3

CHAPTER

HYDROSTATIC FORCES ON SURFACES

▸ 3.1 INTRODUCTION

This chapter deals with the fluids (*i.e.*, liquids and gases) at rest. This means that there will be no relative motion between adjacent or neighbouring fluid layers. The velocity gradient, which is equal to the change of velocity between two adjacent fluid layers divided by the distance between the layers, will be zero or $\frac{du}{dy} = 0$. The shear stress which is equal to $\mu \frac{\partial u}{\partial y}$ will also be zero. Then the forces acting on the fluid particles will be :

1. due to pressure of fluid normal to the surface,
2. due to gravity (or self-weight of fluid particles).

▸ 3.2 TOTAL PRESSURE AND CENTRE OF PRESSURE

Total pressure is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surfaces. This force always acts normal to the surface.

Centre of pressure is defined as the point of application of the total pressure on the surface. There are four cases of submerged surfaces on which the total pressure force and centre of pressure is to be determined. The submerged surfaces may be :

1. Vertical plane surface,
2. Horizontal plane surface,
3. Inclined plane surface, and
4. Curved surface.

▸ 3.3 VERTICAL PLANE SURFACE SUBMERGED IN LIQUID

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown in Fig. 3.1.

Let A = Total area of the surface

$\bar{h}$ = Distance of C.G. of the area from free surface of liquid

G = Centre of gravity of plane surface

P = Centre of pressure

h^* = Distance of centre of pressure from free surface of liquid.

(*a*) **Total Pressure (F).** The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips. The force on small strip is then calculated and the total pressure force on the whole area is calculated by integrating the force on small strip.

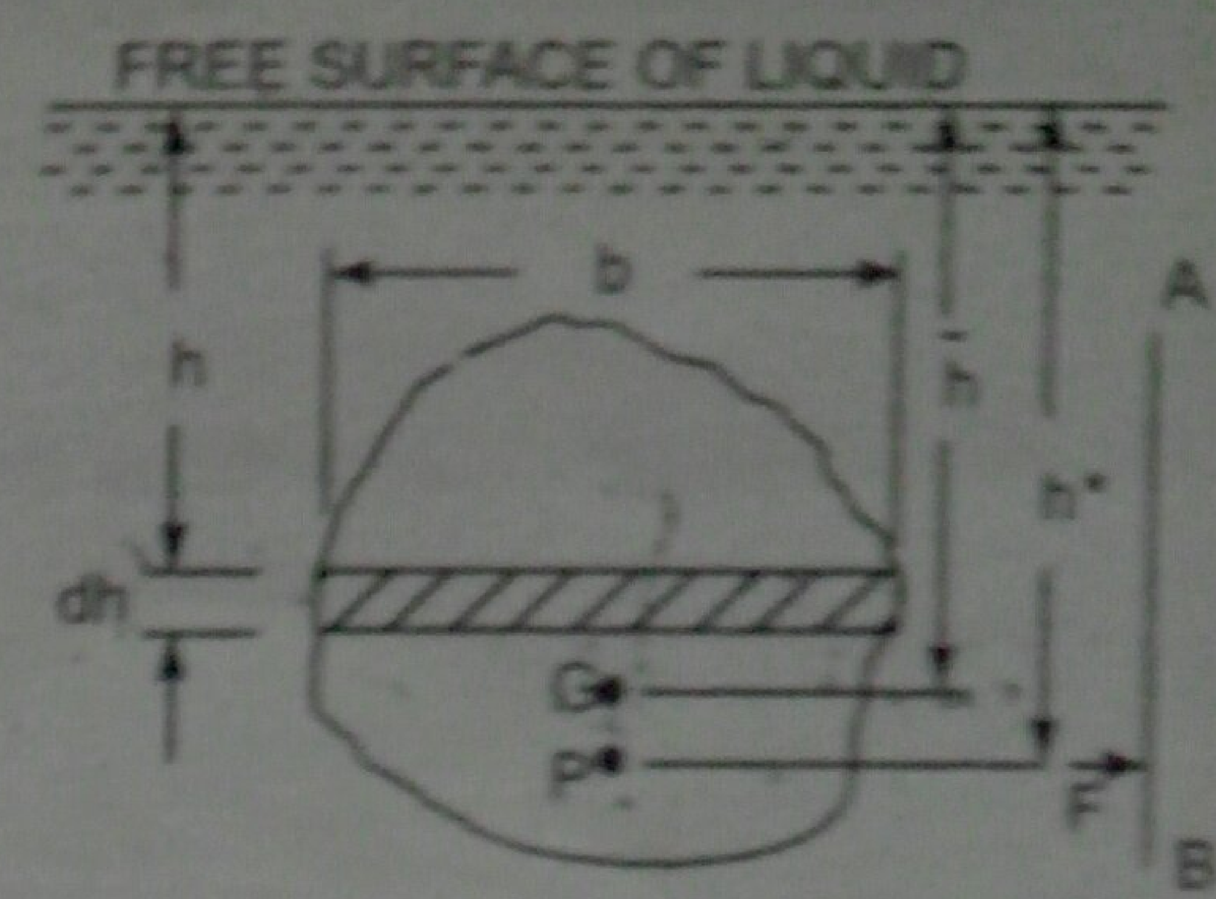


Fig. 3.1

Consider a strip of thickness dh and width b at a depth of h from free surface of liquid as shown in Fig. 3.1

Pressure intensity on the strip, $p = \rho g h$

(See equation 2.5)

Area of the strip, $dA = b \times dh$

Total pressure force on strip, $dF = p \times \text{Area}$

$= \rho g h \times b \times dh$

$\therefore$ Total pressure force on the whole surface,

$$F = \int dF = \int \rho g h \times b \times dh = \rho g \int b \times h \times dh$$

But $\int b \times h \times dh = \int h \times dA$

= Moment of surface area about the free surface of liquid

= Area of surface × Distance of C.G. from free surface

$= A \times \bar{h}$

$$\therefore \qquad F = \rho g A \bar{h} \qquad \text{...(3.1)}$$

For water the value of $\rho = 1000$ **kg/m³** and $g = 9.81$ **m/s²**. The force will be in Newton.

(*b*) **Centre of Pressure (h*).** Centre of pressure is calculated by using the "Principle of Moments", which states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

The resultant force F is acting at P, at a distance h^* from free surface of the liquid as shown in Fig. 3.1. Hence moment of the force F about free surface of the liquid $= F \times h^*$ ...(3.2)

Moment of force dF, acting on a strip about free surface of liquid

$= dF \times h$ $\{\because dF = \rho g h \times b \times dh\}$

$= \rho g h \times b \times dh \times h$

Sum of moments of all such forces about free surface of liquid

$$= \int \rho g h \times b \times dh \times h = \rho g \int b \times h \times h dh$$

$$= \rho g \int b h^2 dh = \rho g \int h^2 dA \qquad (\because b dh = dA)$$

But $\int h^2 dA = \int b h^2 dh$

= Moment of Inertia of the surface about free surface of liquid

$= I_0$

$\therefore$ Sum of moments about free surface

$$= \rho g I_0 \qquad \text{...(3.3)}$$

Equating (3.2) and (3.3), we get

$$F \times h^* = \rho g I_0$$

But $$F = \rho g A\bar{h}$$

$\therefore$ $$\rho g A\bar{h} \times h^* = \rho g I_0$$

or $$h^* = \frac{\rho g I_0}{\rho g A\bar{h}} = \frac{I_0}{A\bar{h}} \quad \text{...(3.4)}$$

By the theorem of parallel axis, we have

$$I_0 = I_G + A \times \bar{h}^2$$

where I_G = Moment of Inertia of area about an axis passing through the C.G. of the area and parallel to the free surface of liquid.

Substituting I_0 in equation (3.4), we get

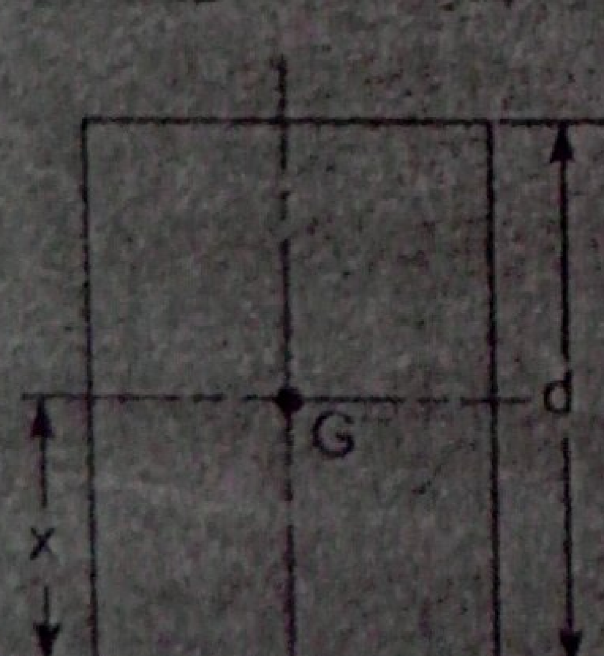
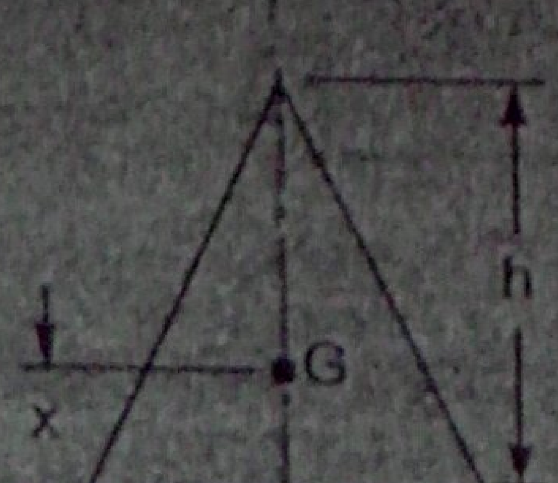
$$h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h} \quad \text{...(3.5)}$$

In equation (3.5), $\bar{h}$ is the distance of C.G. of the area of the vertical surface from free surface of the liquid. Hence from equation (3.5), it is clear that :

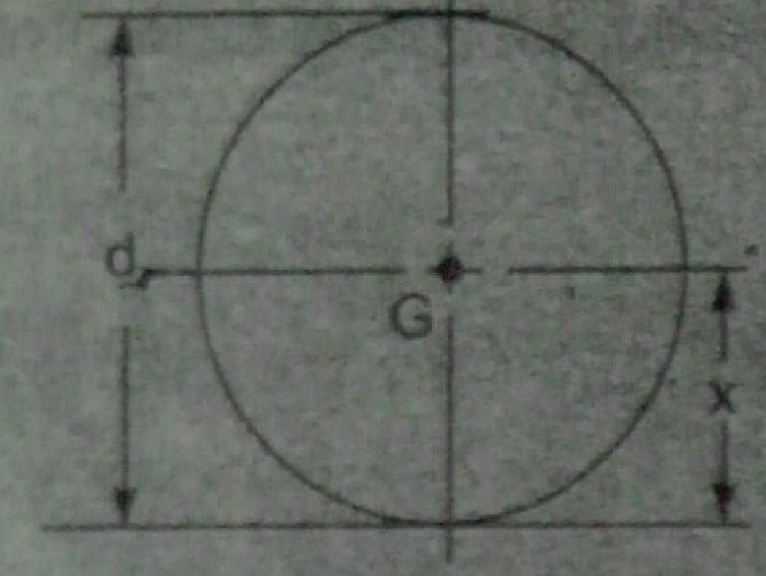
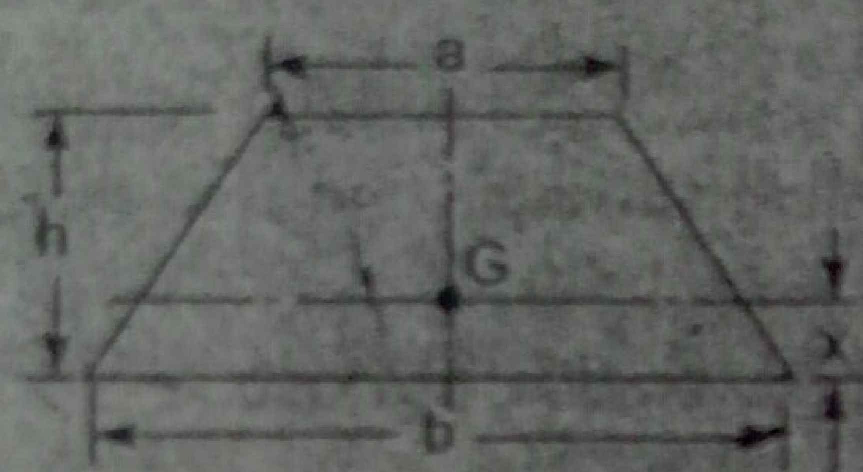
(i) Centre of pressure (*i.e.*, h^*) lies below the centre of gravity of the vertical surface.

(ii) The distance of centre of pressure from free surface of liquid is independent of the density of the liquid.

Table 3.1 The moments of inertia and other geometric properties of some important plane surfaces

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base* (I_G)	*Moment of inertia about base* (I_0)
1. Rectangle 	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
2. Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$

Contd...

Plane surface	C.G. from the base	Area	Moment of inertia about an axis passing through C.G. and parallel to base (I_G)	Moment of inertia about base (I_0)
3. Circle 	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—
4. Trapezium 	$x = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\frac{(a+b)}{2} \times h$	$\left(\frac{a^2+4ab+b^2}{36(a+b)}\right) \times h^3$	—

Problem 3.1 *A rectangular plane surface is 2 m wide and 3 m deep. It lies in vertical plane in water. Determine the total pressure and position of centre of pressure on the plane surface when its upper edge is horizontal and (a) coincides with water surface, (b) 2.5 m below the free water surface.*

Solution. Given :

Width of plane surface, $b = 2$ m

Depth of plane surface, $d = 3$ m

(*a*) **Upper edge coincides with water surface** (**Fig. 3.2**). **Total pressure is given by equation** (3.1) as

$$F = \rho g A \bar{h}$$

where $\rho = 1000$ kg/m^3, $g = 9.81$ m/s^2

$$A = 3 \times 2 = 6 \text{ m}^2, \bar{h} = \frac{1}{2}(3) = 1.5 \text{ m}.$$

$$\therefore \quad F = 1000 \times 9.81 \times 6 \times 1.5$$

$$= 88290 \text{ N. Ans.}$$

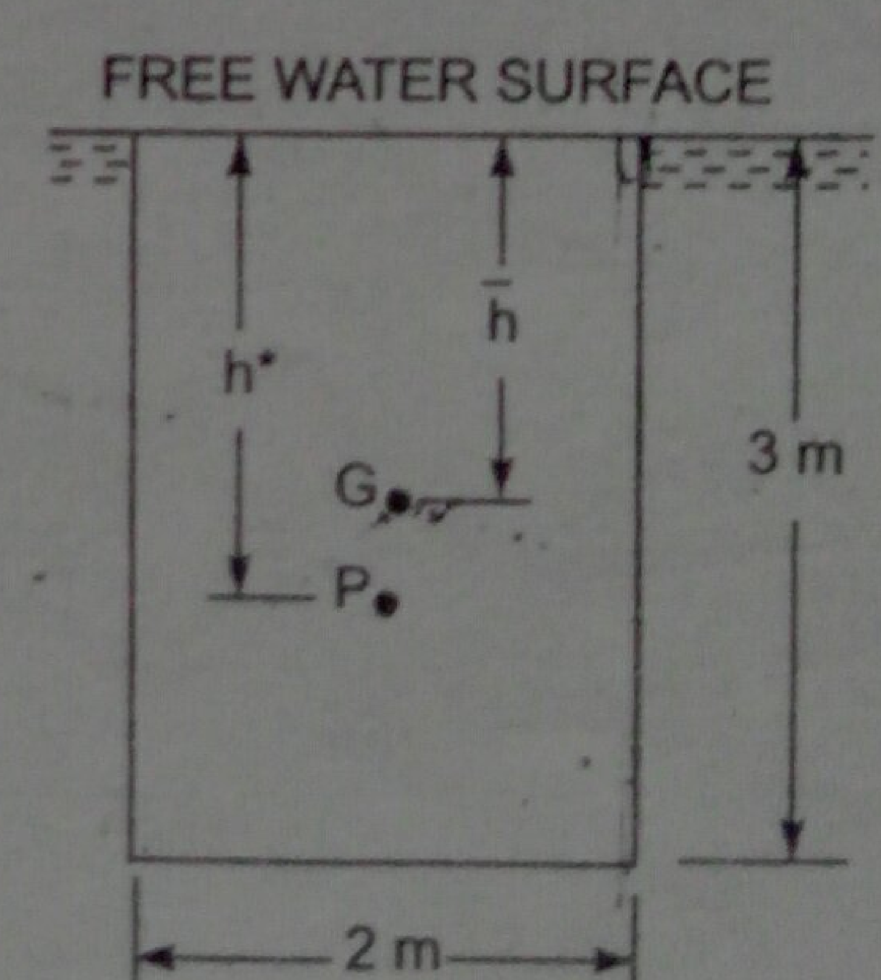


Fig. 3.2

Depth of centre of pressure is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. about C.G. of the area of surface

$$= \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 \text{ m}^4$$

$$\therefore \quad h^* = \frac{4.5}{6 \times 1.5} + 1.5 = 0.5 + 1.5 = \mathbf{2.0\ m.\ Ans.}$$

(*b*) **Upper edge is 2.5 m below water surface (Fig. 3.3).** Total pressure (F) is given by (3.1)

$$F = \rho g A \bar{h}$$

where $\bar{h}$ = Distance of C.G. from free surface of water

$$= 2.5 + \frac{3}{2} = 4.0 \text{ m}$$

$$\therefore \quad F = 1000 \times 9.81 \times 6 \times 4.0$$

$$= \mathbf{235440\ N.\ Ans.}$$

Centre of pressure is given by $h^* = \dfrac{I_G}{A\bar{h}} + \bar{h}$

where $I_G = 4.5$, $A = 6.0$, $\bar{h} = 4.0$

$$\therefore \quad h^* = \frac{4.5}{6.0 \times 4.0} + 4.0$$

$$= 0.1875 + 4.0 = 4.1875 = \mathbf{4.1875\ m.\ Ans.}$$

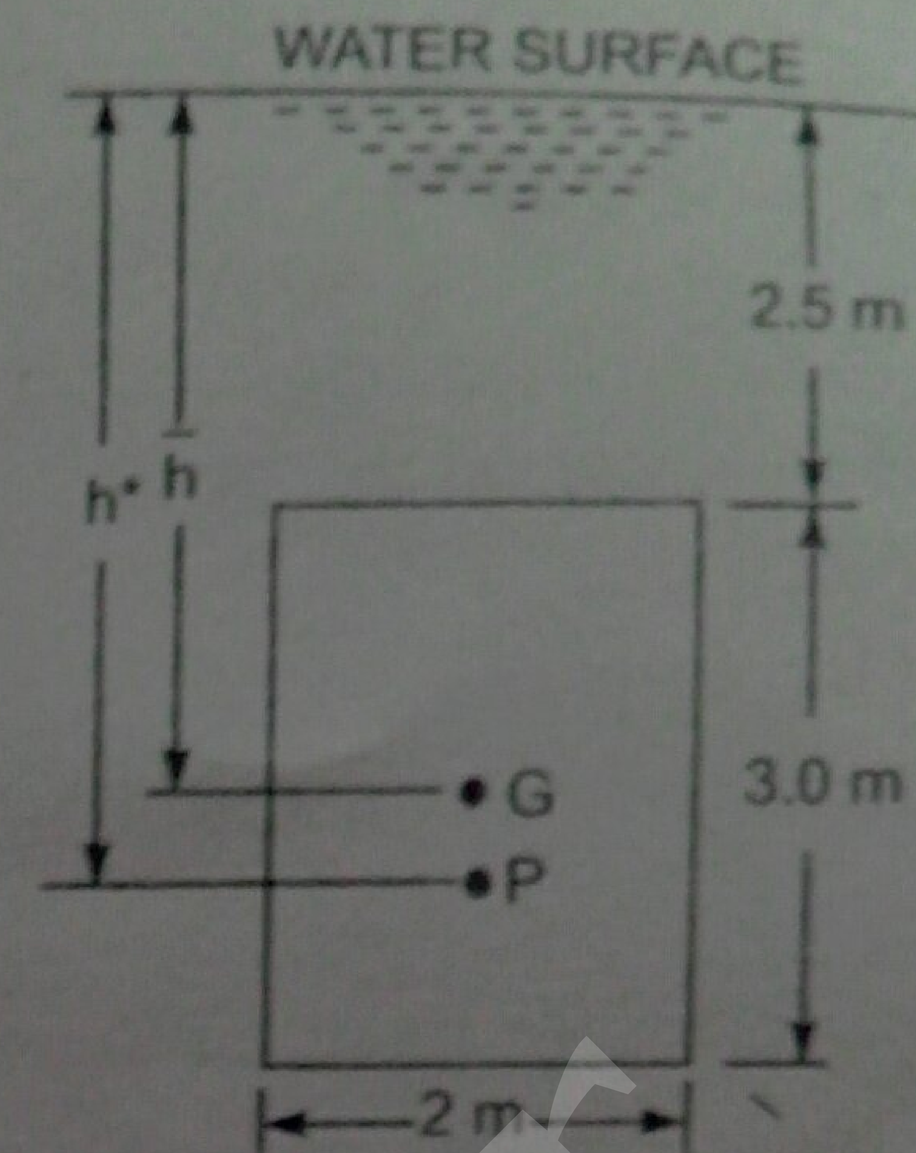


Fig. 3.3

Problem 3.2 *Determine the total pressure on a circular plate of diameter 1.5 m which is placed vertically in water in such a way that the centre of the plate is 3 m below the free surface of water. Find the position of centre of pressure also.*

Solution. Given : Dia. of plate, $d = 1.5$ m

$\therefore$ Area, $A = \dfrac{\pi}{4}(1.5)^2 = 1.767 \text{ m}^2$

$$\bar{h} = 3.0 \text{ m}$$

Total pressure is given by equation (3.1),

$$F = \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times 1.767 \times 3.0 \text{ N}$$

$$= \mathbf{52002.81\ N.\ Ans.}$$

Position of centre of pressure (h^*) is given by equation (3.5),

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where $I_G = \dfrac{\pi d^4}{64} = \dfrac{\pi \times 1.5^4}{64} = 0.2485 \text{ m}^4$

$$\therefore \quad h^* = \frac{0.2485}{1.767 \times 3.0} + 3.0 = 0.0468 + 3.0$$

$$= \mathbf{3.0468\ m.\ Ans.}$$

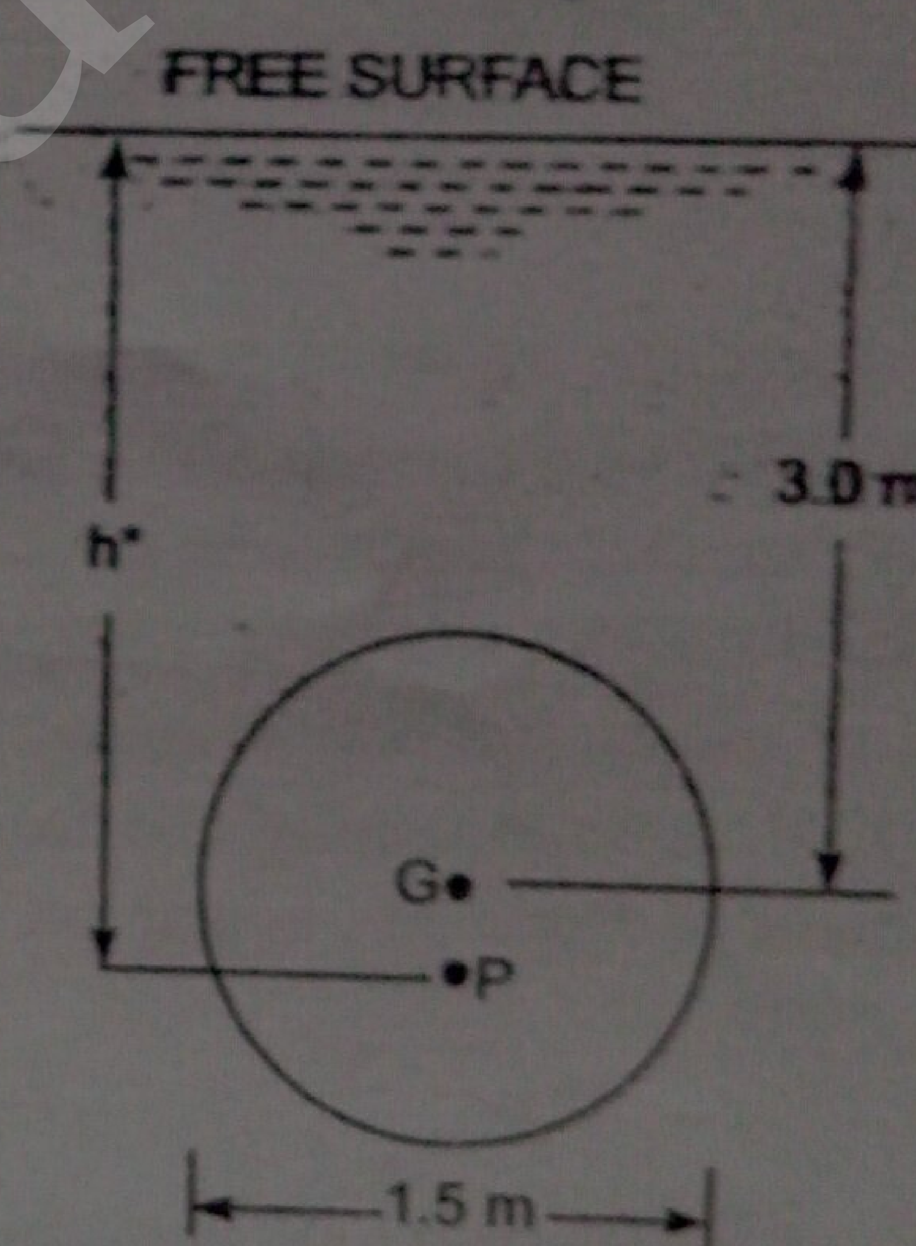


Fig. 3.4

Problem 3.3 *A rectangular sluice gate is situated on the vertical wall of a lock. The vertical side of the sluice is 'd' metres in length and depth of centroid of the area is 'p' m below the water surface. Prove that the depth of pressure is equal to* $\left(p+\frac{d^2}{12p}\right)$.

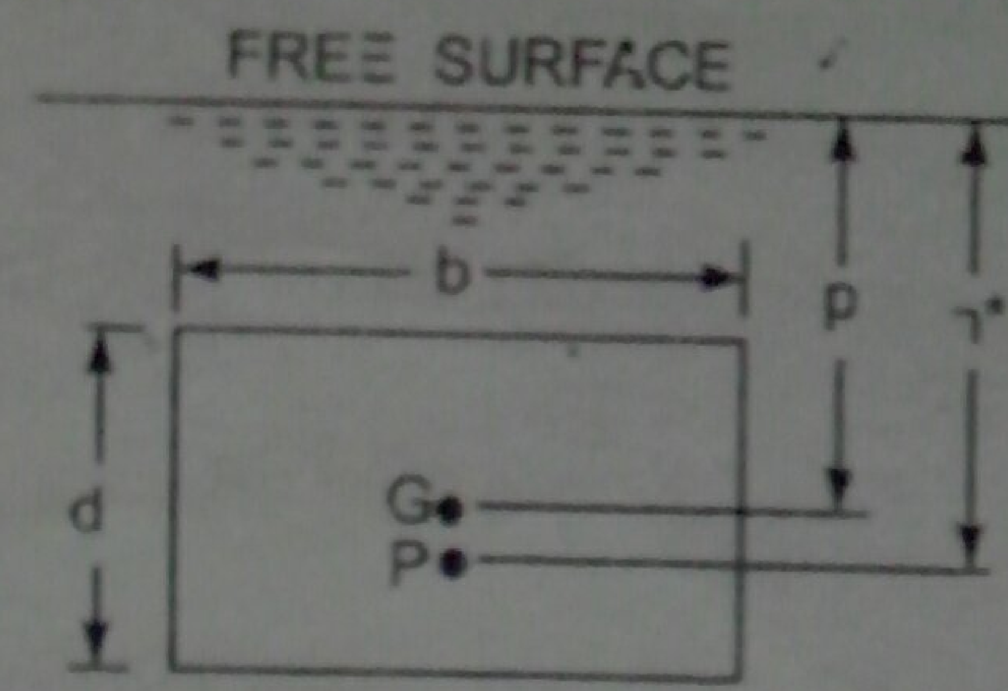


Fig. 3.5

Solution. Given :

Depth of vertical gate $= d$ m

Let the width of gate $= b$ m

∴ Area, $A = b \times d \text{ m}^2$

Depth of C.G. from free surface

$$\bar{h} = p \text{ m.}$$

Let h^* is the depth of centre of pressure from free surface, which is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}, \text{ where } I_G = \frac{bd^3}{12}$$

$$\therefore \quad h^* = \left(\frac{bd^3}{12} \Big/ b \times d \times p\right) + p = \frac{d^2}{12p} + p \quad \text{or} \quad p + \frac{d^2}{12p}. \text{ Ans.}$$

Problem 3.4 *A circular opening, 3 m diameter, in a vertical side of a tank is closed by a disc of 3 m diameter which can rotate about a horizontal diameter. Calculate :*

(i) the force on the disc, and

(ii) the torque required to maintain the disc in equilibrium in the vertical position when the head of water above the horizontal diameter is 4 m.

