current local sidereal time (LST) is exactly 12 hours away from RA of the northern galactic pole (NGP), it will be at the Lower culmination. In figure 12.1, P is the NCP, G is NGP and line EW marks the galactic equator. Plane NWS shows the horizon where letters represent directions. In the equation 2.1, ϵ is basically the angle between the poles of the two co-ordinates. Thus, it will be replaced by angle between ZG. Also, from the figure, it is clear that the altitude of galactic equator will be maximum at $A = 0°$ (North) and minimum at $A = 180°$ (South).

Note that $A = 90°$ at the point where the galactic equator meets the horizon. As compared to this, R.A. is zero at the meeting point of ecliptic. Also as the NGP is below horizon, the directions of increasing coordinates is opposite. Thus, $\sin\alpha$ should be replaced by $-\cos A$.

$$ZG = PG + PZ \tag{12.39}$$
$$= 90° - \delta_G + 90 - \phi$$
$$= 180° - (\delta_G + \phi) \tag{12.40}$$
$$h = \tan^{-1}[-\cos A \tan(180 - (\delta_G + \phi))]$$
$$= \tan^{-1}[\cos A \tan(\delta_G + \phi)] \tag{12.41}$$
$$\phi + \delta_G = 49°39' + 27°08'$$
$$= 74°.7 \tag{12.42}$$
$$h = \tan^{-1}[\cos A \tan 74°.7] \tag{12.43}$$

11. One should always remember that the precession does not change ecliptic latitudes of the stars.
At North Hemisphere, declination of the non-rising stars $\delta < \varphi - 90°$, where φ stands for latitude, and declination of the non-setting stars $\delta > 90° - \varphi$. So in Krakow,

$$\delta_{non-rise} < -39.9° \tag{12.44}$$
$$\delta_{non-set} > 39.9° \tag{12.45}$$

Stars with intermediate declinations will rise and set. As an appproximate solution, we notice that both the stars are reasonably close to 6^h R.A. Let us assume their R.A. to be exactly 6 hours. Currently at this R.A., declination of ecliptic (δ_{ec}) is +23.5°, which is highest possible. If it changes to lowest possible value, i.e. $\delta_{ec} = -23.5°$, then, declination of both the stars will decrease by 47°.

$$\therefore \delta_{Sirius} \approx -63° \tag{12.46}$$
$$\delta_{Canopus} \approx -80° \tag{12.47}$$

It is not possible for declination of canopus to be much higher than its current declination. So it will never be possible for it to rise at Krakow. However, the new declination of Sirius is well inside never-rising region of the sky for Krakow. So it can become a never-rising star.
One can solve this more rigourously using the spherical trigonometric formula given below. However, as the new declinations by approximate method are far away from boundary of never-rising region, such detailed treatment is not necessary

$$\sin\beta = \sin\delta\cos\epsilon - \cos\delta\sin\epsilon\sin\alpha \tag{12.48}$$

12. Sunny solstice in southern hemisphere is December 23. On that day, declination of the Sun is $\delta = -23.45°$. Thus, altitude of the Sun at this island on that noon will be,

$$a_0 = 90° + \varphi - \delta \tag{12.49}$$
$$= 90° - 66.55° - (-23.45°)$$
$$a_0 = 46.9° \tag{12.50}$$

At the Sunset, the centre of the Sun will just touch horizon and then start rising again. i.e. the lowest point of the declination circle along which the Sun travels on that day is just touching the horizon. The locus of tip of the shadow will be symmetric on the either side of the prime meridian (North - Zenith - South). Thus, as shown in the figure, the equation of the locus will be that of a parabola, with origin at the focus. Let us take a generalised parabola equation and find its coefficients.

$$y = px^2 + qx + r \tag{12.51}$$

Firstly, as the focus of the parabola is at the origin, q will be zero. We will take coordinates at noon as (x_0, y_0), and the coordinates when the Sun is at prime vertical (East - Zenith - West), say pre-noon, be (x_1, y_1). To find the second set, we have to find altitude of the Sun when its when it crosses prime vertical. We use the fact that the plane declination circle is inclined to the horizon at ϵ degrees. Let us say R is radius of the imaginary celestial sphere and z' be the projected height of the Sun on z-axis. Therefore,

$$x_0 = 0 \tag{12.52}$$
$$y_1 = 0 \tag{12.53}$$
$$y_0 = r = h_{Christ} tan a_0 \tag{12.54}$$
$$= 39.60 \times \tan 46.9$$
$$\therefore r = -37.06m \tag{12.55}$$
$$z' = R \sin a_1 \tag{12.56}$$
$$\tan \epsilon = \frac{z'}{R} = \sin a_1 \tag{12.57}$$
$$\therefore a_1 = \sin^{-1}(\tan \epsilon) = 25°42' \tag{12.58}$$
$$\tan a_1 = \frac{h_{Christ}}{x_1} \tag{12.59}$$
$$\therefore x_1 = \sqrt{\frac{-r}{p}} = \frac{h_{Christ}}{\tan a_1} \tag{12.60}$$
$$= 82.26$$
$$\therefore p = \frac{-r}{x_1^2} \tag{12.61}$$
$$p = 0.0063 \tag{12.62}$$
$$\therefore y = 0.0063x^2 - 37.06 \tag{12.63}$$

13. It is imporant to understand what is asked in the question. It is as if you are standing inside a rotating, huge sphere and various stars are attached to the sphere. Now one of these stars at the given location comes loose at the given time and flies off tangentially to infinity. We would like to know final apparent coordinates of this star and also time required for him to travel certain distance.

    Let R be the radius of the Celestial sphere and (x, y, z) a coordinate system with origin at the observer, z is the down-up axis, y is the east-west axis and x is the south-north axis. At the superior culmination, we have,

$$a = \varphi + \delta = 7.41° + 49.35° \tag{12.64}$$
$$a = 56.76° \tag{12.65}$$
$$\vec{v} = v_0 \hat{y} = 2\pi R \hat{y} \ /day \tag{12.66}$$
$$\vec{r_0} = R(-\cos a \hat{x} + \sin a \hat{z}) \tag{12.67}$$

Now the object flies off at this stage. At any point in future, its location will be given by,

$$\overrightarrow{r(t)} = \overrightarrow{r_0} + \overrightarrow{v}t \tag{12.68}$$

$$a(t) = \tan^{-1}\left(\frac{z}{\sqrt{x^2+y^2}}\right) \tag{12.69}$$

When $t \to \infty$, the x and z coordinates of the star remain constant, while $y \to \infty$ and we have

$$a = \tan^{-1}\left(\frac{z}{\infty}\right) = \tan^{-1} 0 = 0° \tag{12.70}$$

Similarly,the final azimuth is the West direction (A $= 90° or A = 270°$, depending on the system used). For the magnitude,

$$m_0 - m_6 = -2.5\log\left(\frac{F_0}{F_6}\right) = -5\log\left(\frac{R_6}{R_0}\right) \tag{12.71}$$

$$R_6 = 10^{\frac{(6-0.45)}{5}} R$$

$$R_6 = 12.88R_0 \tag{12.72}$$

$$|\overrightarrow{r(t)}| = \sqrt{x^2+y^2+z^2} \tag{12.73}$$

$$= \sqrt{x_0^2 + (2\pi Rt)^2 + z_0^2}$$

$$= \sqrt{R^2 + 4\pi^2R^2t^2}$$

$$= R\sqrt{1+4\pi^2t^2} = 12.88R \tag{12.74}$$

$$\therefore 12.88^2 = 1 + 4\pi^2t^2$$

$$t = \sqrt{\frac{12.88^2 - 1}{4\pi^2}}$$

$$t = 2.044 days \tag{12.75}$$

14. As seen from an inertial reference frame outside the Earth, the observer must describe a circle parallel to the ecliptic, with constant speed, in a period of one sidereal day. Since the ecliptic poles will now be at zero latitude, this circle must be a great circle.

    But in the reference frame of the Earth, the total motion will be the motion in inertial frame (as described above) plus motion to compensate for the rotation of the Earth. In other words, the displacement of the observer is the vectorial addition of rotation through some angle φ along ecliptic and rotation through same angle φ around the Earth's rotation axis. But this is exactly the same construction which would describe analemma of the Sun, if the Earth would have been in exact circular orbit around the Sun. So the resultant path of the observer will appear like an analemma for circular orbit (symmetrical figure of 8).

It must be
– closed curve.
– should be bound by latitudes = $\pm\epsilon$.
– it should be symmetric around the the equator as well as a chosen longitude.
– It should have the shape of 8.

As the observer starts on the southern hemisphere, he will cross the equator for the first time in the direction south-north. Its velocity relative to an inertial referential will have intensity $\upsilon = \frac{2\pi R}{T}$ and azimuth (the angle with the north-south direction) $A_1 = 90° - \epsilon$. The velocity of the Earth's surface will have the same intensity and azimuth $A_2 = 90°$. Subtracting the vectors,

$$\upsilon_{combined} = \upsilon\sqrt{2(1 - \cos\epsilon)} \tag{12.76}$$

$$A_{combined} = -\frac{\epsilon}{2} = -11.72° \tag{12.77}$$

12.2 Data Analysis

1. This is an interesting problem as there is a lot of data but only a small part of it will be actually used to solve the question. Thus, key part of the solution is to first understand which data should be picked up from the table.

   (a) We start by around the dates of solstices and equinoxes, position of object A are as follows:

		R.A.			Dec.			
		h	m	s		°	'	"
Mar	21	23	59	51.47	-	0	1	1.44
Mar	22	0	3	30.11	+	0	22	40.3
Jun	21	5	56	41.18	+	23	26	13.51
Jun	22	6	0	50.78	+	23	26	20.99
Sep	23	11	58	23.75	+	0	10	27.2
Sep	24	12	1	59.32	-	0	12	54.79
Dec	22	17	58	42.17	-	23	26	27
Dec	23	18	3	8.7	-	23	26	21.28

   This matches exactly with expected positions of the Sun on these dates. Thus, **object A** is the **Sun**.

Now, we see how other objects move with respect to the Sun. If we scan through the entire table, we notice that the difference between R.A. of the Sun and that of object C is never more than 3 hours. Thus, it is an inner planet. The elongation of this planet can be found by calculating difference between the R.A. of the two. We note that on Jan 1, the difference in R. A. is

$$\begin{aligned} e &= 18^h 44m 7^s.11 - 15^h 28^m 49^s.55 \\ &= 3^h 15^m 17^s.56 \\ &\approx 48°49' \end{aligned} \tag{12.78}$$

Here the implicit assumption is the planet is exactly on the ecliptic and hence difference in R.A. will correctly give the elongation. Now in case of the Mercury, the orbital radius is about 0.4 A. U. Thus, its maximum elongation is will be about 28°. Orbital radius of the Venus is about 0.72 A.U. and the maximum elongation (assuming circular orbits) is about 47°. Thus, we note that elongation of object C is way more than maximum elongation of the Mercury but reasonably close to the maximum elongation of the Venus. The small difference can be purely due to our approximations (measuring only difference along R.A. and assuming circular orbits). We can safely conclude that the **object C** is the **Venus**.

A clear sign of outer planet is the fact that during the course of the year, it can be seen sometime at the opposition. In the terms of coordinates, it means the difference between R.A. of the object and R.A. of the Sun should be 12 hours. Both object B (February 3) and object D (August 31) can be seen at opposition. Thus, they are outer planets. We next note that object D is changing coordinates much faster than object B. Let us concentrate on the object B first.

These objects are changing coordinates from day to day. However, we must remember that this change is combined effect of the changing position of the Earth and changing position of the object. In usual cases, it will be difficult to separate the two contributions. However, as the table gives coordinates over the entire year, we can say that the orbital position of the Earth as on Jan. 1 will be nearly the same as that on Dec. 31. Thus difference in coordinates of the objects between these two dates, will be entirely because of changing position of the object.

For object B, difference in R.A. over one year is nearly 2^h i.e. it will take nearly 12 years to go through full circle (24^h). We can recall that synodic period of the Jupiter is 12 years and conclude that the **object B** is the **Jupiter**. As object D is moving faster than object D, we conclude the **object D** is the **Mars**.

(b) To be visible for the longest duration at night, the object should rise at the sunset and set nearly at the sunrise. This again means that it should be at opposition. We have already found dates of opposition of the Jupiter and the Mars above. Next we note that the winter nights are longer than summer nights. As the observer is based in the northern hemisphere, we can say, February nights will be longer than August nights. Thus, we will conclude that the **Jupiter** will be seen for the longest duration at some night.

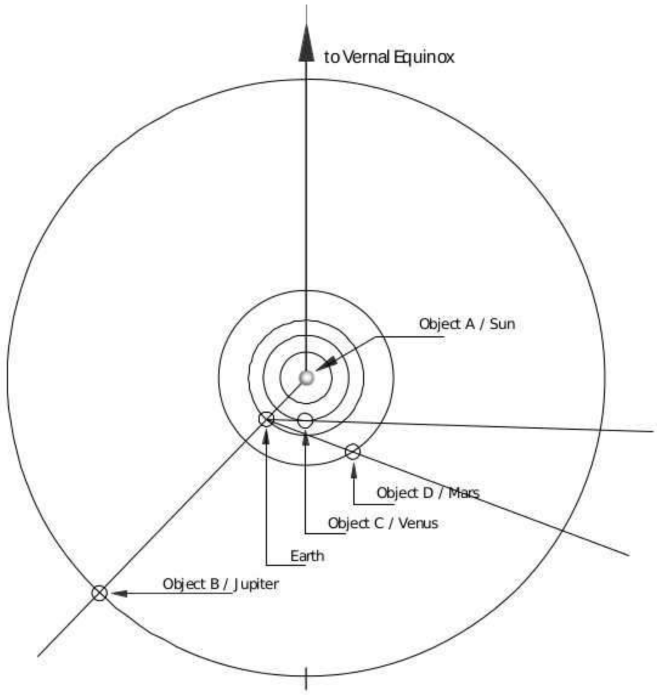
(c) The corresponding date will be the date of opposition i.e. **February 3**.

(d) To place all objects in the orbit diagram as on February 3, we have to take R.A. of each of these objects. As we are viewing the solar system perfectly face-on, declination is not relevant here. To place the earth, we take R.A. of the Sun. Orbit diagram shows direction of the vernal equinox. If the Earth was exactly opposite to that direction (lower part of the paper), observer will see R.A. of the Sun as 0^h. The radial vector of the Earth currently makes

$$\begin{aligned} e &= 24^h 0^m 0^s - 21^h 4^m 47^s.25 \\ &= 2^h 55^m 12^s.75 \\ &\approx 44^\circ \end{aligned} \tag{12.79}$$

angle with the vertical line. When viewed from the top, the planets move anti-clockwise. As we are few days *before* vernal equinox, the Earth's position will be to the left of the vertical line. The four orbits denote Venus, Earth, Mars and Jupiter. So we place the Earth making angle of 44° with the vertical line on the second (from inside) orbit on the lower left part.

The Jupiter is at opposition on that date. So it will be placed in the last orbit exactly along the Sun-Earth line. Next we note that vernal equinox direction points to reference at infinity. Hence, direction of 0^h as drawn from the Earth will also be exactly vertically up. From that direction, as we go anti-clockwise, the R.A. will go on increasing. We will place the Venus and the Mars in resepctive orbits as per their R.A.


to Vernal Equinox
Object A / Sun
Object D / Mars
Object C / Venus
Earth
Object B / Jupiter

Chapter 13

Solutions: Geometric Astronomy and Time

1. The apperant angular separation between the Sun and the Venus as seen from the Earth is maximum, when $\measuredangle SVE = 90°$. See figure.

$$\therefore d_{SV} = d_{\odot-\oplus} \sin 46° \tag{13.1}$$
$$= (1A.U.) \times 0.719$$
$$\approx 0.72A.U. \tag{13.2}$$

2. During a full Moon we see the whole face of the Moon. Hence

$$\text{angle in radians} = \frac{\text{diameter of moon}}{\text{distance to moon}} \tag{13.3}$$
$$\therefore \text{distance to moon} = \frac{\text{diameter of moon}}{\text{angle in radians}}$$
$$= \frac{2 \times 1.7374 \times 10^6 m}{(0.46\pi)/180}$$
$$= \frac{2 \times 1.7374 \times 10^6 \times 180}{0.46\pi}$$
$$= 4.3 \times 10^8 m \tag{13.4}$$

3. From the very definition of unit parsec,

$$\text{angle in arc seconds} = \frac{\text{diameter of orbit in A.U.}}{\text{distance in parsec}} \tag{13.5}$$
$$= \frac{2}{100} = 0.02'' \tag{13.6}$$

4. By Kepler's third Law,

$$\frac{T_M^2}{T_E^2} = \frac{a_M^3}{a_E^3} \tag{13.7}$$

$$\therefore T_M^2 = 1.52^3 \times (365.2564)^2$$

$$T_M = 1.52^{1.5} \times 365.2564$$

$$T_M = 684.48 \text{ days} \tag{13.8}$$

To find time betwen successive oppositions, we have to find the synodic period of the Mars.

$$\frac{1}{T_{Sy}} = \frac{1}{T_E} - \frac{1}{T_M} \tag{13.9}$$

$$\frac{1}{T_{Sy}} = \frac{1}{365.2564} - \frac{1}{684.48}$$

$$T_{Sy} = 783.18 \text{ days} \tag{13.10}$$

Thus the time between successive oppositions (synodic period) of Mars is 783 days.

5. First we find orbital period and synodic period of the Mars.

$$T_M = \sqrt{\frac{R_M^3}{R_\oplus^3}} T_\oplus \tag{13.11}$$

$$= \sqrt{(1.524)^3}$$

$$= 1.8814\ years$$

$$= 687.18\ days \tag{13.12}$$

$$\frac{1}{T_{sy}} = \frac{1}{T_\oplus} - \frac{1}{T_M} \tag{13.13}$$

$$= \frac{T_\oplus T_M}{T_M - T_\oplus} \tag{13.14}$$

$$= \frac{1.8814}{1.8814 - 1}\ years \tag{13.15}$$

$$= 779.65\ days \tag{13.16}$$

This means there is an opposition of Mars about every 780 days. To find the dats of great opposition in the year 2018,

$$15 \times 365 + 4 = 5479\ days \tag{13.17}$$

$$\frac{5479}{779.65} = 7.0275 \tag{13.18}$$

It means that there will be 7 oppositions between Aug. 28, 2003 and Aug. 28, 2018. So the date for the great opposition in 2018 should be

$$5479 - 7 \times 779.65 = 21.38 days \tag{13.19}$$

i.e. 21.4 days before Aug. 28, 2018. It is on Aug. 7, 2018.

6. The length of solar day is exactly 24 hrs. But sidereal day is a little less than solar day because of the yearly motion of the Sun.
   Now, in one solar year the Earth completes 365.25 solar days but it completes one additional rotation around itself. Thus number of sidereal days will be 366.25.

$$T_{sid} = T_{Sol} \times \frac{n}{n+1} \tag{13.20}$$

$$= 24^h \times \frac{365.25}{365.25 + 1}$$

$$= 23^h 56^m 4^s \tag{13.21}$$

   Now, if Earth rotates in opposite direction then exactly opposite thing will happen i.e. the Earth will complete one rotation less than number of solar days in a solar year.
   On the other hand, the Sidereal day will remain the same as it doesn't depends on the rotation direction. (neglecting the precession)

$$T'_{Sol} = T_{sid} \times \frac{n-1}{n} \tag{13.22}$$

$$= T_{Sol} \times \frac{n}{n+1} \times \frac{n-1}{n}$$

$$= 24^h \times \frac{365.25 - 1}{365.25 + 1}$$

$$= 23^h 52^m 8^s \tag{13.23}$$

7. In the Figure 13.1, the observer is located at O and the satellite is at

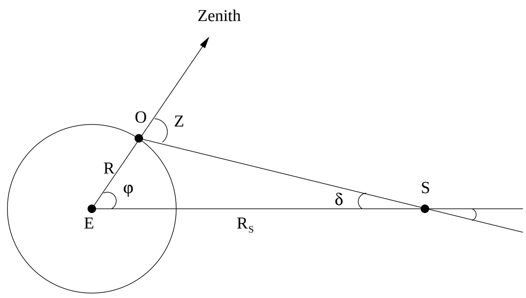


Figure 13.1 – Geocentric parallax of the satellite

S.

$$\angle EOS = 180 - z = 180 - 46.0$$
$$= 134° \quad (13.24)$$
$$\delta = 180 - \varphi - \measuredangle\angle EOS$$
$$= 10.4° \quad (13.25)$$
$$\frac{R_s}{\sin(\measuredangle EOS)} = \frac{R_\oplus}{\sin\delta} \quad (13.26)$$
$$\frac{R_s}{R_\oplus} = \frac{\sin(\measuredangle EOS)}{\sin\delta}$$
$$= \frac{\sin 134°}{\sin 10.4°}$$
$$\therefore R_s = 3.98 R_\oplus \quad (13.27)$$

The angle δ is called geocentric parallax of any near-Earth object, such as the satellite.

8. If the observer was at the centre of the Mars (C), Phobos would have been visible above horizon for exactly half the period. However, the observer is on martian surface (O), thus as shown in the figure 13.2, the Phobos will set at point P.

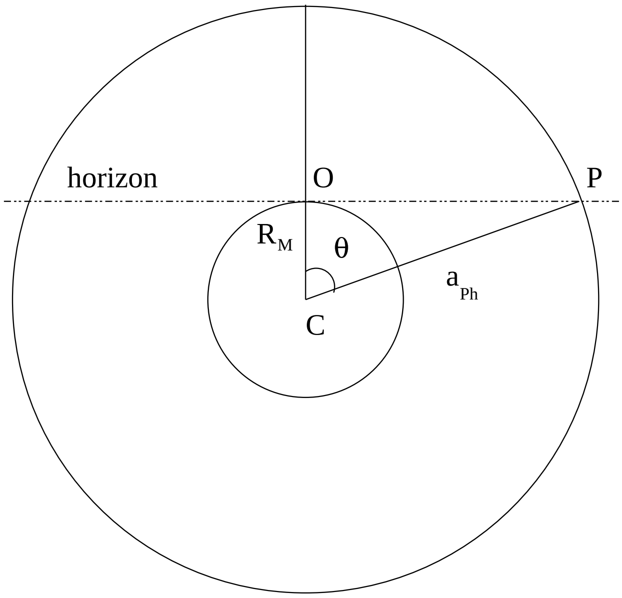


Figure 13.2 – Horizon for martian observer

$$T_{ph} = 2\pi\sqrt{\frac{a_{ph}^3}{GM_{Mars}}} \tag{13.28}$$

$$= 2\pi\sqrt{\frac{(9.38\times10^6)^3}{6.6726\times10^{-11}\times6.421\times10^{23}}}$$

$$= 2.76\times10^4 sec$$

$$T_{ph} \approx 7.660 hrs \tag{13.29}$$

$$\frac{1}{T_{syn}} = \frac{1}{T_{ph}} - \frac{1}{T_{Mars}} \tag{13.30}$$

$$T_{syn} = \frac{T_{Mars}T_{ph}}{T_{Mars}-T_{ph}}$$

$$= \frac{24.623\times7.660}{24.623-7.660}$$

$$= 11.12 hrs \tag{13.31}$$

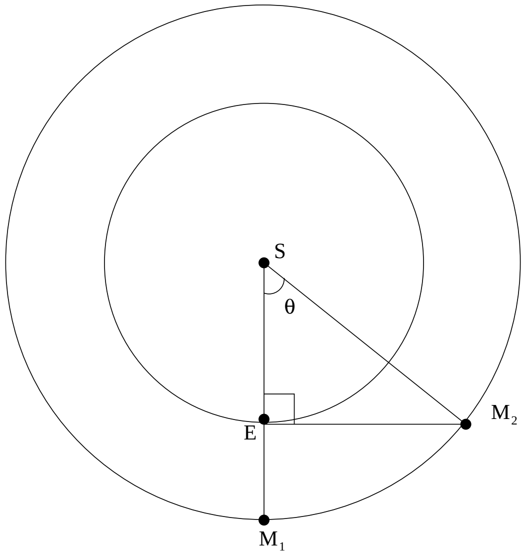


Figure 13.3 – Mars at the opposition and at the quadrature

This is synodic period of Phobos for a martian observer. Now,

$$\theta = \cos^{-1}\left(\frac{R_{Mars}}{R_{Ph}}\right) \tag{13.32}$$

$$\begin{aligned} t &= \frac{2\theta}{360^\circ} \times T_{syn} \\ &= \frac{T_{syn}}{180^\circ} \cos^{-1}\left(\frac{R_{Mars}}{R_{Ph}}\right) \quad (13.33) \\ &= \frac{11.12}{180} \cos^{-1}\left(\frac{3393}{9380}\right) \\ &= 4.25 hrs \quad (13.34) \end{aligned}$$

Hence Phobos will be visible in the martian sky for about 4 hours and 15 minutes

9. If we go into the reference frame of the Earth, the Mars will complete one revolution in its synodic period. In the figure 13.3, length SM_2 is orbital radius of the Mars (a_m) and SE is orbital radius of the Earth

($a_\oplus$). Angle between the two positions is θ.

$$\frac{1}{T_{sy}} = \frac{1}{T_\oplus} - \frac{1}{T_m} \quad (13.35)$$

$$\therefore T_{sy} = \frac{T_m T_\oplus}{(T_m - T_\oplus)} \quad (13.36)$$

$$= \frac{687 \times 365}{(687 - 365)}$$

$$\therefore T_{sy} = 778.7 days \quad (13.37)$$

$$\theta = \frac{t}{T_{sy}} \times 360^\circ = \frac{106}{779} \times 360^\circ \quad (13.38)$$

$$= 49.00^\circ \quad (13.39)$$

$$a_\oplus = a_m \cos\theta \quad (13.40)$$

$$a_m = \frac{1}{\cos 49.00^\circ}$$

$$a_m = 1.52 A.U. \quad (13.41)$$

10. (a) Drawing scaled diagram is impossible. Rough sketch is accepted.

    (b) There are 10 days and nights per taris year. The obliquity is 3°, which means that the planet's rotation is in the same direction as its orbit. Thus, total number of rotations per year is 10 + 1 = 11.

    Note: The obliquity is positive (similar to the Earth / Mars / Jupiter). This means, we have ADD one rotation. Subtracting one rotation by assuming opposite rotation (like the Venus) is not correct.

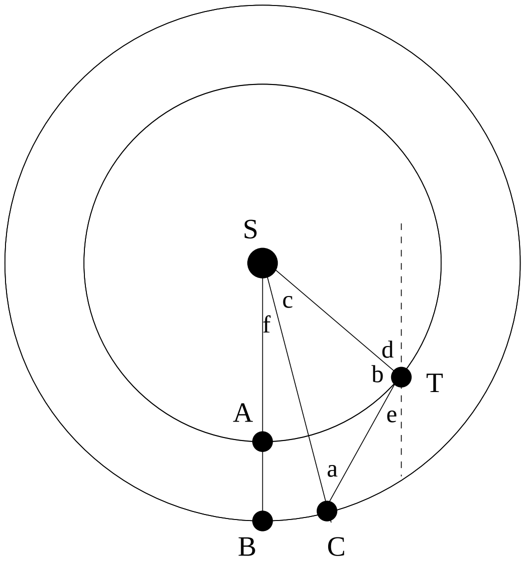
    (c) By Kepler's third law,

$$\frac{T_{en}^2}{R_{en}^3} = \frac{T_{ex}^2}{R_{ex}^3} \quad (13.42)$$

$$R_{ex}^3 = \frac{1.6^2 R_{en}^3}{0.2^2}$$

$$R_{ex} = \sqrt[3]{64}$$

$$= 4 \text{ endorlengths} \quad (13.43)$$



(d) Using same logic as above

$$\frac{T_C^2}{R_C^3} = \frac{T_T^2}{R_T^3} \tag{13.44}$$

$$T_C^2 = \frac{9^3 R_T^3 T_T^2}{R_T^3}$$

$$T_C = \sqrt{729}$$

$$= 27 \text{ tarisyears} \tag{13.45}$$

(e) As Corulus is in Opposition, Sola - Taris - Corulus form straight line (in that order).
Distance = 9 - 1 = 8 tarislengths.

