2 CHAPTER PRESSURE AND ITS MEASUREMENT

▸ 2.1 FLUID PRESSURE AT A POINT

Consider a small area dA in large mass of fluid. If the fluid is stationary, then the force exerted by the surrounding fluid on the area dA will always be perpendicular to the surface dA. Let dF is the force acting on the area dA in the normal direction. Then the ratio of $\frac{dF}{dA}$ is known as the intensity of pressure or simply pressure and this ratio is represented by p. Hence mathematically the pressure at a point in a fluid at rest is

$$p = \frac{dF}{dA}.$$

If the force (F) is uniformly distributed over the area (A), then pressure at any point is given by

$$p = \frac{F}{A} = \frac{\text{Force}}{\text{Area}}.$$

$\therefore$ Force or pressure force, $F = p \times A$.

The units of pressure are : (*i*) kgf/m^2 and kgf/cm^2 in MKS units, (*ii*) Newton/m^2 or N/m^2 and N/mm^2 in SI units. N/m^2 is known as Pascal and is represented by Pa. Other commonly used units of pressure are :

$$\text{kPa} = \text{kilo pascal} = 1000 \text{ N/m}^2$$
$$\text{bar} = 100 \text{ kPa} = 10^5 \text{ N/m}^2.$$

▸ 2.2 PASCAL'S LAW

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions. This is proved as :

The fluid element is of very small dimensions *i.e.*, dx, dy and ds.

Consider an arbitrary fluid element of wedge shape in a fluid mass at rest as shown in Fig. 2.1. Let the width of the element perpendicular to the plane of paper is unity and p_x,

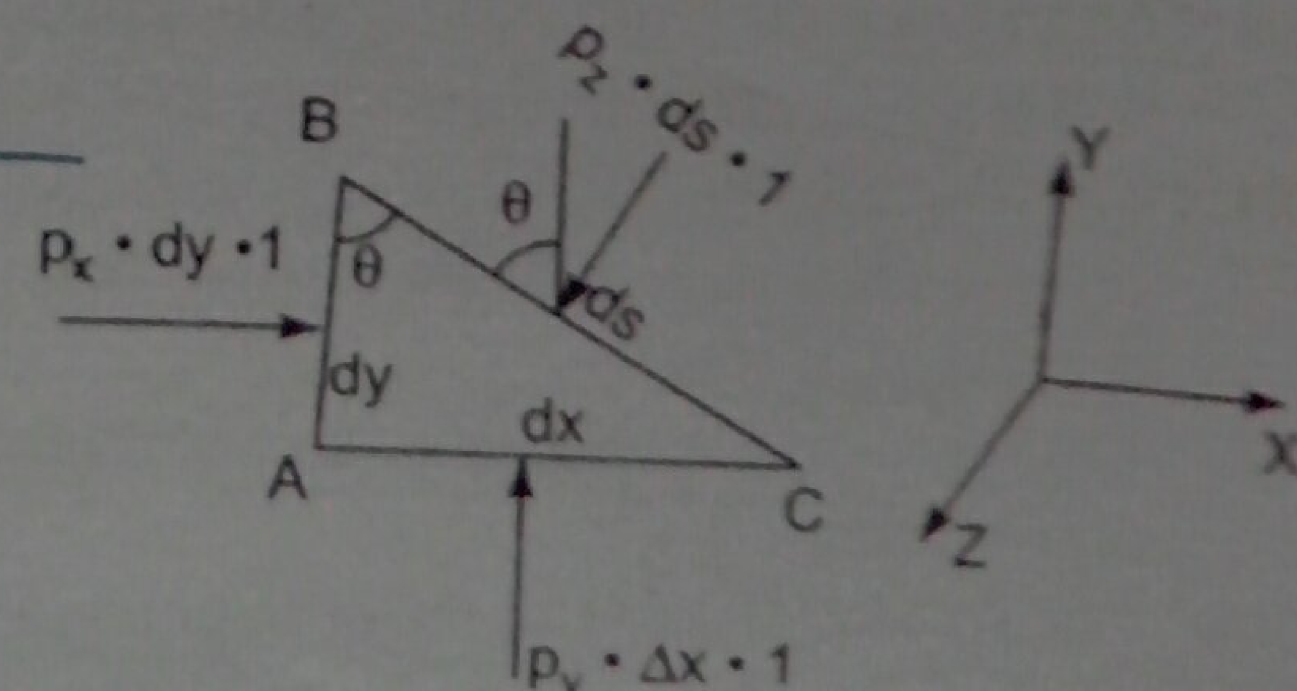

Fig. 2.1 *Forces on a fluid element.*

p_y and p_z are the pressures or intensity of pressure acting on the face AB, AC and BC respectively. Let $\angle ABC = \theta$. Then the forces acting on the element are :

