

LECTURE VII

Harvard Econ 2416
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OUTLINE

- 1 MICRO-TO-MACRO: THE PROBLEM
- 2 MICRO-TO-MACRO: SOLUTIONS
- 3 CONCLUSION

INTRODUCTION

Often have granular or disaggregated data where you can causally estimate impact of treatment in the cross-section but want to know aggregate impact of treatment. Some examples:

- Some firms borrow from good banks, others from bad banks.
 - ▶ Example: Chodorow-Reich (QJE 2014).
- House price growth increases collateral values for firms which own their plant but not for other firms.
 - ▶ Example: Chaney, Sraer, Thesmar (AER 2012).
- Government spending rises more in some regions than others.
 - ▶ Example: Chodorow-Reich (2019).
- House prices boom and bust more in some regions than others.
 - ▶ Example: Mian, Sufi (ECMA 2014).
- Others...

THREE AGGREGATION ISSUES

- ① Micro spillovers.
- ② Macro spillovers.
- ③ Endogenous responses of aggregate variables.

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$$ATE = E[Y_1(i) - Y_0(i)] = E[Y(i)|T = 1] - E[Y(i)|T = 0].$$

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 - ① Whether Jane takes drug affects Joe's blood pressure.
 - ② Offering investment incentive to firm 1 affects investment by firm 2.
- May transmit through prices (wages, interest rate, cost of capital) or not (agglomeration spillovers).

MACRO SPILLOVERS

- Suppose continuum of infinitesimal firms choose investment X and a single firm i receives an investment incentive.
- What firm i does has no (formally, measure 0) effect on average investment \bar{X} (why?).
- Estimated effect of incentive is $X_i - \bar{X}$.
- Suppose aggregate supply of capital goods is fixed. Then potentially large effect on firm i but zero effect if policy applied to all firms.
- Typically transmits through prices.

ENDOGENOUS RESPONSES

- Endogenous variables determined at the aggregate level react to aggregate variables.
- Example: monetary policy reacts to national GDP but not to GDP in one region. Government spending nationally may trigger monetary policy response whereas government spending in one region will not.
- Feature or bug? Maybe we want to know effect of policy or shock separate from effect of policy response.

FORMAL FRAMEWORK: POTENTIAL OUTCOMES

- Following Chodorow-Reich (JEDC, 2020).
- Rubin (1978) potential outcome:

$$Y_{i,t}^{\text{obs}} = Y_{i,t}(\mathbf{W}_t', W_{\text{agg},t}).$$

- ▶ $W_{i,t}$: treatment in unit (“region”) i .
 - ▶ $\mathbf{W}_t = (W_{1,t} \ \dots \ W_{N,t})'$.
 - ▶ $W_{\text{agg},t}$: uniform (“aggregate”) treatment.
 - ▶ $Y_{i,t}^{\text{obs}}$: actual outcome in unit i .
 - ▶ $Y_{i,t}(\cdot)$: potential outcome function.
- Dependence of $Y_{i,t}$ on $W_{j,t}$: SUTVA violation.
 - Let $\mathbf{1}_N$ be vector of ones and \mathbf{e}_i be unit vector with 1 in row i .

TREATMENT EFFECTS AND ESTIMANDS

- ① Aggregate effect of aggregate treatment:

$$\beta^{\text{agg}} = \sum_{j=1}^N \frac{Y_{j,t} ((\mathbf{W}_t + \iota_N \Delta)', \tilde{W}_{\text{agg},t}) - Y_{j,t} (\mathbf{W}_t', W_{\text{agg},t})}{N\Delta}.$$

- ② Aggregate effect of treatment in i :

$$\beta^{\text{all regions}} = \sum_{j=1}^N \frac{Y_{j,t} ((\mathbf{W}_t + \iota_i \Delta)', W_{\text{agg},t}) - Y_{j,t} (\mathbf{W}_t', W_{\text{agg},t})}{\Delta}.$$

- ③ Effect on i of treatment in i :

$$\beta^{\text{micro}} = \frac{Y_{i,t} ((\mathbf{W}_t + \iota_i \Delta)', W_{\text{agg},t}) - Y_{i,t} (\mathbf{W}_t', W_{\text{agg},t})}{\Delta}.$$

- ④ Diff-in-diff estimator:

$$\hat{\beta}^{\text{DiD}} = \frac{\left[Y_{i,t+1}^{\text{obs}} - Y_{i,t}^{\text{obs}} \right] - \frac{1}{N-1} \sum_{j \neq i}^N \left[Y_{j,t+1}^{\text{obs}} - Y_{j,t}^{\text{obs}} \right]}{\Delta}.$$

β^{agg} VERSUS $\beta^{\text{all regions}}$

- Claim: with linear treatment effects and symmetry across regions, that is, if $Y_{i,t} = \alpha_i + \eta W_{i,t} + \gamma \sum_{j \neq i}^N W_{j,t} + \delta W_{\text{agg},t}$, then $\beta^{\text{all regions}}$ differs from β^{agg} only because of the change in $W_{\text{agg},t}$.
- Proof:

$$\begin{aligned}\beta^{\text{agg}} &= \frac{1}{N\Delta} \sum_{i=1}^N \left[\left(\alpha_i + \eta(W_{i,t} + \Delta) + \gamma \sum_{j \neq i} (W_{j,t} + \Delta) + \delta \tilde{W}_{\text{agg},t} \right) \right. \\ &\quad \left. - \left(\alpha_i + \eta W_{i,t} + \gamma \sum_{j \neq i}^N W_{j,t} + \delta W_{\text{agg},t} \right) \right] \\ &= \frac{1}{N\Delta} \sum_{i=1}^N [\eta\Delta + \gamma(N-1)\Delta + \delta(\tilde{W}_{\text{agg},t} - W_{\text{agg},t})] \\ &= \underbrace{\eta + \gamma(N-1)}_{\beta^{\text{all regions}}} + \delta \left(\frac{\tilde{W}_{\text{agg},t} - W_{\text{agg},t}}{\Delta} \right).\end{aligned}$$

- Economics: spending in one region only “scales up.”

β^{DiD} VERSUS β^{MICRO}

- Assume:
 - ① Potential outcome function remains stable between t and $t+1$ across regions (unconfounded of treatment).
 - ② Separability in potential outcome between \mathbf{W}_t and $W_{\text{agg},t}$.

$$\text{Then } \hat{\beta}^{\text{DiD}} - \beta^{\text{micro}} = -\frac{1}{(N-1)\Delta} \sum_{j \neq i}^N \left[Y_{j,t} \left((\mathbf{W}_t + \iota_j \Delta)' , W_{\text{agg},t} \right) - Y_{j,t} \left((\mathbf{W}_t)' , W_{\text{agg},t} \right) \right].$$

- Proof: using $W_{i,t+1} = W_{i,t} + \Delta$ and $W_{j,t+1} = W_{j,t}$,
$$\hat{\beta}^{\text{DiD}} \Delta = \left[Y_{i,t+1} \left((\mathbf{W}_t + \iota_i \Delta)' , W_{\text{agg},t+1} \right) - Y_{i,t} \left((\mathbf{W}_t)' , W_{\text{agg},t} \right) \right] - \frac{1}{N-1} \sum_{j \neq i}^N \left[Y_{j,t+1} \left((\mathbf{W}_t + \iota_j \Delta)' , W_{\text{agg},t+1} \right) - Y_{j,t} \left((\mathbf{W}_t)' , W_{\text{agg},t} \right) \right].$$
Unconfoundedness $\Rightarrow Y_{i,t+1}(\cdot) = Y_{i,t}(\cdot)$.
- Economics: β^{micro} estimable in the data if *average spillover* \rightarrow zero.

