LAKSHYA JEE

LAKSHYA KO HAR HAAL ME PAANA HAI

Electric Potential & Capacitance

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Today's GOALS!

Capacitor

Capacitors

Q. What is capacitor?

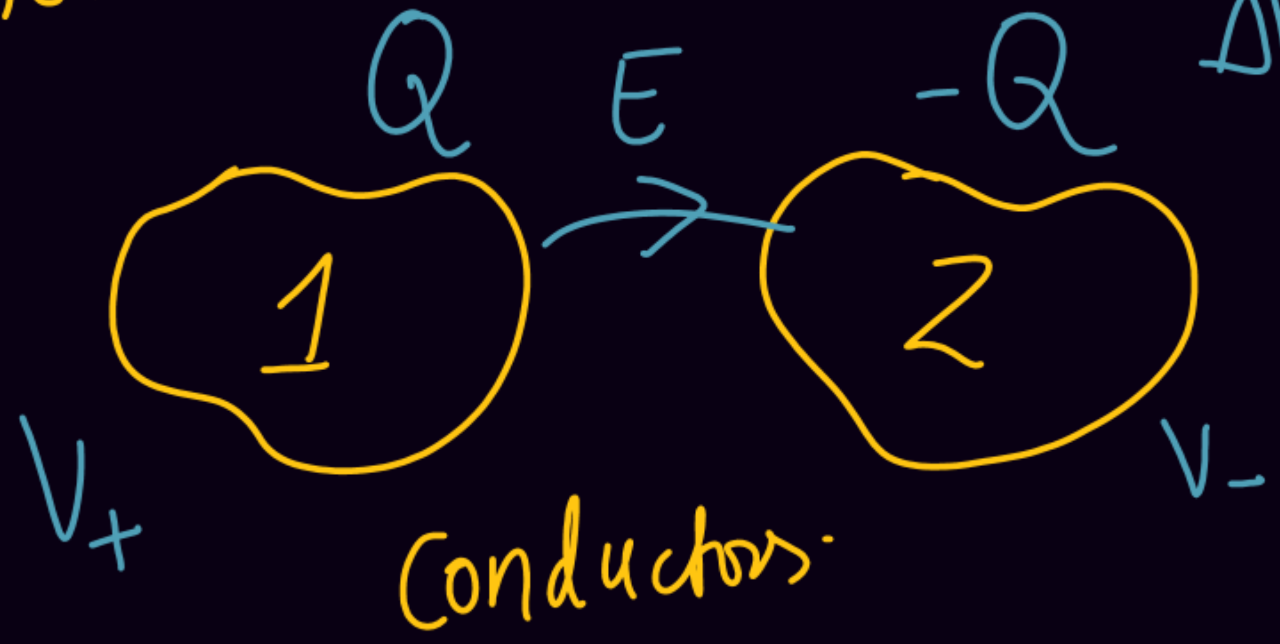
When two conductors are placed close to each other, they form a capacitor.

Capacitance

$$C = \frac{Q}{\Delta V}$$

$$Q = C\Delta V$$

Capacitance of a capacitor is the charge on the positive plate per unit potential difference across the capacitor.



