Model Question Paper SSC-I

Mathematics(Science Group)

(2ⁿᵈ Set)SOLUTION

SECTION-A

1	D	2	A	3	D	4	A	5	D	6	A	7	A	8	A
9	A	10	B	11	A	12	B	13	D	14	B	15	D		

SECTION-B

Question-2(i)

$$BC = \begin{bmatrix} -5 & -3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -18 & -11 \\ 7 & 4 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 0 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -18 & -11 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 14 & 8 \\ -61 & -37 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -5 & -3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ -17 & -10 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} 4 & 2 \\ -17 & -10 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 14 & 8 \\ -61 & -37 \end{bmatrix}$$

From above $A(BC) = (AB)C$

Question-2(ii)

Let $x = \dfrac{\sqrt[3]{46.3}\,(0.05)^2}{\sqrt{8.54}}$

Taking log of both sides

$$logx = log\sqrt[3]{46.3} + log(0.05)^2 - log\sqrt{8.54}$$

$$logx = log(46.3)^{1/3} + log(0.05)^2 - log(8.54)^{1/2}$$

$$logx = \frac{1}{3}log46.3 + 2log0.05 - \frac{1}{2}log8.54$$

$$logx = \frac{1}{3}(1.6660) + 2(\overline{2}.6990) - \frac{1}{2}(0.9315)$$

$$logx = 0.5553 + 2(-2 + 0.6990) - 0.4658$$

$$logx = 0.5553 - 4 + 1.3980 - 0.4658$$

$$logx = -2.5125 = \overline{2}.5125$$

$$x = antilog\,\overline{2}.5125$$

$$x = 0.03256$$

Question-2(iii)

$$\left(\frac{15m^3n^{-2}p^{-1}}{25m^{-2}n^{-9}}\right)^{-3}$$

$$= \left(\frac{3m^{3+2}n^{-2+9}p^{-1}}{5}\right)^{-3}$$

$$= \left(\frac{3m^5n^7}{5p}\right)^{-3} = \left(\frac{5p}{3m^5n^7}\right)^3$$

$$= \left(\frac{5^3p^3}{3^3m^{5\times3}n^{7\times3}}\right)$$

$$= \left(\frac{125p^3}{27m^{15}n^{21}}\right)$$

Question-2(iv)

$$(1+i)^3(x+yi) = (4+5i)$$

$$(1+3i+3i^2+i^3)(x+yi) = 4+5i$$

$$(1+3i-3-i)(x+yi) = 4+5i$$

$$(2i-2)(x+yi) = 4+5i$$

$$2xi + 2yi^2 - 2x - 2yi = 4+5i$$

$$2xi - 2y - 2x - 2yi = 4+5i$$

$$(-2x-2y) + (2x-2y)i = 4+5i$$

Equating the real and imaginary parts

$$-2x - 2y = 4 \ \rightarrow eqn-I \qquad\qquad 2x - 2y = 5 \ \rightarrow eqn-II$$

Adding equations I and II

$$-2x - 2y + 2x - 2y = 4+5$$

$$-4y = 9 \qquad\qquad \Rightarrow y = -\frac{9}{4}$$

Subtracting equations II from I

$$-2x - 2y - 2x + 2y = 4-5$$

$$-4x = -1 \qquad\qquad \Rightarrow x = \frac{1}{4}$$

Question-2(v)

$$x - \frac{1}{x} = 7$$

$$\left(x - \frac{1}{x}\right)^3 = 7^3$$

$$x^3 + \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = 343$$

$$x^3 + \frac{1}{x^3} - 3(7) = 343 \qquad\qquad \because x - \frac{1}{x} = 7$$

$$x^3 + \frac{1}{x^3} = 343 + 21$$

$$x^3 + \frac{1}{x^3} = 364$$

Question-2(vi)

(a) $\quad x = -3 + \sqrt{2}$

$$\frac{1}{x} = \frac{1}{-3 + \sqrt{2}}$$

$$\frac{1}{x} = \frac{1}{-3 + \sqrt{2}} \times \frac{-3 - \sqrt{2}}{-3 - \sqrt{2}}$$

$$\frac{1}{x} = \frac{-3 - \sqrt{2}}{(-3)^2 - \left(\sqrt{2}\right)^2} = \frac{-3 - \sqrt{2}}{7} = -\frac{3 + \sqrt{2}}{7}$$

(b) $\quad x + \frac{1}{x} = -3 + \sqrt{2} - \frac{3 + \sqrt{2}}{7}$

$$x + \frac{1}{x} = \frac{-21 + 7\sqrt{2} - 3 - \sqrt{2}}{7} = \frac{-24 + 6\sqrt{2}}{7}$$