EXAMPLE: LABOR ALLOCATION WITH SUBSIDIES

- Revenue function: $Y_i = (1 + \tau_i)L_i^{1-\frac{1}{\gamma}}$.
- Labor perfectly mobile and wage equals marginal product:
$$L_i^{\text{obs}} = L_i \left(\tau_1 \quad \dots \quad \tau_N \right) = \frac{(1+\tau_i)^\gamma}{\sum_{j=1}^N (1+\tau_j)^\gamma}$$
- True elasticity of labor response in i : $\beta^{\text{micro}} = \frac{d \ln L_i}{d \tau_i} = \gamma(N-1)/N$.
- Difference-in-difference estimator: $\hat{\beta}^{\text{DiD}} = \frac{d \ln L_i}{d \tau_i} - \frac{1}{N-1} \sum_{j \neq i}^N \frac{d \ln L_j}{d \tau_i} = \gamma$.
- Difference due to SUTVA-micro violation: $\frac{d \ln L_j}{d \tau_i} \neq 0$.
- SUTVA violation $\rightarrow 0$ as $N \rightarrow \infty$ (infinitesimal areas).
- $\beta^{\text{all regions}}$ different from β^{micro} :
$$\beta^{\text{all regions}} = \frac{d \ln \left(\sum_{j=1}^N L_j \right)}{d \tau_i} = \left(\frac{N-1}{N} \right) \gamma - (N-1) \left(\frac{1}{N} \right) \gamma = 0$$
- Summing many *de minimis* spillovers results in measurable effect.
- Economics: labor supply to individual region (nearly) perfectly elastic, while labor supply in aggregate perfectly inelastic.

EXAMPLE: KEYNESIAN SPENDING MULTIPLIER

- N regions of size $1/N$.
- Nominal output, consumption, government purchases in (level) deviation from steady state: y_i, c_i, g_i .
- Representative household in each region allocates $1 - \alpha$ of expenditure to local output, $\alpha/(N - 1)$ to each other region.
- Marginal propensity to consume of ρ .
- Bold denotes vector across regions.
- Allocation:

$$\mathbf{y} = \mathbf{B}\mathbf{g},$$

where:

$$b_{i,j} = \begin{cases} \frac{1}{m} + \frac{1}{Nm - \frac{N-1}{\alpha\rho} m^2}, & i = j, \\ \frac{1}{Nm - \frac{N-1}{\alpha\rho} m^2}, & i \neq j, \end{cases}$$

$$m = 1 - \rho(1 - \alpha) + \alpha\rho/(N - 1).$$

- Potential outcome function: $y_i^{\text{obs}} = y_i(g_1 \dots g_N) = \sum_{j=1}^N b_{i,j} g_j$.

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- Potential outcome function: $y_i^{\text{obs}} = y_i(g_1 \dots g_N) = \sum_{j=1}^N b_{i,j} g_j$.
- Linear-symmetric property.
- $N \rightarrow \infty$:
 - ▶ $\beta^{\text{micro}} = \frac{1}{1-\rho(1-\alpha)}$.
 - ▶ SUTVA-micro holds: $\hat{\beta}^{\text{DiD}} \rightarrow \beta^{\text{micro}}$.
 - ▶ $\beta^{\text{all regions}} = \sum_{i=1}^N b_{i,j} g_j = \frac{1}{1-\rho} g_j = \beta^{\text{agg}}$.

SUMMARY

- Effect of local treatment on local region (β^{micro}) estimable with standard applied micro toolkit.
 - ▶ But beware non-negligible SUTVA violations in estimating β^{micro} . May not want too fine fixed effects.
- Under benchmark conditions, $\beta^{\text{all regions}} = \beta^{\text{agg}}$ with fixed treatment.
 - ▶ How reasonable in real world conditions?
- Silent so far on β^{micro} versus $\beta^{\text{all regions}}$.

RELATIONSHIP TO REFLECTION PROBLEM

- Manski (1993) eq. (1): $Y_i = \alpha + \beta E[Y_j] + \gamma E[W_j] + \eta W_i + \epsilon_i$.
 - ▶ “Linear-in-means” model.
 - ▶ Same as previous if $\beta Y = \delta W_{\text{agg},t}$.
 - ▶ Assume away correlated errors problem ($E[\epsilon_i \epsilon_j] \neq 0$) in Manski.
- Take expectations of both sides: $E[Y_j] = \frac{\alpha}{1-\beta} + \frac{\gamma+\eta}{1-\beta} E[W_j]$.
- Substitute back: $Y_i = \frac{\alpha}{1-\beta} + \frac{\gamma+\beta\eta}{1-\beta} E[W_j] + \eta W_i + \epsilon_i$.
- Reflection problem: β, γ not separately identified. Big deal in peer effects (why?).
- In our case, η identifiable in cross-section and we want to learn about $(\gamma + \beta\eta)/(1 - \beta)$.
- Peer effects literature has not really penetrated micro-to-macro.

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- **Direct summation**
- Estimate spillovers
- Network aggregation
- Use theory to derive GE bound
- Recover PE elasticity
- Local nontradables versus tradables
- Look for sufficient statistic
- Restrict elasticity in aggregate system
- Match moment in structural model

3 CONCLUSION

DIRECT SUMMATION OF EMPIRICAL RESPONSES

- Suppose you estimate regression:

$$y_i = \beta_0 + \beta_1 x_i + e_i.$$

- ▶ x_i : continuous variable with $E[x_i e_i] = 0$.
- Let \bar{x} denote the level of x at which treatment is satiated.
- Let $y_i(w)$ denote potential outcome for i with treatment $x_i = w$.
- Under SUTVA-micro:

$$y_i(x_i) - y_i(\bar{x}) = \begin{cases} \beta_1 (x_i - \bar{x}), & x_i \leq \bar{x}, \\ 0, & x_i > \bar{x}. \end{cases}$$

- Sum over i to get direct effect:

$$\text{Direct} = \sum_{x_i < \bar{x}} \beta_1 \times (x_i - \bar{x}).$$

- β_0 contains average outcome for untreated observations. If no other shocks, then β_0 estimates spillover (“missing intercept”).

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SPILLOVER SPECIFICATION

- For units $i = 1, \dots, N$:

$$y_i^{obs} = \beta_0 + \beta_1 x_i + \sum_{j \neq i} \delta_j x_j + e_i.$$

Do $\{\delta_j\}$ measure the spillovers? What is the problem?

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- N observations and $N + 1$ parameters.
- Need to constrain δ s.

