Rough Outline

• Bin-to-Bin Distances.

• Cross-Bin Distances:
  – Quadratic-Form (aka Mahalanobis) / Quadratic-Chi.
  – The Earth Mover’s Distance.

• Perceptual Color Differences.

• Hands-On Code Example.
Distance ?
Metric

- Non-negativity:
  \[ D(P, Q) \geq 0 \]

- Identity of indiscernibles:
  \[ D(P, Q) = 0 \iff P = Q \]

- Symmetry:
  \[ D(P, Q) = D(Q, P) \]

- Subadditivity (triangle inequality):
  \[ D(P, Q) \leq D(P, K) + D(K, Q) \]
Non-negativity:
\[ D(P, Q) \geq 0 \]

Property changed to:
\[ D(P, Q) = 0 \text{ if } P = Q \]

Symmetry:
\[ D(P, Q) = D(Q, P) \]

Subadditivity (triangle inequality):
\[ D(P, Q) \leq D(P, K) + D(K, Q) \]
Metric

- Non-negativity:
  \[ D(P, Q) \geq 0 \]
- Identity of indiscernibles:
  \[ D(P, Q) = 0 \text{ iff } P = Q \]

An object is most similar to itself
Metric

- **Symmetry:**
  \[ D(P, Q) = D(Q, P) \]

- **Subadditivity (triangle inequality):**
  \[ D(P, Q) \leq D(P, K) + D(K, Q) \]

Useful for many algorithms
Minkowski-Form Distances

\[ L_p(P, Q) = \left( \sum_i |P_i - Q_i|^p \right)^{\frac{1}{p}} \]
Minkowski-Form Distances

\[ L_1(P, Q) = \sum_i |P_i - Q_i| \]

\[ L_2(P, Q) = \sqrt{\sum_i (P_i - Q_i)^2} \]

\[ L_\infty(P, Q) = \max_i |P_i - Q_i| \]
Kullback-Leibler Divergence

\[ KL(P, Q) = \sum_i P_i \log \frac{P_i}{Q_i} \]

- Information theoretic origin.
- Non symmetric.
- \( Q_i = 0 \) ?
Jensen-Shannon Divergence

\[ JS(P, Q) = \frac{1}{2} KL(P, M) + \frac{1}{2} KL(Q, M) \]

\[ M = \frac{1}{2} (P + Q) \]

\[ JS(P, Q) = \frac{1}{2} \sum_i P_i \log \frac{2P_i}{P_i + Q_i} + \frac{1}{2} \sum_i Q_i \log \frac{2Q_i}{P_i + Q_i} \]
Jensen-Shannon Divergence

\[ JS(P, Q) = \frac{1}{2} KL(P, M) + \frac{1}{2} KL(Q, M) \]

\[ M = \frac{1}{2}(P + Q) \]

- Information Theoretic origin.
- Symmetric.
- \( \sqrt{JS} \) is a metric.
Jensen-Shannon Divergence

• Using Taylor extension and some algebra:

\[ JS(P, Q) = \sum_{n=1}^{\infty} \frac{1}{2n(2n-1)} \sum_{i} \frac{(P_i - Q_i)^{2n}}{(P_i + Q_i)^{2n-1}} \]

\[ = \frac{1}{2} \sum_{i} \frac{(P_i - Q_i)^2}{(P_i + Q_i)} + \frac{1}{12} \sum_{i} \frac{(P_i - Q_i)^4}{(P_i + Q_i)^3} + \ldots \]
Histogram Distance

\[ \chi^2(P, Q) = \frac{1}{2} \sum_i \frac{(P_i - Q_i)^2}{(P_i + Q_i)} \]

- Statistical origin.
- Experimentally results are very similar to JS.
- Reduces the effect of large bins.
- \( \sqrt{\chi^2} \) is a metric
$\chi^2$ Histogram Distance