Solution. Given :

Dia. of opening, $d = 3$ m

∴ Area, $A = \frac{\pi}{4} \times 3^2 = 7.0685 \text{ m}^2$

Depth of C.G., $\bar{h} = 4$ m

(i) Force on the disc is given by equation (3.1) as

$$F = \rho g A \bar{h} = 1000 \times 9.81 \times 7.0685 \times 4.0$$

$$= \mathbf{277368 \text{ N} = 277.368 \text{ kN. Ans.}}$$

(ii) To find the torque required to maintain the disc in equilibrium, first calculate the point of application of force acting on the disc, *i.e.*, centre of pressure of the force F. The depth of centre of pressure (h^*) is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{\frac{\pi}{64} d^4}{\frac{\pi}{4} d^2 \times 4.0} + 4.0 \qquad \left\{\because I_G = \frac{\pi}{64} d^4\right\}$$

$$= \frac{d^2}{16 \times 4.0} + 4.0 = \frac{3^2}{16 \times 4.0} + 4.0 = 0.14 + 4.0 = 4.14 \text{ m}$$

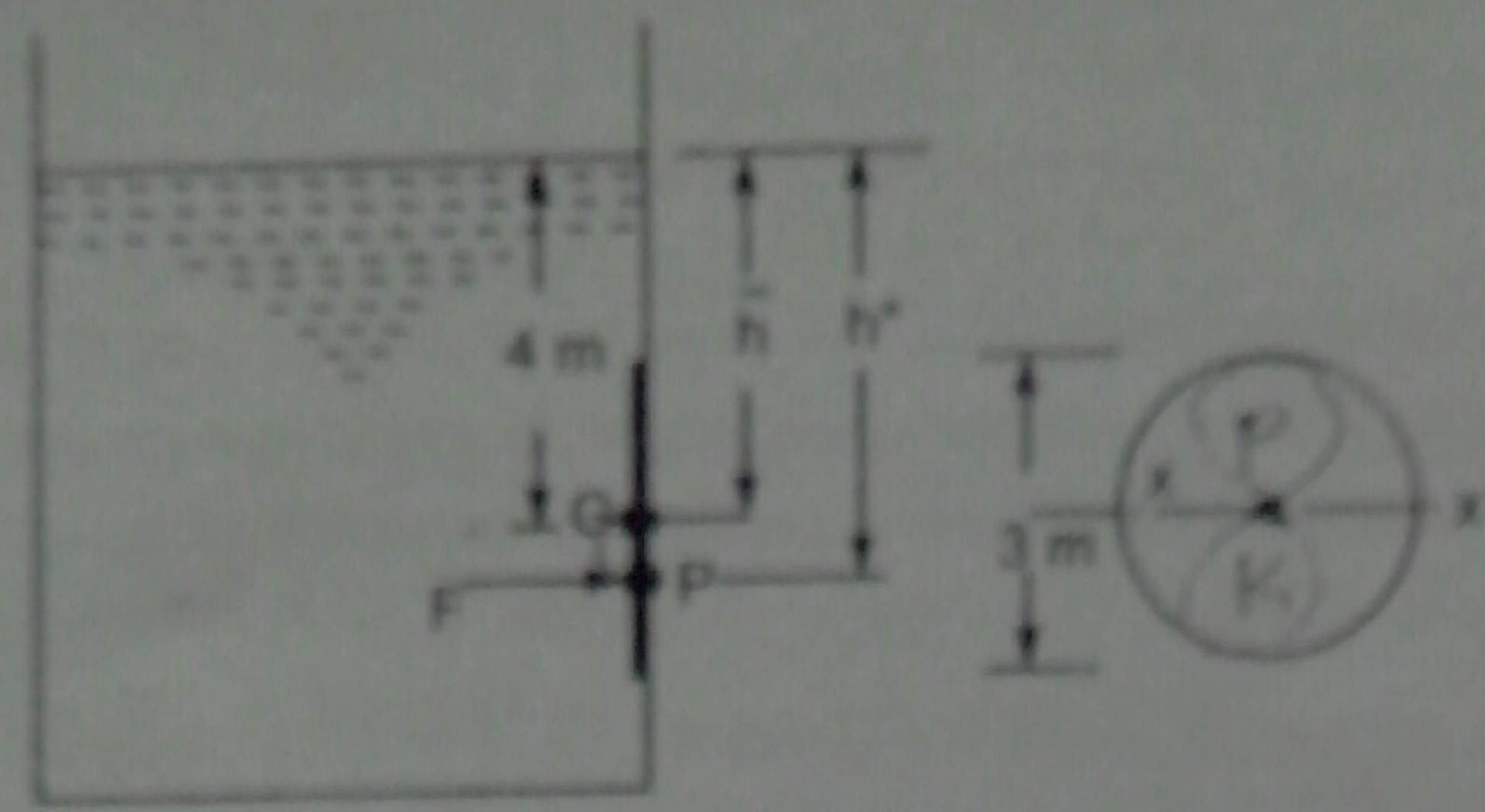


Fig. 3.6

The force F is acting at a distance of 4.14 m from free surface. Moment of this force about horizontal diameter X-X

$$= F \times \left(h^* - \bar{h}\right) = 277368\ (4.14 - 4.0) = \mathbf{38831\ Nm.\ Ans.}$$

Hence a torque of 38831 Nm must be applied on the disc in the clockwise direction.

Problem 3.5 *A pipe line which is 4 m in diameter contains a gate valve. The pressure at the centre of the pipe is 19.6 N/cm². If the pipe is filled with oil of sp. gr. 0.87, find the force exerted by the oil upon the gate and position of centre of pressure.*

Solution. Given :

Dia. of pipe, $d = 4$ m

Fig. 3.7

$\therefore$ Area, $A = \frac{\pi}{4} \times 4^2 = 4\pi\ m^2$

Sp. gr. of oil, $S = 0.87$

$\therefore$ Density of oil, $\rho_0 = 0.87 \times 1000 = 870\ kg/m^3$

$\therefore$ Weight density of oil, $w_0 = \rho_0 \times g = 870 \times 9.81\ N/m^3$

Pressure at the centre of pipe, $p = 19.6\ N/cm^2 = 19.6 \times 10^4\ N/m^2$

$$\therefore \text{ Pressure head at the centre} = \frac{p}{w_0} = \frac{19.6 \times 10^4}{870 \times 9.81} = 22.988\ m$$

$\therefore$ The height of equivalent free oil surface from the centre of pipe = 22.988 m.

The depth of C.G. of the gate valve from free oil surface $\bar{h} = 22.988$ m.

(*i*) Now the force exerted by the oil on the gate is given by

$$F = \rho g A \bar{h}$$

where ρ = density of oil = 870 kg/m³

$$F = 870 \times 9.81 \times 4\pi \times 22.988 = \mathbf{2465500\ N = 2.465\ MN.\ Ans.}$$

(*ii*) Position of centre of pressure (h^*) is given by (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{\frac{\pi}{64}d^4}{\frac{\pi}{4}d^2 \times \bar{h}} + \bar{h} = \frac{d^2}{16\,\bar{h}} + \bar{h} = \frac{4^2}{16 \times 22.988} + 22.988$$

$$= 0.043 + 22.988 = \mathbf{23.031\ m.\ Ans.}$$

Or centre of pressure is below the centre of the pipe by a distance of 0.043 m. **Ans.**

Problem 3.6 *Determine the total pressure and centre of pressure on an isosceles triangular plate of base 4 m and altitude 4 m when it is immersed vertically in an oil of sp. gr. 0.9. The base of the plate coincides with the free surface of oil.*

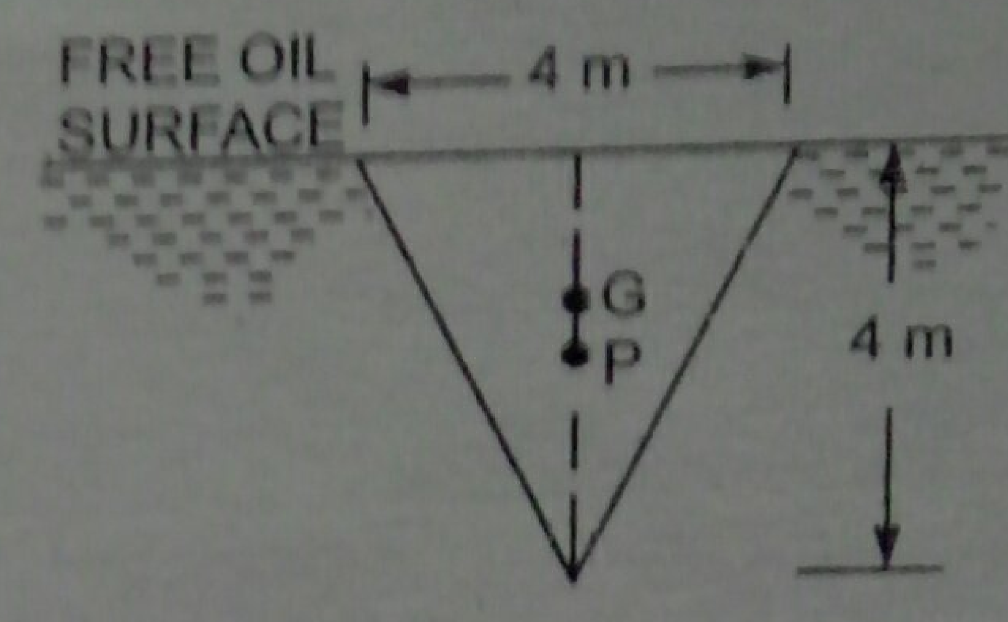


Fig. 3.8

Solution. Given :

Base of plate, $b = 4$ m

Height of plate, $h = 4$ m

∴ Area, $A = \frac{b \times h}{2} = \frac{4 \times 4}{2} = 8.0\ m^2$

Sp. gr. of oil, $S = 0.9$

∴ Density of oil, $\rho = 900\ kg/m^3$.

The distance of C.G. from free surface of oil,

$$\bar{h} = \frac{1}{3} \times h = \frac{1}{3} \times 4 = 1.33\ m.$$

Total pressure (F) is given by $F = \rho g A\bar{h}$

$$= 900 \times 9.81 \times 8.0 \times 1.33\ N = \mathbf{9597.6.\ N.\ Ans.}$$

Centre of pressure (h^*) from free surface of oil is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. of triangular section about its C.G.

$$= \frac{bh^3}{36} = \frac{4 \times 4^3}{36} = 7.11\ m^4$$

∴ $$h^* = \frac{7.11}{8.0 \times 1.33} + 1.33 = 0.6667 + 1.33 = \mathbf{1.99\ m.\ Ans.}$$

Problem 3.7 *A vertical sluice gate is used to cover an opening in a dam. The opening is 2 m wide and 1.2 m high. On the upstream of the gate, the liquid of sp. gr. 1.45, lies upto a height of 1.5 m above the top of the gate, whereas on the downstream side the water is available upto a height touching the top of the gate. Find the resultant force acting on the gate and position of centre of pressure. Find also the force acting horizontally at the top of the gate which is capable of opening it. Assume that the gate is hinged at the bottom.*

Solution. Given :

Width of gate, $b = 2$ m

Depth of gate, $d = 1.2$ m

∴ Area, $A = b \times d = 2 \times 1.2 = 2.4\ m^2$

Sp. gr. of liquid $= 1.45$

$\therefore$ Density of liquid, $\rho_1 = 1.45 \times 1000 = 1450$ kg/m^3

Let F_1 = Force exerted by the fluid of sp. gr. 1.45 on gate

F_2 = Force exerted by water on the gate.

The force F_1 is given by $F_1 = \rho_1 g \times A \times \bar{h}_1$

where $\rho_1 = 1.45 \times 1000 = 1450$ kg/m^2

$\bar{h}_1$ = Depth of C.G. of gate from free surface of liquid

$$= 1.5 + \frac{1.2}{2} = 2.1 \text{ m.}$$

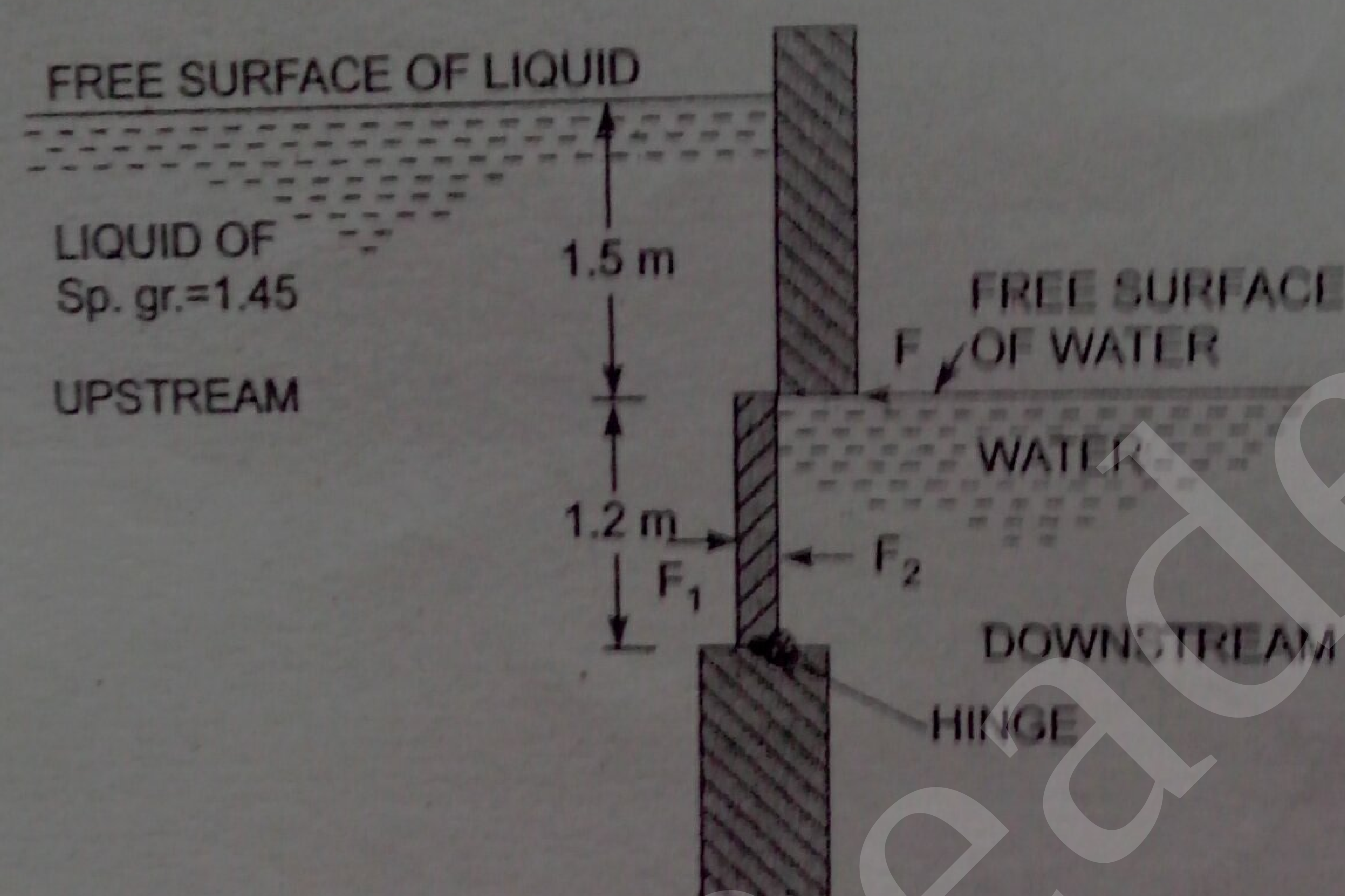


Fig. 3.9

$$\therefore \quad F_1 = 1450 \times 9.81 \times 2.4 \times 2.1 = 71691 \text{ N}$$

Similarly, $F_2 = \rho_2 g . A \bar{h}_2$

where $\rho_2 = 1,000$ kg/m^3

$\bar{h}_2$ = Depth of C.G. of gate from free surface of water

$$= \frac{1}{2} \times 1.2 = 0.6 \text{ m}$$

$$\therefore \quad F_2 = 1000 \times 9.81 \times 2.4 \times 0.6 = 14126 \text{ N}$$

(*i*) **Resultant force on the gate** = $F_1 - F_2$ = 71691 − 14126 = **57565 N. Ans.**

(*ii*) **Position of centre of pressure of resultant force.** The force F_1 will be acting at a depth of h_1^* from free surface of liquid, given by the relation

$$h_1^* = \frac{I_G}{A\bar{h}_1} + \bar{h}_1$$

where $I_G = \frac{bd^3}{12} = \frac{2 \times 1.2^3}{12} = 0.288 \text{ m}^4$

$$\therefore \quad h_1^* = \frac{.288}{2.4 \times 2.1} + 2.1 = 0.0571 + 2.1 = 2.1571 \text{ m}$$

$\therefore$ Distance of F_1 from hinge

$$= (1.5 + 1.2) - h_1^* = 2.7 - 2.1571 = 0.5429 \text{ m}$$

The force F_2 will be acting at a depth of h_2^* from free surface of water and is given by

$$h_2^* = \frac{I_G}{A\bar{h}_2} + \bar{h}_2$$

where $I_G = 0.288\ m^4$, $\bar{h}_2 = 0.6$ m, $A = 2.4\ m^2$,

$$h_2^* = \frac{0.288}{2.4 \times 0.6} + 0.6 = 0.2 + 0.6 = 0.8 \text{ m}$$

Distance of F_2 from hinge $= 1.2 - 0.8 = 0.4$ m

The resultant force 57565 N will be acting at a distance given by

$$= \frac{71691 \times .5429 - 14126 \times 0.4}{57565}$$

$$= \frac{38921 - 5650.4}{57565} \text{ m above hinge}$$

= 0.578 m above the hinge. Ans.

(*iii*) **Force at the top of gate which is capable of opening the gate.** Let F is the force required on the top of the gate to open it as shown in Fig. 3.9. Taking the moments of F, F_1 and F_2 about the hinge, we get

$$F \times 1.2 + F_2 \times 0.4 = F_1 \times .5429$$

or

$$F = \frac{F_1 \times .5429 - F_2 \times 0.4}{1.2}$$

$$= \frac{71691 \times .5429 - 14126 \times 0.4}{1.2} = \frac{38921 - 5650.4}{1.2}$$

= 27725.5 N. Ans.

Problem 3.8 *A caisson for closing the entrance to a dry dock is of trapezoidal form 16 m wide at the top and 10 m wide at the bottom and 6 m deep. Find the total pressure and centre of pressure on the caisson if the water on the outside is just level with the top and dock is empty.*

Solution. Given :

Width at top $= 16$ m

Width at bottom $= 10$ m

Depth, $d = 6$ m

Area of trapezoidal $ABCD$,

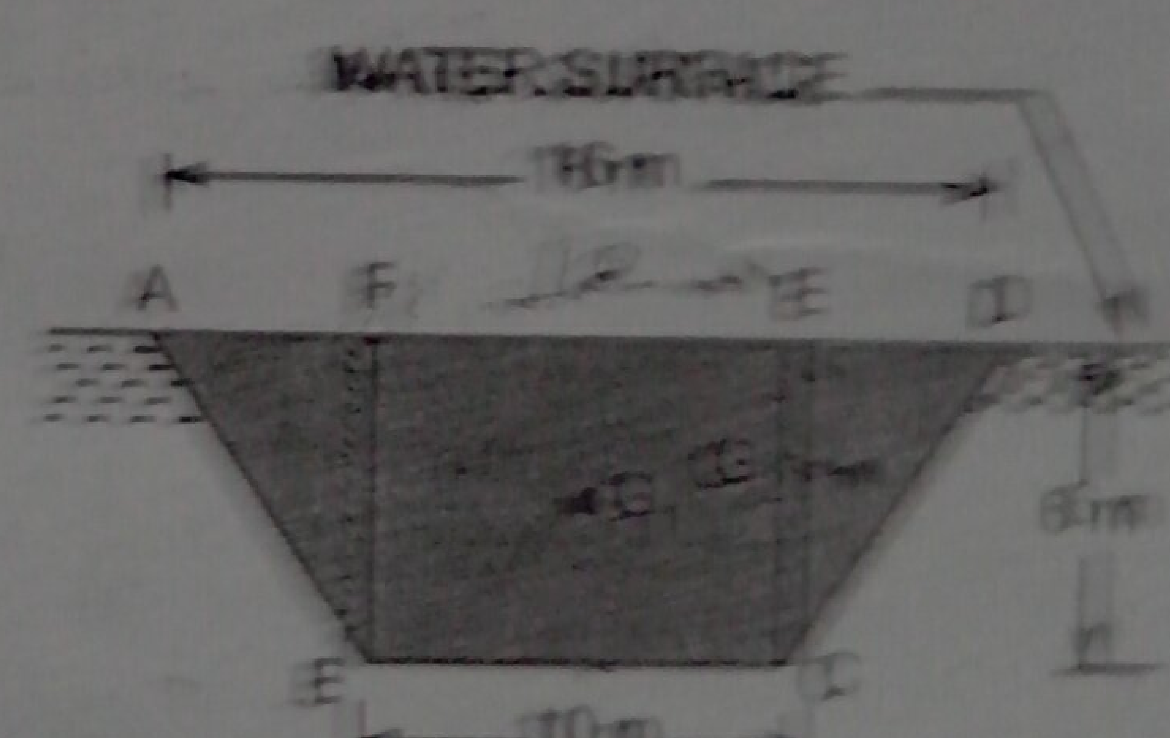


Fig. 3.10

$$A = \frac{(BC + AD)}{2} \times d$$

$$= \frac{(10 + 16)}{2} \times 6 = 78 \text{ m}^2$$

Depth of C.G. of trapezoidal area $ABCD$ from free surface of water,

$$\bar{h} = \frac{10 \times 6 \times 3 + \frac{(16-10)}{2} \times 6 \times \frac{1}{3} \times 6}{78}$$

$$= \frac{180 + 36}{78} = 2.769 \text{ m from water surface.}$$

(*i*) **Total Pressure (F).** Total pressure, F is given by

$$F = \rho g A\bar{h} = 1000 \times 9.81 \times 78 \times 2.769 \text{ N}$$
$$= 2118783 \text{ N} = \mathbf{2.118783 \text{ MN. Ans.}}$$

(*ii*) **Centre of Pressure (h*).** Centre of pressure is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. of trapezoidal *ABCD* about its C.G.

Let I_{G_1} = M.O.I. of rectangle *FBCE* about its C.G.

I_{G_2} = M.O.I. of two ∆s *ABF* and *ECD* about its C.G.

Then $$I_{G_1} = \frac{bd^3}{12} = \frac{10 \times 6^3}{12} = 180 \text{ m}^4$$

I_{G_1} is the M.O.I. of the rectangle about the axis passing through G_1.

∴ M.O.I. of the rectangle about the axis passing through the C.G. of the trapezoidal I_{G_1} + Area of rectangle × x_1^2

where x_1 is distance between the C.G. of rectangle and C.G. of trapezoidal

$$= (3.0 - 2.769) = 0.231 \text{ m}$$

∴ M.O.I. of *FBCE* passing through C.G. of trapezoidal

$$= 180 + 10 \times 6 \times (0.231)^2 = 180 + 3.20 = 183.20 \text{ m}^4$$

Now I_{G_2} = M.O.I. of ∆*ABD* in Fig. 3.11 about $G_2 = \frac{bd^3}{36}$

$$= \frac{(16-10) \times 6^3}{36} = 36 \text{ m}^4$$

The distance between the C.G. of triangle and C.G. of trapezoidal

$$= (2.769 - 2.0) = 0.769$$

∴ M.O.I. of the two ∆s about an axis passing through C.G. of trapezoidal

$$= I_{G_2} + \text{Area of triangles} \times (.769)^2$$

$$= 36.0 + \frac{6 \times 6}{2} \times (.769)^2$$

$$= 36.0 + 10.64 = 46.64$$

∴ I_G = M.O.I. of trapezoidal about its C.G.

= M.O.I. of rectangle about the C.G. of trapezoidal

+ M.O.I. of triangles about the C.G. of the trapezoidal

$= 183.20 + 46.64 = 229.84 \text{ m}^4$

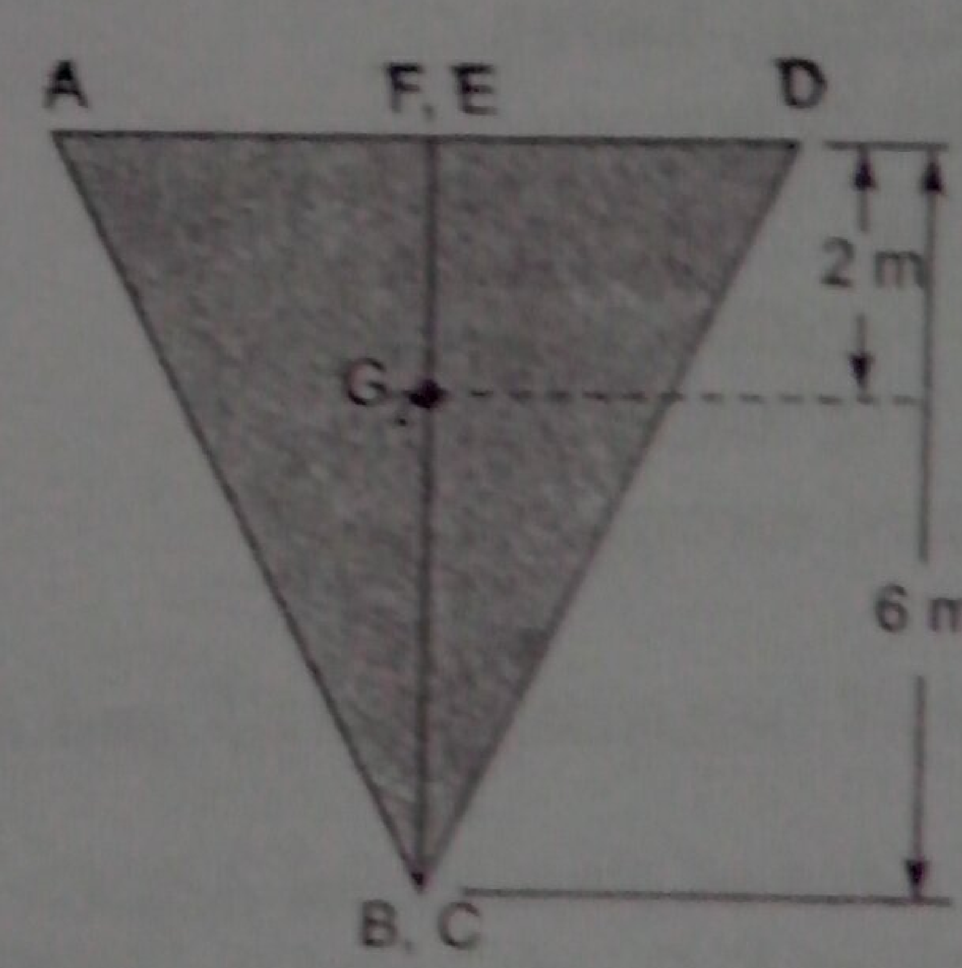


Fig. 3.11

∴ $$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where $A = 78$, $\bar{h} = 2.769$

$$h^* = \frac{229.84}{78 \times 2.769} + 2.769 = 1.064 + 2.769 = \mathbf{3.833 \text{ m. Ans.}}$$

Alternate Method

The distance of the C.G. of the trapezoidal channel from surface *AD* is given by (refer to Table 3.1 on page 71)

$$x = \frac{(2a+b)}{(a+b)} \times \frac{h}{3}$$

$$= \frac{(2 \times 10 + 16)}{(10+16)} \times \frac{6}{3} \qquad (\because \; a = 10, b = 16 \text{ and } h = 6)$$

$$= \frac{36}{26} \times 2 = 2.769 \text{ m}$$

This is also equal to the distance of the C.G. of the trapezoidal from free surface of water.

$$\bar{h} = 2.769 \text{ m}$$

$\therefore$ Total pressure, $F = \rho g A \bar{h}$ $\qquad (\because \; A = 78)$

$$= 1000 \times 9.81 \times 78 \times 2.769 \text{ N} = \mathbf{2118783 \text{ N. Ans.}}$$

Centre of Pressure, $(h^*) = \dfrac{I_G}{A\bar{h}} + \bar{h}$

Now I_G from Table 3.1 is given by,

$$I_G = \frac{(a^2 + 4ab + b^2)}{36(a+b)} \times h^3 = \frac{(10^2 + 4 \times 10 \times 16 + 16^2)}{36(10+16)} \times 6^3$$

$$= \frac{(100 + 640 + 256)}{36 \times 26} \times 216 = 229.846 \text{ m}^4$$

$$\therefore \quad h^* = \frac{229.846}{78 \times 2.769} + 2.769 \qquad (\because \; A = 78 \text{ m}^2)$$

$$= \mathbf{3.833 \text{ m. Ans.}}$$

Problem 3.9 *A trapezoidal channel 2 m wide at the bottom and 1 m deep has side slopes 1 : 1. Determine :*

(i) the total pressure, and

(ii) the centre of pressure on the vertical gate closing the channel when it is full of water.

