The Welfare Effects of Transportation Infrastructure Improvements

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Motivation

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- Complicated problem:
  - Agents choose their routes: trade cost between any 2 locations possibly affected by infrastructure in between any 2 other nodes
  - General equilibrium effects on equilibrium trade flows, income, and population in all locations.

- State of the art: calibrate spatial model, use algorithm (Dijkstra, FMM) to calculate change in least cost routes from a given infrastructure investment, re-calculate equilibrium.

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Overview of this paper

- This paper: Propose a new analytically tractable procedure to determine the welfare impacts of transportation infrastructure investment.

1. Geography component:
   - Develop a new technique for modeling the endogenous trade costs that arise from trading over an infrastructure network.

2. Economic component:
   - Embed in GE spatial frameworks to derive analytical expression for welfare elasticity to changes in network.

3. Empirical component:
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• Use shipment-level data across U.S. cities to assess welfare impact of improving each section of U.S. Interstate Highway System.
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  - Eaton Kortum ’02 Frechet + graph theory = closed form mapping from infrastructure network to endogenous trade costs.

- Intuitive analytical expressions for:
  - Probability a trader going from $i$ to $j$ passes through the link from $k$ to $l$.
  - Elasticity of trade costs from $i$ to $j$ to an infrastructure improvement in link from $k$ to $l$.
  - Mean of distance traveled from $i$ to $j$ across traders (crucial for estimation).
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- Standard (Armington) general equilibrium spatial model.
  - Consider two variants: trade model with immobile labor and economic geography model with mobile labor.

- New methodology: Use dual "planner" formulation of the equilibrium conditions, apply the envelope theorem.
  - Combine with the geographic component to derive elasticity of aggregate welfare to any change in the transportation infrastructure network.
  - Simple and intuitive analytical expression: welfare elasticity equal to flows over an infrastructure link.
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- Use shipment level trade data between 67 U.S. cities to estimate structural parameters of the model.
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- Assess the welfare effect of improving each link in the U.S. Interstate Highway System.

- Most important highway in the U.S.: I-95 South from New York to Philadelphia. Reducing the travel time by 30 minutes would increase aggregate U.S. welfare by 0.02%.
Related literature

- Quantitative spatial models with exogenous trade costs:
  - Trade models: Anderson ’79, Krugman ’80, Eaton and Kortum ’02, Anderson and van Wincoop ’03, Dekle, Eaton and Kortum ’08
  - Economic geography models: Krugman ’91, Allen and Arkolakis ’14, Redding ’16

- Quantitative evaluation of existing infrastructure projects:
  - Donaldson ’12, Allen and Arkolakis ’14, Ahlfeldt et. al. ’15, Donaldson and Hornbeck ’16, Alder ’16.

- Quantitative optimal policy computationally:

- Designing transportation networks: Fajgelbaum and Schaal ’17.
  - Benefits of AA: analytical solutions, gravity framework \(\Rightarrow\) evaluate existing infrastructure using observed trade data.
  - Benefits of FS: globally concave objective with convex constraints (from congestion) \(\Rightarrow\) optimal transportation network from scratch.
Outline of Talk

Introduction

Geographic component: Endogenous transportation costs
  Setup
  Trade costs along the optimal route
  Properties of endogenous trade costs

Economic component: The welfare effect of improving infrastructure

Empirical component: The Interstate Highway Network

Next steps
Endogenous transportation costs: setup

- $N$ locations arrayed on a weighted network.

- Let $T \equiv \begin{bmatrix} t_{ij} \end{bmatrix}$ be the associated infrastructure matrix.
  - $t_{ij} \geq 1$ is the iceberg trade cost incurred by traveling directly from $i$ to $j$ on the infrastructure network.
  - If $t_{ij} \in [1, \infty)$, we say that $i$ and $j$ are connected and we call the connection a link from $i$ to $j$.

Notes:
- We do not impose symmetry, i.e. it is okay if $t_{ij} \neq t_{ji}$.
- We assume $t_{ii} = \infty$ (to avoid certain cycles).
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Example network
A path \( p \) from \( i \) to \( j \) is a sequence of locations beginning in location \( i \) and ending in location \( j \):

\[
\tau_{ij}(p) = \prod_{k=1}^{K-1} t_{p_k},
\]

Let the length of a path \( p \) be the number of elements of the sequence minus one (example: \( K = 4 \)).

The aggregate trade cost from \( i \) to \( j \) along path \( p = \{ p_0 = i, p_1, \ldots, p_K = j \} \) of length \( K \) is:

Let \( P_{ij}, K \) denote the set of all paths of length \( K \) from \( i \) to \( j \).
Endogenous transportation costs: setup (ctd.)

- A path $p$ from $i$ to $j$ is a sequence of locations beginning in location $i$ and ending in location $j$:
  - Example path from $i = 1$ to $j = 25$: $\{1, 7, 13, 19, 25\}$.
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Let $\mathcal{P}_{ij,K}$ denote the set of all paths of length $K$ from $i$ to $j$. 
Traders

- Assume a unit measure of perfectly competitive traders $\nu \in [0, 1]$ shipping goods from $i$ to $j$. 

- If trader $\nu$ takes path $p$ from $i$ to $j$, she incurs a total trade cost $\tilde{\tau}_{ij}(p) \varepsilon_{ij}(p, \nu)$, where $\varepsilon_{ij}(p, \nu)$ is Frechet distributed across paths with shape parameter $\theta > 0$.

