

Solved Problems on Armature of a DC machine

Example: 1

Determine the number of poles, armature diameter and core length for the preliminary design of a 500kW, 400V, 600 rpm, dc shunt generator assuming an average flux density in the air gap of 0.7 T and specific electric loading of 38400 ampere- conductors per metre. Assume core length/ pole arc = 1.1. Apply suitable checks.

$$\text{Armature current } I_a = I_L + I_{sh} = \frac{\text{Output in watts}}{\text{terminal voltage}} = \frac{500 \times 10^3}{400} = 400 \text{ A}$$

In order that the current/path is not more than 200A, a lap winding is to be used. The number of parallel paths required is

$$A = \frac{1250}{200} = 6.25$$

Since the number of parallel paths cannot be a fraction as well an odd integer, A can be 6 or 8. Let it be 6. (Note: A current/path of 200A need not strictly be adhered)

$$\text{Check: Frequency of the induced emf } f = \frac{PN}{120} = \frac{6 \times 60}{120} = 30 \text{ Hz and can be considered}$$

as the frequency generally lies between 25 and 50 Hz

$$D^2 L = \frac{\text{kW}}{1.64 \times 10^{-4} B_{av} q N}$$

$$= \frac{500}{1.64 \times 10^{-4} \times 0.7 \times 38400 \times 600}$$

$$\approx 0.189 \text{ m}^3$$

Note: Since the capacity is considerable, power developed in the armature can be taken as power output for a preliminary design.

$$L = 1.1 \text{ pole arc} = 1.1 \Psi \tau$$

Since Ψ lies between 0.6 and 0.7, let it be 0.7.

$$\text{Since } \tau = \frac{\pi D}{P}, L = 1.1 \times 0.7 \times \frac{\pi D}{6} = 0.4D$$

$$D^2 \times 0.4D = 0.189$$

$$D^3 = \frac{0.189}{0.4} = 0.473 \text{ m}^3$$

$$D = 0.78 \text{ m and } L = 0.4 \times 0.78 \approx 0.31 \text{ m}$$

$$\text{Check: Peripheral velocity } v = \frac{\pi D N}{60} = \frac{\pi \times 0.78 \times 600}{60} = 24.2 \text{ m/s}$$

and is within the limits, i.e. 30 m/s.

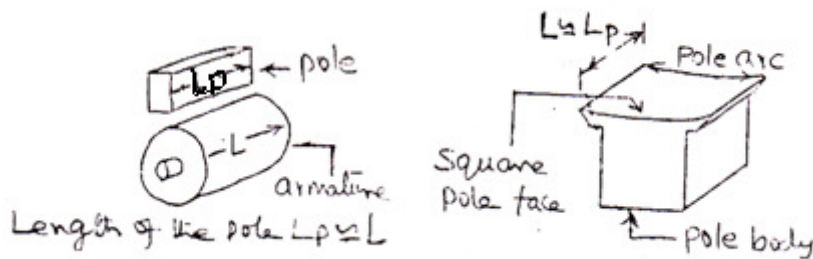
Example: 2

Determine the main dimensions of the armature core, number of conductors, and commutator segments for a 350kW, 500V, 450 rpm, 6pole shunt generator assuming a square pole face with pole arc 70% of the pole pitch. Assume the mean flux density to be 0.7T and ampere-conductors per cm to be 280.

$$D^2 L = \frac{\text{kW}}{1.64 \times 10^{-4} B_{av} q N}$$

$$= \frac{350}{1.64 \times 10^{-4} \times 0.7 \times 280 \times 100 \times 450}$$

$$\approx 0.24 \text{ m}^3$$



For a square pole face, $L = \psi \tau = 0.7 \times \frac{\pi D}{6} = 0.36D$

$$D^3 = \frac{0.24}{0.37} = 0.654 \text{ m}^3$$

Therefore $D = 0.87 \text{ m}$ and $L = 0.37 \times 0.87 \approx 0.32 \text{ m}$

Since $E = \frac{\phi Z N P}{60 A}$, number of armature conductors $Z = \frac{E 60 A}{\phi N P}$

$$\text{Flux per pole} = \frac{B_{av} \pi D L}{P} = \frac{0.7 \times \pi \times 0.87 \times 0.32}{6} = 0.1 \text{ Wb}$$

If a lap winding is assumed, $A=P$ and therefore

$$Z = \frac{60 \times 500 \times 6}{0.1 \times 450 \times 6} = 666.6 \text{ and is not possible as the number of conductors must always be an even integer.}$$

The number of conductors can be 666 or 668. Let it be 666.

Number of coils = Number of commutator segments

$$= \frac{Z}{2 \times \text{turns per coil}} = \frac{666}{2 \times 1} = 333 \text{ if a single turn coil is assumed.}$$

Example: 3

A design is required for a 50kW, 4pole, 600rpm, and 220V dc shunt generator. The average flux density in the air gap and specific electric loading are respectively 0.57T and 30000 ampere-conductors per metre. Calculate suitable dimensions of armature core to lead to a square pole face.