$\Delta V = V_+ - V_- \propto Q$

$Q \propto \Delta V$

$Q = C \Delta V$

$$C = \frac{Q}{\Delta V}$$

$V_+ \propto Q$

$V_- \propto -Q$

$$C = \frac{Q}{\Delta V}$$ ✓✓

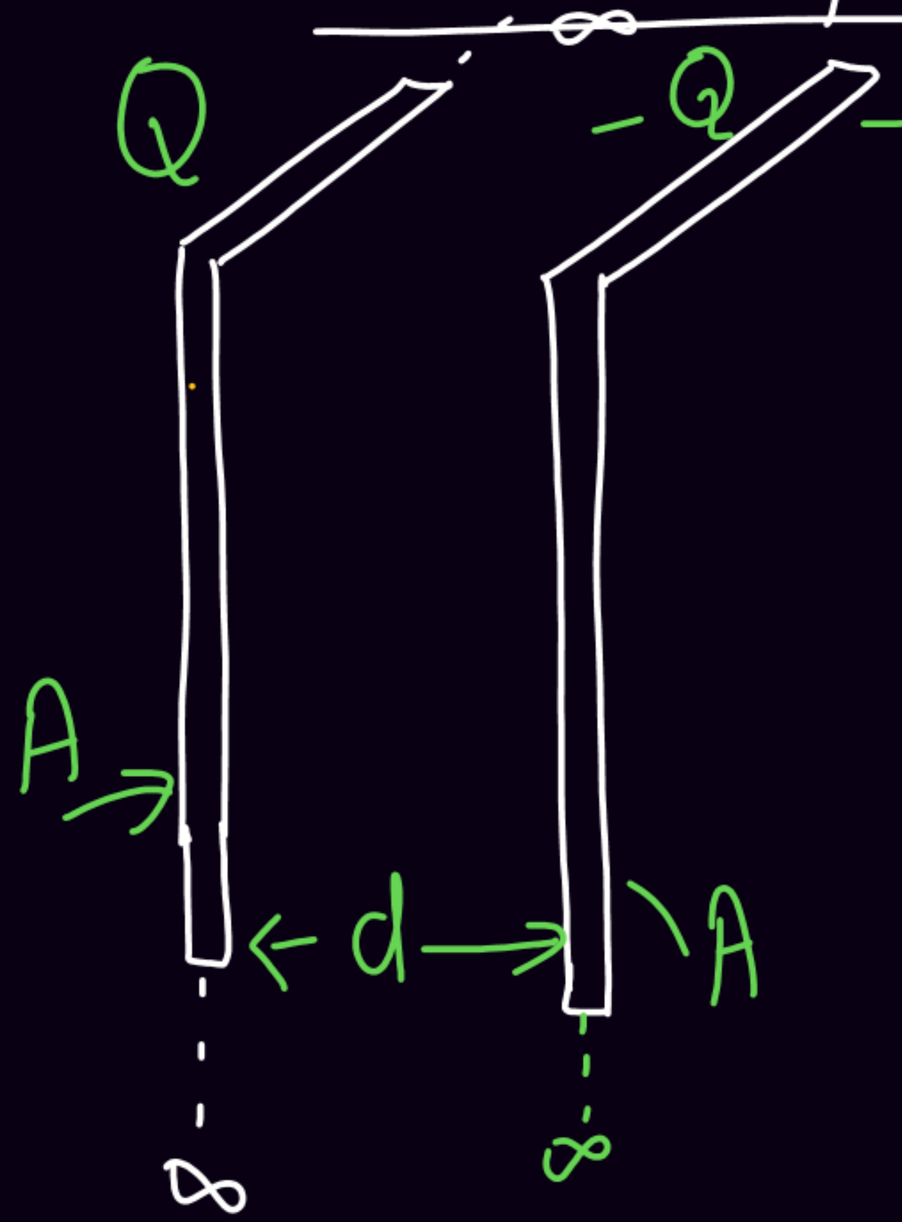
$C \propto Q$ ✗

$C \propto \frac{1}{\Delta V}$ ✗

C depends on

* shape of the conductors.
* Size of conductors.
* Distance between the conductors
* Orientation of conductors.
* Medium.

Parallel plate capacitors

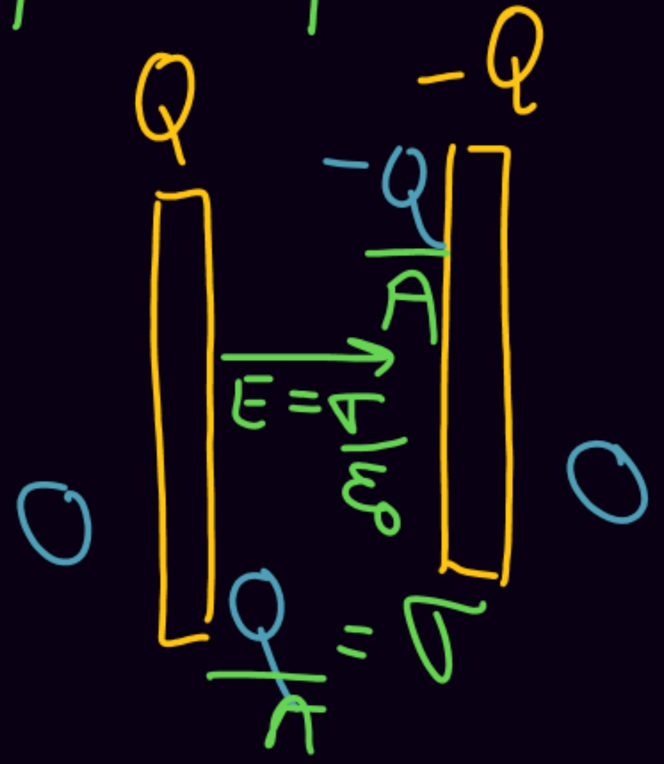


→ infinite conducting sheet.

