2

Three-Phase Systems

2.1. INTRODUCTION

- Generation, transmission and heavy-power utilisation of A.C. electric energy almost invariably involve a type of system or circuit called a *polyphase system or polyphase circuit*. In such a system, each voltage source consists of a group of voltages having relative magnitudes and phase angles. Thus, a *m-phase system will employ voltage sources which, conventionally, consist of m voltages substantially equal in magnitude and successively displaced by a phase angle of 360°/m.*
- A *3-phase system* will employ voltage sources which, conventionally, consist of *three voltages substantially equal in magnitude* and *displaced by phase angles of 120°*. Because it *possesses definite economic and operating advantages, the 3-phase system is by far the most common,* and consequently emphasis is placed on 3-phase circuits.

2.2. ADVANTAGES OF POLYPHASE SYSTEMS

The advantages of polyphase systems *over single-phase systems* are :

1. A polyphase transmission line requires less conductor material than a single-phase line for transmitting the same amount power at the same voltage.

2. For a given frame size a polyphase machine gives a higher output than a single-phase machine. For example, output of a 3-phase motor is 1.5 times the output of single-phase motor of same size.

3. Polyphase motors have a uniform torque where most of the single-phase motors have a pulsating torque.

4. Polyphase induction motors are self-starting and are more efficient. On the other hand single-phase induction motors are not self-starting and are less efficient.

5. Per unit of output, the polyphase machine is very much cheaper.

6. Power factor of a single-phase motor is lower than that of polyphase motor of the same rating.

7. Rotating field can be set up by passing polyphase current through stationary coils.

8. Parallel operation of polyphase alternators is simple as compared to that of single-phase alternators because of pulsating reaction in single-phase alternator.

It has been found that the above advantages are *best realised in the case of three-phase systems*. Consequently, the electric power is generated and transmitted in the form of three-phase system.

2.3. GENERATION OF THREE-PHASE VOLTAGES

- Let us consider an elementary 3-phase 2-pole generator as shown in Fig. 2.1. On the armature are three coils, *ll'*, *mm'*, and *nn'* whose axes are displaced 120° in space from each other.

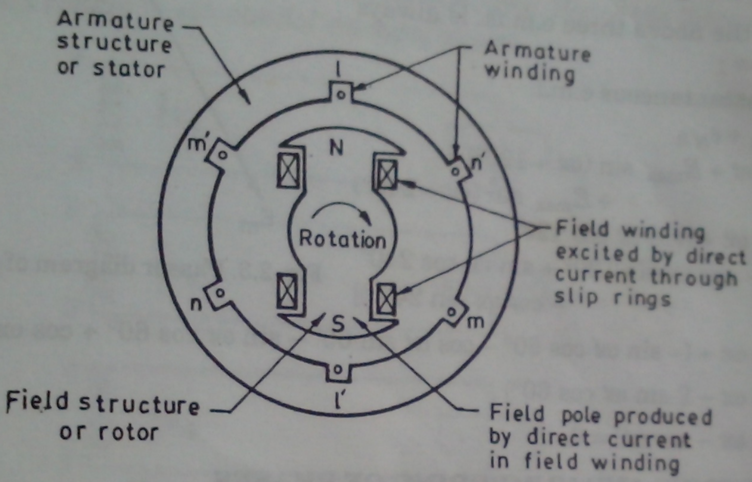


Fig. 2.1. Elementary 3-phase 2-pole generator.

- **When the field is excited and rotated, voltages will be generated in the three phases in accordance with Faraday's law. If the field structure is so designed that the flux is distributed sinusoidally over the poles, the flux linking any phase will vary sinusoidally with time and sinusoidal voltages will be induced in three-phases. These three waves will be displaced 120 electrical degrees (Fig. 2.2) in time as a result of the phases being displaced**

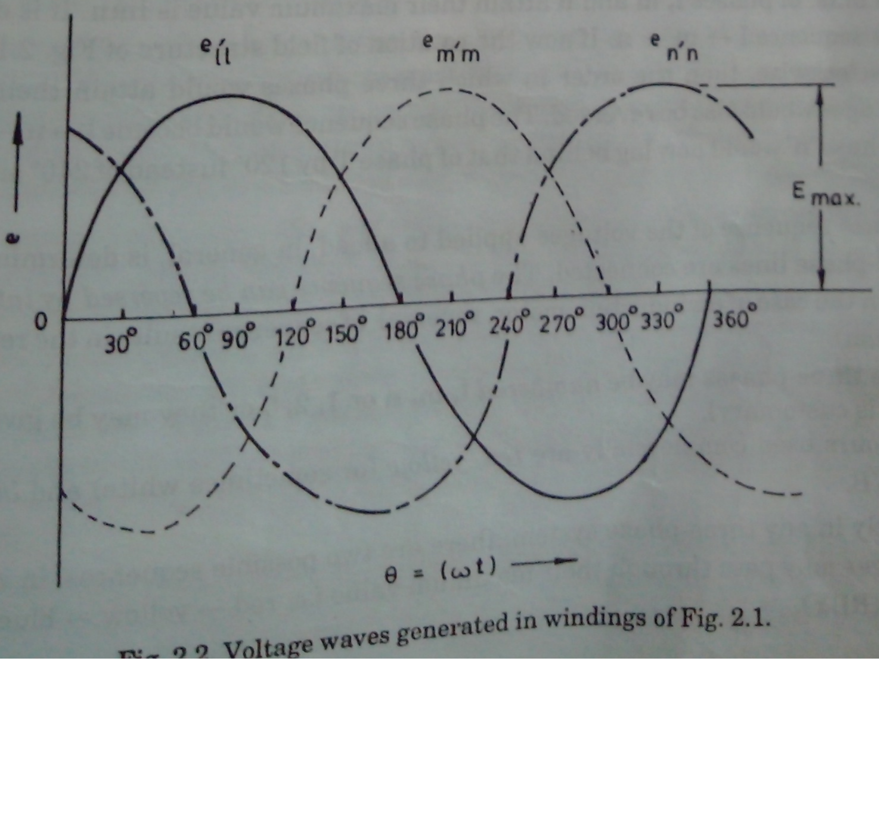


Fig. 2.2. Voltage waves generated in windings of Fig. 2.1.

120° in space. The corresponding phasor diagram is shown in Fig. 2.3. The equations of the instantaneous values of the three voltages (given by Fig. 2.2) are :

$$e_{l'l} = E_{max.} \sin \omega t$$
$$e_{m'm} = E_{max.} \sin (\omega t - 120°)$$
$$e_{n'n} = E_{max.} \sin (\omega t - 240°)$$

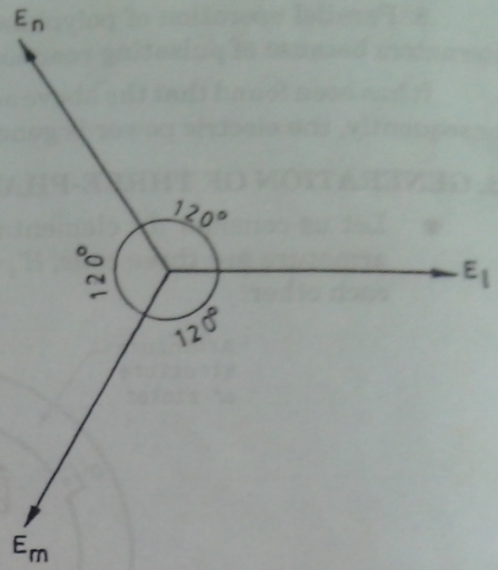


Fig. 2.3. Phasor diagram of generated voltages.

The sum of the above three e.m.fs. is always zero as shown below :

Resultant instantaneous e.m.f.

$$= e_{l'l} + e_{m'm} + e_{n'n}$$
$$= E_{max.} \sin \omega t + E_{max.} \sin (\omega t - 120°) + E_{max.} \sin (\omega t - 240°)$$
$$= E_{max.} [\sin \omega t + (\sin \omega t \cos 120° - \cos \omega t \sin 120° + \sin \omega t \cos 240° - \cos \omega t \sin 240°)]$$
$$= E_{max.} [\sin \omega t + (- \sin \omega t \cos 60° - \cos \omega t \sin 60° - \sin \omega t \cos 60° + \cos \omega t \sin 60°)]$$
$$= E_{max.} (\sin \omega t - 2 \sin \omega t \cos 60°)$$
$$= E_{max.} (\sin \omega t - \sin \omega t) = 0.$$

2.4. PHASE SEQUENCE AND NUMBERING OF PHASES

- *By phase sequence is meant the order in which the three phases attain their peak or maximum.*

In the generation of three-phase e.m.fs. in Fig. 2.2 clockwise rotation of the field system in Fig. 2.1 was assumed. This assumption made the e.m.f. of phase '**m**' lag behind that of '**l**' by 120° and in a similar way, made that of '**n**' lag behind that of '**m**' by 120° (or that of **l** by 240°). Hence, the order in which the e.m.fs. of phases **l, m** and **n** attain their maximum value is **lmn**. It is called the *phase order* or phase sequence **l** → **m** → **n**. If now the rotation of field structure of Fig. 2.1 is reversed *i.e.* made counter-clockwise, then the order in which three phases would attain their corresponding maximum voltages would also be reversed. The phase sequence would become **l** → **n** → **m**. This means that e.m.f. of phase '**n**' would now lag behind that of phase '**l**' by 120° instead of 240° as in the previous case.

The phase sequence of the voltages applied to a load, in general, is determined by the order in which the 3-phase lines are connected. The *phase sequence can be reversed* by *interchanging any pair of lines*. (In the case of an induction motor, reversal of sequence results in the reversed direction of motor rotation).

- The three-phases may be *numbered* **l, m, n** or 1, 2, 3 or they may be given three *colours* (as is customary).

The colours used commercially are *red, yellow* (or sometimes white) and *blue*. In this case sequence is RYB.

Evidently in any three-phase system, there are two possible sequences, in which three coils or phase voltages may pass through their maximum value *i.e.* red → yellow → blue (RYB) or red → blue → yellow (RBY).

By convention :

RYB ...... taken as *positive*.

RBY ...... taken as *negative*.

2.5. INTER-CONNECTION OF THREE PHASES

Each coil of three phases has two terminals [one 'start' (S) and another 'finish' (F)] and if individual phase is connected to a separate load circuit, as shown in Fig. 2.4, we get a non-interlinked 3-phase system. In such a system each circuit will require two conductors, therefore, 6 conductors in all. This makes the whole system *complicated* and *expensive*. Hence the *three phases are generally interconnected which results in substantial saving of copper*.

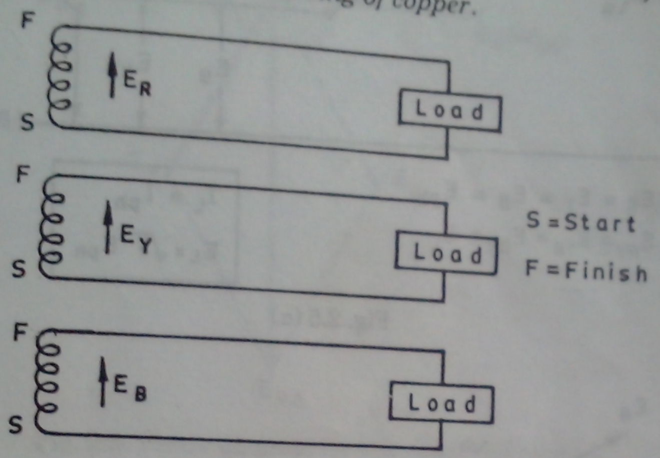


Fig. 2.4. Non-interlinked 3-phase system.