1. Pressure forces normal to the surfaces, and
2. Weight of element in the vertical direction.

The forces on the faces are :

Force on the face AB $= p_x \times$ Area of face AB

$= p_x \times dy \times 1$

Similarly force on the face $AC = p_y \times dx \times 1$

Force on the face BC $= p_z \times ds \times 1$

Weight of element $=$ (Mass of element) $\times g$

$$= (\text{Volume} \times \rho) \times g = \left(\frac{AB \times AC}{2} \times 1\right) \times \rho \times g.$$

where ρ = density of fluid.

Resolving the forces in x-direction, we have

$$p_x \times dy \times 1 - p\,(ds \times 1) \sin (90° - \theta) = 0$$

or $$p_x \times dy \times 1 - p_z\, ds \times 1 \cos \theta = 0.$$

But from Fig. 2.1, $$ds \cos \theta = AB = dy$$

$\therefore$ $$p_x \times dy \times 1 - p_z \times dy \times 1 = 0$$

or $$p_x = p_z \quad \text{...(2.1)}$$

Similarly, resolving the forces in y-direction, we get

$$p_y \times dx \times 1 - p_z \times ds \times 1 \cos (90° - \theta) - \frac{dx \times dy}{2} \times 1 \times \rho \times g = 0$$

or $$p_y \times dx - p_z\, ds \sin \theta - \frac{dxdy}{2} \times \rho \times g = 0.$$

But $ds \sin \theta = dx$ and also the element is very small and hence weight is negligible.

$\therefore$ $$p_y dx - p_z \times dx = 0$$

or $$p_y = p_z \quad \text{...(2.2)}$$

From equations (2.1) and (2.2), we have

$$p_x = p_y = p_z \quad \text{...(2.3)}$$

The above equation shows that the pressure at any point in x, y and z directions is equal.

Since the choice of fluid element was completely arbitrary, which means the pressure at any point is the same in all directions.

▶ 2.3 PRESSURE VARIATION IN A FLUID AT REST

The pressure at any point in a fluid at rest is obtained by the Hydro-static Law which states that the rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid at that point. This is proved as :

Consider a small fluid element as shown in Fig. 2.2

Let ΔA = Cross-sectional area of element

ΔZ = Height of fluid element

p = Pressure on face AB

Z = Distance of fluid element from free surface.