$\chi^2((\ldots, \ldots) < \chi^2((\ldots, \ldots)$
$\chi^2$ Histogram Distance

$L_1((\ldots, \ldots)) > L_1((\ldots, \ldots))$
$\chi^2$ Histogram Distance

$$\chi^2(P, Q) = \frac{1}{2} \sum_i \frac{(P_i - Q_i)^2}{(P_i + Q_i)}$$

- Experimentally better than $L_2$. 
Bin-to-Bin Distances

- Bin-to-Bin distances such as $L_1, L_2, \chi^2$ are sensitive to quantization:

$$L_1\left(\begin{array}{c} \text{bin A} \\ \text{bin B} \end{array}\right) > L_1\left(\begin{array}{c} \text{bin C} \\ \text{bin D} \end{array}\right)$$
Bin-to-Bin Distances

- #bins $\rightarrow$ robustness $\uparrow$ distinctiveness $\downarrow$
- #bins $\uparrow$ $\rightarrow$ robustness $\downarrow$ distinctiveness $\uparrow$

$L_1((\text{\#bins}), (\text{\#bins})) > L_1((\text{\#bins}), (\text{\#bins}))$
Bin-to-Bin Distances

- #bins \(\rightarrow\) robustness \(\uparrow\) distinctiveness \(\downarrow\)
- #bins \(\uparrow\) \(\rightarrow\) robustness \(\downarrow\) distinctiveness \(\rightarrow\)

Can we achieve robustness and distinctiveness?
The Quadratic-Form Histogram Distance

\[ QF^A(P, Q) = \sqrt{(P - Q)^T A (P - Q)} \]

\[ = \sqrt{\sum_{ij} (P_i - Q_i)(P_j - Q_j) A_{ij}} \]

- \( A_{ij} \) is the similarity between bin i and j.
- If \( A \) is the inverse of the covariance matrix, QF is called Mahalanobis distance.
The Quadratic-Form Histogram Distance

\[ QF^A(P, Q) = \sqrt{(P - Q)^T A (P - Q)} = \sqrt{\sum_{ij} (P_i - Q_i)(P_j - Q_j) A_{ij}} \]

\[ A = I \]

\[ \equiv \sqrt{\sum_{ij} (P_i - Q_i)^2} = L_2(P, Q) \]
The Quadratic-Form Histogram Distance

\[ QF^A(P, Q) = \sqrt{(P - Q)^T A (P - Q)} \]

- Does not reduce the effect of large bins.
- Alleviates the quantization problem.
- Linear time computation in \# non zero \( A_{i,j} \).
The Quadratic-Form Histogram Distance

\[ QF^A(P, Q) = \sqrt{(P - Q)^T A (P - Q)} \]

- If A is positive-semidefinite then QF is a pseudo-metric.
Non-negativity:
\[ D(P, Q) \geq 0 \]

Property changed to:
\[ D(P, Q) = 0 \text{ if } P = Q \]

Symmetry:
\[ D(P, Q) = D(Q, P) \]

Subadditivity (triangle inequality):
\[ D(P, Q) \leq D(P, K) + D(K, Q) \]
The Quadratic-Form Histogram Distance

\[ QF^A(P, Q) = \sqrt{(P - Q)^T A (P - Q)} \]

- If $A$ is \textit{positive-definite} then $QF$ is a metric.
The Quadratic-Form Histogram Distance

\[ Q^A_F(P, Q) = \sqrt{(P - Q)^T A (P - Q)} \]

\[
= \sqrt{(P - Q)^T W^T W (P - Q)}
\]

\[
= L_2(WP, WQ)
\]
The Quadratic-Form Histogram Distance

\[ Q^A(P, Q) = \sqrt{(P - Q)^T A (P - Q)} \]

\[ = L_2(WP, WQ) \]

We assume there is a linear transformation that makes bins independent.