The general method of inter-connections are :

1. Star or Wye (Y) connection.
2. Mesh or delta (Δ) connection.

2.6. STAR OR WYE (Y) CONNECTION

- In this method of inter-connection the similar ends either the 'start' or 'finish' are joined together at point N. This common point N [Fig. 2.5 (a)] is called *star point* or *neutral point*. Ordinarily only three wires are carried to the external circuit giving 3-phase, 3-wire start connected system but sometimes a *fourth-wire*, known as *neutral wire* is carried to the neutral point of the external load circuit giving 3-phase, 4 wire star connected system.
- The *voltage between any line and the neutral point* (*i.e.* voltage across the phase winding) is called the **'phase voltage' (E_{ph})** ; while the *voltage available between any pair of terminals* (or outers) is called the **'line voltage' (E_L)**.
- In star connection, as is evident in Fig. 2.5 (a) there are two-phase windings between each pair of terminals, but since their *similar* ends have been joined together, they are in *opposition*. Obviously, the instantaneous value of potential difference between any two terminals is the *arithmetic difference* of the two-phase e.m.fs. concerned. However, the r.m.s. value of this potential difference is given by the *vector difference* of the two-phase e.m.fs.
- Fig. 2.5 (b) shows the vector diagram for phase voltages and currents in a star connection where a *balanced system has been assumed*. [A balanced system is one in which (i) the voltages in all phases are equal in magnitude and differ in phase from one another by

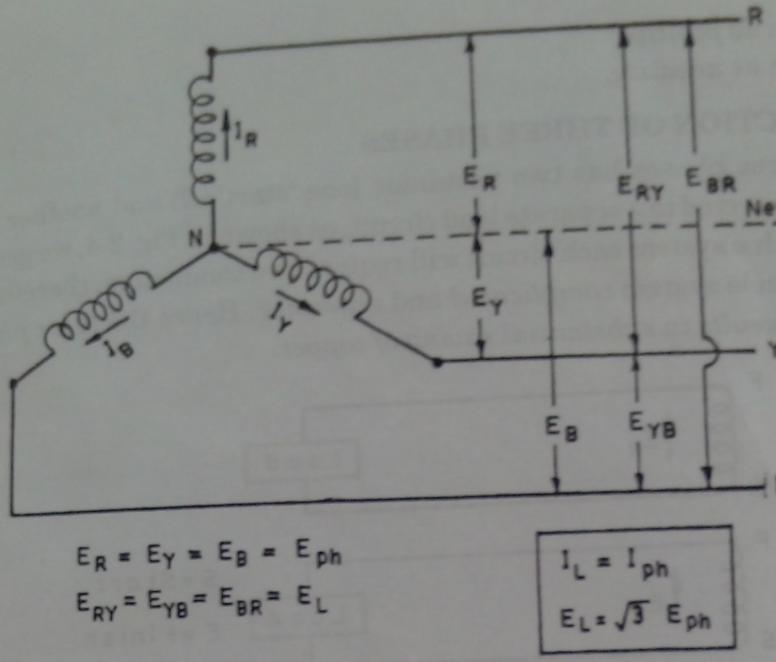


Fig. 2.5 (*a*)

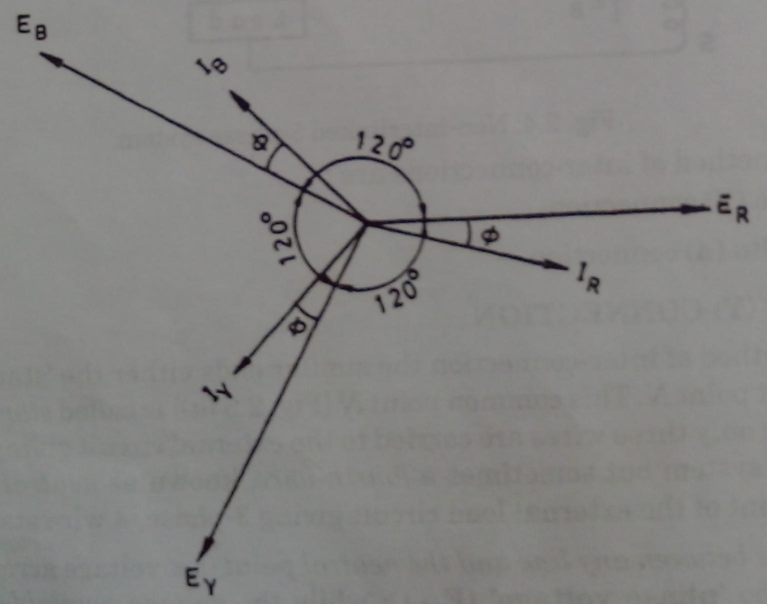


Fig. 2.5. (*b*) Star-connected three-phase net work.

equal angles, in this case, the angle = 360/3 = 120°, (*ii*) the currents in the three phases are equal in magnitude and also differ in phase from one another by equal angles. A three-phase balanced load is that in which the loads connected across the three-phases are *identical*]. Thus, we have

$$E_R = E_Y = E_B = E_{ph} \text{ (phase e.m.f.)}$$

Line voltage, $E_{RY} (= E_L)$ = Vector difference of E_R and E_Y

$$= E_R - E_Y$$

Line voltage, $E_{YB} = E_Y - E_B$

Line voltage, $E_{BR} = E_B - E_R$.

(*a*) **Relation Between Line Voltages and Phase Voltages.** Refer Fig. 2.6.

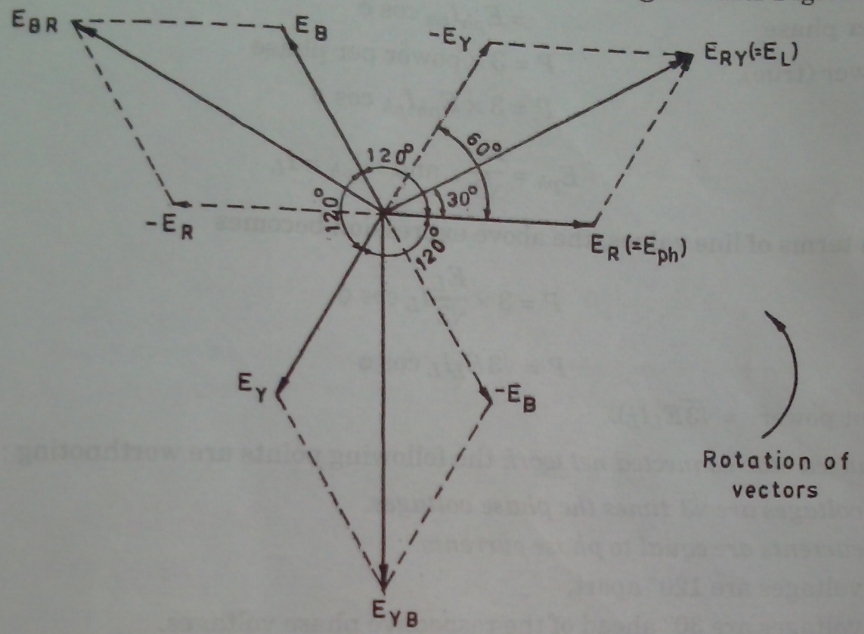


Fig. 2.6. Vector diagram for star connected network.

The potential difference between outers *R* any *Y* is

$$E_{RY} = E_R - E_Y \quad \text{[vector difference]}$$

or

$$E_{RY} = E_R + (-E_Y) \quad \text{[vector sum]}$$

Hence, E_{RY} is found by compounding E_R and E_Y reversed and its value is given by the diagonal of the parallelogram (Fig. 2.6). Obviously the angle between E_R and E_Y reversed is 60° and the value of

$$E_{RY}(\text{or } E_L) = \sqrt{E_R^2 + E_Y^2 + 2E_R E_Y \cos 60^\circ} = \sqrt{E_{ph}^2 + E_{ph}^2 + 2E_{ph} \times E_{ph} \times \frac{1}{2}} = \sqrt{3}E_{ph}$$

Similarly $E_{YB}(= E_L) = E_Y - E_B = \sqrt{3}E_{ph}$

and $E_{BR}(= E_L) = E_B - E_R = \sqrt{3}E_{ph}$

i.e. $E_{RY} = E_{YB} = E_{BR} = E_L = \sqrt{3}E_{ph}$

Hence, $E_L = \sqrt{3}E_{ph}$ ...(2.1)

(*i.e. Line voltage* = $\sqrt{3}$ *phase voltage*).

(*b*) **Relation between Line Currents and Phase Currents.** Since in star-connected system each line conductor is connected to separate phase, so the current flowing through the line and phase are same.

Current in outer (or line) $R = I_R$

Current in outer $Y = I_Y$

Current in outer $B = I_B$

Since $I_R = I_Y = I_B$ = say, I_{ph}—the phase current

∴ Line current, $I_L = I_{ph}$ ...(2.2)

(c) **Power.** *If the phase current has a phase difference of* ϕ *with phase voltage,*

Power per phase $= E_{ph} I_{ph} \cos \phi$

Total power (true), $P = 3 \times$ power per phase

$$P = 3 \times E_{ph} I_{ph} \cos \phi \quad ...(2.3)$$

Now

$$E_{ph} = \frac{E_L}{\sqrt{3}} \quad \text{and} \quad I_{ph} = I_L$$

Hence in terms of line values, the above expression becomes

$$P = 3 \times \frac{E_L}{\sqrt{3}} I_L \cos \phi$$

or

$$P = \sqrt{3} E_L I_L \cos \phi \quad ...(2.4)$$

(Apparent power $= \sqrt{3} E_L I_L$).

In a *balanced star-connected net work* the following points are worthnoting :

(*i*) *Line voltages are* $\sqrt{3}$ *times the phase voltages.*

(*ii*) *Line currents are equal to phase currents.*

(*iii*) Line voltages are 120° apart.

(*iv*) Line voltages are 30° ahead of the respective phase voltages.

(*v*) The angle between line currents and the corresponding line voltages is $(30° \pm \phi)$ + for lagging currents – ve for leading currents.

(*vi*) True power $= \sqrt{3} E_L I_L \cos \phi$, where ϕ is the angle between respective phase current and phase voltage, not between the line current and line voltage.

(*vii*) Apparent power $= \sqrt{3} E_L I_L$.

(*viii*) In balance system, the potential of the neutral or star point is zero.

∴ Potential at neutral (or star) point $= E_{NR} + E_{NY} + E_{NB} = 0$.

STAR OR WYE CONNECTION

Example 2.1. *A balanced star connected load of* $(8 + j6)$ Ω*/phase is connected to a 3-phase, 230 volts, 50 Hz supply. Find the current, p.f., power, volt ampere and reactive power. Draw the phasor diagram for the above circuit.* **(PTU, 1999)**

Solution. *Given* : $R = 8$ Ω ; $X_L = 6$ Ω ; $E_L = 230$ volts, $f = 50$ Hz.

The circuit is shown in Fig. 2.7 (*a*).