Solution. Given :

Width at bottom $= 2$ m

Depth, $d = 1$ m

Side slopes $= 1 : 1$

$\therefore$ Top width, $AD = 2 + 1 + 1 = 4$ m

Area of rectangle $FBEC$, $A_1 = 2 \times 1 = 2 \text{ m}^2$

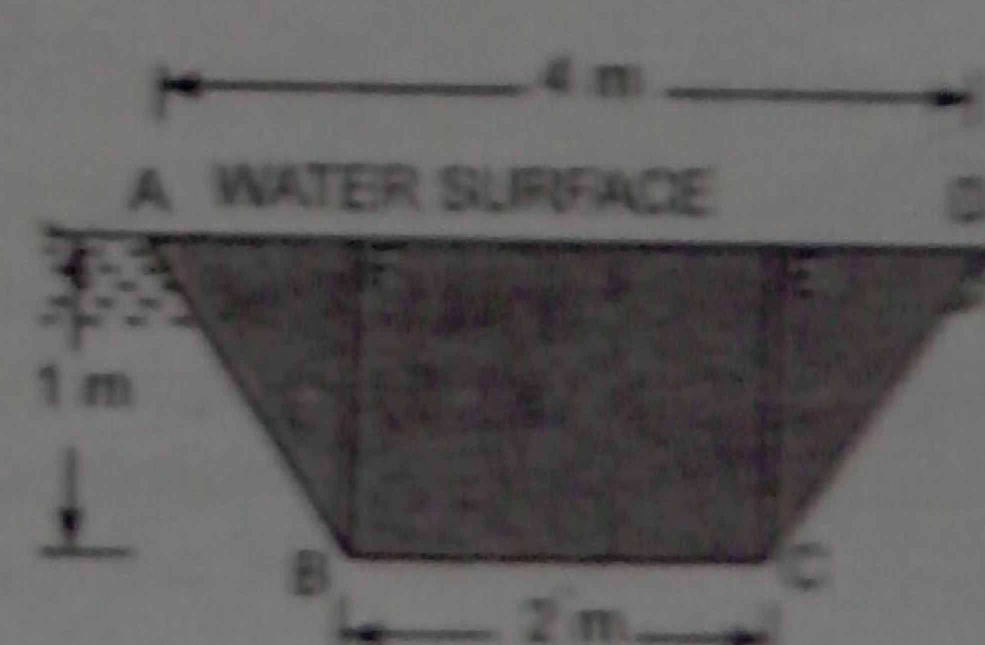


Fig. 3.12

Area of two triangles ABF and ECD, $A_2 = \dfrac{(4-2)}{2} \times 1 = 1 \text{ m}^2$

$\therefore$ Area of trapezoidal $ABCD$, $A = A_1 + A_2 = 2 + 1 = 3 \text{ m}^2$

Depth of C.G. of rectangle $FBEC$ from water surface,

$$\bar{h}_1 = \frac{1}{2} = 0.5 \text{ m}$$

Depth of C.G. of two triangles *ABF* and *ECD* from water surface,

$$\bar{h}_2 = \frac{1}{3} \times 1 = \frac{1}{3} \text{ m}$$

∴ Depth of C.G. of trapezoidal *ABCD* from free surface of water

$$\bar{h} = \frac{A_1 \times \bar{h}_1 + A_2 \times \bar{h}_2}{(A_1 + A_2)} = \frac{2 \times 0.5 + 1 \times 0.33333}{(2+1)} = .44444$$

(*i*) **Total Pressure (F).** Total pressure *F* is given by

$$F = \rho g A \bar{h}$$
$$= 1000 \times 9.81 \times 3.0 \times 0.44444 = \mathbf{13079.9 \text{ N. Ans.}}$$

(*ii*) **Centre of Pressure (h*).** M.O.I. of rectangle *FBCE* about its C.G.,

$$I_{G_1} = \frac{bd^3}{12} = \frac{2 \times 1^3}{12} = \frac{1}{6} \text{ m}^4$$

M.O.I. of *FBCE* about an axis passing through the C.G. of trapezoidal

or
$$I_{G_1}^* = I_{G_1} + A_1 \times [\text{Distance between C.G. of rectangle and C.G. of trapezoidal}]^2$$

$$= \frac{1}{6} + 2 \times [\bar{h}_1 - \bar{h}]^2$$

$$= \frac{1}{6} + 2 \times [0.5 - .4444]^2 = .1666 + .006182 = 0.1727$$

M.O.I. of the two triangles *ABF* and *ECD* about their C.G.,

$$I_{G_2} = \frac{bd^3}{36} = \frac{(1+1) \times 1^3}{36} = \frac{2}{36} = \frac{1}{18} \text{ m}^4.$$

M.O.I. of the two triangles about the C.G. of trapezoidal,

$$I_{G_2}^* = I_{G_1} + A_2 \times [\text{Distance between C.G. of triangles and C.G. of trapezoidal}]^2$$

$$= \frac{1}{18} + 1 \times [\bar{h} - \bar{h}_2]^2 = \frac{1}{18} + 1 \times \left[.4444 - \frac{1}{3}\right]^2$$

$$= \frac{1}{18} + (.1111)^2 = 0.0555 + (.1111)^2$$

$$= .0555 + 0.01234 = 0.06789 \text{ m}^4$$

∴ M.O.I. of the trapezoidal about its C.G.

$$I_G = I_{G_1}^* + I_{G_2}^* = .1727 + .06789 = 0.24059 \text{ m}^4$$

Then centre of pressure (h^*) on the vertical trapezoidal,

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{0.24059}{3 \times .4444} + .4444 = 0.18046 + .4444 = 0.6248$$

$$= \mathbf{0.625 \text{ m. Ans.}}$$

Alternate Method

The distance of the C.G. of the trapezoidal channel from surface AD is given by (refer to Table 3.1 on page 71).

$$x = \frac{(2a+b)}{(a+b)} \times \frac{h}{3} = \frac{(2 \times 2 + 4)}{(2+4)} \times \frac{1}{3} \qquad (\because \ a = 2, b = 4 \text{ and } h = 1)$$

$$= 0.444 \text{ m}$$

$$\therefore \qquad \bar{h} = x = 0.444 \text{ m}$$

$\therefore$ Total pressure,

$$F = \rho g A \bar{h} = 1000 \times 9.81 \times 3.0 \times .444 \qquad (\because \ A = 3.0)$$

$$= \mathbf{13079 \text{ N. Ans.}}$$

Centre of pressure,

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G from Table 3.1 is given by

$$I_G = \frac{(a^2 + 4ab + b^2)}{36(a+b)} \times h^3 = \frac{(2^2 + 4 \times 2 \times 4 + 4^2)}{36(2+4)} \times 1^3 = \frac{52}{36 \times 6} = 0.2407 \text{ m}^4$$

$$\therefore \qquad h^* = \frac{0.2407}{3.0 \times .444} + .444 = \mathbf{0.625 \text{ m. Ans.}}$$

Problem 3.10 *A square aperture in the vertical side of a tank has one diagonal vertical and is completely covered by a plane plate hinged along one of the upper sides of the aperture. The diagonals of the aperture are 2 m long and the tank contains a liquid of specific gravity 1.15. The centre of aperture is 1.5 m below the free surface. Calculate the thrust exerted on the plate by the liquid and position of its centre of pressure.*

Solution. Given : Diagonals of aperture, $AC = BD = 2$ m

$\therefore$ Area of square aperture, A = Area of ΔACB + Area of ΔACD

$$= \frac{AC \times BO}{2} + \frac{AC \times OD}{2} = \frac{2 \times 1}{2} + \frac{2 \times 1}{2} = 1 + 1 = 2.0 \text{ m}^2$$

Sp. gr. of liquid $= 1.15$

$\therefore$ Density of liquid, $\rho = 1.15 \times 1000 = 1150 \text{ kg/m}^3$

Depth of centre of aperture from free surface,

$$\bar{h} = 1.5 \text{ m.}$$

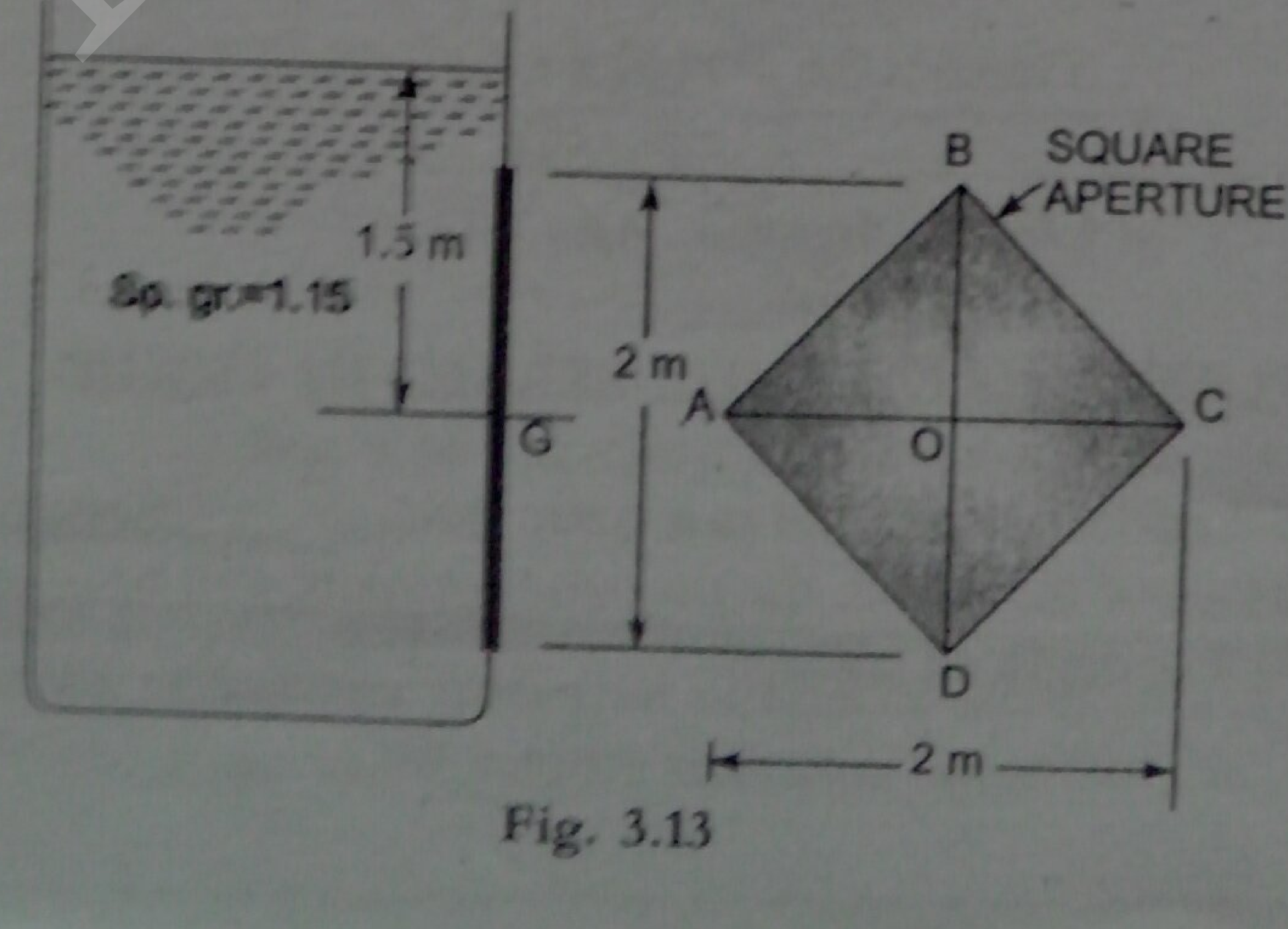


Fig. 3.13

(*i*) The thrust on the plate is given by

$$F = \rho g A \bar{h} = 1150 \times 9.81 \times 2 \times 1.5 = 33844.5.\ \text{Ans.}$$

(*ii*) Centre of pressure (h^*) is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. of $ABCD$ about diagonal AC

= M.O.I. of triangle ABC about AC + M.O.I. of triangle ACD about AC

$$= \frac{AC \times OB^3}{12} + \frac{AC \times OD^3}{12} \qquad \left(\because \text{ M.O.I. of a triangle about its base} = \frac{bh^3}{12}\right)$$

$$= \frac{2 \times 1^3}{12} + \frac{2 \times 1^3}{12} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}\ \text{m}^4$$

$$\therefore \qquad h^* = \frac{\frac{1}{3}}{2 \times 1.5} + 1.5 = \frac{1}{3 \times 2 \times 1.5} + 1.5 = 1.611\ \text{m. Ans.}$$

Problem 3.11 *A tank contains water upto a height of 0.5 m above the base. An immiscible liquid of sp. gr. 0.8 is filled on the top of water upto 1 m height. Calculate :*

(*i*) *total pressure on one side of the tank,*

(*ii*) *the position of centre of pressure for one side of the tank, which is 2 m wide.*

Solution. Given :

Depth of water $= 0.5$ m

Depth of liquid $= 1$ m

Sp. gr. of liquid $= 0.8$

Density of liquid, $\rho_1 = 0.8 \times 1000 = 800$ kg/m^3

Density of water, $\rho_2 = 1000$ kg/m^3

Width of tank $= 2$ m

(*i*) **Total pressure on one side is calculated by drawing pressure diagram, which is shown in Fig. 3.14.**

Intensity of pressure on top, $p_A = 0$

Intensity of pressure on D (or DE), $p_D = \rho_1 g . h_1$

$$= 800 \times 9.81 \times 1.0 = 7848\ \text{N/m}^2$$

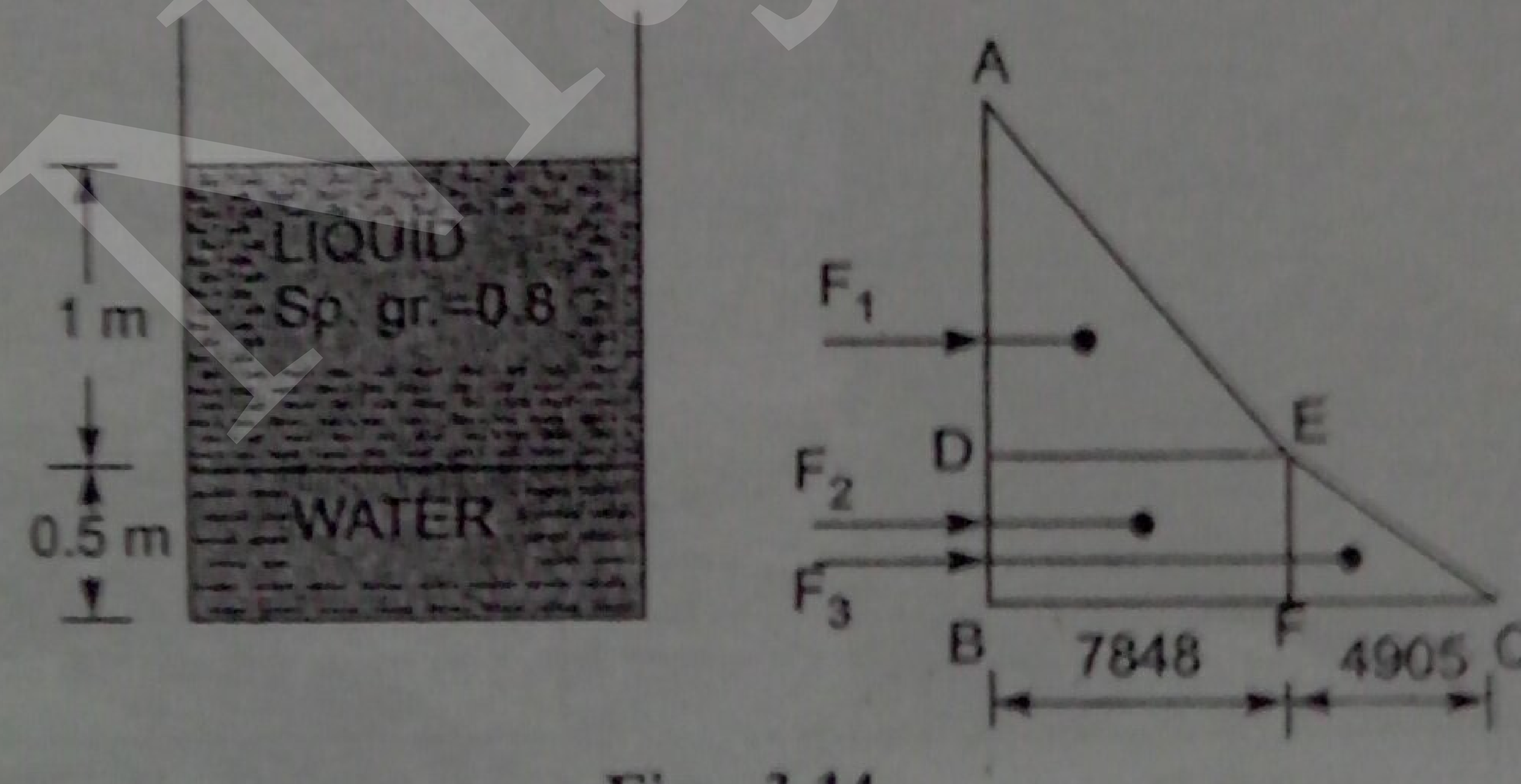


Fig. 3.14

Intensity of pressure on base (or BC), $p_B = \rho_1 g h_1 + \rho_2 g \times 0.5$

$$= 7848 + 1000 \times 9.81 \times 0.5 = 7848 + 4905 = 12753\ \text{N/m}^2$$

Now force F_1 = Area of ΔADE × Width of tank

$$= \frac{1}{2} \times AD \times DE \times 2.0 = \frac{1}{2} \times 1 \times 7848 \times 2.0 = 7848\ \text{N}$$

Force F_2 = Area of rectangle $DBFE$ × Width of tank

$= 0.5 \times 7848 \times 2 = 7848$ N

F_3 = Area of ΔEFC × Width of tank

$$= \frac{1}{2} \times EF \times FC \times 2.0 = \frac{1}{2} \times 0.5 \times 4905 \times 2.0 = 2452.5 \text{ N}$$

∴ Total pressure, $F = F_1 + F_2 + F_3$

$= 7848 + 7848 + 2452.5 =$ **18148.5 N. Ans.**

(*ii*) **Centre of Pressure (h*).** Taking the moments of all force about A, we get

$$F \times h^* = F_1 \times \frac{2}{3} AD + F_2 \left(AD + \frac{1}{2} BD\right) + F_3\left[AD + \frac{2}{3} BD\right]$$

$$18148.5 \times h^* = 7848 \times \frac{2}{3} \times 1 + 7848 \left(1.0 + \frac{0.5}{2}\right) + 2452.5 \left(1.0 + \frac{2}{3} \times .5\right)$$

$$= 5232 + 9810 + 3270 = 18312$$

$$\therefore \quad h^* = \frac{18312}{18148.5} = \textbf{1.009 m from top. Ans.}$$

Problem 3.12 *A cubical tank has sides of 1.5 m. It contains water for the lower 0.6 m depth. The upper remaining part is filled with oil of specific gravity 0.9. Calculate for one vertical side of the tank:*

(a) total pressure, and

(b) position of centre of pressure.

Solution. Given :

Cubical tank of sides 1.5 m means the dimensions of the tank are 1.5 m × 1.5 m × 1.5 m.

Depth of water $= 0.6$ m

Depth of liquid $= 1.5 - 0.6 = 0.9$ m

Sp. gr. of liquid $= 0.9$

Density of liquid, $\rho_1 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

Density of water, $\rho_2 = 1000 \text{ kg/m}^3$

(*a*) **Total pressure** on one vertical side is calculated by drawing pressure diagram, which is shown in Fig. 3.15.

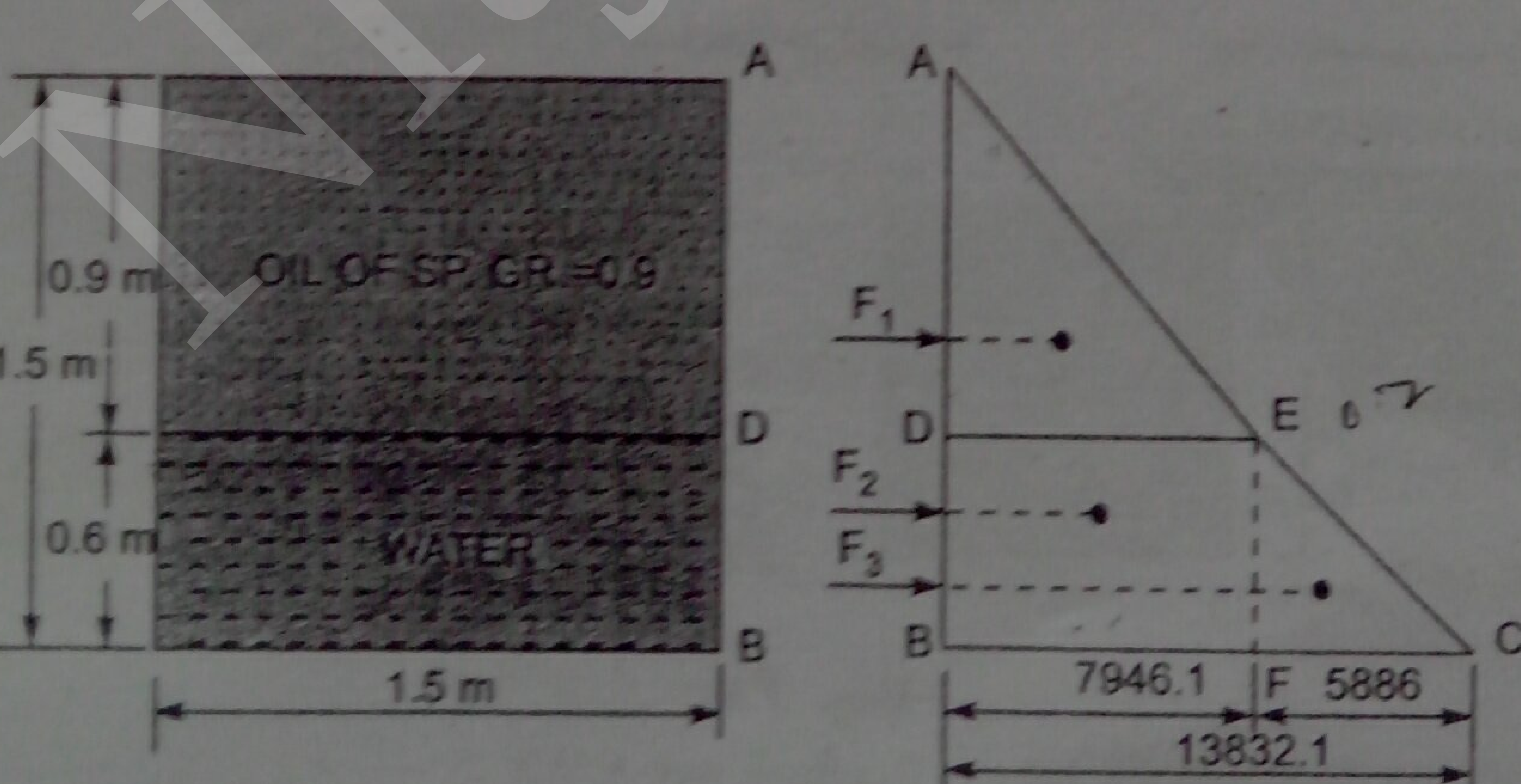


Fig. 3.15

Intensity of pressure at A, $p_A = 0$

Intensity of pressure at D, $p_D = \rho_1 g \times h = 900 \times 9.81 \times 0.9 = 7946.1\ \text{N/m}^2$

Intensity of pressure at B, $p_B = \rho_1 g h_1 + \rho_2 g h_2 = 900 \times 9.81 \times 0.9 + 1000 \times 9.81 \times 0.6$

$$= 7946.1 + 5886 = 13832.1\ \text{N/m}^2$$

Hence in pressure diagram :

$$DE = 7946.1\ \text{N/m}^2,\ BC = 13832.1\ \text{N/m}^2,\ FC = 5886\ \text{N/m}^2$$

The pressure diagram is split into triangle ADE, rectangle $BDEF$ and triangle EFC. The total pressure force consists of the following components :

(i) Force
$$F_1 = \text{Area of triangle } ADE \times \text{Width of tank}$$
$$= \left(\tfrac{1}{2} \times AD \times DE\right) \times 1.5 \qquad (\because \ \text{Width} = 1.5\ \text{m})$$
$$= \left(\tfrac{1}{2} \times 0.9 \times 7946.1\right) \times 1.5\ \text{N} = 5363.6\ \text{N}$$

This force will be acting at the C.G. of the triangle ADE, *i.e.*, at a distance of $\frac{2}{3} \times 0.9 = 0.6$ m below A

(ii) Force
$$F_2 = \text{Area of rectangle } BDEF \times \text{Width of tank}$$
$$= (BD \times DE) \times 1.5 = (0.6 \times 7946.1) \times 1.5 = 7151.5$$

This force will be acting at the C.G. of the rectangle $BDEF$ *i.e.*, at a distance of $0.9 + \frac{0.6}{2} = 1.2$ m below A.

(iii) Force
$$F_3 = \text{Area of triangle } EFC \times \text{Width of tank}$$
$$= \left(\tfrac{1}{2} \times EF \times FC\right) \times 1.5 = \left(\tfrac{1}{2} \times 0.6 \times 5886\right) \times 1.5 = 2648.7\ \text{N}$$

This force will be acting at the C.G. of the triangle EFC, *i.e.*, at a distance of $0.9 + \frac{2}{3} \times 0.6 = 1.30$ m below A.

$\therefore$ **Total pressure force on one vertical face of the tank,**
$$F = F_1 + F_2 + F_3$$
$$= 5363.6 + 7151.5 + 2648.7 = \mathbf{15163.8\ N.\ Ans.}$$

(b) **Position of centre of pressure.**

Let the total force F is acting at a depth of h^* from the free surface of liquid, *i.e.*, from A.

Taking the moments of all forces about A, we get
$$F \times h^* = F_1 \times 0.6 + F_2 \times 1.2 + F_3 \times 1.3$$
or
$$h^* = \frac{F_1 \times 0.6 + F_2 \times 1.2 + F_3 \times 1.3}{F}$$
$$= \frac{5363.6 \times 0.6 + 7151.5 \times 1.2 + 2648.7 \times 1.3}{15163.8}$$
$$= \mathbf{1.005\ m\ from\ A.\ Ans.}$$

▶ 3.4 HORIZONTAL PLANE SURFACE SUBMERGED IN LIQUID

Consider a plane horizontal surface immersed in a static fluid. As every point of the surface is at the same depth from the free surface of the liquid, the pressure intensity will be equal on the entire surface and equal to, $p = \rho g h$, where h is depth of surface.

Let

A = Total area of surface

Then total force, F, on the surface

$$= p \times \text{Area} = \rho g \times h \times A = \rho g A \bar{h}$$

where $\bar{h}$ = Depth of C.G. from free surface of liquid = h

also h^* = Depth of centre of pressure from free surface = h.

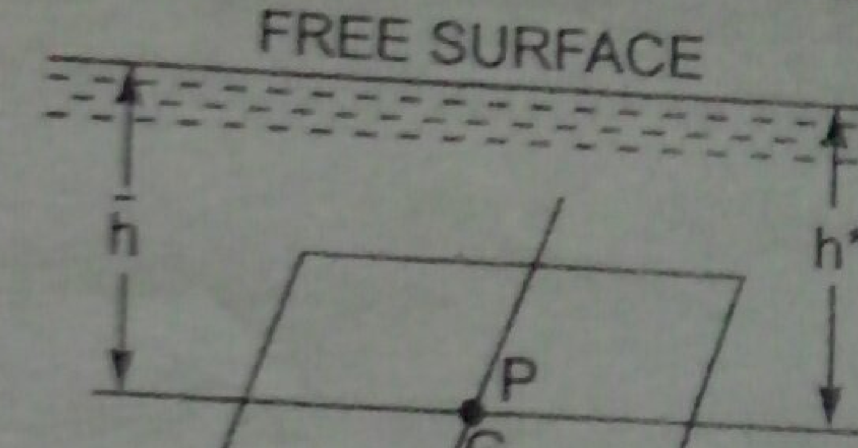


Fig. 3.16

Problem 3.13 *Fig. 3.17 shows a tank full of water. Find :*

(i) Total pressure on the bottom of tank.

(ii) Weight of water in the tank.

(iii) Hydrostatic paradox between the results of (i) and (ii). Width of tank is 2 m.