There are cases where this is not true, e.g., COLOR.
The Quadratic-Form Histogram Distance

$$QF^A(P, Q) = \sqrt{(P - Q)^T A (P - Q)}$$

- Converting distance to similarity (Hafner et. al 95):

$$A_{ij} = 1 - \frac{D_{ij}}{\max_{ij}(D_{ij})}$$
The Quadratic-Form Histogram Distance

\[ QF^A(P, Q) = \sqrt{(P - Q)^T A (P - Q)} \]

- Converting distance to similarity (Hafner et al. 95):

\[ A_{ij} = e^{-\alpha \frac{D(i, j)}{\max_{ij} D(i, j)}} \]

If \( \alpha \) is large enough, 
\( A \) will be positive-definitive
The Quadratic-Form Histogram Distance

\[ QF^A(P, Q) = \sqrt{(P - Q)^T A (P - Q)} \]

- Learning the similarity matrix: part 2 of this tutorial.
The Quadratic-Chi Histogram Distance

\[ QC_{m}^{A}(P, Q) = \sqrt{\sum_{i,j} \frac{(P_i - Q_i)(P_j - Q_j)A_{ij}}{(\sum_{c}(P_{c} + Q_{c})A_{ci})^m(\sum_{c}(P_{c} + Q_{c})A_{cj})^m}} \]

- \(A_{ij}\) is the similarity between bin i and j.
- Generalizes \(QF\) and \(\chi^2\).
- Reduces the effect of large bins.
- Alleviates the quantization problem.
- Linear time computation in \#\) non zero \(A_{ij}\).
The Quadratic-Chi Histogram Distance

\[
QC^A_m(P, Q) = \sqrt{\sum_{ij} \frac{(P_i - Q_i)(P_j - Q_j)A_{ij}}{\left(\sum_c (P_c + Q_c)A_{ci}\right)^m \left(\sum_c (P_c + Q_c)A_{cj}\right)^m}}
\]

- is non-negative if \( A \) is positive-semidefinite.
- Symmetric.
- Triangle inequality unknown.
- If we define \( \frac{0}{0} = 0 \) and \( 0 \leq m < 1 \), QC is continuous.
Similarity-Matrix-Quantization-Invariant Property

\[ D(\begin{pmatrix} \text{column}\end{pmatrix}, \text{column}) = D(\begin{pmatrix} \text{column} & \text{column} & \text{column} \end{pmatrix}, \begin{pmatrix} \text{column} & \text{column} & \text{column} \end{pmatrix}) \]
Sparseness-Invariant

\[ D(\cdot, \cdot) = D(\cdot, \cdot) \]

Empty bins
function dist= QuadraticChi(P,Q,A,m)

Z= (P+Q)*A;
% 1 can be any number as Z_i==0 iff D_i=0
Z(Z==0)= 1;
Z= Z.^m;
D= (P-Q)./Z;
% max is redundant if A is
% positive-semidefinite
dist= sqrt( max(D*A*D',0) );
The Quadratic-Chi Histogram Distance

Time Complexity: $O(\#A_{ij} \neq 0)$

function dist = QuadraticChi(P,Q,A,m)

Z = (P+Q)*A;
% 1 can be any number as Z_i==0 iff D_i=0
Z(Z==0)= 1;
Z= Z.^m;
D= (P-Q)./Z;
% max is redundant if A is
% positive-semidefinite
dist= sqrt( max(D*A*D',0) );
function dist = QuadraticChi(P,Q,A,m)

Z = (P+Q)*A;
% 1 can be any number as Z_i==0 iff D_i=0
Z(Z==0) = 1;
Z = Z.^m;
D = (P-Q)./Z;
% max is redundant if A is % positive-semidefinite
dist = sqrt( max(D*A*D',0) );
What about sparse (e.g. BoW) histograms?

\[ P - Q = \]

\[
\begin{array}{cccccccccccc}
0 & 0 & 0 & -3 & 0 & 0 & 0 & 4 & 5 & 0 & 0 & 0 & 0
\end{array}
\]

Time Complexity: \( O(SK) \)
The Quadratic-Chi Histogram Distance Code

What about sparse (e.g. BoW) histograms?

\[ P - Q = \]

\[
\begin{array}{cccccccccccc}
0 & 0 & 0 & -3 & 0 & 0 & 0 & 4 & 5 & 0 & 0 & 0 & 0
\end{array}
\]

