Math 308: Midterm 2 Review

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1 Matrices

1.1 Linear Transformations

1. Let A be an $n \times m$ matrix and T the linear transformation given by $T(\vec{x}) = A\vec{x}$.
   (a) T is a linear transformation between which two spaces?

   (b) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ 1 & 1 \end{bmatrix}$, then is T one-to-one?

   (c) Using the same A as above, is T onto? What is the range of T?

2. Using two linear transformations, rotate and scale the vector $\vec{e}_1 \in \mathbb{R}^2$ so that it is mapped to the vector $\begin{bmatrix} \sqrt{2} \\ \sqrt{2} \end{bmatrix}$. Write a collection of matrices $C(t)$ that would animate this smoothly.

3. Give an example of a linear transformation from $\mathbb{R}^4 \to \mathbb{R}^3$ that is onto but not one to one.

4. Find a linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ such that $T\left(\begin{bmatrix}1\\2\end{bmatrix}\right) = \begin{bmatrix}2\\1\end{bmatrix}$ and $T\left(\begin{bmatrix}2\\1\end{bmatrix}\right) = \begin{bmatrix}3\\1\end{bmatrix}$. Decide if T is one to one or onto.

5. Explain why translation is not a linear transformation, and why by going up a dimension it can be.

6. The map $T(\vec{x}) = \vec{b} \times \vec{x}$ has matrix $B = \begin{bmatrix}0 & -b_3 & b_2\\ b_3 & 0 & -b_1\\ -b_2 & b_1 & 0\end{bmatrix}$. When augmented with $\begin{bmatrix}x\\y\\z\end{bmatrix}$ and row reduced by a compute we obtain:

$$\left(\begin{array}{ccc|c} 1 & 0 & -\frac{b_1}{b_3} & \frac{y}{b_3} \\ 0 & 1 & -\frac{b_2}{b_3} & -\frac{x}{b_3} \\ 0 & 0 & 0 & \frac{b_1 x}{b_3} + \frac{b_2 y}{b_3} + z \end{array}\right)$$

What is the span of this linear transformation? Describe why this makes sense geometrically. (*Hint: the computer didn't do anything wrong, but row reduced to something correct but a little misleading. Can you correct it?*

1.2 Matrix Algebra

7. Give an example of two 2×2 matrices A and B such that $AB \neq BA$.

8. Explain why the following is true, or why it is false:

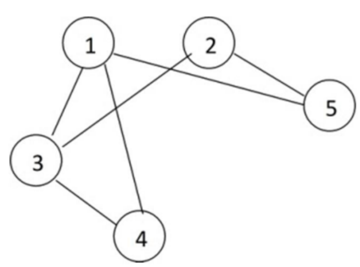
Most linear transformations $S, T\colon \mathbb{R}^2 \to \mathbb{R}^2$ *have the property that* $T \circ S = S \circ T$.

9. Explain why the following is true, or why it is false:

Most linear transformations $S, T\colon \mathbb{R}^2 \to \mathbb{R}^2$ *are invertible.*

10. Give an example of two 3×3 matrices that aren't I and do commute.

11. Write the adjacency matrix for this graph, and without doing any arithmetic write out the adjacency matrix squared.



1.3 Inverses

12. Explain why a non-square matrix cannot have an inverse.

13. What is the inverse of $\begin{bmatrix} 3\cos\theta & -3\sin\theta \\ 3\sin\theta & 3\cos\theta \end{bmatrix}$.

14. Find a 2×3 matrix A and a 3×2 matrix B such that $AB = I$ but $BA \neq I$.

15. Suppose that A is an $n \times n$ matrix and that $A\vec{x} = \vec{0}$ has only the trivial solution $\vec{x} = \vec{0}$. Does this mean A is invertible?