Solution. Given :

Depth of water on bottom of tank

$$h_1 = 3 + 0.6 = 3.6 \text{ m}$$

Width of tank $= 2$ m

Length of tank at bottom $= 4$ m

∴ Area at the bottom, $A = 4 \times 2 = 8 \text{ m}^2$

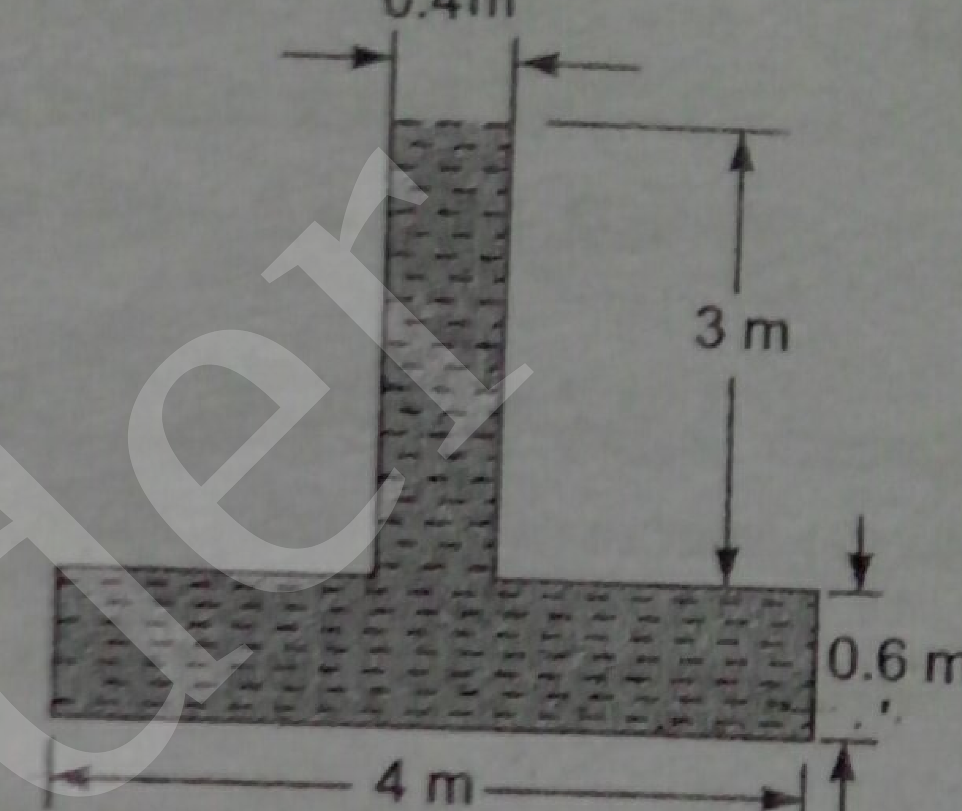


Fig. 3.17

(*i*) Total pressure F, on the bottom is

$$F = \rho g A \bar{h} = 1000 \times 9.81 \times 8 \times 3.6$$

$$= 282528 \text{ N. Ans.}$$

(*ii*) Weight of water in tank $= \rho g \times$ Volume of tank

$$= 1000 \times 9.81 \times [3 \times 0.4 \times 2 + 4 \times .6 \times 2]$$

$$= 1000 \times 9.81 \,[2.4 + 4.8] = 70632 \text{ N. Ans.}$$

(*iii*) From the results of (*i*) and (*ii*), it is observed that the total weight of water in the tank is much less than the total pressure at the bottom of the tank. This is known as Hydrostatic paradox.

▶ 3.5 INCLINED PLANE SURFACE SUBMERGED IN LIQUID

Consider a plane surface of arbitrary shape immersed in a liquid in such a way that the plane of the surface makes an angle θ with the free surface of the liquid as shown in Fig. 3.18.

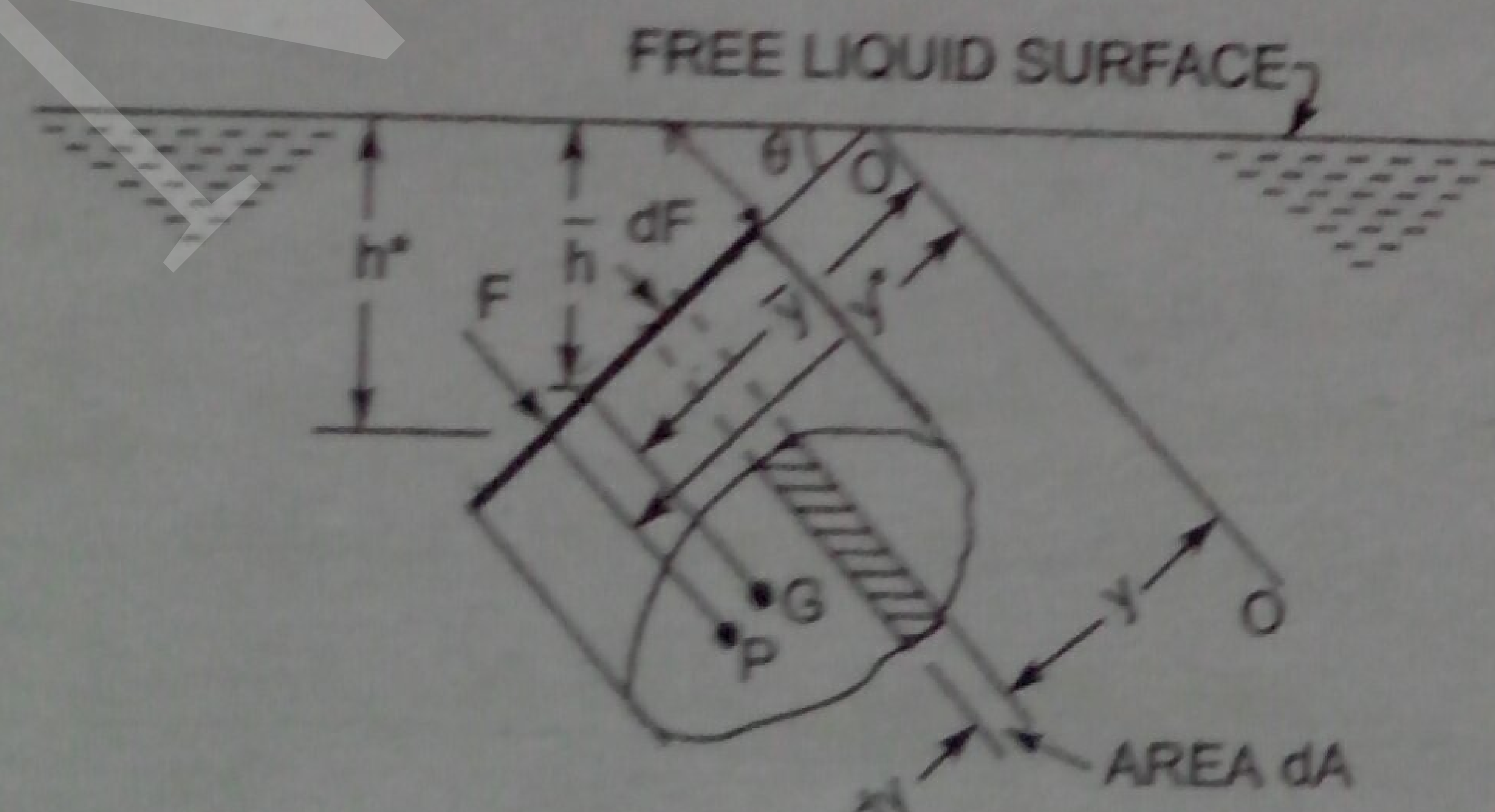


Fig. 3.18 *Inclined immersed surface.*

Let A = Total area of inclined surface

$\bar{h}$ = Depth of C.G. of inclined area from free surface

h^* = Distance of centre of pressure from free surface of liquid

θ = Angle made by the plane of the surface with free liquid surface.

Let the plane of the surface, if produced meet the free liquid surface at O. Then O-O is the axis perpendicular to the plane of the surface.

Let $\bar{y}$ = distance of the C.G. of the inclined surface from O-O

y^* = distance of the centre of pressure from O-O.

Consider a small strip of area dA at a depth 'h' from free surface and at a distance y from the axis O-O as shown in Fig. 3.18.

Pressure intensity on the strip, $p = \rho g h$

$\therefore$ Pressure force, dF, on the strip, $dF = p \times \text{Area of strip} = \rho g h \times dA$

Total pressure force on the whole area, $F = \int dF = \int \rho g h dA$

But from Fig. 3.18,

$$\frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*} = \sin\theta$$

$\therefore$

$$h = y \sin\theta$$

$\therefore$

$$F = \int \rho g \times y \times \sin\theta \times dA = \rho g \sin\theta \int y dA$$

But

$$\int y dA = A\bar{y}$$

where $\bar{y}$ = Distance of C.G. from axis O-O

$\therefore$

$$F = \rho g \sin\theta \, \bar{y} \times A$$

$$= \rho g A \bar{h} \qquad (\because \bar{h} = \bar{y} \sin\theta) \quad ...(3.6)$$

Centre of Pressure (h^*)

Pressure force on the strip, $dF = \rho g h dA$

$$= \rho g y \sin\theta \, dA \qquad [h = y \sin\theta]$$

Moment of the force, dF, about axis O-O

$$= dF \times y = \rho g y \sin\theta \, dA \times y = \rho g \sin\theta \, y^2 dA$$

Sum of moments of all such forces about O-O

$$= \int \rho g \sin\theta \, y^2 dA = \rho g \sin\theta \int y^2 dA$$

But $\int y^2 dA$ = M.O.I. of the surface about O-O = I_0

$\therefore$ Sum of moments of all forces about O-O = $\rho g \sin\theta \, I_0$ ...(3.7)

Moment of the total force, F, about O-O is also given by

$$= F \times y^* \quad ...(3.8)$$

where y^* = Distance of centre of pressure from O-O.

Equating the two values given by equations (3.7) and (3.8)

$$F \times y^* = \rho g \sin\theta \, I_0$$

or

$$y^* = \frac{\rho g \sin\theta \, I_0}{F} \quad ...(3.9)$$

Now

$$y^* = \frac{h^*}{\sin\theta}, \; F = \rho g A \bar{h}$$

and I_0 by the theorem of parallel axis $= I_G + A\bar{y}^2$.

Substituting these values in equation (3.9), we get

$$\frac{h^*}{\sin\theta} = \frac{\rho g \sin\theta}{\rho g A\bar{h}}\left[I_G + A\bar{y}^2\right]$$

$$\therefore \qquad h^* = \frac{\sin^2\theta}{A\bar{h}}\left[I_G + A\bar{y}^2\right]$$

But $\qquad \dfrac{\bar{h}}{\bar{y}} = \sin\theta \quad \text{or} \quad \bar{y} = \dfrac{\bar{h}}{\sin\theta}$

$$\therefore \qquad h^* = \frac{\sin^2\theta}{A\bar{h}}\left[I_G + A \times \frac{\bar{h}^2}{\sin^2\theta}\right]$$

or
$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h} \qquad \text{...(3.10)}$$

If $\theta = 90°$, equation (3.10) becomes same as equation (3.5) which is applicable to vertically plane submerged surfaces.

In equation (3.10), I_G = M.O.I. of inclined surfaces about an axis passing through G and parallel to O-O.

Problem 3.14 (a) *A rectangular plane surface 2 m wide and 3 m deep lies in water in such a way that its plane makes an angle of 30° with the free surface of water. Determine the total pressure and position of centre of pressure when the upper edge is 1.5 m below the free water surface.*

Solution. Given :

Width of plane surface, $b = 2$ m

Depth, $d = 3$ m

Angle, $\theta = 30°$

Distance of upper edge from free water surface = 1.5 m

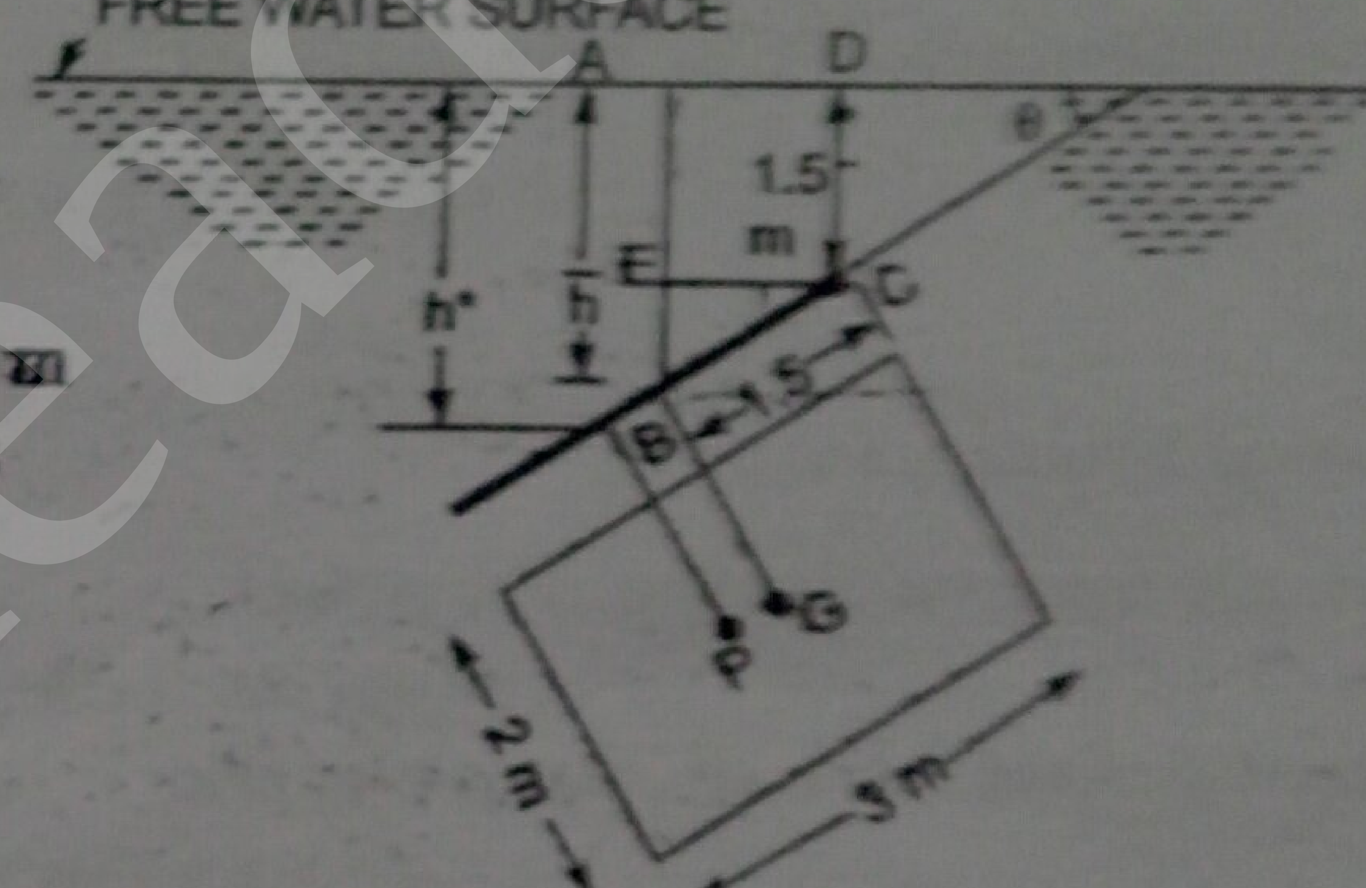


Fig. 3.19

(*i*) Total pressure force is given by equation (3.6) as

$$F = \rho g A\bar{h}$$

where $\rho = 1000$ kg/m^3

$A = b \times d = 3 \times 2 = 6$ m^2

$\therefore \quad \bar{h}$ = Depth of C.G. from free water surface
$= 1.5 + 1.5 \sin 30°$

$[\because \quad \bar{h} = AE + EB = 1.5 + BC \sin 30° = 1.5 + 1.5 \sin 30°]$

$= 1.5 + 1.5 \times \frac{1}{2} = 2.25$ m

$\therefore \qquad F = 1000 \times 9.81 \times 6 \times 2.25 =$ **132435 N. Ans.**

(*ii*) Centre of pressure (h*)

Using equation (3.10), we have

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}, \quad \text{where } I_G = \frac{bd^3}{12} = \frac{2\times 3^3}{12} = 4.5 \text{ m}^4$$

$$\therefore \qquad h^* = \frac{4.5 \times \sin^2 30°}{6 \times 2.25} + 2.25 = \frac{4.5 \times \frac{1}{4}}{6 \times 2.25} + 2.25$$

$= 0.0833 + 2.25 =$ **2.3333 m. Ans.**

Problem 3.14 (*b*) *A rectangular plane surface 3 m wide and 4 m deep lies in water in such a way that its plane makes an angle of 30° with the free surface of water. Determine the total pressure force and position of centre of pressure, when the upper edge is 2 m below the free surface.*

Solution. Given :

$$b = 3 \text{ m},\ d = 4 \text{ m},\ \theta = 30°$$

Distance of upper edge from free surface of water = 2 m

(*i*) **Total pressure force** is given by equation (3.6) as

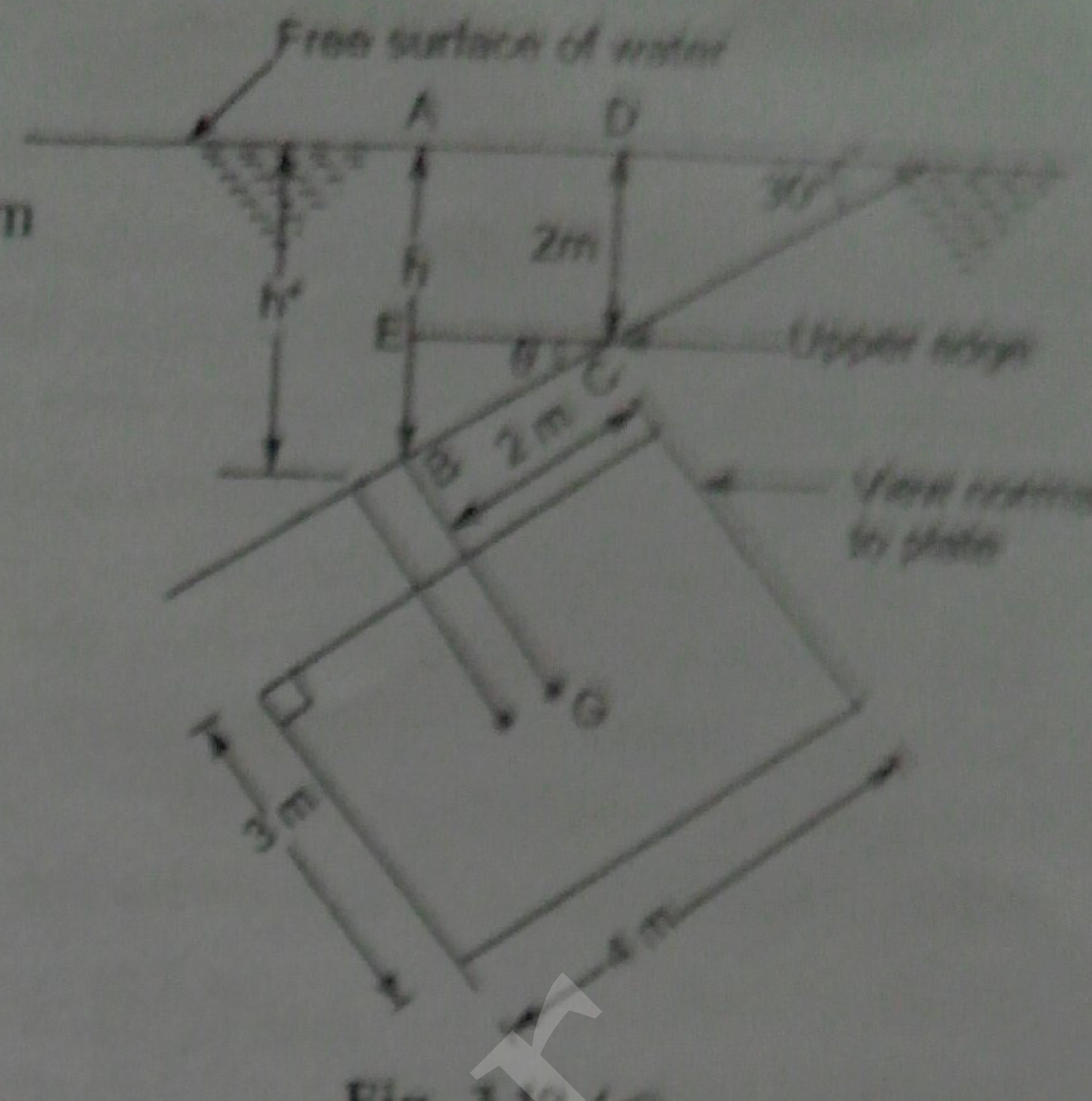


Fig. 3.19 (*a*)

$$F = \rho g A \bar{h}$$

where $\rho = 1000 \text{ kg/m}^3$,

$A = b \times d = 3 \times 4 = 12 \text{ m}^2$

and $\bar{h}$ = Depth of C.G. of plate from free water surface

$= 2 + BE = 2 + BC \sin\theta$

$= 2 + 2 \sin 30° = 2 + 2 \times \frac{1}{2} = 3 \text{ m}$

$\therefore$

$$F = 1000 \times 9.81 \times 12 \times 3 = 353167 \text{ N} = 353.167 \text{ kN. Ans.}$$

(*ii*) **Centre of pressure (h*)**

Using equation (3.10), we have $h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$

where $I_G = \frac{bd^3}{12} = \frac{3 \times 4^3}{12} = 16 \text{ m}^4$

$\therefore$

$$h^* = \frac{16 \times \sin^2 30°}{12 \times 3} + 3 = \frac{16 \times \frac{1}{4}}{36} + 3 = 3.111 \text{ m. Ans.}$$

Problem 3.15 (*a*) *A circular plate 3.0 m diameter is immersed in water in such a way that its greatest and least depth below the free surface are 4 m and 1.5 m respectively. Determine the total pressure on one face of the plate and position of the centre of pressure.*

Solution. Given :

Dia. of plate, $d = 3.0$ m

$\therefore$ Area, $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (3.0)^2 = 7.0685 \text{ m}^2$

Distance $DC = 1.5$ m, $BE = 4$ m

Distance of C.G. from free surface

$= \bar{h} = CD + GC \sin\theta = 1.5 + 1.5 \sin\theta$

But $\sin\theta = \frac{AB}{BC} = \frac{BE - AE}{BC} = \frac{4.0 - DC}{3.0} = \frac{4.0 - 1.5}{3.0}$

$= \frac{2.5}{3.0} = 0.8333$

$\therefore$ $\bar{h} = 1.5 + 1.5 \times .8333 = 1.5 + 1.249 = 2.749$ m

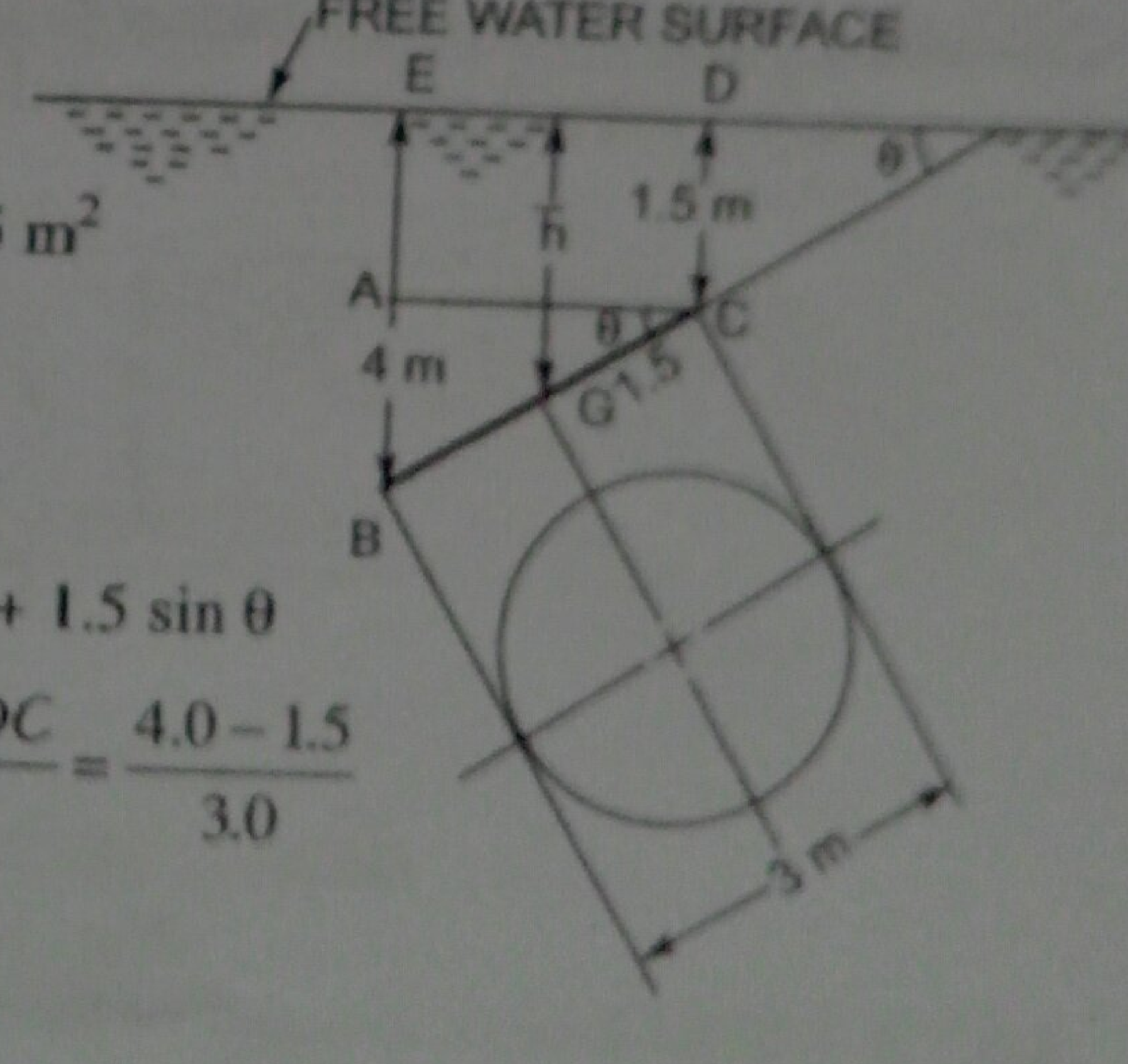


Fig. 3.20

(*i*) **Total pressure (F)**

$$F = \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times 7.0685 \times 2.749 = \textbf{190621 N. Ans.}$$

(*ii*) **Centre of pressure (h*)**

Using equation (3.10), we have $h^* = \dfrac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$

where $I_G = \dfrac{\pi}{64} d^4 = \dfrac{\pi}{64} (3)^4 = 3.976 \text{ m}^4$

$$h^* = \frac{3.976 \times (.8333) \times .8333}{7.0685 \times 2.749} + 2.749 = 0.1420 + 2.749$$

$$= \textbf{2.891 m. Ans.}$$

Problem 3.15 (*b*) *If in the above problem, the given circular plate is having a concentric circular hole of diameter 1.5 m, then calculate the total pressure and position of the centre of pressure on one face of the plate.*

Solution. Given : [Refer to Fig. 3.20 (*a*)]

Dia. of plate, $d = 3.0$ m

$\therefore$ Area of solid plate $= \dfrac{\pi}{4} d^2 = \dfrac{\pi}{4} (3)^2 = 7.0685 \text{ m}^2$

Dia. of hole in the plate, $d_0 = 1.5$ m

$\therefore$ **Area of hole** $= \dfrac{\pi}{4} d_0^2 = \dfrac{\pi}{4} (1.5)^2 = 1.7671 \text{ m}^2$

$\therefore$ **Area of the given plate, A = Area of solid plate − Area of hole**

$= 7.0685 - 1.7671 = 5.3014 \text{ m}^2$

Distance $CD = 1.5$, $BE = 4$ m

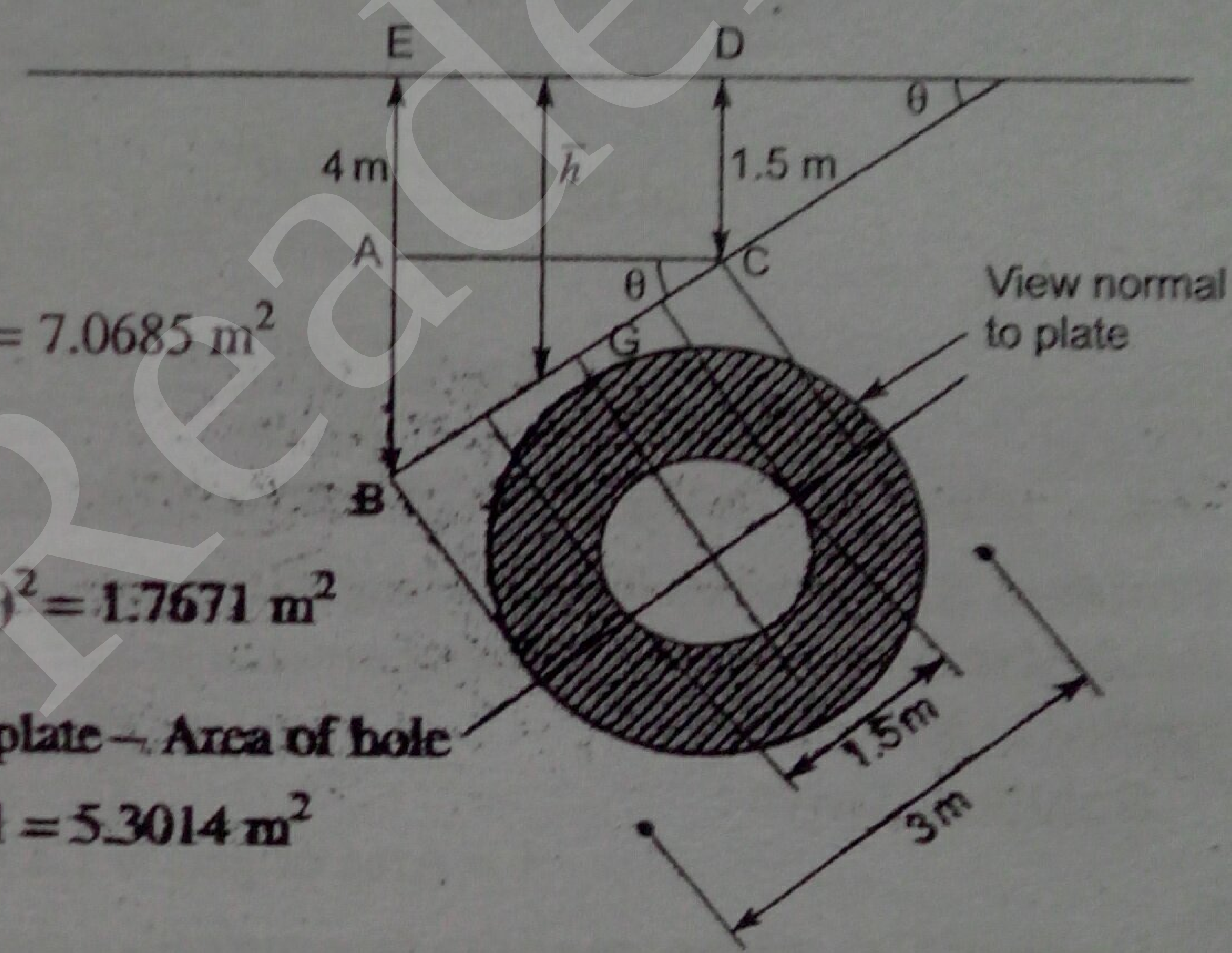


Fig. 3.20 (*a*)

Distance of C.G. from the free surface,

$$\bar{h} = CD + GC \sin\theta$$

$$= 1.5 + 1.5 \sin\theta$$

But $\sin\theta = \dfrac{AB}{BC} = \dfrac{BE - AE}{BC} = \dfrac{4 - 1.5}{3} = \dfrac{2.5}{3}$

$\therefore$ $\bar{h} = 1.5 + 1.5 \times \dfrac{2.5}{3} = 1.5 + 1.25 = 2.75$ m.