- A trader chooses the path from $i$ to $j$ to minimize her total trade cost:

$$\hat{\tau}_{ij}(\nu) = \min_{p \in P_{ij}, K, K \geq 0} \tilde{\tau}_{ij}(p) \varepsilon_{ij}(p, \nu).$$

- The expected trade cost between $i$ and $j$, $\tau_{ij}$, is the expectation of the least cost route across all traders:

$$\tau_{ij} \equiv \mathbb{E}_{\nu} [\hat{\tau}_{ij}(\nu)].$$
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Expected trade costs

- From EK '02, the expected trade cost from $i$ to $j$ is:

\[
\tau_{ij} = c \left( \sum_{K=0}^{\infty} \sum_{p \in \mathbb{P}_{ij,K}} \tilde{\tau}_{ij}(p)^{-\theta} \right)^{-\frac{1}{\theta}},
\]

where $c \equiv \Gamma(\theta - 1)$. Define the weighted adjacency matrix $A \equiv \begin{bmatrix} a_{ij} \equiv t_{ij} - \theta \end{bmatrix}$. Note $a_{ij} \in [0, 1]$, with $a_{ij} = 0 \Leftrightarrow \text{i and j are not connected}$. Use equation (1) to write expected trade costs as:

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- Use equation (1) to write expected trade costs as:

\[
\tau_{ij}^{-\theta} = c^{-\theta} \sum_{K=0}^{\infty} \sum_{p \in P_{ij,K}} \prod_{k=1}^{K} a_{p_{k-1},p_k}
\]
Expected trade costs (ctd.)

- From last slide:

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\tau_{ij}^{\theta} = c^{-\theta} \sum_{K=0}^{\infty} \sum_{p \in P_{ij,K}} \prod_{k=1}^{K} a_{p_{k-1},p_k}
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  \tau_{ij}^{-\theta} = c^{-\theta} \sum_{K=0}^{\infty} \sum_{p \in \mathcal{P}_{ij,K}} \prod_{k=1}^{K} a_{p_{k-1},p_k}
  \]

- Now we explicitly enumerate all possible paths of length \( K \) from \( i \) to \( j \):
  \[
  \left( \frac{\tau_{ij}}{c} \right)^{-\theta} = \sum_{K=0}^{\infty} \left( \sum_{k_1=1}^{N} \ldots \sum_{k_{K-1}=1}^{N} (a_{i,k_1} \times a_{k_{1},k_2} \times \ldots \times a_{k_{K-2},k_{K-1}} \times a_{k_{K-1},j}) \right)
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- Define \( A^K_{ij} \equiv [A^K]_{ij} \), i.e. \( A^K_{ij} \) is the \((i,j)\) element of matrix \( A \) taken to the matrix power \( K \). Then expected trade costs can be written as:

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• Define $A_{ij}^K \equiv [A^K]_{ij}$, i.e. $A_{ij}^K$ is the $(i, j)$ element of matrix $A$ taken to the matrix power $K$. Then expected trade costs can be written as:

$$\tau_{ij}^{-\theta} = c^{-\theta} \sum_{K=0}^{\infty} A_{ij}^K$$

• Note: $A_{ij}^0 = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$, so we allow traders going from $i$ to $i$ to not travel and incur no costs.
Expected trade costs (ctd.)

• Define $\mathbf{B} \equiv (\mathbf{I} - \mathbf{A})^{-1}$ and $b_{ij} \equiv [\mathbf{B}]_{ij}$. If $\rho(\mathbf{A}) < 1$, then:

$$\tau_{ij} = cb_{ij}^{-\frac{1}{\theta}} \quad (2)$$
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- Analytical mapping from transportation infrastructure network to endogenous bilateral trade costs (!)
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- Notes:
  - Sufficient condition for $\rho(A) < 1$:

  $$\sum_{j=1}^{N} t_{ij}^{-\theta} < 1 \ \forall i \in \{1, ..., N\}$$
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- Occurs if trade costs large and/or connections are sufficiently sparse and/or $\theta$ large.
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$$
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$$

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- As $\theta \rightarrow \infty$, $\tau_{ij}$ converges to trade costs of least cost route (generalizes Dijkstra algorithm).
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Properties of endogenous trade costs: Overview

It turns out that there are tractable and intuitive expressions for the following objects:

1. The probability a trader going from $i$ to $j$ uses the connection from $k$ to $l$, $\pi_{kl}^{ij}$.
   - Helpful for intuition.

2. The elasticity of bilateral trade costs between $i$ and $j$ to a change in infrastructure between $k$ and $l$, $\frac{\partial \ln \tau_{ij}}{\partial \ln t_{kl}}$.
   - Necessary for calculating elasticity of welfare to changes in infrastructure.

3. The mean distance traveled from $i$ to $j$, $E[d_{ij}]$.
   - Necessary for estimating the model.
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2. The elasticity of bilateral trade costs between $i$ and $j$ to a change in infrastructure between $k$ and $l$, $\frac{\partial \ln \tau_{ij}}{\partial \ln t_{kl}}$.
   - Necessary for calculating elasticity of welfare to changes in infrastructure.