(f) In the figure 10f, S is Sola, A and B are start of the year positions of Taris and Corulus, T and C are their positions after 'n' days. Angles are named from a to f. The dashed line is parallel to line SB. Triangle(SCT) is used for sine rule as well as answer in the next part. Figure is not to the scale.

$$a + b + c = \pi \tag{13.46}$$
$$b + d + e = \pi \tag{13.47}$$
$$d = f + c \tag{13.48}$$
$$f + c = \frac{2\pi n}{10} \tag{13.49}$$
$$f = \frac{2\pi n}{270} \tag{13.50}$$
$$\sin b = 9 \sin a \tag{13.51}$$
$$e = \pi - b - d \tag{13.52}$$
$$= \pi - b - c - f$$
$$e = a - f \tag{13.53}$$

$$b = \pi - (a + c) \tag{13.54}$$
$$= \pi - \left(a + \frac{2\pi n}{10} - \frac{2\pi n}{270}\right)$$
$$b = \pi - \left(a + \frac{52\pi n}{270}\right) \tag{13.55}$$

$$9 \sin a = \sin\left(\pi - \left(a + \frac{52\pi n}{270}\right)\right) \tag{13.56}$$
$$= \sin\left(a + \frac{52\pi n}{270}\right)$$
$$= \left[\sin a \cos\left(\frac{52\pi n}{270}\right) + \cos a \sin\left(\frac{52\pi n}{270}\right)\right] \tag{13.57}$$
$$9 = \cos\left(\frac{52\pi n}{270}\right) + \cot a \sin\left(\frac{52\pi n}{270}\right)$$
$$\cot a = \frac{9 - \cos\left(\frac{52\pi n}{270}\right)}{\sin\left(\frac{52\pi n}{270}\right)} \tag{13.58}$$
$$a = \tan^{-1}\left[\frac{\sin\left(\frac{52\pi n}{270}\right)}{9 - \cos\left(\frac{52\pi n}{270}\right)}\right] \tag{13.59}$$
$$\lambda = \pi - e \tag{13.60}$$
$$= \pi + f - a$$
$$\lambda = \pi + \frac{2\pi n}{270} - \tan^{-1}\left[\frac{\sin\left(\frac{52\pi n}{270}\right)}{9 - \cos\left(\frac{52\pi n}{270}\right)}\right] \tag{13.61}$$

(g) When n = 1

$$a = \tan^{-1}\left[\frac{\sin\left(\frac{52\pi}{270}\right)}{9-\cos\left(\frac{52\pi}{270}\right)}\right] \tag{13.62}$$
$$= \tan^{-1}\left[\frac{\sin 0.605}{9-\cos 0.605}\right]$$
$$= 0.0756 \quad \text{rad}$$
$$= 4°20' \tag{13.63}$$
$$\sin b = 9\sin a \tag{13.64}$$
$$= 9\sin(4°20')$$
$$= 0.680 \tag{13.65}$$
$$\text{Area} = \frac{1}{2} \times l(ST) \times l(SC) \times \sin b \tag{13.66}$$
$$= \frac{1}{2} \times 1 \times 9 \times 0.680$$
$$= 3.06 \tag{13.67}$$

The area is about 3 $(tarislength)^2$.

11. Since the observer is close to the pole, the affect of the earth's rotation on the transit could be neglected. The Sun's angular size for the observer will be,

$$\theta_\odot = \frac{2R_\odot}{1AU} \tag{13.68}$$
$$= \left(\frac{2 \times 6.955 \times 10^8}{1.496 \times 10^11 1}\right) \times \frac{206265}{60}$$
$$\theta_\odot \approx 32.0' \tag{13.69}$$
$$\theta_{Venus} = \frac{2R_{Venus}}{d_{V-E}} \tag{13.70}$$
$$= \frac{2 \times 0.949 \times 6.371 \times 10^6 \times 206265}{(1-0.723) \times 1.496 \times 10^{11} \times 60}$$
$$\theta_{Venus} \approx 1.00' \tag{13.71}$$

The angular velocity of the Venus around the Sun, with respect to the earth will be,

$$\omega_{sy} = \omega_{Venus} - \omega_{Earth} = \frac{2\pi}{T_{Venus}} - \frac{2\pi}{T_{Earth}} \tag{13.72}$$
$$= \frac{2\pi \times 206265}{86400 \times 60}\left(\frac{1}{224.70} - \frac{1}{365.25}\right)$$
$$\approx 4.28 \times 10^{-4} \text{ arcmin/s} \tag{13.73}$$

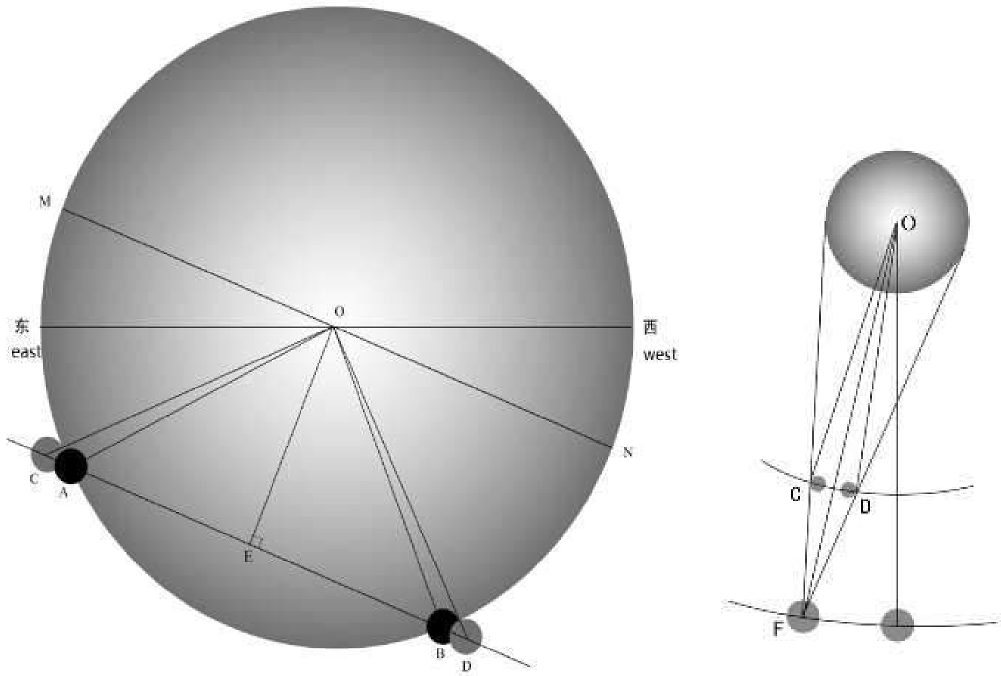


Figure 13.4 – Venus Transit Geometry

For the observer on earth, let us say that the Venus moved through angle θ during the whole transit. Let OE be perpendicular to AB,

$$OA = \theta_{\odot}$$

$$\measuredangle AOB = 90^{\circ}$$

$$MN \parallel AB$$

$$\therefore OE = \theta_{\odot} \sin 45^{\circ}$$

$$= 11.3' \quad (13.74)$$

$$OC = \frac{(\theta_{Venus} + \theta_{\odot})}{2}$$

$$= 16.5' \quad (13.75)$$

$$CE = \sqrt{OC^2 - OE^2} \quad (13.76)$$

$$= \sqrt{16.0^2 - 11.3^2}$$

$$\approx 12.0' \quad (13.77)$$

$$CD = 2CE$$

$$\therefore CD = \measuredangle CFD = \theta = 24.0' \quad (13.78)$$

However CFD is the angle measured from the Earth, which is different from the angle measured from the Sun. Angle measured from the Sun ($\theta' = \measuredangle COD$) is the actual angle that Venus covered during the transit.

$$\frac{\sin(\theta/2)}{\sin(\theta'/2)} = \frac{a_{Venus}}{d_{V_E}} \tag{13.79}$$
$$= \frac{0.723}{(1 - 0.723)}$$
$$\theta' = 2\sin^{-1}\left(\frac{\sin 12'}{2.610}\right) = 9.195' \tag{13.80}$$
$$t_{transit} = \frac{\theta'}{\omega_{sy}} \tag{13.81}$$
$$= \frac{9.195'}{4.28 \times 10^{-4}}$$
$$t_{transit} = 5^h 58^m \tag{13.82}$$

So the transit will finish at about $14^h 58^m$.

12. Let us call the semi-major axis of Moon's orbit as a, its eccentricity as e, the revolution period T, apparent radius r, the distance between Earth and Moon as d, the apparent radius of the Sun as $r_\odot$. When the Moon is at perigee, the total eclipse will be longest and when the Moon is at apogee, the annular eclipse will be longest. By conserving angular momentum,

$$d_{ap}^2 \omega_{ap} = d_{peri}^2 \omega_{peri} \tag{13.83}$$
$$\therefore \frac{\omega_{peri}}{\omega_{ap}} = \left(\frac{a(1+e)}{a(1-e)}\right)^2$$
$$= \frac{(1+e)^2}{(1-e)^2} \tag{13.84}$$

As the Earth's orbit around the Sun is circular, we can assume $\omega_\oplus$ is constant. Thus, time of eclipse at these two points,

$$t_{ap} = \frac{2(r_\odot - r_{ap})}{\omega_{ap}} \tag{13.85}$$
$$t_{peri} = \frac{2(r_{peri} - r_\odot)}{\omega_{peri}} \tag{13.86}$$
$$\therefore \frac{t_{ap}}{t_{peri}} = \frac{\omega_{peri}}{\omega_{ap}} \frac{2(r_\odot - r_{ap})}{2(r_{peri} - r_\odot)}$$
$$= \frac{(1+e)^2}{(1-e)^2} \frac{(r_\odot - r_{ap})}{(r_{peri} - r_\odot)} \tag{13.87}$$

Now we try to get $r_\odot$ in terms of r_{ap}.

$$\frac{r_{ap}}{r_{peri}} = \frac{a(1-e)}{a(1+e)} \tag{13.88}$$

$$\frac{(r_\odot - r_{ap})}{(r_{peri} - r_\odot)} = \frac{t_{ap}}{t_{peri}}\frac{(1-e)^2}{(1+e)^2} = k$$

$$(r_\odot - r_{ap}) = k\left(\frac{(1+e)}{(1-e)}r_{ap} - r_\odot\right)$$

$$(1+k)r_\odot = \left(\frac{(1+e)}{(1-e)}k + 1\right) r_{ap} \tag{13.89}$$

Evaluating,

$$k = \frac{12.5}{7.5} \times \frac{(1-0.0549)^2}{(1+0.0549)^2}$$

$$= 1.667 \times (0.8959)^2$$

$$k = 1.3378 \tag{13.90}$$

$$r_\odot = \left(\frac{(0.8959^{-1} \times 1.3378 + 1)}{(1.3378 + 1)}\right) r_{ap}$$

$$r_\odot = 1.0665 r_{ap} \tag{13.91}$$

$$\tag{13.92}$$

Now, for elliptical orbit, we find θ for which $r_\odot = r$.

$$d_\theta = \frac{a(1-e^2)}{1 - e\cos\theta} \tag{13.93}$$

$$d_{ap} r_{ap} = d_\theta r_\odot \tag{13.94}$$

$$a(1+e)r_{ap} = \frac{a(1-e)(1+e)}{1 - e\cos\theta} 1.0665 r_{ap}$$

$$1 - e\cos\theta = 1.0665(1-e)$$

$$\cos\theta = -(0.0549)^{-1}(0.0665 - 1.0665e)$$

$$= -0.145 \tag{13.95}$$

Thus, annular eclipses will happen for about 98° orbit on either side of apogee. Our assumption here is that the Moon's orbit is in the same plane as the earth. As eccentricity of the moon's orbit is small, we make a further approximation of assuming average ω throughout the motion.

Thus, ratio of number of eclipses,

$$\frac{f_A}{f_T} \approx \frac{\Delta\theta_A}{\Delta\theta_T} \tag{13.96}$$

$$\approx \frac{98.32}{(180 - 98.32)} \tag{13.97}$$

$$\approx 1.2 \tag{13.98}$$

EA: Notes

- Assumption in the last part (constant ω) is a poor assumption. However, calculating exact area swept will require calculus and hence will be deemed beyond the scope of this book.
- We note that the Moon will move slower near apogee and faster near perigee hence this estimate is a lower end estimate.
- Inclination of the Moon's orbit, positions of its nodes and precession of its nodes will complicate the calculation further.

13.1 Data Analysis

1. From the figures of the Moon's phases, we see that the Moon is almost full on June 10 as well as June 11. Thus, we can conclude that the full moon occurs on June 11 (± 0.5 day), 2006. We are going to use this full moon as the reference for calculating the Moon's age on Albert Einstein's birthday (March 14, 1879).
   Approximate time span between June 11, 2006 and March 14, 1879 is 127 years. There are 31 leap years between the two years.
   So, there are $365 \times 127 + 31 = 46386$ days starting from January 1, 2006 to January 1, 1879. As a consequence, there are 46,475 days starting from June 11, 2006 to March 14, 1879.
   Thus number of lunar months will be

$$N = \frac{46475}{29.5306} = 1573.79 \tag{13.99}$$

   This means 1573 lunar months and $0.79 \times 29.5306 = 23.37$ days. As we are going backwards in time, this means 23 and half days before full moon. i.e.

$$p = \frac{29.5306 \times 3}{2} - 23.37 = 23.37 - 14.77 = 8.6 \tag{13.100}$$

   That is 8.6 days before the new moon or $29.53 - 8.6 = 20.9$ days since last the new moon.

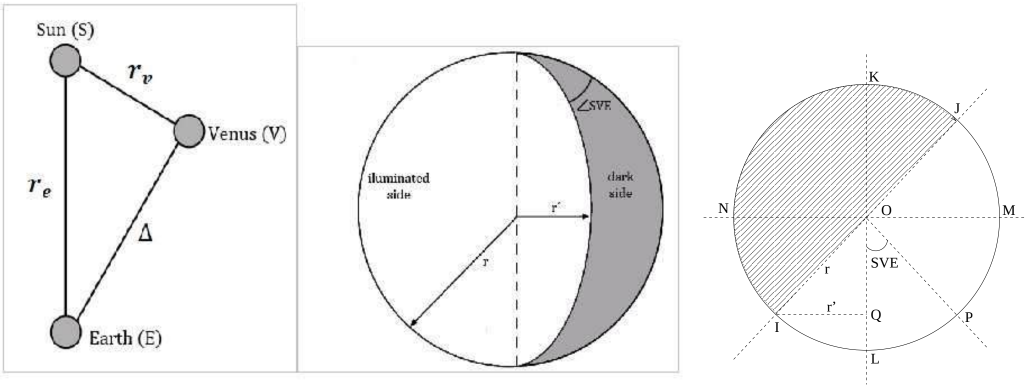


Figure 13.5 – Venus Phase Geometry

We note that our first assumption of exact date of the full moon is accurate only to ±0.5 days. Errors in other quantities taken in this calculation are much smaller. Hence the error in our answer will also be about ±0.5 days.

Thus, the Moon's age on Albert Einstein's birthday (March 14, 1879) is 20.9 ± 0.5 days.

2. (a) See the figure 13.5. In this figure, the Sun-Earth distance is taken as r_e, the Sun-Venus distance is taken as r_v and the Earth-Venus distance is taken as Δ. In the right side figure, the circle represents the Venus as seen from the top. Earth is in the direction of L ($\overrightarrow{KL}$). The Sun is in the direction of P ($\overrightarrow{OP}$). Thus, Region 'IOJKN' will be dark and other half will be illuminated. Region MONILPM will be visible from the earth.

$$\begin{aligned} \angle SVE &= \angle LOP \\ &= \measuredangle ION \quad (13.101) \\ &= \measuredangle OIQ \quad (13.102) \end{aligned}$$

To get $\angle ION$, we have to convert phase information (which gives illuminated area as a percentage) to angle. We can imagine illuminated area as addition of half-circle and half-ellipse (see middle picture of figure 13.5). For circular part, the radius is r and for elliptical part, the semi-major and semi-minor axes are given by r and r' respectively. From the right side figure,

$$l(\overline{OI}) = r$$
$$l(\overline{IQ}) = r'$$
$$\therefore r' = rcos\measuredangle SVE \quad (13.103)$$
$$A_{illu} = \frac{\pi r^2}{2} + \frac{\pi r r'}{2} \quad (13.104)$$
$$A_{tot} = \pi r^2 \quad (13.105)$$
$$\therefore phase = \frac{A_{illu}}{A_{tot}} = \left(\frac{\frac{\pi r^2}{2} + \frac{\pi r r'}{2}}{\pi r^2} \right) \times 100 \quad (13.106)$$
$$= \left(\frac{\frac{\pi r^2}{2} + \frac{\pi r^2 \cos(\angle SVE)}{2}}{\pi r^2} \right) \times 100$$
$$= \frac{100(1 + \cos(\angle SVE))}{2}$$
$$= 100 \cos^2 \left(\frac{\angle SVE}{2} \right)$$
$$\measuredangle SVE = 2 \cos^{-1} \left(\frac{\sqrt{phase}}{10} \right) \quad (13.107)$$

We use this equation to get values of $\measuredangle SVE$ for each value of the phase given in table 3.2. The values are listed in column 2 of table 13.1.

(b) Using sine rule in $\triangle SEV$, we realise that

$$\frac{r_e}{\sin(\angle SVE)} = \frac{r_v}{\sin(\angle SEV)}$$
$$r_v = \frac{r_e \sin(\angle SEV)}{\sin(\angle SVE)} \quad (13.108)$$

With the help of this equation, we comupte the Sun-Venus distance as listed in column 3 of table 13.1.

(c) Column 1 and column 3 of the table 13.1 are plotted in figure 13.6.

(d) From the graph 13.6, we get,

$$r_{v_{max}} = 0.728AU \quad (13.109)$$
$$r_{v_{min}} = 0.718AU \quad (13.110)$$

Date	$\measuredangle SVE$	r_v
20/09/08	39.825	0.7255
10/10/08	47.156	0.7276
20/10/08	50.802	0.7283
30/10/08	54.549	0.7284
09/11/08	58.264	0.7283
19/11/08	62.095	0.7281
29/11/08	66.172	0.7268
19/12/08	74.811	0.7245
29/12/08	79.630	0.7230
18/01/09	90.573	0.7206
07/02/09	104.833	0.7188
17/02/09	114.079	0.7184
27/02/09	125.591	0.7186
19/03/09	157.518	0.7211

Table 13.1 – Venus computations

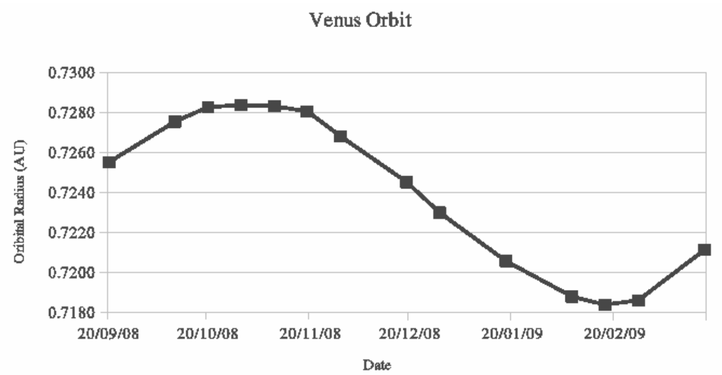


Figure 13.6 – Venus Phase Geometry

(e) Semi-major axis is

$$a = \frac{(r_{v_{max}} + r_{v_{min}})}{2} = 0.723 AU \tag{13.111}$$

(f) Eccentricity could be calculated from both of aphelion and perihelion distances as

$$e = \frac{(r_{v_{max}} - r_{v_{min}})}{2a} = 0.00612 \tag{13.112}$$

Chapter 14

Solutions: Optics and Detectors

14.1 Theory

1. Let us say that diametrically opposite points on the said crater subtend an angle ϕ at our eye. If the minimum separation between these two points to make them resolvable is x, then,

$$\phi \geq 1.22 \frac{\lambda}{D_{eye}} \tag{14.1}$$

$$\phi = \frac{x}{distance\ from\ Earth\ to\ Moon} \tag{14.2}$$

$$\therefore x = \frac{1.22 \times 5 \times 10^{-7} \times 3.78 \times 10^{8}}{5 \times 10^{-3}}$$
$$\approx 46\ km \tag{14.3}$$

As this is smaller than 80 km, the said crater can be resolved. Alternatively,

$$\phi \geq 1.22 \frac{\lambda}{D_{eye}}$$

$$L.H.S. = \phi = \frac{80km}{distance\ from\ Earth\ to\ Moon}$$
$$= \frac{80 \times 10^{3} m}{3.78 \times 10^{8} m} = 2.11 \times 10^{â4} radian \tag{14.4}$$

$$R.H.S. = 1.22 \frac{\lambda}{D_{eye}}$$
$$= \frac{1.22 \times 5 \times 10^{-7}}{5 \times 10^{-3}} = 1.22 \times 10^{-4} radian \tag{14.5}$$

Thus, the ineuqlity is true and it is possible to resolve the 80 km-diameter crater with naked eye. Note the particular choice of wavelength which makes the calculations easier.

2. Since resolution of both the telescopes is same,

$$
\begin{aligned}
1.22\frac{\lambda_1}{D_1} &= 1.22\frac{\lambda_2}{D_2} \\
D_2 &= \frac{D_1\lambda_2}{\lambda_1} && (14.6) \\
&= \frac{0.1 \times 0.01}{5 \times 10^{-7}} \\
&= 2km && (14.7)
\end{aligned}
$$

Answers may vary slightly depending on your choice of optical wavelength. For this reason, the final answer is rounded to single significant digit.

3. One should use angular size of the galactic black hole to find resolution

$$
\begin{aligned}
\theta_{BH} &= \frac{2R_{BH}}{d_{GC}} && (14.8) \\
\theta_{tel} &= \frac{1.22\lambda}{2R_{\oplus}} && (14.9) \\
\theta_{BH} &\geq \theta_{tel} && (14.10) \\
\frac{2R}{D} &\geq \frac{1.22\lambda}{2R_{\oplus}} \\
\lambda &\leq \frac{4R_{BH}R_{\oplus}}{1.22D} && (14.11) \\
&\leq \frac{12M_{BH}R_{\oplus}}{1.22DM_{\odot}} \\
&\leq \frac{12 \times 4 \times 10^9 \times 6.371 \times 10^6}{1.22 \times 8.5 \times 10^3 \times 3.086 \times 10^16} \\
\lambda &\leq 0.96mm && (14.12) \\
\lambda &\leq 0.9mm && (14.13)
\end{aligned}
$$

This means that we need to observe at least at near sub-mm frequencies, which is in radio or far-infrared band. We have rounded limit on λ to 0.9 mm instead of 1.0 mm as the answer is in terms an inequality. Thus, we have to respect direction of inequality.

4. Let us assume the moon is at the maximum possible separation from the ecliptic plane. We know that the displacements of the Earth and moon

are inversely proportional to their masses. Let us call distance between the Earth and the moon as d, semi-amplitude of the Earth's oscillation as x and distance between the centre of the Earth and barycentre of the Earth-Moon system as y.

$$\frac{x}{y} = \tan 5.14° \tag{14.14}$$

$$M_{\oplus} y = M_m(d - y) \tag{14.15}$$

$$\therefore y = \frac{M_m d}{M_{\oplus} + M_m} \tag{14.16}$$

$$x = \frac{M_m d}{M_{\oplus} + M_m} \tan 5.14° \tag{14.17}$$

$$= \frac{7.4377 \times 10^{22} \times 3.84 \times 10^8}{(597.36 + 7.4377) \times 10^{22}} \times 0.0900$$

$$= 4.248 \times 10^5 m \tag{14.18}$$

$$\therefore 2x = 849.6 km \tag{14.19}$$

Note that points L_4 or L_5 are at eaxtly 1 A. U. distance from the earth. Also note that the resolution of the telescope improves with decreasing wavelengths. As the telescope is capable of observing in near UV, we will use a reasonable wavelength in near UV range to determine smallest possible diameter.

$$\alpha = \frac{2x}{d_{L4}} = 1.22 \frac{\lambda}{D} \tag{14.20}$$

$$\therefore D = \frac{1.22 d_{L4} \lambda}{2x}$$

$$= \frac{1.22 \times 1.496 \times 10^{11} \times 350 \times 10^{-9}}{2 \times 4.248 \times 10^5}$$

$$D = 7.5 cm \tag{14.21}$$

5. (a) We note that longest separation one can fit on a CCD in a single frame would be along the diagonal of the CCD.

$$S = \sqrt{765^2 + 510^2} \times 9 \mu m \tag{14.22}$$

$$= 919.4 \times 9 \mu m \approx 8.27 mm \tag{14.23}$$

$$f = 10780 mm \tag{14.24}$$

$$\theta = \frac{S}{f} = \frac{206265 \times 8.27''}{10780} \tag{14.25}$$

$$= 158.3'' = 2.64' \tag{14.26}$$

As the two stars are very close to each other, one can approximate the situation to flat geometry. Hence, angular separation between them is,

$$\gamma = \sqrt{\triangle\ \alpha^2 + \triangle\ \delta^2} \tag{14.27}$$

$$\because \triangle\ \alpha = 20^h 18^m 03^s.3 - 20^h 17^m 38^s.9 = 0^m 24^s.4 \tag{14.28}$$

$$\because \triangle\ \delta = -12°32'41'' + 12°30'30'' = -2'11'' \tag{14.29}$$

$$\therefore \gamma = \sqrt{\left((0^m 24^s.4) \times 15\right)^2 + \left(-2'11''\right)^2} \tag{14.30}$$

$$= \sqrt{\left(366''\right)^2 + \left(131''\right)^2} \tag{14.31}$$

$$\approx 389'' \tag{14.32}$$

This is far larger than 158.3". Thus, the stars will not fit on a single frame.

One can also make an even simpler argument of simply noting the separation in R.A. ($24^s.40$) itself is bigger than the CCD diagonal. It should be noted that the plane trigonometry works only for the stars which are reasonably close to each other. For other cases, spherical trigonometry must be used.

(b) To find position angle, we note that the secondary star is towards the south-west of primary star. As position angle (ϕ) is measured from the north in anti-clockwise fashion, we expect the value to be between 90° and 180°.

$$\phi = \tan^{-1}\left(\frac{\triangle\ \alpha}{\triangle\ \delta}\right) \tag{14.33}$$

$$= \tan^{-1}\left(\frac{366}{131}\right) = \tan^{-1} 2.794 \tag{14.34}$$

$$\approx 109°42' \tag{14.35}$$

6. We first find image scale of the picture in arcseconds per mm.

$$S = \frac{206265}{f} = \frac{206265}{D/F} \tag{14.36}$$

$$= \frac{206265F}{D} = \frac{206265}{1500} \tag{14.37}$$

$$= 137.51''/mm \tag{14.38}$$

Thus, we can get angular diameter and then by dividing physical diameter by the angular diameter, distance can be obtained.

$$\theta_\odot = 137.51 \times 13.817 \tag{14.39}$$
$$= 1900.0'' = 0°31'40'' \tag{14.40}$$
$$d_\odot = \frac{2R_\odot}{\theta_\odot} = \frac{2 \times 6.955 \times 10^8 \times 180}{(0°31'40'') \times \pi} \tag{14.41}$$
$$= 1.510 \times 10^{11} m \tag{14.42}$$
$$\theta_{moon} = 137.51 \times 13.235 \tag{14.43}$$
$$= 1820.0'' = 0°30'20'' \tag{14.44}$$
$$d_{moon} = \frac{2R_{moon}}{\theta_{moon}} = \frac{2 \times 1.7374 \times 10^6 \times 180}{(0°30'20'') \times \pi} \tag{14.45}$$
$$= 3.772 \times 10^8 m \tag{14.46}$$

The percentage of the solar disk covered by the Moon is

$$\left(\frac{13.235}{13.817}\right)^2 \times 100\% = 91.75\% \tag{14.47}$$

7. Focal ratio of telescope is ratio of its aperture and focal length. Thus, $f = 10 \times 0.1 = 1$ m. Next we find angular distance (on solar disk) of the active region form the centre of the disk.

$$\alpha = \cos^{-1}(\cos 40° \cos 8°) \qquad \text{using cosine rule} \tag{14.48}$$
$$= 40°40'' \tag{14.49}$$

The tangential plane of active region is inclined at 40°40″ to normal plane. Let $\theta_{1,2}$ be the Angular Diameters of active region seen from the Earth and let S be its size seen on CCD. Let $d_{1,2}$ be the actual size and

A denote area of the active region.

$$S = (5 \times 4) \times 9\mu m \tag{14.50}$$
$$S = 45\mu m \times 36\mu \tag{14.51}$$
$$S = f\theta \tag{14.52}$$
$$\theta = \frac{45\mu m \times 36\mu m}{1m} \times 206265'' \tag{14.53}$$
$$= 9.3'' \times 7.4'' \tag{14.54}$$
$$\theta_{1,2} = \frac{d_{1,2}\cos\alpha}{d_\odot} \tag{14.55}$$
$$\text{Also, } \frac{32' \times 60}{206265} = \frac{2R_\odot}{d_\odot} \tag{14.56}$$
$$d_{1,2} = \frac{\theta_1 \times \theta_2 \times 2R_\odot \times 206265}{32 \times 60 \times \cos\alpha} \tag{14.57}$$
$$= \frac{(9.3 \times 7.4)}{960 \times 0.7585} R_\odot \tag{14.58}$$
$$= (0.0128R_\odot \times 0.0102R_\odot \tag{14.59}$$
$$A = (0.0128 \times 0.0102)R_\odot^2 \tag{14.60}$$
$$\text{Area of Hemisphere} = 2\pi R_\odot^2 \tag{14.61}$$
$$\frac{A}{A_{\text{hemi}}} = 0.00001446 \approx 14.46\ msh \tag{14.62}$$

Thus, area of the active region is 14.46 msh.