The forces acting on the fluid element are

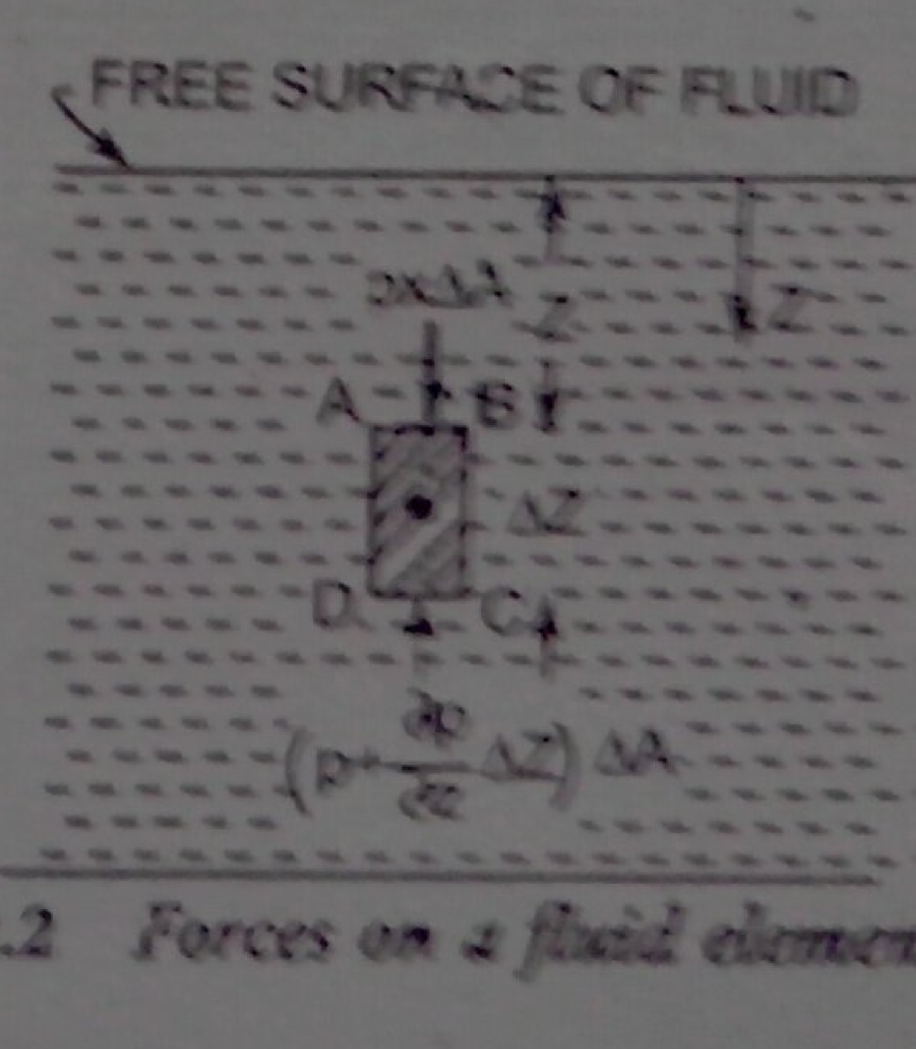

Fig. 2.2 *Forces on a fluid element.*

1. Pressure force on $AB = p \times \Delta A$ and acting perpendicular to face AB in the downward direction.

2. Pressure force on $CD = \left(p + \frac{\partial p}{\partial Z}\Delta Z\right) \times \Delta A$, acting perpendicular to face CD, vertically upward direction.

3. Weight of fluid element = Density × g × Volume = $\rho \times g \times (\Delta A \times \Delta Z)$.

4. Pressure forces on surfaces BC and AD are equal and opposite. For equilibrium of fluid element, we have

$$p\Delta A - \left(p + \frac{\partial p}{\partial Z}\Delta Z\right)\Delta A + \rho \times g \times (\Delta A \times \Delta Z) = 0$$

or
$$p\Delta A - p\Delta A - \frac{\partial p}{\partial Z}\Delta Z\Delta A + \rho \times g \times \Delta A \times Z = 0$$

or
$$-\frac{\partial p}{\partial Z}\Delta Z\Delta A + \rho \times g \times \Delta A\Delta Z = 0$$

or
$$\frac{\partial p}{\partial Z}\Delta Z\Delta A = \rho \times g \times \Delta A\Delta Z \quad \text{or} \quad \frac{\partial p}{\partial Z} = \rho \times g \text{ [cancelling } \Delta A\Delta Z \text{ on both sides]}$$

$$\therefore \quad \frac{\partial p}{\partial Z} = \rho \times g = w \qquad (\because \rho \times g = w) \qquad \ldots(2.4)$$

where w = Weight density of fluid.

Equation (2.4) states that rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point. This is **Hydrostatic Law**.

By integrating the above equation (2.4) for liquids, we get

$$\int dp = \int \rho g dZ$$

or
$$p = \rho g Z \qquad \ldots(2.5)$$

where p is the pressure above atmospheric pressure and Z is the height of the point from free surfaces.

From equation (2.5), we have $\quad Z = \frac{p}{\rho \times g} \qquad \ldots(2.6)$

Here Z is called pressure head.