(c) $\quad x - \frac{1}{x} = -3 + \sqrt{2} + \frac{3 + \sqrt{2}}{7}$

$$x - \frac{1}{x} = \frac{-21 + 7\sqrt{2} + 3 + \sqrt{2}}{7} = \frac{-18 + 8\sqrt{2}}{7}$$

(d) $\quad x^2 + \frac{1}{x^2} = \left(-3 + \sqrt{2}\right)^2 + \left(-\frac{3 + \sqrt{2}}{7}\right)^2$

$$x^2 + \frac{1}{x^2} = 9 + 2 - 6\sqrt{2} + \frac{9 + 2 + 6\sqrt{2}}{49}$$

$$x^2 + \frac{1}{x^2} = 11 - 6\sqrt{2} + \frac{11 + 6\sqrt{2}}{49}$$

$$x^2 + \frac{1}{x^2} = \frac{550 - 288\sqrt{2}}{49}$$

Question-2(vii)
$$2x^2 + 7x + \frac{6}{x}$$

$2x^2$	$4x^4 + 28x^3 + 49x^2 + 24x + 84 + \frac{36}{x^2}$
	$\pm 4x^4$
$4x^2 + 7x$	$28x^3 + 49x^2 + 24x + 84 + \frac{36}{x^2}$
	$\pm 28x^3 \pm 49x^2$

$$4x^2 + 14x + \frac{6}{x} \qquad\qquad 24x + 84 + \frac{36}{x^2}$$

$$\pm 24x \pm 84 \pm \frac{36}{x^2}$$

_____

$$0$$

$$\sqrt{4x^4 + 28x^3 + 49x^2 + 24x + 84 + \frac{36}{x^2}} = \pm\left(2x^2 + 7x + \frac{6}{x}\right)$$

Question-2(viii)

$$x^2 + 4x - 12 = x^2 + 6x - 2x - 12 = x(x + 6) - 2(x + 6) = (x + 6)(x - 2)$$

$$x^2 - 4 = (x + 2)(x - 2)$$

$$x^3 - 2^3 = (x - 2)(x^2 + 2x + 4)$$

H.C.F= $(x - 2)$

Question-2(ix)

$$P(x) = x^3 - 2x^2 - 5x + 6$$

At $x = 1$

$$P(1) = 1 - 2 - 5 + 6 = 0$$

$x - 1$ is a factor of $P(x)$

$$
\begin{array}{r|l|l}
x-1 & x^3 - 2x^2 - 5x + 6 & x^2 - x - 6 \\
& \underline{\pm x^3 \mp x^2} & \\
& -x^2 - 5x + 6 & \\
& \underline{\mp x^2 \pm x} & \\
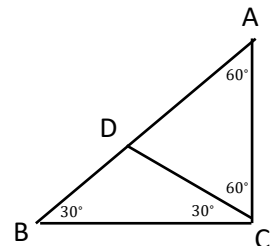& -6x + 6 & \\
& \underline{\mp 6x \pm 6} & \\
\end{array}
$$

$$P(x) = (x - 1)(x^2 - x - 6)$$

$$P(x) = (x - 1)(x - 3)(x + 2)$$

Question-2(x)

Given: In $\triangle ABC$, $\angle C = 90°$, $\angle A = 60°$

To Prove: $\overline{BC} = \frac{1}{2}\overline{AB}$

Construction: At C construct $\angle BCD = 30°$. Let $\overline{CD}$ cuts $\overline{AB}$ at D.

Proof:

<u>Statements</u>   <u>Reasons</u>

In $\triangle ADC$, $\angle A = 60°$ Given

$\angle CDA = 60°$ $m\angle BCD + m\angle CDA = 90°$

$\therefore \triangle ADC$ is equilateral

$\overline{AC} = \overline{CD} = \overline{AD}$

$\overline{AB} = \overline{BD} + \overline{AD}$ $\triangle BDC$ is   isosceles

$\overline{AB} = \overline{DC} + \overline{AC}$ $\overline{BD} = \overline{DC}$ & $\overline{AD} = \overline{AC}$

$\overline{AB} = \overline{AC} + \overline{AC}$

$\overline{AB} = 2\overline{AC}$

$\overline{AC} = \dfrac{1}{2}\overline{AB}$

Question-2(xi)

$A(1,1), B(3,1), C(4,3)$

$\left|\overline{AB}\right|^2 = (3-1)^2 + (1-1)^2 = 2$

$\left|\overline{BC}\right|^2 = (4-3)^2 + (3-1)^2 = 5$

$\left|\overline{AC}\right|^2 = (4-1)^2 + (3-1)^2 = 13$

Since $\left|\overline{AB}\right|^2 + \left|\overline{BC}\right|^2 = 2 + 5 = 7 \neq \left|\overline{AC}\right|^2$

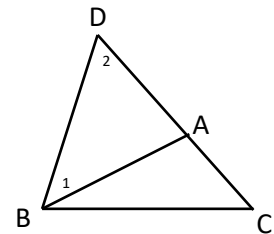Hence ABC is not a right angled triangle.