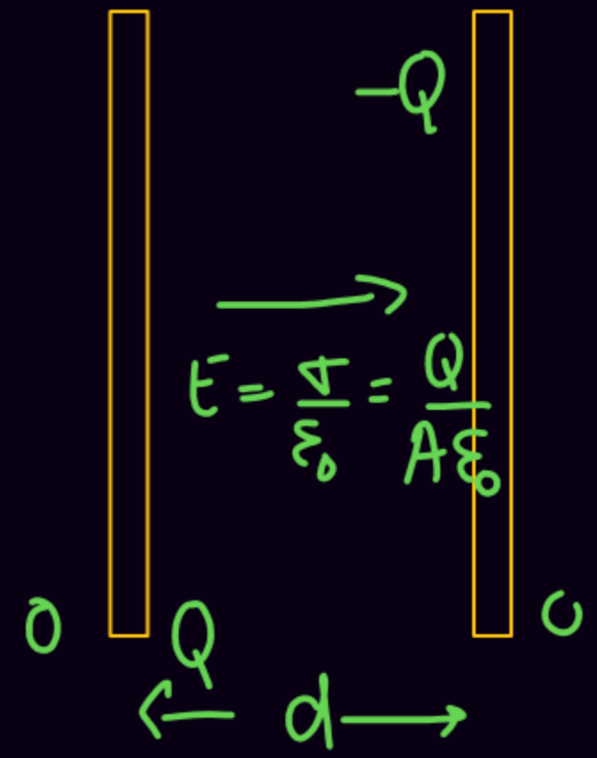
⇓

Very large — Relative term

length & breadth of the plates are very large in front of the distance between the plates.



What is the total charge = Q − Q = 0



For uniform field

$$|\Delta V| = Ed$$

$$\Delta V = \frac{\sigma}{\varepsilon_0} d$$

$$\Delta V = \frac{Q}{A\varepsilon_0} d$$

$$C = \frac{Q}{\Delta V}$$

$$C = \frac{Q}{\frac{Qd}{A\varepsilon_0}} = \frac{\varepsilon_0 A}{d}$$

*

$$C = \frac{\varepsilon_0 A}{d}$$

* school !!

Symbol.

