Name: **Quiz 1 Solutions** September 25, 2012

1. Differentiate the following functions:

   (a) $f(x) = \sqrt{x^2+1}$

$$\frac{2x}{2\sqrt{x^2+1}}$$

   (b) $g(x) = \frac{\sin x}{\ln x}$.

$$\frac{\cos x \ln x - \frac{1}{x}\sin x}{(\ln x)^2}.$$

   (c) $h(x) = e^x \tan^{-1}(x)$.

$$e^x \tan^{-1} x + \frac{e^x}{1+x^2}.$$

2. Find the general antiderivative, $F(x)$, of for $f(x) = \frac{4}{x} - \frac{2}{1+x^2} + 3\cos x - \sqrt{x^3}$.

$$F(x) = -\frac{4}{x^2} - 2\arctan x + 3\sin x - \frac{3}{5}x^{5/3}.$$

3. (For any hotshots in the class, but **don't worry if you cannot do these problems yet!**)

   (a) Let $f(x)$ be a differentiable function. Compute $\displaystyle\int \frac{\ln(f(x))f'(x)}{f(x)}\,dx$.

make $u$-substitution $u = \ln(f(x))$ and get as answer $\frac{1}{2}\ln(f(x))^2 + C$.

   (b) Compute $\displaystyle\int_2^\infty \frac{\ln x}{x^4}\,dx$

Use integration by parts with

$$u = \ln x, \quad v = -(1/5)\frac{1}{x^5}$$

$$du = \frac{1}{x}dx, \quad dv = \frac{1}{x^4}dx.$$

to write

$$\int_2^\infty \frac{\ln x}{x^4}\,dx = \lim_{b\to\infty} -\frac{\ln x}{5x^5}\Big|_2^b + \frac{1}{5}\int_2^\infty \frac{1}{x^6}dx$$
$$= \frac{\ln 2}{5\cdot 2^5} + \frac{1}{5}\lim_{b\to\infty} -(1/7)\frac{1}{x^7}\Big|_2^b$$
$$= \frac{\ln 2}{5\cdot 2^5} + \frac{1}{35\cdot 2^7}$$

Where we use L'Hospital to conclude that

$$\lim_{b\to\infty}\frac{\ln b}{5b^5} = 0$$

and we use general know-how to conclude that

$$\lim b \to \infty \frac{1}{b^7} = 0.$$