Time Complexity: \( O(SK) \)

\[ \#(P \neq 0) + \#(Q \neq 0) \]
The Quadratic-Chi Histogram Distance Code

What about sparse (e.g. BoW) histograms?

\[ P - Q = \]

\[
\begin{array}{cccccccccccc}
0 & 0 & 0 & -3 & 0 & 0 & 0 & 4 & 5 & 0 & 0 & 0 & 0
\end{array}
\]

Time Complexity: \( O(SK) \)

Average of non-zero entries in each row of A
The Earth Mover’s Distance
The Earth Mover’s Distance

- The Earth Mover’s Distance is defined as the minimal cost that must be paid to transform one histogram into the other, where there is a “ground distance” between the basic features that are aggregated into the histogram.
The Earth Mover’s Distance
The Earth Mover’s Distance
The Earth Mover’s Distance

\[ \text{EMD}^D(P, Q) = \min_{F = \{F_{ij}\}} \sum_{i,j} F_{ij} D_{ij} \]

s.t.: \[ \sum_j F_{ij} = P_i \quad \sum_i F_{ij} = Q_j \]
\[ \sum_{i,j} F_{ij} = \sum_i P_i = \sum_j Q_j = 1 \]
\[ F_{ij} \geq 0 \]
The Earth Mover’s Distance

\[ EMDD(P, Q) = \min_{F=\{F_{ij}\}} \frac{\sum_{i,j} F_{ij} D_{ij}}{\sum_i F_{ij}} \]

s.t : \[ \sum_j F_{ij} \leq P_i \quad \sum_i F_{ij} \leq Q_j \]

\[ \sum_{i,j} F_{ij} = \min(\sum_i P_i, \sum_j Q_j) \]

\[ F_{ij} \geq 0 \]
The Earth Mover’s Distance

- Pele and Werman 08 – $\overline{EMD}$, a new EMD definition.
Definition:

\[
\text{EMD}_D^C(P, Q) = \min_{F=\{F_{ij}\}} \sum_{i,j} F_{ij} D_{ij} + \left| \sum_i P_i - \sum_j Q_j \right| \times C
\]

s.t.: \[
\sum_j F_{ij} \leq P_i \quad \sum_i F_{ij} \leq Q_j \\
\sum_{i,j} F_{ij} = \min(\sum_i P_i, \sum_j Q_j) \quad F_{ij} \geq 0
\]
\[ EMD \]

\[ \sum \text{Demander} \leq \sum \text{Supplier} \]

\[ D_{ij} = 0 \]
\[ \sum \text{Demander} \leq \sum \text{Supplier} \]

\[ D_{ij} = C \]
When to Use $EMD$

- When the total mass of two histograms is important.

$$EMD\left( \begin{array} \text{Blue} \\ \text{Orange} \end{array} \right) = EMD\left( \begin{array} \text{Blue} \\ \text{Orange} \end{array} \right)$$
When to Use $\text{EMD}$

- When the total mass of two histograms is important.

\[
\text{EMD}(\text{histogram}_1, \text{histogram}_2) = \text{EMD}(\text{histogram}_3, \text{histogram}_4)
\]

\[
\text{EMD}(\text{histogram}_5, \text{histogram}_6) < \text{EMD}(\text{histogram}_7, \text{histogram}_8)
\]
When to Use $EMD$

- When the difference in total mass between histograms is a distinctive cue.

\[ EMD(\text{histogram}_1, \text{histogram}_2) = EMD(\text{histogram}_1, \text{histogram}_3) = 0 \]
When to Use $\overline{EMD}$

- When the difference in total mass between histograms is a distinctive cue.

\[
\overline{EMD}(\text{histogram 1}, \text{histogram 2}) = \overline{EMD}(\text{histogram 3}, \text{histogram 4}) = 0
\]

\[
\overline{EMD}(\text{histogram 5}, \text{histogram 6}) < \overline{EMD}(\text{histogram 7}, \text{histogram 8})
\]
When to Use $\text{EMD}$