Assume that full load armature drop is 3% of the rated voltage and the field current is 1% of rated full load current. Ratio pole arc to pole pitch is 0.67.

$$D^2 L = \frac{\text{kW}}{1.64 \times 10^{-4} B_{av} q N}$$

Note: Since data is available in the problem to calculate the power developed in the armature, the same may be obtained to substitute for kW.

$$\text{Power developed in the armature in kW} = E I_a \times 10^{-3}$$

$$E = V + \text{voltage drop in the armature circuit} = 220 + 0.03 \times 220 = 226.6 \text{ V}$$

$$I_a = I_L + I_{Sh}$$

$$I_L = \frac{\text{Output in watts}}{\text{terminal voltage}} = \frac{50 \times 10^3}{220} = 227.3 \text{ A}$$

$$I_a = 227.3 + 0.01 \times 227.3 = 229.6 \text{ A}$$

$$\text{kW} = 226.6 \times 229.6 \times 10^{-3} = 52$$

$$D^2L = \frac{52}{1.64 \times 10^{-4} \times 0.57 \times 300 \times 600}$$

$$\approx 0.031\text{m}^3$$

For a square pole face, $L = \Psi \tau = 0.67 \times \pi D/4 = 0.53D$

$$D^3 = \frac{0.031}{0.53} \text{m}^3 \text{ and therefore } D=0.39\text{m and } L=0.53 \times 0.39 \approx 0.21\text{m}$$

Example: 4

Determine the diameter and length of the armature core for a 55kW, 110V, 1000rpm, and 4pole dc shunt generator. Assume:

Specific magnetic loading 0.5T, Specific electric loading 13000 ampere –turns,

Pole arc 70% of pole pitch and length of core about 1.1 times the pole arc, Allow 10A for field current and a voltage drop of 4V for the armature circuit.

Determine also the number of armature conductors and slots.

$$D^2L = \frac{\text{kW}}{1.64 \times 10^{-4} B_{av} q N}$$

Power developed in the armature in kW = $E I_a \times 10^{-3}$

$E = V + \text{voltage drop in the armature circuit} = 110 + 4 = 114\text{V}$

$$I_a = I_L + I_{Sh} = \frac{55 \times 10^3}{110} + 10 = 510\text{A}$$

$$\text{kW} = 114 \times 510 \times 10^{-3} = 58.14$$

$$q = 13000 \times 2 = 26000$$

as q must be substituted in ampere conductors and one turn corresponds 2 conductors

$$D^2L = \frac{58.14}{1.64 \times 10^{-4} \times 0.5 \times 26000 \times 1000}$$

$$\approx 0.027\text{m}^3$$

since $L = 1.1$ times pole arc, $L = \Psi \tau = 1.1 \times 0.7 \times \pi D/4 = 0.4D$

$$D^3 = \frac{0.027}{0.6} \text{m}^3 \text{ and therefore } D=0.36\text{m and } L=0.6 \times 0.36 \approx 0.22\text{m}$$

$$\text{Number of armature conductors } Z = \frac{60EA}{\phi NP}$$

$$\text{Flux per pole } \phi = \frac{B_{av} \pi D L}{P} = \frac{0.5 \times \pi \times 0.36 \times 0.22}{4} = 0.03 \text{ Wb}$$

If a lap winding is assumed, $A=P$ and therefore

$$Z = \frac{60 \times 114 \times 4}{0.03 \times 1000 \times 4} \approx 228$$

Since the minimum number of slots per pole is 9 from commutation point of view, minimum number of slots for the machine is

$$= 9 \times 4 = 36 \text{ -----(1)}$$

Since slot pitch $\lambda_s = \frac{\pi D}{S}$, lies between 2.5 and 3.5 cm, the number of slots

$$\text{lies between } \frac{\pi \times 36}{3.5} \approx 32 \text{ and } \frac{\pi \times 36}{2.5} \approx 45 \text{ -----(2)}$$

From 1 and 2 the number of slots lies between 36 and 45.

Since a lap winding is assumed and for a lap winding the number of slots may be multiple of number of poles i.e. 6 or multiple of number of pair poles i.e. 3, let the number of slots $S = 40$.

Note: Since preliminary number of slots and conductors are known, actual number of conductors per slot and number of conductors can be calculated as detailed below.

$$\text{Conductors per slot} = \frac{228}{40} = 5.7 \text{ and is not possible as Conductors per slot must always be}$$

an even integer for a double layer winding. Since only double layer winding is for dc machines,

let the number Conductors per slot be 6. Therefore $Z_{\text{revised}} = 40 \times 6 = 240$.