MULTI-LAYERED TREATMENT

- Unit i in group g : $y_{i,g}^{obs} = \beta_1 x_{i,g} + \delta \bar{x}_g + e_{i,g}$, where \bar{x}_g is average treatment in group g .
- If $x_{i,g}$ random within g and \bar{x}_g random across g , then estimable.
- What is δ ? The local spillover.
- Requires $E[e_{i,g} e_{j,g}] \perp \bar{x}_g$ (Manski, 1993).
- Referred to in experimental literature as partial interference (Sobel, JASA 2006).
- Practical concern if $x_{i,g}$ measured with error (Moffitt, 2001).
- Generalization: $y_{i,g}^{obs} = \beta_1 x_{i,g} + \delta \bar{x}_g + \gamma x_{i,g} \bar{x}_g + e_{i,g}$.
- See e.g. Baird, Bohren, McIntosh, Ozler (RESTAT 2018); Huber (AER, 2018); Berg, Streitz (WP); Egger, Haushofer, Miguel, Niehaus, Walker (WP).

Figure 1: Study design and timeline

(a) Randomization

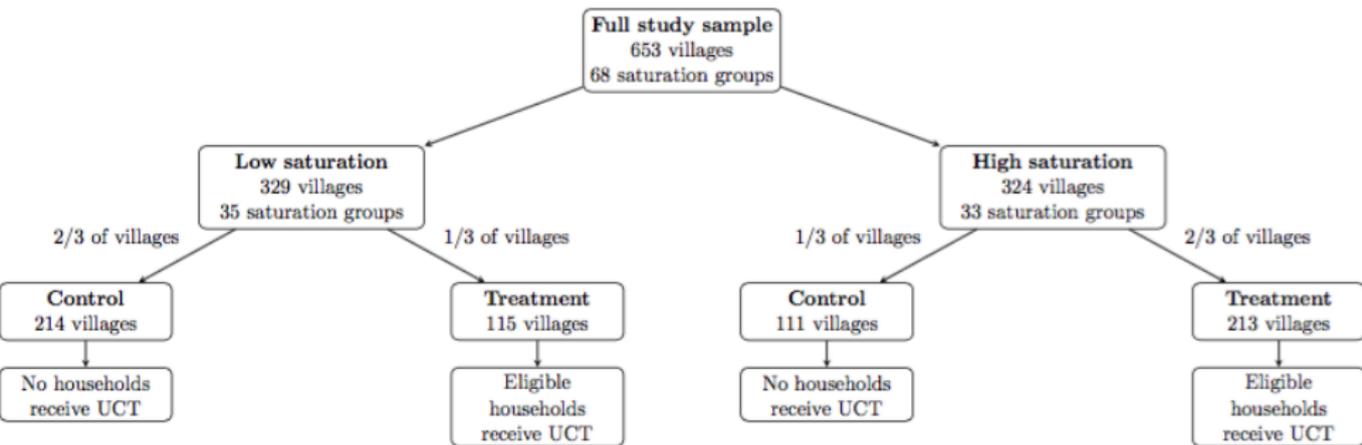


Figure 2: Study area

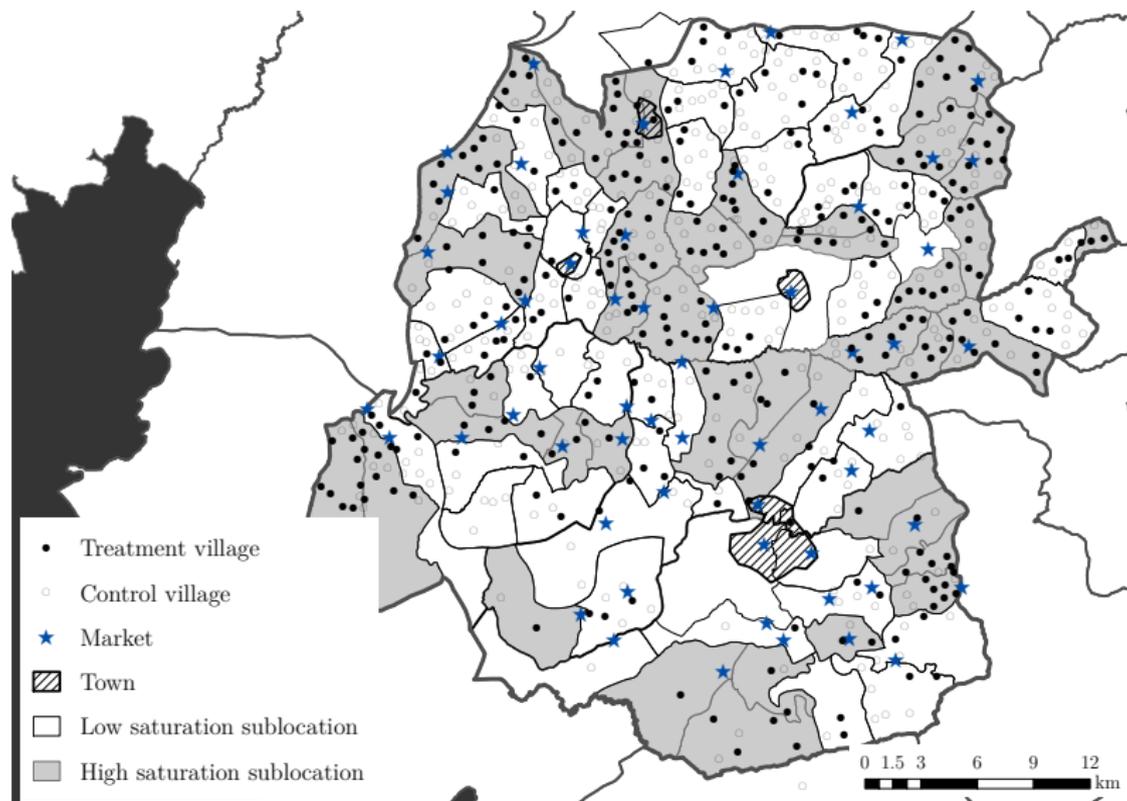


Table 1: Expenditures, Savings and Income

	(1)	(2)	(3)	(4)
	Treated Households		Untreated Households	
	1 (Treat village)	Total Effect	Total Effect	Control, low saturation
	Reduced form	IV	IV	mean (SD)
<i>Panel A: Expenditure</i>				
Household expenditure, annualized	292.98*** (60.09)	343.34*** (112.02)	333.66*** (123.22)	2,536.86 (1,934.09)
Non-durable expenditure, annualized	174.99*** (55.41)	211.90** (96.75)	288.46*** (111.44)	2,402.43 (1,801.59)
Food expenditure, annualized	71.61* (36.93)	138.57** (66.75)	132.81** (58.57)	1,578.43 (1,072.31)
Temptation goods expenditure, annualized	6.51 (5.79)	4.48 (9.17)	-0.71 (6.50)	37.10 (123.59)
Durable expenditure, annualized	95.18*** (12.64)	106.29*** (21.44)	8.40 (12.50)	59.44 (230.90)

ALTERNATIVE

- ① $\delta_j \propto d(i,j)$ where $d(i,j)$ measures “distance” between units i and j .
- ② Related to spatial economics literature.
- ③ Distance metric should be theoretically grounded.