14.2 Data Analysis

1. (a) Refer to map 14.1. Five bright stars are marked in the map. Note that as compared to the photograph, the map is rotated by 90°counter-clockwise.

   (b) The boundaries of photograph are drawn on map 14.1. By measuring the field size by a measuring scale and converting it as per the coordinates of the grid, we get that the size of field is about 26 arc minutes. Note that declination of the point at the centre of the field is roughly 7°. Strictly speaking, we have to correct the field size measured along R.A. by the $\cos\delta$ factor. But, our placement of field boundaries is only approximate and based on visual estimation. Thus, for such small declination, we can ignore the declination correction. One can check that field size measured along declination is also same within the error bars.

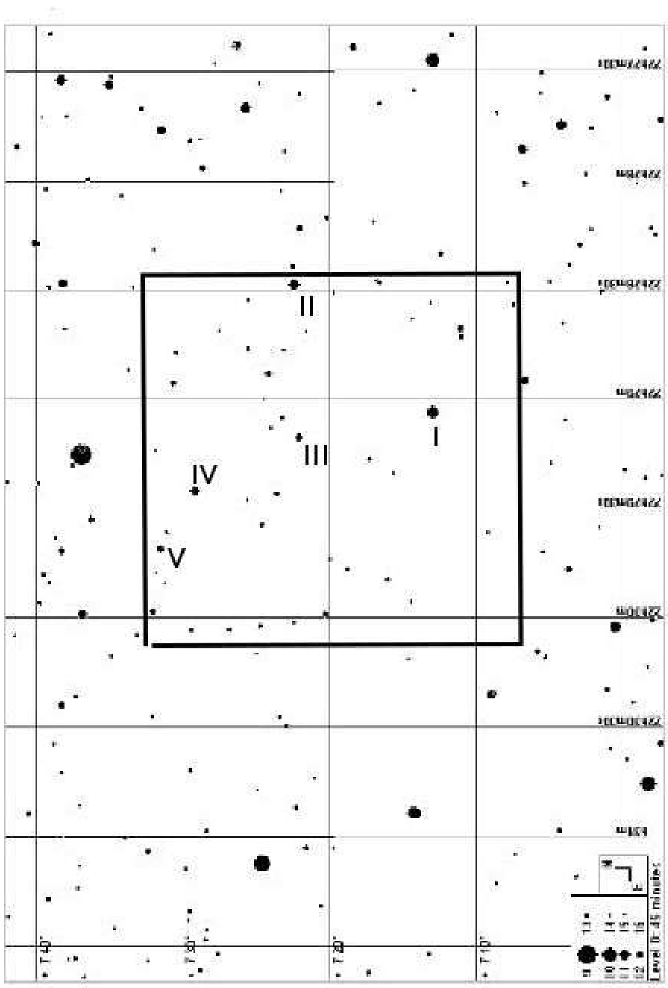


Figure 14.1 – Sky map with region of the sky and bright stars marked

(c) One can find physical size of CCD chip and hence size of each pixel.

$$
\begin{aligned}
s &= f\theta && (14.63)\\
&= \frac{3.180 \times 26 \times 60}{206265} &&\\
s &= 24.1mm && (14.64)\\
\therefore s_{px} &= \frac{24.1}{1024} &&\\
s_{px} &\approx 23\mu m && (14.65)
\end{aligned}
$$

(d) If we see images, size of the star is roughly 10 pixels. This is 3.5 times the original FWHM. Thus, orignial FWHM of the star is slightly smaller than 3 pixels. Now, as field size of 26 arc minutes corresponds to 1024 pixels,

$$\theta_{px} = \frac{26 \times 60}{1024} \approx 1.5'' \qquad (14.66)$$

Thus, three pixels will correspond to about 4.5". This is the size of seeing disk.

(e) Theoretical diameter of diffraction disc will be

$$\theta_{diff} = \frac{2 \times 1.22\lambda}{D} \approx \frac{2.44 \times 5 \times 10^{-7} \times 206265}{0.61} = 0.4'' \qquad (14.67)$$

Thus, diffraction limit (0.4") is much smaller than the seeing disk.

(f) If seeing was 1", the FWHM will be roughly $\frac{2}{3}^{\text{rd}}$ of a pixel. Thus, if we increase contrast by 3.5 times, the disk will appear to be about 3 pixels wide.

(g) We have to calculate velocities of both the objects. For object 1, one can simply measure length of the trail (in arcseconds) during the exposure time. For second object, we have to measure motion of its centre in number of pixels in given time difference between two photos.

$$\begin{aligned}
&Object\ \ 1\\
&l \approx 107'' \qquad (14.68)\\
&\therefore \omega_1 = \frac{l}{t} = \frac{107}{300} \approx 0.36''/s\\
&\omega_1 = 1.73 \times 10^{-6} rad/s \qquad (14.69)\\
&Object\ \ 2\\
&\omega_2 = \frac{pixel \times \theta_{px}}{\Delta t} \qquad (14.70)\\
&= \frac{8 \times 1.5}{17^m 27^s} = 0.011''/sec\\
&\omega_2 = 5.6 \times 10^{-8} rad/s \qquad (14.71)
\end{aligned}$$

Probable causes of different angular velocities are different orbital velocities or different distances from the earth.

2. This problem closely simulates the process of converting CCD reading to magnitudes of stars. At the start, we notice, that there are five regions with high pixel count. These are clearly the stars. Rest of the field is empty sky. Problem explicitly states that we should approximate the stars as squares. We can see that differnt stars have different sizes. Biggest of these (star 3) can be properly included in a 5×5 pixel square.
   We have to realise, that even dark sky is giving us some non-zero pixel count. So the total pixel count of the square containing the stars also includes this contribution from overall sky brightness. To get flux of the star, we have to subtract total pixel count from the equivalent area of the dark sky. As the pixel count in dark sky appears random, we should measure pixcel counts from squares over dark sky at more than one places and then take average. Now, if we take different square sizes for different stars, we will add our own work as we have to measure total count from dark sky separately for each square size. Thus, for the sake of convenience, it makes sense to use same size square for all the stars. One should be worried about dark sky pixels getting included in the square as those contributions will automatically get subtracted when we subtract average count of dark sky.
   As mentioned above, we should take minimum size of the square as 5×5 pixels. We can even go for a bigger square, but it will unnecessarily increase our labour. Further, as we want the centre of the star to be at the centre of our square, the side of the square should be taken as a odd number. Thus, the choice is restricted to 5×5 pixels (correct size), 3×3 pixels (too small) or 7×7 pixels (too large).

(a) We take five random 5×5 pixel squares of dark sky. Each region should be relatively far from any of the stars in the field. We get counts as 121, 113, 115, 123 and 113. The average of these five counts is 117 (for 25 pixels.

We note that instrumental magnitude (m_I), is log of flux recieved from the star. Now, when we talk about stellar magnitude, it is implicitly assumed that the integration time for the flux is of the order of 1 second. However, in this image, CCD was exposed for 450 seconds. Thus,

$$m_I = -2.5 \log \left(\frac{count_{st} - count_{sky}}{t_{exp}} \right) \tag{14.72}$$

Star	$count_{st}$	$count_{st} - count_{sky}$	m_I	m_t
1	7389	7272	-3.021	9.03
2	3313	3196	-2.128	—
3	98520	98403	-5.849	6.22
4	18668	18551	-4.038	8.02
5	949	832	-0.667	—

(b) We correct for exinction next.

$$\begin{aligned}
m_I &= m_t + KX - Z_{mag} \\
Z_{mag_1} &= m_{t_1} - m_{I_1} + KX \\
&= 9.03 - (-3.02) + 0.3 \times \sec(25^\circ) \\
&= 9.03 + 3.02 + 0.3 \times 1.10 \\
&= 12.05 + 0.33 \\
Z_{mag_1} &= 12.38 \qquad (14.73) \\
\therefore Z_{mag_3} &= 12.40 \qquad (14.74) \\
\therefore Z_{mag_4} &= 12.39 \qquad (14.75)
\end{aligned}$$

Average Z_{mag} is therefore 12.39.

(c) For stars 2 and 5, we can calculate true magnitudes (m_t).

$$\begin{aligned}
m_I &= m_t + KX - Z_{mag} \\
\therefore m_t &= m_I - KX + Z_{mag} \\
m_{t_2} &= -2.13 - 0.33 + 12.39 \\
m_{t_2} &= 9.93 \qquad (14.76) \\
m_{t_5} &= -0.67 - 0.33 + 12.39 \\
m_{t_5} &= 11.39 \qquad (14.77)
\end{aligned}$$

(d) Pixel scale for this CCD is calculated as

$$S = f\theta \tag{14.78}$$

$$\theta = \frac{25 \times 10^{-6} \times 206265}{1.2}$$
$$= 4.3'' \tag{14.79}$$

(e) Average sky brightness of the dark sky is measured as magnitude per square arcseconds. As sky brightness is isotropic intensity perceived by the observer, we need not worry about the atmospheric extinction.

$$S_{sky} = m_{sky} + 2.5 \log A \tag{14.80}$$
$$= -2.5 \log \left(\frac{count_{sky}}{25 t_{exp}} \right) + Z_{mag} + 2.5 \log \theta^2$$
$$= -2.5 \log \left(\frac{count_{sky}}{25 t_{exp} \theta^2} \right) + Z_{mag} \tag{14.81}$$
$$= -2.5 \log \left(\frac{117}{25 \times 450 \times 4.3^2} \right) + 12.39$$
$$S_{sky} = 20.5 \ mag/arcsec^2 \tag{14.82}$$

(f) To estimate astronomical seeing, we have to know FWHM (full width at half Maximum) size of stellar disk. First we plot pixel values along vertical line as well as horizontal line for one of the bright stars in the image. As Figure shows, the FWHM of pixel values which is plotted for star 3, is 1 pixel, hence astronomical seeing is equal to $seeing \cong 4''$.

We should not estimate seeing in terms of fractional number of pixels as that is not physically possible.

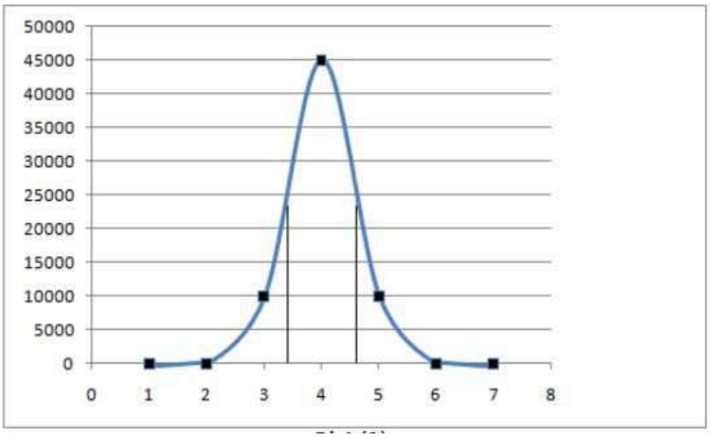


Figure 14.2 – Astronomical seeing

Chapter 15

Solutions: Physics of Stars and Planets

15.1 Theory

1. From the Wien's displacement law

$$\lambda_{max} = \frac{2.898 \times 10^{-3}}{4000} m \tag{15.1}$$
$$= 7.245 \times 10^{-7} m$$
$$\lambda_{max} = 724.5 \quad nm \tag{15.2}$$

2. By Wien's Law

$$\lambda_{max} T = 0.002898 \tag{15.3}$$
$$T = \frac{0.002898}{\lambda_{max}} \tag{15.4}$$
$$= \frac{0.2898}{5 \times 10^{-7}} = 5800K$$
$$\approx 6000K \tag{15.5}$$

The temperature is rounded as the peak wavelength has only one significant digit.

3.

$$L_1 = 4\sigma\pi R_1^2 T_1^4 \tag{15.6}$$
$$L_2 = 4\sigma\pi R_2^2 T_2^4 \tag{15.7}$$
$$\Delta m = 2.5\log\left(\frac{L_1}{L_2}\right) \tag{15.8}$$
$$2 = 2.5\log\left(\frac{4\sigma\pi R_1^2 T_1^4}{4\sigma\pi R_2^2 T_2^4}\right)$$
$$2 = 5\log(\frac{R_1 T_1^2}{R_2 T_2^2}) \tag{15.9}$$
$$\therefore \frac{R_1 T_1^2}{R_2 T_2^2} = 10^{0.4} \tag{15.10}$$
$$\frac{R_1}{R_2} = 2.512\left(\frac{T_2}{T_1}\right)^2 \tag{15.11}$$

As both the stars are on the main sequence, we can assume that the star with higher temperature is the Brighter star.

$$\frac{R_1}{R_2} = 2.512\left(\frac{5000}{6000}\right)^2 \tag{15.12}$$
$$\frac{R_1}{R_2} = 1.74 \tag{15.13}$$

4. The bolometric magnitude of the star remaining constant, which means its luminosity remains constant. As stars can be approximated as black bodies,

$$L_\star = 4\pi {R_\star}^2 \sigma {T_\star}^4 \tag{15.14}$$
$$4\pi {R_1}^2 \times \sigma {T_1}^4 = 4\pi {R_2}^2 \times \sigma {T_2}^4$$
$${R_1}^2 {T_1}^4 = {R_2}^2 {T_2}^4$$
$$\frac{R_1}{R_2} = \frac{{T_2}^2}{T_1} \tag{15.15}$$
$$\frac{R_{5000K}}{R_{30000K}} = \left(\frac{30000}{5000}\right)^2$$
$$= 36 \tag{15.16}$$

5. From the Stefan-Boltzmann law,

$$L = \sigma A T^4 = \sigma(4\pi R^2)T^4 \tag{15.17}$$

$$\therefore \frac{L}{L_\odot} = \frac{R^2T^4}{R_\odot^2 T_\odot^4} = \left(\frac{R}{R_\odot}\right)^2 \left(\frac{L}{L_\odot}\right)^4$$

$$= (2.5)^2 \left(\frac{7500}{5800}\right)^4 = 17.47$$

$$\therefore L \approx 17L_\odot \tag{15.18}$$

6. As the escape velocity exceeds speed of light,

$$\sqrt{\frac{2GM_\odot}{R}} > c \tag{15.19}$$

$$R_{object} < \frac{2GM_\odot}{c^2} \tag{15.20}$$

$$R_{object} < \frac{2 \times 6.6726 \times 10^{-11} \times 1.9891 \times 10^30}{(2.9979 \times 10^8)^2}$$

$$R_{object} < 2953.6m \tag{15.21}$$

7. The radius of the event horizon is given by,

$$R = \frac{2GM_\odot}{c^2} \tag{15.22}$$

$$R = \frac{2 \times 6.672 \times 10^{\hat{a}11} \times 1.989 \times 10^{30}}{(2.99792458 \times 10^2)} m$$

$$= 2953 \ m$$

$$R = 2.953 \ km \tag{15.23}$$

8. Schwarzschild radius of a black hole with mass M is $R = \frac{2GM}{c^2}$

$$\bar{\rho} = \frac{M}{V} = \frac{3M}{4\pi R_{sch}^3} \tag{15.24}$$

$$= \frac{3Mc^6}{4\pi(2GM)^3} = \frac{3c^6}{32\pi G^3 M^2}$$

$$= \frac{3 \times (3 \times 10^8)^6}{32\pi(6.672 \times 10^{-11})^3 \times (10^8 \times 1.989 \times 10^30)^2}$$

$$= 1.85 \times 10^3 \ Kg/m^3 \tag{15.25}$$

$$\approx 2 \times 10^3 Kg/m^3 \tag{15.26}$$

9. As the Earth is in thermal equilibrium, Power of radiation received by the Earth from the Sun will be equal to the power transmitted by the earth to outer space.

$$P_r = F_\odot A t_1 = \frac{L_\odot}{4\pi d_{\oplus-\odot}^2}\pi R_\oplus^2 t_1 \tag{15.27}$$

$$= \frac{4\pi R_\odot^2 \sigma T_\odot^4}{4\pi d_{\oplus-\odot}^2}\pi R_\oplus^2 t_1 \tag{15.28}$$

$$P_r = \left(\frac{R_\odot R_\oplus}{d_{\oplus-\odot}}\right)^2 \pi\sigma T_\odot^4 t_1 \tag{15.29}$$

$$P_t = \sigma T_\oplus^4 4\pi R_\oplus^2 t_2 \tag{15.30}$$

$$P_t = P_r \tag{15.31}$$

$$\sigma T_\oplus^4 4\pi R_\oplus^2 t_2 = \left(\frac{R_\odot R_\oplus}{d_{\oplus-\odot}}\right)^2 \pi\sigma T_\odot^4 t_1 \tag{15.32}$$

$$T_\oplus^4 = \left(\frac{R_\odot}{d_{\oplus-\odot}}\right)^2 \times \frac{t_1}{4t_2} T_\odot^4 \tag{15.33}$$

$$T_\oplus = \sqrt{\frac{R_\odot}{2d_{\oplus-\odot}}\sqrt{\frac{t_1}{t_2}}} T_\odot \tag{15.34}$$

10. We can find mass defect by

$$\Delta m = 2M_{He3} - M_{He} - 2M_H \tag{15.35}$$

$$= 2808.30 \times 2 - 3727.40 - 938.27 \times 2$$

$$\Delta m = 12.66\ MeV/c^2 \tag{15.36}$$

$$\therefore \frac{\Delta m}{2M_{He3}} = \frac{12.66}{2 \times 2808.30} \tag{15.37}$$

$$\text{Mass fraction} = 2.254 \times 10^{-3} = 0.2254\% \tag{15.38}$$

As masses are expressed in energy units, the energy released will be 12.66 MeV and mass fraction loss is 0.23%.

11. The mass of three helium nuclei = 3×4.002603 amu = 12.007809 amu. The mass converted to energy is the difference between the sum of the masses of three helium nuclei and the mass of the resulting carbon nucleus:
$12.007809\ amu - 12.000000\ amu = 7.809 \times 10^{-3} amu$
This represents a fractional loss of mass of the original mass.
$m_{frac} \frac{7.809 \times 10^{-3}}{12.007809} = 6.503 \times 10^{-4}$

Let the stellar mass be M, luminosity L and helium-burning time t.

$$t = \frac{E}{0.3L} = \frac{\triangle\ mc^2}{0.3 \times 100 L_\odot} \quad (15.39)$$
$$= \frac{0.2 M m_{frac} c^2}{0.3 \times 100 L_\odot} = \frac{0.2 \times 20 M_\odot m_{frac} c^2}{0.3 \times 100 L_\odot} \quad (15.40)$$
$$= \frac{0.2 \times 20 \times 1.989 \times 10^{30} \times 6.503 \times 10^{-4} (3 \times 10^8)^2}{0.3 \times 100 \times 3.826 \times 10^{26}} \quad (15.41)$$
$$= 4.06 \times 10^{16} \quad seconds \quad (15.42)$$
$$= 1.29 \times 10^9 \quad years \quad (15.43)$$

Thus, it will take 1.3 billion years for the star to burn all that helium. However, in real stars not all helium is available for burning and luminosity of a massive star in a helium-burning phase would be much higher. Thus, helium-buring time scales are much shorter.

12. To keep same brightness, the flux of the star should remain the same. Using the radius-luminosity-temperature relation,

$$L \propto R^2 T^4 \quad (15.44)$$
$$\frac{L_G}{L_{MS}} = \frac{R_G^2 T_G^4}{R_{MS}^2 T_{MS}^4} \quad (15.45)$$
$$= \left(\frac{R_G}{R_{MS}}\right)^2 \left(\frac{T}{T_{MS}}\right)^4 \quad (15.46)$$
$$= (100)^2 \left(\frac{1}{3}\right)^4 \approx 11.1^2 = 123.5 \quad (15.47)$$
$$\text{Now} \quad F = \frac{L}{4\pi d^2} \quad (15.48)$$
$$\frac{F_G}{F_{MS}} = \frac{L_G}{L_{MS}} \left(\frac{d_{MS}}{d_G}\right)^2 = 1 \quad (15.49)$$
$$d_G = \sqrt{\frac{L_G}{L_{MS}}} \times 20pc \quad (15.50)$$
$$\approx \sqrt{11.1^2} \times 20pc \quad (15.51)$$
$$\approx 220pc \quad (15.52)$$

Thus, at 220 pc it would be barely visible.

13. We can compare energy of each fusion reaction to the Solar constant.

$$N_{neutrino} = 2N_{fusion}$$
$$= 2\frac{S}{E_{fusion}} = \frac{2 \times 1366}{26.8 \times 1.6 \times 10^{-13}}$$
$$N_{neutrino} = 6.4 \times 10^{14}\ neutrinos\ m^{-2}s^{-1} \tag{15.53}$$

14. (a) Radiation flux of the star at the planet's surface is given by

$$F = \frac{L}{4\pi d^2} \tag{15.54}$$

(b) We assume that out of total flux incident, fraction α is reflected and rest of the flux is absorbed. Thus, absorption rate of the planet is given by,

$$A = (1-\alpha)\pi R^2 F = (1-\alpha)\frac{LR^2}{4d^2} \tag{15.55}$$

(c) Energy reflected by the planet per unit time is

$$L_{pl-refl} = \alpha\pi R^2 I = \frac{\alpha L R^2}{4d^2} \tag{15.56}$$

(d) Here, we will neglect the planet's internal source of energy, if any. Let T be the black-body temperature of the planet's surface in Kelvins. Since the planet is rotating fast, we may assume that its surface is being heated up uniformly to approximately the same temperature T. At equilibrium, the total amount of black-body radiation emitted by the planet's surface (given by the Stefan's law) must be equal to the absorption rate in equation 15.55.

$$4\pi R^2 \sigma T^4 = (1-\alpha)\frac{LR^2}{4d^2} \tag{15.57}$$
$$\therefore T = \sqrt[4]{\frac{(1-\alpha)L}{16\pi\sigma d^2}} \tag{15.58}$$

(e) In this case, only one side of planet will act like an ideal black body. The emitted black-body radiation is mostly from the planet's surface facing the star. The emitting surface area is now only $2\pi R^2$ and not $4\pi R^2$. Hence, the surface temperature is given by T', where

$$2\pi R^2 \sigma (T')^4 = (1-\alpha)\frac{LR^2}{4d^2}$$
$$T' = \sqrt[4]{(1-\alpha)\frac{L}{8\pi\sigma d^2}} \tag{15.59}$$
$$= \sqrt[4]{2}T \approx (1.19)T \tag{15.60}$$

(f) Using equation 15.58.

$$T = \sqrt[4]{\frac{(1-\alpha)L}{16\pi\sigma d^2}}$$
$$T = \sqrt[4]{\times\frac{(1-0.25)\times 3.826\times 10^{26}}{16\pi\times 5.67\times 10^{-8}\times(1.523\times 1.496\times 10^{11})^2}}$$
$$= 209.9 \simeq 210K \tag{15.61}$$

15. As the charged particle moves through enters a uniform magnetic field, it will start gaining velocity component perpendicular to both the initial velocity and the magnetic field. The motion of this particle is similar to motion in a cyclotron. If the thickness of magnetic field is too large, then eventually the particle will turn back. Let τ be the thickness of the magnetic field.

$$\frac{mv^2}{r} = q|\vec{v}\times\vec{B}|$$
$$\therefore r = \frac{mv}{qB} = \frac{E}{cqB} \tag{15.62}$$
$$\text{Now} r \geq \tau$$
$$E \geq \tau cqB \tag{15.63}$$
$$E_{min} = 10^7\times 3\times 10^8\times 1.602\times 10^{-19}\times 30\times 10^{-6}$$
$$= 1.4\times 10^{-8}J \tag{15.64}$$

16. Note that when the star will collapse to a Neutron star, the magnetic field lines on the surface of the star, will remain trapped and hence magnetic flux will be conserved.

$$B \propto \frac{1}{R^2}$$
$$4\pi R^2 B = 4\pi R_n^2 B_n \tag{15.65}$$
$$B_n = \left(\frac{R}{R_n}\right)^2 B \tag{15.66}$$
$$= \left(\frac{4\times 6.955\times 10^5}{20}\right)^2\times 10^{-4} \tag{15.67}$$
$$B_n = 2\times 10^6 T \tag{15.68}$$

17. We find total energy available and calculate the time for which it can

sustain current solar luminosity.

$$E = mc^2 = 0.008 M_\odot c^2 \tag{15.69}$$
$$E = L_\odot t \tag{15.70}$$
$$\therefore t = \frac{M_\odot c^2}{L_\odot} \tag{15.71}$$
$$= \frac{0.008 \times 1.9891 \times 10^{30} \times (2.9979 \times 10^8)^2}{3.826 \times 10^{26}}$$
$$= 3.74 \times 10^{18} sec$$
$$\approx 1.2 \times 10^{11} \ years \tag{15.72}$$

18. The mass loss of the Sun can be calculated from the energy required for the Sun to sustain solar luminosity,

$$L_\odot = -\frac{\Delta E}{\Delta t} = -\frac{\Delta M c^2}{\Delta t} \tag{15.73}$$
$$\Delta M = -\frac{L_\odot \Delta t}{c^2} \tag{15.74}$$
$$\tag{15.75}$$

Assuming the mass loss rate to be constant for t = 100 years. Note that angular momentum of radiation is zero. From conservation of angular momentum,

$$M_\oplus v_1 r_1 = M_\oplus v_2 r_2 \tag{15.76}$$
$$\sqrt{\frac{GM_{\odot,1}}{r_1}} r_1 = \sqrt{\frac{GM_{\odot,2}}{r_2}} r_2 \tag{15.77}$$
$$\frac{M_{\odot,1}}{r_1} r_1^2 = \frac{M_{\odot,2}}{r_2} r_2^2$$
$$\therefore M_{\odot,1} r_1 = M_{\odot,2} r_2 \tag{15.78}$$
$$M_{\odot,1}(r_2 - \Delta r) = (M_{\odot,1} - \Delta M) r_2 \tag{15.79}$$
$$1 - \frac{\Delta M}{M_\odot} = 1 - \frac{\Delta r}{a_\oplus}$$
$$\Delta r = \frac{\Delta M a_\oplus}{M_\odot} = \frac{a_\oplus}{M_\odot} \times \frac{L_\odot \Delta t}{c^2} \tag{15.80}$$
$$= \frac{1.496 \times 10^{11} \times 3.826 \times 10^{26} \times 100 \times 3.1557 \times 10^7}{(3.00 \times 10^8)^2 \times 1.989 \times 10^{30}}$$
$$= 1.01 m \tag{15.81}$$

Thus, the radius increases by about 1 metre.

