LAKSHYA (JEE)

Electrostatic Potential & Capacitance

DPP-06

1. Two equal negative charge $-q$ are fixed at the fixed points $(0, a)$ and $(0, -a)$ on the Y-axis. A positive charge Q is released from rest at the point $(2a, 0)$ on the X-axis. The charge Q will

(a) Execute simple harmonic motion about the origin
(b) Move to the origin and remain at rest
(c) Move to infinity
(d) Execute oscillatory but not simple harmonic motion

2. An electric line of force in the xy plane is given by equation $x^2 + y^2 = 1$. A particle with unit positive charge, initially at rest at the point $x = 1, y = 0$ in the xy plane

(a) Not move at all
(b) Will move along straight line
(c) Will move along the circular line of force
(d) Information is insufficient to draw any conclusion

3. A solid metallic sphere has a charge $+3Q$. Concentric with this sphere is a conducting spherical shell having charge $-Q$. The radius of the sphere is a and that of the spherical shell is $b(b > a)$. What is the electric field at a distance $R(a < R < b)$ from the center

(a) $\frac{Q}{2\pi\varepsilon_0 R}$ (b) $\frac{3Q}{2\pi\varepsilon_0 R}$

(c) $\frac{3Q}{4\pi\varepsilon_0 R^2}$ (d) $\frac{4Q}{4\pi\varepsilon_0 R^2}$

4. If on the concentric hollow spheres of radii r and $R(> r)$ the charge Q is distributed such that their surface densities are same then the potential at their common centre is

(a) $\frac{Q(R^2 + r^2)}{4\pi\varepsilon_0 (R + r)}$ (b) $\frac{QR}{R + r}$

(c) Zero (d) $\frac{Q(R + r)}{4\pi\varepsilon_0 (R^2 + r^2)}$

5. Two equal charges q of opposite sign separated by a distance $2a$ constitute an electric dipole of dipole moment p. If P is a point at a distance R from the centre of the dipole and the line joining the centre of the dipole to this point makes an angle θ with the axis of the dipole, then the potential at p is given by $(r >> 2a)$ (Where $p = 2qa$)

(a) $V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$ (b) $V = \frac{p\cos\theta}{4\pi\varepsilon_0 r}$

(c) $V = \frac{p\sin\theta}{4\pi\varepsilon_0 r}$ (d) $V = \frac{p\cos\theta}{2\pi\varepsilon_0 r^2}$

6. A point charge q is placed at a distance $a/2$ directly above the centre of a square of side a. The electric flux through the square is

(a) $\frac{q}{\varepsilon_0}$ (b) $\frac{q}{\pi\varepsilon_0}$

(c) $\frac{q}{4\varepsilon_0}$ (d) $\frac{q}{6\varepsilon_0}$

7. Two infinitely long parallel wires having linear charge densities λ_1 and λ_2 respectively are placed at a distance of R metres. The force per unit length on either wire will be $\left(K = \frac{1}{4\pi\varepsilon_0}\right)$

(a) $K\frac{2\lambda_1\lambda_2}{R^2}$ (b) $K\frac{2\lambda_1\lambda_2}{R}$

(c) $K\frac{\lambda_1\lambda_2}{R^2}$ (d) $K\frac{\lambda_1\lambda_2}{R}$

8. A non-conducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its centre

(a) Increases as r increases for $r < R$
(b) Decreases as r increases for $0 < r < \infty$
(c) Decreases as r increases for $R < r < \infty$
(d) Is discontinuous at $r = R$

9. Two identical thin rings each of radius R meters are coaxially placed at a distance R meters apart. If Q_1 coulomb and Q_2 coulomb are respectively the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to that of other is

(a) Zero

(b) $\frac{q(Q_1 - Q_2)(\sqrt{2}-1)}{\sqrt{2}.4\pi\varepsilon_0 R}$

(c) $\frac{q\sqrt{2}(Q_1 + Q_2)}{4\pi\varepsilon_0 R}$

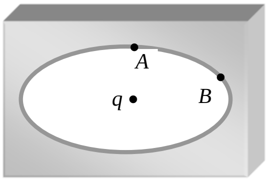
(d) $\frac{q(Q_1 + Q_2)(\sqrt{2}+1)}{\sqrt{2}.4\pi\varepsilon_0 R}$

10. A negatively charged plate has charge density of 2×10^{-6} C/m^2. The initial distance of an electron which is moving toward plate, cannot strike the plate, if it is having energy of 200 eV

(a) $1.77\,mm$ (b) $3.51\,mm$

(c) $1.77\,cm$ (d) $3.51\,cm$

11. An ellipsoidal cavity is carved within a perfect conductor. A positive charge q is placed at the centre of the cavity. The points A and B are on the cavity surface as shown in the figure. Then



(a) Electric field near A in the cavity = Electric field near B in the cavity

(b) Charge density at A = Charge density at B

(c) Potential at A = Potential at B

(d) Total electric field flux through the surface of the cavity is q/ε_0

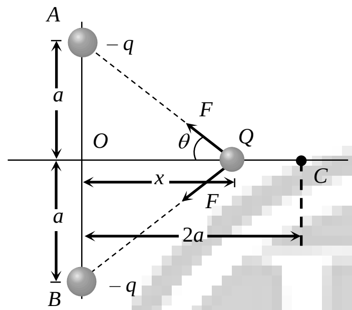
ANSWER KEY

1. **(d)**
2. **(c)**
3. **(c)**
4. **(d)**
5. **(a)**
6. **(d)**
7. **(b)**
8. **(a, c)**
9. **(b)**
10. **(a)**
11. **(c, d)**