Phase voltage,
$$E_{ph} = \frac{E_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132.8 \text{ V}$$

Impedance,
$$Z = \sqrt{R^2 + X_L^2} = \sqrt{8^2 + 6^2} = 10 \ \Omega$$

Current,
$$I_{ph} = I_L = \frac{E_{ph}}{Z} = \frac{132.8}{10} = 13.28 \text{ A. } \textbf{(Ans.)}$$

Power factor,
$$\cos \phi = \frac{R}{Z} = \frac{8}{10} = 0.8. \ \textbf{(Ans.)} \quad (\because \ \phi = \cos^{-1}(0.8) = 36.87°)$$

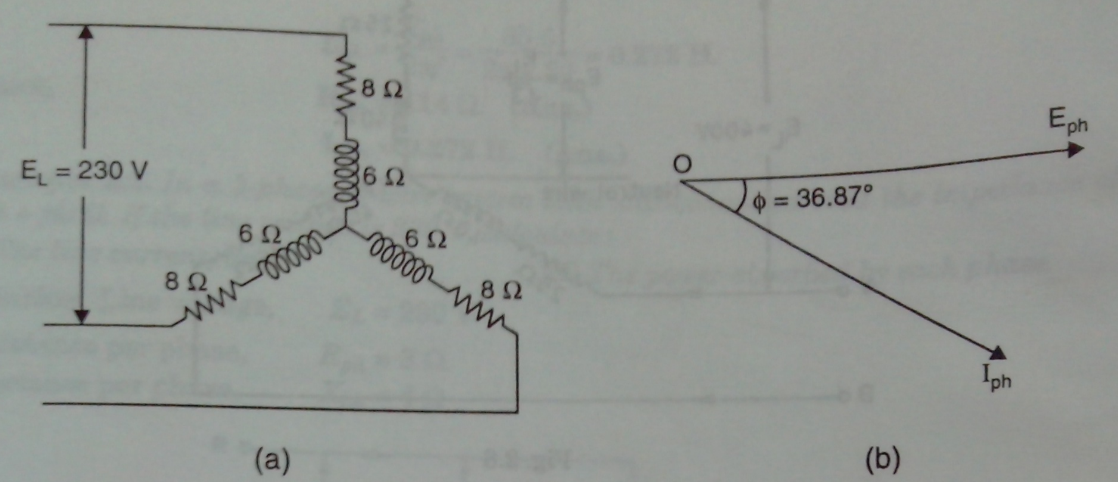


Fig. 2.7

Power, $P = \sqrt{3}E_L I_L \cos\phi$

$= \sqrt{3} \times 230 \times 13.28 \times 0.8 =$ **4342.3 W. (Ans.)**

Apparent power $= \sqrt{3}E_L I_L = \sqrt{3} \times 230 \times 13.28 =$ **5290.4 VA. (Ans.)**

Reactive power $= \sqrt{3}E_L I_L \sin\phi = \sqrt{3} \times 230 \times 13.28 \times \sin(36.87°)$

= **3174 VAR. (Ans.)**

The phasor diagram is shown in Fig. 2.7 (*b*).

Example 2.2. *Three equal impedances each having a resistance of 25 Ω and reactance of 40 Ω are connected in star to a 400 V, 3-phase, 50 Hz system. Calculate :*

(i) The line current *(ii) Power factor, and*
(iii) Power consumed.

Solution. Resistance per phase, $R_{ph} = 25\ \Omega$

Reactance per phase, $X_{ph} = 40\ \Omega$

Line voltage, $E_L = 400$ V

Line current, I_L :

Power factor, $\cos\phi$:

Power consumed, P :

Refer Fig. 2.8.

Impedance per phase, $Z_{ph} = \sqrt{R_{ph}^2 + X_{ph}^2}$

$Z_{ph} = \sqrt{25^2 + 40^2} = 47.17\ \Omega$

Phase voltage, $E_{ph} = \dfrac{E_L}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} = 231$ V

Phase current, $I_{ph} = \dfrac{E_{ph}}{Z_{ph}} = \dfrac{231}{47.17} = 4.9$ A (app.)

(*i*) **Line current,** I_L = phase current, I_{ph}

$I_L =$ **4.9 A. (Ans.)**

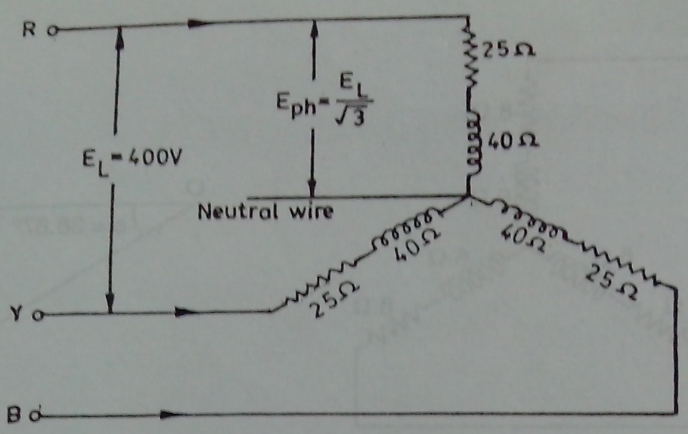


Fig. 2.8

(*ii*) **Power factor,** $\cos\phi = \dfrac{R_{ph}}{Z_{ph}} = \dfrac{25}{47.17} = \mathbf{0.53\ (lag).}$ **(Ans.)**

(*iii*) **Power consumed,** $P = \sqrt{3}E_L I_L \cos\phi = \sqrt{3} \times 400 \times 4.9 \times 0.53$

$= \mathbf{1800\ W\ (app.).}$ **(Ans.)**

[or $P = 3I_{ph}^2 R_{ph} = 3 \times 4.9^2 \times 25 = 1800$ W].

Example 2.3. *Three identical coils are connected in star to a 400 V (line voltage), 3-phase A.C. supply and each coil takes 300 W. If the power factor is 0.8 (lagging). Calculate :*

(*i*) *The line current,* (*ii*) *Impedance, and*

(*iii*) *Resistance and inductance of each coil.*

Solution. Line voltage, $E_L = 400$ V

Power taken by each coil, $P_{ph} = 300$ W

Power factor, $\cos\phi = 0.8$ (lagging)

$\mathbf{I_L ; Z ; R_{ph} ; L_{ph}}$ **:**

Phase voltage, $E_{ph} = \dfrac{E_L}{\sqrt{3}} = \dfrac{400}{\sqrt{3}}$ V

Also $P_{ph} = E_{ph} I_{ph} \cos\phi$

$300 = \dfrac{400}{\sqrt{3}} \times I_{ph} \times 0.8$

$I_{ph} = \dfrac{300 \times \sqrt{3}}{400 \times 0.8} = 1.62$ A.

(*i*) **Line current,** I_L = phase current, I_{ph}

$\mathbf{I_L = 1.62\ A.}$ **(Ans.)**

(*ii*) **Coil impedance,** $Z_{ph} = \dfrac{E_{ph}}{I_{ph}} = \dfrac{\frac{400}{\sqrt{3}}}{1.62} = 142.5\ \Omega$

$\mathbf{Z_{ph} = 142.5\ \Omega.}$ **(Ans.)**

(*iii*) $R_{ph} = Z_{ph} \cos\phi = 142.5 \times 0.8 = 114\ \Omega$

Coil reactance, $X_{ph} = Z_{ph} \sin\phi = 142.5 \times 0.6 = \mathbf{85.5\ \Omega}$ **(Ans.)**

But $X_{ph} = 2\pi f L_{ph}$

$$L_{ph} = \frac{X_{ph}}{2\pi f} = \frac{85.5}{2\pi \times 50} = 0.272 \text{ H.}$$

Hence, $\mathbf{R_{ph} = 114\ \Omega.}$ **(Ans.)**

and $\mathbf{L_{ph} = 0.272\ H.}$ **(Ans.)**

Example 2.4. *In a 3-phase, 3-wire system with star-connected load the impedance of each phase is (3 + j4) Ω. If the line voltage is 230 V, calculate :*

(i) The line current, and *(ii) The power absorbed by each phase.*

Solution. Line voltage, $E_L = 230$ V

Resistance per phase, $R_{ph} = 3\ \Omega$

Reactance per phase, $X_{ph} = 4\ \Omega$

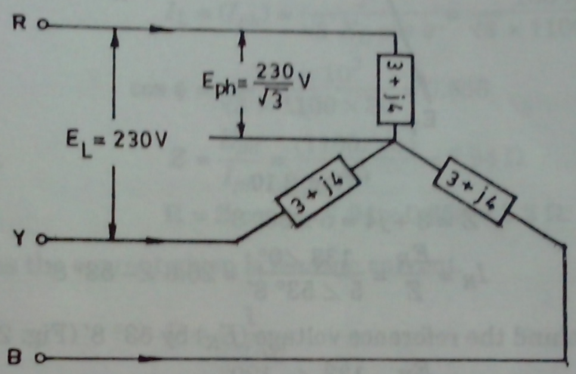


Fig. 2.9

$\mathbf{I_L}$ **;** $\mathbf{P_{ph}}$ **:**

Phase voltage, $E_{ph} = \frac{E_L}{\sqrt{3}} = \frac{230}{\sqrt{3}}$ V

Impedance per phase, $Z_{ph} = \sqrt{R_{ph}^2 + X_{ph}^2} = \sqrt{3^2 + 4^2} = 5\ \Omega$

Power factor, $\cos\phi = \frac{R}{Z} = \frac{3}{5} = 0.6$

Phase current, $I_{ph} = \frac{E_{ph}}{Z_{ph}} = \frac{230}{\sqrt{3} \times 5} = 26.56$ A

Line current, $\mathbf{I_L = I_{ph} = 26.56\ A.}$ **(Ans.)**

Power absorbed by each phase,

$$\mathbf{P_{ph}} = E_{ph} I_{ph} \cos\phi = \frac{230}{\sqrt{3}} \times 26.56 \times 0.6 = \mathbf{2116\ W\ (Ans.)}$$

$$[P_{ph} = I_{ph}^2 R_{ph} = 26.56^2 \times 3 = 2116 \text{ W}]$$

Solution by Symbolic Notation. In Fig. 2.10 E_R, E_Y and E_B are the phase voltages whereas I_R, I_Y and I_B are phase currents.

Taking *E_R as the reference vector*, we get

$$E_R = \frac{230}{\sqrt{3}} \angle 0° = 133 \angle 0° = 133 + j0 \text{ volt}$$

$$E_Y = 133 \angle -120° = 133\,(-0.5 - j0.866) = (-66.5 - j115) \text{ volts}$$

$$E_B = 133 \angle 120° = 133\,(-0.5 + j0.866) = (-66.5 + j115) \text{ volts}$$

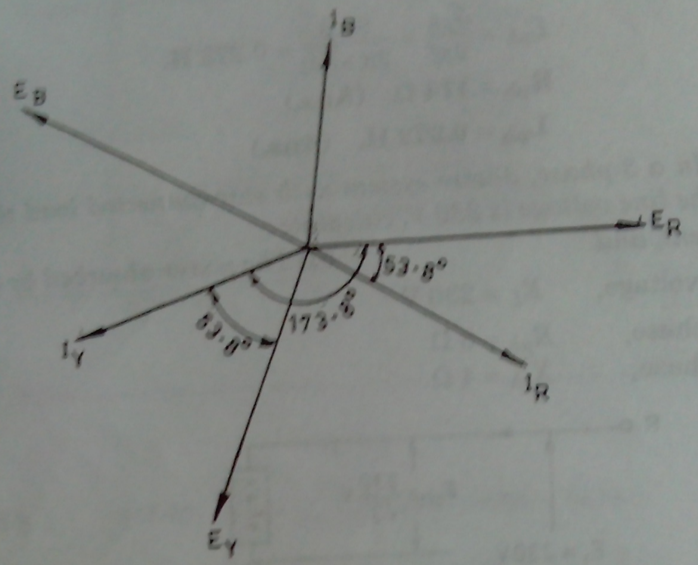


Fig. 2.10

$$Z = 3 + j4 = 5 \angle 53° \, 8'$$

$$I_R = \frac{E_R}{Z} = \frac{133 \angle 0°}{5 \angle 53° \, 8'} = 26.6 \angle -53° \, 8'$$

This current lags behind the reference voltage (E_R) by 53° 8′ (Fig. 2.10).

$$I_Y = \frac{E_Y}{Z} = \frac{133 \angle -120°}{5 \angle 53° \, 8'} = 26.6 \angle -173° \, 8'$$

It lags the reference vector *i.e.*, E_R by 173° 8′ which amounts to lagging behind its phase voltage E_Y by 53° 8′,

$$I_B = \frac{E_B}{Z} = \frac{133 \angle 120°}{5 \angle 53° \, 8'} = 26.6 \angle 66° \, 52'$$

This current leads E_R by 66° 52′ which is the same as *lagging* behind its phase voltage by 53° 8′.