(*i*) **Total pressure force (F)**

$$F = \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times 5.3014 \times 2.75$$

$$= 143018 \text{ N} = \textbf{143.018 kN. Ans.}$$

(ii) Position of centre of pressure (h*)

Using equation (3.10), we have

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

where $I_G = \frac{\pi}{64}[d^4 - d_0^4] = \frac{\pi}{64}[3^4 - 1.5^4]\ \text{m}^4$

$A = \frac{\pi}{4}[d^2 - d_0^2] = \frac{\pi}{4}[3^2 - 1.5^2]\ \text{m}^2$

$\sin\theta = \frac{2.5}{3}$ and $\bar{h} = 2.75$

$$\therefore \quad h^* = \frac{\frac{\pi}{64}\left[3^4 - 1.5^4\right] \times \left(\frac{2.5}{3}\right)^2}{\frac{\pi}{4}\left[3^2 - 1.5^2\right] \times 2.75} + 2.75$$

$$= \frac{\frac{1}{16}\left[3^2 + 1.5^2\right] \times \left(\frac{2.5}{3}\right)^2}{2.75} + 2.75 = \frac{1 \times 11.25 \times 6.25}{16 \times 2.75 \times 9} + 2.75$$

$= 0.177 + 2.75 =$ **2.927 m. Ans.**

Problem 3.16 *A circular plate 3 metre diameter is submerged in water as shown in Fig. 3.21. Its greatest and least depths are below the surfaces being 2 metre and 1 metre respectively. Find : (i) the total pressure on front face of the plate, and (ii) the position of centre of pressure.*

Solution. Given :

Dia. of plate, $d = 3.0$ m

$\therefore$ Area, $A = \frac{\pi}{4}(3.0)^2 = 7.0685\ \text{m}^2$

Distance, $DC = 1$ m, $BE = 2$ m

In ΔABC, $\sin\theta = \frac{AB}{AC} = \frac{BE - AE}{BC} = \frac{BE - DC}{BC} = \frac{2.0 - 1.0}{3.0} = \frac{1}{3}$

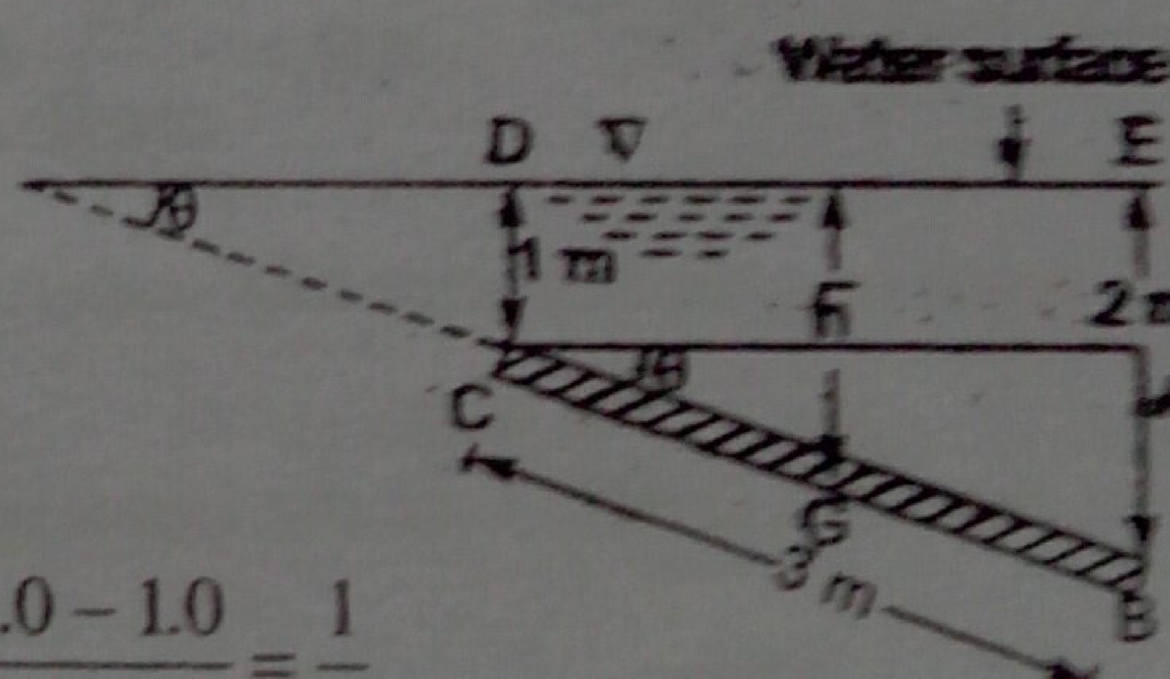


Fig. 3.21

The centre of gravity of the plate is at the middle of BC, *i.e.*, at a distance 1.5 m from C.

The distance of centre of gravity from the free surface of the water is given by

$$\bar{h} = CD + CG \sin\theta = 1.0 + 1.5 \times \frac{1}{3} \qquad \left(\because \sin\theta = \frac{1}{3}\right)$$

$= 1.5$ m.

(*i*) Total pressure on the front face of the plate is given by

$$F = \rho g A \bar{h}$$

$= 1000 \times 9.81 \times 7.0685 \times 1.5 =$ **104013 N. Ans.**

(*ii*) Let the distance of the centre of pressure from the free surface of the water be h^*. Then using equation (3.10), we have

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

where $I_G = \frac{\pi}{64} d^4 = \frac{\pi}{64} (3)^4$, $A = \frac{\pi}{4} d^2$, $\bar{h} = 1.5$ m and $\sin\theta = \frac{1}{3}$

Substituting the values, we get

$$h^* = \frac{\frac{\pi}{64} d^4 \times \left(\frac{1}{3}\right)^2}{\frac{\pi}{4} d^2 \times 1.5} + 1.5 = \frac{d^2}{16} \times \frac{1}{9 \times 1.5} + 1.5$$

$$= \frac{3^2}{16 \times 9 \times 1.5} + 1.5 = .0416 + 1.5 = \mathbf{1.5416\ m.\ Ans.}$$

Problem 3.17 *A rectangular gate 5 m × 2 m is hinged at its base and inclined at 60° to the horizontal as shown in Fig. 3.22. To keep the gate in a stable position, a counter weight of 5000 kgf is attached at the upper end of the gate as shown in figure. Find the depth of water at which the gate begins to fall. Neglect the weight of the gate and friction at the hinge and pulley.*

Solution. Given :

Length of gate $= 5$ m

Width of gate $= 2$ m

$\theta = 60°$

Weight, $W = 5000$ kgf
$= 5000 \times 9.81$ N
$= 49050$ N $\quad (\because 1 \text{ kgf} = 9.81 \text{ N})$

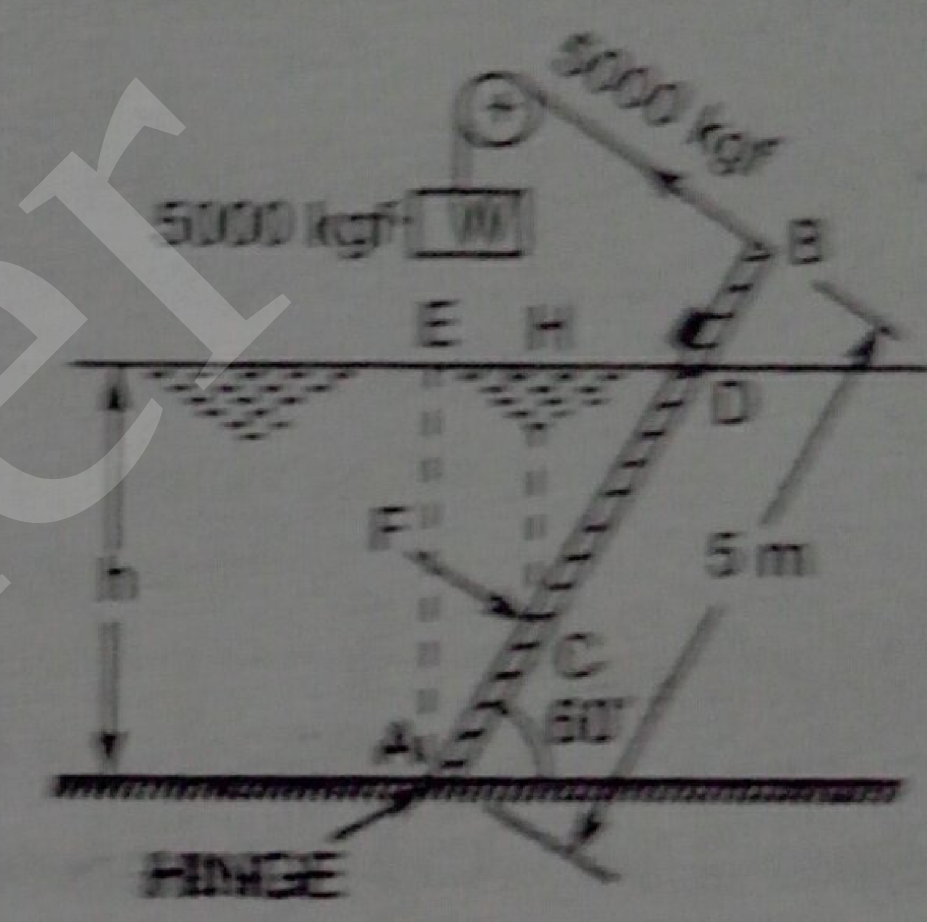


Fig. 3.22

As the pulley is frictionless, the force acting at $B = 49050$ N. First find the total force F acting on the gate AB for a given depth of water.

From figure, $$AD = \frac{AE}{\sin\theta} = \frac{h}{\sin 60°} = \frac{5}{\sqrt{3}/2} = \frac{2h}{\sqrt{3}}$$

$\therefore$ Area of gate immersed in water, $A = AD \times \text{Width} \times \frac{2h}{\sqrt{3}} \times 2 = \frac{4h}{\sqrt{3}}$ m²

Also depth of the C.G. of the immersed area $= \bar{h} = \frac{h}{2} = 0.5\,h$

$\therefore$ Total force F is given by $F = \rho g A \bar{h} = 1000 \times 9.81 \times \frac{4h}{\sqrt{3}} \times \frac{h}{\sqrt{2}} = \frac{19620}{\sqrt{3}} h^2$N

The centre of pressure of the immersed surface, h^* is given by

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. of the immersed area

$$= \frac{b \times (AD)^3}{12} = \frac{2}{12} \times \left(\frac{2h}{\sqrt{3}}\right)^3 \qquad \left\{\because AD = \frac{2h}{\sqrt{3}}\right\}$$

$$= \frac{16h^3}{12 \times 3 \times \sqrt{3}} = \frac{4h^3}{9 \times \sqrt{3}} \text{ m}^4$$

$$\therefore \quad h^* = \frac{4h^3}{9\times\sqrt{3}} \times \frac{\left(\frac{\sqrt{3}}{2}\right)^2}{\frac{4h}{\sqrt{3}}\times\frac{h}{2}} + \frac{h}{2} = \frac{3h^3}{18h^2} + \frac{h}{2} = \frac{h}{6} + \frac{h}{2} = \frac{h+3h}{6} = \frac{2h}{3}$$

Now in the ΔCHD, $\quad CH = h^* = \frac{2h}{3}$, $\angle CDH = 60°$

$$\therefore \quad \frac{CH}{CD} = \sin 60°$$

$$\therefore \quad CD = \frac{CH}{\sin 60°} = \frac{h^*}{\sin 60°} = \frac{2h}{3\times\frac{\sqrt{3}}{2}} = \frac{4h}{3\times\sqrt{3}}$$

$$\therefore \quad AC = AD - CD = \frac{2h}{\sqrt{3}} - \frac{4h}{3\sqrt{3}} = \frac{6h-4h}{3\sqrt{3}} = \frac{2h}{3\sqrt{3}} \text{ m}$$

Taking the moments about hinge, we get

$$49050 \times 5.0 = F \times AC = \frac{19620}{\sqrt{3}} h^2 \times \frac{2h}{3\sqrt{3}}$$

or

$$245250 = \frac{39240\, h^3}{3\times 3}$$

$$\therefore \quad h^3 = \frac{9\times 245250}{39240} = 56.25$$

$$\therefore \quad h = (56.25)^{1/3} = 3.83 \text{ m. Ans.}$$

Problem 3.18 *An inclined rectangular sluice gate AB, 1.2 m by 5 m size as shown in Fig. 3.23 is installed to control the discharge of water. The end A is hinged. Determine the force normal to the gate applied at B to open it.*

Solution. Given :

$$A = \text{Area of gate} = 1.2 \times 5.0 = 6.0 \text{ m}^2$$

Depth of C.G. of the gate from free surface of the water $= \bar{h}$

$$= DG = BC - BE$$

$$= 5.0 - BG \sin 45°$$

$$= 5.0 - 0.6 \times \frac{1}{\sqrt{2}} = 4.576 \text{ m}$$

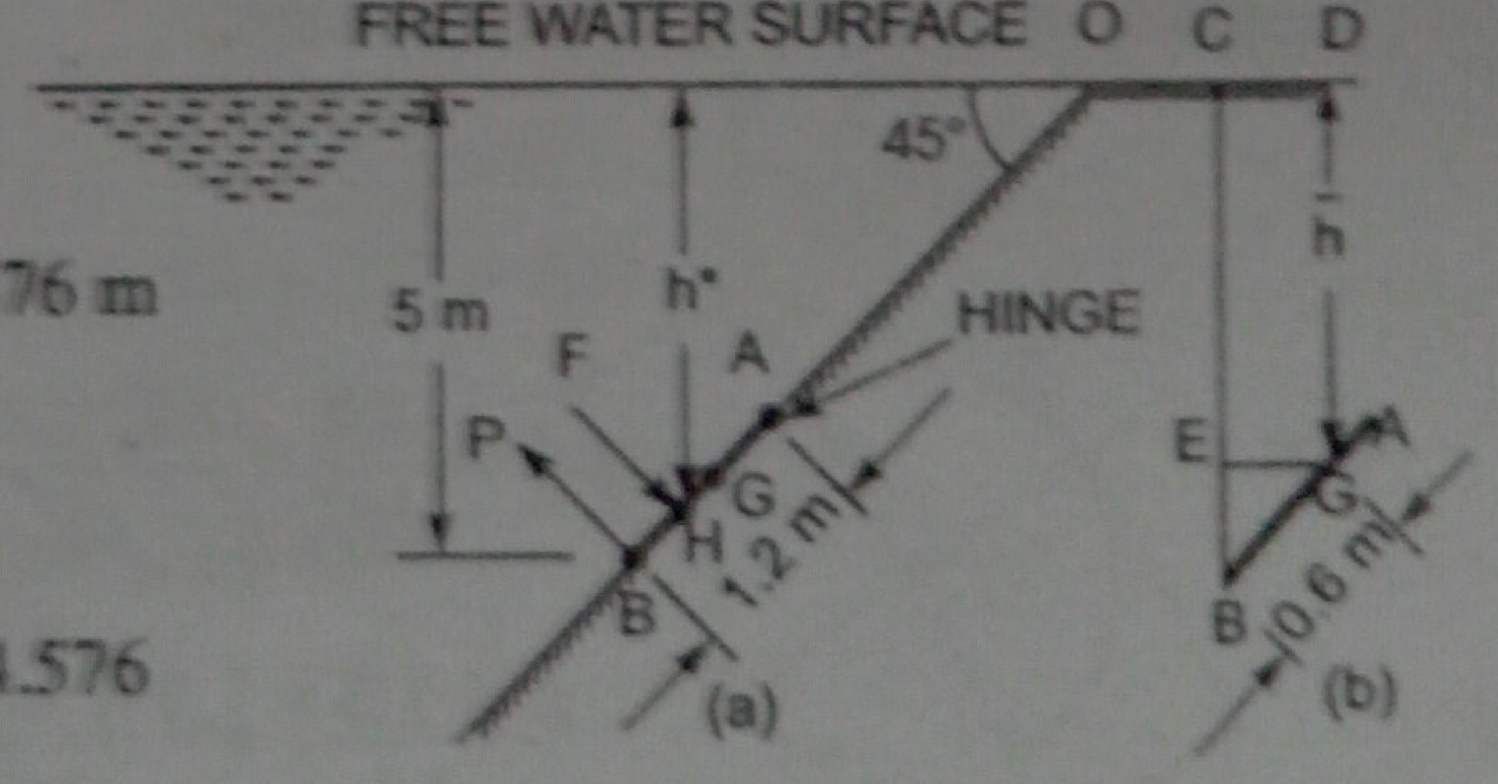


Fig. 3.23

The total pressure force (F) acting on the gate,

$$F = \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times 6.0 \times 4.576$$

$$= 269343 \text{ N}$$

This force is acting at H, where the depth of H from free surface is given by

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. of gate $= \dfrac{bd^3}{12} = \dfrac{5.0 \times 1.2^3}{12} = 0.72$ m

$\therefore$ Depth of centre of pressure $h^* = \dfrac{0.72 \times \sin^2 45°}{6 \times 4.576} + 4.576 = .013 + 4.576 = 4.589$ m

But from Fig. 3.23 (*a*), $\dfrac{h^*}{OH} = \sin 45°$

$\therefore$ Distance, $OH = \dfrac{h^*}{\sin 45°} = \dfrac{4.589}{\frac{1}{\sqrt{2}}} = 4.589 \times \sqrt{2} = 6.489$ m

Distance, $BO = \dfrac{5}{\sin 45°} = 5 \times \sqrt{2} = 7.071$ m

Distance, $BH = BO - OH = 7.071 - 6.489 = 0.582$ m

$\therefore$ Distance $AH = AB - BH = 1.2 - 0.582 = 0.618$ m

Taking the moments about the hinge *A*

$$P \times AB = F \times (AH)$$

where *P* is the force normal to the gate applied at *B*

$$\therefore \quad P \times 1.2 = 269343 \times 0.618$$

$$\therefore \quad P = \frac{269343 \times 0.618}{1.2} = \mathbf{138708\ N.\ Ans.}$$

Problem 3.19 ***A gate supporting water is shown in Fig. 3.24. Find the height h of the water so that the gate tips about the hinge. Take the width of the gate as unity.***

Solution. Given : $\theta = 60°$

Distance, $AC = \dfrac{h}{\sin 60°} = \dfrac{2h}{\sqrt{3}}$

where h = Depth of water.

The gate will start tipping about hinge *B* if the resultant pressure force acts at *B*. If the resultant pressure force passes through a point which is lying from *B* to *C* anywhere on the gate, the gate will tip over the hinge. Hence limiting case is when the resultant force passes through *B*. But the resultant force passes through the centre of pressure. Hence for the given position, point *B* becomes the centre of pressure. Hence depth of centre of pressure,

$$h^* = (h - 3) \text{ m}$$

But h^* is also given by $= \dfrac{I_G \sin^2 \theta}{A\bar{h}} + \bar{h}$

Taking width of gate unity. Then

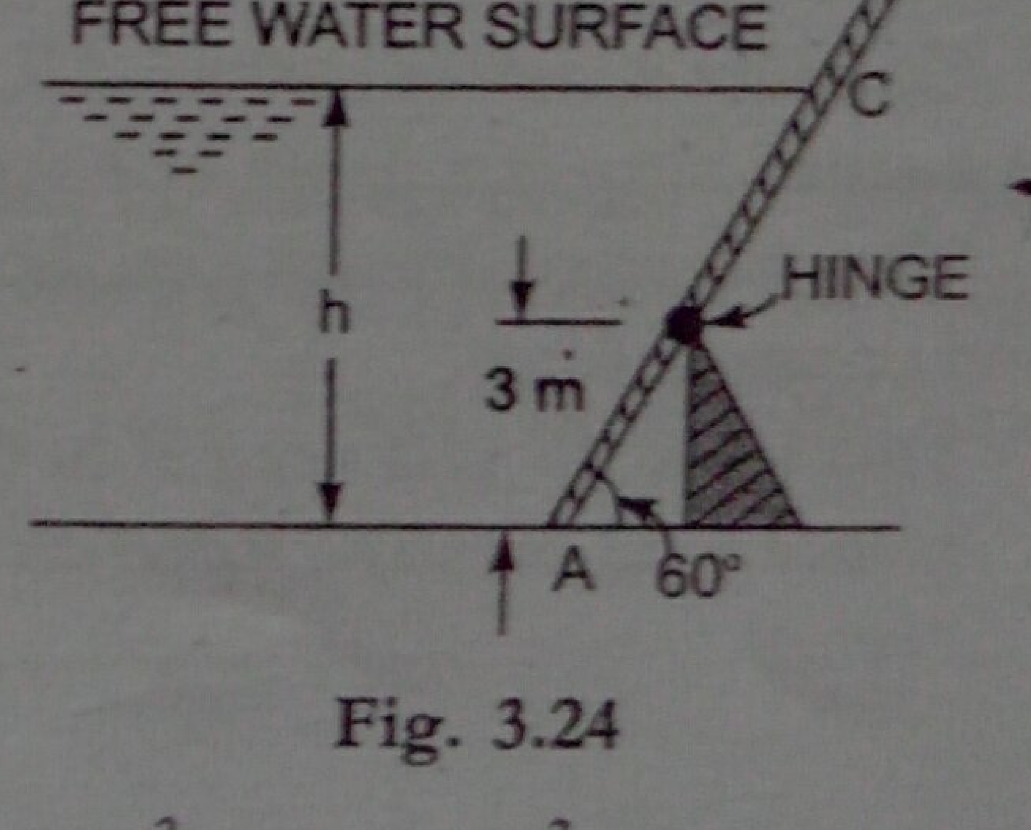


Fig. 3.24

Area, $A = AC \times 1 = \dfrac{2h}{\sqrt{3}} \times 1 \; ; \; \bar{h} = \dfrac{h}{2}$

$$I_G = \frac{bd^3}{12} = \frac{1 \times AC^3}{12} = \frac{1 \times \left(\frac{2h}{\sqrt{3}}\right)^3}{12} = \frac{8h^3}{12 \times 3 \times \sqrt{3}} = \frac{2h^3}{9 \times \sqrt{3}}$$

$$\therefore \quad h^* = \frac{2h^3}{9 \times \sqrt{3}} \times \frac{\sin^2 60^\circ}{\frac{2h}{\sqrt{3}} \times \frac{h}{2}} + \frac{h}{2} = \frac{2h^3 \times \frac{3}{4}}{9h^2} + \frac{h}{2} = \frac{h}{6} + \frac{h}{2} = \frac{2h}{3}$$

Equating the two values of h^*,

$$h - 3 = \frac{2h}{3} \quad \text{or} \quad h - \frac{2h}{3} = 3 \quad \text{or} \quad \frac{h}{3} = 3$$

$$\therefore \quad h = 3 \times 3 = 9 \text{ m}$$

$\therefore$ Height of water for tipping the gate = **9 m. Ans.**

Problem 3.20 *A rectangular sluice gate AB, 2 m wide and 3 m long is hinged at A as shown in Fig. 3.25. It is kept closed by a weight fixed to the gate. The total weight of the gate and weight fixed to the gate is 343350 N. Find the height of the water 'h' which will just cause the gate to open. The centre of gravity of the weight and gate is at G.*

Solution. Given :

Width of gate, $b = 2$ m ; Length of gate $L = 3$ m

$\therefore$ Area, $A = 2 \times 3 = 6 \text{ m}^2$

Weight of gate and $W = 343350$ N

Angle of inclination, $\theta = 45°$

Let h is the required height of water.