Capacitor | Battery

Parallel plate capacitor $\Rightarrow$ $\boxed{C = \frac{\varepsilon_0 A}{d}}$

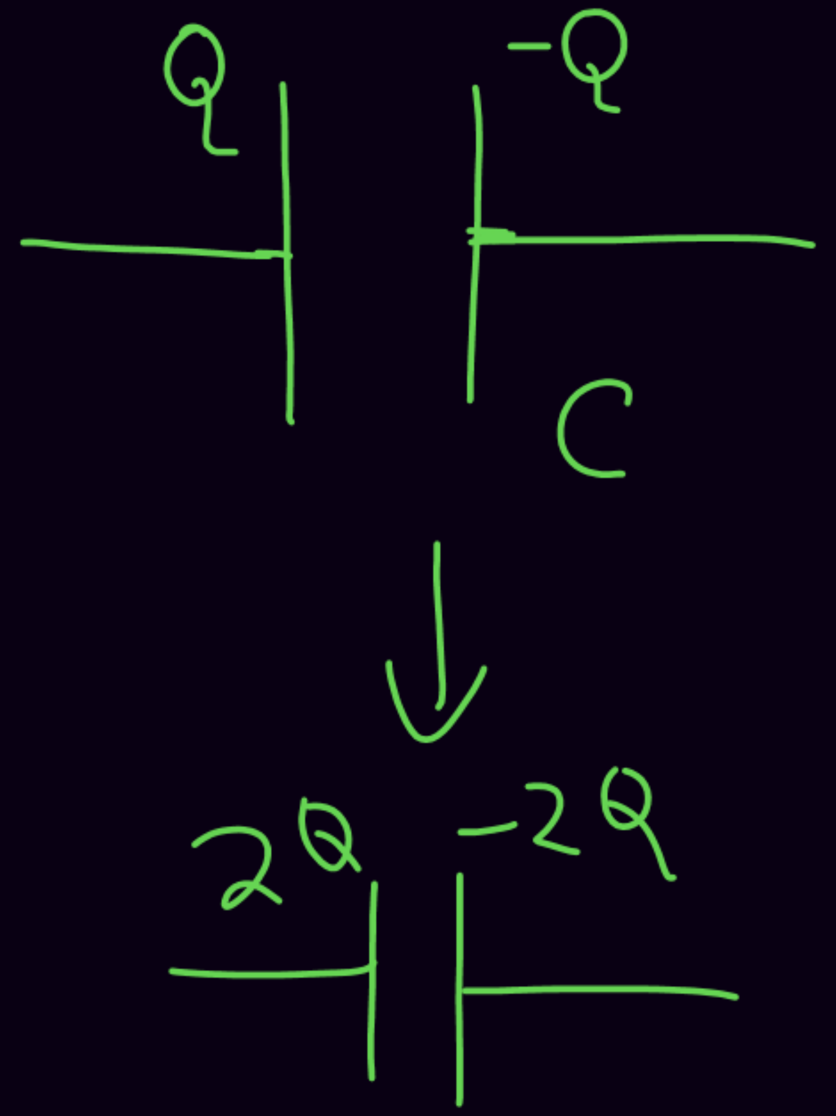
$\underline{\underline{Q}}$

If the distance between the plates is doubled find the new capacitance?

$$C = \frac{\varepsilon_0 A}{d}$$

$$C' = \frac{\varepsilon_0 A}{2d} = \frac{C}{2}$$

$$C \propto \frac{1}{d}$$



Now the charge on the capacitor is made double.

Find the new capacitance of the capacitor.

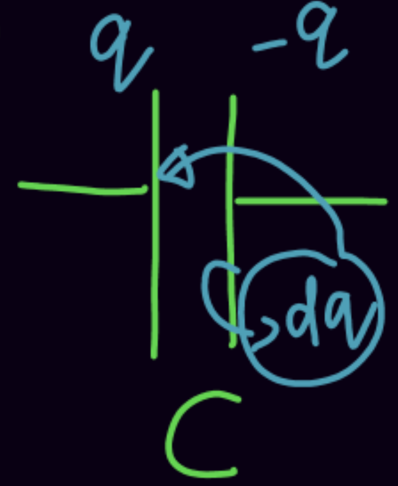
$C = \frac{Q}{\Delta V}$ ✗

No change in C

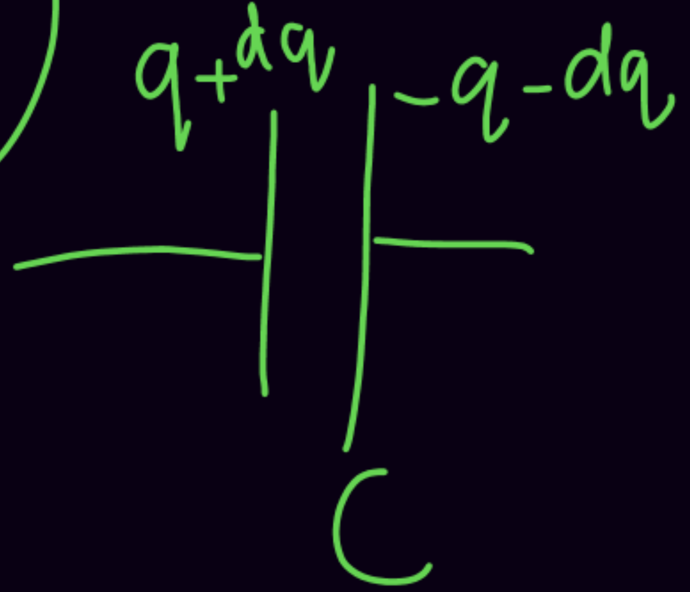
Energy in side a capacitor

$dW = \text{Charge} \times \text{pot. diff}$

$= \left(dq \, \frac{q}{C}\right)$



$V = \frac{q}{C}$ → potential diff.