Let us consider R-phase for calculation of power

$$E_R = (133 + j0) \; ; \; I_R = 26.6 \, (0.6 - j0.8) = (15.96 - j21.28)$$

Using method of conjugates, we get

$$P_{VA} = (133 - j0)\,(15.96 - j21.28) = 2116 - j2830$$

∴ Real power absorbed/phase = 2116 W ...(As before)

Example 2.5. *A star-connected, 6000 V, 3-phase alternator is supplying 4000 kW at a power factor of 0.8. Calculate the active and reactive components of the current in each phase.*

Solution. Line voltage, $E_L = 6000$ V

Power supplied, $P = 4000$ kW

Power factor, $\cos \phi = 0.8$

Active and reactive components of current :

We know that, $P = \sqrt{3} \, E_L I_L \cos \phi$

$$4000 \times 1000 = \sqrt{3} \times 6000 \times I_L \times 0.8$$

i.e., $$I_L = \frac{4000 \times 1000}{\sqrt{3} \times 6000 \times 0.8} = 481 \text{ A}$$

$\therefore$ $$I_{ph} = I_L = 481 \text{ A}$$

Active component $= I_{ph} \cos \phi = 481 \times 0.8 = 384.8$ **A. (Ans.)**

Reactive component $= I_{ph} \sin \phi = 481 \times 0.6 = 288.6$ **A. (Ans.)**

Example 2.6. *A balanced 3-phase star connected load of 100 kW takes a leading current of 80 A, when connected across a 3-phase, 1100 V, 50 Hz supply. Find the circuit constants of the load per phase.*

Solution. *Given* : $P = 100$ kW ; $I_{ph} (= I_L) = 80$ A ; $E_L = 1100$ V ; $f = 50$ Hz

Circuit constants of the load per phase, R, C :

As the 3-ϕ load is balanced and star connected, line or phase current,

$$I_L = (I_{ph}) = \frac{P}{\sqrt{3}\, E_L \cos \phi} = \frac{100 \times 10^3}{\sqrt{3} \times 1100 \times \cos \phi}$$

or $$\cos \phi = \frac{100 \times 10^3}{\sqrt{3} \times 1100 \times 80} = 0.656$$

Load impedance, $$Z = \frac{E_{ph}}{I_{ph}} = \frac{(1100/\sqrt{3})}{80} = 7.94\ \Omega$$

$$\mathbf{R} = Z \cos \phi = 7.94 \times 0.656 = \mathbf{5.2\ \Omega.} \ \textbf{(Ans.)}$$

Now $X_C = \frac{1}{2\pi f C}$ as the current given is *leading* current.

$$C = \frac{1}{2\pi f X_C}$$

But $$X_C = Z \sin \phi = 7.94 \times 0.755 = 5.99\ \Omega$$

$$\mathbf{C} = \frac{1}{2\pi \times 50 \times 5.99} \text{ F}$$

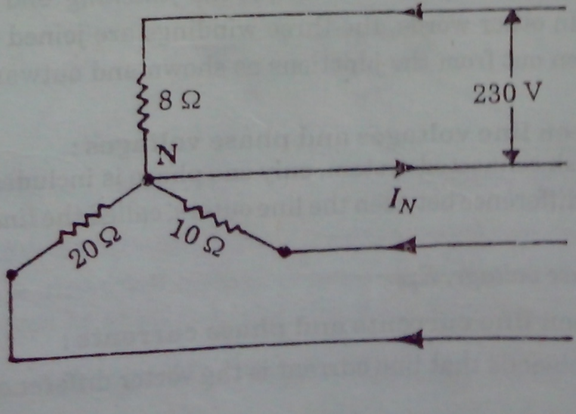
$$= \frac{10^6}{2\pi \times 50 \times 5.99}\ \mu\text{F} = \mathbf{531.4\ \mu F.} \ \textbf{(Ans.)}$$

Example 2.7. *A 3-phase, star-connected system with 230 V between each phase and neutral has resistances of 8, 10 and 20 Ω respectively in three phases, calculate :*

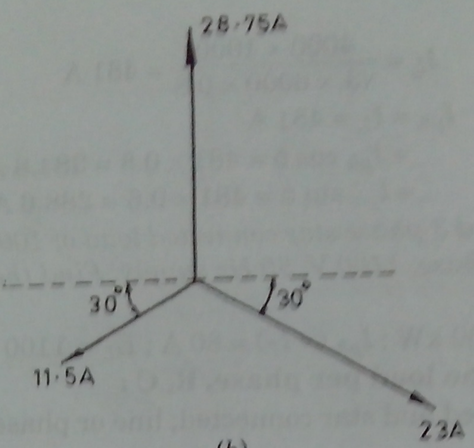
(i) The current flowing in each phase, *(ii) The neutral current, and*

(iii) The total power absorbed.

Solution. Refer Figs. 2.11 (*a*) and (*b*).



(*a*)



(b)

Fig. 2.11

Phase voltage, $E_{ph} = 230$ V

(i) Current in 8 Ω resistor $= \frac{230}{8} = 28.75$ **A. (Ans.)**

Current in 10 Ω resistor $= \frac{230}{10} = 23$ **A. (Ans.)**

Current in 12 Ω resistor $= \frac{230}{20} = 11.5$ **A. (Ans.)**

(ii) The above currents are mutually displaced by 120°. The neutral current I_N is the vector sum of these three currents.

I_N can be found by splitting up these three-phase currents into their X-components and Y-components and then by combining them together.

ΣX-components $= 23 \cos 30° - 11.5 \cos 30° = 11.5 \cos 30° = 9.96$ A

ΣY-components $= 28.75 - 23 \sin 30° - 11.5 \sin 30° = 28.75 - 34.5 \sin 30° = 11.5$ A

∴ **Neutral current,** $I_N = \sqrt{(9.96)^2 + (11.5)^2} = 15.21$ **A. (Ans.)**

(iii) **Total power absorbed,**

$$P = 230\,(28.75 + 23 + 11.5) = 14547.5 \text{ W. (Ans.)}$$

2.7. DELTA (Δ) OR MESH CONNECTION

In a delta or mesh connection the *dissimilar* ends of the three-phase windings are joined together *i.e.* the '*starting*' end of one phase is joined to the '*finishing*' end of the other phase and so on as shown in Fig. 2.12. In other words, the three windings are joined in series to form a closed mesh. Three leads are taken out from the junctions as shown and outward directions are taken as positive.

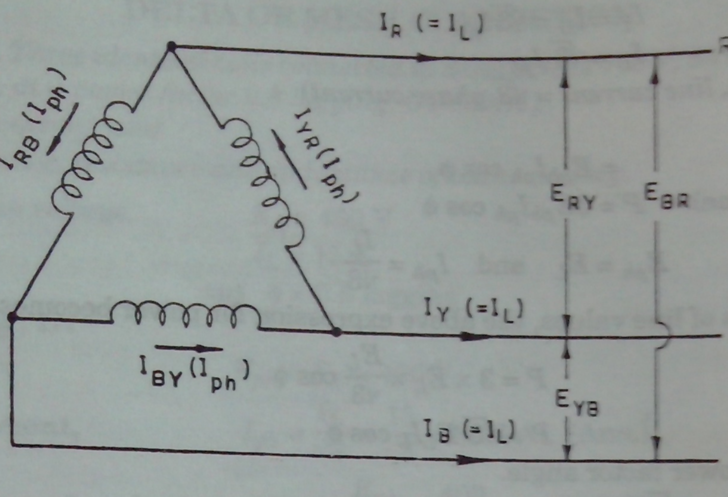
(a) **Relation between line voltages and phase voltages :**

Since in delta or mesh connected system, only one phase is included between any pair of line outers, therefore potential difference between the line outers, called the *line voltage*, is equal to phase voltage.

i.e., Line voltage, E_L = *phase voltage,* E_{ph}.

(b) **Relation between line currents and phase currents :**

From Fig. 2.12 it is obvious that line current is the vector difference of phase currents of two phases concerned.



$E_{RY} = E_{YB} = E_{BR} = E_{ph} = E_L$

$I_R = I_Y = I_B = I_L \; (= \sqrt{3}\, I_{ph})$

Fig. 2.12. Delta or mesh connected diagram.

Thus, line current, $I_R = I_{YR} - I_{RB}$ (Vector difference)

$= I_{YR} + (-I_{RB})$ (Vector sum)

Similarly, $I_Y = I_{BY} - I_{YR}$ and $I_B = I_{RB} - I_{BY}$

Refer Fig. 2.13. Since phase angle between phase current I_{YR} and $-I_{RB}$ is 60°,

$$\therefore \quad I_R = \sqrt{I_{YR}^2 + I_{RB}^2 + 2 I_{YR} I_{RB} \cos 60^\circ}$$

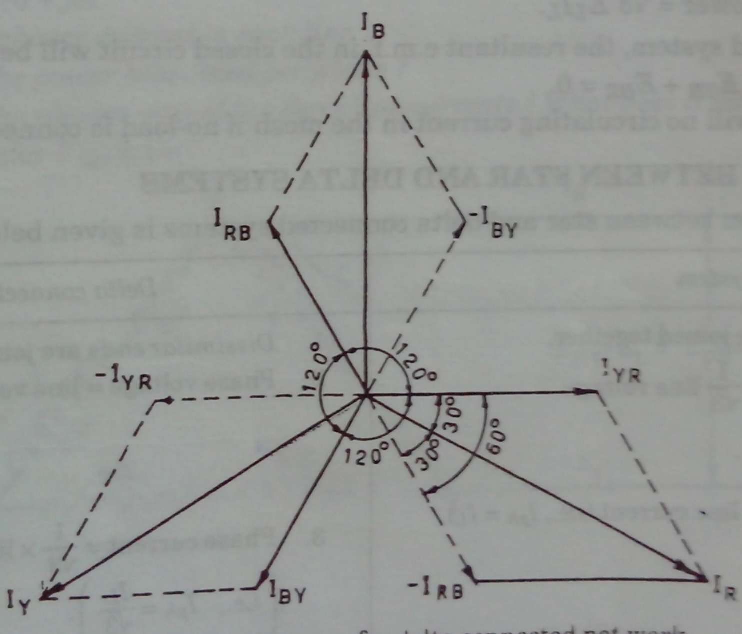


Fig. 2.13. Vector diagram for delta connected net work.