Depth of C.G. of the gate and weight $= \bar{h}$

From Fig. 3.25 (*a*),

$$\bar{h} = h - ED = h - (AD - AE)$$

$$= h - (AB \sin\theta - EG \tan\theta) \qquad \left\{\because \tan\theta = \frac{AE}{EG} \therefore AE = EG \tan\theta\right\}$$

$$= h - (3 \sin 45° - 0.6 \tan 45°)$$

$$= h - (2.121 - 0.6) = (h - 1.521) \text{ m}$$

The total pressure force, F is given by

$$F = \rho g A\bar{h} = 1000 \times 9.81 \times 6 \times (h - 1.521)$$

$$= 58860 (h - 1.521) \text{ N.}$$

The total force F is acting at the centre of pressure as shown in Fig. 3.25 (*b*) at H. The depth of H from free surface is given by h^* which is equal to

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}, \text{ where } I_G = \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = \frac{54}{12} = 4.5 \text{ m}^4$$

$$\therefore \quad h^* = \frac{4.5 \times \sin^2 45°}{6 \times (h - 1.521)} + (h - 1.521) = \frac{0.375}{(h - 1.521)} + (h - 1.521) \text{ m}$$

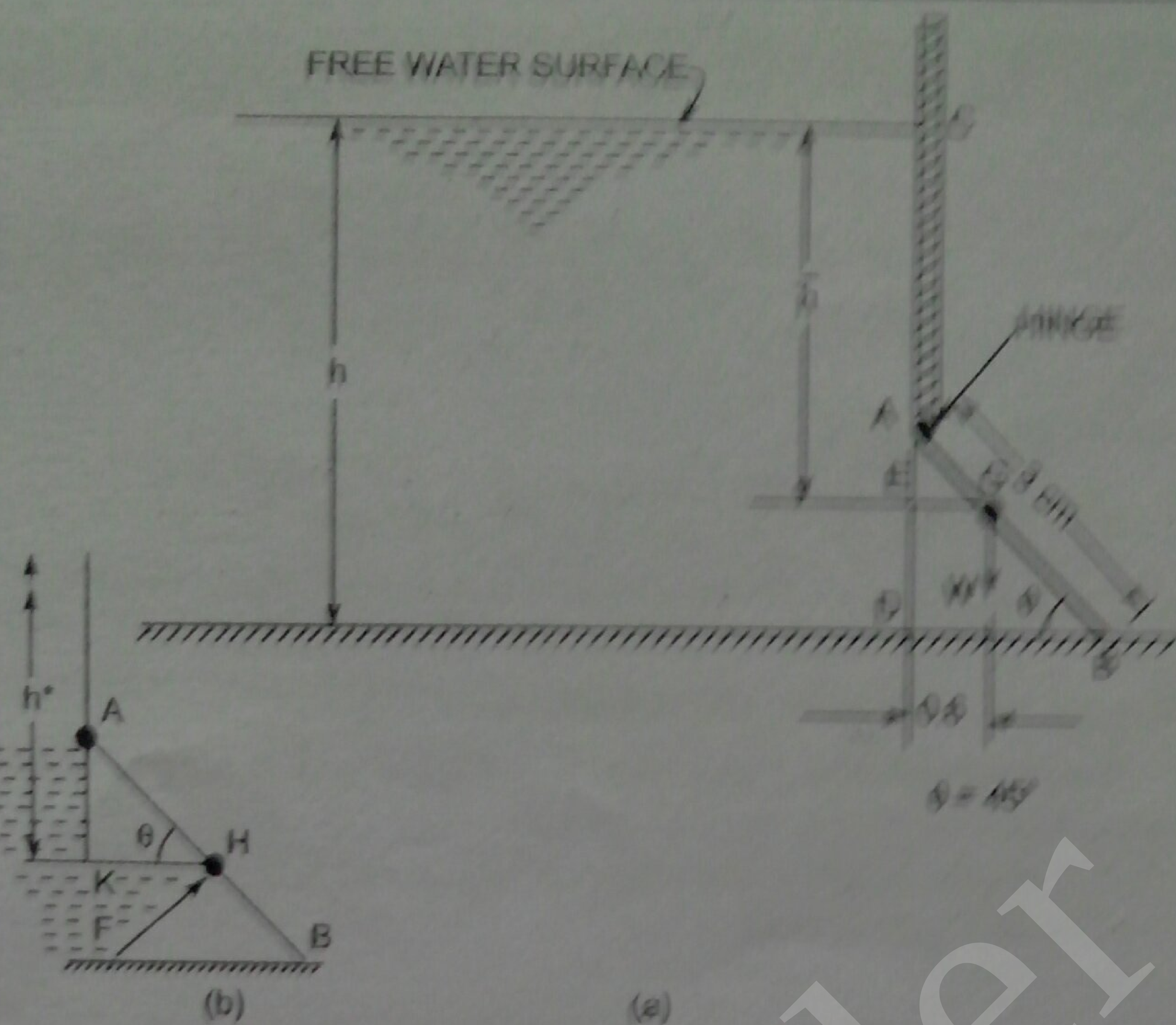


Fig. 3.26

Now taking moments about hinge A, we get

$$343350 \times EG = F \times AH$$

or
$$343350 \times 0.6 = F \times \frac{AK}{\sin 45^\circ}$$

$$\left[\text{From } \Delta AKH, \text{ Fig. 3.25 } (b) \; AK = AH \sin\theta = AH \sin 45^\circ \;\therefore\; AH = \frac{AK}{\sin 45^\circ}\right]$$

$$= \frac{58860\,(h - 1.521) \times AK}{\sin 45^\circ}$$

$$\therefore \quad AK = \frac{343350 \times 0.6 \times \sin 45^\circ}{58860\,(h - 1.521)} = \frac{0.3535 \times 7}{(h - 1.521)} \qquad \text{...(i)}$$

But
$$AK = h^* - AC = \frac{.375}{(h - 1.521)} + (h - 1.521) - AC \qquad \text{...(ii)}$$

But
$$AC = CD - AD = h - AB \sin 45^\circ = h - 3 \times \sin 45^\circ = h - 2.121$$

∴ Substituting this value in (*ii*), we get

$$AK = \frac{.375}{h - 1.521} + (h - 1.521) - (h - 2.121)$$

$$= \frac{.375}{h - 1.521} + 2.121 - 1.521 = \frac{.375}{h - 1.521} + 0.6 \qquad \text{...(iii)}$$

Equating the two values of AK from (*i*) and (*iii*)

$$\frac{0.3535 \times 7}{h - 1.521} = \frac{0.375}{h - 1.521} + 0.6$$

or $\quad 0.3535 \times 7 = 0.375 + 0.6\,(h - 1.521) = 0.375 + 0.6\,h - 0.6 \times 1.521$

or $\quad 0.6h = 2.4745 - .375 + 0.6 \times 1.521 = 2.0995 + 0.9126 = 3.0121$

$$\therefore \quad h = \frac{3.0121}{0.6} = \mathbf{5.02\ m.\ Ans.}$$

Problem 3.21 *Find the total pressure and position of centre of pressure on a triangular plate of base 2 m and height 3 m which is immersed in water in such a way that the plane of the plate makes an angle of 60° with the free surface of the water. The base of the plate is parallel to water surface and at a depth of 2.5 m from water surface.*

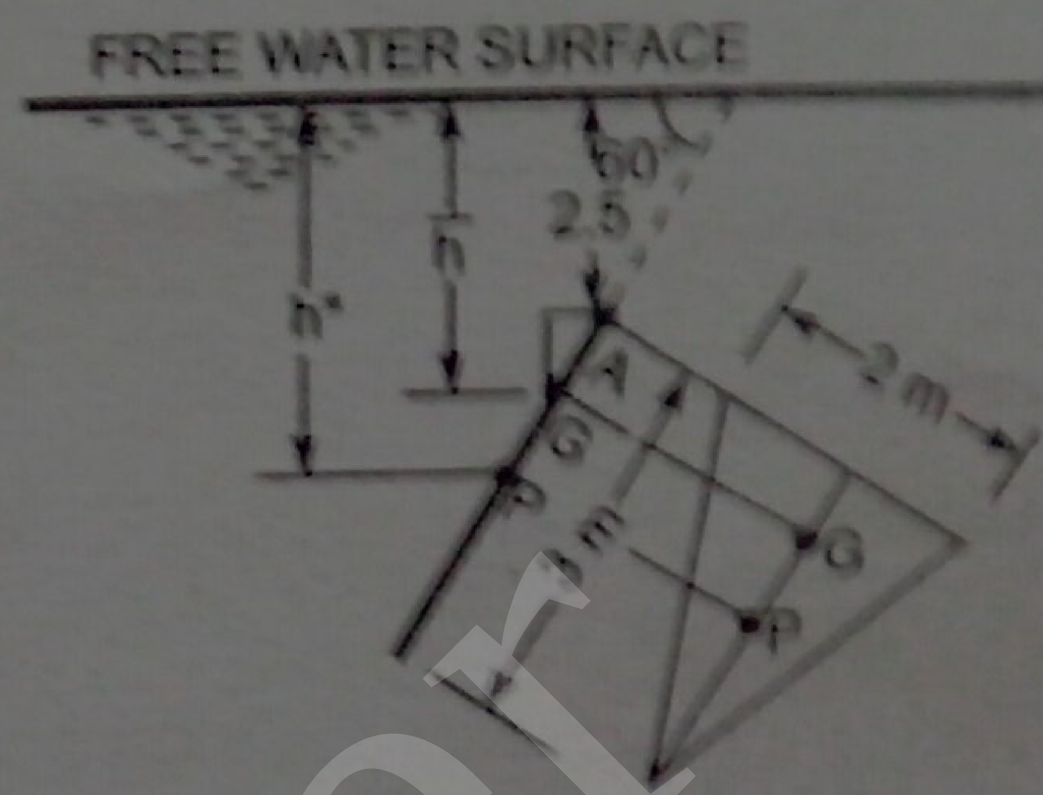


Fig. 3.26

Solution. Given :

Base of plate, $\quad b = 2$ m

Height of plate, $\quad h = 3$ m

$\therefore$ Area, $\quad A = \frac{b \times h}{2} = \frac{2 \times 3}{2} = 3\ m^2$

Inclination, $\quad \theta = 60°$

Depth of centre of gravity from free surface of water,

$$\bar{h} = 2.5 + AG \sin 60°$$

$$= 2.5 + \frac{1}{3} \times 3 \times \frac{\sqrt{3}}{2} \qquad \left\{ \because AG = \frac{1}{3} \text{ of height of triangle} \right\}$$

$$= 2.5 + .866\ m = 3.366\ m$$

(i) Total pressure force (F)

$$F = \rho g A \bar{h} = 1000 \times 9.81 \times 3 \times 3.366 = \mathbf{99061.38\ N.\ Ans.}$$

(ii) Centre of pressure (h*). Depth of centre of pressure from free surface of water is given by

$$h^* = \frac{I_G \sin^2 \theta}{A\bar{h}} + \bar{h}$$

where $\quad I_G = \frac{ba^3}{36} = \frac{2 \times 3^3}{36} = \frac{3}{2} = 1.5\ m^4$

$$\therefore \quad h^* = \frac{1.5 \times \sin^2 60°}{3 \times 3.366} + 3.366 = 0.111 + 3.366 = \mathbf{3.477\ m.\ Ans.}$$

▶ 3.6 CURVED SURFACE SUB-MERGED IN LIQUID

Consider a curved surface *AB*, sub-merged in a static fluid as shown in Fig. 3.27. Let *dA* is the area of a small strip at a depth of *h* from water surface.

Then pressure intensity on the area *dA* is $= \rho g h$

and pressure force, $\quad dF = p \times \text{Area} = \rho g h \times dA \quad$ ...(3.11)

This force *dF* acts normal to the surface.

Hence total pressure force on the curved surface should be

$$F = \int \rho g h\, dA \qquad ...(3.12)$$

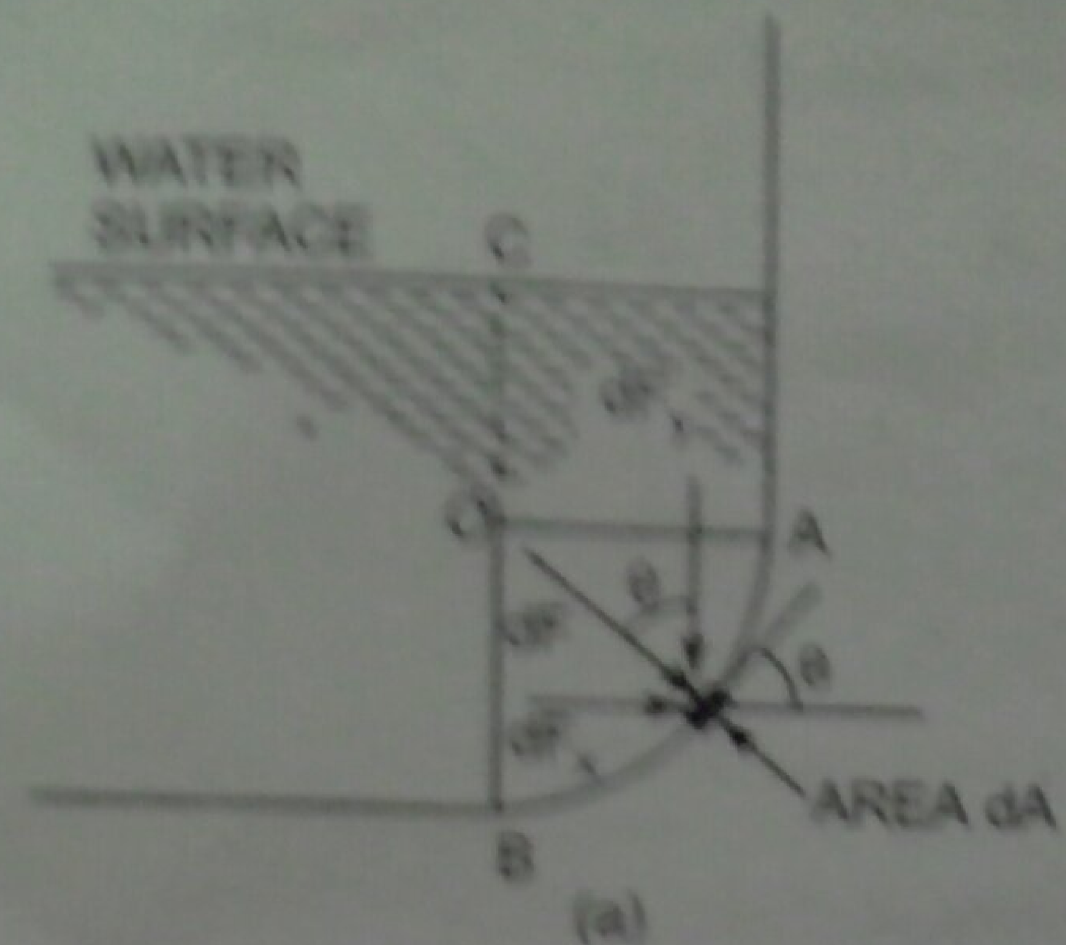


Fig. 3.27

But here as the direction of the forces on the small areas are not in the same direction, but varies from point to point. Hence integration of equation (3.11) for curved surface is impossible. The problem can, however, be solved by resolving the force dF in two components dF_x and dF_y in the x and y directions respectively. The total force in the x and y directions, *i.e.*, F_x and F_y are obtained by integrating dF_x and dF_y. Then total force on the curved surface is

$$F = \sqrt{F_x^2 + F_y^2} \qquad \text{...(3.13)}$$

and inclination of resultant with horizontal is $\tan \phi = \dfrac{F_y}{F_x}$ ...(3.14)

Resolving the force dF given by equation (3.11) in x and y directions :

$$dF_x = dF \sin\theta = \rho g h dA \sin\theta \qquad \{\because dF = \rho g h dA\}$$

and
$$dF_y = dF \cos\theta = \rho g h dA \cos\theta$$

Total forces in the x and y direction are :

$$F_x = \int dF_x = \int \rho g h dA \sin\theta = \rho g \int h dA \sin\theta \qquad \text{...(3.15)}$$

and
$$F_y = \int dF_y = \int \rho g h dA \cos\theta = \rho g \int h dA \cos\theta \qquad \text{...(3.16)}$$

Fig. 3.27 (*b*) shows the enlarged area dA. From this figure, *i.e.*, ΔEFG,

$$EF = dA$$
$$FG = dA \sin\theta$$
$$EG = dA \cos\theta$$

Thus in equation (3.15), $dA \sin\theta = FG$ = Vertical projection of the area dA and hence the expression $\rho g \int h dA \sin\theta$ represents the total pressure force on the projected area of the curved surface on the vertical plane. Thus

F_x = Total pressure force on the projected area of the curved surface on vertical plane. ...(3.17)

Also $dA \cos\theta = EG$ = horizontal projection of dA and hence $h dA \cos\theta$ is the volume of the liquid contained in the elementary area dA upto free surface of the liquid. Thus $\int h dA \cos\theta$ is the total volume contained between the curved surface extended upto free surface.

Hence $\rho g \int h dA \cos\theta$ is the total weight supported by the curved surface. Thus

$$F_y = \rho g \int h dA \cos\theta$$

= weight of liquid supported by the curved surface upto free surface of liquid. ...(3.18)

In Fig. 3.28, the curved surface AB is not supporting any fluid. In such cases, F_y is equal to the weight of the imaginary liquid supported by AB upto free surface of liquid. The direction of F_y will be taken in upward direction.

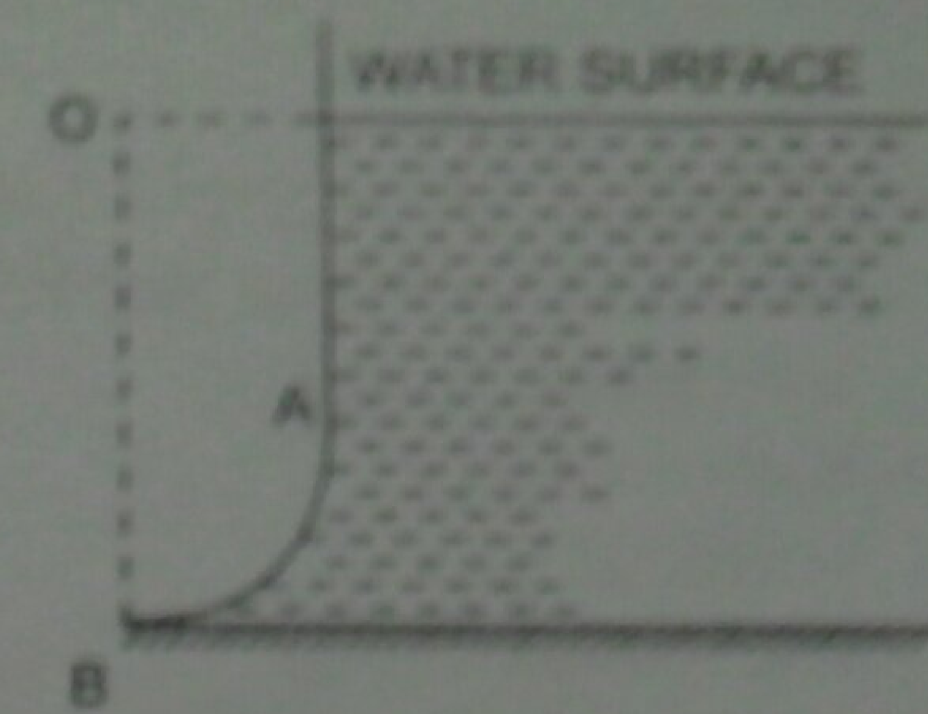


Fig. 3.28

Problem 3.22 *Compute the horizontal and vertical components of the total force acting on a curved surface AB, which is in the form of a quadrant of a circle of radius 2 m as shown in Fig. 3.29. Take the width of the gate as unity.*

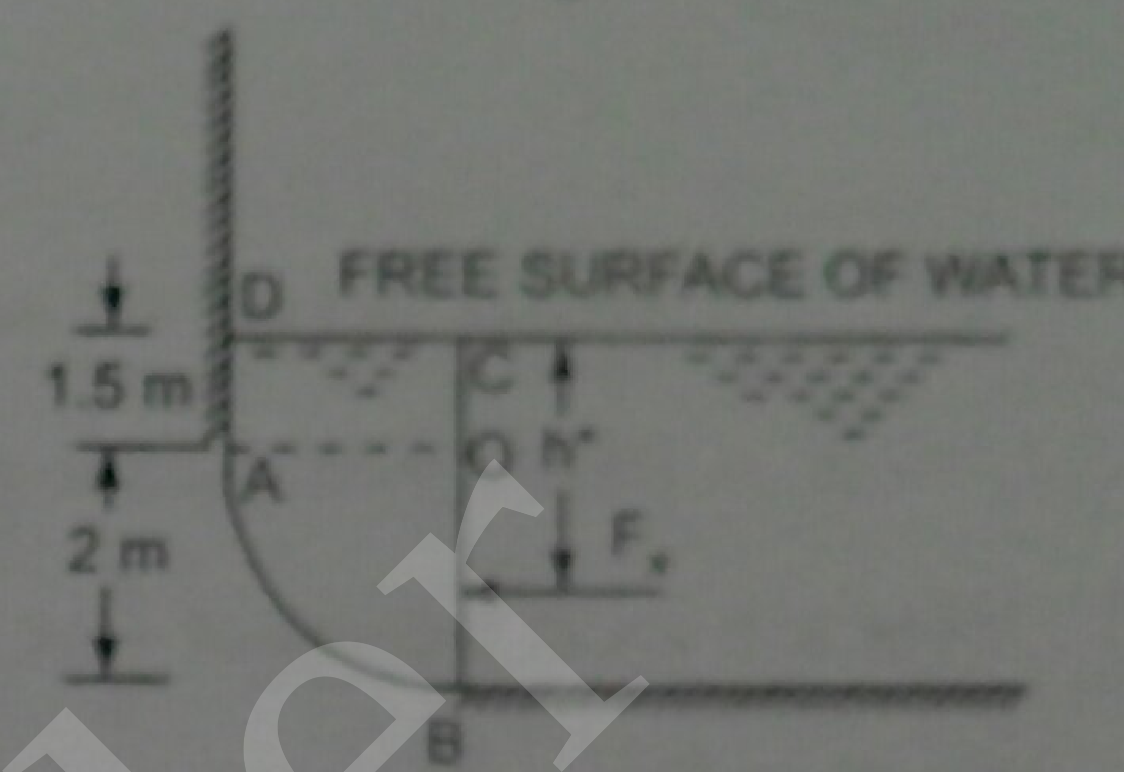


Fig. 3.29

Solution. Given :

Width of gate $= 1.0$ m

Radius of the gate $= 2.0$ m

$\therefore$ Distance $AO = OB = 2$ m

Horizontal force, F_x exerted by water on gate is given by equation (3.17) as

F_x = Total pressure force on the projected area of curved surface AB on vertical plane

= Total pressure force on OB

{projected area of curved surface on vertical plane $= OB \times 1$}

$$= \rho g A\bar{h}$$

$$= 1000 \times 9.81 \times 2 \times 1 \times \left(1.5 + \frac{2}{2}\right)$$

{$\because$ **Area of $OB = A = BO \times 1 = 2 \times 1 = 2$.**

$\bar{h}$ = Depth of C.G. of OB from free surface $= 1.5 + \frac{2}{2}$}

$$F_x = 9.81 \times 2000 \times 2.5 = 49050 \text{ N. Ans.}$$

The point of application of F_x is given by $h^* = \dfrac{I_G}{A\bar{h}} + \bar{h}$

where I_G = M.O.I. of OB about its C.G. $= \dfrac{bd^3}{12} = \dfrac{1 \times 2^3}{12} = \dfrac{2}{3}$ m^4

$$\therefore \quad h^* = \frac{\frac{2}{3}}{2 \times 2.5} + 2.5 = \frac{1}{7.5} + 2.5 \text{ m}$$

$= 0.1333 + 2.5 = 2.633$ m from free surface.

Vertical force, F_y, exerted by water is given by equation (3.18)

F_y = Weight of water supported by AB upto free surface

= Weight of portion $DABOC$

= Weight of $DAOC$ + Weight of water AOB

$= \rho g$ [Volume of $DAOC$ + Volume of AOB]

$$= 1000 \times 9.81 \left[AD \times AO \times 1 + \frac{\pi}{4}(AO)^2 \times 1\right]$$

$$= 1000 \times 9.81 \left[1.5 \times 2.0 \times 1 + \frac{\pi}{4} \times 2^2 \times 1\right]$$

$$= 1000 \times 9.81\,[3.0 + \pi]\text{N} = \mathbf{60249.1\ N.\ Ans.}$$

Problem 3.23 *Fig. 3.30 shows a gate having a quadrant shape of radius 2 m. Find the resultant force due to water per metre length of the gate. Find also the angle at which the total force will act.*

Solution. Given :

Radius of gate $= 2$ m

Width of gate $= 1$ m

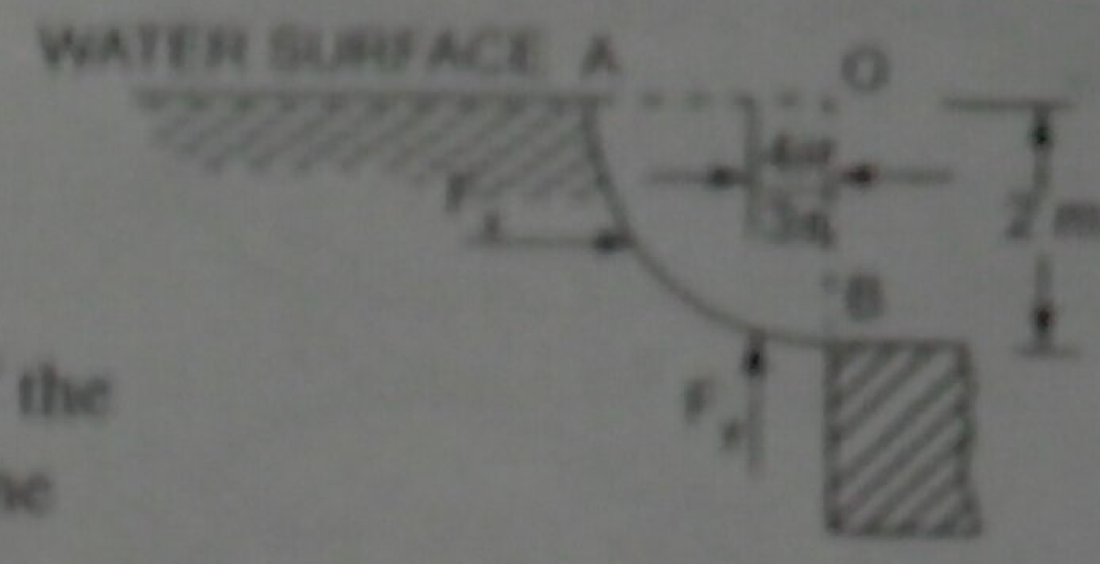


Fig. 3.30

Horizontal Force

F_x = Force on the projected area of the curved surface on vertical plane

$= \text{Force on } BO = \rho g A \bar{h}$

where $A = \text{Area of } BO = 2 \times 1 = 2\ \text{m}^2,\ \bar{h} = \frac{1}{2} \times 2 = 1\ \text{m}$;

$$F_x = 1000 \times 9.81 \times 2 \times 1 = 19620\ \text{N}$$

This will act at a depth of $\frac{2}{3} \times 2 = \frac{4}{3}$ m from free surface of liquid.

Vertical Force, F_y

F_y = Weight of water (imagined) supported by AB

$= \rho g \times \text{Area of } AOB \times 1.0$

$$= 1000 \times 9.81 \times \frac{\pi}{4}(2)^2 \times 1.0 = 30819\ \text{N}$$

This will act at a distance of $\frac{4R}{3\pi} = \frac{4 \times 2.0}{3\pi} = 0.848$ m from OB.

$\therefore$ **Resultant force, F is given by**

$$F = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{19620 + 30819} = \sqrt{384944400 + 949810761}$$

$$= \mathbf{36534.4\ N.\ Ans.}$$

The angle made by the resultant with horizontal is given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{30819}{19620} = 1.5708$$

$$\therefore \quad \theta = \tan^{-1} 1.5708 = \mathbf{57^\circ\ 31'.\ Ans.}$$

Problem 3.24 *Find the magnitude and direction of the resultant force due to water acting on a roller gate of cylindrical form of 4.0 m diameter, when the gate is placed on the dam in such a way that water is just going to spill. Take the length of the gate as 8 m.*

Solution. Given :

Dia. of gate $= 4$ m

$\therefore$ Radius, $R = 2$ m

Length of gate, $l = 8$ m

Horizontal force, F_x acting on the gate is

$F_x = \rho g A \bar{h}$ = Force on projected area of curved surface ACB on vertical plane

= Force on vertical area AOB

where A = Area of $AOB = 4.0 \times 8.0 = 32.0 \text{ m}^2$

$\bar{h}$ = Depth of C.G. of $AOB = 4/2 = 2.0$ m

$\therefore \quad F_x = 1000 \times 9.81 \times 32.0 \times 2.0$

$= 627840$ N.

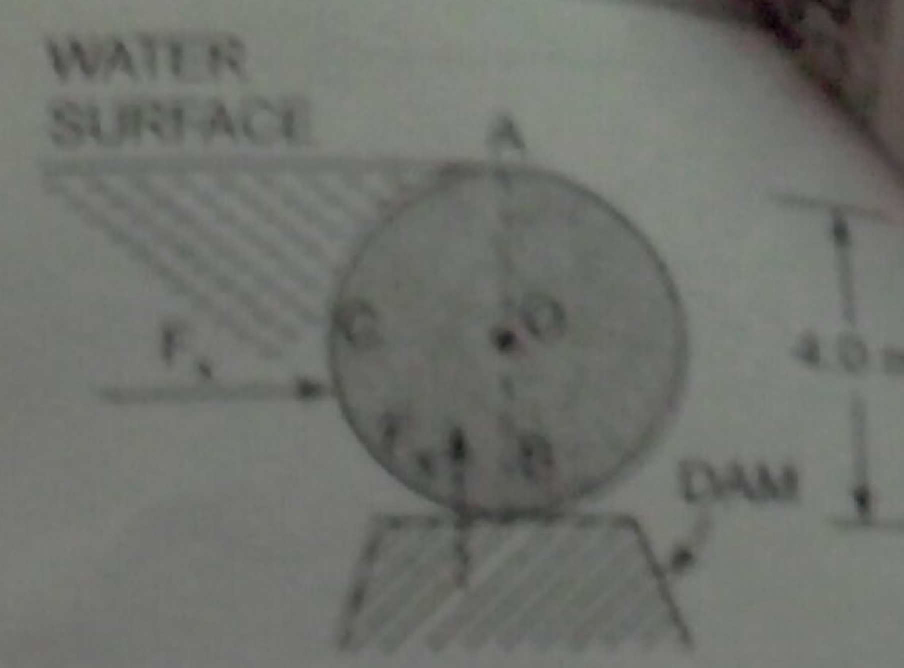


Fig. 3.31

Vertical force, F_y is given by

F_y = Weight of water enclosed or supported (actually or imaginary) by the curved surface ACB

$= \rho g \times$ Volume of portion ACB

$= \rho g \times$ Area of $ACB \times l$

$$= 1000 \times 9.81 \times \frac{\pi}{2}(R)^2 \times 8.0 = 9810 \times \frac{\pi}{2}(2)^2 \times 8.0 = 493104 \text{ N}$$

It will be acting in the upward direction.