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OVERVIEW

- Use explicit theory of relationships among units to aggregate.
- Many recent theory papers: Acemoglu, Carvalho, Ozdaglar, Tahbaz-Salehi (AER, 2012); Baquee and Farhi (QJE, 2020), etc.
- Following from Carvalho, Nirei, Saito, Tahbaz-Salehi (QJE, 2021), “Supply Chain Disruptions: Evidence from the Great East Japan Earthquake.”

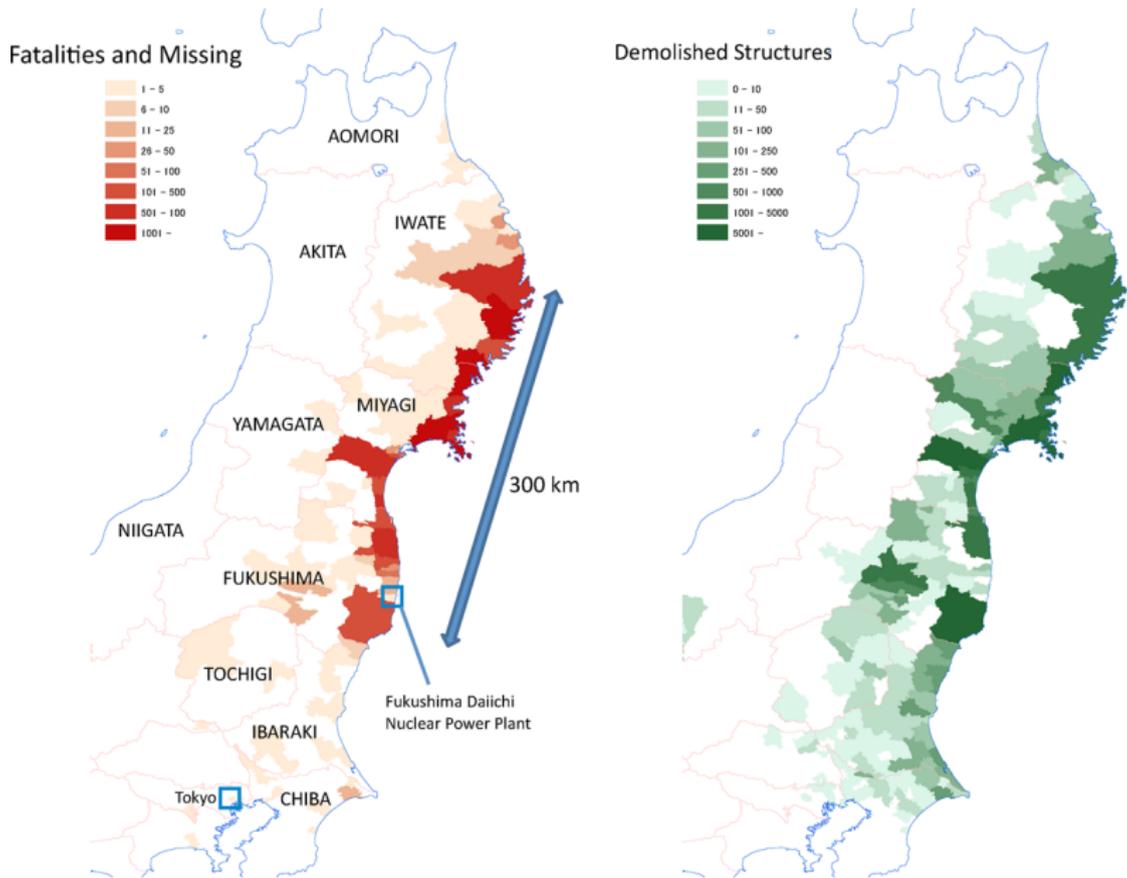


FIGURE I

Geographical Distribution of Losses and Damages in Northeast Japan

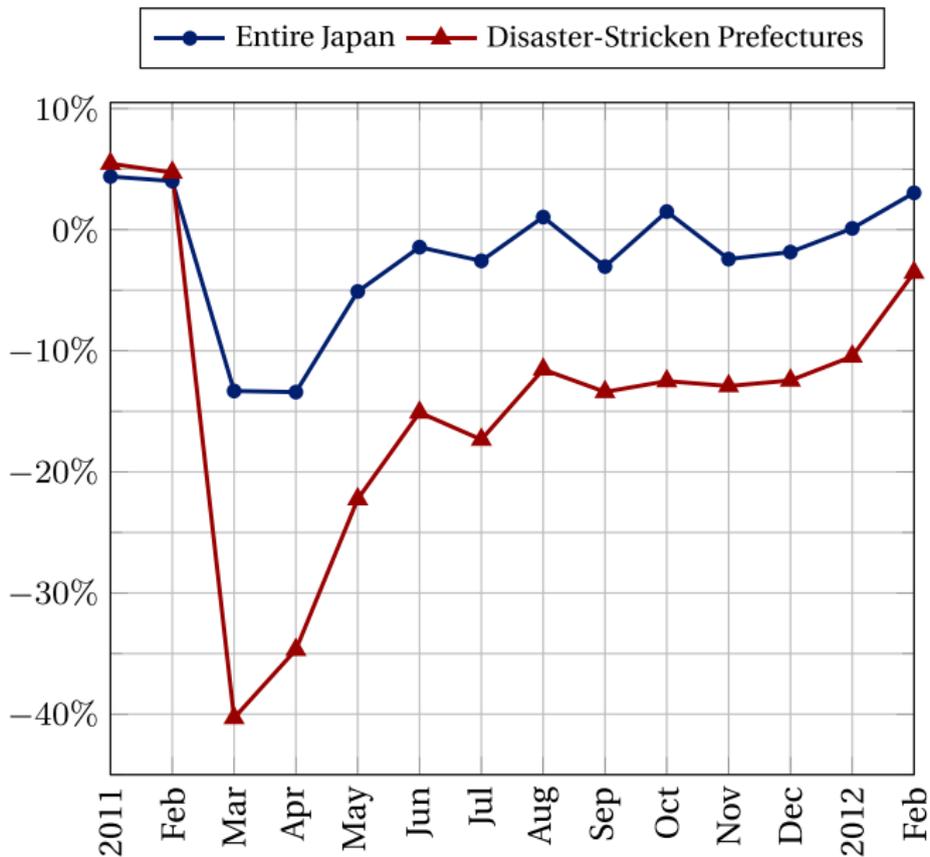


FIGURE II

Growth Rate of Index of Industrial Production

DATA AND REDUCED-FORM SPECIFICATION

- Tokyo Shoko Research (TSR): credit agency firm-level data containing location, sales, employees, industry, etc.
- Binary listing of most important customers and suppliers.
- Other countries: VAT network of customers and suppliers.
- Specification:
$$y_{ipst} = \gamma_i + \gamma_{pst} + \sum_{k=1}^4 \sum_{\tau \neq 2011} \beta_{k,\tau}^{\text{down}} \times \text{Down}_i^{(k)} \times \text{year}_\tau + \sum_{k=1}^4 \sum_{\tau \neq 2011} \beta_{k,\tau}^{\text{up}} \times \text{Up}_i^{(k)} \times \text{year}_\tau + \text{Controls} + \epsilon_{ispt}.$$
 - ▶ y_{ipst} : log sales of firm i in prefecture p and industry s in year t .
 - ▶ $\text{Down}_i^{(k)}$: firms is k links downstream from firm in earthquake zone.
 - ▶ $\text{Up}_i^{(k)}$: firms is k links upstream from firm in earthquake zone.