Assuming the delta connected system or network be balanced, the phase current in each winding is equal and let each be equal to I_{ph} (*i.e.*, $I_{YR} = I_{BY} = I_{RB} = I_{ph}$)

$$\therefore \quad I_R (= I_L) = \sqrt{I_{ph}^2 + I_{ph}^2 + 2 I_{ph} I_{ph} \cos 60^\circ} = \sqrt{2 I_{ph}^2 + 2 I_{ph}^2 \times \frac{1}{2}} = \sqrt{3}\, I_{ph}$$

$$I_Y = I_B = \sqrt{3} I_{ph}$$

Similarly,

Hence, $I_L = \sqrt{3}\ I_{ph}$ ...(2.5)

(*i.e., line current* = $\sqrt{3}$ *phase current*)

(*c*) **Power :**

Power/phase $= E_{ph} I_{ph} \cos \phi$

Total power (true) $P = 3E_{ph} I_{ph} \cos \phi$

But $E_{ph} = E_L$ and $I_{ph} = \frac{I_L}{\sqrt{3}}$

Hence, in terms of line values, the above expression for power becomes

$$P = 3 \times E_L \times \frac{E_L}{\sqrt{3}} \cos \phi$$

or $$P = \sqrt{3}\ E_L I_L \cos \phi \quad ...(2.6)$$

where ϕ = the phase power factor angle.

(*Apparent power* = $\sqrt{3} E_L I_L$)

In case of delta or mesh connected system the *following points are worthnoting* :

(*i*) *Line voltages are equal to phase voltages.*

(*ii*) Line currents are $\sqrt{3}$ times phase currents.

(*iii*) Line currents are 120° apart.

(*iv*) Line currents are 30° behind their respective phase currents.

(*v*) The angle between line currents and corresponding line voltages is (30° ± ϕ) as in the star system.

(*vi*) True power = $\sqrt{3}\ E_L I_L \cos \phi$, where ϕ is the phase angle between respective phase current and phase voltage.

(*vii*) Apparent power = $\sqrt{3}\ E_L I_L$.

(*viii*) In balanced system, the resultant e.m.f. in the closed circuit will be zero.

i.e., $E_{RY} + E_{YB} + E_{BR} = 0$.

Hence, there will no circulating current in the mesh if no-load is connected to the lines.

2.8. COMPARISON BETWEEN STAR AND DELTA SYSTEMS

The comparison between star and delta connected systems is given below :

Star connected system	*Delta connected system*
1. *Similar ends* are joined together.	1. *Dissimilar ends* are joined.
2. Phase voltage = $\frac{1}{\sqrt{3}}$ line voltage $\left(i.e., E_{ph} = \frac{E_L}{\sqrt{3}}\right)$.	2. Phase voltage = line voltage (*i.e.*, $E_{ph} = E_L$).
3. Phase current = line current (*i.e.*, $I_{ph} = I_L$).	3. Phase current = $\frac{1}{\sqrt{3}}$ × line current $\left(i.e.,\ I_{ph} = \frac{I_L}{\sqrt{3}}\right)$.
4. Possible to carry neutral to the load.	4. Neutral wire not available.
5. Provides 3-phase 4-wire arrangement.	5. Provides 3-phase 3-wire arrangement.
6. Can be used for lighting as well as power load.	6. Can be used for power loads only.
7. Neutral wire of a star connected alternator can be connected to earth, so relays and protective devices can be provided in the star connected alternators for safety.	7. Not possible. Delta connected system is mostly used in transformer for running of small low voltage 3-phase motors and *best suited for rotary converters.*

DELTA OR MESH CONNECTION

Example 2.8. *Three identical coils connected in delta across 400 V, 50 Hz, 3-phase supply take a line current of 15 A at a power factor 0.8 lagging. Calculate :*

(i) The phase current, and

(ii) The impedance, resistance and inductance of each winding.

Solution. Line voltage, $E_L = 400$ V

Line current, $I_L = 15$ A

Power factor, $\cos\ \phi = 0.8$ lagging

$\mathbf{I_{ph}}$; $\mathbf{Z_{ph}}$; $\mathbf{R_{ph}}$; **L :**

Phase voltage, $E_{ph} = E_L = 400$ V

(i) **Phase current,** $I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{15}{\sqrt{3}} = \mathbf{8.66\ A.}$ **(Ans.)**

(ii) **Impedance of each phase,** $\mathbf{Z_{ph}} = \frac{E_{ph}}{I_{ph}} = \frac{400}{8.66} = \mathbf{46.19\ \Omega.}$ **(Ans.)**

Resistance of each phase, $\mathbf{R_{ph}} = Z_{ph}\cos\phi = 46.19 \times 0.8 = \mathbf{36.95\ \Omega.}$ **(Ans.)**

Reactance of each phase, $X_{ph} = Z_{ph}\sin\phi = 46.19\sqrt{(1-\cos^2\phi)}$

$$= 46.19\sqrt{1-(0.8)^2} = 27.71\Omega$$

∴ **Inductance,** $\mathbf{L} = \frac{X_{ph}}{2\pi f} = \frac{27.71}{2\pi \times 50} = \mathbf{0.088\ H.}$ **(Ans.)**

Example 2.9. *A 220 V, 3-phase voltage is applied to a balanced delta-connected 3-phase load of phase impedance (6 + j8).*

(i) Find the phasor current in each line.

(ii) What is the power consumed per phase ?

(iii) What is the phasor sum of the three line currents ? What does it have this value ?

Solution. Refer Fig. 2.14.

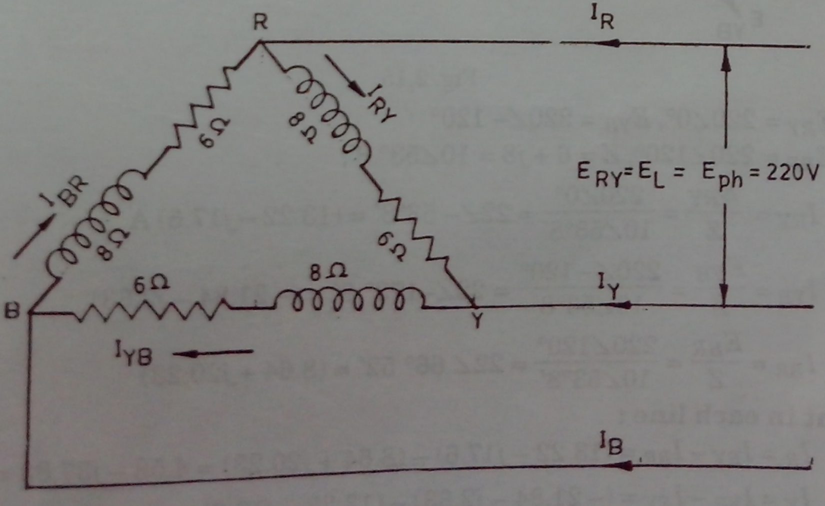


Fig. 2.14

Resistance per phase, $R_{ph} = 6\ \Omega$

Reactance per phase, $X_{ph} = 8\ \Omega$

$E_L = E_{ph} = 220$ V

Impedance per phase, $Z_{ph} = \sqrt{R_{ph}^2 + X_{ph}^2} = \sqrt{6^2 + 8^2} = 10\ \Omega$

(*i*) Phase current, $I_{ph} = \dfrac{E_{ph}}{Z_{ph}} = \dfrac{220}{10} = 22$ A

∴ Line current, $I_L = \sqrt{3} \times 22 =$ **38.1 A. (Ans.)**

(*ii*) **Power consumed per phase,**

$$P_{ph} = I_{ph}^2 \times R_{ph} = 22^2 \times 6 = \mathbf{2904\ W.}\ \ \textbf{(Ans.)}$$

(*iii*) Phasor sum would be zero because the three currents are equal in magnitudes and have a mutual phase difference of 120°.

Solution by Symbolic Notation. Let E_{RY} is taken as a reference vector (Fig. 2.15).

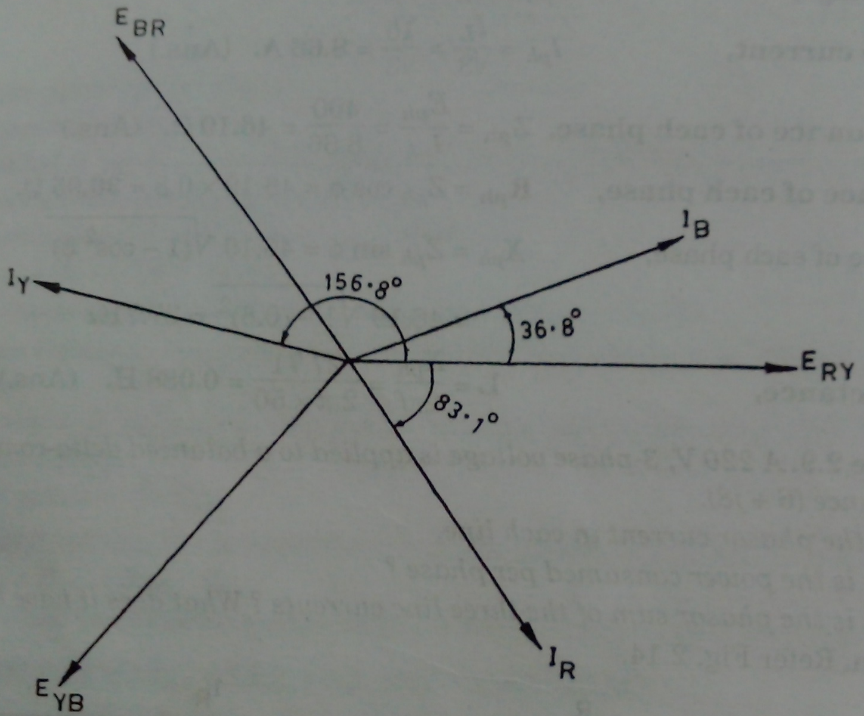


Fig. 2.15

$E_{RY} = 220\angle 0°$, $E_{YB} = 220\angle -120°$

$E_{BR} = 220\angle 120°$, $Z = 6 + j8 = 10\angle 53°\ 8'$,

$$I_{RY} = \frac{E_{RY}}{Z} = \frac{220\angle 0°}{10\angle 53°8'} = 22\angle -53°8' = (13.22 - j17.6)\ \text{A}$$

$$I_{YB} = \frac{E_{YB}}{Z} = \frac{220\angle -120°}{10\angle 53°8'} = 22\angle -173°\ 8' = (-21.84 - j2.63)$$

$$I_{BR} = \frac{E_{BR}}{Z} = \frac{220\angle 120°}{10\angle 53°8'} = 22\angle\ 66°\ 52' = (8.64 + j20.23)$$

(*i*) **Current in each line :**

$I_R = I_{RY} - I_{BR} = (13.22 - j17.6) - (8.64 + j20.23) = 4.58 - j37.83 = \mathbf{38.1\angle 83.1°}$. **(Ans.)**

$I_Y = I_{YB} - I_{RY} = (-21.84 - j2.63) - (13.22 - j17.6)$

$= -21.84 - j2.63 - 13.22 + j17.60 = -35.06 + j14.97 = \mathbf{38.12\angle 156.8°}$. **(Ans.)**

$I_B = I_{BR} - I_{YB} = (8.64 + j20.23) - (-21.84 + j2.63) = 8.64 + j20.23 + 21.84 + j2.63$

$= 30.48 + j22.86 = \mathbf{38.1\angle 36.8°}$. **(Ans.)**

(*ii*) **Power consumed per phase :**

Using conjugate of voltage, we get for *R*-phase

$$P_{VA} = E_{RY} \cdot I_{RY} = (220 - j0)(13.22 - j17.6) = (2908.4 - j3872) \text{ volt ampere}$$