HINTS AND SOLUTIONS

1. (d)

By symmetry of problem the components of force on Q due to charges at A and B along y-axis will cancel each other while along x-axis will add up and will be along CO. Under the action of this force charge Q will move towards O. If at any time charge Q is at a distance x from O. Net force on charge Q



$$F_{net} \Rightarrow 2F\cos\theta = 2\frac{1}{4\pi\varepsilon_0}\frac{-qQ}{(a^2+x^2)}\times\frac{x}{(a^2+x^2)^{1/2}}$$

i.e., $F_{net} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{2qQx}{\left(a^2+x^2\right)^{3/2}}$

As the restoring force F_{net} is not linear, motion will be oscillatory (with amplitude $2a$) but not simple harmonic.

2. (c)

Charge will move along the circular line of force because $x^2+y^2=1$ is the equation of circle in xy-plane.

3. (c)

Electric field at a distance R is only due to sphere because electric field due to shell inside it is always zero. Hence electric field

$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{3Q}{R^2}$

4. (d)

$q_1+q_2=Q$ and $\frac{q_1}{4\pi r^2}=\frac{q_2}{4\pi R^2}$ (given)

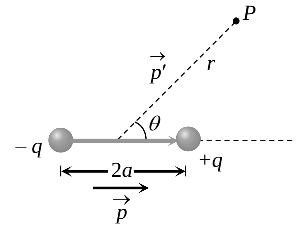
$q_1=\frac{Qr^2}{R^2+r^2}$ and $q_2=\frac{QR^2}{R^2+r^2}$

Potential at common centre

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{Qr^2}{(R^2+r^2)r}+\frac{QR^2}{(R^2+r^2)R}\right]=\frac{Q(R+r)}{4\pi\varepsilon_0(R^2+r^2)}$$

5. (a)

For the given situation, diagram can be drawn as follows



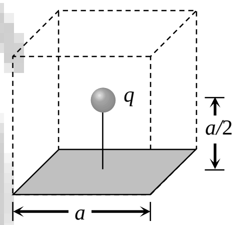
As shown in figure component of dipole moment along the line OP will be $p' = p\cos\theta$.

Hence electric potential at point P will be

$V=\frac{1}{4\pi\varepsilon_0}\cdot\frac{p\cos\theta}{r^2}$

6. (d)

An imaginary cube can be made by considering charge q at the centre and given square is one of it's face.

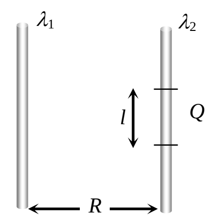


So flux from given square (i.e. one face)

$\phi=\frac{q}{6\varepsilon_0}$

7. (b)

Force on l length of the wire 2 is



$F_2=QE_1=(\lambda_2 l)\frac{2k\lambda_1}{R}$

$\Rightarrow \frac{F_2}{l}=\frac{2k\lambda_1\lambda_2}{R}$

Also $\frac{F_1}{l}=\frac{F_2}{l}=\frac{F}{l}=\frac{2k\lambda_1\lambda_2}{R}$

8. (a, c)

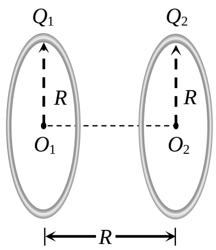
For non-conducting solid sphere $E_{in}\propto r$

and $E_{out}\propto\frac{1}{r^2}$

i.e. for $r<R$; E increases as r increases

and for $R<r<\infty$; E decreases as r increases

9. **(b)**



$W = q(V_{O_2} - V_{O_1})$

where $V_{O_1} = \frac{Q_1}{4\pi\varepsilon_0 R} + \frac{Q_2}{4\pi\varepsilon_0 R\sqrt{2}}$

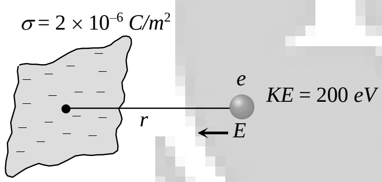
and $V_{O_2} = \frac{Q_2}{4\pi\varepsilon_0 R} + \frac{Q_1}{4\pi\varepsilon_0 R\sqrt{2}}$

$\Rightarrow V_{O_2} - V_{O_1} = \frac{(Q_2 - Q_1)}{4\pi\varepsilon_0 R}\left[1 - \frac{1}{\sqrt{2}}\right]$

So, $W = \frac{q.(Q_2 - Q_1)}{4\pi\varepsilon_0 R}\frac{(\sqrt{2}-1)}{\sqrt{2}}$

10. **(a)**

Let an electron is projected towards the plate from the r distance as shown in fig.



It will not strike the plate if and only if $KE \leq e(E \cdot r)$ (where E = Electric field due to charge plate $= \frac{\sigma}{2\varepsilon_0}$)

$\Rightarrow r \geq \frac{KE}{eE}$. Hence minimum value of r is given by

$$r = \frac{KE}{eE} = \frac{200\ eV}{e \times \frac{\sigma}{2\varepsilon_0}}$$

11. **(c, d)**

Under electrostatic condition, all points lying on the conductor are in same potential. Therefore, potential at A = potential at B.

From Gausss theorem, total flux through the surface of the cavity will be q/ε_0.

Note : ❑ Instead of an elliptical cavity, if it would had been a spherical cavity then options (a) and (b) were also correct.