- If ground distance is a metric:
  - $\text{EMD}$ is a metric only for normalized histograms.
  - $\text{EMD}$ is a metric for all histograms ($C \geq \frac{1}{2} \Delta$).
\( \overline{EMD} \) - a Natural Extension to \( L_1 \)

- \( \overline{EMD} = L_1 \) if:

\[
D_{ij} = \begin{cases} 
0 & \text{if } i = j \\
2 & \text{otherwise}
\end{cases}
\]

\[ C = 1 \]

\[
\overline{EMD}_C^D(P, Q) = \min_{F = \{F_{ij}\}} \sum_{i,j} F_{ij} D_{ij} + \left| \sum_i P_i - \sum_j Q_j \right| \times C
\]
The Earth Mover’s Distance Complexity Zoo

- General ground distance: $O(N^3 \log N)$
  - Orlin 88

- $L_1$ normalized 1D histograms $O(N)$
  - Werman, Peleg and Rosenfeld 85
The Earth Mover’s Distance Complexity Zoo

- $L_1$ normalized 1D cyclic histograms $O(N)$
  - Pele and Werman 08 (Werman, Peleg, Melter, and Kong 86)
The Earth Mover’s Distance Complexity Zoo

- $L_1$ Manhattan grids $O(N^2 \log N(D + \log N))$
  - Ling and Okada 07
What about N-dimensional histograms with a cyclic dimensions?
What about N-dimensional histograms with a cyclic dimensions?
What about N-dimensional histograms with a cyclic dimensions?
The Earth Mover’s Distance Complexity Zoo

- $L_1$ general histograms $O(N^2 \log^{2D-1} N)$
  - Gudmundsson, Klein, Knauer and Smid 07
The Earth Mover’s Distance Complexity Zoo

- $\min(L_1, 2)$ 1D linear/cyclic $\{1, 2, \ldots, \Delta\}$ $O(N)$ histograms
  - Pele and Werman 08
• $\min(L_1, 2)$ 1D linear/cyclic $\{1, 2, \ldots, \Delta\}$ $O(N)$
histograms
  – Pele and Werman 08
The Earth Mover’s Distance Complexity Zoo

- $\min(L_1, 2)$ general $\{1, 2, \ldots, \Delta\}^D$ histograms

$K$ is the number of edges with cost 1 $O(N^2 K \log(\frac{N}{K}))$

- Pele and Werman 08
The Earth Mover’s Distance Complexity Zoo

- Any thresholded distance $O(N^2 \log N (K + \log N))$
  - Pele and Werman 09

$O(N^2 \log^2 N) \leftarrow K = O(\log N)$

number of edges with cost different from the threshold
Thresholded Distances

- EMD with a thresholded ground distance is **not** an approximation of EMD.
- It has better performance.
The Flow Network Transformation

Original Network

Simplified Network
The Flow Network Transformation

Original Network

Simplified Network
The Flow Network Transformation

Flowing the Monge sequence
(if ground distance is a metric, zero-cost edges are a Monge sequence)
The Flow Network Transformation

Removing Empty Bins and their edges
The Flow Network Transformation

We actually finished here....
Combining Algorithms

- EMD algorithms can be combined.
- For example $L_1$: 

![Diagram showing a network of connected nodes and arrows, possibly representing an EMD algorithm combination.]
Combining Algorithms

- EMD algorithms can be combined.
- For example, **thresholded** $L_1$:
The Earth Mover’s Distance Approximations
The Earth Mover’s Distance Approximations

- Charikar 02, Indyk and Thaper 03 – approximated EMD on \( \{1, \ldots, \Delta\}^d \) by embedding it into the \( L_1 \) norm.

Time complexity: \( O(TN d \log \Delta) \)

Distortion (in expectation): \( O(d \log \Delta) \)
The Earth Mover’s Distance Approximations

• Grauman and Darrell 05 – Pyramid Match Kernel (PMK) same as Indyk and Thaper, replacing $L_1$ with histogram intersection.