Example: 5

For a preliminary design of a 50hp, 230V, 1400 rpm dc motor, calculate the armature diameter and core length, number of poles and peripheral speed. Assume specific magnetic loading 0.5T, specific electric loading 25000 ampere- conductors per meter, efficiency 0.9.

$$I_a = I_L - I_{sh} \approx I_L = \frac{\text{Input in watts}}{\text{Voltage}} = \frac{hp \times 746 / \eta}{V} = \frac{50 \times 747}{0.9 \times 230} = 180.2 \text{ A}$$

For this armature current, a lap or wave winding can be used. Since minimum number

of paths and poles is two, 2 poles are sufficient for the machine. However, to gain more advantages of more number of poles, let $P=4$.

$$f = \frac{PN}{120} = \frac{4 \times 1400}{120} = 46.7 \text{ Hz, within the limits.}$$

$$D^2L = \frac{\text{kW}}{1.64 \times 10^{-4} B_{av} q N}$$

Power developed in the armature

$$= \left(\frac{1 + 2\eta}{3\eta} \right) \text{output power}$$

$$= \left(\frac{1 + 2 \times 0.9}{3 \times 0.9} \right) 50 \times 0.746 = 38.7 \text{ kW}$$

$$D^2L = \frac{38.7}{1.64 \times 10^{-4} \times 0.5 \times 25000 \times 1400} = 0.0134 \text{ m}^3$$

Since $\frac{L}{\tau}$ lies between 0.55 and 1.1, let it be 1.0.

$$\text{Therefore } L = \tau = \frac{\pi D}{4} = 0.785 D$$

$$D^3 = \frac{0.0134}{0.785} = 0.017 \text{ m}^3 \quad \text{and} \quad D = 0.26 \text{ m}, \quad L = 0.785 \times 0.26 \approx 0.2 \text{ m}$$

$$\text{Peripheral velocity } v = \frac{\pi D N}{60} = \frac{\pi \times 0.26 \times 1400}{60} \approx 19 \text{ m/s}$$

Example: 6

Calculate the armature diameter and core length for a 7.5kW, 4pole, 1000rpm, and 220V shunt motor. Assume:

Full load efficiency = 0.83, field current is 2.5% of rated current. The maximum efficiency occurs at full load.

$$D^2L = \frac{\text{kW}}{1.64 \times 10^{-4} B_{av} q N}$$

Power developed in the armature

= Electrical input to the motor – field and armature copper losses

= Electrical output of the motor + Iron, friction and windage losses

$$\text{Electrical input to the motor} = \frac{\text{output}}{\eta} = \frac{7.5 \times 10^3}{0.83} \approx 9036 \text{ W}$$

$$\text{Losses at full load} = \left(\frac{1-\eta}{\eta} \right) \text{output} = \left(\frac{1-0.83}{0.83} \right) \times 7.5 \times 10^3 = 1536 \text{ W}$$

Since efficiency is maximum at full load and at maximum efficiency, the variable loss is equal to constant loss,

$$\text{Variable loss} = \text{armature copper loss} = \frac{1536}{2} \approx 768 \text{ W}$$

$$\text{Field copper loss} = (I_{sh})^2 R_{sh} \text{ or } V I_{sh}$$

$$I_{sh} = 0.025 \text{ full load current} = 0.025 \times \frac{7.5 \times 10^3}{0.83 \times 220} = 1.03 \text{ A}$$

$$\text{Field copper loss} = 220 \times 1.03 = 226.6 \text{ W}$$

$$\text{Power developed in the armature} = 9036 - 768 - 226.6 = 8041.4 \text{ W}$$

OR

$$\text{Iron, friction and windage losses} = \text{Constant losses} - \text{field copper loss}$$

$$= 768 - 226.6 = 541.4 \text{ W}$$

$$\text{OR Power developed in the armature} = 7500 + 541.4 = 8041.4 \text{ W}$$

Since specific magnetic loading lies between 0.45 and 0.75 T, let it be 0.6 T.

Since specific electric loading lies between 15000 to 50000 ampere-conductors, let it be 30000.

$$D^2 L = \frac{8.041}{1.64 \times 10^{-4} \times 0.6 \times 30000 \times 1000} = 0.0027 \text{ m}^3$$

Since $\frac{L}{\tau}$ lies between 0.55 and 1.1, let it be 1.0.

$$\text{Therefore } L = \tau = \frac{\pi D}{4} = 0.785 D$$

$$D^3 = \frac{0.0027}{0.785} = 0.0343 \text{ m}^3 \quad \text{and} \quad D = 0.33 \text{ m}, \quad L = 0.785 \times 0.33 \approx 0.26 \text{ m}$$

Example: 7

List the design details of armature winding suitable for a 35hp, 4pole, 500rpm, and 230V, motor. Flux per pole 0.028Wb.

Details of the winding : Type of winding, number of slots, number of conductors, cross-sectional area of conductor, number of coils, back pitch, front pitch etc.

$$I_a \approx \frac{\text{Input in watts}}{\text{Voltage}} = \frac{hp \times 746 / \eta}{V}$$

$$= \frac{35 \times 747}{0.9 \times 230} = 126.1A \text{ with the assumption, efficiency is 0.9.}$$

For the armature current of 126.1A, a lap or wave winding can be used. Let a wave winding be used.

$$\text{Number of armature conductors } Z = \frac{60EA}{\phi NP} = \frac{60 \times 230 \times 2}{0.028 \times 500 \times 4} \approx 492$$

Since the minimum number of slots per pole is 9 from commutation point of view, minimum number of slots for the machine is $9 \times 4 = 36$.