Question-2(xii)

Given: In $\triangle ABC$

To Prove: (i) $m\overline{AB} + m\overline{AC} > m\overline{BC}$

              (ii) $m\overline{AB} + m\overline{BC} > m\overline{AC}$

              (iii) $m\overline{BC} + m\overline{AC} > m\overline{AB}$

Construction: Take a point D on $\overline{CA}$ such that $\overline{AD} = \overline{AB}$. Join B to D.

Proof:

Statements	Reasons
In $\triangle ABD$	Given
$\angle 1 \cong \angle 2$	Construction
$m\angle DBC > m\angle 1$ eqn(i)	$m\angle DBC + m\angle 1 + m\angle ABC$
$m\angle DBC > m\angle 2$ eqn(ii)	from (i) and (ii)
In $\triangle DBC$ eqn(iii)	

$$m\overline{DC} > m\overline{BC} \qquad\qquad \text{from (iii)}$$

$$m\overline{AD} + m\overline{AC} > m\overline{BC} \qquad m\overline{CD} = m\overline{AD} + m\overline{AC}$$

$$\text{Hence } m\overline{AB} + m\overline{AC} > m\overline{BC} \qquad m\overline{AD} = m\overline{AB}$$

Similarly

$$m\overline{AB} + m\overline{BC} > m\overline{AC}$$

$$m\overline{BC} + m\overline{AC} > m\overline{AB}$$

Question-2(xiii)

$$A(2,4),\ B(4,4),\ C(-1,3),\ D(-3,3)$$

$$\left|\overline{AB}\right| = \sqrt{(4-2)^2 + (4-4)^2} = 2$$

$$\left|\overline{DC}\right| = \sqrt{(-1+3)^2 + (3-3)^2} = 2$$

$$\left|\overline{AD}\right| = \sqrt{(-3-2)^2 + (3-4)^2} = \sqrt{26}$$

$$\left|\overline{BC}\right| = \sqrt{(-1-4)^2 + (3-4)^2} = \sqrt{26}$$

Since $\left|\overline{AB}\right| = \left|\overline{DC}\right|$ and $\left|\overline{AD}\right| = |BC|$
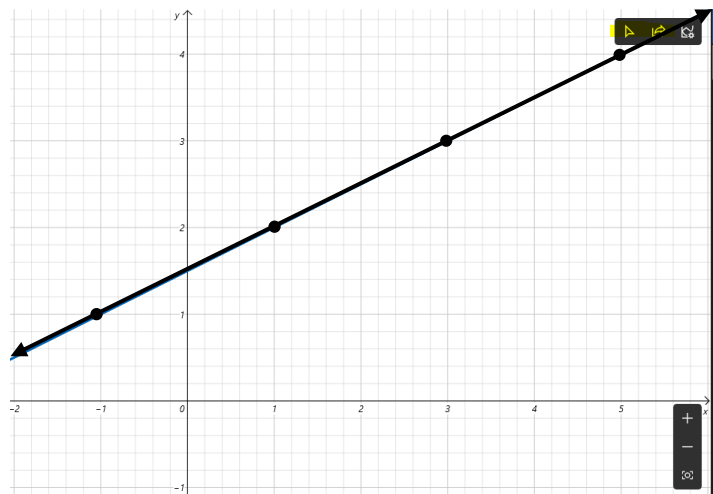
i.e. opposite sides of the quadrilateral ABCD are equal.

Hence ABCD is a parallelogram.