19. We need to be along the direction of the shaded region to be able to detect the beam. For each of the two strips,

$$A_{shaded} = R \sin\theta_1 \times 2\pi \times R d\theta_2 \tag{15.82}$$

Here θ_1 is angle between rotation axis and the emission axis and $d\theta_2$ is the opening angle. Remember there are two such strips. As the pulsar distribution is random, the probability will be,

$$p = \frac{2A_{shaded}}{A_{total}} = \frac{4\pi R^2 \sin\theta_1 d\theta_2}{4\pi R^2} \tag{15.83}$$

$$= \sin\theta_1 d\theta_2 \tag{15.84}$$

$$= 4^\circ \times \frac{\pi}{180^\circ} \times \sin(30^\circ) = \frac{\pi}{360}$$

$$p = 8.7 \times 10^{-3} \tag{15.85}$$

Now let us assume we are along this shaded region. Now emission is not isotropic it is only along two small regions each of $d\theta_2/2$ radius. Thus, solid angle is given by,

$$\alpha = 4\pi(1 - \cos 2^\circ) \tag{15.86}$$

$$= 4\pi \times 6.09 \times 10^{-4} \tag{15.87}$$

$$I_{pulsar} = \frac{L}{4\pi(1 - \cos 2^\circ)d^2} \tag{15.88}$$

$$I_\odot = \frac{L_\odot}{4\pi d_0^2} \tag{15.89}$$

$$M_{bol} - M_{Sun} = -2.5 \log\left(\frac{I}{I_0}\right) \tag{15.90}$$

$$= -2.5 \log\left(\frac{L}{4\pi(1 - \cos 2^\circ)d^2} \times \frac{4\pi d_0^2}{L_\odot}\right)$$

$$= -2.5 \log\left(\frac{10000 L_\odot}{6.09 \times 10^{-4} \times 1000^2} \times \frac{10^2}{L_\odot}\right)$$

$$= -2.5 \log\left(\frac{1}{6.09 \times 10^{-4}}\right)$$

$$= -8.04 \tag{15.91}$$

$$M_{bol} = M_{Sun} - 8.04$$

$$= 4.72 - 8.04$$

$$M_{bol} = -3.32 \tag{15.92}$$

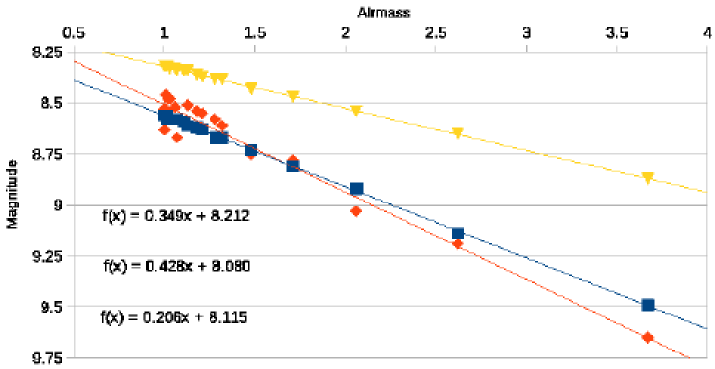


Figure 15.1 – Extinction Coefficient Determination for the three nights

15.2 Data Analysis

1. (a) In the figure 15.1, square symbols represent night A, dimond shaped ones night B and triangular ones night C.

   (b) The graph of magnitude v/s airmass is a straight line for each line, the resulting equations are shown on the graph 15.1. One can calculate β manually using equation 5.4.

Night	$\sum_{i=1}^{n} x_i$	$\sum_{i=1}^{n} x_i^2$	$\sum_{i=1}^{n} y_i$	$\sum_{i=1}^{n} x_i y_i$	β
A	25.96	45.83	156.88	229.19	0.349 mag/airmass
B	25.96	45.83	156.56	229.39	0.428 mag/airmass
C	25.96	45.83	151.41	220.09	0.206 mag/airmass

   One can see that a small change in numbers will lead to significantly different extinction coefficient.

   (c) As the plotted data for night B is different than other nights, clearly Night B was affected by clouds.

   (d) Using equations 5.2 and 5.5,

$$M_{ast} = m_{ast} + A - \beta X \qquad (15.93)$$

   Corrected values for asteroid are shown in table 15.1. Figure 15.2 shows variation of corrected asteroid magnitude with time on differnt nights.

   (e) Table below lists rotation period as measured on the three nights.

Δt	Air	M_{ast}		
(hr)	mass	Night A	Night B	Night C
0	1.28	6.98	10.98	13.00
0.44	1.18	6.96	10.91	12.99
0.89	1.11	6.94	10.97	12.92
1.33	1.06	6.90	10.92	12.91
1.77	1.02	6.95	11.03	12.92
2.21	1	6.97	11.25	12.96
2.66	1	6.97	11.13	12.98
3.1	1.01	6.94	10.98	12.98
3.54	1.03	6.90	10.91	12.94
3.99	1.07	6.88	11.06	12.95
4.43	1.13	6.90	10.91	12.93
4.87	1.21	6.94	11.02	12.91
5.31	1.32	6.95	11.09	12.96
5.76	1.48	6.96	11.13	13.02
6.2	1.71	6.98	11.01	13.01
6.64	2.06	6.96	11.03	12.99
7.09	2.62	6.94	10.90	12.92
7.53	3.67	6.92	10.94	12.93

Table 15.1 – Corrected Magnitudes of the asteroid for three nights at different times

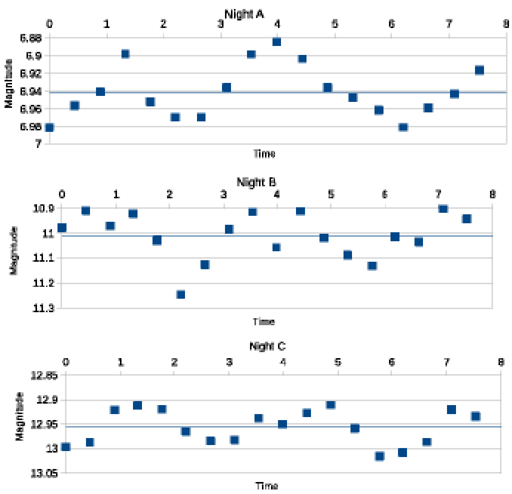


Figure 15.2 – Variation of the asteroid brightness for the three nights

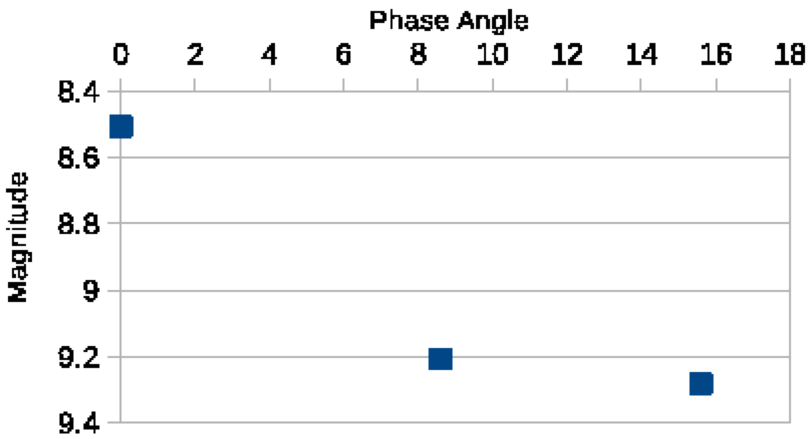


Figure 15.3 – Magnitude of asteroid at different phase angles

Night	Time Between Maxima	Time Between Minima	Average semi-period	Average period	Amplitude
A	3.6 hrs	2.4 hrs	3.0 hrs	6.0 hrs	0.10 mag
B	3.2 hrs	2.9 hrs	3.05 hrs	6.1 hrs	0.21 mag
C	3.1 hrs	3.2 hrs	3.15 hrs	6.3 hrs	0.10 mag

(f) Peak-to-peak amplitude as determined from the curves is mentioned in the table above.

Night	D	R	$\langle M \rangle$	m_r	φ
A	0.36	1.35	6.94	8.51	0.00
B	1.15	2.13	11.15	9.20	8.6
C	2.70	1.89	12.82	9.28	15.6

(g) The plot can be seen in figure 15.3.

(h) If you take the two points of non-zero phase angle, the angular coefficient will turn out to be,

$$\text{Slope} = \frac{(9.28 - 9.20)}{(15.6 - 8.6)} = 0.011\ mag/degree \tag{15.94}$$

(i) As per the angular coefficient, without regoliths one would expect the asteroid to brighten up to 9.11 magnitude, when the phase angle is zero. However, in reality it is much brighter (magnitude = 8.51). Thus, it is clearly a regolith covered asteroid.

2. (a) From $andD(t) = N_0 - N(t)$, the we have

$$\begin{aligned}
N(t) &= N_0 e^{-\lambda t} \\
\therefore N_0 &= N(t)e^{\lambda t} \\
D(t) &= N_0 - N(t) \\
D(t) &= N(t)e^{\lambda t} - N(t) \\
&= N(t)(e^{\lambda t} - 1) \qquad (15.95) \\
\frac{D(t)}{N(t)} &= e^{\lambda t} - 1 \\
\lambda t &= \ln\left(\frac{D(t)}{N(t)} + 1\right) and \\
t &= \frac{1}{\lambda}\ln\left(\frac{D(t)}{N(t)} + 1\right) \qquad (15.96)
\end{aligned}$$

(b) After one half-life, half the parent nuclides will turn into daughter nuclides. Thus, from eq. 15.96,

$$\begin{aligned}
D(t) &= N(t) \\
T_{1/2} &= \frac{\ln 2}{\lambda} = \frac{\ln 2}{1.42 \times 10^{-11}} \qquad (15.97) \\
T_{1/2} &= 48.81 Gyr \qquad (15.98)
\end{aligned}$$

(c) As it is possible that some ^{87}Sr was present in the meteorite from the beginning, we can use a simple linear regression equation:

$$\begin{aligned}
D(t) &= D_0 + (e^{\lambda t} - 1)N(t) \qquad (15.99) \\
{}^{87}Sr &= {}^{87}Sr_0 + (e^{\lambda t} - 1)^{87}Rb \\
\frac{^{87}Sr}{^{86}Sr} &= \left(\frac{^{87}Sr}{^{86}Sr}\right)_0 + (e^{\lambda t} - 1)\frac{^{87}Rb}{^{86}Sr} \qquad (15.100) \\
\therefore a &= \left(\frac{^{87}Sr}{^{86}Sr}\right)_0 \qquad (15.101) \\
\& b &= (e^{\lambda t} - 1) \qquad (15.102)
\end{aligned}$$

Now to find regression coefficients for each type of meteorite we separate samples of type A and type B. Calculation table for type A is shown below.

Sample	$\frac{^{87}Rb}{^{86}Sr}$ (X_i)	$\frac{^{87}Sr}{^{86}Sr}$ (Y_i)	X_i^2	Y_i^2	$X_i \cdot Y_i$
1	0.0101351	0.6993243	0.0001027	0.4890545	0.0070877
4	0.1741294	0.7114428	0.0303210	0.5061508	0.1238831
5	0.0203046	0.7005076	0.0004123	0.4907109	0.0142235
7	0.1197605	0.7065868	0.0143426	0.4992649	0.0846212
9	0.0816327	0.7040816	0.0066639	0.4957309	0.0574761
Total	0.4059622	3.5219432	0.0518425	2.4809122	0.2872916

Using these totals,

$$SS_{xx} = \sum_{i=1}^{n} X_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right)^2$$

$$= 0.0518425 - \frac{0.4059622^2}{5}$$

$$SS_{xx} = 0.0188814 \tag{15.103}$$

$$SS_{yy} = \sum_{i=1}^{n} Y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} Y_i\right)^2 \tag{15.104}$$

$$= 2.4809122 - \frac{3.5219432^2}{5}$$

$$SS_{yy} = 0.0000954 \tag{15.105}$$

$$SS_{xy} = \sum_{i=1}^{n} X_i Y_i - \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right)\left(\sum_{i=1}^{n} Y_i\right)$$

$$= 0.2872916 - \frac{0.4059622 \times 3.5219432}{5}$$

$$SS_{xy} = 0.0013364 \tag{15.106}$$

$$b = \frac{SS_{xy}}{SS_{xx}} = \frac{0.0013364}{0.0188814}$$

$$b = 0.0707786 \tag{15.107}$$

$$a = \overline{y} - b\overline{x}$$

$$= 0.6986419 \tag{15.108}$$

Thus, linear regression equation for type A meteorite is,

$$\frac{^{87}Sr}{^{86}Sr} = 0.6986419 + 0.0707786\frac{^{87}Rb}{^{86}Sr} \tag{15.109}$$

Doing similar analysis for Type B,

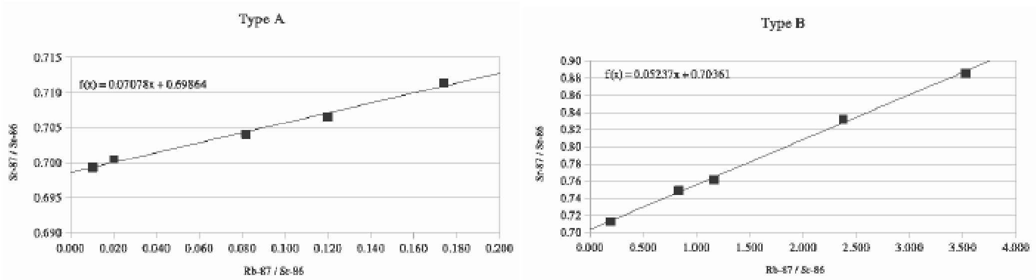


Figure 15.4 – Regression plots for Meteorite A (left) and B (right)

Sample	$\frac{^{87}Rb}{^{86}Sr}$ (X_i)	$\frac{^{87}Sr}{^{86}Sr}$ (Y_i)	X_i^2	Y_i^2	$X_i \cdot Y_i$
2	1.1669506	0.7614991	1.3617737	0.5798810	0.8886319
3	0.1940701	0.7129380	0.0376632	0.5082806	0.1383599
6	0.8337731	0.7493404	0.6951776	0.5615110	0.6247798
8	3.5234899	0.8859060	12.4149813	0.7848295	3.1214810
10	2.3783784	0.8324324	5.6566837	0.6929438	1.9798393
Total	8.0966621	3.9421160	20.1662795	3.1274458	6.7530920

$$
\begin{aligned}
SS_{xx} &= 20.1662795 - \frac{8.0966621^2}{5} \\
SS_{xx} &= 7.0550921 \qquad (15.110) \\
SS_{yy} &= 3.1274458 - \frac{3.9421160^2}{5} \\
SS_{yy} &= 0.0193901 \qquad (15.111) \\
SS_{xy} &= 6.7530920 - \frac{8.0966621 \times 3.9421160}{5} \\
SS_{xy} &= 0.3694958 \qquad (15.112) \\
b &= \frac{0.3694958}{7.0550921} \\
&= 0.0523729 \qquad (15.113) \\
a &= 0.7039330 \qquad (15.114)
\end{aligned}
$$

Thus, linear regression equation for type B meteorite is,

$$\frac{^{87}Sr}{^{86}Sr} = 0.7039330 + 0.0523729 \frac{^{87}Rb}{^{86}Sr} \qquad (15.115)$$

(d) Following are the plots for each type of meteorite.

(e) For type A meteorite,

$$
\begin{aligned}
b &= (e^{\lambda t} - 1) \\
t &= \frac{\ln(b+1)}{\lambda} \qquad (15.116) \\
&= \frac{\ln(1 + 0.0707786)}{1.42 \times 10^{-11}} \\
t_A &= 4.815919 Gyr \qquad (15.117)
\end{aligned}
$$

Next we find errors in the estimations

$$
\begin{aligned}
S_b &= \sqrt{\frac{SS_{yy} - \frac{SS_{xy}^2}{SS_{xx}}}{(n-2)SS_{xx}}} \\
&= \sqrt{\frac{0.0000954 - \frac{0.0013364^2}{0.0188814}}{3 \times 0.0188814}} \\
&= 0.0037848 \qquad (15.118) \\
S_a &= S_b \sqrt{\sum i = 1nX_i^2} \\
&= S_b \sqrt{0.0518425} \\
&= 0.0008618 \qquad (15.119) \\
\frac{S_t}{t} &= \frac{\partial t}{\partial b} \times \frac{S_b}{t} \qquad (15.120) \\
S_t &= \frac{S_b t}{(b+1)\ln(b+1)} \qquad (15.121) \\
&= \frac{0.0037848 \times 4.815919}{1.0707786 \ln 1.0707786} \\
&= 0.25 Gyr \qquad (15.122)
\end{aligned}
$$

For type B meteorite,

$$t = \frac{\ln(1 + 0.0523729)}{1.42 \times 10^{-11}} \tag{15.123}$$

$$t_B = 3.594896Gyr \tag{15.124}$$

$$S_b = \sqrt{\frac{SS_{yy} - \dfrac{SS_{xy}^2}{SS_{xx}}}{(n-2)SS_{xx}}}$$

$$= \sqrt{\frac{0.0193901 - \dfrac{0.3694958^2}{7.0550921}}{3 \times 7.0550921}}$$

$$= 0.0013491 \tag{15.125}$$

$$S_a = S_b\sqrt{\sum i = 1nX_i^2}$$

$$= 0.0013491 \times \sqrt{20.1662795}$$

$$= 0.0060586 \tag{15.126}$$

$$S_t = \frac{S_b t}{(b+1)\ln(b+1)} \tag{15.127}$$

$$= \frac{0.0013491 \times 3.594896}{1.0523729 \ln 1.0523729}$$

$$= 0.09Gyr \tag{15.128}$$

(f) Answers already found above.
Final Answers:

– For meteorite type A:

$$\frac{^{87}Sr}{^{86}Sr} = (0.699 \pm 0.001) + (0.071 \pm 0.004)\frac{^{87}Rb}{^{86}Sr} \tag{15.129}$$

$$\left(\frac{^{87}Sr}{^{86}Sr}\right)_0 = (0.699 \pm 0.001) \tag{15.130}$$

$$t = (4.8 \pm 0.3)Gyr \tag{15.131}$$

– For meteorite type B:

$$\frac{^{87}Sr}{^{86}Sr} = (0.704 \pm 0.006) + (0.052 \pm 0.001)\frac{^{87}Rb}{^{86}Sr} \tag{15.132}$$

$$\left(\frac{^{87}Sr}{^{86}Sr}\right)_0 = (0.704 \pm 0.006) \tag{15.133}$$

$$t = (3.59 \pm 0.09)Gyr \tag{15.134}$$

Chapter 16

Solutions: Stellar Observations

16.1 Theory

1. Let F_1, F_2 and F_c be the flux of the first star, the second star and the combined flux, respectively.

$$\delta m = -2.5 \log \left(\frac{F_1}{F_2} \right) \tag{16.1}$$

$$\frac{F_1}{F_2} = 10^{\left(\frac{2-1}{2.5}\right)}$$

$$= 10^{0.4} \tag{16.2}$$

$$F_c = F_1 + F_2 \tag{16.3}$$

$$m_c = m_2 - 2.5 \log \left(\frac{F_1 + F_2}{F_2} \right) \tag{16.4}$$

$$= 2 - 2.5 \log \left(1 + \frac{F_1}{F_2} \right)$$

$$= 2 - 2.5 \log \left(1 + 10^{0.4} \right)$$

$$= 2.0 - 1.36$$

$$m_c = 0.64 \ mag \tag{16.5}$$

2. Let d be the distance to that supernova. Then the flux density on Earth

would be

$$f = \frac{10^{10} L_{\odot}}{4\pi d^2} = \frac{L_{\odot}}{4\pi d^2_{\oplus-\odot}} \tag{16.6}$$

$$\frac{10^{10}}{d^2} = \frac{1}{(1A.U.)^2}$$

$$\therefore d = 10^{\frac{10}{2}} A.U. = 10^5 A.U.$$

$$= 1.496 \times 10^{16} m = \frac{1.496}{3.0856} pc$$

$$\approx 0.5 \;\; pc \tag{16.7}$$

3. The flux ratio can be calculated as,

$$m_1 - m_2 = -2.5 \log \left(\frac{f_1}{f_2}\right) \tag{16.8}$$

$$\therefore \frac{f_1}{f_2} = 10^{\frac{(m_2 - m_1)}{2.5}}$$

$$\frac{f_{min}}{f_{max}} = 10^{\frac{6.0-(-1.5)}{2.5}} = 10^{\frac{7.5}{2.5}}$$

$$= 1000 \tag{16.9}$$

Thus, the brightest star is 1000 times brighter than the faintest star visible to the naked eye.

4. Let the magnitude of full moon for current magnitude be m_0 and for the mmon with albedo $(\alpha_1) = 1$ be m_1 and let F be total flux incident on the moon's surface.

$$m_1 - m_0 = -2.5 \log \left(\frac{\alpha_0 F}{\alpha_1 F}\right) \tag{16.10}$$

$$m_1 = m_0 - 2.5 \log \left(\frac{1}{0.14}\right)$$

$$= -12.74 - 2.5 \log(7.14)$$

$$m_1 = -14.87 \tag{16.11}$$

5. Assuming the radiation of the star is isotropic,

$$\text{flux density} = \frac{\text{luminosity}}{4\pi D^2} \tag{16.12}$$

$$\therefore D = \sqrt{\frac{0.4 \times L_\odot}{4\pi \text{ (flux density)}}}$$

$$= \sqrt{\frac{0.4 \times 3.826 \times 10^{26}}{4\pi \times 6.23 \times 10^{-14}}}$$

$$= 1.4 \times 10^{19} m = \frac{1.4 \times 10^{19}}{3.0856 \times 10^{16}} pc$$

$$D \approx 450 \ \ pc \tag{16.13}$$

6. We take Sun as a G2 star. if N photons reach to our eye per second from this star, then

$$m_{st} - m_\odot = -2.5 \log \frac{f_{st}}{f_\odot} \tag{16.14}$$

$$6 - (-26.72) = -2.5 \log \frac{f_{st}}{1366}$$

$$f_{st} = 1.12 \times 10^{-10} W/m^2 \tag{16.15}$$

$$f_{st} = \frac{b_{st}}{D_{eye}^2} \tag{16.16}$$

$$b_{st} = 1.12 \times 10^{-10} \times 0.006^2$$

$$= 4.02 \times 10^{-15} W \tag{16.17}$$

$$N = \frac{b_s t}{E_{photon}} = \frac{b_{st}\lambda}{hc} \tag{16.18}$$

$$= \frac{4.02 \times 10^{-15} \times 550 \times 10^{-9}}{6.62 \times 10^{-34} \times 3 \times 10^8} \tag{16.19}$$

$$\approx 1 \times 10^4 \tag{16.20}$$

7. Let f be the flux emitted per unit area by the Sun's disk. When the Earth passes in front, it blocks a small area as seen from the distance. As the star and exoplanet are very far, their mutual physical separation does not matter and their angular sizes are proportional to their cross-

section area.

$$B_{max} = \pi R_{\odot}^2 f \quad (16.21)$$
$$B_{min} = \pi(R_{\odot}^2 - R_{\oplus}^2)f \quad (16.22)$$
$$\frac{B_{max}}{B_{min}} = \frac{\pi R_{\odot}^2 f}{\pi(R_{\odot}^2 - R_{\oplus}^2)f}$$
$$= \frac{R_{\odot}^2}{R_{\odot}^2 - R_e^2}$$
$$= \frac{(695.5 \times 10^6)^2}{(695.5 \times 10^6)^2 - (6.371 \times 10^6)^2}$$
$$\frac{B_{max}}{B_{min}} = 1.000084 \quad (16.23)$$

8.

$$m_{star} - m_{Vega} = -2.5 \log\left(\frac{F_{star}}{F_{Vega}}\right) \quad (16.24)$$
$$22 - 0 = -2.5 \log\left(\frac{F_{star}}{8.3 \times 10^{-12}}\right)$$
$$\therefore F_{star} = 8.3 \times 10^{-12} \times 10^{-\frac{22}{2.5}}$$
$$= 1.32 \times 10^{-20}\ Wm^{-2}nm^{-1} \quad (16.25)$$

For the star, the number of photons detected per unit wavelength per unit area is the flux divided by the energy of a photon with the effective wavelength

$$N_{star} = \frac{F_{star}}{E_{photon}} \times (Area) \times QE \times \Delta\lambda \quad (16.26)$$
$$= \frac{F_{star}}{hc/\lambda_{eff}} \times \pi\frac{D^2}{4} \times QE \times \Delta\lambda \quad (16.27)$$
$$= \frac{1.32 \times 10^{-20} \times 8 \times 10^{-7}}{6.6261 \times 10^{-34} \times 2.9979 \times 10^8} \times \pi\frac{8^2}{4} \times 0.4 \times 24$$
$$= 25.7$$
$$N_{star} \approx 30\ photons/s \quad (16.28)$$

9. The oscillatory motion of the Sun will be primarily due to the Jupiter as the centre of mass of the this system will not coincide with the centre of Sun. We can neglect contribution of all other planets. The magnitude

of Sun's oscillation will be given by distance between the centre of the Sun and the centre of mass of the solar system.

$$\begin{aligned}
d_\odot &= \frac{d_J M_J}{(M_\odot + M_J)} && (16.29)\\
&= \frac{5.204 \times 1.898 \times 10^{27}}{(1.989 \times 10^{30} + 1.898 \times 10^{27})} AU\\
&= 4.961 \times 10^{-3} AU && (16.30)\\
d_\odot &= 2.405 \times 10^{-8} pc\\
\alpha &= \frac{2 d_\odot}{d_{Barnarad}} = \frac{2 \times 2.405 \times 10^{-8}}{1.83} && (16.31)\\
&= 2.628 \times 10^{-8} rad\\
\alpha &= 0.00542'' && (16.32)
\end{aligned}$$

Period of oscillation will be same as the period of revolution of Jupiter.

$$\begin{aligned}
T &= 2\pi \sqrt{\frac{a_J^3}{G(M_\odot + M_J)}} && (16.33)\\
&= 2\pi \sqrt{\frac{(5.204 \times 1.496 \times 10^{11})^3}{6.672 \times 10^{-11}(1.989 \times 10^{30} + 1.898 \times 10^{27})}}\\
&= 3.745 \times 10^8 s\\
T &= 11.87 yr && (16.34)
\end{aligned}$$

10. Let B_0, V_0, U_0 be the intrinsic absolute magnitudes of the white dwarf. Let A_B, A_V, A_U be the interstellar extinction coefficients per kpc. Let A_{neb_B}, A_{neb_V}, A_{neb_U} be the total extinction for the planetary nebula around the first WD, for light travelling from the centre of the nebula to the edge.

$$\begin{aligned}
(U - B)_0 &= (U - V)_0 - (B - V)_0 && (16.35)\\
&= 0.330 - 0.300\\
(U - B)_0 &= 0.030 && (16.36)\\
(U - V)_1 &= (U - B)_1 + (B - V)_1 && (16.37)\\
&= 0.038 + 0.327\\
(U - V)_1 &= 0.365 && (16.38)
\end{aligned}$$

For, WD inside the nebula,

$$(B-V)_1 = (B-V)_0 + (A_B - A_V)d + (A_{neb_B} - A_{neb_V}) \quad (16.39)$$

$$(A_{neb_B} - A_{neb_V}) = (B-V)_1 - (B-V)_0 - (A_B - A_V)d$$
$$= 0.327 - 0.300 - (1.23 - 1.00) \times 0.05$$
$$(A_{neb_B} - A_{neb_V}) = 0.0155 \quad (16.40)$$
$$(A_{neb_U} - A_{neb_V}) = (U-V)_1 - (U-V)_0 - (A_U - A_V)d \quad (16.41)$$
$$= 0.365 - 0.330 - (1.50 - 1.00) \times 0.05$$
$$(A_{neb_U} - A_{neb_V}) = 0.0100 \quad (16.42)$$
$$(A_{neb_U} - A_{neb_B}) = (U-B)_1 - (U-B)_0 - (A_U - A_B)d \quad (16.43)$$
$$= 0.038 - 0.030 - (1.50 - 1.23) \times 0.05$$
$$(A_{neb_U} - A_{neb_B}) = -0.0055 \quad (16.44)$$

For second white dwarf, the extinction in planetary nebula will be doubled as extinction the light from this pulsar has to cross the entire diameter of the nebula. Overall distance of this pular is 3 times larger.

$$(B-V)_2 = (B-V)_0 + 3(A_B - A_V)d + 2(A_{neb_B} - A_{neb_V}) \quad (16.45)$$
$$= 0.300 + 3 \times (1.23 - 1.00) \times 0.05 + 2 \times 0.0155$$
$$(B-V)_2 = 0.3655 \approx 0.366 \quad (16.46)$$
$$(U-V)_2 = (U-V)_0 + 3(A_U - A_V)d + 2(A_{neb_U} - A_{neb_V}) \quad (16.47)$$
$$= 0.330 + 3 \times (1.50 - 1.00) \times 0.05 + 2 \times 0.0100$$
$$(U-V)_2 = 0.425 \quad (16.48)$$
$$(U-B)_2 = (U-B)_0 + 3(A_U - A_B)d + 2(A_{neb_U} - A_{neb_B}) \quad (16.49)$$
$$= 0.030 + 3 \times (1.50 - 1.23) \times 0.05 - 2 \times 0.0055$$
$$(U-B)_2 = 0.0595 \approx 0.060 \quad (16.50)$$

