LAKSHYA (JEE)

Electrostatic Potential & Capacitance

DPP-07

1. Electric potential is given by

$$V = 6x - 8xy^2 - 8y + 6yz - 4z^2$$

Then electric force acting on $2C$ point charge placed on origin will be

(a) $2N$ (b) $6N$

(c) $8N$ (d) $20\,N$

2. The electric field in a region is radially outward with magnitude $E = A\gamma_0$. The charge contained in a sphere of radius γ_0 centered at the origin is

(a) $\frac{1}{4\pi\varepsilon_0}A\gamma_0^3$ (b) $4\pi\varepsilon_0 A\gamma_0^3$

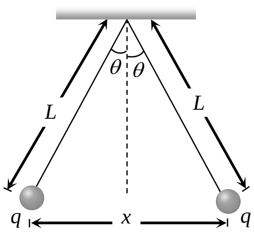
(c) $\frac{4\pi\varepsilon_0 A}{\gamma_0}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{A}{\gamma_0^3}$

3. Charge q is uniformly distributed over a thin half ring of radius R. The electric field at the centre of the ring is

(a) $\frac{q}{2\pi^2\varepsilon_0 R^2}$ (b) $\frac{q}{4\pi^2\varepsilon_0 R^2}$

(c) $\frac{q}{4\pi\varepsilon_0 R^2}$ (d) $\frac{q}{2\pi\varepsilon_0 R^2}$

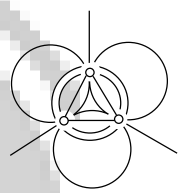
4. In the given figure two tiny conducting balls of identical mass m and identical charge q hang from non-conducting threads of equal length L. Assume that θ is so small that $\tan\theta \approx \sin\theta$, then for equilibrium x is equal to

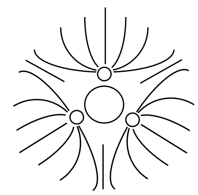


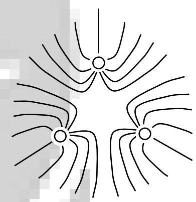
(a) $\left(\frac{q^2L}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$ (b) $\left(\frac{qL^2}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$

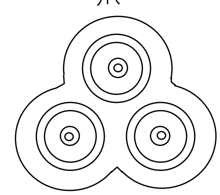
(c) $\left(\frac{q^2L^2}{4\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$ (d) $\left(\frac{q^2L}{4\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$

5. Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in

(a) 

(b) 

(c) 

(d) 

6. A uniform electric field pointing in positive x-direction exists in a region. Let A be the origin, B be the point on the x-axis at $x = +1$ cm and C be the point on the y-axis at $y = +1$ cm. Then the potentials at the points A, B and C satisfy

(a) $V_A < V_B$ (b) $V_A > V_B$

(c) $V_A < V_C$ (d) $V_A > V_C$

7. There is a uniform electric field of strength $10^3\,V/m$ along y-axis. A body of mass $1g$ and charge $10^{-6}C$ is projected into the field from origin along the positive x-axis with a velocity 10 m/s. Its speed in m/s after $10s$ is (Neglect gravitation)

(a) 10 (b) $5\sqrt{2}$

(c) $10\sqrt{2}$ (d) 20

8. The electric potential at a point (x, y) in the $x-y$ plane is given by $V = -kxy$. The field intensity at a distance r from the origin varies as

(a) r^2 (b) r

(c) $\frac{1}{r}$ (d) $\frac{1}{r^2}$

9. Two equal charges are separated by a distance d. A third charge placed on a perpendicular bisector at x distance will experience maximum coulomb force when

(a) $x = \frac{d}{\sqrt{2}}$ (b) $x = \frac{d}{2}$

(c) $x = \frac{d}{2\sqrt{2}}$ (d) $x = \frac{d}{2\sqrt{3}}$

10. Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of Q, when it is displaced by a small distance x along the x-axis, is approximately proportional to

(a) x (b) x^2

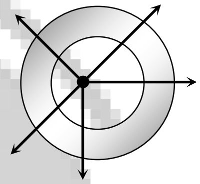
(c) x^3 (d) $1/x$

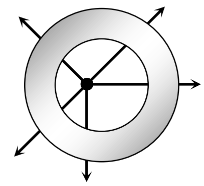
11. An elementary particle of mass m and charge $+e$ is projected with velocity v at a much more massive particle of charge Ze, where $Z > 0$. What is the closest possible approach of the incident particle

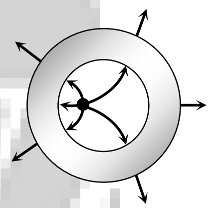
(a) $\frac{Ze^2}{2\pi\varepsilon_0 mv^2}$ (b) $\frac{Ze}{4\pi\varepsilon_0 mv^2}$