$\therefore$ Resultant force, $\quad F = \sqrt{F_x^2 + F_y^2} = \sqrt{627840 + 493104} = 798328$ **N. Ans.**

Direction of resultant force is given by $\tan\theta = \dfrac{F_y}{F_x} = \dfrac{493104}{627840} = 0.7853$

$\therefore \quad$ **$\theta = 31° 8'$. Ans.**

Problem 3.25 *Find the horizontal and vertical component of water pressure acting on the face of a tainter gate of 90° sector of radius 4 m as shown in Fig. 3.32. Take width of gate unity.*

Solution. Given :

Radius of gate, $\quad R = 4$ m

Horizontal component of force acting on the gate is

F_x = Force on area of gate projected on vertical plane

= Force on area ADB

$= \rho g A \bar{h}$

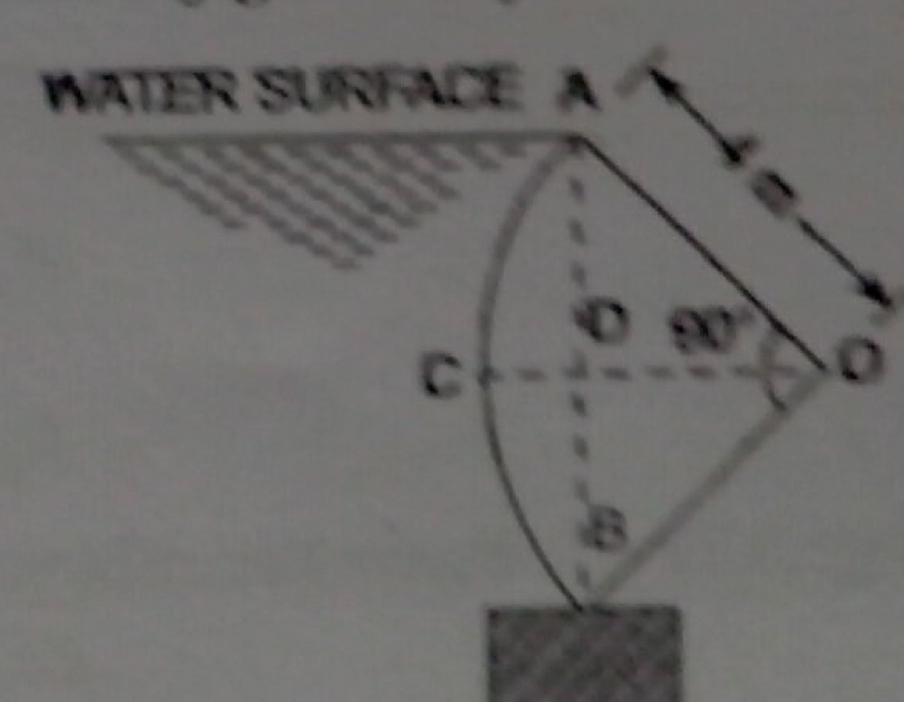


Fig. 3.32

where $A = AB \times$ Width of gate

$= 2 \times AD \times 1 \qquad (\because AB = 2AD)$

$= 2 \times 4 \times \sin 45° = 8 \times .707 = 5.656 \text{ m}^2 \qquad \{\because AD = 4 \sin 45°\}$

$$\bar{h} = \frac{AB}{2} = \frac{5.656}{2} = 2.828 \text{ m}$$

$\therefore \quad F_x = 1000 \times 9.81 \times 5.656 \times 2.828$ N = **156911 N. Ans.**

Vertical component

F_y = Weight of water supported or enclosed by the curved surface

= Weight of water in portion $ACBDA$

$= \rho g \times$ Area of $ACBDA \times$ Width of gate

$= 1000 \times 9.81 \times$ [Area of sector $ACBOA$ − Area of ΔABO] $\times 1$

$$= 9810 \times \left[\frac{\pi}{4}R^2 - \frac{AO \times BO}{2}\right] \qquad [\because \Delta AOB \text{ is a right angled}]$$

$$= 9810 \times \left[\frac{\pi}{4}4^2 - \frac{4 \times 4}{2}\right] = \mathbf{44796\ N.\ Ans.}$$

Problem 3.26 *Calculate the horizontal and vertical components of the water pressure exerted on a tainter gate of radius 8 m as shown in Fig. 3.33. Take width of gate unity.*

Solution. The horizontal component of water pressure is given by

$F_x = \rho g A \bar{h}$ = Force on the area projected on vertical plane

= Force on the vertical area of BD

where $A = BD \times$ Width of gate $= 4.0 \times 1 = 4.0$ m

$\bar{h} = \frac{1}{2} \times 4 = 2$ m

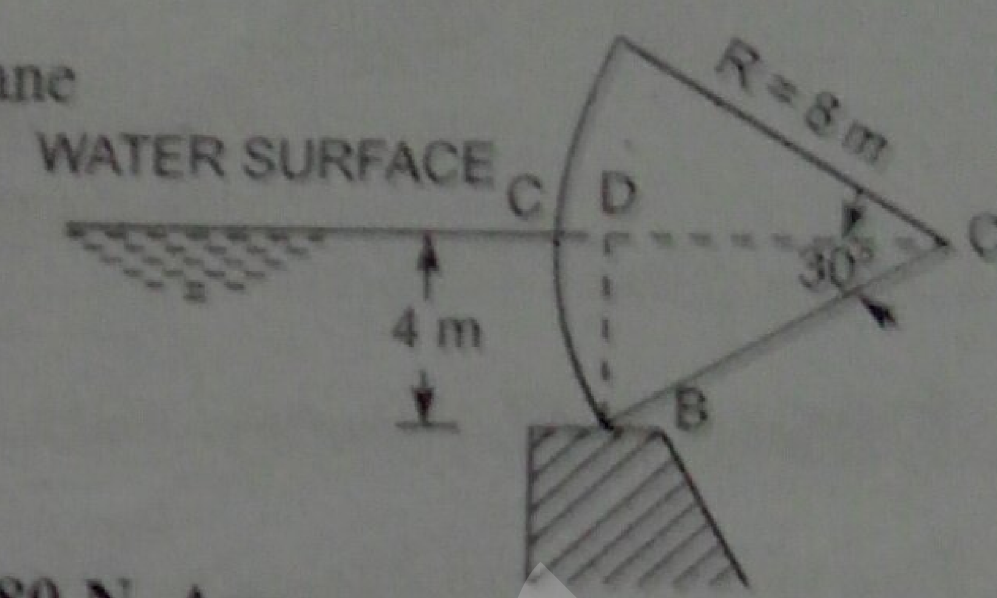


Fig. 3.33

$$F_x = 1000 \times 9.81 \times 4.0 \times 2.0 = \mathbf{78480\ N.\ Ans.}$$

Vertical component of the water pressure is given by

F_y = Weight of water supported or enclosed (imaginary) by curved surface CB

= Weight of water in the portion $CBDC$

$= \rho g \times$ [Area of portion $CBDC$] $\times$ Width of gate

$= \rho g \times$ [Area of sector CBO – Area of the triangle BOD] $\times 1$

$$= 1000 \times 9.81 \times \left[\frac{30}{360} \times \pi R^2 - \frac{BD \times DO}{2}\right]$$

$$= 9810 \times \left[\frac{1}{12}\pi \times 8^2 - \frac{4.0 \times 8.8 \cos 30°}{2}\right]$$

$\{\because DO = BO \cos 30° = 8 \times \cos 30°\}$

$$= 9810 \times [16.755 - 13.856] = \mathbf{28439\ N.\ Ans.}$$

Problem 3.27 *A cylindrical gate of 4 m diameter 2 m long has water on its both sides as shown in Fig. 3.34. Determine the magnitude, location and direction of the resultant force exerted by the water on the gate. Find also the least weight of the cylinder so that it may not be lifted away from the floor.*

Solution. Given :

Dia. of gate = 4 m

Radius = 2 m

(i) The forces acting on the left side of the cylinder are :

The horizontal component, F_{x_1}

where F_{x_1} = Force of water on area projected on vertical plane

= Force on area AOC

$= \rho g A \bar{h}$

$= 1000 \times 9.81 \times 8 \times 2$

$= 156960$ N

where $A = AC \times$ Width $= 4 \times 2$

$= 8\ m^2$

$= \bar{h} = \frac{1}{2} \times 4 = 2$ m

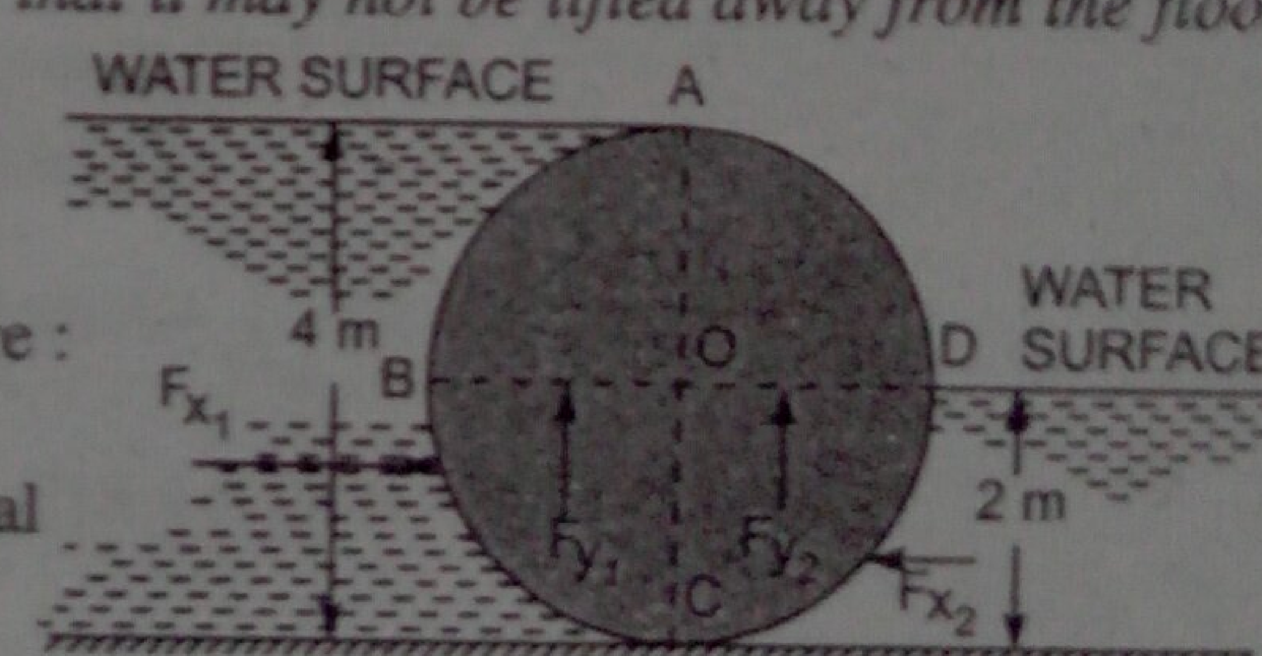


Fig. 3.34

$$F_{y_1} = \text{weight of water enclosed by } ABCQA$$

$$= 1000 \times 9.81 \times \left[\frac{\pi}{2} R^2\right] \times 2.0 = 9810 \times \frac{\pi}{2} \times 2^2 \times 2.0 = 123276 \text{ N.}$$

Right Side of the Cylinder

$$F_{x_2} = \rho g A_2 \bar{h}_2 = \text{Force on vertical area } CQ$$

$$= 1000 \times 9.81 \times 2 \times 2 \times \frac{2}{2} \quad \left\{ A_2 = CQ \times 1 = 2 \times 1 = 2 \text{ m}^2, \bar{h}_2 = \frac{2}{2} = 1.0 \right\}$$

$$= 39240 \text{ N}$$

$$F_{y_2} = \text{Weight of water enclosed by } DOCD$$

$$= \rho g \times \left[\frac{\pi}{4} R^2\right] \times \text{Width of gate}$$

$$= 1000 \times 9.81 \times \frac{\pi}{4} \times 2^2 \times 2 = 61638 \text{ N}$$

∴ Resultant force in the direction of x,

$$F_x = F_{x_1} - F_{x_2} = 156960 - 39240 = 117720 \text{ N}$$

Resultant force in the direction of y,

$$F_y = F_{y_1} + F_{y_2} = 123276 + 61638 = 184914 \text{ N}$$

(*i*) **Resultant force,** F is given as

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(117720)^2 + (184914)^2} = 219206 \text{ N. Ans.}$$

(*ii*) Direction of resultant force is given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{184914}{117720} = 1.5707$$

∴ $\theta = 57° \, 31'$. **Ans.**

(*iii*) Location of the resultant force

Force, F_{x_1} acts at a distance of $\frac{2 \times 4}{3} = 2.67$ m from the top surface of water on left side, while F_{x_2} acts at a distance of $\frac{2}{3} \times 2 = 1.33$ m from free surface on the right side of the cylinder. The resultant force F_x in the direction of x will act at a distance of y from the bottom as

$$F_x \times y = F_{x_1} [4 - 2.67] - F_{x_2} [2 - 1.33]$$

or

$$117720 \times y = 156960 \times 1.33 - 39240 \times .67 = 208756.8 - 26290.8 = 182466$$

∴ $$y = \frac{182466}{117720} = 1.55 \text{ m from the bottom.}$$

Force F_{y_1} acts at a distance $\frac{4R}{3\pi}$ from AOC or at a distance $\frac{4 \times 2.0}{3\pi} = 0.8488$ m from AOC towards left of AOC.

Also F_{y_2} acts at a distance $\frac{4R}{3\pi} = 0.8488$ m from AOC towards the right of AOC. The resultant force F_y will act at a distance x from AOC which is given by

$$F_y \times x = F_{y_1} \times .8488 - F_{y_2} \times .8488$$

or $$184914 \times x = 123276 \times .8488 - 61638 \times .8488 = .8488\,[123276 - 61638] = 52318.4$$

$$\therefore \quad x = \frac{52318.4}{184914} = 0.2829 \text{ m from } AOC.$$

(*iv*) **Least weight of cylinder.** The resultant force in the upward direction is

$$F_y = 184914 \text{ N}$$

Thus the weight of cylinder should not be less than the upward force F_y. Hence least weight of cylinder should be at least

$$= \mathbf{184914\ N.}\ \text{Ans.}$$

Problem 3.28 *Fig. 3.35 shows the cross-section of a tank full of water under pressure. The length of the tank is 2 m. An empty cylinder lies along the length of the tank on one of its corner as shown. Find the horizontal and vertical components of the force acting on the curved surface ABC of the cylinder.*

Solution. Radius, $R = 1$ m

Length of tank, $l = 2$ m

Pressure, $p = 0.2 \text{ kgf/cm}^2 = 0.2 \times 9.81 \text{ N/cm}^2$

$= 1.962 \text{ N/cm}^2 = 1.962 \times 10^4 \text{ N/m}^2$

$\therefore$ Pressure head, $h = \dfrac{p}{\rho g} = \dfrac{1.962 \times 10^4}{1000 \times 9.81} = 2$ m

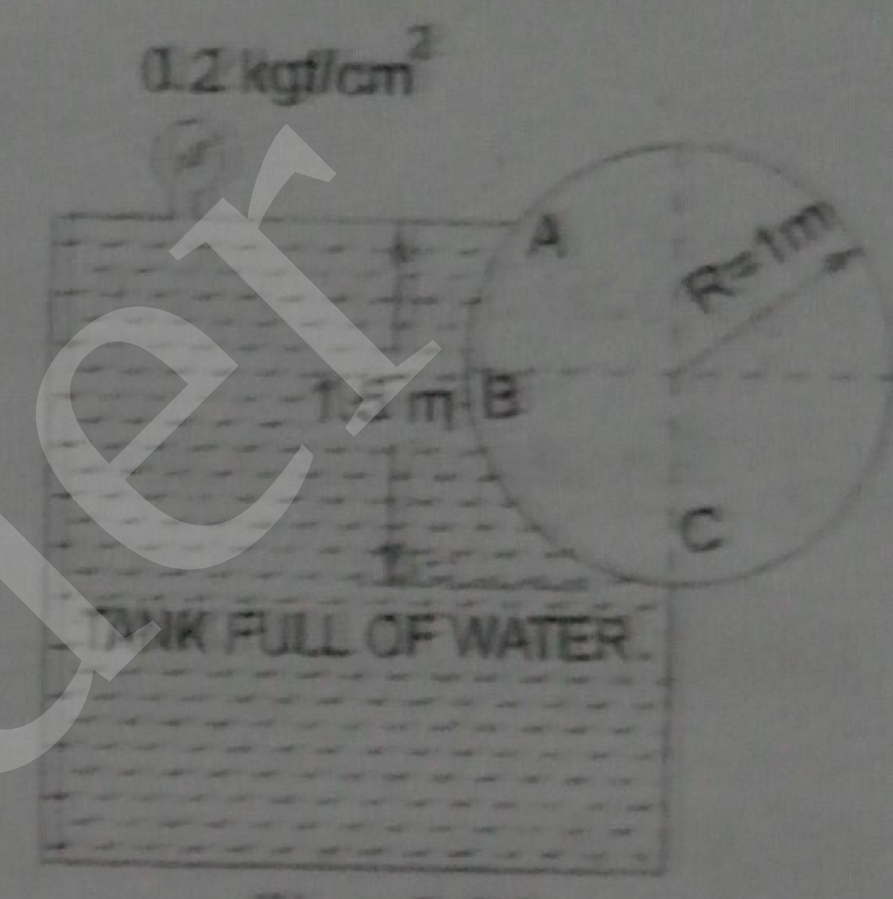


Fig. 3.35

$\therefore$ Free surface of water will be at a height of 2 m from the top of the tank.

$\therefore$ Fig. 3.36 shows the equivalent free surface of water.

(*i*) Horizontal Component of Force

$$F_x = \rho g A \bar{h}$$

where A = Area projected on vertical plane

$= 1.5 \times 2.0 = 3.0 \text{ m}^2$

$$\bar{h} = 2 + \frac{1.5}{2} = 2.75$$

$\therefore$ $F_x = 1000 \times 9.81 \times 3.0 \times 2.75$

$= \mathbf{80932.5\ N.}$ Ans.

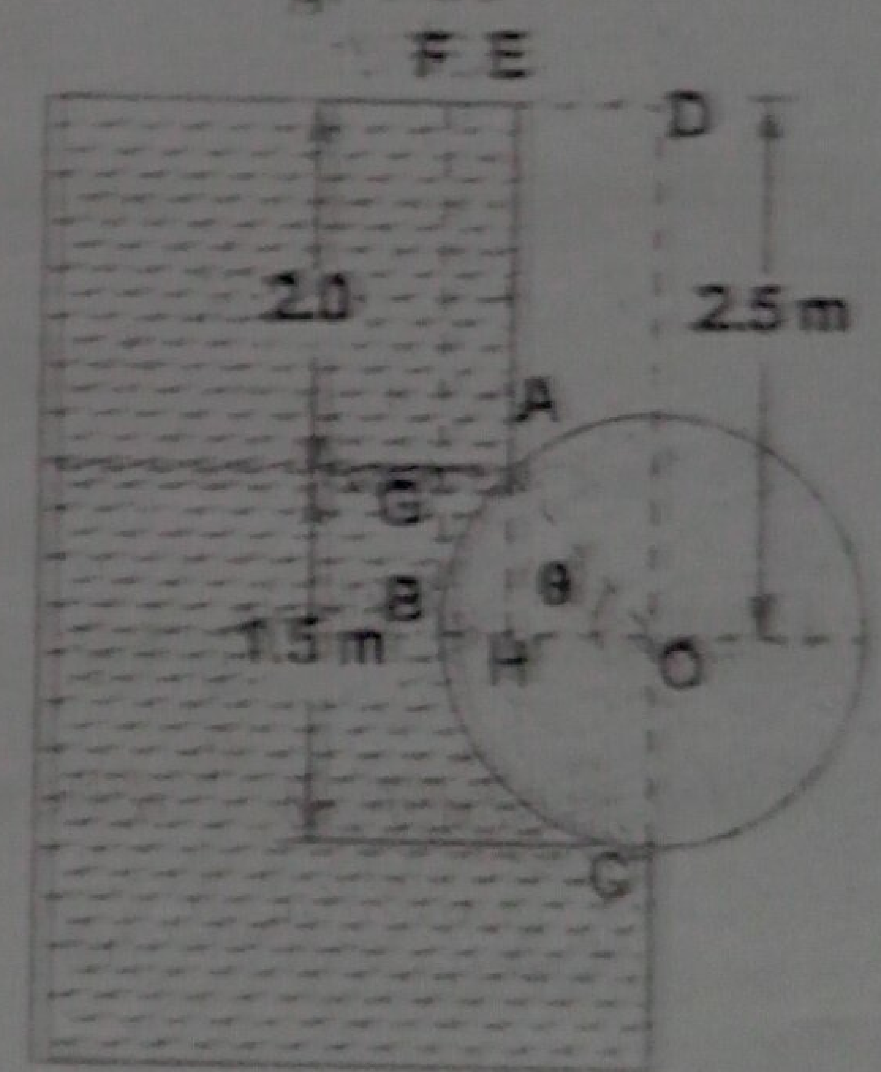


Fig. 3.36

(*ii*) **Vertical Component of Force**

F_y = Weight of water enclosed or supported actually or imaginary by curved surface ABC

= Weight of water in the portion $CODE\,ABC$

= Weight of water in $CODFBC$ – Weight of water in $AEFB$

But weight of water in $CODFBC$

= Weight of water in $[COB + ODFBO]$

$$= \rho g \left[\frac{\pi R^2}{4} + BO \times OD\right] \times 2 = 1000 \times 9.81 \left[\frac{\pi}{4} \times 1^2 + 1.0 \times 2.5\right] \times 2$$

$= 64458.5$ N

Weight of water in $AEFB = \rho g$ [Area of $AEFB$] $\times 2.0$

$= 1000 \times 9.81$ [Area of $(AEFG + AGBH - AHB)$] $\times 2.0$

In ΔAHO, $\sin\theta = \frac{AH}{AO} = \frac{0.5}{1.0} = 0.5 \quad \therefore \quad \theta = 30^\circ$

$BH = BO - HO = 1.0 - AO\cos\theta = 1.0 - 1 \times \cos 30^\circ = 0.134$

Area, ABH = Area ABO – Area AHO

$$= \pi R^2 \times \frac{30}{360} - \frac{AH \times HO}{2.0} = \frac{\pi R^2}{12} - \frac{0.5 \times .866}{2} = 0.0453$$

$\therefore$ Weight of water in $AEFB$

$$= 9810 \times [AE \times AG + AG \times AH - 0.0453] \times 2.0$$
$$= 9810 \times [2.0 \times .134 + .134 \times .5 - .0453] \times 2.0$$
$$= 9810 \times [.268 + .067 - .0453] \times 2.0 = 5684 \text{ N}$$

$\therefore$ $F_y = 64458.5 - 5684 =$ **58774.5 N. Ans.**

Problem 3.29 *Find the magnitude and direction of the resultant water pressure acting on a curved face of a dam which is shaped according to the relation $y = \frac{x^2}{9}$ as shown in Fig. 3.37. The height of the water retained by the dam is 10 m. Consider the width of the dam as unity.*

Solution. Equation of curve AB is

$$y = \frac{x^2}{9} \quad \text{or} \quad x^2 = 9y$$

$\therefore$ $x = \sqrt{9y} = 3\sqrt{y}$

Height of water, $h = 10$ m

Width, $b = 1$ m

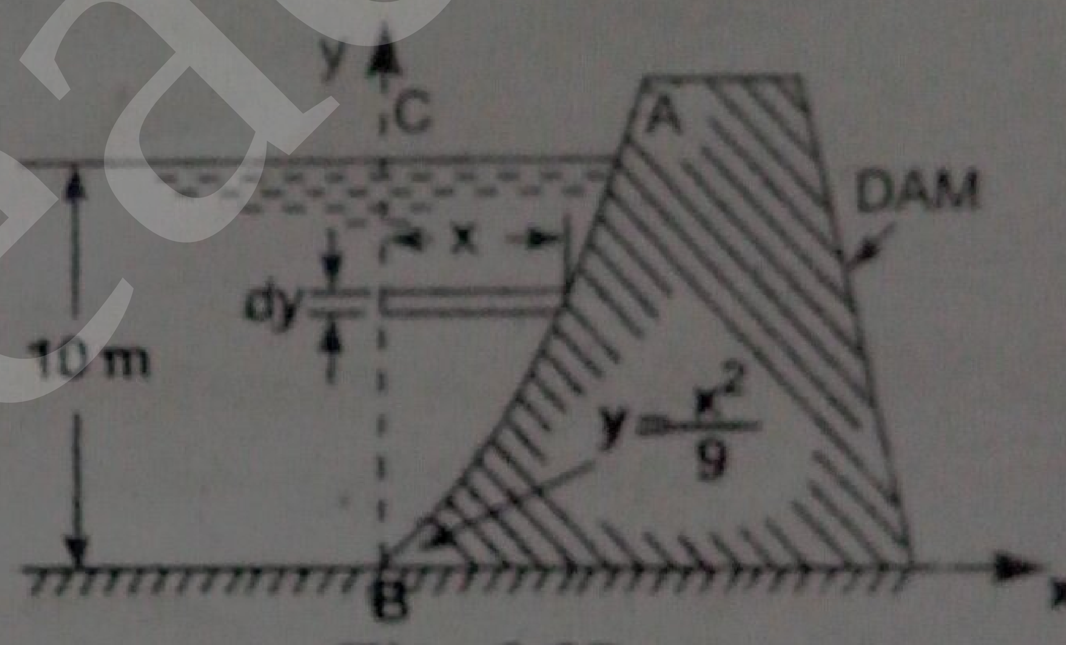


Fig. 3.37

The horizontal component, F_x is given by

F_x = Pressure due to water on the curved area projected on vertical plane

= Pressure on area BC

$= \rho g A\bar{h}$

where $A = BC \times 1 = 10 \times 1$ m^2, $\bar{h} = \frac{1}{2} \times 10 = 5$ m

$F_x = 1000 \times 9.81 \times 10 \times 5 = 490500$ N

Vertical component, F_y is given by

F_y = Weight of water supported by the curve AB

= Weight of water in the portion ABC

$= \rho g$[Area of ABC] $\times$ Width of dam

$$= \rho g \left[\int_0^{10} x \times dy\right] \times 1.0 \qquad \left\{\text{Area of strip} = xdy \quad \therefore \text{Area } ABC = \int_0^{10} xdy\right\}$$

$$= 1000 \times 9.81 \times \int_0^{10} 3\sqrt{y}\, dy \qquad \{\because x = 3\sqrt{y}\}$$

$$= 29430 \left[\frac{y^{3/2}}{3/2}\right]_0^{10} = 29430 \times \frac{2}{3}\left[y^{3/2}\right]_0^{10} = 19620\,[10^{3/2}]$$

$$= 19620 \times 31.622 = 620439 \text{ N}$$

$\therefore$ Resultant water pressure on dam

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(490500)^2 + (620439)^2}$$
$$= 790907 \text{ N} = \mathbf{790.907 \text{ kN. Ans.}}$$

Direction of the resultant is given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{620439}{490500} = 1.265$$

$\therefore$ $\theta = \mathbf{51° 40'}$. **Ans.**

Problem 3.30 *A dam has a parabolic shape* $y = y_0\left(\frac{x}{x_0}\right)^2$ *as shown in Fig. 3.38 below having* $x_0 = 6$ *m and* $y_0 = 9$ *m. The fluid is water with density = 1000 kg/m³. Compute the horizontal, vertical and the resultant thrust exerted by water per metre length of the dam.*

Solution. Given :

Equation of the curve *OA* is

$$y = y_0\left(\frac{x}{x_0}\right)^2 = 9\left(\frac{x}{6}\right)^2 = 9 \times \frac{x^2}{36} = \frac{x^2}{4}$$

or $x^2 = 4y$

$\therefore$ $x = \sqrt{4y} = 2y^{1/2}$

Width of dam, $b = 1$ m.