11. The physical radius of the cluster R will be given by,

$$R = \frac{D\alpha}{2} \quad (16.51)$$

As the cluster contains N stars, its stellar density will be,

$$\rho = \frac{N}{V} = \frac{3N}{4\pi}\left(\frac{2}{D\alpha}\right)^3$$
$$\rho = \frac{6N}{\pi D^3 \alpha^3} \quad (16.52)$$

Now for the astronomer, all the stars are roughly at the same distance but for biologist, the stars are located in differnt shells around him. Let the luminosity of each star be L_0. Suppose if we have a shell of thickness ΔR at a distance R' from the centre of the cluster, its flux will be given by,

$$\Delta V(R', \Delta R) = 4\pi R'^2 \Delta R$$

$$\Delta N(R', \Delta R) = \Delta V \rho = \frac{24NR'^2}{D^3\alpha^3}\Delta R \tag{16.53}$$

$$F_B(R', \Delta R) = \frac{\Delta N L_0}{4\pi R'^2} \tag{16.54}$$

$$= \frac{24NR'^2}{D^3\alpha^3} \times \frac{L_0}{4\pi R'^2}\Delta R$$

$$F_B(R', \Delta R) = \frac{6NL_0}{\pi D^3\alpha^3}\Delta R \tag{16.55}$$

Note that this expression is independent of R'. Now if we say,

$$\Delta R = R/n$$

$$F_B = nF_B(\Delta R) \tag{16.56}$$

$$= \frac{D\alpha}{2\Delta R}\frac{6NL_0}{\pi D^3\alpha^3}\Delta R$$

$$\therefore F_B = \frac{3NL_0}{\pi D^2\alpha^2} \tag{16.57}$$

$$F_A = \frac{NL_0}{4\pi D^2} \tag{16.58}$$

(a) Difference in apparent magnitude will be

$$m_A - m_B = -2.5\log\left(\frac{F_A}{F_B}\right) \tag{16.59}$$

$$= -2.5\log\left(\frac{\pi D^2\alpha^2}{3NL_0}\frac{NL_0}{4\pi D^2}\right)$$

$$= -2.5\log\left(\frac{\alpha^2}{12}\right) \tag{16.60}$$

$$\therefore m_A - m_B = 2.5\log\left(\frac{12}{\alpha^2}\right) \tag{16.61}$$

(b) For the same brightness, light gathering power should be equal.

$$\therefore F_A \times \pi(D_A)^2 = F_B \times \pi(D_B)^2 \tag{16.62}$$

$$D_A = D_B\sqrt{\frac{F_B}{F_A}} = D_B\sqrt{\frac{12}{\alpha^2}}$$

$$= 6 \times \frac{2\sqrt{3}}{\alpha} \tag{16.63}$$

$$D_A \approx \frac{21}{\alpha}\ mm \tag{16.64}$$

(c) α is very small. As the field of view of the biologist is reduced, the flux will also get reduced. The solid angle seen will be

$$\omega = 2\pi(1 - \cos\frac{\alpha}{2}) \tag{16.65}$$

$$= 2\pi\frac{\alpha^2}{8} = \frac{\pi\alpha^2}{4} \tag{16.66}$$

$$\therefore F'_B = \frac{F_B}{4\pi} \times \omega \tag{16.67}$$

$$= \frac{F_B}{4\pi} \times \frac{\pi\alpha^2}{4}$$

$$F'_B = \frac{\alpha^2}{16}F_B \tag{16.68}$$

$$m_A - m'_B = -2.5\log\left(\frac{F_A}{F'_B}\right) \tag{16.69}$$

$$= -2.5\log\left(\frac{F_A}{F_B} \times \frac{16}{\alpha^2}\right)$$

$$= -2.5\log\left(\frac{\alpha^2}{12} \times \frac{16}{\alpha^2}\right) = -2.5\log\left(\frac{4}{3}\right)$$

$$m_A - m'_B = -0.31 \tag{16.70}$$

12. (a) Let intrinsic magnitudes of the star be U_0, B_0, V_0 respectively. Let

E denote colour excess and let A denote value of extinction.

$$
\begin{aligned}
U - B &= 8.15 - 8.50 = -0.35 && (16.71)\\
B - V &= 8.50 - 8.14 = 0.36 && (16.72)\\
E(U - B) &= (U - B) - (U - B)_0 && (16.73)\\
&= -0.35 - (-0.45) && (16.74)\\
&= 0.1 && (16.75)\\
E(U - B) &= 0.72E(B - V) && (16.76)\\
\therefore E(B - V) &= \frac{0.1}{0.72} = 0.14 && (16.77)\\
A_V &= R_V E(B - V) && (16.78)\\
&= 3.2 \times 0.14 && (16.79)\\
A_V &= 0.45 && (16.80)\\
V_0 &= V - A_V = 8.14 - 0.45 = 7.69 && (16.81)\\
E(B - V) &= (B - V) - (B - V)_0 && (16.82)\\
0.14 &= 0.36 - (B_0 - V_0) && (16.83)\\
B_0 &= 0.36 - 0.14 + 7.69 && (16.84)\\
\therefore B_0 &= 7.9 && (16.85)\\
U_0 &= B_0 + (U - B)_0 && (16.86)\\
&= 7.9 - 0.45 && (16.87)\\
\therefore U_0 &= 7.45 && (16.88)
\end{aligned}
$$

(b) Star's absolute bolometric magnitude = - 0.25

Sun's absolute bolometric magnitude = 4.72

$$L = \sigma T^4 4\pi R^2 \quad (16.89)$$

$$\frac{L}{L_\odot} = (\frac{T}{T_\odot})^4 (\frac{R}{R_\odot})^2 \quad (16.90)$$

$$\log\left(\frac{L}{L_\odot}\right) = 4\log\left(\frac{T}{T_\odot}\right) + 2\log\left(\frac{R}{R_\odot}\right) \quad (16.91)$$

$$\because -2.5\log\left(\frac{L}{L_\odot}\right) = M - M_\odot \quad (16.92)$$

$$M_\odot - M = 10\log\left(\frac{T}{T_\odot}\right) + 5\left(\frac{R}{R_\odot}\right) \quad (16.93)$$

$$4.72 - (-0.25) = 10\log\left(\frac{T}{T_\odot}\right) + 5\log 2.3 \quad (16.94)$$

$$\log\left(\frac{T}{T_\odot}\right) = 0.1(4.97 - 1.8086) = 0.3161 \quad (16.95)$$

$$T = 2.07 T_\odot \quad (16.96)$$

$$T \approx 12000K \quad (16.97)$$

(c)

$$BC = -0.15 \quad (16.98)$$

$$M_V = M_{bol} + BC \quad (16.99)$$

$$= -0.25 - 0.15 \quad (16.100)$$

$$= -0.4 \quad (16.101)$$

$$m_v - M_V = 5\log\left(\frac{r}{10}\right) \quad (16.102)$$

$$7.69 - (-0.4) = 5\log\left(\frac{r}{10}\right) \quad (16.103)$$

$$\log\left(\frac{r}{10}\right) = 1.618 \quad (16.104)$$

$$r = 415pc \quad (16.105)$$

16.2 Data Analysis

1. (a) This part specifiaclly asks you to plot magnitudes of KZ Hyd w.r.t. comparison star. i.e. simply plot ΔV and ΔR against HJD*. The plots will look as shown in figure 16.1:

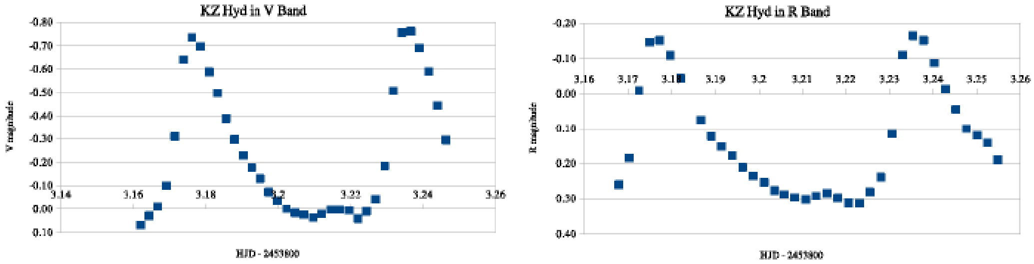


Figure 16.1 – Light Curves of KZ Hydrae

(b) For the check star, average of all 36 readings may be taken as the average magnitude.

For KZ Hydrae, It is not correct to simply take average of all readings in each column. The readings contain one complete period plus some readings before and after that. Thus, if you take average of all 36 readings, the same will be biased. Also note that the light curve is not symmetric and star's magnitude is close to minima for many readings. Thus simple average of maximum and minimum magnitude will also not be correct. To find reasonably correct average we should take readings for one complete period (say maxima to maxima) and calculate average for those.

$$\Delta V_{av} = -0.208 \tag{16.106}$$
$$\Delta R_{av} = 0.152 \tag{16.107}$$
$$\Delta V_{ch-av} = 4.388 \tag{16.108}$$
$$\Delta R_{ch-av} = 2.793 \tag{16.109}$$

EA: Even this average will be correct only if the readings are equispaced in time. Otherwise, ideally, we have to integrate the light curve and find average magnitude such that area bounded by the light curve and the horizontal line showing average magnitude is equal above and below the line. By doing that, we get values as $\Delta V_{av} = -0.205$ and $\Delta R_{av} = 0.148$. However, this is not expected from students who are finding average magnitudes manually.

(c) Assuming that comparison and check stars do not have variable magnitudes, the standard deviation in each band will give the photometric precisions in respective band.

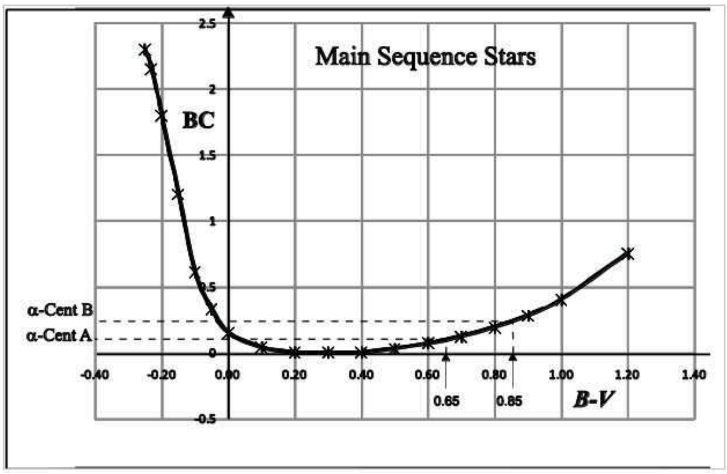
$$\sigma_V = \sqrt{\frac{\sum (\Delta V_{ch} - \Delta V_{ch-av})^2}{N}} \tag{16.110}$$
$$= 0.083 \tag{16.111}$$
$$\sigma_R = \sqrt{\frac{\sum (\Delta R_{ch} - \Delta R_{ch-av})^2}{N}} \tag{16.112}$$
$$= 0.011 \tag{16.113}$$

(d) From the graph 16.1,
Pulsation period in V band, $T_V = 0.058$ days $= 1.4$ hours
Pulsation period in R band, $T_R = 0.058$ days $= 1.4$ hours

(e) From the graph 16.1,
Pulsation amplitude in V band, $A_V = 0.83$ mag
Pulsation amplitude in R band, $A_R = 0.48$ mag

(f) Within the uncertainities, there is no phase difference between the light curves in two bands.

2. (a) Graph is drawn with $(B-V)_0$ on x-axis and BC on y-axis.



(b) From the graph

$$BC_A = 0.09$$
$$m_{bol-A} = m_v - BC = -0.01 - 0.09$$
$$= -0.1 \tag{16.114}$$
$$BC_B = 0.23$$
$$m_{bol_B} = m_v - BC = 1.34 - 0.23$$
$$= 1.11 \tag{16.115}$$

(c) Orbital period of $\alpha-$ Centauri is $T = 79.24$ years and its angular semi-major axis is $\alpha = 17.59''$. If d is the distance to the star in parsec, M_A and M_B are masses of two components in solar masses and a is their orbital radius in A. U., using Kepler's third law we get,

$$T^2 = \frac{a^3}{(M_A + M_B)} \tag{16.116}$$

$$= \frac{(\alpha d)^3}{(M_A + M_B)}$$

$$d^3 = \frac{(M_A + M_B)T^2}{\alpha^3} \tag{16.117}$$

The implicit trick here is the fact that the Earth goes around one solar mass star in one year with orbital radius of 1 A.U. So by taking quantities in those units will automatically eliminate $\frac{4\pi^2}{G}$. From here we can proceed by two methods:

i. We find the masses by iterative method. *1*$^{\text{st}}$ *iteration:* We first assume

$$M_A + M_B = 2M_\odot \tag{16.118}$$

$$d^3 = \frac{(M_A + M_B)T^2}{\alpha^3}$$

$$d = \sqrt[3]{\frac{2 \times 79.24^2}{17.59^3}}$$

$$= \sqrt[3]{2.3074}$$

$$d = 1.3214pc \tag{16.119}$$

We determine the absolute bolometric magnitude.

$$M_{bol} = m_{bol} + 5 - 5\log d \tag{16.120}$$

$$M_{bol-A} = -0.1 + 5 - 5\log(1.3214)$$

$$M_{bol-A} = 4.9 - 0.6052 = 4.29 \tag{16.121}$$

$$M_{bol-B} = 1.11 + 5 - 5\log(1.3214)$$

$$M_{bol-B} = 6.11 - 0.6052 = 5.50 \tag{16.122}$$

Now, we determine mass of both stars using luminosity mass relation.

$$\begin{aligned}
M_{bol} &= -10.2 \log\left(\frac{M}{M_\odot}\right) + 4.9 \\
\therefore M &= 10^{\frac{(4.9-M_{bol})}{10.2}} M_\odot && (16.123)\\
M_A &= 10^{\frac{4.9-4.29}{10.2}} M_\odot \\
&= 10^{\frac{0.61}{10.2}} M_\odot \\
M_A &= 1.1476 M_\odot && (16.124)\\
M_B &= 10^{\frac{(4.9-5.50)}{10.2}} M_\odot \\
&= 10^{\frac{-0.6}{10.2}} M_\odot \\
M_B &= 0.8733 M_\odot && (16.125)\\
\therefore M_A + M_B &= (1.1476 + 0.8733) M_\odot \\
M_A + M_B &= 2.0209 M_\odot && (16.126)
\end{aligned}$$

2[nd] *iteration:* Repeating the steps above and using the previous resulting total mass from the first iteration,

$$\begin{aligned}
d &= \sqrt[3]{\frac{2.0209 \times (79.24)^2}{(17.59)^3}} \\
d &= 1.3260 pc && (16.127)\\
M_{bol-A} &= -0.1 + 5 - 5\log(1.3260) \\
M_{bol-A} &= 4.29 && (16.128)\\
M_{bol-B} &= 1.11 + 5 - 5\log(1.3260) \\
M_{bol-B} &= 5.50 && (16.129)
\end{aligned}$$

We can note that within the accuracy limits of magnitudes, the absolute bolometric magnitudes in the second iteration are exactly same as the ones in the first iteration. Thus, masses determined would again be exactly the same. Thus,

$$\begin{aligned}
\therefore M_A &= 1.15 M_\odot && (16.130)\\
\& \ M_B &= 0.87 M_\odot && (16.131)
\end{aligned}$$

ii. Alternatively, we can substitute d in terms of mass.

$$d^3 = \frac{(M_A + M_B)T^2}{\alpha^3}$$

$$\therefore \frac{(M_A + M_B)}{d^3} = \frac{\alpha^3}{T^2} = \frac{17.59^3}{79.24^2} \tag{16.132}$$

$$= 0.8668 M_\odot / pc^3 \tag{16.133}$$

$$M_{bol} = m_{bol} + 5 - 5\log d$$

$$M_{bol} = -10.2 \log\left(\frac{M}{M_\odot}\right) + 4.9$$

$$\therefore 10.2 \log\left(\frac{M}{M_\odot}\right) = 5\log d - m_{bol} - 5 + 4.9$$

$$\log M = \frac{5\log d - m_{bol} - 0.1}{10.2} \tag{16.134}$$

$$\log M_A = 0.49 \log d \tag{16.135}$$

$$\log M_B = 0.49 \log d - \frac{1.21}{10.2} \tag{16.136}$$

Combining the two equations, we get,

$$= \log M_A - \frac{1.21}{10.2}$$

$$\therefore M_B = 10^{\frac{-1.21}{10.2}} M_A$$

$$M_B = 0.7610 M_A \tag{16.137}$$

$$M_A + M_B = 1.7610 M_A$$

$$\therefore 0.8668 d^3 = 1.7610 d^{0.49} \tag{16.138}$$

$$d = \left(\frac{1.7610}{0.8668}\right)^{\frac{1}{2.51}}$$

$$d = 1.3260 pc \tag{16.139}$$

$$M_A = d^{0.49}$$

$$= 1.15 M_\odot \tag{16.140}$$

$$M_B = 0.7610 \times 1.1483 M_\odot$$

$$M_B = 0.87 M_\odot \tag{16.141}$$

Chapter 17

Solutions: Binaries and Variables

17.1 Theory

1. As the orbital plane is perpendicular to the line of sight, the largest angular separation corresponds to the apastron position and the smallest angular separation corresponds to the periastron position. Thus,

$$a = \frac{1}{2} \times (7+1) \times 10 \tag{17.1}$$

$$a = 40AU \tag{17.2}$$

$$M_{\odot} = \frac{4\pi^2}{G} \frac{(1\ AU)^3}{(1\ year)^2} \tag{17.3}$$

$$m_1 + m_2 = \frac{4\pi^2}{G} \frac{a^3}{T^2} \tag{17.4}$$

$$= \frac{40^3}{100^2} M_{\odot}$$

$$= 6.4 M_{\odot} \tag{17.5}$$

$$a_2 = a - a_1 = 4'' - 3''$$

$$= 1'' \tag{17.6}$$

$$\because \frac{m_1}{m_2} = \frac{a_2}{a_1} = \frac{1}{3} \tag{17.7}$$

$$\therefore m_1 = 1.6 M_{\odot} \tag{17.8}$$

$$m_2 = 4.8 M_{\odot} \tag{17.9}$$

2. Assuming brightness is proportional to the area of stars visible to us.

$$B_{max} \propto 2 \times (\text{area of elliptic cross} - \text{section})$$
$$\therefore B_{max} \propto 2\pi ab \tag{17.10}$$
$$B_{min} \propto (\text{area of circular cross} - \text{section})$$
$$B_{min} \propto \pi b^2 \tag{17.11}$$
$$\frac{B_{max}}{B_{min}} = \frac{2\pi ab}{\pi b^2}$$
$$= \frac{2a}{b} = 4 \tag{17.12}$$
$$\Delta m = 2.5 \log \left(\frac{B_{max}}{B_{min}} \right) \tag{17.13}$$
$$= 2.5 \log 4 \tag{17.14}$$
$$\Delta m = 1.5 \tag{17.15}$$

3. (a) From conservation of angular momentum,

$$\omega r^2 = constant \tag{17.16}$$
$$\omega r^2 = \langle \omega \rangle ab \tag{17.17}$$

Thus, when $\omega < \langle \omega \rangle$, clearly $r > \sqrt{ab}$. From the image,

$$2a = 6.2\ cm \tag{17.18}$$
$$2b = 5.0\ cm \tag{17.19}$$
$$\sqrt{ab} \approx 2.8\ cm\ < r \tag{17.20}$$

We can measure r from M at various points to mark the regions.

(b) Let V_t be the tangential velocity at distance r. Now at peri-astron, the total velocity of m is maximum and is purely tangential. Hence that is the point where $v_t = v_{t_{max}}$. Similarly at ap-astron, $v_t = v_{t_{min}}$.

(c) For maximum radial velocity, one can note that radial accelaration changes sign at the end points of latus rectum. Thus, they are the points where radial velocity will be maximum (when receding from the observer) or minimum (when approaching the observer).

4. During the secondary minimum, the brighter (and larger) star is eclipsing the fainter (and smaller) star. Therefore, during the primary minimum we will have an annular eclipse - the cooler and smaller component is visible against the larger one, only obscuring it partially.

We use subscripts:
0 - data concerning the sum of both components (when there is no eclipse),
1 - data concerning the brighter component,
2 - data concerning the fainter component,
p - data concerning the primary minimum,
s - data concerning the primary minimum,
Let R_is be radii, T_is be effective temperatures, L_is be luminosities and f_is be the luminosities per unit surface area of the star.

Remember that the eclipse starts (first contact) when two stars' edges seen to be touching from the outside. At the start of totality phase (second contact), the edges touch from the inside. At the end of totality (third contact) edges again touch from the inside and at the end of eclipse (fourth contact) edges again touch from the outside.

Thus, between first and second contact, the fainter star moves distance of $2R_2$.
Between first and fourth contact (corresponding to D), the fainter star moves distance of $2(R_1 + R_2)$.
between second and third contact (corresponding to d), the fainter star moves distance of $2(R_1 - R_2)$.
From the graph,

$$\frac{2(R_1+R_2)}{2(R_1-R_2)} = \frac{D}{d} = 9 \tag{17.21}$$

$$\therefore \frac{R_2}{R_1} = \frac{4}{5} \tag{17.22}$$

$$L_0 = L_1 + L_2 = R_1^2 f_1 + R_2^2 f_2 \tag{17.23}$$

$$L_s = L_1 = R_1^2 f_1 \tag{17.24}$$

$$L_p = R_2^2 f_2 + (R_1^2 - R_2^2) f_1 \tag{17.25}$$

$$m_0 - m_s = -2.5\log\left(\frac{L_0}{L_s}\right) = -2.5\log\left(\frac{R_1^2 f_1 + R_2^2 f_2}{R_1^2 f_1}\right) \tag{17.26}$$

$$= -2.5\log\left(1 + \frac{R_2^2 f_2}{R_1^2 f_1}\right) \tag{17.27}$$

Let the bracket be $(1+x)$.

$$-0.33 = -2.5\log(1+x)$$

$$\therefore 1 + x = 10^{\frac{0.33}{2.5}}$$

$$1 + x = 1 + \frac{R_2^2 f_2}{R_1^2 f_1} = 1.3552 \tag{17.28}$$

$$m_p - m_0 = -2.5\log\left(\frac{L_p}{L_0}\right) = -2.5\log\left(\frac{R_2^2 f_2 + (R_1^2 - R_2^2) f_1}{R_1^2 f_1 + R_2^2 f_2}\right) \tag{17.29}$$

$$= -2.5\log\left(\frac{\frac{R_2^2 f_2}{R_1^2 f_1} + 1 - \frac{R_2^2}{R_1^2}}{1 + \frac{R_2^2 f_2}{R_1^2 f_1}}\right)$$

$$= -2.5\log\left(\frac{1 + x - \frac{R_2^2}{R_1^2}}{1+x}\right)$$

$$= -2.5\log\left(1 - \frac{1}{1+x}\left(\frac{R_2}{R_1}\right)^2\right) \tag{17.30}$$

$$= -2.5\log\left(1 - \frac{1}{1.3552}\left(\frac{4}{5}\right)^2\right)$$

$$m_p - m_0 = 0.694 \tag{17.31}$$

This is 2.1 times the magnitude difference between m_0 and m_s. Hence we can draw the lightcurve at primary minima.

5. (a) The total angular momentum of the system is

$$L = I\omega = (M_1 r_1^2 + M_2 r_2^2)\omega \tag{17.32}$$

As there is quasi-static equilibrium, $M_1 r_1 = M_2 r_2$ and $D = r_1 + r_2$, which yields,

$$r_1 = \frac{M_2 D}{M_1 + M_2} \tag{17.33}$$

$$\therefore L = \frac{M_1 M_2}{M_1 + M_2} D^2 \omega \tag{17.34}$$

The kinetic energy of the system is

$$\begin{aligned} K.E. &= \frac{1}{2} M_1 (r_1 \omega)^2 + \frac{1}{2} M_2 (r_2 \omega)^2 \\ &= \frac{1}{2}(M_1 r_1^2 + M_2 r_2^2)\omega^2 = \frac{1}{2} L\omega \\ &= \frac{1}{2} \frac{M_1 M_2}{M_1 + M_2} D^2 \omega^2 \end{aligned} \tag{17.35}$$

(b) Equating the centripetal accelaration to the gravitational accelaration,

$$M_1 \omega^2 r_1 = \frac{G M_1 M_2}{D^2} \tag{17.36}$$

The equations 17.33 and 17.36 together yield

$$\omega^2 = \frac{G(M_1 + M_2)}{D^3} \tag{17.37}$$

(c) As the system is isolated,

$$M_1 + M_2 = constant$$

Since there is no external torque acting on the system, the total angular momentum must be conserved.

$$\begin{aligned} L = \frac{M_1 M_2}{M_1 + M_2} D^2 \omega &= \text{ constant} \\ \therefore M_1 M_2 D^2 \omega &= \text{ constant} \end{aligned} \tag{17.38}$$

Now, after the mass transfer,

$$\omega \rightarrow \omega + \Delta\omega$$

$$M1 \rightarrow M_1 + \Delta M_1 \tag{17.39}$$

$$M2 \rightarrow M_2 - \Delta M_1 \tag{17.40}$$

$$D \rightarrow D + \Delta D$$

$$\therefore M_1 M_2 D^2 \omega = (M_1 + \Delta M_1)(M_2 - \Delta M_1)(D + \Delta D)^2 (\omega + \Delta\omega) \tag{17.41}$$

$$1 = (1 + \frac{\Delta M_1}{M_1})(1 - \frac{\Delta M_1}{M_2})(1 + \frac{\Delta D}{D})^2 (1 + \frac{\Delta\omega}{\omega})$$

After using the approximation $(1+x)^n \sim (1+nx)$ and rearranging, we get

$$1 = 1 + \frac{\Delta M_1}{M_1} - \frac{\Delta M_1}{M_2} + \frac{2\Delta D}{D} + \frac{\Delta \omega}{\omega}$$
$$\frac{\Delta \omega}{\omega} + 2\frac{\Delta D}{D} = \left(\frac{M_1 - M_2}{M_1 M_2}\right) \Delta M_1 \tag{17.42}$$

From equation 17.37, $\omega^2 D^3$ is also constant. That is,

$$\omega^2 D^3 = (\omega + \Delta\omega)^2 (D + \Delta D)^3$$
$$\frac{\Delta D}{D} = -\frac{2}{3}\frac{\Delta \omega}{\omega} \tag{17.43}$$
$$\therefore \Delta\omega = -\left(\frac{3(M_1 - M_2)}{M_1 M_2}\right) \omega \Delta M_1 \tag{17.44}$$

(d) We first find $\Delta\omega$ by

$$\Delta\omega = \omega_2 - \omega_1 = \frac{2\pi}{T_2} - \frac{2\pi}{T_1}$$
$$\approx \frac{2\pi(T_1 - T_2}{T^2} = \frac{-\omega}{T}\Delta T \tag{17.45}$$
$$\therefore \Delta M_1 = -\left(\frac{M_1 M_2}{3(M_1 - M_2)}\right) \frac{\Delta\omega}{\omega}$$
$$\Delta M_1 = \left(\frac{M_1 M_2}{3(M_1 - M_2)}\right) \frac{\Delta T}{T} \tag{17.46}$$
$$\frac{\Delta M_1}{M_1 \Delta t} = \frac{1.4 \times 20}{3 \times (2.9 - 1.4) \times 2.49 \times 86400 \times 100}$$
$$\approx 2.9 \times 10^{-7} \text{per year} \tag{17.47}$$

(e) In equation 17.39 above, we added ΔM_1 to M_1. As the final sign of ΔM_1 is positive, mass is flowing from M_2 to M_1.