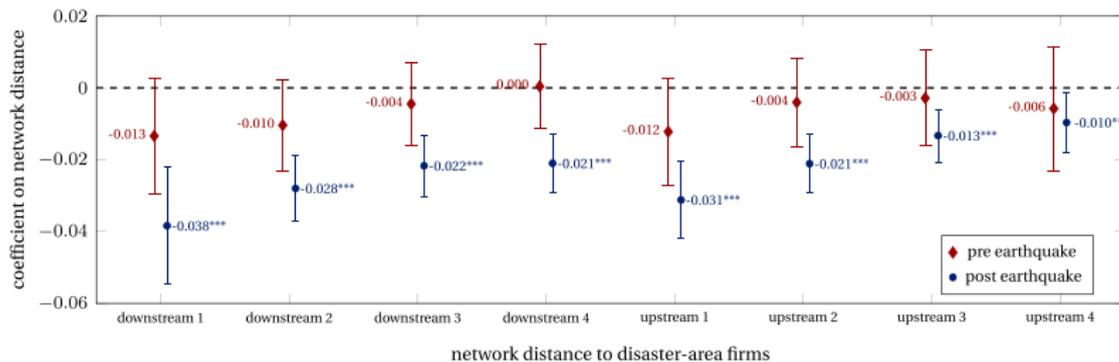


FIGURE V

Propagation of the Shock over the Production Network: Baseline Specification

This figure is based on a single panel regression, in which the dependent variable is the log of firms' annual sales. The plotted point estimates are the coefficients on network distance dummies interacted with annual dummy variables. The diamonds (in red) and circles (in blue) indicate the coefficients on network distance dummies interacted with 2010 and 2012 time dummies, respectively. The vertical error bars indicate 95% confidence intervals based on two-way clustered standard errors at the level of prefecture and industry. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. The regression includes firm and prefecture-industry-year fixed effects and a set of control variables consisting of the log of the number of transaction partners, age, logarithm of the number of employees, and distance to the disaster area. All control variables are measured for the year 2010 and interacted with 2010 and 2012 time dummies.

MODEL

Prod fxn:
$$y_i = \left[\chi(1 - \mu)^{\frac{1}{\sigma}} \left((z_i k_i)^\alpha l_i^{1-\alpha} \right)^{\frac{\sigma-1}{\sigma}} + \mu^{\frac{1}{\sigma}} M_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

Materials:
$$M_i = \left[\sum_{j=1}^n a_{ij}^{\frac{1}{\xi}} x_{ij}^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}$$

I-O matrix: $\mathbf{A} = \{a_{ij}\}$

Leontief inv: $\mathbf{L} = (\mathbf{I} - \mu\mathbf{A})^{-1} = \{\ell_{ij}\}$

Domar weight: $\lambda_i = p_i y_i / GDP$

Other elements: final good expenditure weight β_i , labor fixed. Note that z_i is a firm-specific capital shock, e.g. earthquake-induced depreciation.

PROPOSITION 1. The impact of a shock to firm j on the sales share of firm i is given by

$$\begin{aligned}
 \frac{d \log \lambda_i}{d \log z_j} &= (\sigma - 1) \sum_{h=1}^n \alpha \mu (1 - \mu) \frac{\lambda_h}{\lambda_i} \\
 &\times \left[\left(\sum_{r=1}^n a_{hr} \ell_{rj} \right) \left(\sum_{s=1}^n a_{hs} \ell_{si} \right) - \ell_{hj} \sum_{r=1}^n a_{hr} \ell_{ri} \right] \\
 &+ (\xi - 1) \sum_{h=1}^n \alpha \mu (1 - \mu) \frac{\lambda_h}{\lambda_i} \\
 (3) \quad &\times \left[\sum_{r=1}^n a_{hr} \ell_{ri} \ell_{rj} - \left(\sum_{r=1}^n a_{hr} \ell_{rj} \right) \left(\sum_{s=1}^n a_{hs} \ell_{si} \right) \right],
 \end{aligned}$$

$$\begin{aligned}\log(p_i y_i) &= \log(p_i^{\text{ss}} y_i^{\text{ss}}) + (\log \text{GDP} - \log \text{GDP}^{\text{ss}}) \\ &\quad + (\sigma - 1)\Sigma_i + (\xi - 1)\Xi_i,\end{aligned}$$

where GDP^{ss} and $p_i^{\text{ss}} y_i^{\text{ss}}$ are the steady-state (i.e., pre-shock) levels of aggregate output and sales of firm i , respectively, and Σ_i and Ξ_i denote the i th elements of vectors

$$(5) \quad \Sigma = \alpha\mu(1 - \mu)\Lambda^{-1}\mathbf{L}'(\mathbf{A}'\Lambda\mathbf{A} - \mathbf{A}'\Lambda)\mathbf{L} \Delta\log z$$

$$(6) \quad \Xi = \alpha\mu(1 - \mu)\Lambda^{-1}\mathbf{L}'(\text{diag}(\mathbf{A}'\Lambda\mathbf{1}) - \mathbf{A}'\Lambda\mathbf{A})\mathbf{L} \Delta\log z,$$

and $\Lambda = \text{diag}(\lambda_1^{\text{ss}}, \dots, \lambda_n^{\text{ss}})$ is the diagonal matrix of firms' pre-shock Domar weights. Therefore, provided we can measure the vectors in [equations \(5\) and \(6\)](#), the elasticities of substitution σ and ξ can be estimated using the following specification:

$$(7) \quad \log(p_{it}y_{it}) = \gamma_i + \gamma_t + \beta_1(\Sigma_i \times \text{year}_{2012}) + \beta_2(\Xi_i \times \text{year}_{2012}) + \varepsilon_{it},$$

TABLE III
ESTIMATION OF ELASTICITIES OF SUBSTITUTION

	Baseline (1)	Capital destruction rate		Intermediate input share		Uniform- weighted network (6)
		Low (2)	High (3)	Low (4)	High (5)	
σ	0.595*** (0.062)	0.400*** (0.091)	0.694*** (0.047)	0.556*** (0.077)	0.639*** (0.051)	0.570*** (0.062)
ξ	1.183*** (0.034)	1.271*** (0.051)	1.138*** (0.026)	1.287*** (0.049)	1.111*** (0.024)	1.155*** (0.034)

Σ, Ξ : Λ from sales shares; α, μ from input and capital income shares; \mathbf{A}, \mathbf{L} from TSR + aggregate input-output; dz_i from estimates of capital destruction in earthquake regions.