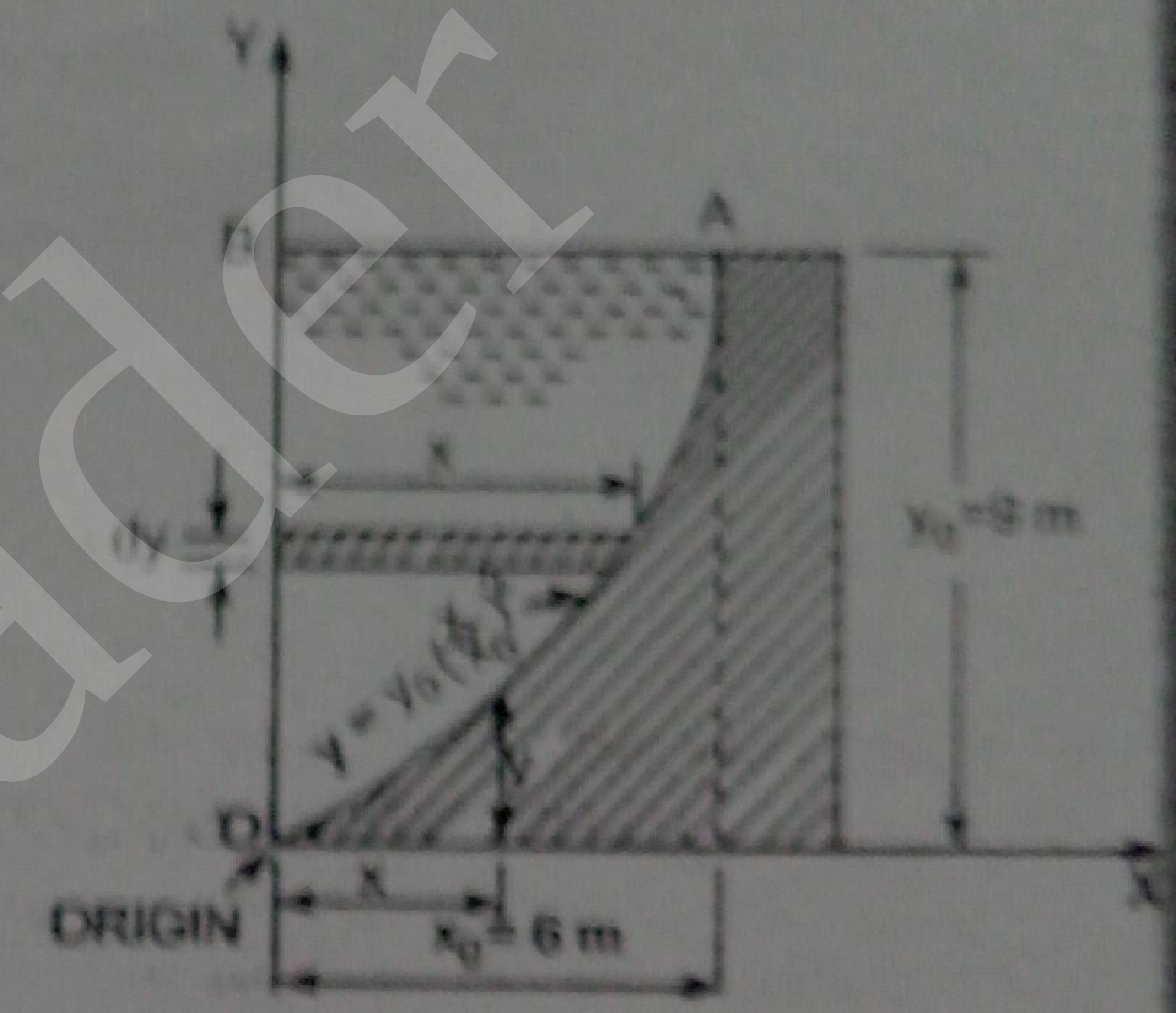


Fig. 3.38

(i) Horizontal thrust exerted by water

F_x = Force exerted by water on vertical surface *OB*, *i.e.*, the surface obtained by projecting the curved surface on vertical plane

$= \rho g A \bar{h}$

$= 1000 \times 9.81 \times (9 \times 1) \times \frac{9}{2} = \mathbf{397305 \text{ N. Ans.}}$

(ii) Vertical thrust exerted by water

F_y = Weight of water supported by curved surface *OA* upto free surface of water

= Weight of water in the portion *ABO*

= ρg × Area of *OAB* × Width of dam

$$= 1000 \times 9.81 \times \left[\int_0^9 x \times dy\right] \times 1.0$$

$$= 1000 \times 9.81 \times \left[\int_0^9 2y^{1/2} \times dy\right] \times 1.0 \qquad (\because x = 2y^{1/2})$$

$$= 19620 \times \left[\frac{y^{3/2}}{(3/2)}\right]_0^9 = 19620 \times \frac{2}{3}\,[9^{3/2}]$$

$$= 19620 \times \frac{2}{3} \times 27 = \mathbf{353160 \text{ N. Ans.}}$$

(*iii*) **Resultant thrust exerted by water**

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{397305 + 353160} = 531574 \text{ N. Ans.}$$

Direction of resultant is given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{353160}{397305} = 0.888$$

$$\theta = \tan^{-1} 0.888 = \mathbf{41.63^\circ.\ Ans.}$$

Problem 3.31 *A cylinder 3 m in diameter and 4 m long retains water on one side. The cylinder is supported as shown in Fig. 3.39. Determine the horizontal reaction at A and the vertical reaction at B. The cylinder weighs 196.2 kN. Ignore friction.*

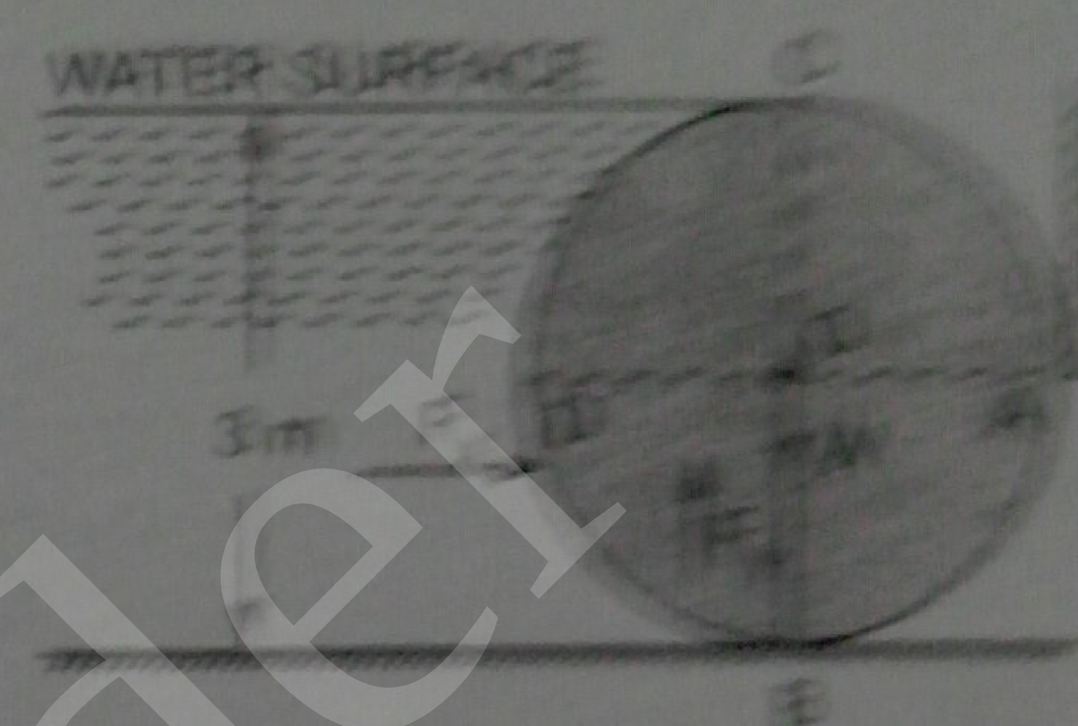


Fig. 3.39

Solution. Given :

Dia. of cylinder $= 3$ m

Length of cylinder $= 4$ m

Weight of cylinder, $W = 196.2 \text{ kN} = 196200 \text{ N}$

Horizontal force exerted by water

$$F_x = \text{Force on vertical area } BOC$$
$$= \rho g A \bar{h}$$

where $A = BOC \times l = 3 \times 4 = 12 \text{ m}^2,\ \bar{h} = \frac{1}{2} \times 3 = 1.5 \text{ m}$

$$F_x = 1000 \times 9.81 \times 12 \times 1.5 = 176580 \text{ N}$$

The vertical force exerted by water

$$F_y = \text{Weight of water enclosed in } BDCOB$$

$$= \rho g \times \left(\frac{\pi}{2} R^2\right) \times l = 1000 \times 9.81 \times \frac{\pi}{2} \times (1.5)^2 \times 4 = 138684 \text{ N}$$

Force F_y is acting in the upward direction.

For the equilibrium of cylinder

Horizontal reaction at $A = F_x = 176580$ N

Vertical reaction at $B = \text{Weight of cylinder} - F_y$

$= 196200 - 138684 = 57516$ **N. Ans.**

▸ 3.7 TOTAL PRESSURE AND CENTRE OF PRESSURE ON LOCK GATES

Lock gates are the devices used for changing the water level in a canal or a river for navigation. Fig. 3.40 shows plan and elevation of a pair of lock gates. Let AB and BC be the two lock gates. Each gate is supported on two hinges fixed on their top and bottom at the ends A and C. In the closed position, the gates meet at B.

Let F = Resultant force due to water on the gate AB or BC acting are right angles to the gate

R = Reaction at the lower and upper hinge

P = Reaction at the common contact surface of the two gates and acting perpendicular to the contact surface.

Let the force P and F meet at O. Then the reaction R must pass through O as the gate AB is in the equilibrium under the action of three forces. Let θ is the inclination of the lock gate with the normal to the side of the lock.

In $\angle ABO$, $\angle OAB = \angle ABO = \theta$.

Resolving all forces along the gate AB and putting equal to zero, we get

$$R \cos \theta - P \cos \theta = 0 \text{ or } R = P \quad ...(3.19)$$

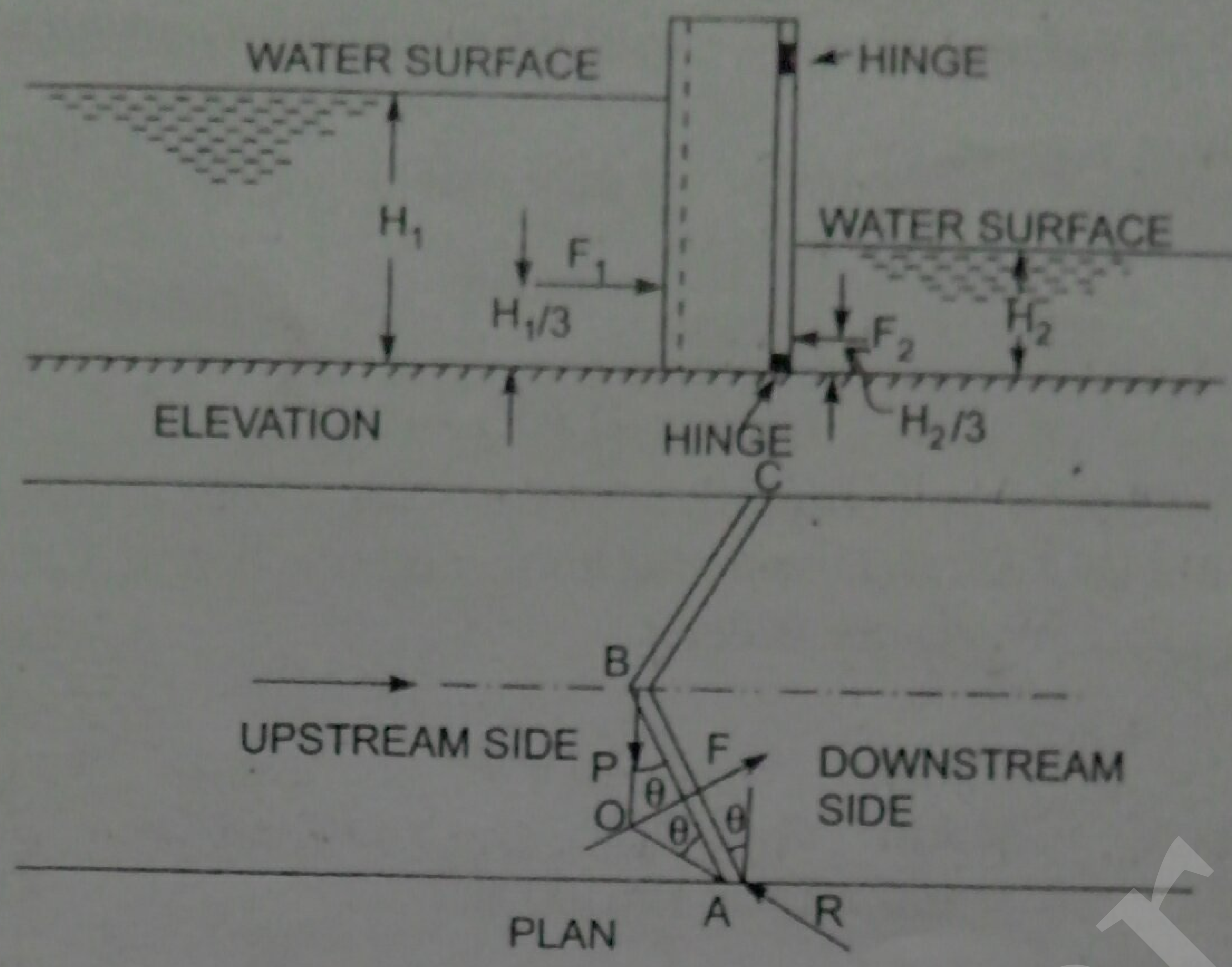


Fig. 3.40

Resolving forces normal to the gate AB

$$R \sin \theta + P \sin \theta - F = 0$$

or

$$F = R \sin \theta + P \sin \theta = 2P \sin \theta \quad [\because R = P]$$

$\therefore$

$$P = \frac{F}{2 \sin \theta} \quad ...(3.20)$$

To calculate P and R

In equation (3.20), P can be calculated if F and θ are known. The value of θ is calculated from the angle between the lock gates. The angle between the two lock gate is equal to $180° - 2\theta$. Hence θ can be calculated. The value of F is calculated as :

Let

H_1 = Height of water on the upstream side
H_2 = Height of water on the downstream side
F_1 = Water pressure on the gate on upstream side
F_2 = Water pressure on the gate on downstream side of the gate
l = Width of gate

Now

$$F_1 = \rho g A_1 \bar{h}_1$$
$$= \rho g \times H_1 \times l \times \frac{H_1}{2}$$
$$= \rho g l \frac{H_1^2}{2} \quad \left[\because A_1 = H_1 \times l, \bar{h}_1 = \frac{H_1}{2}\right]$$

Similarly,

$$F_2 = \rho g A_2 \bar{h}_2 = \rho g \times (H_2 \times l) \times \frac{H_2}{2} = \frac{\rho g l H_2^2}{2}$$

$\therefore$ Resultant force

$$F = F_1 - F_2 = \frac{\rho g l H_1^2}{2} - \frac{\rho g l H_2^2}{2}$$

Substituting the value of θ and F in equation (3.20), the value of P and R can be calculated.

Reactions at the top and bottom hinges

Let R_t = Reaction of the top hinge

R_b = Reaction of the bottom hinge

Then $R = R_t + R_b$

The resultant water pressure F acts normal to the gate. Half of the value of F is resisted by the hinges of one lock gates and other half will be resisted by the hinges of other lock gate. Also F_1 acts at a distance of $\frac{H_1}{3}$ from bottom while F_2 acts at a distance of $\frac{H_2}{3}$ from bottom.

Taking moments about the lower hinge

$$R_t \times \sin\theta \times H = \frac{F_1}{2} \times \frac{H_1}{3} - \frac{F_2}{2} \times \frac{H_2}{3} \quad \text{...(i)}$$

where H = Distance between two hinges

Resolving forces horizontally

$$R_t \sin\theta + R_b \sin\theta = \frac{F_1}{2} - \frac{F_2}{2} \quad \text{...(ii)}$$

From equations (i) and (ii), we can find R_t and R_b.

Problem 3.32 *Each gate of a lock is 6 m high and is supported by two hinges placed on the top and bottom of the gate. When the gates are closed, they make an angle of 120°. The width of lock is 5 m. If the water levels are 4 m and 2 m on the upstream and downstream sides respectively, determine the magnitude of the forces on the hinges due to water pressure.*

Solution. Given :

Height of lock = 6 m

Width of lock = 5 m

Width of each lock gate = AB

or $l = \frac{AD}{\cos 30°} = \frac{2.5}{\cos 30°}$

= 2.887 m

Angle between gates = 120°

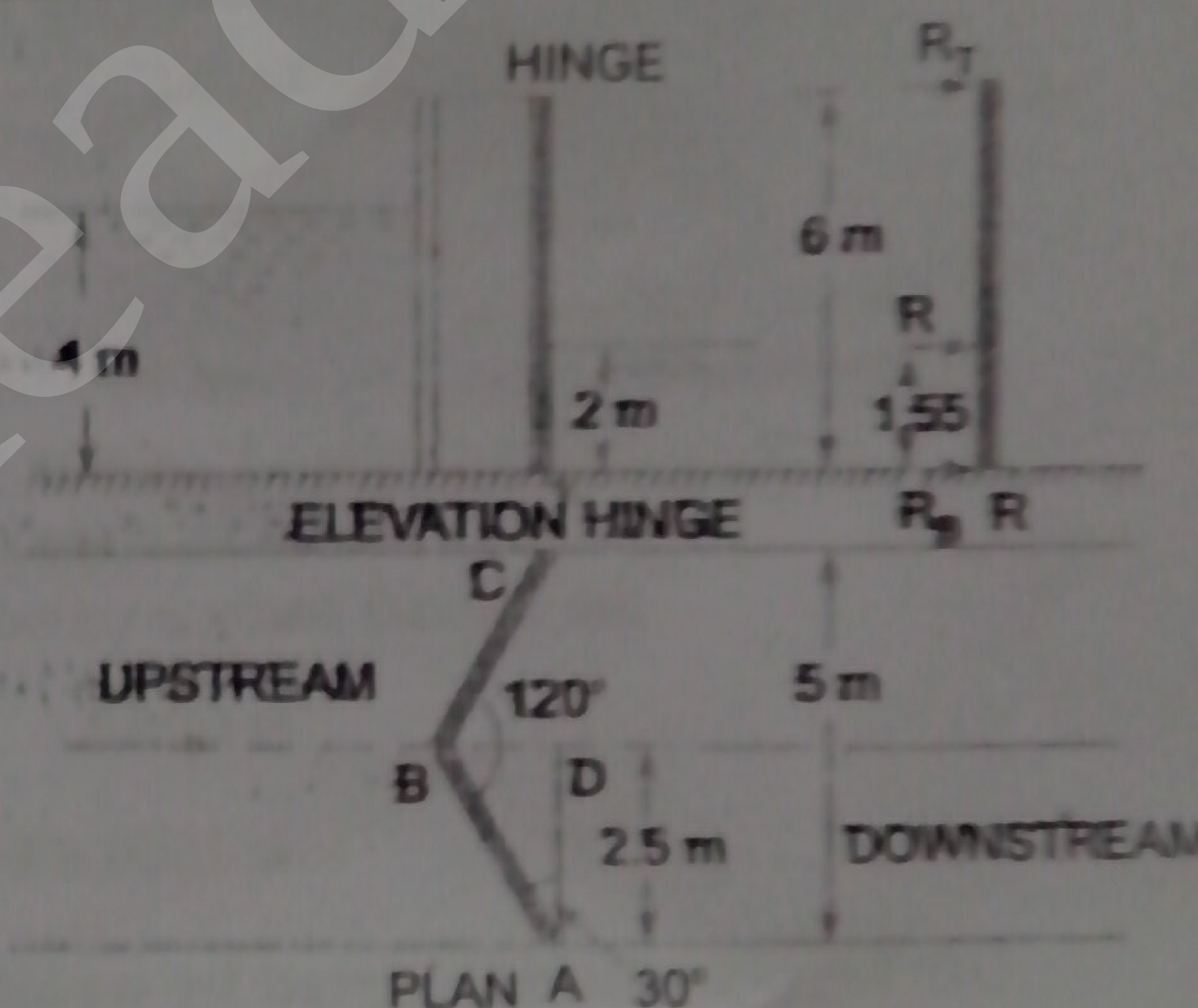


Fig. 3.41

∴ $\theta = \frac{180° - 120°}{2} = \frac{60°}{2} = 30°$

Height of water on upstream side

$H_1 = 4$ m

and $H_2 = 2$ m

∴ Total water pressure on upstream side

$$F_1 = \rho g A_1 \bar{h}_1, \text{ where } A_1 = H_1 \times l = 4.0 \times 2.887 \text{ m}^2$$

$$= 1000 \times 9.81 \times 4 \times 2.887 \times 2.0 \qquad \left\{\bar{h}_1 = \frac{H_1}{2} = \frac{4}{2} = 2.0 \text{ m}\right\}$$

$$= 226571 \text{ N}$$

Force F_1 will be acting at a distance of $\frac{H_1}{3} = \frac{4}{3} = 1.33$ m from bottom.

Similarly, total water pressure on the downstream side

$$F_2 = \rho g A_2 \bar{h}_2, \text{ where } A_2 = H_2 \times l = 2 \times 2.887 \text{ m}^2$$

$$= 1000 \times 9.81 \times 2 \times 2.887 \times 1.0 \qquad \bar{h}_2 = \frac{H_2}{2} = \frac{2}{2} = 1.0 \text{ m}$$

$= 56643$ N

F_2 will act at a distance of $\frac{H_2}{3} = \frac{2}{3} = 0.67$ m from bottom,

Resultant water pressure on each gate

$$F = F_1 - F_2 = 226571 - 56643 = 169928 \text{ N}.$$

Let x is height of F from the bottom, then taking moments of F_1, F_2 and F about the bottom, we have

$$F \times x = F_1 \times 1.33 - F_2 \times 0.67$$

or

$$169928 \times x = 226571 \times 1.33 - 56643 \times 0.67$$

$$\therefore \quad x = \frac{226571 \times 1.33 - 56643 \times 0.67}{169928} = \frac{301339 - 37950}{169928} = 1.55 \text{ m}$$

From equation (3.20), $P = \frac{F}{2 \sin \theta} = \frac{169928}{2 \sin 30} = 169928$ N.

From equation (3.19), $R = P = 169928$ N.

If R_T and R_B are the reactions at the top and bottom hinges, then $R_T + R_B = R = 169928$ N.

Taking movements of hinge reactions R_T, R_B and R about the bottom hinges, we have

$$R_T \times 6.0 + R_B \times 0 = R \times 1.55$$

$$\therefore \quad R_T = \frac{169928 \times 1.55}{6.0} = 43898 \text{ N}$$

$$\therefore \quad R_B = R - R_T = 169928 - 43898 = 126030 \text{ N. Ans.}$$

Problem 3.33 *The end gates ABC of a lock are 9 m high and when closed include an angle of 120°. The width of the lock is 10 m. Each gate is supported by two hinges located at 1 m and 6 m above the bottom of the lock. The depths of water on the two sides are 8 m and 4 m respectively. Find:*

(i) Resultant water force on each gate,

(ii) Reaction between the gates AB and BC, and

(iii) Force on each hinge, considering the reaction of the gate acting in the same horizontal plane as resultant water pressure.

Solution. Given :

Height of gate $= 9$ m

Inclination of gate $= 120°$

$$\therefore \quad \theta = \frac{180° - 120°}{2} = 30°$$

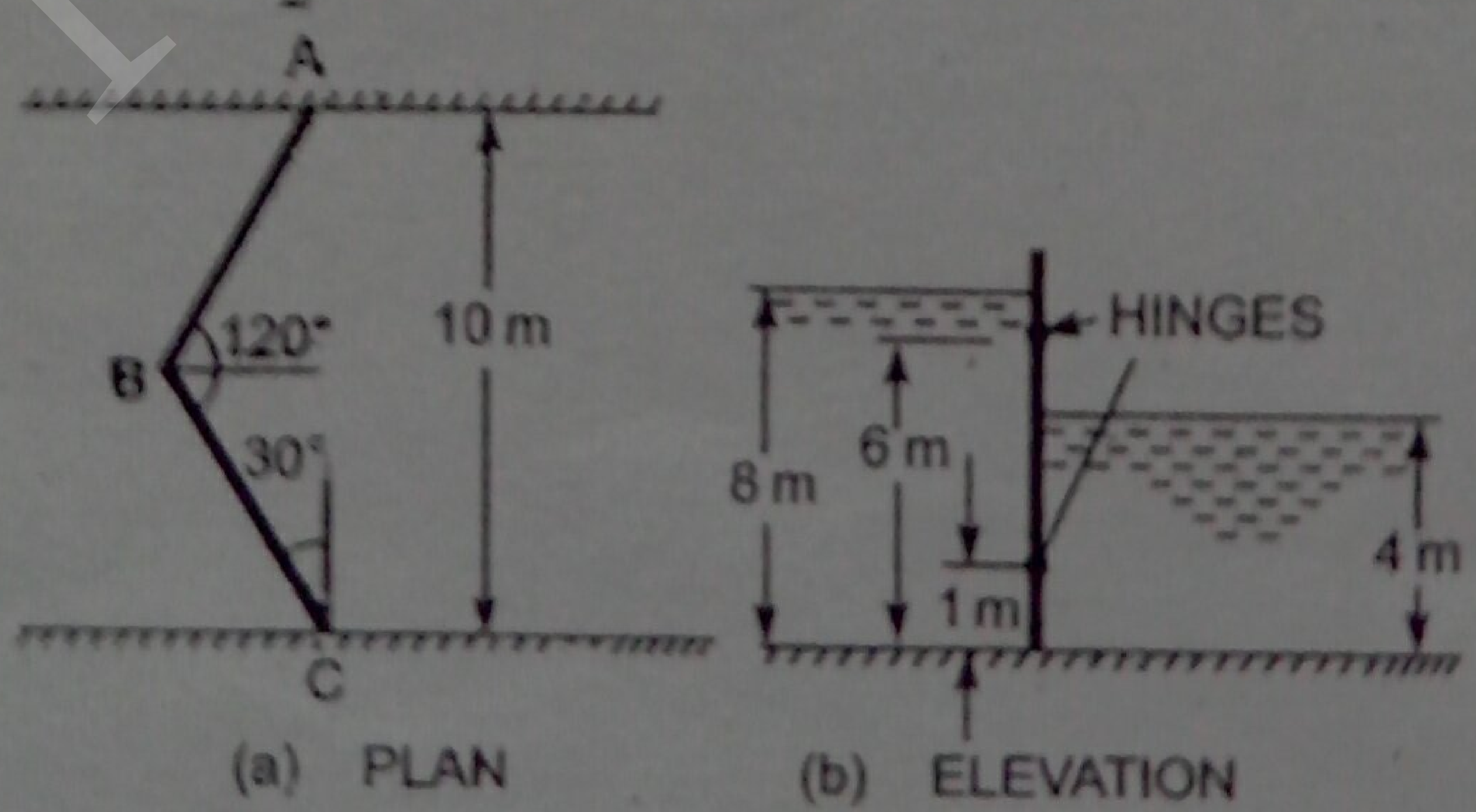


Fig. 3.42

Width of lock $= 10$ m

$\therefore$ Width of each lock $= \dfrac{5}{\cos 30^\circ}$ or $l = 5.773$ m

Depth of water on upstream side, $H_1 = 8$ m

Depth of water on downstream side, $H_2 = 4$ m

(*i*) **Water pressure on upstream side**

$$F_1 = \rho g A_1 \bar{h}_1$$

where $A_1 = l \times H_1 = 5.773 \times 8 = 46.184$ m, $\bar{h}_1 = \dfrac{H_1}{2} = \dfrac{8}{2} = 4.0$ m

$$F_1 = 1000 \times 9.81 \times 46.184 \times 4.0 = 1812260 \text{ N} = 1812.26 \text{ kN}$$

Water pressure on downstream side,

$$F_2 = \rho g A_2 \bar{h}_2$$

where $A_2 = l \times H_2 = 5.773 \times 4 = 23.092$ m, $\bar{h}_2 = \dfrac{4}{2} = 2.0$

$$F_2 = 1000 \times 9.81 \times 23.092 \times 2.0 = 453065 \text{ N} = 453.065 \text{ kN}$$

$\therefore$ Resultant water pressure

$$= F_1 - F_2 = 1812.26 - 453.065 = 1359.195 \text{ kN}$$

(*ii*) **Reaction between the gates AB and BC.** The reaction (P) between the gates AB and BC is given by equation (3.20) as

$$F = \frac{F}{2\sin\theta} = \frac{1359.195}{2 \times \sin 30^\circ} = \mathbf{1359.195\ kN.\ Ans.}$$

(*iii*) **Force on each hinge.** If R_T and R_B are the reactions at the top and bottom hinges then

$$R_T + R_B = R$$

But from equation (3.19), $R = P = 1359.195$

$$\therefore \quad R_T + R_B = 1359.195$$

The force F_1 is acting at $\dfrac{H_1}{3} = \dfrac{8}{3} = 2.67$ m from bottom and F_2 at $\dfrac{H_2}{3} = \dfrac{4}{3} = 1.33$ m from bottom.

The resultant force F will act at a distance x from bottom is given by

$$F \times x = F_1 \times 2.67 - F_2 \times 1.33$$

or

$$x = \frac{F_1 \times 2.67 - F_2 \times 1.33}{F} = \frac{1812.26 \times 2.67 - 453.065 \times 1.33}{1359.195}$$

$$= \frac{4838.734 - 602.576}{1359.195} = 3.116 = 3.11 \text{ m}$$

Hence R is also acting at a distance 3.11 m from bottom.

Taking moments of R_T and R about the bottom hinge

$$R_T \times [6.0 - 1.0] = R \times (x - 1.0)$$

$$\therefore \quad R_T = \frac{R \times (x - 1.0)}{5.0} = \frac{1359.195 \times 2.11}{5.0} = 573.58 \text{ N}$$

$$\therefore \quad R_B = R - R_T = 1359.195 - 573.58$$

$$= \mathbf{785.615\ kN.\ Ans.}$$