Problem 2.1 *A hydraulic press has a ram of 30 cm diameter and a plunger of 4.5 cm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 500 N.*

Solution. Given :

Dia. of ram, $D = 30 \text{ cm} = 0.3 \text{ m}$

Dia. of plunger, $d = 4.5 \text{ cm} = 0.045 \text{ m}$

Force on plunger, $F = 500 \text{ N}$

Find weight lifted $= W$

Area of ram, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.3)^2 = 0.07068 \text{ m}^2$

Area of plunger, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.045)^2 = .00159 \text{ m}^2$

Pressure intensity due to plunger

$$= \frac{\text{Force on plunger}}{\text{Area of plunger}} = \frac{F}{a} = \frac{500}{.00159} \text{ N/m}^2.$$

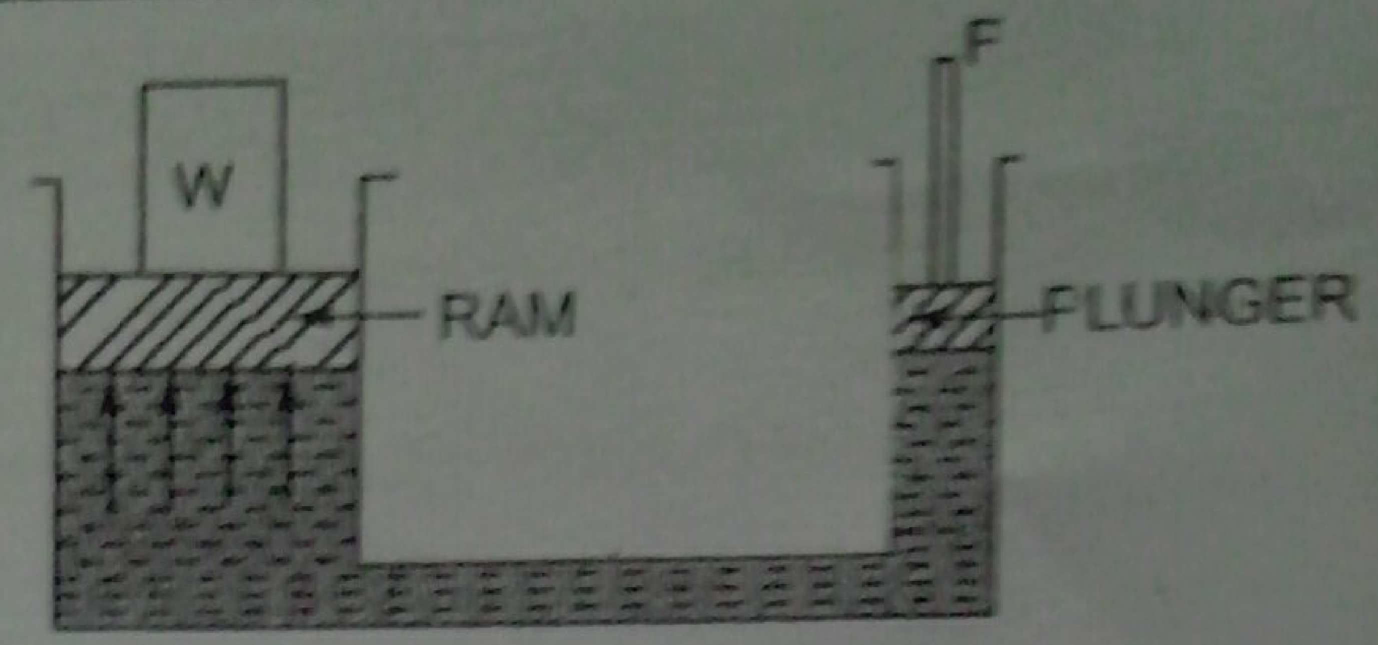

Fig. 2.3

Due to Pascal's law, the intensity of pressure will be equally transmitted in all directions. Hence the pressure intensity at the ram

$$= \frac{500}{.00159} = 314465.4 \text{ N/m}^2$$

But pressure intensity at ram $= \frac{\text{Weight}}{\text{Area of ram}} = \frac{W}{A} = \frac{W}{.07068} \text{ N/m}^2$

$$\frac{W}{.07068} = 314465.4$$

$\therefore$ Weight $= 314465.4 \times .07068 = 22222 \text{ N} =$ **22.222 kN. Ans.**

Problem 2.2 *A hydraulic press has a ram of 20 cm diameter and a plunger of 3 cm diameter. It is used for lifting a weight of 30 kN. Find the force required at the plunger.*

Solution. Given :

Dia. of ram, $D = 20 \text{ cm} = 0.2 \text{ m}$

$\therefore$ Area of ram, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (.2)^2 = 0.0314 \text{ m}^2$

Dia. of plunger $d = 3 \text{ cm} = 0.03 \text{ m}$

$\therefore$ Area of plunger, $a = \frac{\pi}{4} (.03)^2 = 7.068 \times 10^{-4} \text{ m}^2$

Weight lifted, $W = 30 \text{ kN} = 30 \times 1000 \text{ N} = 30000 \text{ N}.$

See Fig. 2.3.