$$dU = dq \, V$$

$$\int_0^U dU = \int_0^Q \frac{q}{C} \, dq$$

$$U = \frac{1}{C} \frac{q^2}{2} \Big|_0^Q = \frac{Q^2}{2C}$$

$$U = \frac{1}{2}\frac{Q^2}{C}$$

$$= \frac{1}{2}\frac{(CV)^2}{C}$$

$$U = \frac{1}{2}CV^2$$

$$U = \frac{1}{2}(CV)V$$

$$U = \frac{1}{2}QV$$

Q. Two capacitors of capacitance C & 3C have equal charge. Find the ratio of the energy stored in them?

$$U = \frac{1}{2}\frac{Q^2}{C} \Rightarrow U \propto \frac{1}{C} \qquad \frac{U_1}{U_2} = \frac{C_2}{C_1} = \frac{3}{1}$$

Q. Two capacitors of capacitance C & 4C have equal potential diff. Find the ratio of the energy stored in them?

$$U = \frac{1}{2}CV^2 \Rightarrow U \propto C \Rightarrow \frac{U_1}{U_2} = \frac{C_1}{C_2} = \frac{1}{4}$$

Energy density

$$\text{Energy density } \boxed{\varepsilon_d} = \frac{\text{Energy}}{\text{Volume}} = \frac{\frac{1}{2}\frac{q^2}{C}}{Ad} = \frac{\frac{1}{2}q^2}{Ad\,\frac{\varepsilon_0 A}{d}}$$

Kiski energy hai!

electric field

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

$$= \frac{1}{2}\frac{q^2}{A^2\varepsilon_0}$$

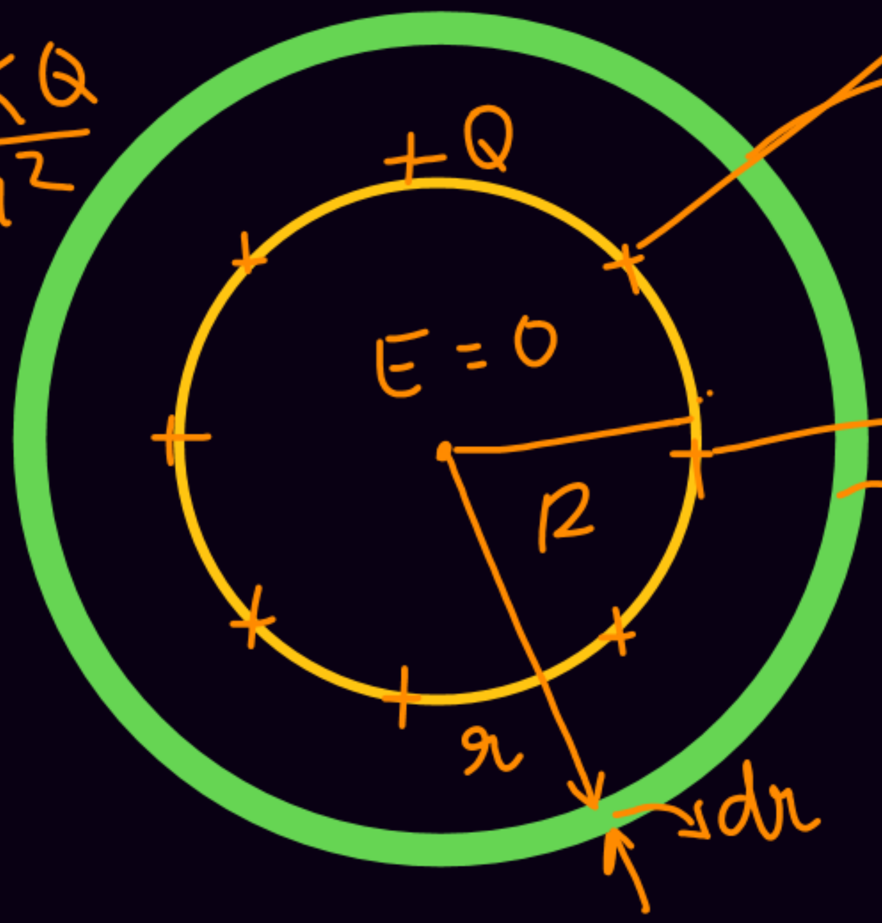
$$= \frac{1}{2}\frac{\sigma^2}{\varepsilon_0}\frac{\varepsilon_0}{\varepsilon_0} = \frac{1}{2}\varepsilon_0\frac{\sigma^2}{\varepsilon_0^2}$$