3. The mean distance traveled from $i$ to $j$, $E[d_{ij}]$.
   - Necessary for estimating the model.
Probability of a trader going from $i$ to $j$ using link $kl$

**Proposition**

The probability of a trader going from $i$ to $j$ using link $kl$, $\pi_{ij}^{kl}$ is:

$$
\pi_{ij}^{kl} = \left( \frac{1}{c \frac{\tau_{ij}}{\tau_{ik} t_{kl} \tau_{lj}}} \right)^\theta.
$$
Probability of a trader going from $i$ to $j$ using link $kl$

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$$\pi_{ij}^{kl} = \left( \frac{1}{c \frac{\tau_{ij}}{\tau_{ik} t_{kl} \tau_{lj}}} \right)^{\theta}.$$ 

• Intuition:
  • Denominator $\tau_{ik} t_{kl} \tau_{lj}$ is the expected least cost from $i$ to $j$ going through link $kl$. 
Proposition

The probability of a trader going from \( i \) to \( j \) using link \( kl \), \( \pi_{ij}^{kl} \) is:

\[
\pi_{ij}^{kl} = \left( \frac{1}{c \ \tau_{ik} t_{kl} \tau_{lj}} \right)^{\theta} \frac{\tau_{ij}}{\tau_{ik} t_{kl} \tau_{lj}}.
\]

• Intuition:
• Denominator \( \tau_{ik} t_{kl} \tau_{lj} \) is the expected least cost from \( i \) to \( j \) going through link \( kl \).
• \( \frac{\tau_{ij}}{\tau_{ik} t_{kl} \tau_{lj}} \in [0, 1] \) since \( \tau_{ij} \) is the unconditional expected least cost.
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The probability of a trader going from $i$ to $j$ using link $kl$, $\pi_{ij}^{kl}$ is:

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- Intuition:
  - Denominator $\frac{\tau_{ij}}{\tau_{ik} t_{kl} \tau_{lj}}$ is the expected least cost from $i$ to $j$ going through link $kl$.
  - $\frac{\tau_{ij}}{\tau_{ik} t_{kl} \tau_{lj}} \in [0, 1]$ since $\tau_{ij}$ is the unconditional expected least cost.
  - The less “out of the way” link $kl$ is on the path from $i$ to $j$, the larger $\frac{\tau_{ij}}{\tau_{ik} t_{kl} \tau_{lj}}$ and hence the higher the probability.
Example path probability

Probability of traveling across each link from i=1 to j=25
Example path probability #2

Probability of traveling across each link from $i=1$ to $j=15$
Proof (sketch)

• Probability of taking path $p$ of length $K$ from $i$ to $j$ is:

$$\pi_{ij}(p) = \frac{\tilde{\tau}_{ij}(p)^{-\theta}}{\sum_{K=0}^{\infty} \sum_{p' \in P_{ij,K}} \tilde{\tau}_{ij}(p')^{-\theta}} = \frac{1}{b_{ij}} \prod_{k=1}^{K} a_{p_{k-1},p_k}.$$
Proof (sketch)

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\]

- Let $\mathbb{P}_{ij,K}^{kl}$ be the set of all paths from $i$ to $j$ going through link $kl$ of length $K$. Then:

\[
\pi_{ij}^{kl} = \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{p \in \mathbb{P}_{ij,K}^{kl}} \prod_{k=1}^{K} a_{p_{k-1},p_{k}}.
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$$\pi_{ij}^{kl} = \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{p \in P_{ij,K}^{kl}} \prod_{k=1}^{K} a_{p_{k-1},p_{k}}.$$ 

• Enumerate all paths in set $P_{ij,K}^{kl}$:

$$\pi_{ij}^{kl} = \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{B=0}^{K-1} \left( \sum_{k_{1}=1}^{N} \cdots \sum_{k_{B-1}=1}^{N} a_{i,k_{1}} \times \cdots \times a_{k_{B-1},k} \right) \times a_{k_{l}} \times \left( \sum_{k_{1}=1}^{N} \cdots \sum_{k_{K-B-1}=1}^{N} a_{l,k_{1}} \times \cdots \times a_{k_{K-B-1},j} \right).$$
Proof (sketch, ctd.)

• Write as matrix powers:

\[ \pi_{ij}^{kl} = \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{B=0}^{K-1} A_{ik}^B \times a_{kl} \times A_{lj}^{K-B-1}. \]
Proof (sketch, ctd.)

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\]

- Prove using matrix calculus that for any matrix \( C \) we have:

\[
\sum_{K=0}^{\infty} \sum_{B=0}^{K-1} A^B C A^{K-B-1} = (I - A)^{-1} C (I - A)^{-1}
\]
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- Write as matrix powers:

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- Set \( C \equiv \begin{cases} a_{kl} & \text{if } i = k, j = l \\ 0 & \text{otherwise} \end{cases} \), yielding the desired result:

\[ \pi_{kl} = \frac{b_{ik} a_{kl} b_{lj}}{b_{ij}} = \left( \frac{1}{c} \frac{\tau_{ij}}{\tau_{ik} t_{kl} \tau_{lj}} \right)^{\theta}. \]
Elasticity of trade costs from $i$ to $j$ to a change in $t_{kl}$

Proposition

The elasticity of trade costs between $i$ and $j$ to a change in the infrastructure link $kl$ is:

$$\frac{\partial \ln \tau_{ij}}{\partial \ln t_{kl}} = \left( \frac{1}{c} \frac{\tau_{ij}}{\tau_{ik} t_{kl} \tau_{lj}} \right)^\theta.$$

Note: trade cost $\tau_{ij}$ elasticity to infrastructure improvements in link $kl$ is equal to the probability traders from $i$ to $j$ use the link.

Intuition:

• "Out of the way" links are less likely to be used and have smaller effects on bilateral trade flows.
• Similar to EK'02, where price distribution of goods purchased are equalized across source countries.
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Proof (sketch)

- Recall $\tau_{ij} = c b_{ij}^{-\theta}$ and $a_{ij} = t_{ij}^{-\theta}$, so that $\frac{\partial \ln \tau_{ij}}{\partial \ln t_{kl}} = \frac{\partial \ln b_{ij}}{\partial \ln a_{kl}}$.

- Since $B \equiv (I - A)^{-1}$, we have:
  $$\frac{\partial \ln b_{ij}}{\partial \ln a_{kl}} = \left[ d \left( I - A_{kl}(t) \right)^{-1} dt \right]_{ij} a_{kl} b_{ij},$$
  where $A_{kl}(t) = \begin{cases} a_{ij} & \text{if } k \neq i \text{ or } l \neq j \\ t & \text{if } k = i \text{ and } l = j \end{cases}$.