Question-2(xiv)

$$2y - x - 3 = 0$$

x	-1	1	3	5
y	1	2	3	4

SECTION-C

Q3. (a) $\left|\dfrac{x+8}{12}\right| = \dfrac{x-1}{5}$

$$\frac{x+8}{12} = + \left(\frac{x-1}{5}\right)$$ 　　　　　$$\frac{x+8}{12} = - \left(\frac{x-1}{5}\right)$$

$12(x-1) = 5(x+8)$ 　　　　　　　$12(x-1) = -5(x+8)$

$12x - 12 = 5x + 40$ 　　　　　　$12x - 12 = -5x - 40$

$12x - 5x = 40 + 12$ 　　　　　　$12x + 5x = 12 - 40$

$7x = 52$ 　　　　　　　　　　　　$17x = -28$

$x = \dfrac{52}{7}$ 　　　　　　　　　　$x = -\dfrac{28}{17}$

Solution set$= \left\{\dfrac{52}{7}, -\dfrac{28}{17}\right\}$

(b) $2 \le \dfrac{2}{3} - 4x < 3 - 5x$

Multiply by 3

$6 \le 2 - 12x < 9 - 15x$

$6 \le 2 - 12x$ 　　　　　　$2 - 12x < 9 - 15x$

$12x \le 2 - 6$ 　　　　　　$15x - 12x < 9 - 2$

$12x \le -4$ 　　　　　　　$3x < 7$

$x \le -\dfrac{1}{3}$ 　　　　　　　$x < \dfrac{7}{3}$
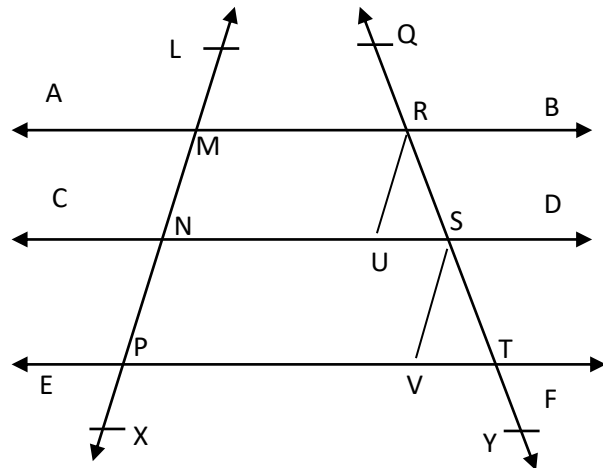
Solution Set $= \left\{x \mid -\dfrac{1}{3} \ge x < \dfrac{7}{3}\right\}$

Q4

Statement: If three or more parallel lines make congruent segments on transversal, they also intercept congruent segments on any other

line that cuts them.

Figure:

Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$

The transversal $\overleftrightarrow{LX}$ intersects $\overleftrightarrow{AB}, \overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ at the points M, N and P respectively, such that $\overline{MN} \cong \overline{NP}$. The transversal $\overleftrightarrow{QY}$ intersects them at points R, S and T respectively.

To Prove: 　　$\overline{RS} = \overline{ST}$

Construction: From R, draw $\overline{RU} \parallel \overline{LX}$, which meets $\overline{CD}$ at U. From S, draw $\overline{SV} \parallel \overline{LX}$ which meets $\overline{EF}$ at V. As shown in the figure let the angles be labelled as $\angle 1, \angle 2, \angle 3$ and $\angle 4$.

Proof:

Statements	Reasons
MNUR is a parallelogram.	$\overrightarrow{RU} \parallel \overrightarrow{LX}$, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$
$\overline{MN} \cong \overline{RU} \quad \longrightarrow (i)$	Opposite sides of a $\parallel gram$
Similarly,	
$\overline{NP} \cong \overline{SV} \quad \longrightarrow (ii)$	
But $\overline{MN} \cong \overline{NP} \longrightarrow (iii)$	Given
$\therefore \overline{RU} \cong \overline{SV}$	
Also $\overline{RU} \parallel \overline{SV}$	From (i), (ii) and (iii)
$\therefore \angle 1 \cong \angle 2$ and $\quad\quad \angle 3 \cong \angle 4$	each is $\parallel \overleftrightarrow{LX}$
In $\Delta RUS \longleftrightarrow \Delta SVT$	Corresponding angles
$\overline{RU} \cong \overline{SV}$	
$\angle 1 \cong \angle 2$	Proved
$\angle 3 \cong \angle 4$	Proved
$\Delta RUS \cong \Delta SVT$	Proved
Hence $\overline{RS} = \overline{ST}$	S.A.A$\cong$S.A.A
	Corresponding sides of congruent triangles.

Q5.