[Note: Since the diameter of the armature is not known, number of slots from the slot pitch consideration cannot be fixed. It is not very much essential to determine D either from D^2L product or peripheral velocity and then fix number of the slots. Even the condition minimum, 9 slots per pole need not be satisfied, as it is only a guiding figure in fixing the number of slots for the machine. For a preliminary design, a number, around the minimum number of slots can be selected.]

Since for a wave winding, the number of slots should not be a multiple of pair of poles i.e. $p = 2$, let the number of slots be 43.

Conductors per slot = $\frac{492}{43} \approx 11.5$ and is not possible. Let the number Conductors per slot be 12.

$$\text{Therefore } Z_{\text{revised}} = 12 \times 43 = 516$$

$$\text{Commutator or average pitch } Y_c \text{ for simplex wave winding} = \frac{C \pm 1}{p}$$

and must be an integer

$$\text{Number of coils } C = \frac{Z}{2 \times \text{turns per coil}} = \frac{516}{2 \times 1} = 258 \text{ with single turn coils (assumed).}$$

$C = 258$ leads to an asymmetrical, unbalanced or a wave winding with a dummy coil because 258 is a multiple of number of pair of poles.

If a two turn coil is assumed, then the number coils will be $\frac{516}{2 \times 2} = 129$ and leads to a

symmetrical, balanced or a wave winding without a dummy coil because 129 is not a multiple of number of pair of poles.

$$\text{Therefore } Y_C = \frac{C \pm 1}{p} = \frac{129 \pm 1}{2} = 65 \text{ or } 64$$

If $Y_C = 65$, $Y_B = 65$ and $Y_F = 65$

If $Y_C = 64$, $Y_B = Y_C \pm 1 = 64 \pm 1 = 63 \text{ or } 65$

With $Y_B = 63$, $Y_F = Y_B \pm 2 = 63 \pm 2 = 65$ so that $\frac{Y_B + Y_F}{2} = Y_C = 64$

With $Y_B = 65$, $Y_F = Y_B \pm 2 = 65 \pm 2 = 63$ so that $\frac{Y_B + Y_F}{2} = Y_C = 64$

$$\text{Cross-sectional area of the conductor } a = \frac{I_a}{A \delta} = \frac{126.1}{2 \times 5} = 12.61 \text{ mm}^2$$

For the above area a round or rectangular conductor can be used.

Example: 8

For a preliminary design of a 1500kW, 275V, 300rpm, dc shunt generator determine the number of poles, armature diameter and core length, number of slots and number of conductors per slot. Assume:

Average flux density over the pole arc as 0.85T, Output coefficient 276, Efficiency 0.91. Slot loading should not exceed 1500A.

$$I_a = I_L + I_{Sh} \approx I_L = \frac{1500 \times 10^3}{275} = 5454.5 \text{ A}$$

In order that the current per path is not more than about 200A, a simplex wave winding cannot be used. Obviously, a lap winding has to be used having number of parallel paths

$$A = \frac{5454.5}{200} \approx 27.27.$$

Since the number of parallel paths must be an even integer, it can be 26 or 28. Let $A = 28$. Therefore, with a simplex lap winding considered $P = 28$.

Check: $f = \frac{PN}{120} = \frac{28 \times 300}{120} = 70\text{Hz}$ and is on the higher side as frequency generally considered between 25 and 50Hz. In order to reduce the frequency and to have $A = 28$, a duplex lap winding can be used with $P = 14$ and $f = 35\text{Hz}$.

$$D^2L = \frac{\text{kW}}{1.64 \times 10^{-4} B_{av} q N} = \frac{\text{kW}}{C_o N}$$

$$\text{Power developed in the armature in kW} = \frac{\text{output}}{\eta} = \frac{1500}{0.91} = 1648.1$$

[Note: a) When the speed is in rpm in the expression, $D^2L = \frac{\text{kW}}{C_o N}$, the output coefficient

$$C_o = 1.64 \times 10^{-4} B_{av} q \text{ lies between}$$

$$1.64 \times 10^{-4} \times 0.45 \times 15000 = 1.0 \text{ and } 1.64 \times 10^{-4} \times 0.75 \times 50000 = 6.0.$$

On the other hand if the speed is in rpm, $C_o = \pi^2 \times 10^{-3} B_{av} q$ lies between

$$\pi^2 \times 10^{-3} \times 0.45 \times 15000 = 60 \text{ and } \pi^2 \times 10^{-3} \times 0.75 \times 50000 = 360.$$

Since the given value of C_o lies in the range of 60 and 360, the speed must be in rps when substituted in the output equation.]

$$D^2L = \frac{1648.4}{276 \times \frac{300}{60}} \approx 1.2\text{m}^3$$

[Note: In order to split up D^2L product into D and L , a value for $\frac{L}{\tau}$ ratio or peripheral velocity has to be assumed.]

$$\text{Let } L = \tau, \text{ Therefore, } L = \frac{\pi D}{14} = 0.23D$$

$$D^3 = \frac{1.2}{0.23} = 5.2\text{m}^3 \text{ and therefore } D = 1.7\text{m and } L = 0.23 \times 1.7 \approx 0.39\text{m}$$

Since the minimum number of slots per pole is 9 from commutation point of view, minimum number of slots for the machine is

$$= 9 \times 44 = 126 \text{ -----(1)}$$

Since slot pitch $\lambda_s = \frac{\pi D}{S}$, lies between 2.5 and 3.5 cm, the number of slots

lies between $\frac{\pi \times 170}{3.5} \approx 152$ and $\frac{\pi \times 170}{2.5} \approx 214$ -----(2)

From 1 and 2 the number of slots lies between 152 and 214.