- Show that:
  $$d \left( I - A_{kl}(t) \right)^{-1} dt = -BE_{kl}B,$$
  where $E_{kl} = \begin{cases} 0 & \text{if } k \neq i \text{ or } l \neq j \\ 1 & \text{if } k = i \text{ and } l = j \end{cases}$.

- Hence
  $$\frac{\partial \ln b_{ij}}{\partial \ln a_{kl}} = b_{ik} a_{kl} b_{lj} b_{ij} = \left( 1 - \frac{\tau_{ik} \tau_{lj}}{\tau_{ij}} \right) \theta.$$
Proof (sketch)

• Recall \( \tau_{ij} = c b_{ij}^{-\theta} \) and \( a_{ij} = t_{ij}^{-\theta} \), so that \( \frac{\partial \ln \tau_{ij}}{\partial \ln t_{kl}} = \frac{\partial \ln b_{ij}}{\partial \ln a_{kl}} \).

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\[
\frac{\partial \ln b_{ij}}{\partial \ln a_{kl}} = \left[ \frac{d (I - A_{kl}(t))^{-1}}{dt} \right]_{ij} \times \frac{a_{kl}}{b_{ij}},
\]

where \( A_{kl}(t) = \begin{cases} a_{ij} & \text{if } k \neq i \text{ or } l \neq j \\ t & \text{if } k = i \text{ and } l = j \end{cases} \).
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• Hence $\frac{\partial \ln b_{ij}}{\partial \ln a_{kl}} = \frac{b_{ik}a_{kl}b_{lj}}{b_{ij}} = \left( \frac{1}{c \tau_{ij} t_{kl} \tau_{lj}} \right)^{\theta}$. 
Mean of distance traveled by traders

Proposition

The expected distance traveled from $i$ to $j$ is:

$$E[d_{ij}] = \sum_{k=1}^{N} \sum_{l=1}^{N} d_{kl} \left( \frac{1}{c \tau_{ik} \tau_{kl} \tau_{lj}} \right)^\theta.$$

Proof.

See paper.
Outline of Talk

Introduction

Geographic component: Endogenous transportation costs

Economic component: The welfare effect of improving infrastructure
  Setup and Equilibrium
  Elasticity of welfare to changes in infrastructure

Empirical component: The Interstate Highway Network

Next steps
Economic component: overview

- Goal: analytical formula for the elasticity of equilibrium welfare to a change in infrastructure network.
- Consider both “trade” and “economic geography” versions of the Armington model.
Economic component: overview

- Goal: analytical formula for the elasticity of equilibrium welfare to a change in infrastructure network.
  - Consider both “trade” and “economic geography” versions of the Armington model.
- New technique to derive the elasticity of aggregate welfare to changes in (endogenous) bilateral trade flows.
  - Takes the same form in both model variants.
Economic component: overview

• Goal: analytical formula for the elasticity of equilibrium welfare to a change in infrastructure network.
  • Consider both “trade” and “economic geography” versions of the Armington model.

• New technique to derive the elasticity of aggregate welfare to changes in (endogenous) bilateral trade flows.
  • Takes the same form in both model variants.

• We then combine with the results above to derive the welfare effects of changes in infrastructure, as desired.
Model Setup

- Exogenous measure $\bar{L}$ of agents inhabit the world.
Model Setup

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• Production:
  • Labor is the only factor of production; perfect competition.
  • Each location $i \in \{1, \ldots, N\}$ produces a differentiated variety.
  • Agent in location $i$ supplies her unit of labor inelastically, produces $A_i$ products, and is compensated with wage $w_i$.

• Consumption:
  • Agents use their wage income to purchase a CES bundle of differentiated varieties with elasticity of substitution $\sigma$.
  • To purchase goods, agents randomly matched with traders (so trade costs are equal to expected trade costs $\tau_{ij}$ above).
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Equilibrium

- Perfect competition + CES preferences yield following expression for value of trade flows from $i$ to $j$:

$$X_{ij} = r_{ij}^{1-\sigma} A_i^{\sigma-1} w_i^{1-\sigma} P_j^{\sigma-1} E_j,$$

- $P_j$ is the Dixit Stiglitz price index and $E_j$ is total expenditure by agents in location $j$. 
**Equilibrium**

- Perfect competition + CES preferences yield following expression for value of trade flows from $i$ to $j$:

  \[ X_{ij} = \tau_{ij}^{1-\sigma} A_i^{\sigma-1} w_i^{1-\sigma} P_j^{\sigma-1} E_j, \]

- $P_j$ is the Dixit Stiglitz price index and $E_j$ is total expenditure by agents in location $j$.

- Equilibrium conditions:
  - Income is equal to total sales: $Y_i = \sum_{j=1}^{N} X_{ij}$ for all $i$.
  - Expenditure is equal to total purchases: $E_i = \sum_{j=1}^{N} X_{ji}$ for all $i$.
  - Budget constraint: Income equals expenditure equals wage $\times$ labor: $Y_i = E_i = w_i L_i$. 

- Welfare of agent in location $i$ is $W_i \equiv w_i P_i u_i$, where $u_i$ is an amenity value.
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- Welfare of agent in location $i$ is $W_i \equiv \frac{w_i}{P_i} u_i$, where $u_i$ is an amenity value.
Equilibrium (ctd.)