• PMK approximates EMD with partial matching.

• PMK is a mercer kernel.

• Time complexity & distortion – same as Indyk and Thaper (proved in Grauman and Darrell 07).
Lazebnik, Schmid and Ponce 06 – used PMK in the spatial domain (SPM).
The Earth Mover’s Distance Approximations

level 0

level 1

level 2

$\times \frac{1}{8}$

$\times \frac{1}{4}$

$\times \frac{1}{2}$
The Earth Mover’s Distance Approximations
The Earth Mover’s Distance Approximations

- Shirdhonkar and Jacobs 08 - approximated EMD using the sum of absolute values of the weighted wavelet coefficients of the difference histogram.

\[
\begin{align*}
\sum_{j=0}^{1} & \times 2^{-2 \times 0} \\
& + \times 2^{-2 \times 1}
\end{align*}
\]
The Earth Mover’s Distance Approximations

- Khot and Naor 06 – any embedding of the EMD over the $d$-dimensional Hamming cube into $L_1$ must incur a distortion of $\Omega(d)$.
- Andoni, Indyk and Krauthgamer 08 - for sets with cardinalities upper bounded by a parameter $s$ the distortion reduces to $O(\log s \log d)$.
- Naor and Schechtman 07 - any embedding of the EMD over $\{0, 1, \ldots, \Delta\}^2$ must incur a distortion of $\Omega(\sqrt{\log \Delta})$. 
Robust Distances

- Very high distances $\leftrightarrow$ outliers $\leftrightarrow$ same difference.
Robust Distances

- With colors, the natural choice.
Robust Distances

\[ \Delta E_{00}(\text{blue, red}) = 56 \]
\[ \Delta E_{00}(\text{blue, yellow}) = 102 \]
Robust Distances - Exponent

- Usually a negative exponent is used:
- Let $d(a, b)$ be a distance measure between two features - $a, b$.
- The negative exponent distance is:

$$d_e(a, b) = 1 - e^{-\frac{d(a, b)}{\sigma}}$$
Robust Distances - Exponent

- Exponent is used because (Ruzon and Tomasi 01):
  robust, smooth, monotonic, and a metric

Input is always discrete anyway ...
Robust Distances - Thresholded

- Let $d(a, b)$ be a distance measure between two features - $a, b$.
- The thresholded distance with a threshold of $t > 0$ is:

$$d_t(a, b) = \min(d(a, b), t).$$
Thresholded Distances

- Thresholded metrics are also metrics (Pele and Werman ICCV 2009).
- Better results.
- Pele and Werman ICCV 2009 algorithm computes EMD with thresholded ground distances much faster.
- Thresholded distance corresponds to sparse similarities matrix -> faster QC / QF computation.

\[ A_{ij} = 1 - \frac{D_{ij}}{\max_{ij}(D_{ij})} \]
Thresholded Distances

- **Thresholded vs. exponent:**
  - Fast computation of cross-bin distances with a **thresholded** ground distance.
  - **Exponent** changes small distances – can be a problem (e.g. color differences).
Thresholded Distances

- Color distance should be thresholded (robust).

\[ \Delta E_{00} \]
\[ \min(\Delta E_{00}, 10) \]
\[ 10 \left( 1 - e^{-\frac{\Delta E_{00}}{4}} \right) \]
\[ 10 \left( 1 - e^{-\frac{\Delta E_{00}}{5}} \right) \]
Thresholded Distances

Exponent changes small distances

\[ \Delta E_{00} \]

\[ \min(\Delta E_{00}, 10) \]

\[ 10 \left(1 - e^{-\frac{-\Delta E_{00}}{4}}\right) \]

\[ 10 \left(1 - e^{-\frac{-\Delta E_{00}}{5}}\right) \]
A Ground Distance for SIFT

The ground distance between two SIFT bins \((x_i, y_i, o_i)\) and \((x_j, y_j, o_j)\):

\[
d_R = \|(x_i, y_i) - (x_j, y_j)\|_2 + \min(|o_i - o_j|, M - |o_i - o_j|) \\
d_T = \min(d_R, T)
\]
A Ground Distance for Color Image