$$\boxed{\varepsilon_d = \frac{1}{2}\varepsilon_0 E^2} \; *$$

$$\varepsilon_d = \frac{1}{2}\varepsilon_0 E^2 \Rightarrow$$

Self energy of hollow sphere

$E = \frac{KQ}{r^2}$



$$\Rightarrow dU = \varepsilon_d \times dV$$

$$\rightarrow dV = 4\pi r^2 dr \qquad \infty$$

$$dU = \frac{1}{2}\varepsilon_0 E^2 \times 4\pi r^2 dr$$

$$= \int_R^{\infty} \frac{1}{2}\varepsilon_0 \left(\frac{KQ}{r^2}\right)^2 \times 4\pi r^2 dr$$

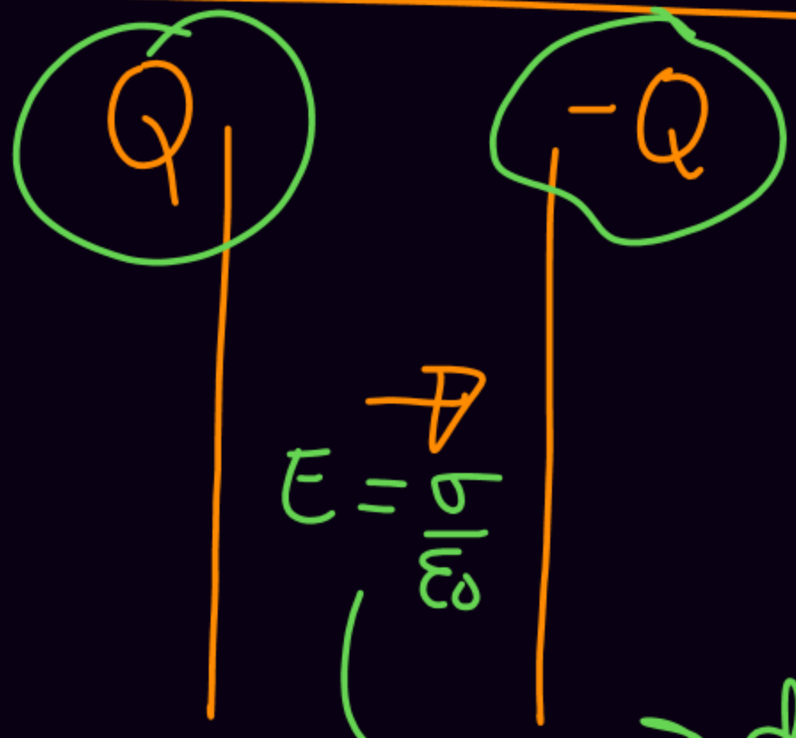
$$\int_R^\infty \frac{2\pi\varepsilon_0 K^2 Q^2 r^2 dr}{r^4}$$

$$\frac{2\pi\varepsilon_0 \frac{1}{4\pi\varepsilon_0}}{2} K Q^2 \int_R^\infty \frac{1}{r^2} dr$$

$$\frac{KQ^2}{2}\left(-\frac{1}{r}\right)_R^\infty \Rightarrow \frac{KQ^2}{2}\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

$$= \frac{KQ^2}{2R}$$

Force between plates of capacitors



→ due to all charges

Force on $-Q = E_{+Q} \times -Q$

$$|F_{-Q}| = |-Q\,E_Q| = \left|\frac{\sigma}{2\varepsilon_0} \times (-Q)\right|$$

F attractive

$$= \frac{\sigma Q}{2\varepsilon_0} = \frac{Q}{A}\frac{Q}{2\varepsilon_0} = \frac{Q^2}{2A\varepsilon_0}$$

$$\boxed{F = \frac{Q^2}{2A\varepsilon_0}} \quad *$$

Thank You Lakshyians