(f) From equations 17.43 and 17.45:

$$\frac{\Delta D}{D} = -\frac{2}{3}\frac{\Delta\omega}{\omega} = +\frac{2}{3}\frac{\Delta T}{T}$$
$$\therefore \frac{\Delta D}{D\Delta t} = +\frac{2}{3}\frac{\Delta T}{T\Delta t}$$
$$= \frac{2 \times 20}{3 \times 2.49 \times 86400 \times 100}$$
$$= 6.2 \times 10^{-7} \text{per year} \tag{17.48}$$

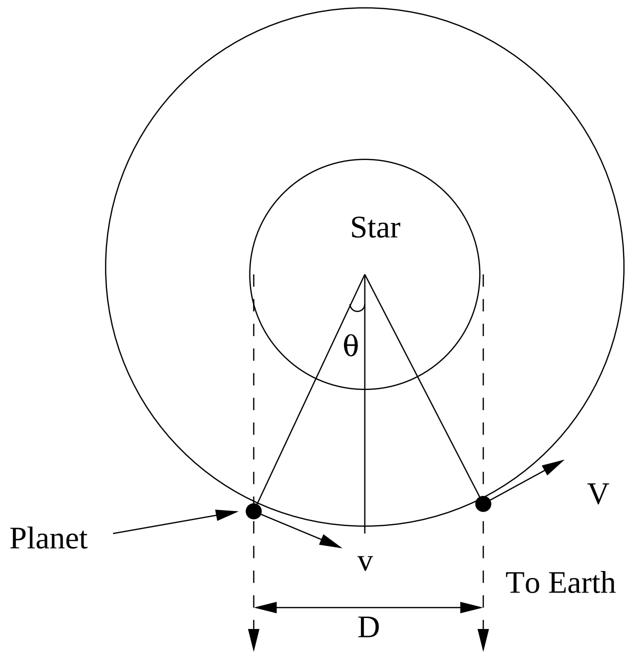


Figure 17.1 – Transit of Exoplanet

6. Refer to figure 17.1. As the planet is going around the star, there will be a component of its velocity along the line of sight of the terristrial observer. As shown in the figure, The difference in radial velocity of planet between begining and end of the transit is due to change in radial velocity of planet due to revolution. Any velocity due to rotation of planet will get cancelled.

$$2\theta = \frac{t_{transit}}{T} \times 360 \tag{17.49}$$

$$\theta = \frac{180}{2 \times 84 \times 60} \times 360$$

$$= 12.86° \tag{17.50}$$

$$V_{r1} = -V \sin\theta$$

$$V_{r2} = V \sin\theta$$

$$\Delta V_r = 2V \sin(\theta) \tag{17.51}$$

$$\therefore V = \frac{\Delta V_r}{2 \sin\theta} = \frac{30}{2 \times \sin 12.85}$$

$$V = 134\ km/s \tag{17.52}$$

This orbital velocity will allow us to calculate orbital radius, assuming circular orbit.

$$V = \frac{2\pi a}{T} \tag{17.53}$$

$$a = \frac{134 \times 10^3 \times 84 \times 3600}{2\pi}$$

$$= \frac{VT}{2\pi}$$

$$a = 6.45 \times 10^9 m \tag{17.54}$$

$$M = \frac{4\pi^2 a^3}{GT^2} \tag{17.55}$$

$$= \frac{4\pi^2 \times (6.45 \times 10^9)^3}{6.6726 \times 10^{-11} \times (84 \times 3600)^2}$$

$$M = 1.74 \times 10^{30} kg \tag{17.56}$$

$$2R_{st} = tV \tag{17.57}$$

$$R_{st} = \frac{tV}{2} = \frac{180 \times 60 \times 134 \times 10^3}{2} \tag{17.58}$$

$$R = 7.2 \times 10^8 m. \tag{17.59}$$

7. We first calculate angular separation between the stars.

$$\Delta\alpha = 14^h 39^m 39.39^s - 14^h 29^m 44.95^s$$

$$= 9^m 54.44^s = 2.4768^\circ \tag{17.60}$$

$$\Delta\delta - 6050'22.10'' - (-62^\circ 40' 46.14'')$$

$$= 1^\circ 50' 24.04'' = 1.8400^\circ \tag{17.61}$$

$$\Delta\alpha \cos\delta_{av} = 2.4768 \times \cos(61^\circ 45' 34.12'')$$

$$= 1.17196 \tag{17.62}$$

$$\therefore \Delta\theta = \sqrt{\Delta\delta^2 + (\Delta\alpha\cos\delta)^2} \tag{17.63}$$

$$= \sqrt{1.8400^2 + 1.17196^2} \tag{17.64}$$

$$= 2.1815 \tag{17.65}$$

Next we find physical separation between the two stars.

$$a^2 = d_1^2 + d_2^2 - 2d_1 d_2 \cos\theta \tag{17.66}$$

$$a = \sqrt{1.2953^2 + 1.3475^2 - 2 \times 1.2953 \times 1.3475 \cos 2.1815}$$

$$a = 0.07249 pc$$

$$= 2.237 \times 10^{15} m \tag{17.67}$$

Next we find relative velocity of one star w.r.t. another.

$$\Delta\mu_\alpha = 0.176'' \quad (17.68)$$
$$\Delta\mu_\delta = 0.18'' \quad (17.69)$$
$$V_{rel} = d_{av}\sqrt{(\Delta\mu_\delta)^2 + (\Delta\mu_\alpha)^2} \quad (17.70)$$
$$= 1.3214 \times \sqrt{(0.18^2 + 0.176^2)} \times \frac{\pi}{180 \times 3600} \times \frac{3.0856 \times 10^{16}}{3.1557 \times 10^7}$$
$$V_{rel} = 1.58\ Km/s \quad (17.71)$$

The Keplerian orbital velocity for this orbital radius, where the central mass is about 1-2 solar masses,

$$v_{Kep} = \sqrt{\frac{GM_\odot}{a}} \quad (17.72)$$
$$= \sqrt{\frac{6.6726 \times 10^{-11} \times 1.9891 \times 10^{30}}{2.237 \times 10^{15}}}$$
$$\approx (250 - 500)\ m/s \quad (17.73)$$

Thus, clearly, $V_{rel} > V_Kep$, which means that the two stars are NOT gravitationally bound.

8. Since the orbits are circular, magnitude of velocity of stars remains constant at any point in their orbit. Let $a_{1,2}$ indicate orbital radii, $M_{1,2}$ the masses, $V_{1,2}$ velocities and $R_{1,2}$ be the stellar radii.

$$T = 30 \text{ days} = 2.59 \times 10^6 s \quad (17.74)$$
$$\omega = \frac{2 \times \pi}{T} = \frac{2 \times \pi}{2.59 \times 10^6} \quad (17.75)$$
$$= 2.424 \times 10^{-6} rad/s \quad (17.76)$$
$$\because V = r\omega \quad (17.77)$$
$$a_1 = \frac{V_1}{\omega} = \frac{30 \times 10^3}{2.424 \times 10^{-6}} m \quad (17.78)$$
$$a_1 = 1.24 \times 10^{10} m \quad (17.79)$$
$$a_2 = \frac{V_2}{\omega} = 1.65 \times 10^{10} m \quad (17.80)$$
$$a = a_1 + a_2 \quad (17.81)$$
$$= (1.24 + 1.65) \times 10^{10} m \quad (17.82)$$
$$a = 2.89 \times 10^{10} m \quad (17.83)$$
$$(17.84)$$

This total separation between the two stars will be used in Kepler's third law. Next we find the masses by noting that moments around the centre of mass of the system should be the same.

$$\because M_1 a_1 = M_2 a_2 \tag{17.85}$$

$$\frac{M_2}{M_1} = \frac{a_1}{a_2} = \frac{V_1}{V_2} = \frac{3}{4} \tag{17.86}$$

$$T = 2\pi\sqrt{\frac{a^3}{G(M_1 + M_2)}} \tag{17.87}$$

$$2.59 \times 10^6 = 2\pi\sqrt{\frac{(2.89 \times 10^{10})^3}{6.6726 \times 10^{-11} \times (M_1 + M_2)}} \tag{17.88}$$

$$M_1 + M_2 = \frac{4\pi^2 \times (2.89 \times 10^{10})^3}{6.6726 \times 10^{-11} \times (2.59 \times 10^6)^2} \tag{17.89}$$

$$= 2.13 \times 10^{30} kg \tag{17.90}$$

$$M_1 + M_2 = 1.06 M_\odot \tag{17.91}$$

$$\therefore M_1 = \frac{4}{7}(M_1 + M_2) = \frac{4 \times 1.06}{7} M_\odot \tag{17.92}$$

$$M_1 = 0.61 M_\odot \tag{17.93}$$

$$M_2 = \frac{3}{7}(M_1 + M_2) \tag{17.94}$$

$$M_2 = 0.45 M_\odot \tag{17.95}$$

If we assume primary star to be stationary, then secondary star appears to move in front of it with relative velocity of

$$V_r = V_1 + V_2 = 70 km/s \tag{17.96}$$

During A to D distance covered is $2(R_1 + R_2)$ and during B to C is $2(R_1 - R_2)$. This assumes that the star is moving transverse to the line joining stars to Earth as, time period of eclipse $\ll$ time period of orbit.

$$2(R_1 + R_2) = 70 \times 10^3 \times 8 \text{ hours} \tag{17.97}$$

$$= 70 \times 10^3 \times 2.88 \times 10^4 \tag{17.98}$$

$$R_1 + R_2 = 1.008 \times 10^9 m = 1.448 R_\odot \tag{17.99}$$

$$2(R_1 - R_2) = 70 \times 10^3 \times 78 \text{ min} \tag{17.100}$$

$$= 70 \times 10^{30} \times 4.68 \times 10^3 \tag{17.101}$$

$$R_1 - R_2 = 1.638 \times 10^8 m = 0.235 R_\odot \tag{17.102}$$

$$\therefore R_1 = 0.841 R_\odot \tag{17.103}$$

$$\& \ R_2 = 0.607 R_\odot. \tag{17.104}$$

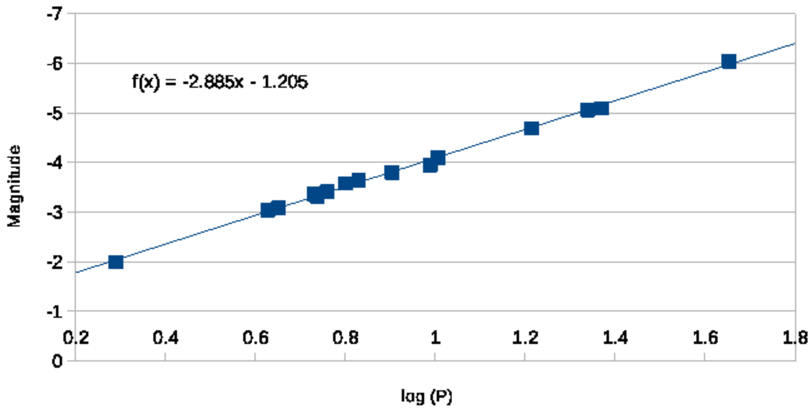


Figure 17.2 – Period - Luminosity graph of cepheids

Thus, the bigger star has mass of $0.61M_\odot$ and radius of $0.841R_\odot$. The smaller star has mass of $0.45M_\odot$ and radius of $0.607R_\odot$.

17.2 Data Analysis

1. (a) The plot can be seen in figure 17.2.

(b) The best fit can be found by,

$$\sum_{i=1}^{n} x_i = 13.90 \tag{17.105}$$

$$\left(\sum_{i=1}^{n} x_i\right)^2 = 193.21$$

$$\sum_{i=1}^{n} x_i^2 = 14.57 \tag{17.106}$$

$$\sum_{i=1}^{n} y_i = -58.18 \tag{17.107}$$

$$\sum_{i=1}^{n} x_i y_i = -58.78 \tag{17.108}$$

$$\beta = \frac{\sum_{i=1}^{n} x_i y_i - \frac{1}{n} \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2}$$

$$= \frac{-58.78 - \frac{1}{15} \times (13.90) \times (-58.18)}{14.57 - \frac{1}{15} \times 193.21}$$

$$\beta = -2.88 \tag{17.109}$$

$$\langle M_v \rangle = -1.21 - 2.88(\log_{10} P_0) \tag{17.110}$$

(c) Approximate period of cepheid 1 is (48 ± 1) days. Thus, as per figure, 17.2, it will have absolute magnitude of about -6.06. Its observed mean magnitude and hence its distance (including uncertainty) is given by

$$\langle m \rangle = 25.30 \pm 0.1 \tag{17.111}$$

$$m = M + 5 \log d - 5 \tag{17.112}$$

$$d = 10^{\frac{(m-M+5)}{5}} \tag{17.113}$$

$$d_1 = 10^{\frac{(25.30+6.06+5)}{5}}$$

$$= 1.86 \times 10^7 pc$$

$$d_1 = 18.7 \; Mpc \tag{17.114}$$

For a simplified calculation of uncertainty, we will assume that the coefficients determined in the previous part have a high accuracy and primary source of error is observations of these two distant cepheids. Thus,

$$\frac{\Delta d}{d} = \frac{\Delta P}{P^2} + \frac{\Delta m}{m} \tag{17.115}$$

$$\Delta d = 18.6 \times \left(\frac{1}{48^2} + \frac{0.1}{25.3}\right)$$
$$= 0.1\ Mpc \tag{17.116}$$

Similarly, for the second cepheid,

$$p = 23 \pm 1\ days \tag{17.117}$$
$$\therefore M_2 = -5.11 \tag{17.118}$$
$$\langle m \rangle = 26.30 \pm 0.1 \tag{17.119}$$
$$d_2 = 10^{\frac{(26.30+5.11+5)}{5}}$$
$$d_2 = (19.1 \pm 0.1)\ Mpc \tag{17.120}$$

(d) Assuming both the cepheids lie along the same of line of sight, we note even after inclding error margins, they are separated by atleast 200 kpc. This distance is much bigger than size of a typical spiral galaxy [1]. Thus, prima facie it is unlikely that both the cepheids lie in the same galaxy.
EA: In reality, both the cepheids used here are from the same galaxy in the Virgo cluster. Light from the second cepheid is heavily extincted due to interstellar matter in that galaxy. That is the prime reason for variation in the distance estimate.

2. The errors in the last column of table 7.2 are merely indicative of the fact that observations are not exact. Including them in the calculations will unnecessarlily complicate the solution.

   (a) The graph shows that the primary and secondary minima are almost similar. Thus, the reported minima can be either primary or secondary. Further, from the graph we note that the secondary minima happens exactly at the phase 0.5. Next we note that there are some pairs of minima which are reported for the same night. As observations in the same night are continuous we can assume they are consecutive minima. One should remember that JD changes at noon UT but civil date changes at midnight UT. Hence consecutive minima during same night may appear on different civil dates but will be on same JD. We find five pairs of consecutive minima in table 7.2 as,

1. Diameter of the milky way is 30 kpc.

Sr.	between	difference (days)
1	1 and 2	0.1367 ± 0.0006
2	3 and 4	0.1372 ± 0.0010
3	5 and 6	0.1364 ± 0.0020
4	10 and 11	0.1373 ± 0.0005
5	11 and 12	0.1359 ± 0.0005

Average of these values gives the initial estimate of the half-period of V 1107 CAS as 0.1367 days. Hence the initial estimate of the period of V 1107 CAS as 0.2734 days.

Coincidently, difference between the 1st and 2nd observations is the same as the average. However, it is a sheer chance event and it is necessary to take average of all five pairs. It is also important to get all 5 pairs in the average. If you miss one of the pairs, your average will be little farther from the true period. The small errror in each period will keep accumulating and final answers will be affected.

(b) We can assume first reading to be M_0. Thus, $M_0 = 2454092.4111$. Now, given the initial period (P), we can count approximately how many periods have elpased since the initial moment (E). This gives us "expected" or "calculated" instance of the minima. We can then compare it with the observed instance of minima and find the difference (O-C). This difference should be plotted against the number of epochs to get the "O-C" graph.

E	M_{cal}	M_{obs}	$M_{obs} - M_{cal}$
0	2454092.4111	2454092.4111	0.0000
0.5	2454092.5478	2454092.5478	0.0000
1005.5	2454367.3148	2454367.3284	0.0136
1006	2454367.4515	2454367.4656	0.0141
1083	2454388.5033	2454388.5175	0.0142
1083.5	2454388.6400	2454388.6539	0.0139
2240	2454704.8271	2454704.8561	0.0290
2502	2454776.4579	2454776.4901	0.0322
2717	2454835.2389	2454835.2734	0.0345
2761	2454847.2685	2454847.3039	0.0354
2761.5	2454847.4052	2454847.4412	0.0360
2762	2454847.5419	2454847.5771	0.0352

(c) "O-C" diagram shows difference between observed and calculated moments of minima. The difference is present because our initial estimate of P and M_0 may not be exact. The linear fit to all points

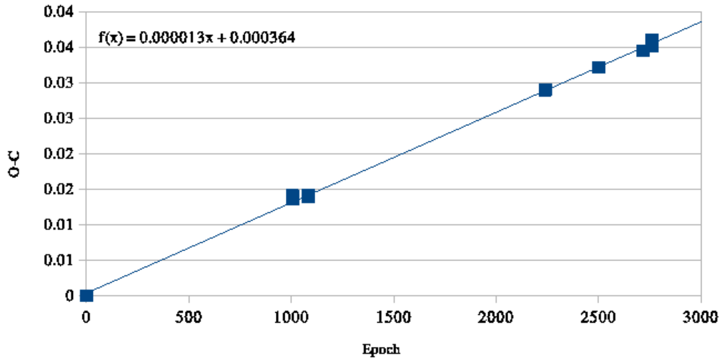


Figure 17.3 – O-C diagram for V1107 Cas

in the graph will help us eliminate these errors. The y-intercept of this graph will give you error in M_0. To get error in initial period, realise that slope of this graph is nothing but average (per period) difference between "observed" and "calculated" moments. This is exactly the error in initial period.

From the graph, the slope is 0.000013 and y-intercept is 0.000364. Thus corrected period and initial moments will be,

$$P = 0.2734 + 0.000013$$
$$\therefore P = 0.2734 \text{ days} \quad (17.121)$$
$$M_0 = 2454092.4111 + 0.000364$$
$$\therefore M_0 = 2454092.4115 \text{ HJD} \quad (17.122)$$

However, we will retain the additional decimal digits for calculations in next part.

(d) To jump to 1st September 2011, we use the fact that we know HJD for 23rd September 2007. Each Julian year will be exactly 365.25 days long. Thus, Julian Date for 1st - 2nd September 2011, midnight UT will be,

$$JD_{obs} = 2454367.5 + 365.25 \times 4 - 22 \quad (17.123)$$
$$= 2455806.5HJD \quad (17.124)$$

Thus, we are looking for minima between HJD 2455806.2917 and HJD 2455806.5833. Using corrected period ($P = 0.273413$)and initial moment ($M_0 = 2454092.411464$) we find that there are three instances of minima in this window. They are,

$$
\begin{array}{lclcl}
1^{\text{st}} \text{ minima} & = & 2455806.3009 \text{ HJD} & = & 19^h\ 13^m \text{ UT} \\
2^{\text{nd}} \text{ minima} & = & 2455806.4376 \text{ HJD} & = & 22^h\ 30^m \text{ UT} \\
3^{\text{rd}} \text{ minima} & = & 2455806.5743 \text{ HJD} & = & 1^h\ 47^m \text{ UT}
\end{array}
$$

Chapter 18

Solutions: Galactic Astrophysics

18.1 Theory

1. The difference in frequencies is due to the relativistic Doppler shift. Since the observed frequency of emission from the gas cloud is higher than the laboratory frequency ν_0, the gas cloud must be approaching the observer.

$$\nu = \nu_0 \sqrt{\frac{c+v}{c-v}} \tag{18.1}$$

$$\begin{aligned} \frac{v}{c} &= \frac{\left(\frac{\nu}{\nu_0}\right)^2 - 1}{\left(\frac{\nu}{\nu_0}\right)^2 + 1} \\ &= \frac{(1.000876)^2 - 1}{(1.000876)^2 + 1} \\ &= \frac{0.001752379}{2.001752379} = 0.0008754224 \\ v &= 0.0008754224 \times 2.99792458 \times 10^8 \\ &= 2.62426 \times 10^5 m \\ v &= 262.426 \;\; km.s^{-1} \end{aligned} \tag{18.2}$$

2. Rest frequency $\nu_{21} = 1420.41$ MHz.
Detected frequency $\nu = 1421.23$ MHz

$$\Delta\nu = \nu_{21} - \nu = 1420.41 - 1421.23 \tag{18.3}$$
$$= -0.82 MHz \tag{18.4}$$
$$v = \frac{\triangle\ \nu}{\nu_{21}} c = \frac{-0.82}{1420.41} \times 3 \times 10^8 \tag{18.5}$$
$$= -1.73 \times 10^5 m/s \tag{18.6}$$

Since $\nu > \nu_{21}$, then the cloud is moving toward us.
If M is mass of black hole, v is the speed of the cloud and R is the orbital radius of cloud, then

$$v^2 = \frac{GM}{R} \tag{18.7}$$
$$\therefore M = \frac{v^2 R}{G} \tag{18.8}$$
$$= \frac{(1.73 \times 10^5)^2 \times 0.2 \times 3.0856 \times 10^{16}}{6.672 \times 10^{-11}} \tag{18.9}$$
$$= 2.77 \times 10^{36} Kg = \frac{2.77 \times 10^{36}}{1.989 \times 10^{30}} M_\odot \tag{18.10}$$
$$= 1.4 \times 10^6 M_\odot \tag{18.11}$$

Thus, mass of the black hole is 1.4 billion solar masses and the cloud is coming towards us with velocity of 173 km/s.

3. The vertical density profile will be,

$$\rho(h) = \rho_0 e^{-h/h_0} \tag{18.12}$$

where $\rho(h)$ = density at a distance h from the mid plane and h_0 is the scale height.

$$\rho(h) = \rho_0 e^{-h/h_0} \tag{18.13}$$
$$\rho(0.5) = \rho_0 e^{-0.5/0.3}$$
$$= 0.189\rho_0 \tag{18.14}$$
$$\rho(1.5) = \rho_0 e^{-1.5/0.3}$$
$$= 0.0064\rho_0 \tag{18.15}$$
$$\rho(0.5) = 28\rho(1.5) \tag{18.16}$$

4. In the figure 18.1, S is the location of the Sun, C is the location of the galactic centre and P is possible location of the star. Let $R_\odot$ be the

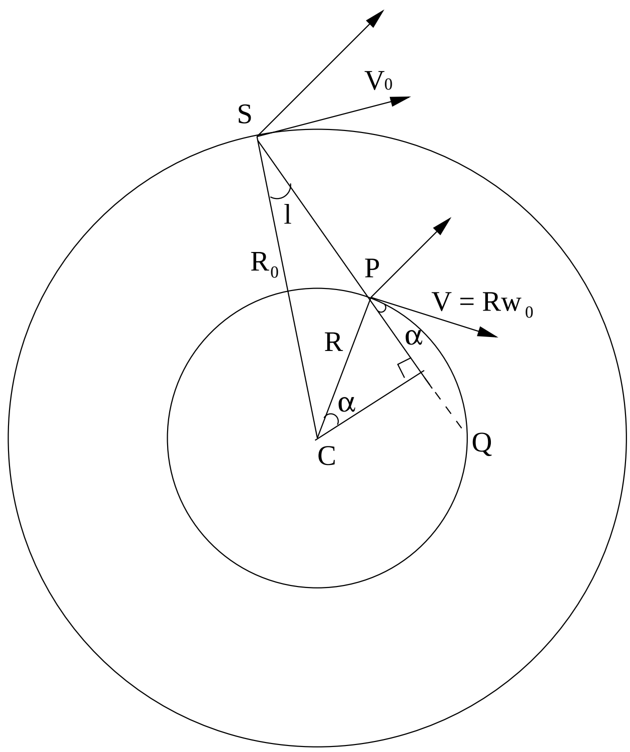


Figure 18.1 – Galactic position of the star as seen from galactic south pole

distance of the Sun from the centre of the Galaxy. Let the same for the star be denoted by R. Thus, Next, the radial velocity of star with respect to the Sun is

$$V_r = V_{st}\cos\alpha - V_\odot \sin l \quad (18.17)$$
$$= V_0(\cos\alpha - \sin l) \quad (18.18)$$

In the triangle, △ SCP,

$$\measuredangle CPS = 180 - (90 - \alpha) \quad (18.19)$$
$$= 90 + \alpha \quad (18.20)$$
$$\therefore \frac{\sin l}{R} = \frac{\sin(90+\alpha)}{R_\odot} \quad (18.21)$$
$$\cos\alpha = \frac{R_\odot}{R}\sin l \quad (18.22)$$
$$V_r = V_0(\frac{R_0}{R} - 1)\sin l \quad (18.23)$$
$$V_r R = V_0 R_0 \sin l - V_0 R \sin l \quad (18.24)$$
$$R = \frac{V_0 \sin l}{V_r + V_0 \sin l} R_0 \quad (18.25)$$
$$R = \frac{250\sin 15°}{100 + 250\sin 15°} \times 8.3 \quad (18.26)$$
$$R = 3.1 kpc \quad (18.27)$$

Note this distance of 3.1 kpc is the distance to point P from the Sun. However, there is another possible solution which is the star located at point Q. One may argue that the radial velocity given in the problem implicitly includes positive sign and hence Q may be ignored.

5. (a) We find maximum distance as, Relation between the apparent and absolute magnitude is given by

$$m = M + 5\log d - 5 \quad (18.28)$$
$$18 = -0.2 + 5\log d - 5$$
$$\therefore d = 10^{4.64}$$
$$= 4.37 \times 10^4 \ pc$$
$$d = 43.7 \ kpc \quad (18.29)$$

(b) Adding the term for the extinction, will make the stars dimmer.

Thus, the new expression will be,

$$m = M + 5\log\left(\frac{1000x}{10}\right) + 0.7x \quad (18.30)$$

$$= M + 10 + 5\log x + 0.7x \quad (18.31)$$

$$8.2 = 5\log x + 0.7x \quad (18.32)$$

$$\text{Let } e(x) = 5\log x + 0.7x - 8.2 \quad (18.33)$$

where x is given in terms of kilo parsec. As it is not possible to solve this equation analytically, we try different values of x.

x	5.0	6.0	6.5	6.2	6.1
$e(x)$	-1.205	-0.109	0.415	0.102	-0.003

Thus, the distanc reduces to approximately 6.1 kpc.

(c) For a solid angle Ω, the number of observed red giant stars at the distance in the range of d and $(d + \Delta d)$ is given by

$$\Delta N = \text{volume } \times \text{number density } \times \text{ fraction of red giant stars}$$

$$= \Omega x^2 \Delta x f n(r) \quad (18.34)$$

$$\therefore \frac{\Delta N}{\Delta x} = \Omega x^2 f n(r) \quad (18.35)$$

$$m = M + 5\log x + 10 \quad (18.36)$$

$$m + \Delta M = M + 5\log(x + \Delta x) + 10 \quad (18.37)$$

$$\therefore \Delta m = 5\log\left(\frac{x + \Delta x}{x}\right) \quad (18.38)$$

$$= 5\log\left(1 + \frac{\Delta x}{x}\right)$$

$$\Delta m = \frac{5}{\ln 10}\left(\frac{\Delta x}{x}\right) \quad (18.39)$$

$$\frac{\Delta x}{\Delta m} = \frac{x\ln 10}{5} \quad (18.40)$$

Finally, combining equations 18.35 and 18.40,

$$\frac{\Delta N}{\Delta m} = \frac{\Delta N}{\Delta x} \times \frac{\Delta x}{\Delta m} \quad (18.41)$$

$$= \Omega x^2 f n(r) \times \frac{x\ln 10}{5} \quad (18.42)$$

$$= \frac{n_0 f \ln 10}{5}\Omega x^3 e^{-\left(\frac{r-R_0}{R_d}\right)} \quad (18.43)$$

We have two distance variables, x and r. They can be related to each other by,

$$x = R_0 - r \qquad \text{for } x < R_0$$
$$x = R_0 + r \qquad \text{for } x > R_0$$

Finally we write x in terms of apparent magnitude.

$$x = 10^{\frac{m-9.78}{5}} \tag{18.44}$$

For $x < R_0$

$$\frac{\Delta N}{\Delta m} = \frac{n_0 f \ln 10}{5} \Omega x^3 e^{\left(\frac{x}{R_d}\right)} \tag{18.45}$$

For $43.7 > d > R_0$

$$\frac{\Delta N}{\Delta m} = \frac{n_0 f \ln 10}{5} \Omega x^3 e^{\left(\frac{(2R_0 - x)}{R_d}\right)} \tag{18.46}$$

6. Since the galaxy is edge-on, there will be no correction for inclination.
   (a) We draw first line 1st line going through points (0,0) and (16,214). The scales on axes are kly and km/s respectively. Second line can be drwan through (16,214) and (60, 200). Thus, the function becomes,

$$V(D) = 13.375D \qquad 0 < D < 16$$
$$= 214 - \frac{14}{45}D \qquad D > 16 \tag{18.47}$$

   where V is in km/s and D in kly.