The ground distances between two LAB image bins $(x_i, y_i, L_i, a_i, b_i)$ and $(x_j, y_j, L_j, a_j, b_j)$ we use are:

\[ dc_T = \min((\|(x_i, y_i) - (x_j, y_j)\|_2) + \Delta_00((L_i, a_i, b_i), (L_j, a_j, b_j)), T) \]
Perceptual Color Differences
Perceptual Color Differences

- Euclidean distance on $L^*a^*b^*$ space is widely considered as perceptual uniform.
Perceptual Color Differences

The diagram shows a graph with the title "Perceptual Color Differences". The x-axis represents the distance from blue, and the y-axis represents the perceptual color differences. The graph illustrates that purples are perceived before blues, as indicated by the annotation "Purples before blues".
Perceptual Color Differences

- $\Delta E_{00}$ on L*a*b* space is better.

Luo, Cui and Rigg 01. Sharma, Wu and Dalal 05.
Perceptual Color Differences

- $\Delta E_{00}$ on $L^*a^*b^*$ space is better.
Perceptual Color Differences

- $\Delta E_{00}$ on L*a*b* space is better.
Perceptual Color Differences

- $\Delta E_{00}$ on L*a*b* space is better.
- But still has major problems.

\[
\Delta E_{00}(\text{blue, red}) = 56
\]

\[
\Delta E_{00}(\text{blue, yellow}) = 102
\]

- Color distance should be thresholded (robust).
Perceptual Color Differences

- Color distance should be saturated (robust).

\[
\Delta E_{00} = \min(\Delta E_{00}, 10) \\
10(1 - e^{-\frac{\Delta E_{00}}{4}}) \\
10(1 - e^{-\frac{\Delta E_{00}}{5}})
\]
Thresholded Distances

\[ \Delta E_{00} \]

\[ \min(\Delta E_{00}, 10) \]

\[ 10(1 - e^{-\frac{\Delta E_{00}}{4}}) \]

\[ 10(1 - e^{-\frac{\Delta E_{00}}{5}}) \]

Exponent changes small distances
Perceptual Color Descriptors

- 11 basic color terms. Berlin and Kay 69.
Perceptual Color Descriptors

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Perceptual Color Descriptors

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Image copyright by Eric Rolph. Taken from: 
http://upload.wikimedia.org/wikipedia/commons/5/5c/Double-alaskan-rainbow.jpg
Perceptual Color Descriptors

- 11 basic color terms. Berlin and Kay 69.
How to give each pixel an “11-colors” description?
Perceptual Color Descriptors

Perceptual Color Descriptors

Perceptual Color Descriptors

Perceptual Color Descriptors

- For each color returns a probability distribution over the 11 basic colors.

white  black  red  green  yellow  blue  brown  purple  pink  orange  grey
Perceptual Color Descriptors

- Outperformed state of the art color descriptors.
Perceptual Color Descriptors

• Using illumination invariants – black, gray and white are the same.

• “Too much invariance” happens in other cases (Local features and kernels for classification of texture and object categories: An in-depth study - Zhang, Marszalek, Lazebnik and Schmid. IJCV 2007, Learning the discriminative power-invariance trade-off - Varma and Ray. ICCV 2007).

• To conclude: Don’t solve imaginary problems.
This method is still not perfect.

11 color vector for purple \((255,0,255)\) is:

![Color vector diagram]

In real world images there are no such over-saturated colors.
Open Questions

- EMD variant that reduces the effect of large bins.
- Learning the ground distance for EMD.
- Learning the similarity matrix and normalization factor for QC.
Hands-On Code Example

http://www.cs.huji.ac.il/~ofirpele/FastEMD/code/

http://www.cs.huji.ac.il/~ofirpele/QC/code/
Tutorial:

http://www.cs.huji.ac.il/~ofirpele/DFML_ECCV2010_tutorial/

Or “Ofir Pele”