Since a lap winding is assumed and for a lap winding the number of slots may be multiple of number of poles i.e. 14 or multiple of number of pair poles i.e. 7, let the number of slots $S = 196$.

Number of armature conductors $Z = \frac{E60A}{\phi NP}$

$$\begin{aligned} \text{Flux per pole } \phi &= \frac{B_{av} \pi D L}{P} = \frac{\psi B_g \pi D L}{P} \\ &= \frac{0.7 \times 0.85 \times \pi \times 1.7 \times 0.39}{4} = 0.089 \text{ Wb with the assumption } \Psi = 0.7 \end{aligned}$$

$$Z = \frac{60 \times 275 \times 28}{0.089 \times 300 \times 14} \approx 1236$$

Conductors per slot = $\frac{1236}{196} = 6.3$ and is not possible as Conductors per slot must always be

an even integer for a double layer winding. Let the number Conductors per slot be 6.

Therefore $Z_{\text{revised}} = 196 \times 6 = 1176$.

Slot loading = conductors per slot \times current through the conductor i.e. $\frac{I_a}{A}$

$$= 6 \times \frac{5454.5}{28} = 1168.8 < 1500 \text{ and the condition is satisfied.}$$

Example: 9

A 150hp, 500V, 6pole, 450rpm, dc shunt motor has the following data.

Armature diameter = 54cm, length of armature core = 24.5cm, average flux density in the air gap = 0.55T, number of ducts = 2, width of each duct = 1.0cm, stacking factor = 0.92.

Obtain the number of armature slots and work the details of a suitable armature winding. Also determine the dimensions of the slot. The flux density in the tooth at one third height from the root should not exceed 2.1T.

$$I_a = \frac{hp \times 746}{\eta \times V} = \frac{150 \times 746}{0.9 \times 500} = 248.7 \text{ A}$$

For this armature current, a lap or wave winding may be used. Let a lap winding be used.

Since the minimum number of slots per pole is 9 from commutation point of view, minimum number of slots for the machine is

$$= 9 \times 6 = 54 \text{ -----(1)}$$

Since slot pitch $\lambda_s = \frac{\pi D}{S}$, lies between 2.5 and 3.5 cm, the number of slots

$$\text{lies between } \frac{\pi \times 54}{3.5} \approx 48 \text{ and } \frac{\pi \times 54}{2.5} \approx 68 \text{ -----(2)}$$

From 1 and 2 the number of slots lies between 54 and 68.

Since a lap winding is assumed and for a lap winding the number of slots may be multiple of number of poles i.e. 6 or multiple of number of pair poles i.e. 3, let the number of slots $S = 60$.

Details of winding: Type of winding (already fixed), number of slots (also fixed), number of conductors, cross-sectional area of the conductor, back pitch, front pitch etc.

$$\text{Flux per pole } \phi = \frac{B_{av} \pi D L}{P} = \frac{0.55 \times \pi \times 0.54 \times 0.245}{6} = 0.038 \text{ Wb}$$

$$\text{Since a lap winding is assumed, } Z = \frac{60 \times 500 \times 6}{0.038 \times 450 \times 6} \approx 1754$$

$$\text{Conductors per slot} = \frac{1754}{60} = 29.2 \text{ and is not possible}$$

let the number Conductors per slot be 30. Therefore $Z_{\text{revised}} = 30 \times 60 = 1800$.

$$\text{Back pitch } Y_B = \frac{Z}{P} \pm K = \frac{1800}{6} \pm K = \frac{1800}{6} \pm 1 = 301 \text{ say.}$$

$$\text{Front pitch } Y_F = Y_B \pm 2 = 301 \pm 2 = 299 \text{ for a progressive winding}$$