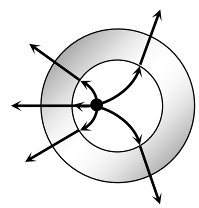
(c) $\frac{Ze^2}{8\pi\varepsilon_0 mv^2}$ (d) $\frac{Ze}{8\pi\varepsilon_0 mv^2}$

12. A metallic shell has a point charge 'q' kept inside its cavity. Which one of the following diagrams correctly represents the electric lines of forces

(a) 

(b) 

(c) 

(d) 

ANSWER KEY

1. (d)
2. (b)
3. (a)
4. (a)
5. (c)
6. (b)
7. (c)
8. (b)
9. (c)
10. (b)
11. (a)
12. (c)

HINTS AND SOLUTIONS

1. (d)

$$E_x = -\frac{dV}{dx} = -(6-8y^2),$$

$$E_y = -\frac{dV}{dy} = -(-16xy - 8 + 6z)$$

$$E_z = -\frac{dV}{dz} = -(6y - 8z)$$

At origin $x = y = z = 0$ so, $E_x = -6, E_y = 8$ and $E_z = 0$

$$\Rightarrow E = \sqrt{E_x^2 + E_y^2} = 10\,N/C.$$

Hence force $F = QE = 2 \times 10 = 20\,N$

2. (b)

Flux linked with the given sphere $\varphi = \frac{Q}{\varepsilon_o}$;

where Q = Charge enclosed by the sphere.

Hence $Q = \phi\varepsilon_0 = (EA)\varepsilon_0$

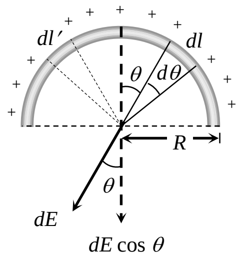
$\Rightarrow Q = 4\pi(\gamma_0)^2 \times A\gamma_0\varepsilon_0 = 4\pi\varepsilon_0 A\gamma_0^3$.

3. (a)

From figure $dl = R\,d\theta$;

Charge on $dl = \lambda R\,d\theta$ $\quad \left\{\lambda = \frac{q}{\pi R}\right\}$

Electric field at centre due to dl is $dE = k.\frac{\lambda R d\theta}{R^2}$.



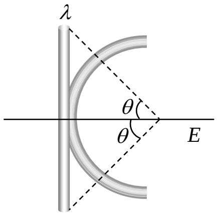
We need to consider only the component $dE\cos\theta$, as the component dE sin θ will cancel out because of the field at C due to the symmetrical element dl'.

Total field at centre $= 2\int_0^{\pi/2} dE\cos\theta$

$$= \frac{2k\lambda}{R}\int_0^{\pi/2}\cos\theta\, d\theta = \frac{2k\lambda}{R} = \frac{q}{2\pi^2\varepsilon_0 R^2}$$

Alternate method : As we know that electric field due to a finite length charged wire on it's perpendicular bisector is given by $E = \frac{2k\lambda}{R}\sin\theta$.

If it is bent in the form of a semicircle then θ = 90°

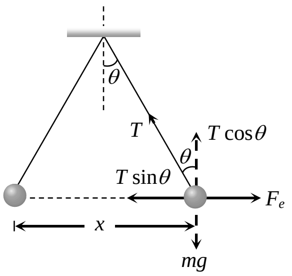


$$\Rightarrow E = \frac{2k\lambda}{R}$$

$$= 2 \times \frac{1}{4\pi\varepsilon_0}\left(\frac{q/\pi R}{R}\right)$$

$$= \frac{q}{2\pi^2\varepsilon_0 R^2}$$

4. (a)



In equilibrium $F_e = T\sin\theta$ .... (i)

$mg = T\cos\theta$ .... (ii)

$$\tan\theta = \frac{F_e}{mg} = \frac{q^2}{4\pi\varepsilon_o x^2 \times mg}$$

also $\tan\theta \approx \sin\theta = \frac{x/2}{L}$

Hence $\frac{x}{2L} = \frac{q^2}{4\pi\varepsilon_o x^2 \times mg}$

$$\Rightarrow x^3 = \frac{2q^2L}{4\pi\varepsilon_o mg} \Rightarrow x = \left(\frac{q^2L}{2\pi\varepsilon_o mg}\right)^{1/3}$$

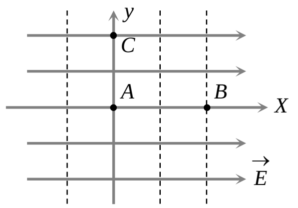
5. (c)

Option (a) shows lines of force starting from one positive charge and terminating at another. Option (b) has one line of force making closed loop. Option (d) shows all lines making closed loops. All these are not correct. Only option (c) is correct.