AGGREGATE IMPACT 2ND ORDER APPROXIMATION

$$\begin{aligned} \Delta \log \text{GDP} &= \alpha(1 - \mu)\mathbf{1}' \left(\frac{\mathbf{\Lambda} + \mathbf{\Lambda}^*}{2} \right) \Delta \log z + \frac{1}{2} \alpha^2 \mu(1 - \mu)(\sigma - 1) \\ (8) \quad &\times \Delta \log z' \mathbf{\Lambda}(\mathbf{I} - \mathbf{A})\mathbf{L} \Delta \log z, \end{aligned}$$

OUTLINE

1 MICRO-TO-MACRO: THE PROBLEM

2 MICRO-TO-MACRO: SOLUTIONS

- Direct summation
- Estimate spillovers
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3 CONCLUSION

USE THEORY TO DERIVE GE BOUND

- Sometimes theory offers guidance on whether macro elasticity is larger or smaller than micro elasticity.
- If inequality is the "right sign", then cross-sectional evidence provides interesting bound for aggregate effect.
- Example: cross-sectional fiscal multiplier as in Nakamura and Steinsson (2014).

THEORETICAL MAPPING

- Two differences between cross-sectional multiplier and closed economy multiplier:
 - ① In (almost all) cross-sectional studies, spending in local area does not require raising local taxes. E.g. spending paid by federal government.
 - ② Financed local spending in one region of currency union different from spending in closed economy.
- Thought experiment: local government issues debt to pay for spending. Then federal government buys the debt and cancels it.
- Argument in Chodorow-Reich (2019): transfer multiplier is small. Currency union multiplier lower bound if comparison to closed economy zero lower bound multiplier.

ARGUMENT OVERVIEW

- Closed economy consists of unit continuum of areas.
- At time t a new path of government spending is announced for local area s : $\Delta G_{s,t+j} = e^{-\rho j} \Delta G_{s,t}$.
- Cross-sectional multiplier:

$$\beta_h^{xs} = \frac{(Y_{s,t+h} - Y_{s,t}) - (Y_{t+h} - Y_t)}{\Delta G_{s,t}}.$$

- Claim: β_h^{xs} lower bound for closed economy, passive monetary policy, temporary, deficit-financed multiplier.
 - ① Relation between $\beta_h^{xs,transfer-financed}$ and $\beta_h^{xs,deficit-financed}$.
 - ② Relation between $\beta_h^{xs,deficit-financed}$ and closed economy multiplier.
- Formal treatment as in Shoag (2015); Nakamura and Steinsson (2014); Farhi and Werning (2016).

OUTSIDE-FINANCED MULTIPLIERS

- Outside-financed spending = locally-financed spending + transfer.
- Corollary: $\beta_h^{xs, transfer-financed} \neq \beta_h^{xs, deficit-financed}$ to the extent agents react to transfer.
 - ① Ricardian case: reaction small if spending is transitory.
 - ② Non-Ricardian case: no reaction to transfer.

RICARDIAN ENVIRONMENT

- Representative household in each local area with preferences:

Intertemporal preferences:
$$U_t = \int_t^{\infty} e^{-r\tau} \left(\ln C_\tau - \frac{1}{1+\phi} L_\tau^{1+\phi} \right) d\tau,$$

Home consumption:
$$C_t = C_{H,t}^{1-\alpha} M_{H,t}^\alpha,$$

Imported consumption:
$$\ln M_{H,t} = \int_0^1 \ln M_{H,t}^j dj.$$

- Local area budget constraint:

NFA:
$$\dot{N}_t = (P_{H,t}(Y_{H,t} - G_{H,t}) - P_t C_t) + i_t N_t.$$

- Nominal interest rate fixed at rate of time preference: $i_t = r.$
- Experiment: one-time transfer at date t to pay for spending.

RICARDIAN TRANSFER MULTIPLIER

- Present value of transfer: $V = 1/(r + \rho)\Delta G_{s,t}$.

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- Alternative derivation: domestic income \uparrow by $rV + \left(\frac{1 - \alpha}{\alpha}\right) rV = \frac{1}{\alpha} rV$, exactly enough to induce residents purchase rV of foreign output.

RICARDIAN TRANSFER SUMMARY

- Response of expenditure on local output to transfer:

$$\beta^{transfer, nominal} V = \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{r}{r+\rho} \right) \Delta G_{s,t}.$$

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- Implication: $\beta_h^{xs,transfer-financed} \approx \beta_h^{xs,deficit-financed}.$

NON-RICARDIAN AGENTS

- Non-ricardian agent does not care about source of financing as long as it does not come from current taxes.
 - ▶ Life-cycle agents.
 - ▶ Borrowing-constrained or hand-to-mouth.
 - ▶ Myopic.
- Important to compare $\beta_h^{xs,transfer-financed}$ to $\beta_h^{xs,deficit-financed}$ rather than $\beta_h^{xs,tax-financed}$.
- Exact equivalence between other future agents paying for spending and other current agents paying for spending.

LOCAL VERSUS NATIONAL MULTIPLIER: MONETARY POLICY

- Well known that government spending multiplier depends on reaction of monetary policy (Woodford, 2011; Christiano, Eichenbaum, Rebelo, 2011).
- Monetary policy does not react to spending in local area.
- Proper comparison of local multiplier is to national, fixed-nominal-interest-rate multiplier.
- Important leading case: zero lower bound multiplier.

LOCAL VERSUS NATIONAL MULTIPLIER: EXPENDITURE-SWITCHING AND INCOME EFFECTS

- Expenditure switching: increase in price of domestic output shifts expenditure toward output produced in other areas.
- Income: increase in local income causes domestic residents to increase expenditure, which partly “leaks” abroad.
- Neither effect present for increased government spending in closed economy.
- Reduce local multiplier relative to national multiplier.

LOCAL VERSUS NATIONAL MULTIPLIER: FACTOR MOBILITY

- Higher local area income can cause in-migration from other areas.
- Effect not present for closed economy.
- Raises local multiplier relative to national multiplier.
- Quantitatively small for temporary spending due to fixed costs of moving (Shoag, 2015; Nakamura and Steinsson, 2014).

SUMMARY

- Main reasons local multiplier differs from closed economy multiplier:
 - ① Monetary policy unreactive \Rightarrow compare to zero lower bound multiplier.
 - ② Expenditure switching and income effects: local multiplier $<$ closed economy multiplier.
 - ③ Migration: local multiplier $>$ closed economy multiplier but small for transitory spending.
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- Chodorow-Reich (2019): mean cross-study cross-sectional output multiplier is 1.8.
- Theoretically, empirically, and policy relevant result.
- Lower bound result applies to many settings with regional shocks and tradable goods.