Pressure intensity developed due to plunger $= \frac{\text{Force}}{\text{Area}} = \frac{F}{a}$.

By Pascal's Law, this pressure is transmitted equally in all directions

Hence pressure transmitted at the ram $= \frac{F}{a}$

$\therefore$ Force acting on ram = Pressure intensity $\times$ Area of ram

$$= \frac{F}{a} \times A = \frac{F \times .0314}{7.068 \times 10^{-4}} \text{ N}$$

But force acting on ram = Weight lifted = 30000 N

$$\therefore \quad 30000 = \frac{F \times .0314}{7.068 \times 10^{-4}}$$

$$\therefore \quad F = \frac{30000 \times 7.068 \times 10^{-4}}{.0314} = 675.2 \text{ N. Ans.}$$

Problem 2.3 *Calculate the pressure due to a column of 0.3 of (a) water, (b) an oil of sp. gr. 0.8, and (c) mercury of sp. gr. 13.6. Take density of water, $\rho = 1000 \text{ kg/m}^3$.*

Solution. Given :

Height of liquid column, $Z = 0.3 \text{ m}.$

The pressure at any point in a liquid is given by equation (2.5) as

$$p = \rho g Z$$

(a) For water, $\rho = 1000 \text{ kg/m}^3$

$$\therefore \quad p = \rho g Z = 1000 \times 9.81 \times 0.3 = 2943 \text{ N/m}^2$$

$$= \frac{2943}{10^4} \text{ N/cm}^2 = \mathbf{0.2943 \text{ N/cm}^2. \text{ Ans.}}$$

(b) For oil of sp. gr. 0.8,

From equation (1.1A), we know that the density of a fluid is equal to specific gravity of fluid multiplied by density of water.

$\therefore$ Density of oil, $\rho_0 = \text{Sp. gr. of oil} \times \text{Density of water}$ $\quad (\rho_0 = \text{Density of oil})$

$$= 0.8 \times \rho = 0.8 \times 1000 = 800 \text{ kg/m}^3$$

Now pressure, $p = \rho_0 \times g \times Z$

$$= 800 \times 9.81 \times 0.3 = 2354.4 \frac{\text{N}}{\text{m}^2} = \frac{2354.4}{10^4} \frac{\text{N}}{\text{cm}^2}.$$

$$= \mathbf{0.2354 \frac{N}{cm^2}. \text{ Ans.}}$$

(c) For mercury, sp. gr. $= 13.6$

From equation (1.1A) we know that the density of a fluid is equal to specific gravity of fluid multiplied by density of water

$\therefore$ Density of mercury, $\rho_s = \text{Specific gravity of mercury} \times \text{Density of water}$

$$= 13.6 \times 1000 = 13600 \text{ kg/m}^3$$

$$\therefore \quad p = \rho_s \times g \times Z$$

$$= 13600 \times 9.81 \times 0.3 = 40025 \frac{\text{N}}{\text{m}^2}$$

$$= \frac{40025}{10^4} = \mathbf{4.002 \frac{N}{cm^2}. \text{ Ans.}}$$

Problem 2.4 *The pressure intensity at a point in a fluid is given 3.924 N/cm². Find the corresponding height of fluid when the fluid is : (a) water, and (b) oil of sp. gr. 0.9.*

Solution. Given :

Pressure intensity, $p = 3.924 \frac{\text{N}}{\text{cm}^2} = 3.924 \times 10^4 \frac{\text{N}}{\text{m}^2}.$