- Re-write gravity equation as a function of welfare:

\[ X_{ij} = \left( \frac{\tau_{ij}}{A_i u_j} \right)^{1-\sigma} w_i^{1-\sigma} W_j^{1-\sigma} w_j^\sigma L_j. \]
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- Equilibrium can be written as:

\[
\begin{align*}
    w_i L_i &= \sum_{j=1}^{N} \left( \frac{\tau_{ij}}{A_i u_j} \right)^{1-\sigma} W_i^{1-\sigma} W_j^{1-\sigma} w_j^{\sigma} L_j \\
    w_i L_i &= \sum_{j=1}^{N} \left( \frac{\tau_{ji}}{A_j u_i} \right)^{1-\sigma} W_j^{1-\sigma} W_i^{1-\sigma} w_i^{\sigma} L_i
\end{align*}
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Equilibrium (ctd.)

- Re-write gravity equation as a function of welfare:

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\[ w_i L_i = \sum_{j=1}^{N} \left( \frac{\tau_{ji}}{A_j u_i} \right)^{1-\sigma} w_j^{1-\sigma} W_i^{1-\sigma} w_i^{\sigma} L_i \tag{4} \]

- Two alternative setups:
Equilibrium (ctd.)

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- Two alternative setups:
  - Trade framework: Labor is perfectly immobile, \( L_i \) is exogenous, solve equations (3) and (4) for \( \{w_i, W_i\}_i \).
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• Re-write gravity equation as a function of welfare:

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• Equilibrium can be written as:

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\[ w_i L_i = \sum_{j=1}^{N} \left( \frac{\tau_{ji}}{A_j u_i} \right)^{1-\sigma} w_j^{1-\sigma} W_i^{1-\sigma} w_i^\sigma L_i \quad (4) \]

• Two alternative setups:
  • Trade framework: Labor is perfectly immobile, \( L_i \) is exogenous, solve equations (3) and (4) for \( \{w_i, W_i\}_i \).
  • Economic geography framework: Labor is perfectly mobile, \( L_i \) is endogenous, solve equations (3) and (4) for \( \{\{w_i, L_i\}_i, W\} \).
Outline of Talk

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Next steps
Trade framework

- Consider a “planner” problem that maximizes weighted (log) welfare:

\[
\max_{\{w_i, W_i\}_i} \sum_{i=1}^{N} \omega_i \ln W_i
\]
Trade framework

- Consider a “planner” problem that maximizes weighted (log) welfare:

\[
\max_{\{w_i, W_i\}_i} \sum_{i=1}^{N} \omega_i \ln W_i
\]

- subject to aggregate labor income equaling aggregate trade income:

\[
\sum_{i=1}^{N} w_i L_i = \sum_{i=1}^{N} \sum_{j=1}^{N} \left( \frac{\tau_{ij}}{A_i u_j} \right)^{1-\sigma} w_i^{1-\sigma} W_j^{1-\sigma} w_j^\sigma L_j
\]

- where \(\omega_i = \frac{w_i L_i}{\sum_{i=1}^{N} w_i L_i} = \frac{Y_i}{Y}W\)

- FOCs of this maximization problem yield (3) and (4).

- A direct application of the envelope theorem yields:

\[
- \sum_{i=1}^{N} \left( \frac{Y_i}{Y}W \right) \frac{\partial \ln W_i}{\partial \ln \tau_{ij}} = X_{ij} Y W
\]

- New method, old result (Atkeson and Burstein ’10, Fan, Lai, and Qi ’13).
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- where \( \omega_i = \frac{w_i L_i}{\sum_{i=1}^{N} w_i L_i} = \frac{Y_i}{Y_W} \) are the Pareto weights.

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\[
\sum_{i=1}^{N} w_i L_i = \sum_{i=1}^{N} \sum_{j=1}^{N} \left( \frac{T_{ij}}{A_i u_j} \right)^{1-\sigma} w_i^{1-\sigma} W_j^{1-\sigma} w_j^\sigma L_j,
\]

- where \(\omega_i = \frac{w_i L_i}{\sum_{i=1}^{N} w_i L_i} = \frac{Y_i}{Y} \) are the Pareto weights.

- FOCs of this maximization problem yield (3) and (4).

\[
- \sum_{i=1}^{N} (Y_i Y W)^2 \frac{\partial \ln W_i}{\partial \ln \tau_{ij}} = X_{ij} Y W
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Trade framework

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  \[
  \sum_{i=1}^{N} w_i L_i = \sum_{i=1}^{N} \sum_{j=1}^{N} \left( \frac{\tau_{ij}}{A_i u_j} \right)^{1-\sigma} w_i^{1-\sigma} W_j^{1-\sigma} w_j^{\sigma} L_j,
  \]
  where \(\omega_i = \frac{w_i L_i}{\sum_{i=1}^{N} w_i L_i} = \frac{Y_i}{Y^W}\) are the Pareto weights.

- FOCs of this maximization problem yield (3) and (4).

- A direct application of the envelope theorem yields:
  \[
  - \sum_{i=1}^{N} \left( \frac{Y_i}{Y^W} \right) \frac{\partial \ln W_i}{\partial \ln \tau_{ij}} = \frac{X_{ij}}{Y^W}
  \]
Trade framework

- Consider a “planner” problem that maximizes weighted (log) welfare:

\[
\max \{w_i, W_i\}_i \sum_{i=1}^N \omega_i \ln W_i
\]

- subject to aggregate labor income equaling aggregate trade income:

\[
\sum_{i=1}^N w_i L_i = \sum_{i=1}^N \sum_{j=1}^N \left( \frac{\tau_{ij}}{A_i u_j} \right)^{1-\sigma} w_i^{1-\sigma} W_j^{1-\sigma} w_j^{\sigma} L_j,
\]

- where \(\omega_i = \frac{w_i L_i}{\sum_{i=1}^N w_i L_i} = \frac{Y_i}{YW}\) are the Pareto weights.