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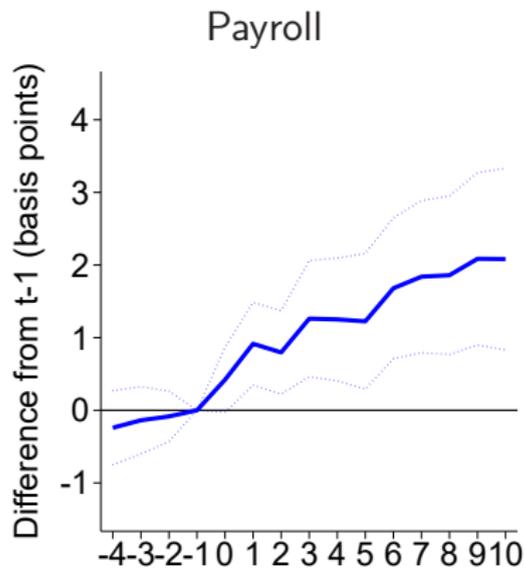
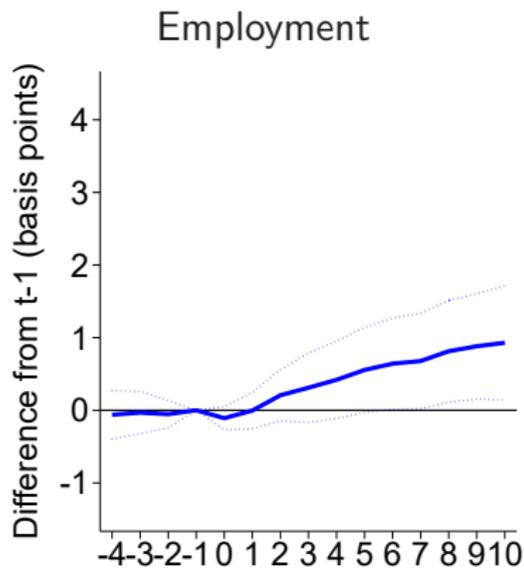
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DECOMPOSITION

- Somewhat generically, local outcome = partial equilibrium response \times local multiplier.
- Example (Guren, McKay, Nakamura, Steinsson, RESTUD, forthcoming): local consumption response to house price change equals MPC out of housing wealth \times local multiplier.
- Example (Chodorow-Reich, Nenov, Simsek, AER, forthcoming): local employment response in nontradable sector equals MPC out of stock wealth \times labor market factor \times local multiplier.
- Key insight: local multiplier is *same* as local government purchases multiplier.
- Wolf (WP) formalizes relationship in dynamic setting. Equality requires same fiscal as private impulse, same financing, same monetary policy response, etc.

CRNS: ALL INDUSTRIES IRF TO STOCK RETURN



THEORY OVERVIEW

- Infinite horizon: $t = 0$ is the "short run" and $t \geq 1$ the "long run".
- Continuum of areas. Capital/labor. **Nontradables/tradables.**
- Capital ownership (stock wealth) is heterogeneous across areas.
- Areas have stock holders (mass $1 - \theta$) and hand-to-mouth (mass θ .)
- Capital price determined by expected discounted capital productivity.
- Monetary policy sets "rstar" to stabilize aggregate employment.

PRODUCTION

- Areas a . Period 0 is short run and $t \geq 1$ is long run.
- Two factors: labor (region-specific in period 0) and capital (mobile).
- Two household types, $i \in \{s, h\}$.
- Two goods: Nontradables and tradables. Production technologies:

$$C_{a,t}^i = \left(C_{a,t}^{i,N} / \eta \right)^\eta \left(C_{a,t}^{i,T} / (1 - \eta) \right)^{1 - \eta}$$

$$Y_{a,t}^N = \left(K_{a,t}^N / \alpha^N \right)^{\alpha^N} \left(L_{a,t}^N / (1 - \alpha^N) \right)^{1 - \alpha^N}$$

$$Y_t^T = \left(\int_a \left(Y_{a,t}^T \right)^{\frac{\varepsilon - 1}{\varepsilon}} da \right)^{\frac{\varepsilon}{\varepsilon - 1}},$$

$$\text{where } Y_{a,t}^T = \left(K_{a,t}^T / \alpha^T \right)^{\alpha^T} \left(L_{a,t}^T / (1 - \alpha^T) \right)^{1 - \alpha^T}.$$

PRODUCTION (CNTD.)

- Starting from $t = 1$ onwards,

$$\tilde{Y}_t^T = D^{1-\alpha^T} \tilde{K}_t^T.$$

- $t = 0$ capital price, Q_0 , driven by **future capital productivity**, D .
- Capital ownership is heterogeneous, $\{1 + x_{0,a}\}_a$ with $\int_a x_{0,a} da = 0$.

CONSUMPTION AND LABOR

- Stockholders have exogenous labor supply $L_{a,t}^s = \bar{L}$ for each a .

- Utility function

$$\sum_{t=0}^{\infty} (1 - \rho)^t \log C_{a,t}^s,$$

- Standard consumption-saving and portfolio choice problem.

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- Standard consumption-saving and portfolio choice problem.
- Hand-to-mouth households are myopic.
- Choose labor according to GHH preferences (no wealth effects) or have fixed wages: [▶ details](#)

Log linear labor supply:

$$w_{a,0} = \lambda (p_{a,0} + \varphi l_{a,0}).$$

- ▶ $z_{a,0} = \log Z_{a,0} / \bar{Z}$ for $Z \in \{W, P, L\}$.
- ▶ φ : inverse labor supply elasticity.
- ▶ $\lambda \subseteq [0, 1]$: decreasing in wage stickiness.

HETEROGENEOUS WEALTH: EMPIRICAL PREDICTIONS

Log-linearized equilibrium around common-wealth benchmark ($\epsilon = 1$):

$$\text{Total payroll: } \Delta(w_{a,0} + l_{a,0}) = \mathcal{M} (1 - \alpha^N) \eta \rho \frac{x_{a,0} \Delta Q_0}{WL_0},$$

where:

$$\mathcal{M} = \frac{1}{1 - (1 - \alpha^N) \eta \left\{ \frac{\theta \kappa + 1}{\kappa + 1} + \rho \frac{\kappa(1 - \theta)}{\kappa + 1} \right\}},$$
$$\kappa = \frac{\lambda \varphi}{1 - \lambda \eta (1 - \alpha^N)}.$$

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CALIBRATION USING TWO MODEL EQUATIONS ($h = 7$)

- 1 Calibrate ρ using the nontradables equation for $\varepsilon = 1$:

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- ▶ Robust to $\varepsilon \neq 1$ because bound on κ disciplines trade effects.

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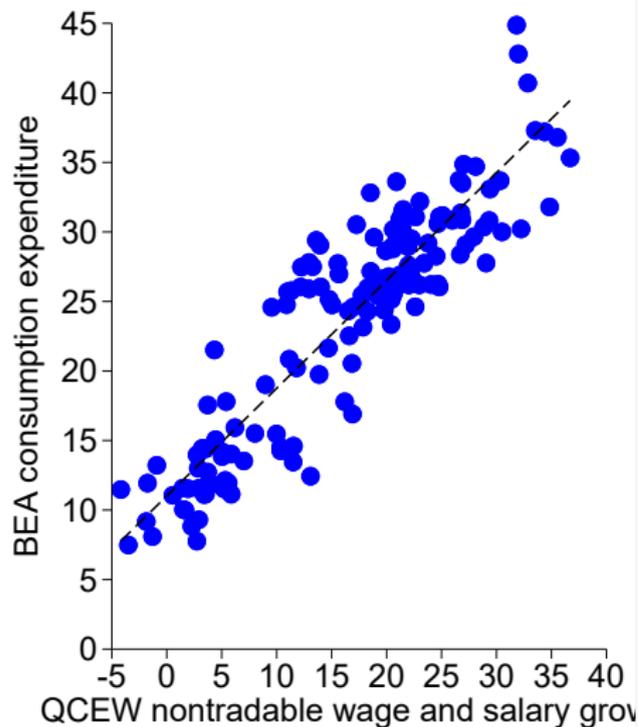
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USE THEORY TO PREDICT SECTORAL RESPONSE