The corresponding height, Z, of the fluid is given by equation (2.6) as

$$Z = \frac{p}{\rho \times g}$$

(a) For water, $\rho = 1000 \text{ kg/m}^3$

$$\therefore \quad Z = \frac{p}{\rho \times g} = \frac{3.924 \times 10^4}{1000 \times 9.81} = \mathbf{4 \text{ m of water. Ans.}}$$

(b) For oil, sp. gr. $= 0.9$

$\therefore$ Density of oil $\rho_0 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

$$\therefore \quad Z = \frac{p}{\rho_0 \times g} = \frac{3.924 \times 10^4}{900 \times 9.81} = \mathbf{4.44 \text{ m of oil. Ans.}}$$

Problem 2.5 *An oil of sp. gr. 0.9 is contained in a vessel. At a point the height of oil is 40 m. Find the corresponding height of water at the point.*

Solution. Given :

Sp. gr. of oil, $S_0 = 0.9$

Height of oil, $Z_0 = 40$ m

Density of oil, $\rho_0 =$ Sp. gr. of oil × Density of water $= 0.9 \times 1000 = 900$ kg/m³

Intensity of pressure, $p = \rho_0 \times g \times Z_0 = 900 \times 9.81 \times 40 \frac{N}{m^2}$

$$\therefore \text{ Corresponding height of water} = \frac{p}{\text{Density of water} \times g}$$

$$= \frac{900 \times 9.81 \times 40}{1000 \times 9.81} = 0.9 \times 40 = \textbf{36 m of water. Ans.}$$

Problem 2.6 *An open tank contains water upto a depth of 2 m and above it an oil of sp. gr. 0.9 for a depth of 1 m. Find the pressure intensity (i) at the interface of the two liquids, and (ii) at the bottom of the tank.*

Solution. Given :

Height of water, $Z_1 = 2$ m

Height of oil, $Z_2 = 1$ m

Sp. gr. of oil, $S_0 = 0.9$

Density of water, $\rho_1 = 1000$ kg/m³

Density of oil, $\rho_2 =$ Sp. gr. of oil × Density of water $= 0.9 \times 1000 = 900$ kg/m³

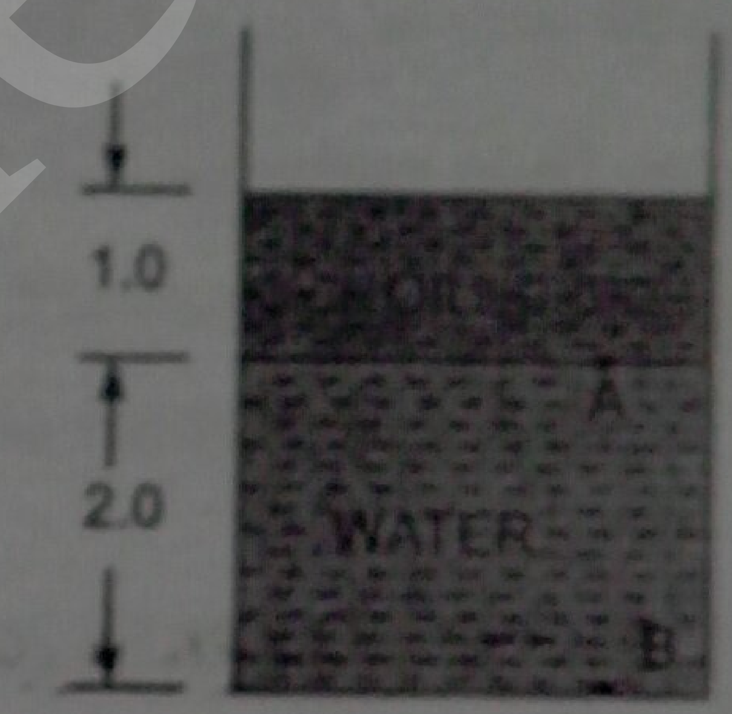

Fig. 2.4

Pressure intensity at any point is given by

$$p = \rho \times g \times Z.$$

(*i*) At interface, *i.e.*, at *A*

$$p = \rho_2 \times g \times 1.0$$

$$= 900 \times 9.81 \times 1.0$$

$$= 8829 \frac{N}{m^2} = \frac{8829}{10^4} = \textbf{0.8829 N/cm}^2\textbf{. Ans.}$$