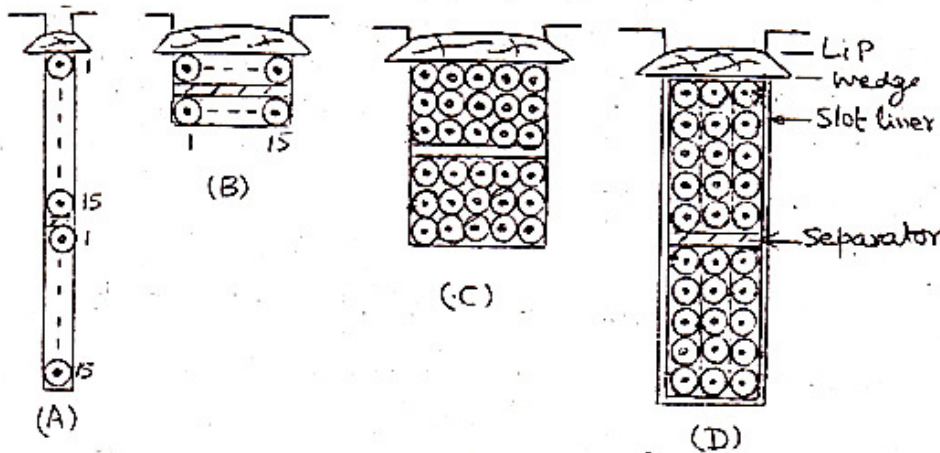
Since the current density lies between 4.5 and 7 A/mm², let it be 5 A/mm²

$$\text{Cross-sectional area of the conductor } a = \frac{I_a}{A \delta} = \frac{248.7}{6 \times 5} = 8.3 \text{ mm}^2$$

Since a is less than 10 mm², let a round conductor be used of bare diameter

$$\sqrt{\frac{4a}{\pi}} = \sqrt{\frac{4 \times 8.3}{\pi}} = 3.25\text{mm}$$

Since only double layer winding is used for a dc machine, number of conductor per layer is $(30/2) = 15$ and can be arranged in any one of the following ways.



Conductor of 3.25mm diameter.
Insulation on the conductor, enamel coating of 0.075mm thick.

Insulation on the coil side

(A)- 15 conductors are arranged one below the other in each layer. Slot is not proportionate.

(B)- 15 conductors are arranged one beside the other in each layer. Slot is not proportionate.

(C) and (D) - arrangement of conductors as shown in (C) or (D), leads to proportionate slots

If the conductors are arranged as shown in (D), then,

Slot width $b_s = (\text{diameter of the conductor} + \text{insulation on it}) \times \text{number of conductors along the}$

slot width + (insulation over the coil side or group of coil sides + slot liner
+ clearance)

$$= (3.25 + 2 \times 0.075)3 + 1.5$$

$$= 11.7\text{mm}$$

Slot depth $h_t = (\text{diameter of the conductor} + \text{insulation on it}) \times \text{number of conductors along the slot depth} + (\text{insulation over the coil side or group of coil sides} + \text{slot liner}$

+ separator + clearance) + wedge 3 to 5 mm + lip 1 or 2mm.

$$= (3.25 + 2 \times 0.075)10 + 4 + 4 + 2$$

$$= 44\text{mm}$$

Flux density in the tooth at one third height from the root of the tooth $B_{t_{1/3}} = \frac{\varphi}{b_{t_{1/3}} \times L_i \times \frac{S}{P}}$

width of the tooth at $\frac{1}{3}$ height from the root of the tooth $b_{t_{1/3}}$

$$= \frac{\pi \left(D - \frac{4h_t}{3} \right)}{S} - b_s = \frac{\pi \left(54 - \frac{4 \times 4.4}{3} \right)}{60} - 1.17 = 1.35\text{cm}$$

Net iron length $L_i = K_i(L - n_v b_v) = 0.9(24.5 - 2 \times 1) = 20.7\text{cm}$

$$\text{Therefore } B_{t_{1/3}} = \frac{0.038}{0.0135 \times 0.207 \times \frac{60}{6}} = 1.36T.$$

$B_{t_{1/3}} = 1.36T$ is the average flux density in the tooth

with assumption, that the flux per pole is uniformly distributed in all the teeth under one pole.

However because of higher value of flux under pole arc, the flux density in the tooth will be more than the average value. Thus maximum value of the average flux density ($B_{t_{1/3}}\bigg|_{\text{max}}$)

can be taken as $\frac{\left(B_{t_{1/3}} \right)_{\text{ave}}}{\text{Field form factor or per unit enclosure}}$

If per unit enclosure ψ is taken as 0.7, then, $(B_{t_{1/3}})_{\max} = \frac{1.36}{0.7} = 1.94\text{T}$ and less than 2.1T

Example:10

A 250kW, 500V, 6pole, 600rpm, dc generator is built with an armature of 0.75m and core length of 0.3m. The lap connected armature has 720 conductors. Using the data obtained from this machine, determine the armature diameter, core length, number of armature slots, armature conductors and commutator segments for a 350kW, 440V, 720 rpm, 6pole dc generator.

Assume a square pole face with ratio pole arc to pole pitch equal to 0.66. The full load efficiency is 0.91 and the internal voltage drop is 4% of rated voltage. The diameter of commutator is 0.7 of armature diameter. The pitch of the commutator segments should not be less than 4mm. The voltage between adjacent segments should not exceed 15V at no load.

[Note: In order to determine the details of 350kW machine, B_{av} and q are required. Since the data of 250kW machine is to be used for the second machine, B_{av} , q and C_o are same for both the machines. The data square pole, efficiency, voltage drop can assumed to be applicable to both machines.]