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\[
- \sum_{i=1}^N \left( \frac{Y_i}{YW} \right) \frac{\partial \ln W_i}{\partial \ln \tau_{ij}} = \frac{X_{ij}}{YW}
\]

- New method, old result (Atkeson and Burstein ’10, Fan, Lai, and Qi ’13).
Economic geography framework

- Consider an alternative “planner” problem that maximizes (log) welfare:

\[
\max \quad \ln W \\
\left\{ \{w_i, L_i\}_i, W \right\}
\]

- Again, FOCs of this maximization problem yield (3) and (4) (with welfare equalized).

- Implication: A direct application of the envelope theorem yields:

\[
\frac{\partial \ln W}{\partial \ln \tau_{ij}} = X_{ij} Y W
\]

- New method, new result!
Economic geography framework

- Consider an alternative “planner” problem that maximizes (log) welfare:

\[
\max \ln W \quad \left\{ \{w_i, L_i\}_i, W \right\}
\]

- subject to aggregate labor income equaling aggregate trade income (and welfare being equalized):

\[
\sum_{i=1}^{N} w_i L_i = \sum_{i=1}^{N} \sum_{j=1}^{N} \left( \frac{\tau_{ij}}{A_i u_j} \right)^{1-\sigma} w_i^{1-\sigma} W^{1-\sigma} w_j^{\sigma} L_j.
\]

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\]

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\[
\max \ln \mathcal{W} \quad \text{subject to aggregate labor income equaling aggregate trade income (and welfare being equalized)}:
\]

\[
\sum_{i=1}^{N} w_i L_i = \sum_{i=1}^{N} \sum_{j=1}^{N} \left( \frac{\tau_{ij}}{A_i u_j} \right)^{1-\sigma} w_i^{1-\sigma} W^{1-\sigma} w_j^{\sigma} L_j.
\]

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\[
- \frac{\partial \ln \mathcal{W}}{\partial \ln \tau_{ij}} = \frac{X_{ij}}{Y^W}.
\]
Economic geography framework

- Consider an alternative “planner” problem that maximizes (log) welfare:

\[
\max \{ \begin{array}{l}
\ln W \\
\{(w_i, L_i)_i, W\}
\end{array}
\]

- subject to aggregate labor income equaling aggregate trade income (and welfare being equalized):

\[
\sum_{i=1}^{N} w_i L_i = \sum_{i=1}^{N} \sum_{j=1}^{N} \left( \frac{\tau_{ij}}{A_i u_j} \right)^{1-\sigma} w_i^{1-\sigma} W^{1-\sigma} w_j^\sigma L_j.
\]

- Again, FOCs of this maximization problem yield (3) and (4) (with welfare equalized).

- Implication: A direct application of the envelope theorem yields:

\[
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\]

- New method, new result!
Elasticity of welfare to changes in infrastructure

- Elasticity of welfare to changes in infrastructure:

\[
\frac{\partial \ln W}{\partial \ln t_{ij}} = \sum_{k=1}^{N} \sum_{l=1}^{N} \frac{\partial \ln W}{\partial \ln \tau_{kl}} \frac{\partial \ln \tau_{kl}}{\partial \ln t_{ij}}
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Elasticity of welfare to changes in infrastructure

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\frac{\partial \ln W}{\partial \ln t_{ij}} = \sum_{k=1}^{N} \sum_{l=1}^{N} \frac{\partial \ln W}{\partial \ln \tau_{kl}} \frac{\partial \ln \tau_{kl}}{\partial \ln t_{ij}}
\]

- “Matrix notation”:

\[
- \frac{\partial \ln W}{\partial \ln t_{ij}} = \sum_{k=1}^{N} \sum_{l=1}^{N} X_{kl} b_{ki} a_{ij} b_{jl} \frac{\partial \ln W}{\partial b_{kl}}
\]

Simple intuition: Welfare elasticity equal to trade value flowing through a connection!
Elasticity of welfare to changes in infrastructure

- Elasticity of welfare to changes in infrastructure:

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  \frac{\partial \ln W}{\partial \ln t_{ij}} = \sum_{k=1}^{N} \sum_{l=1}^{N} \frac{\partial \ln W}{\partial \ln \tau_{kl}} \frac{\partial \ln \tau_{kl}}{\partial \ln t_{ij}}
  \]

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  \[
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  \]

- “Trade notation”:

  \[
  - \frac{\partial \ln W}{\partial \ln t_{ij}} = c^\theta \sum_{k=1}^{N} \sum_{l=1}^{N} \frac{X_{kl}}{Y^W} \left( \frac{\tau_{kl}}{\tau_{ki} t_{ij} \tau_{jl}} \right)^\theta
  \]
Elasticity of welfare to changes in infrastructure

- Elasticity of welfare to changes in infrastructure:

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\frac{\partial \ln W}{\partial \ln t_{ij}} = \sum_{k=1}^{N} \sum_{l=1}^{N} \frac{\partial \ln W}{\partial \ln \tau_{kl}} \frac{\partial \ln \tau_{kl}}{\partial \ln t_{ij}}
\]

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\]

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Example welfare elasticity to changes in infrastructure

Welfare effect of improving a link
Outline of Talk

Introduction

Geographic component: Endogenous transportation costs

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Empirical component: The Interstate Highway Network
  The Interstate Highway Network
  Estimation
  The welfare effects of improving the IHS

Next steps
The Interstate Highway Network: Overview

- We assess the welfare impact of improving each segment of the U.S. Interstate Highway Network (IHS).
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The Interstate Highway Network: Overview