True power per phase = **2.908 kW. (Ans.)**

(*iii*) **Phase sum of the three line currents**

$$= I_R + I_Y + I_B$$
$$= (4.58 - j37.83) + (-35.06 + j14.96) + (30.48 + j22.86) = 0$$

Hence, **the phasor sum of three line currents drawn by a 'balanced load' is zero. (Ans.)**

Example 2.10. *A delta-connected balanced 3-phase load is supplied from a 3-phase, 400 V supply. The line current is 30 A and the power taken by the load is 12 kW. Find :*

(*i*) *Impedance in each branch ; and*

(*ii*) *The line current, power factor and power consumed if the same load is connected is star.*

Solution. Delta-connection :

$$E_{ph} = E_L = 400 \text{ V}$$
$$I_L = 30 \text{ A}$$

$$\therefore \quad I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{30}{\sqrt{3}} = 17.32 \text{ A.}$$

(*i*) **Impedance per phase**

$$\mathbf{Z_{ph}} = \frac{E_{ph}}{I_{ph}} = \frac{400}{17.32} = \mathbf{23.09\ \Omega.} \quad \textbf{(Ans.)}$$

Now $P = \sqrt{3} E_L I_L \cos \phi$

$$12000 = \sqrt{3} \times 400 \times 30 \times \cos \phi.$$

or $$\cos \phi \text{ (power factor)} = \frac{12000}{\sqrt{3} \times 400 \times 30} = 0.577$$

(*ii*) **Star-connection**

$$E_{ph} = \frac{E_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

$$\mathbf{I_L} = I_{ph} = \frac{E_{ph}}{Z_{ph}} = \frac{231}{23.09} = \mathbf{10\ A.} \quad \textbf{(Ans.)}$$

Power factor, $\cos \phi = 0.577$ (since impedance is same)

Power consumed $= \sqrt{3}\, E_L I_L \cos \phi = \sqrt{3} \times 400 \times 10 \times 0.577 = \mathbf{3997.6\ W.}$ **(Ans.)**

Example 2.11. *Three 50 Ω non-inductive resistances are connected in (i) star, (ii) delta across a 400 V, 50 Hz., 3-phase mains. Calculate the power taken from the supply system in each case. In the event of one of the three resistances getting opened, what would be the value of the total power taken from the mains in each of the two cases.*

Solution. Star connection :

Phase voltage, $$E_{ph} = \frac{E_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

Phase current, $$I_{ph} = \frac{E_{ph}}{R_{ph}} = \frac{231}{50} = 4.62 \text{ A}$$

Power consumed, $$P = 3I_{ph}^2 R_{ph} = 3 \times 4.62^2 \times 50 = \mathbf{3200\ W.} \quad \textbf{(Ans.)}$$

$$[\text{or } P = \sqrt{3} E_L I_L \cos \phi = \sqrt{3} \times 400 \times 4.62 \times 1 = 3200 \text{ W}]$$

Delta connection :

Phase voltage, $E_{ph} = E_L = 400$ V

Phase current, $I_{ph} = \frac{E_{ph}}{R_{ph}} = \frac{400}{50} = 8$ A

Power consumed, $P = 3I_{ph}^2 R_{ph} = 3 \times 8^2 \times 50 =$ **9600 W. (Ans.)**

When one of the resistances is disconnected :

(*i*) **Star connection.** Refer Fig. 2.16.

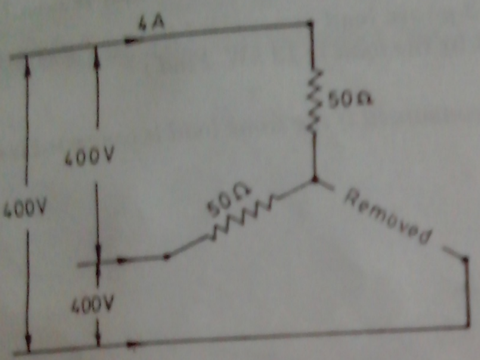


Fig. 2.16

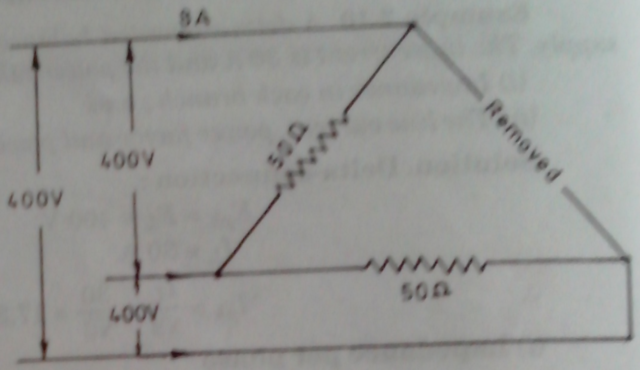


Fig. 2.17

When one of the resistances is disconnected, the circuit is no longer 3-phase but converted into single-phase circuit, having two resistances each of 50 ohm connected in series across supply of 400 V.

Hence line current, $I_L = \frac{E_L}{2R_{ph}} = \frac{400}{2 \times 50} = 4$ A

Power consumed, $P = 4^2 (50 + 50) =$ **1600 W. (Ans.)**

[or $P = VI \cos \phi = 400 \times 4 \times 1 = 1600$ W].

(*ii*) **Delta connection.** Refer Fig. 2.17.

Potential difference across each resistance, $E_L = 400$ V

Current in each resistance $= \frac{400}{50} = 8$ A

Power consumed in both resistances $= 2 \times 8^2 \times 50 =$ **6400 W. (Ans.)**

[or $P = 2 \times E_{ph} I_{ph} \cos \phi = 2 \times 400 \times 8 \times 1 = 6400$ W].

Example 2.12. *The secondary of a 3-phase star-connected transformer, which has a phase voltage of 230 V feeds a 3-phase delta connected load ; each phase of which has a resistance of 30 Ω and an inductive reactance of 40 Ω. Draw the circuit diagram of the system and calculate :*

(*i*) *The voltage across each phase of load,*

(*ii*) *The current in each phase of load,*

(*iii*) *The current in the transformer secondary windings, and*

(*iv*) *The total power taken from the supply and its power factor.*

Solution. Refer Fig. 2.18.

Resistance per phase, $R_{ph} = 20$ Ω

Reactance per phase, $X_{ph} = 40$ Ω

Phase voltage across transformer secondary,

$E_{ph} = 230$ V

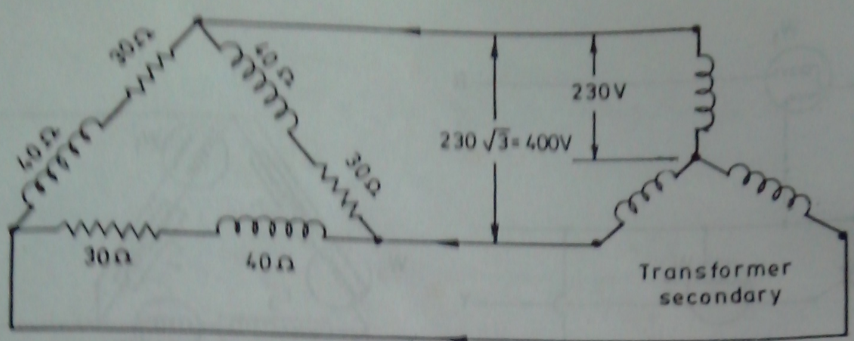


Fig. 2.18

Line voltage across delta connected load = line voltage across transformer secondary

$= \sqrt{3} \times 230 = 400$ V (app.)

(*i*) **Voltage across each phase of the load,**

$E_{ph} = E_L =$ **400 V. (Ans.)**

(*ii*) **Current in each phase of the load,**

$$I_{ph} = \frac{E_{ph}}{Z_{ph}} = \frac{400}{\sqrt{R_{ph}^2 + X_{ph}^2}} = \frac{400}{\sqrt{30^2 + 40^2}} = \frac{400}{50} = \mathbf{8\ A.\ (Ans.)}$$

(*iii*) **Current in the transformer secondary**

$= \text{line current of load} = \sqrt{3} \times I_{ph} = \sqrt{3} \times 8 =$ **13.86 A. (Ans.)**

(*iv*) **Power factor,** $\cos\phi = \frac{R_{ph}}{Z_{ph}} = \frac{30}{50} =$ **0.6. (Ans.)**

Total power consumed, $P = \sqrt{3} E_L I_L \cos\phi = \sqrt{3} \times 400 \times 13.86 \times 0.6 =$ **5761.5 W. (Ans.)**

2.9. MEASUREMENT OF POWER IN 3-PHASE CIRCUIT

- The power in 3-phase load can be measured by using the following methods :
  1. Three wattmeters method.
  2. Two wattmeters method.
  3. One wattmeter method.
- A wattmeter consists of two coils : Refer Fig. 2.19.
  1. *Current coil*—possesses a *low resistance*.
  2. *Pressure or potential coil*—possesses a *high resistance*.

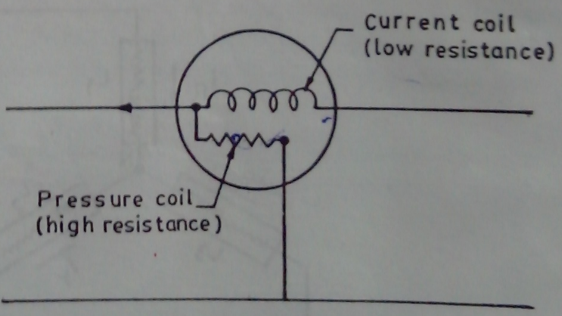


Fig. 2.19. Connections of a wattmeter.

The '*current coil*' is connected in *series with the line carrying the current* and the '*pressure coil*' *is connected across the two points whose potential difference is to be measured*. A wattmeter shows a reading which is proportional to the *product* of the *current* through its current coil, the *potential difference* across its pressure coil and *cosine of the angle* between this voltage and current.

2.9.1. Three-wattmeter Method. Figs. 2.20 and 2.21 show the connection diagram for star-connected and delta-connected loads respectively. As indicated in the figures three wattmeters are connected in each of the three phases of the load whether star or delta connected. The current coil of each wattmeter carries the current of one phase only and the pressure coil measures the

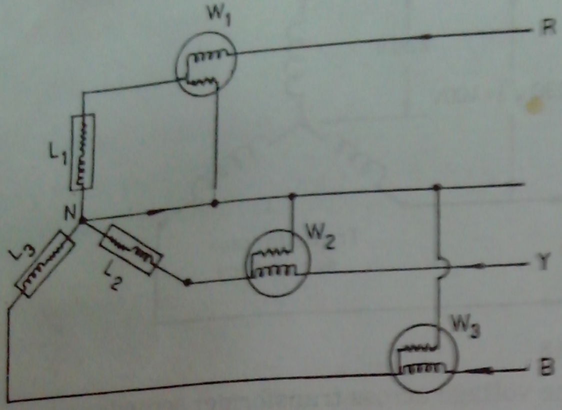


Fig. 2.20. Star-connected load.