I machine:

$$q = \frac{I_a Z}{A \pi D}$$

$$I_a = \frac{\text{Power developed in the armature}}{\text{induced emf}}$$

$$\text{Power developed in the armature} = \frac{\text{Output}}{\text{Efficiency}} = \frac{250}{0.91} = 274.7\text{kW}$$

$$\text{emf induced } E = 500 + 0.04 \times 500 = 520\text{V}$$

$$I_a = \frac{274.7 \times 10^3}{520} = 528.3\text{A}$$

$$q = \frac{528.3 \times 720}{6 \times \pi \times 0.75} \approx 26906 \text{ ampere-conductors per meter}$$

$$B_{av} = \frac{P\phi}{\pi DL}$$

$$\phi = \frac{60EA}{ZNP} = \frac{60 \times 520 \times 6}{720 \times 600 \times 6} \approx 0.072\text{Wb}$$

$$B_{av} = \frac{6 \times 0.072}{\pi \times 0.75 \times 0.3} = 0.61\text{T}$$

OR

$$B_{av} = \frac{kW}{1.64 \times 10^{-4} q N D^2 L} = \frac{274.7}{1.64 \times 10^{-4} \times 26906 \times 600 \times 0.75^2 \times 0.3} = 0.61T$$

II machine:

$$D^2 L = \frac{kW}{1.64 \times 10^{-4} B_{av} q N} = \frac{350/0.91}{1.64 \times 10^{-4} \times 0.61 \times 26906 \times 720} \approx 0.195$$

For a square pole face, $L = \Psi \tau = 0.66 \times \pi D/6 = 0.35D$

$$D^3 = \frac{0.195}{0.35} = 0.564 \text{m}^3$$

Therefore $D = 0.83\text{m}$ and $L = 0.35 \times 0.83 \approx 0.29\text{m}$

Since the minimum number of slots per pole is 9 from commutation point of view, minimum number of slots for the machine is

$$= 9 \times 6 = 54 \text{ -----(1)}$$

Since slot pitch $\lambda_s = \frac{\pi D}{S}$, lies between 2.5 and 3.5 cm, the number of slots

$$\text{lies between } \frac{\pi \times 83}{3.5} \approx 74 \text{ and } \frac{\pi \times 83}{2.5} \approx 104 \text{ -----(2)}$$

From 1 and 2 the number of slots lies between 74 and 104.

If a lap winding is assumed then for a lap winding the number of slots may be multiple of number of poles i.e. 6 or multiple of number of pair poles i.e. 3, let the number of slots $S = 78$

$$\text{Number of armature conductors } Z = \frac{E 60 A}{\phi N P}$$

.

$$\text{emf induced } E = 440 + 0.04 \times 440 = 457.6V$$

$$\text{Flux per pole } \phi = \frac{B_{av} \pi D L}{P} = \frac{0.61 \times \pi \times 0.83 \times 0.29}{6} = 0.078 \text{Wb}$$

$$Z = \frac{60 \times 457.6 \times 6}{0.078 \times 720 \times 6} \approx 488$$

$$\text{Conductors per slot} = \frac{488}{78} = 6.2 \text{ and is not possible. Let it be 6}$$

$$Z_{\text{revised}} = 78 \times 6 = 468$$

$$\text{Conductors /layer} = \frac{6}{3} = 3$$

Three conductors /layer is possible only with 3 coil sides/layer of single turn coil or one coil

Side/layer of 3 turn coil. If a single turn coil is assumed, then the number of coils = $\frac{468}{2} = 234$.

There the number of segments = 234.

Check:

$$\text{Commutator segment pitch } \tau_c = \frac{\pi D_c}{\text{number of segments}}$$

$$\text{Commutator diameter } D_c = 0.7 \times 83 = 58.1 \text{ cm}$$

$$\tau_c = \frac{\pi \times 58.1}{234} = 0.78 \text{ cm} > 4 \text{ mm}$$

$$\text{Voltage between segments pitch} = \frac{\text{Open circuit voltage}}{\text{number of } \frac{\text{segments}}{\text{pole}}} = \frac{457.6}{234/6} = 11.7 \text{ V} < 15 \text{ V}$$

Example:11

$$\text{Prove that the output equation } W = \frac{30 a E_b q v}{p N} \text{ watts}$$

where a = number of parallel paths in pairs

E_b = average voltage between commutator segments

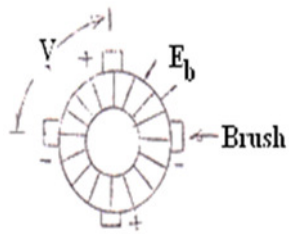
q = ampere – conductors per metre of periphery

v = periphery velocity in m/s

p = number of pairs of poles

N = speed in rpm

Assume the machine to be lap wound with single turn coils.