Let cost of chair $= x$

Let cost of Table $= y$

According to First condition

$$x = \frac{y}{2} + 3$$

$$\Longrightarrow 2x = y + 6 \quad \Longrightarrow 2x - y = 6 - - - (1)$$

According to 2nd condition

$$3x + y = 54 - - - (2)$$

These equations can be written in form of matrices as ;

$$\begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 54 \end{bmatrix}$$

Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 54 \end{bmatrix}$

$$\therefore AX = B \Longrightarrow X = A^{-1}B - - - (3)$$

Now $|A| = \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} = 2(1) - (-1 \times 3) = 5 \neq 0 - - - (4)$

$$Adj\ A = \begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix} - - - (5)$$

$$A^{-1} = \frac{Adj\ A}{|A|} \quad ------(6)$$

Using Equation (4) and Equation (5) in Equation (6)

$$A^{-1} = \frac{1}{5}\begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix} ---(6)$$

Putting the values in Equation (3)

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ 54 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 6+54 \\ -18+108 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 60 \\ 90 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 18 \end{bmatrix}$$

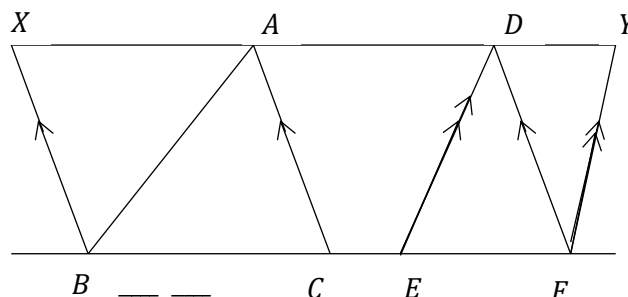By definition of equal matrices their corresponding elements are equal

So   x= 12 , y =18

∴ Cost of chair= Rs 12   and Cost of table=Rs 18

Q6.

Statement: Triangles on equal bases and of equal altitudes are equal in area.

Figure:

**Given:** As ABS, DEF on equal bases $\overline{BC}, \overline{EF}$ and having equal altitudes.

To prove: Area of $\triangle ABC =$ Area of $\triangle DEF$

Construction: Place the $\triangle s$ ABC and DEF so that their equal bases $\overline{BC}$ and $\overline{EF}$ are on the same straight lines $BCEF$ and their vertices on the same side of it. Draw $\overline{BX} \parallel \overline{CA}$ and $\overline{FY} \parallel \overline{ED}$ meeting AD produces in X and Y, respectively.

Proof:

Statements	Reasons
$\triangle ABC, \triangle DEF$ are between the same parallels	Their altitudes are equal (Given).
$\therefore XADY$ is parallel to $BCEF$	
$\therefore$ Area of $\parallel$gm $BCAX =$ Area of $\parallel$gm $EFYD \rightarrow (i)$	These $\parallel$gms are on equal bases and between the same parallel lines.
But, Area of $\triangle ABC = \frac{1}{2}$(Area of $\parallel$gm $BCAX) \rightarrow (ii)$	Diagonals of a $\parallel$gm bisect it.
And Area of $\triangle DEF = \frac{1}{2}$(Area of $\parallel$gm$EFYD) \rightarrow (iii)$	
$\therefore$ Area of $\triangle ABC =$ Area of $\triangle DEF$	From $(i), (ii)$ and (iii)

Q7 $m\overline{AB} = 3cm$, $m\angle B = 120°$, $m\overline{BC} = 5.8cm$
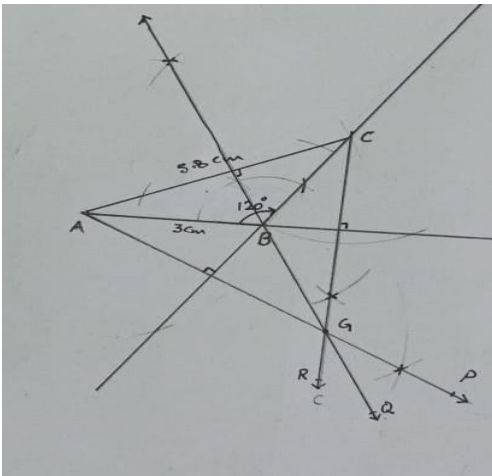
Construction Steps:

a.   Draw $m\overline{AB} = 3cm$

b.   Using pair of compasses to draw $m\angle B = 120°$.

c.   With A as centre draw an arc of radius 5.8 cm that cuts $m\angle B$ at C.

d. $\triangle ABC$ is completed.

d.   Construct $\overline{AP}$ altitude from vertex A.

e.   Construct $\overline{BQ}$ altitude from vertex B.

f.   Construct $\overline{CR}$ altitude from vertex C.

g.   The altitudes intersect at point G.

               i.e. altitudes of $\triangle ABC$ are concurrent.