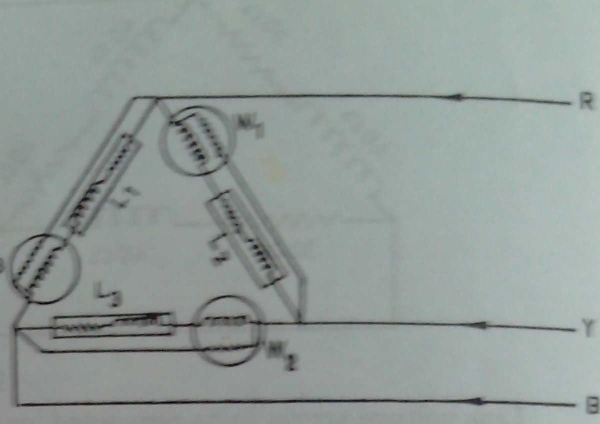


Fig. 2.21. Delta-connected load.

phase-voltage of the phase. Hence, *each wattmeter measures the power in a single phase. The total power in the load is given by the algebraic sum of the readings of the three wattmeters.*

While using this method following *difficulty* is met with :

- In case of star-connected load it is not always possible to get at neutral point which is required for connections (Fig. 2.20).
- In case of delta-connected load, under ordinary conditions it is not generally feasible to break into the phases of the load.

To measure power it is not necessary to use three wattmeters, two wattmeters can be used for the purpose as explained in the Article 2.9.2.

2.9.2. Two-wattmeter Method. Balanced or unbalanced load, Figs. 2.22 and 2.23 show connection diagrams for star-connected and delta-connected loads respectively. In this method the current coils of the two wattmeters are inserted in *any two* lines and the pressure (or potential) coil of each joined to the *third line.*

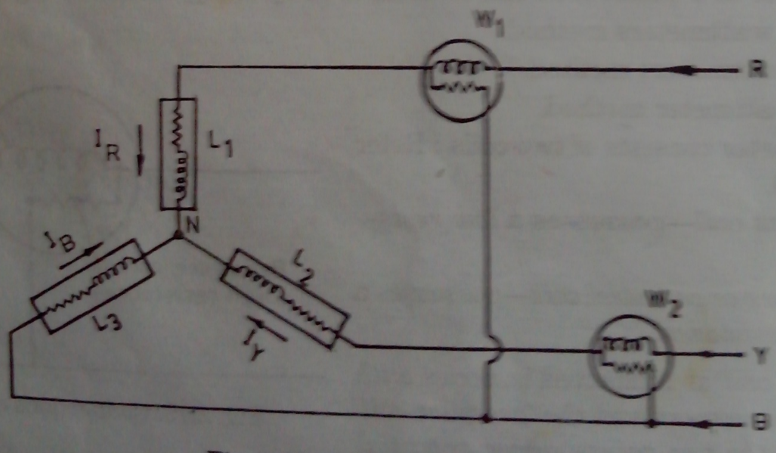


Fig. 2.22. Star-connected load.

It can be proved that the sum of the instantaneous powers indicated by W_1 and W_2 gives the instantaneous power absorbed by the three loads L_1, L_2 and L_3. Let us consider a star-connected load (although it can be equally applied to a delta-connected load which can always be replaced by an equivalent star-connected load).

Keeping in mind that it is important to take *the direction of the voltage through the circuit as the same as that taken for the current when establishing the readings of the two wattmeters.*

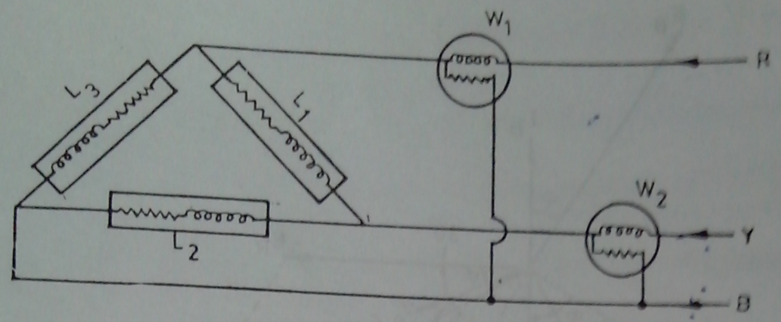


Fig. 2.23. Delta-connected load.

Instantaneous current through $W_1 = i_R$

Instantaneous potential difference across $W_1 = e_{RB} = e_R - e_B$

Instantaneous power read by $W_1 = i_R(e_R - e_B)$

Instantaneous current through $W_2 = i_Y$

Instantaneous potential difference across $W_2 = e_{YB} = e_Y - e_B$

Instantaneous power read by $W_2 = i_Y(e_Y - e_B)$

$$\therefore \quad W_1 + W_2 = i_R(e_R - e_B) + i_Y(e_Y - e_B) = i_Re_R + i_Ye_Y - e_B(i_R + i_Y)$$

Now according to Kirchhoff's point law

$$i_R + i_Y + i_B = 0$$

$$\therefore \quad i_R + i_Y = -i_B$$

$$\therefore \quad W_1 + W_2 = i_Re_R + i_Ye_Y + i_Be_B = p_1 + p_2 + p_3$$

where p_1 = power absorbed by load L_1, p_2 = power absorbed by load L_2, and p_3 = power absorbed by load L_3.

$$\therefore \quad W_1 + W_2 = \text{total power absorbed.}$$

This proof is true whether the load is *balanced* or *unbalanced*.

In case the load is star-connected, then it should have no neutral connection (*i.e.*, 3-phase, 3-wire connected) and if it has a neutral connection (*i.e.*, 3-phase, 4-wire connected) that it should be exactly balanced so that in each case there is no neutral current i_N otherwise Kirchhoff's point law will give $i_R + i_Y + i_B + i_N = 0$.

In the above derivation we have considered the *instantaneous* readings. In fact the moving system wattmeter, due to its inertia, cannot quickly follow the variations taking place in cycle, hence it indicates the *average power*.

$$\therefore \text{ Total power } = W_1 + W_2$$

$$= \frac{1}{T}\int_0^T i_Re_{RB}\,dt + \frac{1}{T}\int_0^T i_Ye_{YB}\,dt. \qquad ...(2.7)$$

Two-wattmeter Method—Balanced load. The total power consumed by a *balanced load* can be found by using two wattmeters (Figs. 2.22 and 2 23) When load is assumed inductive in Fig. 2.20, the vector diagram for such a balanced star-connected load is shown in Fig. 2.24.

Let us consider the problem in terms of r.m.s. values (instead of instantaneous values).

Let E_R, E_Y, E_B = r.m.s. values of the three phase voltages,

and I_R, I_Y, I_B = r.m.s. values of the currents.

Since these voltages and currents are assumed sinusoidal, they can be represented by vectors, the currents lagging behind their respective phase voltages by ϕ.

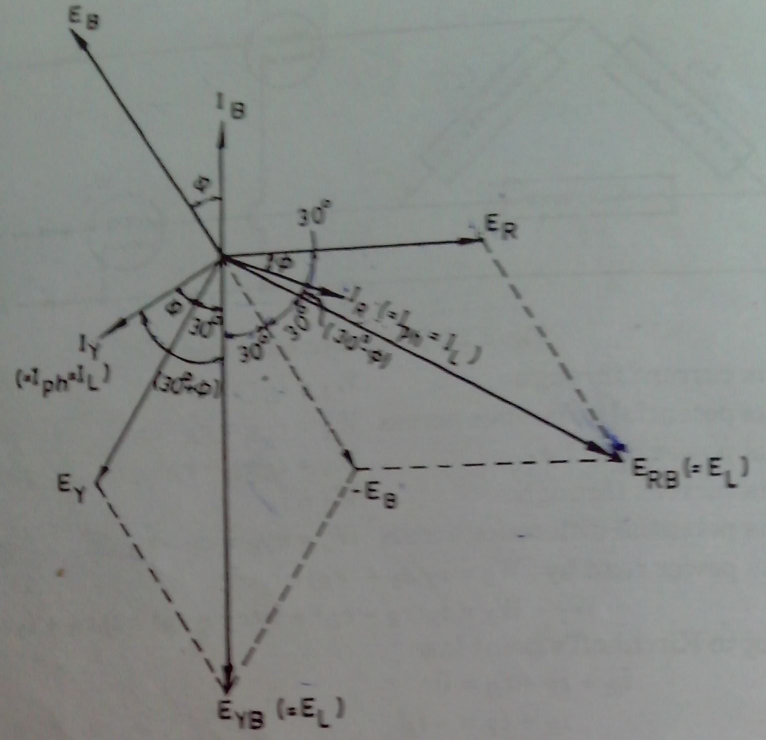


Fig. 2.24. Vector diagram—two wattmeters method.

Refer Fig. 2.24.

Current through wattmeter $W_1 = I_R$

Potential difference across pressure coil of wattmeter

$$W_1 = E_{RB} = E_R - E_B \qquad \text{(Vectorially)}$$

The value of E_{RB} is found by compounding E_R and E_B reversed as shown in Fig. 2.24. It may be observed that phase difference between E_{RB} and $I_R = (30° - \phi)$.

$\therefore$ Reading of wattmeter $W_1 = E_{RB} I_R \cos (30° - \phi)$ ...(2.8)

Similarly, current through wattmeter $W_2 = I_Y$

Potential difference across pressure coil of wattmeter

$$W_2 = E_{YB} = E_Y - E_B \qquad \text{(Vectorially)}$$

The value of E_{YB} is found by compounding E_Y and E_B reversed as shown in Fig. 2.24. The phase difference between E_{YB} and $I_Y = (30° + \phi)$.

$\therefore$ Reading of wattmeter $W_2 = E_{YB} I_Y \cos (30° + \phi)$ ...(2.9)

Since the load is balanced, $E_{RB} = E_{YB} = E_L$ (Line voltage)

and $I_R = I_Y = I_L$ (Line current)

$\therefore$ $W_1 = E_L I_L \cos (30° - \phi)$

and $W_2 = E_L I_L \cos (30° + \phi)$

$\therefore$ Total power,

$$\begin{aligned} P &= W_1 + W_2 \\ &= E_L I_L \cos (30° - \phi) + E_L I_L \cos (30° + \phi) \\ &= E_L I_L [\cos (30° - \phi) + \cos (30° + \phi)] \end{aligned}$$

$$= E_L I_L [\cos 30° \cos \phi + \sin 30° \sin \phi + \cos 30° \cos \phi - \sin 30° \sin \phi]$$

$$= E_L I_L (2 \cos 30° \cos \phi) = E_L I_L \times 2 \times \frac{\sqrt{3}}{2} \cos \phi = \sqrt{3} E_L I_L \cos \phi$$

i.e., $$P = \sqrt{3} E_L I_L \cos \phi$$

Hence the *sum of the readings of the two wattmeters give the total power consumption in the 3-phase load.*

It is worth noting that in the above case the phase sequence of RYB has been assumed, the readings of the two wattmeters will change if the phase sequence is reversed.

Variations in wattmeter readings. As shown above that for a *lagging power factor*

$$W_1 = E_L I_L \cos (30° - \phi)$$

and $$W_2 = E_L I_L \cos (30° + \phi)$$

From above it is evident that individual readings of the wattmeters *not only depend on the load but also upon its power factor.* Let us take up the following cases :

(*i*) When $\phi = 0$

i.e., power factor is *unity* (*i.e.,* load is *resistive*)

Then $$W_1 = W_2 = E_L I_L \cos 30°$$

The reading of each wattmeter will be equal and opposite (*i.e.,* up-scale reading).

(*ii*) When $\phi = 60°$

i.e., power factor = 0.5 (lagging)

Then $$W_2 = E_L I_L \cos (30° + 60°) = 0$$

Hence, the power is measured by W_1 above.