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  • “Increase federal infrastructure funding by $275 billion over a five-year period” (Hillary Clinton)
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- Widespread consensus of the pressing need for greater investment:
  - “Increase federal infrastructure funding by $275 billion over a five-year period” (Hillary Clinton)
  - “We need much more money to rebuild our infrastructure... I would say at least double her numbers” (Donald Trump)
The Interstate Highway Network
Data

- Use recently released public use microdata from 2012 Commodity Flow Survey.
Data

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- Observe 696,021 shipments between 67 metropolitan statistical areas (MSAs) shipped by for-hire trucks.
  - Comprises 60% of value of commodity flows in the U.S.
  - Observe origin MSA, destination MSA, value of shipment, and distance traveled.
Data

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- Observe 696,021 shipments between 67 metropolitan statistical areas (MSAs) shipped by for-hire trucks.
  - Comprises 60% of value of commodity flows in the U.S.
  - Observe origin MSA, destination MSA, value of shipment, and distance traveled.

- For each pair of MSAs, determine if the shortest path via IHS goes through another city.
  - If so, then the two MSAs are not connected.
  - If not, then the two MSAs are connected. Calculate distance $d_{ij}$ and travel time $time_{ij}$ using Google Maps.
The IHS: A graphical representation
Outline of Talk

Introduction

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Next steps
Estimation

- Assume components of infrastructure matrix depend on travel time along the direct route:

\[ t_{ij} = \exp (\kappa \times \text{time}_{ij}) \]

- Unknown parameters: cost of travel time (\( \kappa \)), trader heterogeneity (\( \theta \)), elasticity of substitution (\( \sigma \)).

- Estimate parameters via GMM using three sets of moments:
  - Value of trade flows between each origin MSA - destination MSA:
    \[ \ln X_{ij} = (\sigma - 1) \ln \left[ I - \left[ \exp (-\theta \kappa \text{time}_{ij}) \right] \right] - 1_{ij} + \ln \gamma_i + \ln \delta_j + \varepsilon_{ij} \]
  - Mean distance traveled between each origin MSA - destination MSA:
    \[ \bar{d}_{ij} = \frac{1}{N} \sum_{k,l=1}^{N} d_{kl} \left[ I - \left[ \exp (-\theta \kappa \text{time}_{ij}) \right] \right] - 1_{ki} \times \exp (-\theta \kappa \text{time}_{ij}) \times \left[ I - \left[ \exp (-\theta \kappa \text{time}_{ij}) \right] \right] - 1_{jl} \times \left[ I - \left[ \exp (-\theta \kappa \text{time}_{ij}) \right] \right] - 1_{kl} \]
  - Average trade cost of 20% ad valorem equivalent (e.g. Anderson and van Wincoop '04). [work in progress].
Estimation

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    \[
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    \]
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  • Mean distance traveled between each origin MSA - destination MSA:
    \[ \bar{d}_{ij} = \sum_{k,l=1}^{N} d_{kl} \left( \frac{[I - \left[ \exp (-\theta \kappa \text{time}_{ij}) \right]^{-1} \times \exp (-\theta \kappa \text{time}_{ij}) \times [I - \left[ \exp (-\theta \kappa \text{time}_{ij}) \right]^{-1} \times \exp (-\theta \kappa \text{time}_{ij}) \times [I - \left[ \exp (-\theta \kappa \text{time}_{ij}) \right]^{-1}]}{[I - \left[ \exp (-\theta \kappa \text{time}_{ij}) \right]^{-1}\times \exp (-\theta \kappa \text{time}_{ij}) \times [I - \left[ \exp (-\theta \kappa \text{time}_{ij}) \right]^{-1} \times \exp (-\theta \kappa \text{time}_{ij}) \times [I - \left[ \exp (-\theta \kappa \text{time}_{ij}) \right]^{-1}}{[I - \left[ \exp (-\theta \kappa \text{time}_{ij}) \right]^{-1}}} \right) \]
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  - Mean distance traveled between each origin MSA - destination MSA:
    \[ \bar{d}_{ij} = \sum_{k,l=1}^{N} d_{kl} \frac{\left[ I - \left[ \exp (-\theta \kappa time_{ij}) \right] \right]^{-1}_{ki} \times \exp (-\theta \kappa time_{ij}) \times \left[ I - \left[ \exp (-\theta \kappa time_{ij}) \right] \right]^{-1}_{ji}}{\left[ I - \left[ \exp (-\theta \kappa time_{ij}) \right] \right]^{-1}_{kl}} \]
  - Average trade cost of 20% ad valorem equivalent (e.g. Anderson and van Wincoop ’04). [work in progress].
### Table: Estimation

**Model parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Moment</th>
<th>Estimated value</th>
<th>Correlation: predicted vs. observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effect of distance on direct trade cost</td>
<td>$\kappa$</td>
<td>Mean path distance</td>
<td>0.0108</td>
<td>0.997</td>
</tr>
<tr>
<td>Trader heterogeneity</td>
<td>$\theta$</td>
<td>Trade flows</td>
<td>136.13</td>
<td>0.737</td>
</tr>
<tr>
<td>Elasticity of substitution</td>
<td>$\sigma$</td>
<td>Trade costs</td>
<td>7.9237</td>
<td>1</td>
</tr>
</tbody>
</table>

**Implied trade costs**

<table>
<thead>
<tr>
<th>Mean</th>
<th>25% perc.</th>
<th>Median</th>
<th>75% perc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2 (calibrated)</td>
<td>1.104</td>
<td>1.169</td>
<td>1.271</td>
</tr>
</tbody>
</table>

*Notes:* This table reports the estimated parameter values and summary statistics for the implied bilateral trade costs. The three parameters were estimated to most closely match the three reported moments; note that the estimation procedure simultaneously estimated all parameters by minimizing all moments, so the assignment of parameter to moment above is heuristic.
Model fit