   (b) For the spiral arms to take one turn around the galaxy, we note that angular velocity of the spiral wave is the difference between angular velocities of the fastest and the slowest points of the pressure wave.

$$\omega_{wave}(D) = \frac{1}{2}\omega_{mass}(D) = \frac{V(D)}{2D} \tag{18.48}$$
$$\omega_{spiral} = \omega_{wave}^{max} - \omega_{wave}^{min} \tag{18.49}$$
$$\omega_{spiral} = \frac{1}{2}\left[\left(\frac{214}{16} - \frac{14}{45}\right) - \left(\frac{214}{80} - \frac{14}{45}\right)\right]$$
$$= 1.78 \times 10^{-8} rad/yr \tag{18.50}$$
$$P_{spiral} = \frac{2\pi}{\omega_{rel}} \tag{18.51}$$
$$= 3.52 \times 10^{8}\ years \tag{18.52}$$

(c) The maximum velocity difference will be given by,

$$\Delta V = 2V_{max}(D) = 2 \times 230km/s \quad (18.53)$$
$$\Delta V = 460km/s \quad (18.54)$$
$$L = 0.317 \times L_{\odot} \times (460)^4 \quad (18.55)$$
$$L = 1.42 \times 10^{10} L_{\odot} \quad (18.56)$$
$$M_{sun} - M_{galaxy} = 2.5\log(1.42 \times 10^{10})$$
$$M_{galaxy} = 4.72 - 25.38$$
$$M_{galaxy} = -20.66 \quad (18.57)$$
$$m_{galaxy} = 8.5 \quad (18.58)$$
$$m - M = 5\log d - 5 \quad (18.59)$$
$$\log d = \frac{m - M}{5} + 1 = \frac{8.5 + 20.66}{5} + 1$$
$$d = 10^{6.832}$$
$$= 6.80Mpc \quad (18.60)$$

(d) As given in the question, the final observed wavelengths will include effect of Hubble recession velocity.

$$V_H = H_0 d \quad (18.61)$$
$$= 67.8 \times 6.8$$
$$= 460km/s \quad (18.62)$$

Center of the galaxy recedes with this speed.

$$V = V_H \pm V_{max} \quad (18.63)$$
$$= 460 \pm 230km/s$$
$$\frac{\Delta\lambda}{\lambda_0} = \frac{v}{c} \quad (18.64)$$
$$\lambda = \lambda_0 \left(1 + \frac{V_H \pm V_{max}}{c}\right)$$
$$\lambda_{max} = 656.28 \times (\frac{6.9 \times 10^5}{3 \times 10^8})$$
$$\lambda_{max} = 657.79nm \quad (18.65)$$
$$\lambda_{min} = 656.78nm \quad (18.66)$$

Note that both λ_{max} and λ_{min} are greater than λ_0.

(e) At a distance of 30 kpc, velocity is roughly equal to 225 km/s. v = 220 km/s

$$\frac{v^2}{R} = \frac{GM}{R^2} \tag{18.67}$$

$$\begin{aligned} M_{dynamic} &= \frac{V^2 R}{G} \\ &= \frac{(2.25 \times 10^5)^2 \times 3 \times 10^4 \times 3.0856 \times 10^{16}}{6.672 \times 10^{-11}} \\ M_{dynamic} &= 7.02 \times 10^{41}\ Kg \qquad (18.68) \\ &= 3.53 \times 10^{11} M_\odot \qquad (18.69) \end{aligned}$$

(f) Let there be n stars of $M_\odot$ mass.

$$\begin{aligned} nM_\odot &= \frac{M_{bary}}{3} \qquad (18.70) \\ &= \frac{M_{dynamic}}{3} \times \frac{4}{(22+4)} \\ \therefore n &= \frac{2M_{dynamic}}{39 M_\odot} \qquad (18.71) \\ &= \frac{23.53 \times 10^{11}}{39} \\ n &= 1.8 \times 10^{10} \qquad (18.72) \end{aligned}$$

Note that we are estimating stellar mass only till 30 kpc but the galaxy is extended much further.

7. Let the distance to the galaxy be d (pc) and let physical seperation of two objects r AU apart within this galaxy correspond to $\theta = 1''$. Let the total number of stars in area of 1 arcsec^2 be N. It is important to note that as you go further from the galaxy, the light intensity goes down as $1/d^2$. But at the same time, the angular area of the galaxy will also decrease as $1/d^2$. Thus, the surface brightness of the galaxy does not depend on distance to the galaxy. As all stars are sun-like,

magnitude of one star

$$d = r\theta \tag{18.73}$$

$$1\ (arcsec^2) = \left(\frac{r}{d}\right)^2 \tag{18.74}$$

$$m_{star} = M_{\odot} + 5\log d - 5$$

$$= 4.82 + 5\log d - 5$$

$$m_{star} = 5\log d - 0.18 \tag{18.75}$$

$$m_N = m_{star} - 2.5\log N \tag{18.76}$$

$$18 = 5\log d - 0.25 - 2.5\log N$$

$$18.25 = 2.5\log\frac{d^2}{N} \tag{18.77}$$

$$\frac{d^2}{N} = 1.995\times 10^7 \tag{18.78}$$

Now each of the star occupies cross-section of $\pi R_{\odot}^2$. Thus, the fraction will be,

$$f = \frac{A(stars)}{A(total)} = \frac{N\pi R_{\odot}^2}{r^2\theta^2\times(1AU)^2} \tag{18.79}$$

$$= \frac{NR_{\odot}^2\pi}{d^2(1AU)^2}$$

$$= \frac{\pi\times(6.955\times 10^8)^2}{1.995\times 10^7\times(1.496\times 10^11)^2}$$

$$\therefore f = 3.4\times 10^{-12} \tag{18.80}$$

18.2 Data Analysis

1. First we estimate physical length of galaxy per pixel (a'). Given that the distance to the galaxy (d) is 40 Mpc, 1 arcsec will subtends an angle 4 $\times 10^7$ AU at that distance.

$$a' = d\theta \tag{18.81}$$

$$1A.U. = (1pc)\times 1'' \tag{18.82}$$

$$a' = 40\times 10^6\times 0.82\ \text{A.U.}$$

$$= \frac{32.8\times 10^6}{3.262\times 6.324\times 10^4}\ pc$$

$$a' = 159pc \tag{18.83}$$

For the scale on x-axis we note that the two wavelengths which are separated by 2 nm are separated by 20 pixels on the spectrum. Thus each pixel corresponds to 0.1 nm.

Most spiral galaxies have a nearly circular cross section. But the image of NGC 7083 shows that it is somewhat elliptical. This tells you that the plane of the galaxy is inclined at some angle θ to the line of sight. We may estimate θ by measuring the vertical and horizontal dimeters of the galaxy.

$$\sin\theta = \frac{6.7}{13.7} = 0.5323 \tag{18.84}$$

$$\therefore \theta = 29^\circ.28 \tag{18.85}$$

$$\cos\theta = 0.8723 \tag{18.86}$$

Also, in this spectrum, we assume that it has been corrected for Hubble expansion and that the only reason for deviation of λ is rotation of the galaxy.

$$V_{obs} = V_{rot}\cos\theta \tag{18.87}$$

$$\frac{V_{obs}}{c} = \frac{\Delta\lambda}{\lambda} \tag{18.88}$$

$$\therefore V_{rot} = \frac{\Delta\lambda c}{\lambda\cos\theta} \tag{18.89}$$

It can be seen that away from the spherical bulge, of radius r_0, at the centre, the $\Delta\lambda$ has almost constant value and hence V_{rot} is almost constant.

For $d < r_0, \Delta\lambda$ increases nearly linearly with d. As velocity becomes constant about 7-8 pixels away from the central line, we can say

$$r_0 \approx (1.20 \pm 0.07)kpc \tag{18.90}$$

For λ_1 as well as λ_2:

$$|\Delta\lambda(d = +r_0) - \Delta\lambda(d = -r_0)| = 0.9 \pm 0.1nm \tag{18.91}$$

Note that doppler shift for both wavelengths is verified independently. It turns out for the given observational accuracy, the variation in both wavelengths is the same.

$$\Delta\lambda(d = \pm r_0) = -0.45 \pm 0.05nm \tag{18.92}$$

We can find V_{rot} for both wavelengths and take average.

$$V_{rot,1} = \frac{(0.45 \pm 0.05) \times 3 \times 10^8}{656.4 \times 0.8723}$$
$$= (236 \pm 26)\ Km/s \tag{18.93}$$
$$V_{rot,2} = \frac{(0.45 \pm 0.05) \times 3 \times 10^8}{658.4 \times 0.8723}$$
$$= (235 \pm 26)\ Km/s \tag{18.94}$$
$$V_{rot} = (2.4 \pm 0.3) \times 10^5\ m/s \tag{18.95}$$

If mass of the central bulge of the galaxy is M, then for a test mass m outside bulge,

$$\frac{GMm}{(r_0)^2} = \frac{mV_{rot}^2}{r_0} \tag{18.96}$$
$$M = \frac{V_{rot}^2 r_0}{G}$$
$$= \frac{(2.4 \times 10^5)^2 \times 1.20 \times 3.086 \times 10^{19}}{6.672 \times 10^{-11}}$$
$$M = 1.6 \times 10^{10} M_\odot \tag{18.97}$$
$$\frac{\Delta M}{M} = \frac{2\Delta V_{rot}}{V_{rot}} + \frac{\Delta r_0}{r_0} \tag{18.98}$$
$$= \frac{2 \times 0.3}{2.4} + \frac{0.07}{1.20}$$
$$\Delta M = 0.3 \times 10^{10} M_\odot \tag{18.99}$$
$$\therefore M = (1.6 \pm 0.3) \times 10^{10} M_\odot \tag{18.100}$$

EA: Note that there is also a small error due to rounding of constants G and $M_\odot$. But that is much smaller (one part in thousand) than other quantitites and it is hence ignored.

Chapter 19

Solutions: Extragalactic Astrophysics

19.1 Theory

1. By Wien's law,

$$\lambda_{max}T = constant = b$$

$$\lambda_{max} = \frac{b}{T} \tag{19.1}$$

$$\frac{\lambda_{observed}}{\lambda_{emitted}} = 1 + z \tag{19.2}$$

$$\lambda_{observed} = \lambda_{emitted}(1 + z)$$

$$\frac{b}{T_{observed}(1 + z)} = \frac{b(1 + z)}{T_{emitted}}$$

$$T_{emitted} = T_{observed}(1 + z) \tag{19.3}$$

$$= 2.73 \times (1 + 10)$$

$$T_{emitted} = 30.03 \approx 30K \tag{19.4}$$

2. Let recession velocity of the QSO be v, Then

$$\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} = 1+z \tag{19.5}$$

$$\frac{1+\frac{v}{c}}{1-\frac{v}{c}} = (1+z)^2$$

$$\therefore \frac{v}{c} = \frac{(1+z)^2-1}{(1+z)^2+1} \tag{19.6}$$

$$= \frac{(1.2)^2-1}{(1.2)^2+1}$$

$$= \frac{0.44}{2.44}$$

$$= 0.18 \tag{19.7}$$

$$v = H_0 D \tag{19.8}$$

$$\therefore D = \frac{v}{H_0} \tag{19.9}$$

$$= \frac{0.18 \times 3.00 \times 10^5}{68}$$

$$\approx 800\ Mpc \tag{19.10}$$

3. Key to solving this question is the fact that since dark energy is the energy of empty space, its energy density remains constant during expansion of the universe. If a_0 is scale factor today and a_z as the scale factor at the recombination epoch.

$$1+z = \frac{a_0}{a_z} \tag{19.11}$$

$$a_z = \frac{1}{1+z}a_0 = \frac{1}{1+1100}a_0 \tag{19.12}$$

$$= 0.00091a_0 \tag{19.13}$$

Matter densities will change as per size of the universe.

$$\rho_{DM_z} = \frac{\rho_{DM}}{a^3} \tag{19.14}$$

$$= \frac{2.4 \times 10^{-30}}{(9.1 \times 10^{-4})^3} = \frac{2.4 \times 10^{-30}}{7.5 \times 10^{-10}} \tag{19.15}$$

$$= 3.2 \times 10^{-21} g/cm^3 \tag{19.16}$$

$$\frac{\rho_{DM}}{\rho_{DE}} = \frac{3.2 \times 10^{-21}}{7.1 \times 10^{-30}} \tag{19.17}$$

$$= 4.5 \times 10^8 \tag{19.18}$$

4. By Tully - Fischer Relation,

$$M_B = -9.95 \log v_{max} + 3.15 \tag{19.19}$$

$$= -9.95 \log 324 + 3.15 \tag{19.20}$$

$$M_B = -21.83 \tag{19.21}$$

$$logR_{25} = -0.249 M_B - 4.00 \tag{19.22}$$

$$R_{25} = 27.27 kpc \tag{19.23}$$

$$\mu_{25} = \frac{R_{25} V^2}{G} \tag{19.24}$$

$$= \frac{27.27 \times 10^3 \times 3.0856 \times 10^{16} \times (3.24 \times 10^5)^2}{6.672 \times 10^{-11}} \tag{19.25}$$

$$= 1.324 \times 10^{42} kg \tag{19.26}$$

$$\mu_{25} = 6.66 \times 10^{11} M_\odot \tag{19.27}$$

Sun's absolute Magnitude in B-Band is

$$M_{B_\odot} = M_{V_\odot} + [B_\odot - V_\odot] \tag{19.28}$$

$$= 4.82 + 0.64 \tag{19.29}$$

$$= 5.46 \tag{19.30}$$

$$and M_B = -21.83 \tag{19.31}$$

$$M_B - M_{B_\odot} = -2.5 \log \left(\frac{L}{L_\odot} \right) \tag{19.32}$$

$$= -21.83 - 5.46 = -27.29 \tag{19.33}$$

$$L = 10^{\frac{-27.29}{-2.5}} L_\odot \tag{19.34}$$

$$L = 8.24 \times 10^{10} L_\odot \tag{19.35}$$

5. Note that Vega is standard zero magnitude star for Johnson UBV photometry. We know magnitude of the Sun is -26.7 and we know luminosity of the Sun. Combining this information,

$$m_{\odot} - m_{vega} = -2.5\log\left(\frac{F_{\odot}}{F_{vega}}\right) \tag{19.36}$$

$$-26.7 - 0 = 2.5\log\left(\frac{F_{vega}}{F_{\odot}}\right) \tag{19.37}$$

$$F_{vega} = 10^{\frac{-26.7}{2.5}} F_{\odot} = 10^{-10.68} F_{\odot} \tag{19.38}$$

$$= 2.09 \times 10^{-11} F_{\odot} \tag{19.39}$$

$$F_{SN} = 1.6 \times 10^{-7} F_{vega} \tag{19.40}$$

$$= 1.6 \times 10^{-7} \times 2.09 \times 10^{-11} F_{\odot} \tag{19.41}$$

$$F_{SN} = 3.34 \times 10^{-18} F_{\odot} \tag{19.42}$$

$$\frac{L_{SN}}{4\pi d_{SN}^2} = 3.34 \times 10^{-18} \frac{L_{\odot}}{4\pi d_{\odot}^2} \tag{19.43}$$

$$d_{SN}^2 = \frac{L_{SN}}{3.34 \times 10^{-18} L_{\odot}} \times d_{\odot}^2 \tag{19.44}$$

$$d_{SN} = \sqrt{\frac{5.8 \times 10^9}{3.34 \times 10^{-18}}} \times 1.496 \times 10^{11} \tag{19.45}$$

$$= 6.2 \times 10^{24} m \tag{19.46}$$

$$d_{SN} \approx 202 Mpc \tag{19.47}$$

If t_H is the Hubble time, then by combining Hubble's law and approximate relation between redshift and velocity (for low redshift),

$$t_H = \frac{1}{H_0} = \frac{d_{SN}}{v_{SN}} = \frac{d_{SN}}{cz} \tag{19.48}$$

$$= \frac{202 Mpc}{(1lyr\ /\ yr)(0.05)} \tag{19.49}$$

$$= 202 \times 10^6 \times 3.262 \times 20 \text{ years} \tag{19.50}$$

$$= 1.32 \times 10^{10} \text{ years} \tag{19.51}$$

$$t_H = 13.2 \text{ Gyrs} \tag{19.52}$$

You can note clever use of speed of light as one light year per year to get the answer directly in years. The Hubble time thus obtained is for a non-accelerating universe and based on observation of only one galaxy. Thus, we are not surprised by the fact that the answer is lower than actual age of the universe.

6. In a Flat Universe, total energy is zero. For a test particle at the edge of the universe, we can take R to be radius of visible universe and M as its total mass. Let ρ_c be the critical density of the universe.

$$KE = -PE \tag{19.53}$$
$$\frac{1}{2}v^2 = \frac{GM}{R} \tag{19.54}$$
$$\frac{1}{2}(H_0R)^2 = \frac{4\pi R^3\rho_c G}{3R} \tag{19.55}$$
$$\therefore \rho_c = \frac{3H_0^2}{8\pi G} \tag{19.56}$$
$$= \frac{3 \times (67.80 \times 10^3)^2}{8\pi \times 6.672 \times 10^{-11} \times (3.0856 \times 10^{22})^2}$$
$$\rho_c = 8.6378 \times 10^{-27}\ kgm^{-3} \tag{19.57}$$
$$m_{dark} = 0.25 m_{total}$$
$$\therefore \rho_{dark} = 0.25\rho_c \tag{19.58}$$
$$n_\nu = \frac{\rho_{dark}}{m_\nu} = \frac{0.25\rho_c}{10^{-5}m_e} \tag{19.59}$$
$$= \frac{0.25 \times 8.6378 \times 10^{-27}}{10^{-5} \times 9.1 \times 10^{-31}}$$
$$= 2.37 \times 10^8 m^{-3}$$
$$n_\nu \approx 2 \times 10^8 m^{-3} \tag{19.60}$$

Note that the final accuracy is rounded to a single digit due to approximation in the mass of neutrino.

7. If the assumed model of the universe is without dark energy, one cannot simply take present age of the universe as 13.7 Gyrs. One should go back to the definition of the Hubble constant and start from there. Let T, t and a be the temperature, age and scale factor of the Universe. We assume that variation in H is very small and it can be approximated

as a constant.

$$H_0 = \frac{1}{a}\frac{da}{dt} \approx \frac{1}{a}\frac{\Delta a}{\Delta t} \tag{19.61}$$

$$\frac{\Delta a}{\Delta t} = H_0 a \tag{19.62}$$

$$T \propto a^{-1} \tag{19.63}$$

$$\frac{\Delta T}{\Delta t} = -H_0 T_i \tag{19.64}$$

$$\begin{aligned}\Delta t &= -\frac{\Delta T}{H_0 T_i} \\ &= -\frac{(-0.1) \times 3.0856 \times 10^{22}}{67.80 \times 10^3 \times 2.73} \\ &= 1.667 \times 10^{16} s\end{aligned}$$

$$\Delta t = 528 Myr \tag{19.65}$$

Alternatively, one can solve this more rigourously using calculus. In a matter dominated universe,

$$\rho = \zeta a^{-3} \tag{19.66}$$

$$a \propto t^{\frac{2}{3}} \tag{19.67}$$

$$\Omega_0 = \frac{8\pi G\rho}{3H^2} = \frac{8\pi G}{3H^2}\frac{\zeta}{a^3} = 1 \tag{19.68}$$

Again using the definition of Hubble constant and find expression for

a by integration,

$$\begin{aligned}
\frac{da}{dt} &= Ha \\
\left(\frac{da}{dt}\right)^2 &= H^2a^2 = \frac{8\pi G\zeta}{3a} \\
\therefore \frac{da}{dt} &= \sqrt{\frac{8\pi G\zeta}{3a}} \qquad (19.69) \\
\therefore \int \sqrt{a}\ da &= \sqrt{\frac{8}{3}\pi G\zeta} \int dt \qquad (19.70) \\
\frac{2}{3}\left({a_f}^{\frac{3}{2}} - {a_i}^{\frac{3}{2}}\right) &= \sqrt{\frac{8}{3}\pi G\zeta}\Delta t \\
\frac{2}{3}\left(\left(\frac{a_f}{a_i}\right)^{\frac{3}{2}} - 1\right) &= \sqrt{\frac{8\pi G\zeta}{3a_i^3}}\Delta t \\
&= H_i\Delta t \\
\Delta t &= \frac{2}{3}\left(\left(\frac{a_f}{a_i}\right)^{\frac{3}{2}} - 1\right)\frac{1}{H_i} \qquad (19.71) \\
&= \frac{2}{3}\left(\left(\frac{T_i}{T_f}\right)^{\frac{3}{2}} - 1\right)\frac{1}{H_i} \\
&= \frac{2}{3}\left(\left(\frac{2.73}{2.63}\right)^{\frac{3}{2}} - 1\right)\frac{3.0856 \times 10^{22}}{67.80 \times 10^3} \\
&= 1.730 \times 10^{16}s \\
\Delta t &= 548Myr \qquad (19.72)
\end{aligned}$$

8.

$$
\begin{aligned}
1+z &= \frac{a_0}{a_z} = \left(\frac{t_0}{t_z}\right)^{\frac{2}{3}} && (19.73)\\
&= \frac{t_0}{(1+z)^{1.5}} = \frac{1.377\times 10^{10}}{(1+6.03)^{1.5}} \\
t &= 739\times 10^6 \ years && (19.74)\\
t_{form,max} &= t - 560\times 10^6 \\
&= 179\times 10^6 \ years && (19.75)\\
t_{form,min} &= 139\times 10^6 \ years && (19.76)\\
z_{form,min} &= \left(\frac{t_0}{t_f}\right)^{\frac{2}{3}} - 1 && (19.77)\\
&= \left(\frac{13770}{179}\right)^{\frac{2}{3}} - 1 \\
&= 17.1 && (19.78)\\
z_{form,max} &= 20.4 && (19.79)
\end{aligned}
$$

Epoch of star formation was from z = 17.3 to z = 20.8

9. In this question, it helps if you start with a general solution (i.e. $\beta \neq 0$) then converge to special case $\beta = 0$. Refer to figure 9.2. As ϕ, β, and θ are very very small angles, the point of bending is very very close to the lens and almost at distance D_L from the observer.

$$
\begin{aligned}
PS_1 &= PS + SS_1 \\
\theta D_S &= \beta D_S + (D_S - D_L)\phi && (19.80)\\
(\theta-\beta)D_S &= (D_S - D_L)\frac{4GM}{\xi c^2} \\
&= (D_S - D_L)\frac{4GM}{\theta D_L c^2} \\
\therefore \theta(\theta-\beta) &= \frac{4GM}{c^2}\frac{(D_S - D_L)}{D_L D_S} \\
\theta^2 - \beta\theta &= \frac{4GM}{c^2}\frac{D_S - D_L}{D_L D_S} && (19.81)
\end{aligned}
$$

(a) For a perfect alignment in which $\beta = 0$, we have $\theta = \pm\theta_E$, where

$$
\theta_E = \sqrt{\frac{4GM}{c^2}\frac{(D_S - D_L)}{D_L D_S}} \qquad (19.82)
$$

(b) For given parameters,

$$D_L = 50 - 10 = 40kpc \tag{19.83}$$

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{(D_S - D_L)}{D_L D_S}}$$

$$\theta_{E1} = \sqrt{\frac{4 \times 6.672 \times 10^{-11} \times 1.989 \times 10^{30} \times (50-40)}{(3 \times 10^8)^2 \times 50 \times 40 \times 3.0856 \times 10^{19}}}$$

$$= 9.8 \times 10^{-10} rad \tag{19.84}$$

$$\theta_{E1} = 2.0 \times 10^{-4} arcsec \tag{19.85}$$

(c) The resolution of the Hubble space telescope for mean optical wavelength is,

$$\theta_{Hubble} = \frac{1.22\lambda}{D} = 1.22 \times \frac{1.22 \times 5 \times 10^{-7}}{2.4}$$

$$= 2.5 \times 10^{-7} rad \tag{19.86}$$

$$\theta_{Hubble} > \theta_{E1} \tag{19.87}$$

Hence the Hubble telescope can not resolve this Einstein ring.

(d) The quadratic equation 19.81 has two distinct roots, namely,

$$\theta_1 = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \theta_E^2} \tag{19.88}$$

$$\theta_2 = \frac{\beta}{2} - \sqrt{\left(\frac{\beta}{2}\right)^2 + \theta_E^2} \tag{19.89}$$

This implies that there are two images for a single isolated source.

(e) Dividing the above equations by β

$$\theta_{1,2} = \frac{\beta}{2} \pm \sqrt{\left(\frac{\beta}{2}\right)^2 + \theta_E^2}$$

$$\frac{\theta_{1,2}}{\beta} = \frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\theta_E}{\beta}\right)^2}$$

$$\frac{\theta_{1,2}}{\beta} = \frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{1}{\eta^2}}$$

$$= \frac{1}{2}\left(1 \pm \frac{\sqrt{\eta^2 + 4}}{\eta}\right) \tag{19.90}$$

(f) From the equation 19.81,

$$\begin{aligned}\theta^2 - \beta\theta - \theta_E^2 &= 0 \\ (\theta + \Delta\theta)^2 - (\beta + \Delta\beta)(\theta + \Delta\theta) - \theta_E^2 &= 0 \qquad (19.91) \\ (\theta^2 - \beta\theta - \theta_E^2) + 2\theta\Delta\theta - \beta\Delta\theta - \theta\Delta\beta &= 0 \\ (2\theta - \beta)\Delta\theta - \theta\Delta\beta &= 0 \qquad (19.92)\end{aligned}$$

Thus, we get,

$$\begin{aligned}\frac{\Delta\theta}{\Delta\beta} &= \frac{\theta}{2\theta - \beta} \qquad (19.93) \\ \left[\frac{\Delta\theta}{\Delta\beta}\right]_{\theta=\theta_{1,2}} &= \frac{\theta_{1,2}}{2\theta_{1,2} - \beta} \\ &= \frac{\theta_{1,2}/\beta}{2\theta_{1,2}/\beta - 1} \\ &= \frac{\left(\frac{1}{2}\left(1 \pm \frac{\sqrt{\eta^2+4}}{\eta}\right)\right)}{2\left(\frac{1}{2}\left(1 \pm \frac{\sqrt{\eta^2+4}}{\eta}\right)\right) - 1} \\ &= \frac{\eta \pm \sqrt{\eta^2+4}}{2(\eta \pm \sqrt{\eta^2+4} - \eta)} \\ \left[\frac{\Delta\theta}{\Delta\beta}\right]_{\theta=\theta_{1,2}} &= \frac{1}{2}\left(1 \pm \frac{\eta}{\sqrt{\eta^2+4}}\right) \qquad (19.94)\end{aligned}$$

10.

$$F(\lambda_B) = \frac{L_\lambda(\lambda_B)}{4\pi d_L^2} \qquad (19.95)$$

$$\lambda_{emitted} = \frac{\lambda_{observed}}{(1+z)} \qquad (19.96)$$

Let monochromatic flux of the galaxy be denoted by $S(\lambda)$, then for small wavelength interval $\Delta\lambda$,

$$S(\lambda_{ob})\Delta\lambda_{ob} = \frac{L_\lambda(\lambda_{emitted})}{4\pi d_L^2}\Delta\lambda_{emitted}$$

$$= \frac{1}{4\pi d_L^2} L_\lambda\left(\frac{\lambda_{emitted}}{1+z}\right)\frac{\Delta\lambda_{ob}}{1+z} \tag{19.97}$$

$$S_{10}(\lambda) = \frac{L_\lambda(\lambda)}{4\pi d_{10}^2}$$

$$\therefore \frac{S(\lambda)}{S_{10}(\lambda)} = \left(\frac{d_{10}}{d_L}\right)^2 \frac{L_\lambda\left(\frac{\lambda}{1+z}\right)}{L_\lambda(\lambda)}\frac{1}{1+z} \tag{19.98}$$

$$= \left(\frac{d_{10}}{d_L}\right)^2 \left(\frac{1}{1+z}\right)^4 \frac{1}{1+z}$$

$$\frac{S(\lambda)}{S_{10}(\lambda)} = \left(\frac{d_{10}}{d_L}\right)^2 \left(\frac{1}{1+z}\right)^5 \tag{19.99}$$

Here we have used SED approximation, which is justified by the fact that λ_{min} = 250nm for the approximation to be valid and the lower end of B band filter is at 445 - 47 = 398 nm. Thus,

$$\lambda_{SED,min}(1+z) < \lambda_{B,min} \tag{19.100}$$

This, means λ is in the range of SED approximation. Now evaluting,

$$m_B - M_B = -2.5\log\left(\frac{S(\lambda)}{S_{10}(\lambda)}\right) \tag{19.101}$$

$$= -2.5\log\left(\frac{d_{10}}{d_L}\right)^2 - 2.5\log(1+z)^{-5}$$

$$\therefore M_B = m_B - 5\log\left(\frac{d_L}{d_{10}}\right) - 12.5\log(1+z) \tag{19.102}$$

$$= 20.40 - 5\log\left(\frac{2754\times10^6}{10}\right) - 12.5\log(1.5)$$

$$= -2.5\log\frac{f(\lambda_B)4\pi(10pc)^2}{L(\lambda_B)} \tag{19.103}$$

$$= -24.00 \tag{19.104}$$

As normal elliptical galaxies are fainter than this by atleast 2 magnitudes, we conclude that this galaxy is not a member of this cluster but is a forground object.