(*ii*) At the bottom, *i.e.*, at *B*

$$p = \rho_2 \times g Z_2 + \rho_1 \times g \times Z_1 = 900 \times 9.81 \times 1.0 + 1000 \times 9.81 \times 2.0$$

$$= 8829 + 19620 = 28449 \text{ N/m}^2 = \frac{28449}{10^4} \text{ N/cm}^2 = \textbf{2.8449 N/cm}^2\textbf{. Ans.}$$

Problem 2.7 *The diameters of a small piston and a large piston of a hydraulic jack are 3 cm and 10 cm respectively. A force of 80 N is applied on the small piston. Find the load lifted by the large piston when :*

(a) the pistons are at the same level.

(b) small piston is 40 cm above the large piston.

The density of the liquid in the jack is given as 1000 kg/m³.

Solution. Given :

Dia. of small piston, $d = 3$ cm

$\therefore$ Area of small piston, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (3)^2 = 7.068$ cm²

Dia. of large piston, $D = 10$ cm

$\therefore$ Area of larger piston, $A = \frac{\pi}{4} \times (10)^2 = 78.54 \text{ cm}^2$

Force on small piston, $F = 80$ N

Let the load lifted $= W$.

(a) When the pistons are at the same level

Pressure intensity on small piston

$$\frac{F}{a} = \frac{80}{7.068} \text{ N/cm}^2$$

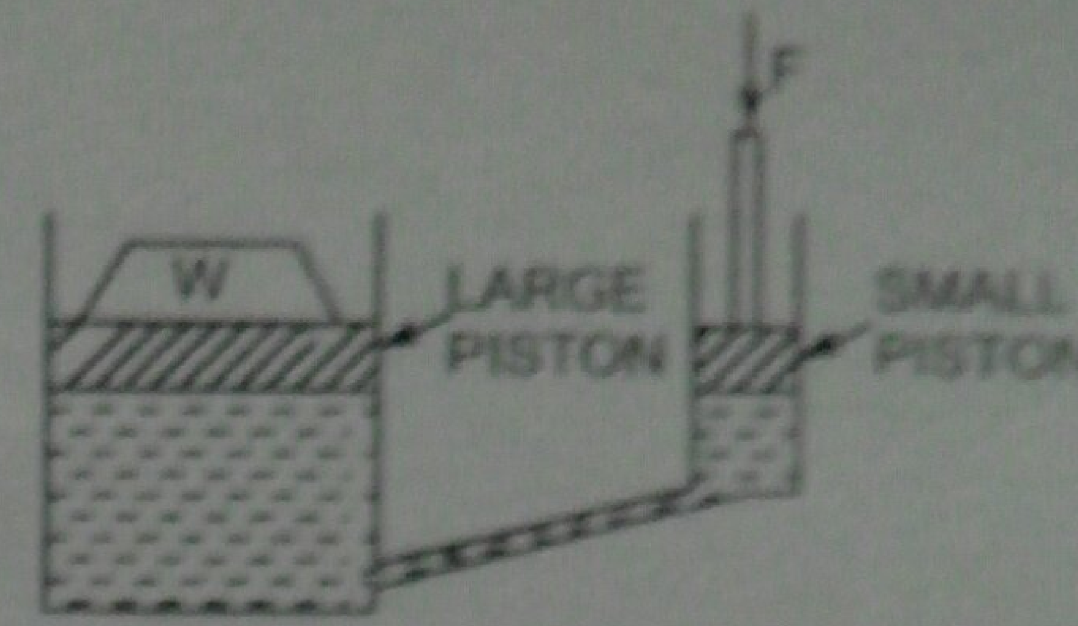

Fig. 2.5

This is transmitted equally on the large piston.