(*iii*) When $90° > \phi > 60°$

i.e., $0.5 > \text{p.f.} > 0$

Then W_1 is still *positive* but reading of W_2 is *reversed*. For a leading p.f., conditions are just the opposite of this. In that case, W_1 will read *negative* because the phase angle between the current and voltage is *more than* 90°. For getting the total power, the reading of W_2 is to be *subtracted* from that of W_1. Under this condition, W_2 will read *'down scale' i.e., backwards*. Hence, *to obtain a reading on* W_2, *it is necessary to reverse either its pressure coil or current coil, usually the former.*

All readings taken after reversal of pressure coil are to be taken as negative.

(*iv*) When $\phi = 90°$

(*i.e.,* p.f. = 0 *i.e.,* pure inductive or capacitive load)

Then $$W_1 = E_L I_L \cos (30° - 90°) = E_L I_L \sin 30°$$

and $$W_2 = E_L I_L \cos (30° + 90°) = - E_L I_L \sin 30°$$

These two readings are equal in magnitude but opposite in sign

$$\therefore \quad W_1 + W_2 = 0.$$

So far we have considered *lagging angles* (taken as *positive*). Now let us discuss how the readings of wattmeters change when the power factor is *leading* one.

— For $\phi = +60°$ (lag) : $W_2 = 0$

— For $\phi = -60°$ (lead) : $W_1 = 0$

Thus we find that for angles of lead the readings of the two wattmeters are interchanged.

Hence, when the power is *leading* :

$$W_1 = E_L I_L \cos (30° + \phi)$$
$$W_2 = E_L I_L \cos (30° - \phi).$$

Power Factor—When the load is 'balanced'. When load is balanced with a *lagging power factor* and the voltage and currents are sinusoidal :

$$W_1 + W_2 = E_L I_L \cos (30° - \phi) + E_L I_L \cos (30° + \phi) = \sqrt{3} E_L I_L \cos \phi \quad ...(2.10)$$

Similarly,

$$W_1 - W_2 = E_L I_L \cos (30° - \phi) - E_L I_L \cos (30° + \phi)$$
$$= E_L I_L (2 \sin \phi \sin 30°) = E_L I_L \sin \phi \quad ...(2.11)$$

Dividing (2.11) by (2.10), we get

$$\tan \phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} \quad ...(2.12)$$

For a *leading power*, this expression becomes

$$\tan \phi = -\frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)} \quad ...(2.13)$$

After finding tan ϕ, hence ϕ, the value of power factor cos ϕ can be found (from trigonometrical tables).

One important point which must be kept in mind is that if W_2 reading has been taken after reversing the pressure coil *i.e.*, if W_2 is *negative*, then the eqn. (2.12) becomes

$$\tan \phi = \sqrt{3}\, \frac{W_1 - (-W_2)}{W_1 + (-W_2)}$$

or

$$\tan \phi = \sqrt{3}\, \frac{W_1 + W_2}{W_1 - W_2} \quad ...(2.14)$$

The power factor may also be expressed in terms of ratio of the readings of the two wattmeters.

Let $\dfrac{\text{Smaller reading}}{\text{Larger reading}} = \dfrac{W_2}{W_1} = \alpha$

Then from eqn. (2.12) above, we have

$$\tan \phi = \frac{\sqrt{3}\left[1 - \left(\dfrac{W_2}{W_1}\right)\right]}{\left[1 + \left(\dfrac{W_2}{W_1}\right)\right]} = \frac{\sqrt{3}\,(1 - \alpha)}{(1 + \alpha)}$$

We know that,

$$\sec^2 \phi = 1 + \tan^2 \phi$$

or

$$\frac{1}{\cos^2 \phi} = 1 + \tan^2 \phi \quad \text{or} \quad \cos^2 \phi = \frac{1}{1 + \tan^2 \phi}$$

or

$$\cos \phi = \frac{1}{\sqrt{1 + \tan^2 \phi}} = \frac{1}{\sqrt{1 + \left[\dfrac{\sqrt{3}\,(1 - \alpha)}{1 + \alpha}\right]^2}}$$

$$= \frac{1}{\sqrt{1 + 3\left(\dfrac{1 - \alpha}{1 + \alpha}\right)^2}} = \frac{1 + \alpha}{\sqrt{(1 + \alpha)^2 + 3\,(1 - \alpha)^2}}$$

$$= \frac{1+\alpha}{\sqrt{1+\alpha^2+2\alpha+3(1+\alpha^2-2\alpha)}} = \frac{1+\alpha}{\sqrt{4+4\alpha^2-4\alpha}}$$

$$= \frac{1+\alpha}{2\sqrt{1-\alpha+\alpha^2}}$$

i.e.,
$$\cos\phi = \frac{1+\alpha}{2\sqrt{1-\alpha+\alpha^2}} \quad ...(2.15)$$

If a curve is plotted between α and $\cos\phi$, then the curve obtained will be as shown in Fig. 2.25, this curve is called **watt-ratio curve.**

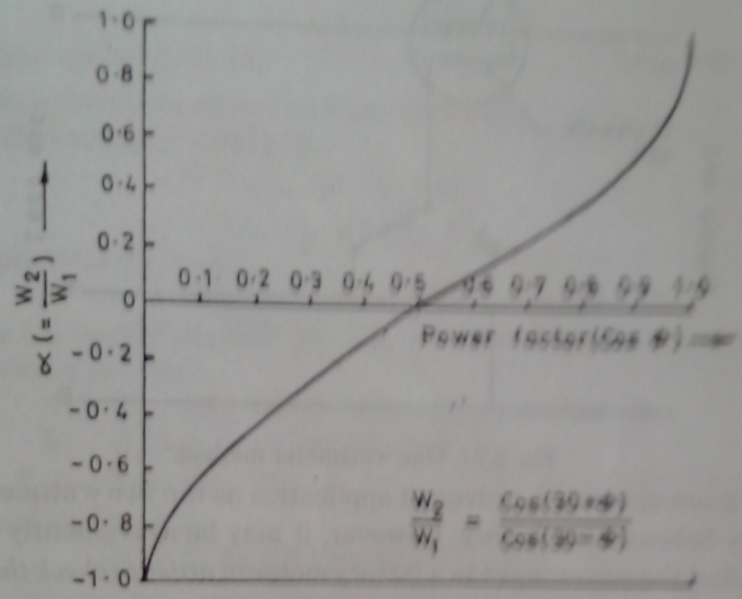


Fig. 2.25. Watt-ratio curve.

Reactive volt amperes *(with two wattmeters)*

We know that $\tan\phi = \dfrac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$

As the tangent of the angle of lag between phase current and phase voltage of a circuit is always equal to the ratio of reactive power to the true power (Fig. 2.26). Hence, in case of a balanced load, the reactive power is given by $\sqrt{3}$ *times the difference of the readings of the two wattmeters* used to measure the power of a 3-phase circuit by two wattmeter method.

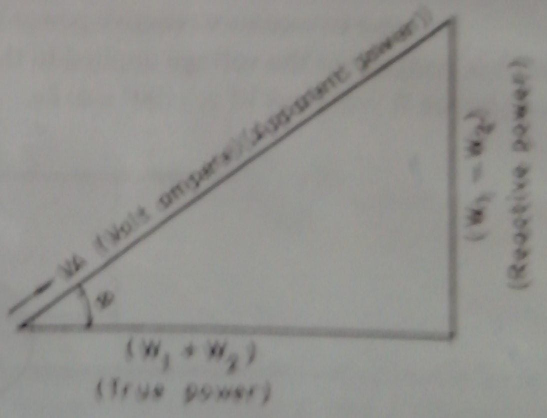


Fig. 2.26

Mathematical proof is as follows :

$$\sqrt{3}(W_1 - W_2) = \sqrt{3}\,[(E_L I_L \cos(30° - \phi) - E_L I_L \cos(30° + \phi)]$$
$$= \sqrt{3}\,E_L I_L\,[(\cos 30° \cos\phi + \sin 30° \sin\phi) - (\cos 30° \cos\phi - \sin 30° \sin\phi)]$$
$$= \sqrt{3}\,E_L I_L\,[\cos 30° \cos\phi + \sin 30° \sin\phi - \cos 30° \cos\phi + \sin 30° \sin\phi]$$

$$= \sqrt{3}\, E_l I_L\, (2 \sin 30° \sin \phi) = \sqrt{3}\, E_l I_L \sin \phi$$

i.e., $$\sqrt{3}\,(W_1 - W_2) = \sqrt{3}\, E_l I_L \sin \phi.$$

2.9.3. One-wattmeter method. In this method the current coil is connected in any one line and the pressure coil is connected alternately between this and the other two lines as shown in Fig. 2.27. The two readings so obtained, for a balanced load, correspond to those obtained by normal two wattmeter method.

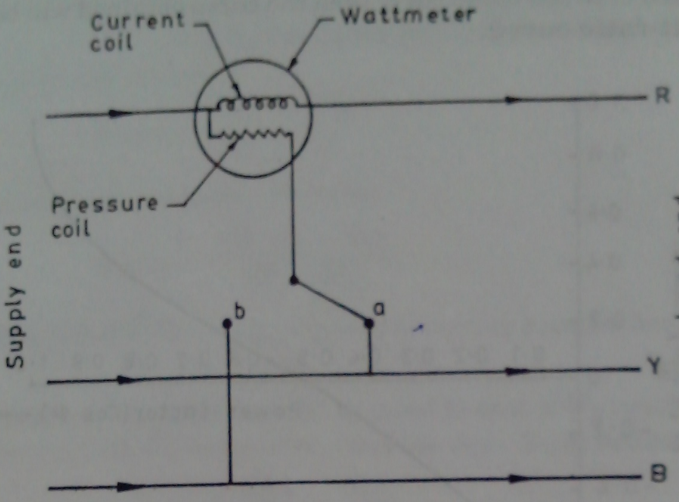


Fig. 2.27. One wattmeter method.

This method is not of as much universal application as the two wattmeter method because it is restricted to *fairly balanced loads only*. However, it may be conveniently applied, for instance, when it is desired to find the power input to a factory motor *in order to check the load upon the motor*.

2.10. MEASUREMENT OF REACTIVE VOLT AMPERES

In order to measure reactive power in a single phase circuit a *compensated wattmeter* is used. In this wattmeter the voltage applied to the pressure coil is *90° out of phase with the actual voltage* and hence it will read $VI \cos (90° - \phi)$ *i.e.,* $VI \sin \phi$, reactive power.

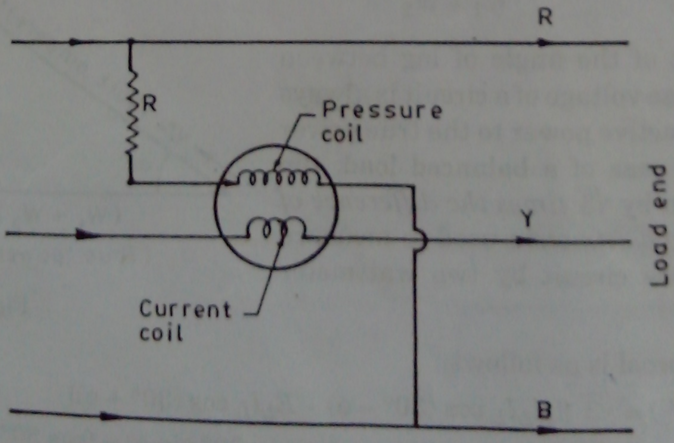


Fig. 2.28. Measurement of reactive volt-amperes.