11. (a) Let θ_0 be the horizon distance and let X_r be the physical distance between two points, which are on mutual horizon at recombination epoch. It would be tempting to relate the two by simple

$$X_r = d\theta_0 = c(t_0 - t_r)\theta_0 \tag{19.105}$$

where d is the distance to CMB horizon. However, it must be remembered that the space itself is continuously expanding. Hence the light from CMB has in fact travelled much shorter distance. The physical distance X_r is the real horizon distance at that epoch.

$$X_r = c(t_0 - t_r)\theta_0 \times \frac{a_r}{a_0} \tag{19.106}$$

$$ct_r = c(t_0 - t_r)\theta_0 \times \frac{T_0}{T_r} \tag{19.107}$$

$$\therefore \theta_0 = \frac{T_r t_r}{T_0(t_0 - t_r)} \approx \frac{T_r t_r}{T_0 t_0} \tag{19.108}$$

$$\approx \frac{3000 \times 3 \times 10^5}{3 \times 1.5 \times 10^{10}} \tag{19.109}$$

$$\theta_0 \approx 0.02 rad = 1.15^\circ \tag{19.110}$$

(b) No. It is not possible. As the given separation is more than θ_0.

(c) We note that magnitude of current age of the universe can be also interpreted as the size of present universe in light years. Let S denote size of the universe at a given epoch.

$$S_0 = 1.5 \times 10^{10} \times 9.46 \times 10^{15} m \tag{19.111}$$

$$S_r = \frac{a_r}{a_0} S_0 = \left(\frac{t_r}{t_0}\right)^{2/3} S_0 \tag{19.112}$$

$$S_i = \frac{a_i}{a_r} S_r = \left(\frac{t_i}{t_r}\right)^{1/2} S_r \tag{19.113}$$

$$= \left(\frac{t_i}{t_r}\right)^{1/2} \left(\frac{t_r}{t_0}\right)^{2/3} S_0 \tag{19.114}$$

$$= \left(\frac{10^{-32}}{3 \times 10^5 \times 3.1557 \times 10^7}\right)^{1/2} \left(\frac{3 \times 10^5}{1.5 \times 10^{10}}\right)^{2/3} S_0 \tag{19.115}$$

$$= 3.25 \times 10^{-23} \times 7.37 \times 10^{-4} S_0 \tag{19.116}$$

$$= 2.40 \times 10^{-26} S_0 \tag{19.117}$$

$$= 2.40 \times 10^{-26} \times 1.5 \times 10^{10} \times 9.46 \times 10^{15} \tag{19.118}$$

$$\approx 3.40 mt \tag{19.119}$$

Thus, the size of the universe was just 3.4 metres at the end of the inflation period.

19.2 Data Analysis

1. **Virgo Cluster**

   (a) Simply substitute the values and find answers.

$$\begin{aligned} d_{avg} &= \frac{\sum_{i=1}^{n} \frac{d_i}{s_i^2}}{\sum_{i=1}^{n} s_i^2} \\ &= \frac{55.943}{3.5485} \\ &= 15.765Mpc \end{aligned} \tag{19.120}$$

   (b) The r.m.s. uncertainity will be given by:

$$s_a = \sqrt{\frac{1}{\sum_{i=1}^{n} s_i^2}} \tag{19.121}$$

$$= \sqrt{\frac{1}{3.5485}} Mpc = 0.53Mpc$$

$$d_{Virgo} = 15.8 \pm .5Mpc \tag{19.122}$$

   (c) The Hubble constant can be found by the Hubble's law

$$H_0 = \frac{\upsilon}{d_{avg}} = \frac{1136km/s}{15.765Mpc} \tag{19.123}$$

$$H_0 = 72.058km/sec/Mpc \tag{19.124}$$

$$s_H = \left(\frac{s_d}{d_{avg}} + \frac{\triangle\,\upsilon}{\upsilon}\right) H_0 \tag{19.125}$$

$$= \left(\frac{0.53}{15.765} + \frac{1}{1136}\right) H_0 \times 72.058$$

$$= 0.0345 \times 72.058 \tag{19.126}$$

$$s_H = 2.49km/sec/Mpc \tag{19.127}$$

$$\therefore H_0 = 72 \pm 2km/sec/Mpc \tag{19.128}$$

   (d) The Hubble time is simply inverse of the Hubble constant. Thus the percentage error remains the same as 19.126.

$$H_0^{-1} = 72.058^-1 \times 3.086 \times 10^{16} \times 10^6/10^3$$
$$= 4.28 \times 10^{17} sec$$
$$T_H = 13.57 Gyr \tag{19.129}$$
$$e_T = 13.57 \times 0.0345$$
$$e_T = 0.47 Gyr \tag{19.130}$$
$$\therefore T_H = 13.6 \pm 0.5 Gyr \tag{19.131}$$

Chapter 20

Solutions: Night Sky Observation

20.1 General Night Sky

1. One is expected to remember stars which are reasonably close to celestial equator and then sweep the laser pointer making an arc passing through the stars. Celestial equator can be remembered something lie this:
   Mintaka (δ Ori), a couple of degrees south of Procyon (α CMi), similarly slight south of Hydra's head, Porrima (γ Vir), slight south of Kelb Al Rai (β Oph), Sadalmalik (α Aqr), slight south of southern fish of Pisces and south of head of Cetus.
2. The vernal equinox is located slightly south of the circlet of Pisces. There is no bright star near its exact location hence it can be identified only through practice.
3. Question self-explanatory.
4. α - And
5. Question is self-explanatory.
6. Question is self-explanatory.
7. The constellations shown were: Cygnus (Cyg), Corona Borealis (Crb), Pisces Australis (Psa), Delphinus (Del), Saggitarius (Sgr)
8. The equatorial constellations visible in the projected sky were: Virgo (Vir), Serpens (Ser), Ophiuchus (Oph), Aquila (Aql), Aquarius (Aqr), Pisces (Psc) and Cetus (Cet)
9. Figure 20.1 shows the missing stars put back at their positions.

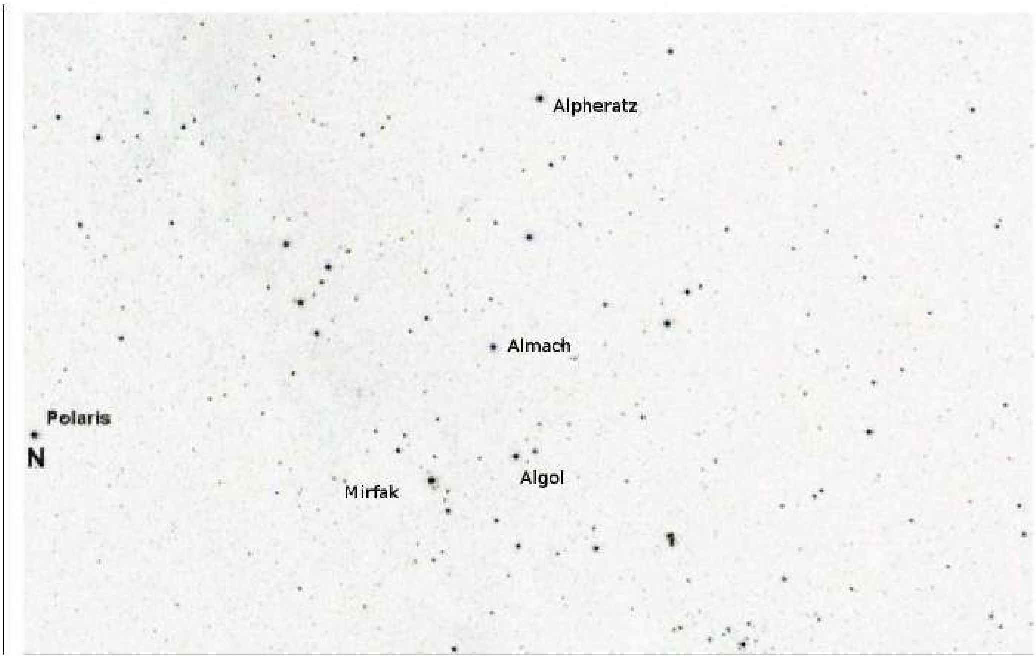


Figure 20.1 – Sky Map for Scorpio and surrounding regions

Table 20.1 – Angular Distance

Pairs of stars	**Angular Distance (degrees)**
1 (Errai) and 2 (Alfirk)	$\approx 11°$
1 (Errai) and 3 (Alderamin)	$\approx 18°$

10. The students were expected to put the missing stars in the proper location and associate correct name with the correct position. Similarly they were expected to draw boundaries of constellations by remembering that all constellations boundries are along constant R.A. or constant Dec.

11. Answers are given on map 20.2.

20.2 Magnitudes and Angular Sizes

1. Mirfak (α- Per). (I07 - O05 - A)

2. 3.7 - 3.8 (acutal answer 3.75).

3. Answers given table 20.1 and table 20.2.

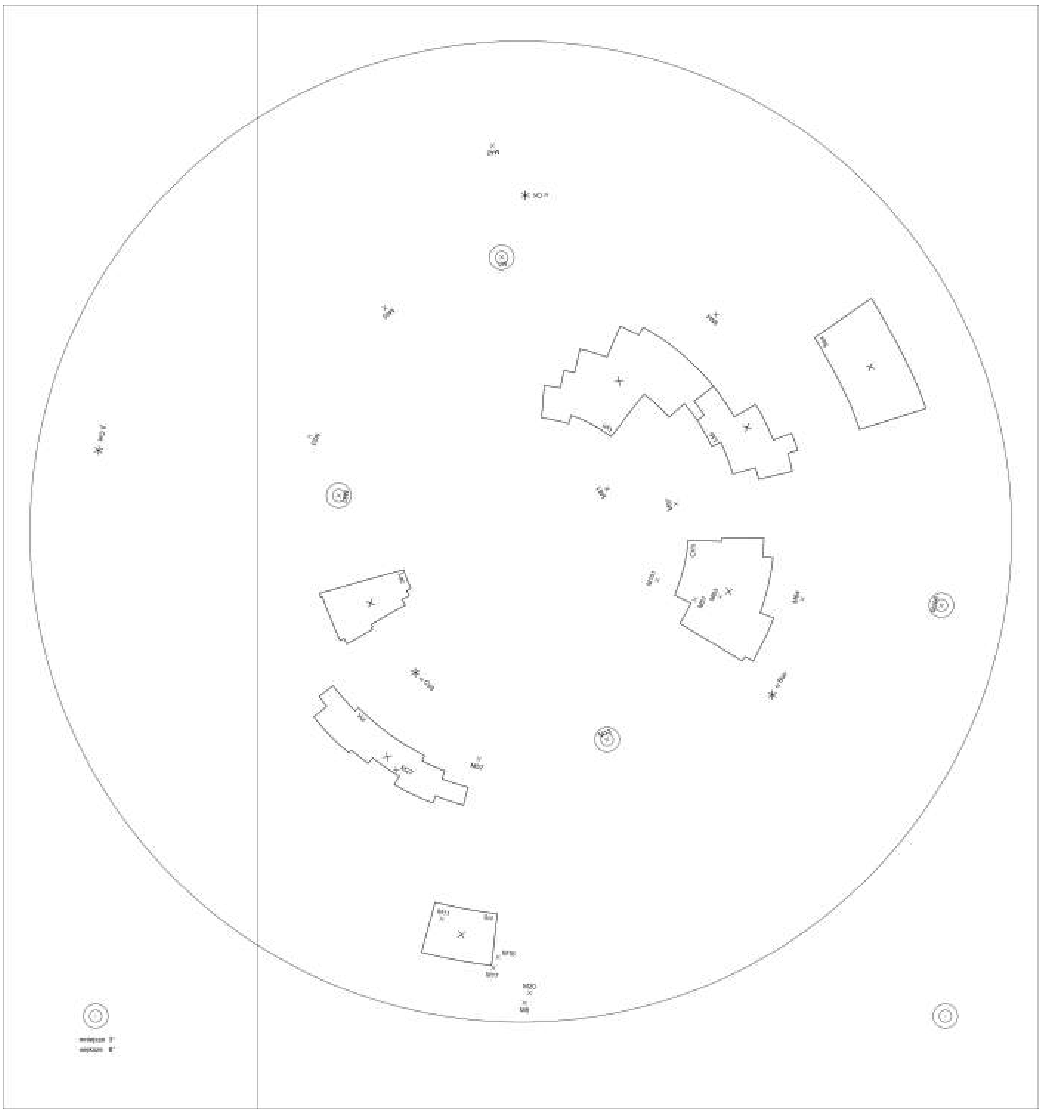


Figure 20.2 – Solution for IOAA 2011 group task 1

Table 20.2 – Star Magnitudes

Star Name	**Apparent Visual Magnitude**
2 (Alfirk)	3.2 - 3.3 (actual answer 3.23)
3 (Alderamin)	2.4 - 2.6 (actual answer 2.47)

Table 20.3 – Parts of A Telescope

	Item name	**Letter**
	(example) Tripod	M
1.	Counterweight	L
2.	Right Ascension Setting Circle (R.A. Scale)	C
3.	Declination Setting Circle (Declination Scale)	K
4.	Right Ascension locking knob	B
5.	Declination locking knob	J
6.	Geographical latitude scale	A
7.	Finder scope	F
8.	Focuser tube	H
9.	Focuser knob	G
10.	Eyepiece	I
11.	Declination Axis	XX
12.	Right Ascension Axis (Polar Axis)	XY
13.	Right Ascension slow motion adjustment	E
14.	Declination flexible slow motion adjustment	D
15.	90° diagonal mirror	T
16.	Azimuth adjustment knobs	P
17.	Altitude adjustment screws	R
18.	Lock screw	O
19.	Spirit level bubble	W
20.	Eyepiece reticle light switch and brightness control	U

20.3 Instrument Aided Observations

1. (a)

   (b) Select and circle the correct answer for each of the questions below:
   21. d
   22. c
   23.1. b
   23.2. a
   24. c
   25. b
   26. c

   (c) Determine the following theoretical instrument parameters:
   27. Magnification : $M = \dfrac{f}{f_{eyepiece}} = 32X$
   28. Focal ratio : 5 or f/5

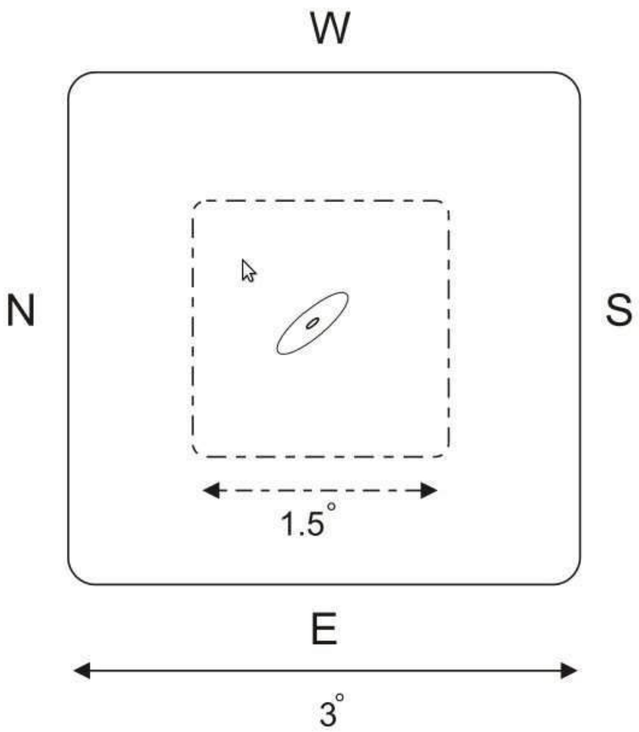


Figure 20.3 – Drawing of Andromeda Galaxy

29. Resolution (in arcseconds) : $r = \dfrac{1.22\lambda}{D} = 1.73$ arcsec
30. Limiting magnitude: 11.5 mag to 12 mag

2. Question is self-explanatory.
3. Brighter: White/blue ( ) Yellow ( ) Red ( )
   Dimmer: White/blue ( ) Yellow ( ) Red ( )
4. Brighter: White ( ) Blue ( ) Yellow ( ) Red ( )
   Dimmer: White ( ) Blue () Yellow () Red ()
5. Night 1: Object 1 (M22) Object 2 (M8) Object 3 (M7) Object 4 (M57)
   Night 2: Object 1 (M4) Object 2 (M8) Object 3 (M6) Object 4 (M57)
6. ν - Scorpii will be resolved into 4 components at this magnification.
7. About 13'.
8. Scale and orientation of M31 should be exact as shown in the figure 20.3.

9. It is critical to note R.A. and Dec. readings for Caph before moving the telescope. This will help in eliminating the zero errors. After pointing the telescope to the next star, note the readings again and hence find R.A. and Dec. of the star. Knowing current date and time, R.A. can be easily converted in to hour angle.
10. About 24'.
11. About 7.8.
12. Solving this question without actual CCD and related softwares is impossible. However, we will briefly state motivation behind the question. With 3 different exposure times, students can choose the best image, identify different stars in the image and by identifying the faintest star in the image, they can get imperical estimate of the limiting magnitude. The particular software used (”The Sky”) includes mangitudes of bright individual stars within the clusters.

20.4 Planetarium Based Questions

1. Planetarium is projecting the sky of Bejing, on some day, just about 1 hour after the sunset. Estimate which month (use 1-12) it should be according to the displayed night sky. What is the age[1] of the moon (in 1-30 days)? (I10 - O03 - A)
2. Simulation of the Earth Sky (I11 - O01 - A)
   (a) In the sky projected, one can notice cresent moon, a nova and a comet. On the map of the sky (use figure 10.3), mark nova with a cross and label it as “N”. Mark the Moon with a Moon symbol and draw the shape and position of the comet.
   (b) In the table below, circle only those objects which are above the astronomical horizon.

M20 - Triffid Nebula	o Cet - Mira	δ CMa - Wezen
α Cyg - Deneb	M57 - Ring Nebula	β Per - Algol
δ Cep - Alrediph	α Boo - Arcturus	M44 - Praesepe (Beehive Cluster)

   (c) Coordinate grid will now be switched on. When it is visible, mark on the map the northern part of the local meridian (from the zenith to the horizon) and the ecliptic north pole (with a cross and marked “P”).

1. Age of the moon is number of days since last new moon.

(d) For the displayed sky, note down:
geographical latitude of the observer (ϕ):
Local Sidereal Time (θ):
approximate time of year (by circling the calendar month):
Jan, Feb, Mar, Apr, May, Jun, Jul, Aug, Sep, Oct, Nov, Dec.

(e) Give the names of the objects, whose approximate horizontal coordinates are :
azimuth $A_1 = 45°$ and altitude $h_1 = 58°$:
azimuth $A_2 = 278°$ and altitude $h_2 = 20°$:
(If you can, use Bayer designations, IAU abbreviations and Messier numbers or English or Latin names.)

(f) Give the horizontal coordinates (azimuth, altitude) of:
Sirius (α CMa): $A_3 =$; $h_3 =$
The Andromeda Galaxy (M31): $A_4 =$; $h_4 =$

(g) Give the equatorial coordinates of the star marked on the sky with a red arrow: $\alpha =$; $\delta =$

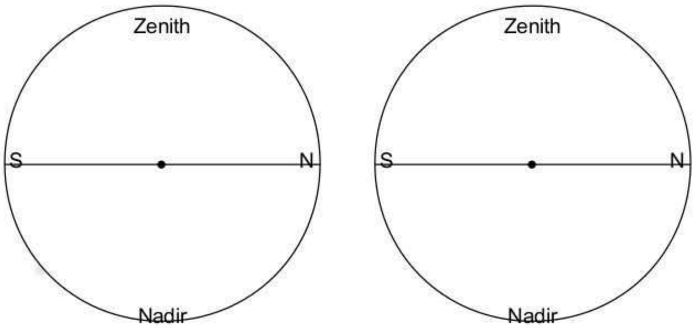
3. Simulation of Martian sky: The sky projected now is as it appears for an observer standing at some distance from a martian base. The martian base is visible on the horizon. (I11 - O02 - A)

(a) Give the areographic (Martian) latitude of the observer : $\varphi =$

(b) Give the altitudes of upper (h_u) and lower (h_l) culmination of:
Pollux (β Gem) : $h_u =$; $h_l =$
Deneb (α Cyg) : $h_u =$; $h_l =$

(c) Give the areocentric (Martian) declination of:
Regulus (α Leo) $\delta=$
Toliman (α Cen) $\delta=$

(d) Sketch diagrams to illustrate your working in the two sub-questions above:

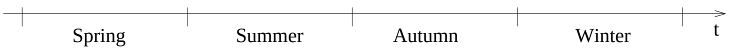


(e) On the map of the sky, mark the Martian celestial North Pole with a cross and label it as "M".

(f) Estimate the azimuth (measured from the South) of the observer as seen from the Martian base visible on the horizon.

(g) Estimate the location of the base on Mars and circle the appropriate description:
a. near the Equator b. near the northern Tropic circle
c. near the northern Arctic circle d. near the North Pole

(h) The time axis below shows the Martian year and the seasons in the northern hemisphere. Mark the date corresponding to the sky on the axis.



Appendix A

Syllabus of IOAA

A.1 General Notes:

1. Extensive contents in basic astronomical concepts are required in theoretical and practical problems.
2. Basic concepts in physics and mathematics at high school level are required in solving the problems. Standard solutions should not involve use of calculus and/or the use of complex numbers and/or solving differential equations.
3. Astronomical software packages may be used in practical and observational problems. The contestants will be informed the list of software packages to be used at least 3 months in advance. The chosen software packages should be preferably freewares or low-cost ones enabling all countries to obtain them easily for practice purpose. The chosen softwares should preferably be available on multiple OSs (Windows / Unix / GNU-Linux / Mac).
4. Concepts and phenomena not included in the Syllabus may be used in questions but sufficient information must be given in the questions so that contestants without previous knowledge of these topics would not be at a disadvantage.
5. Sophisticated practical equipments likely to be unfamiliar to the candidates should not dominate a problem. If such devices are used in the questions, sufficient information must be provided. In such case, students should be given opportunity to familiarise themselves with such equipments.
6. The original texts of the problems have to be set in the SI units, wherever applicable. Participants will be expected to mention appropriate

units in their answers and should be familiar with the idea of correct rounding off and expressing the final result(s) and error(s) with correct number of significant digits.

A.2 Theoretical Part

Symbol (Q) is attached to some topics in the list. It means "qualitative understanding only". Quantitative reasoning / proficiency in these topics is not mandatory.
The following theoretical contents are proposed for the contestants.

A.2.1 Basic Astrophysics

Contents	**Remarks**
Celestial Mechanics	Newton's Laws of Gravitation, Kepler's Laws for circular and non-circular orbits, Roche limit, barycentre, 2-body problem, Lagrange points
Electromagnetic Theory & Quantum Physics	Electromagnetic spectrum, Radiation Laws, Blackbody radiation
Thermodynamics	Thermodynamic equilibrium, Ideal gas, Energy transfer
Spectroscopy and Atomic Physics	Absorption, Emission, Scattering, Spectra of Celestial objects, Doppler effect, Line formations, Continuum spectra, Splitting and Broadening of spectral lines, polarisation
Nuclear Physics	Basic concepts including structure of atom, Mass defect and binding energy Radioactivity, Neutrinos (Q)

A.2.2 Coordinates and Times

Contents	**Remarks**
Celestial Sphere	Spherical trigonometry, Celestial coordinates and their applications, Equinox and Solstice, Circumpolar stars, Constellations and Zodiac
Concept of Time	Solar time, Sidereal time, Julian date, Heliocentric Julian date, Time zone, Universal Time, Local Mean Time, Different definitions of "year", Equation of time

A.2.3 Solar System

Contents	**Remarks**
The Sun	Solar structure, Solar surface activities, Solar rotation, Solar radiation and Solar constant, Solar neutrinos (Q), Sun-Earth relations, Role of magnetic fields (Q), Solar wind and radiation pressure, Heliosphere (Q), Magnetosphere (Q)
The Solar System	Earth-Moon System, precession, nutation, libration, Formation and evolution of the Solar System (Q), Structure and components of the Solar System (Q), Structure and orbits of the Solar System objects, Sidereal and Synodic periods, Retrograde motion, Outer reaches of the solar system (Q)
Space Exploration	Satellite trajectories and transfers, Human exploration of the Solar System (Q), planetary missions (Q), Sling-shot effect of gravity, Space-based instruments (Q)
Phenomena	Tides, Seasons, Eclipses, Aurorae (Q), Meteor Showers

A.2.4 Stars

Contents	**Remarks**
Stellar Properties	Methods of Distance determination, Radiation, Luminosity and magnitude, Color indices and temperature, Determination of radii and masses, Stellar motion, Irregular and regular stellar variabilities â broad classification & properties, Cepheids & period-luminosity relation, Physics of pulsation (Q)
Stellar Interior and Atmospheres	Stellar equilibrium, Stellar nucleosynthesis, Energy transportation (Q), Boundary conditions, Stellar atmospheres and atmospheric spectra
Stellar Evolution	Stellar formation, Hertzsprung-Russell diagram, Pre-Main Sequence, Main Sequence, Post-Main Sequence stars, supernovae, planetary nebulae, End states of stars

A.2.5 Stellar Systems

Contents	Remarks
Binary Star Systems	Different types of binary stars, Mass determination in binary star systems, Light and radial velocity curves of eclipsing binary systems, Doppler shifts in binary systems, interacting binaries, peculiar binary systems
Exoplanets	Techniques used to detect exoplanets
Star Clusters	Classification and Structure, Mass, age, luminosity and distance determination
Milky Way Galaxy	Structure and composition, Rotation, Satellites of Milky Way (Q)
Interstellar Medium	Gas (Q), dust (Q), HII regions, 21cm radiation, nebulae (Q), interstellar absorption, dispersion measure, Faraday rotation
Galaxies	Classifications based on structure, composition and activity, Mass, luminosity and distance determination, Rotation curves
Accretion Processes	Basic concepts (spherical and disc accretion) (Q), Eddington luminosity

A.2.6 Cosmology

Contents	Remarks
Elementary Cosmology	Expanding Universe and Hubbleâs Law, Cluster of galaxies, Dark matter, Dark energy (Q), Gravitational lensing, Cosmic Microwave Background Radiation, Big Bang (Q), Alternative models of the Universe (Q), Large scale structure (Q), Distance measurement at cosmological scale, cosmological redshift

A.2.7 Instrumentation and Space Technologies

Contents	Remarks
Multi-wavelength Astronomy	Observations in radio, microwave, infrared, visible, ultraviolet, X-ray, and gamma-ray wavelength bands, Earthâs atmospheric effects
Instrumentation	Telescopes and detectors (e.g. charge-coupled devices, photometers, spectrographs), Magnification, Focal length, Focal ratio, resolving and light-gathering powers of telescopes, Geometric model of two element interferometer, Aperture synthesis, Adaptive optics, photometry, astrometry

A.3 Practical Part

This part consists of 2 sections: observations and data analysis sections. The theoretical part of the Syllabus provides the basis for all problems in the practical part.

The observations section focuses on contestantâs experience in

1. naked-eye observations,
2. usage of sky maps and catalogues,
3. application of coordinate systems in the sky, magnitude estimation, estimation of angular separation
4. usage of basic astronomical instrumentsâtelescopes and various detectors for observations but enough instructions must be provided to the contestants. Observational objects may be from real sources in the sky or imitated sources in the laboratory. Computer simulations may be used in the problems but sufficient instructions must be provided to the contestants.

The data analysis section focuses on the calculation and analysis of the astronomical data provided in the problems. Additional requirements are as follows:

1. Proper identification of error sources, calculation of errors, and estimation of their influence on the final results.
2. Proper use of graph papers with different scales, e.g., polar and logarithmic papers. Transformation of the data to get a linear plot and finding âBest Fitâ line approximately.

3. Basic statistical analysis of the observational data.
4. Knowledge of the most common experimental techniques for measuring physical quantities mentioned in Part A.