Mean distance

Correlation: 0.997

Log Trade Flows

Correlation: 0.737
Outline of Talk

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Next steps
The welfare effects of improving the IHS

Welfare elasticity

- 0.000367 - 0.001347
- 0.001348 - 0.002593
- 0.002594 - 0.003803
- 0.003804 - 0.004815
- 0.004816 - 0.006455
- 0.006456 - 0.008267
- 0.008268 - 0.010332
- 0.010333 - 0.014548
- 0.014549 - 0.021815
- 0.021816 - 0.044808
### Table: Top 20 Highways

<table>
<thead>
<tr>
<th>Origin City</th>
<th>Destination City</th>
<th>Interstate</th>
<th>Welfare Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>New York</td>
<td>Philadelphia</td>
<td>95 (South)</td>
<td>0.04481</td>
</tr>
<tr>
<td>Baltimore</td>
<td>Washington</td>
<td>95 (South)</td>
<td>0.04477</td>
</tr>
<tr>
<td>Columbus</td>
<td>Dayton</td>
<td>70 (West)</td>
<td>0.0438</td>
</tr>
<tr>
<td>Dayton</td>
<td>Columbus</td>
<td>70 (East)</td>
<td>0.04094</td>
</tr>
<tr>
<td>Dayton</td>
<td>Cincinnati</td>
<td>75 (South)</td>
<td>0.04007</td>
</tr>
<tr>
<td>Philadelphia</td>
<td>Baltimore</td>
<td>95 (South)</td>
<td>0.03961</td>
</tr>
<tr>
<td>Cincinnati</td>
<td>Dayton</td>
<td>75 (North)</td>
<td>0.03691</td>
</tr>
<tr>
<td>Philadelphia</td>
<td>New York</td>
<td>95 (South)</td>
<td>0.03292</td>
</tr>
<tr>
<td>Washington</td>
<td>Baltimore</td>
<td>95 (North)</td>
<td>0.03069</td>
</tr>
<tr>
<td>Washington</td>
<td>Richmond, VA</td>
<td>95 (South)</td>
<td>0.03025</td>
</tr>
<tr>
<td>Beaumont</td>
<td>Houston</td>
<td>10 (West)</td>
<td>0.02874</td>
</tr>
<tr>
<td>Tulsa</td>
<td>Oklahoma City</td>
<td>44 (West)</td>
<td>0.02874</td>
</tr>
<tr>
<td>Pittsburgh</td>
<td>Columbus</td>
<td>70 (West)</td>
<td>0.02868</td>
</tr>
<tr>
<td>Lake Charles, LA</td>
<td>Beaumont</td>
<td>10 (West)</td>
<td>0.02858</td>
</tr>
<tr>
<td>Baltimore</td>
<td>Philadelphia</td>
<td>95 (North)</td>
<td>0.02793</td>
</tr>
<tr>
<td>Columbus</td>
<td>Cincinnati</td>
<td>71 (South)</td>
<td>0.02742</td>
</tr>
<tr>
<td>Cincinnati</td>
<td>Louisville/Jefferson County</td>
<td>71 (South)</td>
<td>0.02662</td>
</tr>
<tr>
<td>Louisville/Jefferson County</td>
<td>Nashville</td>
<td>65 (South)</td>
<td>0.02659</td>
</tr>
<tr>
<td>Columbus</td>
<td>Cleveland</td>
<td>71 (North)</td>
<td>0.02629</td>
</tr>
<tr>
<td>Nashville</td>
<td>Memphis, TN</td>
<td>40 (West)</td>
<td>0.02617</td>
</tr>
</tbody>
</table>

Most important highways are either between major cities or comprise major trade corridors.
Next step: Congestion

- Congestion: the more an infrastructure link is used, the slower the speed along the link.
- Empirically relevant, but usually ignored in general equilibrium spatial models.
- Framework developed above allows us to incorporate congestion in a tractable manner. Suppose:

\[ t_{ij} = \exp (\kappa \times \text{time}_{ij}) \]

\[ \text{time}_{ij} = \bar{\text{time}}_{ij} + \gamma \sum_{k=1}^{N} \sum_{l=1}^{N} X_{kl}^{ij} \]

\[ \text{value of trade along link} \]

- Recall closed form solution for value of trade along link:

\[ \sum_{k=1}^{N} \sum_{l=1}^{N} X_{kl}^{ij} \equiv \sum_{k=1}^{N} \sum_{l=1}^{N} \frac{X_{kl}^{ij} b_{ki} a_{ij} b_{jl}}{b_{kl}}. \]
Next step: Congestion

Congestion parameter $\gamma = 0$

Population

0

0.5

1

1.5

2

2.5

3
Next step: Congestion

Congestion parameter $\gamma = 1$

Population

0
0.5
1
1.5
2
2.5
3
Next step: Congestion

Congestion parameter $\gamma = 5$

Population

0
0.5
1
1.5
2
2.5
3
Next step: Congestion

Congestion parameter $\gamma = 10$

Population

0
0.5
1
1.5
2
2.5
3
Next step: Congestion

Trade flows across infrastructure links

Congestion parameter $\gamma = 0$
Next step: Congestion

Trade flows across infrastructure links

Congestion parameter $\gamma = 1$
Next step: Congestion

Trade flows across infrastructure links

Congestion parameter $\gamma=5$
Next step: Congestion

Trade flows across infrastructure links

Congestion parameter $\gamma=10$
Conclusion

Conclusion


- Intuitive analytical solutions with straightforward mapping to the data.
Conclusion

- Intuitive analytical solutions with straightforward mapping to the data.
- Necessary steps toward the design of the optimal transportation network.