End view of the
commutator

Output W = terminal voltage \times load current

$$\begin{aligned}
 &= E_b \times (\text{number of } E_b' \text{ s between positive and negative brushes} \\
 &\quad \text{or number of segments between brushes or number of segments per pole} \\
 &\quad \text{or number of coils per pole}) \times I_L \\
 &= E_b \times (\text{number coils per pole} \times I_L
 \end{aligned}$$

$$\begin{aligned}
 &= E_b \times \frac{\frac{Z}{(2 \times \text{number coils per pole})}}{P} I_L \\
 &= \frac{E_b}{2p} \times \frac{Z I_L}{2 \times 1} \text{ as } p = \frac{P}{2} \text{ and the turn of the coil is one} \dots \dots \dots (1)
 \end{aligned}$$

$$\text{since } q = \frac{I_a Z}{A \pi D} \approx \frac{I_L Z}{A \pi D}, I_L Z = q 2 a \pi D \text{ as } a = \frac{A}{2} \dots \dots \dots (2)$$

$$v = \frac{\pi D N}{60}, \approx \pi D = \frac{60 v}{N}, \dots \dots \dots (3)$$

Substitution of equations 2 and 3 in 1 lead to

$$W = \frac{E_b}{2p} \times \frac{q 2 a 60 v / N}{2}$$

$$W = \frac{30 a E_b q v}{p N} \text{ watts}$$

Note: In case of lap winding, number of brushes is equal to number of poles.

Example: 12

Determine the main dimensions, number of poles, number of conductors per slot and air gap length subjecting the results to design checks for a 600kW, 500V 900rpm dc generator. Assume:

Average flux density 0.6T, ampere-conductors per metre 35000. The ratio pole arc to pole pitch 0.67, efficiency 91%.

Peripheral velocity should not exceed 40m/s

Armature mmf per pole should be below 7500A

Current per brush arm should not exceed 400A

Frequency of the flux reversal should not exceed 50Hz

The mmf required for air gap is 50% of armature mmf and gap contraction factor is 1.15.

$$\text{Armature current } I_a \approx I_L = \frac{600 \times 10^3}{300} = 1200 \text{ A}$$

In order that current per path is not greater than 200A or current per brush arm is not greater than 400A, a lap winding is to be used having parallel paths,

$$A = \frac{1200}{200} = 6$$

Since for a simplex lap winding, $A=P$, $P=6$

$$\text{Check: Frequency } f = \frac{PN}{120} = \frac{6 \times 900}{120} = 45 < 50 \text{ Hz and the condition is satisfied.}$$

$$D^2 L = \frac{kW}{1.64 \times 10^{-4} B_{av} q N}$$

$$\text{Power developed in kW} = \frac{\text{Output}}{\eta} = \frac{600}{0.91} = 659.4$$

$$D^2 L = \frac{659.4}{1.64 \times 10^{-4} \times 0.6 \times 3500 \times 900} = 0.213 \text{ m}^3$$

Since the peripheral velocity should not be greater than 40 m/s, let it be 36 m/s.

$$\text{Since } v = \frac{\pi D N}{60}, D = \frac{60v}{\pi N} = \frac{60 \times 36}{\pi \times 900} = 0.76 \text{ m}$$

$$L = \frac{0.213}{0.76^2} = 0.28 \text{ m}$$

Since the minimum number of slots per pole is 9 from commutation point of view, minimum number of slots for the machine is

$$= 9 \times 6 = 54 \text{ -----(1)}$$

Since slot pitch $\lambda_s = \frac{\pi D}{S}$, lies between 2.5 and 3.5 cm, the number of slots

$$\text{lies between } \frac{\pi \times 76}{3.5} \approx 68 \text{ and } \frac{\pi \times 76}{2.5} \approx 96 \text{ -----(2)}$$

From 1 and 2 the number of slots lies between 68 and 96.

Since for a lap winding the number of slots may be multiple of number of poles i.e. 6 or multiple of number of pair poles i.e. 3, let the number of slots $S = 72$.

$$\text{Number of armature conductors } Z = \frac{E60A}{\phi NP}$$

$$\phi = \frac{B_{av}\pi DL}{P} = \frac{0.6 \times \pi \times 0.76 \times 0.28}{6} = 0.067 \text{ Wb}$$

$$Z = \frac{60 \times 500 \times 6}{0.067 \times 900 \times 6} \approx 498$$

$$\text{Conductors per slot} = \frac{498}{72} = 6.9 \text{ and is not possible. Let it be 6.}$$

$$Z_{\text{revised}} = 72 \times 6 = 432$$

Check:

$$\text{Armature mmf per pole } AT_a / \text{pole} = \frac{I_a Z}{2AP} = \frac{1200 \times 432}{2 \times 6 \times 6} = 7200 \text{ A} < 7500 \text{ A}$$

and the condition is satisfied.

$$\text{Air gap mmf } AT_g = 800000 l_g K_g B_g = 0.5 AT_a / \text{pole}$$

Carter's gap expansion or contraction coefficient $K_g = 1.15$

$$\text{Maximum gap density } B_g = \frac{B_{av}}{\Psi} = \frac{0.6}{0.67} = 0.895 \text{ T}$$

$$\text{Air gap length } l_g = \frac{0.5 \times 7200}{800000 \times 1.15 \times 0.895} = 0.0043 \text{ m}$$