6. **(b)**

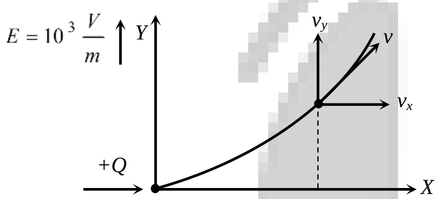
Potential decreases in the direction of electric field. Dotted lines are equipotential lines

$\therefore\ V_A = V_C$ and $V_A > V_B$



7. **(c)**

Body moves along the parabolic path.



For vertical motion : By using $v = u + at$

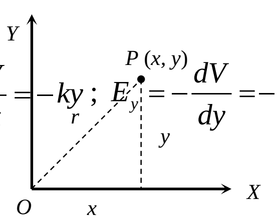
$$\Rightarrow v_y = 0 + \frac{QE}{m}.t = \frac{10^{-6} \times 10^3}{10^{-3}} \times 10 = 10\, m/\text{sec}$$

For horizontal motion – It's horizontal velocity remains the same *i.e.* after 10 *sec*, horizontal velocity of body v_x = 10 *m/sec*.

Velocity after 10 *sec* $v = \sqrt{v_x^2 + v_y^2}$

$= 10\sqrt{2}\, m/sec$

8. **(b)**



$$E_x = -\frac{dV}{dx} = -ky\ ;\ E_y = -\frac{dV}{dy} = -kx$$

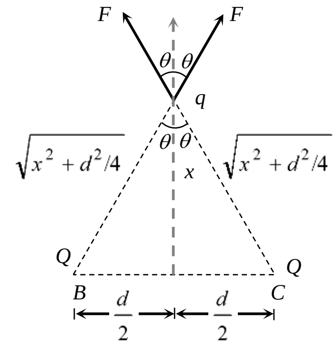
$$\Rightarrow E = \sqrt{E_x^2 + E_y^2} = k\sqrt{x^2 + y^2} = kr \Rightarrow E \propto r$$

9. **(c)**

Suppose third charge is similar to Q and it is q

So net force on it

$F_{net} = 2F\cos\theta$



Where $F = \frac{1}{4\pi\varepsilon_0} . \frac{Qq}{\left(x^2 + \frac{d^2}{4}\right)}$ and

$$\cos\theta = \frac{x}{\sqrt{x^2 + \frac{d^2}{4}}}$$

$$\therefore F_{net} = 2 \times \frac{1}{4\pi\varepsilon_0} . \frac{Qq}{\left(x^2 + \frac{d^2}{4}\right)} \times \frac{x}{\left(x^2 + \frac{d^2}{4}\right)^{1/2}}$$

$$= \frac{2Qqx}{4\pi\varepsilon_0\left(x^2 + \frac{d^2}{4}\right)^{3/2}}$$

For F_{net} to be maximum $\frac{dF_{net}}{dx} = 0$

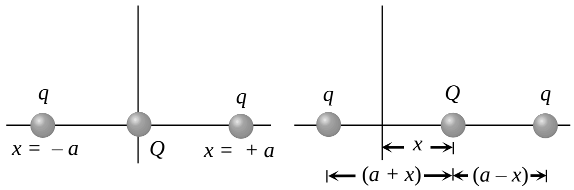
$$i.e.\ \frac{d}{dx}\left[\frac{2Qqx}{4\pi\varepsilon_0\left(x^2 + \frac{d^2}{4}\right)^{3/2}}\right] = 0$$

$$\text{or}\ \left[\left(x^2 + \frac{d^2}{4}\right)^{-3/2} - 3x^2\left(x^2 + \frac{d^2}{4}\right)^{-5/2}\right] = 0$$

$$i.e.\ \ x = \pm\frac{d}{2\sqrt{2}}$$

10. **(b)**

Initially according to figure (i) potential energy of Q is $U_i = \frac{2kqQ}{a}$ ......(i)



(i) (ii)

According to figure (ii) when charge Q is displaced by small distance x then it's potential energy now

$$U_f = kqQ\left[\frac{1}{(a+x)} + \frac{1}{(a-x)}\right] = \frac{2kqQa}{(a^2 - x^2)}$$

.......(ii)

Hence change in potential energy

$$\Delta U = U_f - U_i = 2kqQ\left[\frac{a}{a^2 - x^2} - \frac{1}{a}\right]$$

$$= \frac{2kqQx^2}{(a^2 - x^2)}$$

Since $x << a$ so $\Delta U = \dfrac{2kqQx^2}{a^2} \Rightarrow \Delta U \propto x^2$

11. **(a)**

Suppose distance of closest approach is r, and according to energy conservation applied for elementary charge.

Energy at the time of projection = Energy at the distance of closest approach

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}.\frac{(Ze).e}{r} \Rightarrow r = \frac{Ze^2}{2\pi\varepsilon_0 mv^2}$$

12. **(c)**

Electric field is perpendicular to the equipotential surface and is zero every where inside the metal.