$\therefore$ Pressure intensity on the large piston

$$= \frac{80}{7.068}$$

$\therefore$ Force on the large piston $=$ Pressure $\times$ Area

$$= \frac{80}{7.068} \times 78.54 \text{ N} = \mathbf{888.96 \text{ N. Ans.}}$$

(b) When the small piston is 40 cm above the large piston

Pressure intensity on the small piston

$$= \frac{F}{a} = \frac{80}{7.068} \frac{\text{N}}{\text{cm}^2}$$

$\therefore$ Pressure intensity at section *A-A*

$$= \frac{F}{a} + \text{Pressure intensity due to height of 40 cm of liquid.}$$

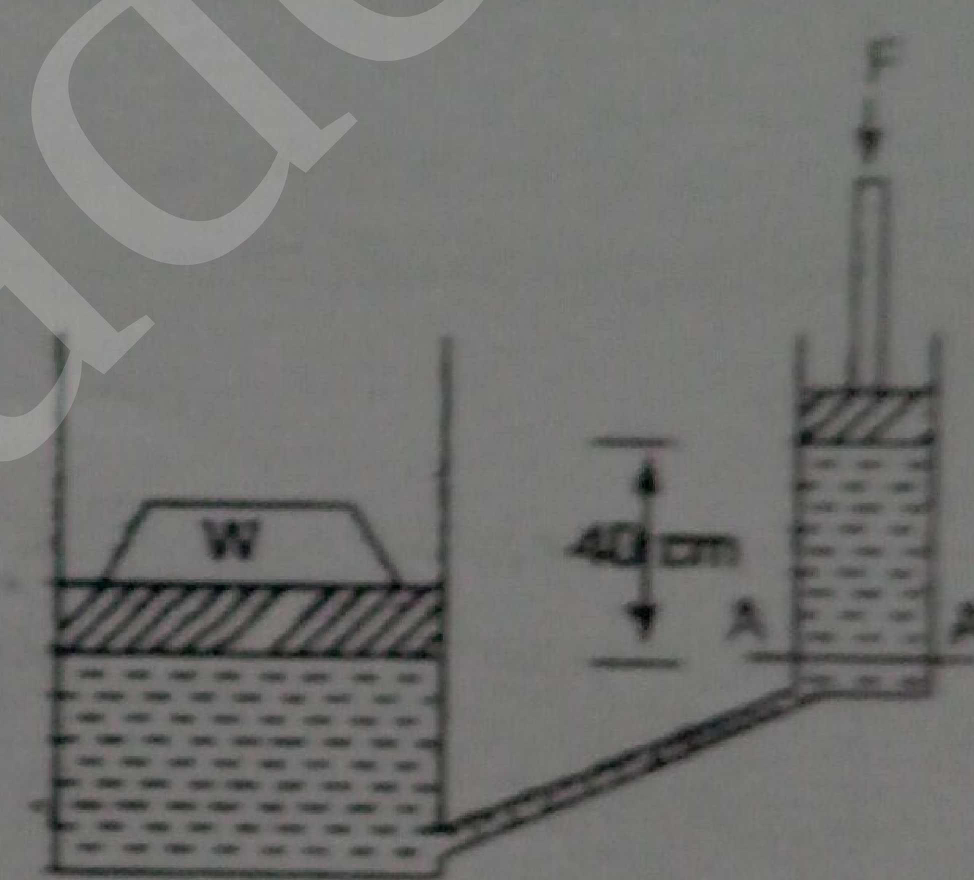

Fig. 2.6

But pressure intensity due to 40 cm of liquid

$$= \rho \times g \times h = 1000 \times 9.81 \times 0.4 \text{ N/m}^2$$

$$= \frac{1000 \times 9.81 \times 40}{10^4} \text{ N/cm}^2 = 0.3924 \text{ N/cm}^2$$

$\therefore$ Pressure intensity at section *A-A*

$$= \frac{80}{7.068} + 0.3924$$

$$= 11.32 + 0.3924 = 11.71 \text{ N/cm}^2$$

$\therefore$ Pressure intensity transmitted to the large piston $= 11.71 \text{ N/cm}^2$

$\therefore$ Force on the large piston $=$ Pressure $\times$ Area of the large piston

$$= 11.71 \times A = 11.71 \times 78.54 = 919.7 \text{ N.}$$

▸ 2.4 ABSOLUTE, GAUGE, ATMOSPHERIC AND VACUUM PRESSURES

The pressure on a fluid is measured in two different systems. In one system, it is measured above the absolute zero or complete vacuum and it is called the absolute pressure and in other system, pressure is measured above the atmospheric pressure and it is called gauge pressure. Thus :

1. **Absolute pressure** is defined as the pressure which is measured with reference to absolute vacuum pressure.

2. **Gauge pressure** is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.