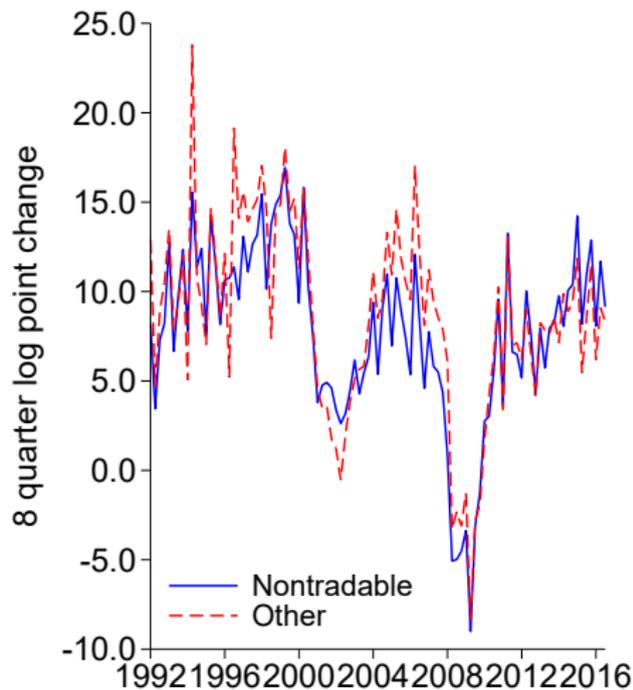
- Local demand shock directly affects local spending on nontradables (Mian, Sufi, ECMA 2014; Chodorow-Reich, Nenov, Simsek, WP).
- In aggregate closed economy, everything is nontradable.
- With homotheticity, *proportional* response of nontradable spending also gives response of tradable spending.

HOMOTHETICITY EVIDENCE

Nontr. payroll versus consumption



Nontr. versus other payroll



STOCK WEALTH EFFECT ON LABOR BILL

- Log-linear expressions:

$$\text{Aggregate labor bill: } \Delta(w_0 + l_0) = \mathcal{M}^A (1 - \alpha) \rho \times S_0^A R_0,$$

$$\text{Local nontr. labor bill: } \Delta(w_{a,0} + l_{a,0}^N) = \mathcal{M} (1 - \alpha) \rho \times S_{a,0} R_0.$$

- $\mathcal{M} < \mathcal{M}^A$: local multipliers lower bound for aggregate multipliers (Chodorow-Reich, 2019):

$$\implies \mathcal{M}^A (1 - \alpha) \rho = \frac{\mathcal{M}^A}{\mathcal{M}} 3.23\% \geq 3.23\%.$$

- One dollar stock wealth increases agg. labor bill by 3.23 cents.

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CHODOROW-REICH (QJE 2014 APPENDIX) MODEL

- Household consumes and supplies labor to firms.
- Firms produce a differentiated product and hold a credit line.
- A firm on “island” s can obtain credit line only from the bank operating on island s .
- Financial crisis: increase in the interest rate charged on the credit line.

HOUSEHOLD

Maximize

$$U = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(C_{\tau}, L_{\tau})$$

subject to

$$P_t C_t + B_t = w_t L_t + (1 + i_{t-1}) B_{t-1} + T_t.$$

HOUSEHOLD

C_t, L_t CES aggregates:

$$C_t = \left[\int_0^1 \int_0^1 \xi_{j,s,t}^{\frac{1}{\sigma}} c_{j,s,t}^{\frac{\sigma-1}{\sigma}} djds \right]^{\frac{\sigma}{\sigma-1}},$$
$$L_t = \left[\int_0^1 \int_0^1 L_{j,s,t}^{\frac{\nu+1}{\nu}} djds \right]^{\frac{\nu}{\nu+1}},$$

with price indexes given by

$$P_t = \left[\int_0^1 \int_0^1 \xi_{j,s,t} p_{j,s,t}^{1-\sigma} djds \right]^{\frac{1}{1-\sigma}},$$
$$w_t = \left[\int_0^1 \int_0^1 w_{j,s,t}^{1+\nu} djds \right]^{\frac{1}{1+\nu}}.$$

HOUSEHOLD FIRST ORDER CONDITIONS

Consumption allocation:

$$c_{j,s,t} = \xi_{j,s,t} \left(\frac{p_{j,s,t}}{P_t} \right)^{-\sigma} C_t.$$

Labor-consumption tradeoff:

$$- \left[\frac{L_{j,s,t}}{L_t} \right]^{\frac{1}{\nu}} \frac{u_{L_t}}{u_{C_t}} = \frac{w_{j,s,t}}{P_t}.$$

Intertemporal Euler equation:

$$u_{C_t} = E_t \left[\beta (1 + i_t) \frac{P_t}{P_{t+1}} u_{C_{t+1}} \right].$$

FIRMS

Production technology

$$y_{j,s,t} = a_{j,s,t} l_{j,s,t}^{1-\gamma}.$$

Simplifying assumption: $\gamma = 0$.

Firms exit with exogenous probability δ each period. Exit *after* production occurs but *before* paying workers. Firms hold letters of credit sufficient to cover their payroll. A firm on island s pays $r_{s,t}$ per unit of coverage.

The firm maximizes:

$$\Pi_{j,s,t} = [1 - \delta] [p_{j,s,t} y_{j,s,t} - (1 + r_{s,t}) w_{j,s,t} l_{j,s,t}] + \delta [0].$$

FIRM DECISION RULES

Price:

$$p_{j,s,t} = \mathcal{M}(1 + r_{s,t}) \frac{w_{j,s,t}}{a_{j,s,t}},$$

where $\mathcal{M} \equiv \frac{\sigma}{\sigma-1}$.

Labor demand:

$$l_{j,s,t} = \left[\frac{\xi_{j,s,t}}{a_{j,s,t}} \left(\frac{\mathcal{M}(1 + r_{s,t}) \frac{w_{j,s,t}}{a_{j,s,t}}}{P_t} \right)^{-\sigma} C_t \right].$$

FINANCIAL FIRMS

- Financial firm on island s provides credit lines to all firms on the island.
- Financial firm diverts a fraction $\zeta_{s,t}$ of profits to the household.
 - ▶ Think of as mortgage writedowns.
- Financial firms earn zero profits (free entry). Hence:

$$r_{s,t} = \frac{\delta}{1-\delta} + \zeta_{s,t}.$$

FINANCIAL CRISIS

- $t = 1, 2, \dots, t_0 - 1$: $\zeta_{s,t} = 0 \forall s$.
- t_0 : $\zeta_{s,t} \geq 0 \forall s$.
- Let $r_s < r_{s'}$ if $s > s'$.
- Assume $\zeta_{1,t} = 0$.
- Suppose $\zeta_{s,t}, r_{s,t}$ unobserved, but $m_{s,t} = \chi \hat{r}_{s,t}$ is observed.

REGRESSION EQUATION

Firm labor equilibrium condition:

$$\hat{l}_{j,s,t} = \hat{C}_t - \frac{\nu\sigma}{\nu + \sigma} [\hat{w}_t - \hat{P}_t] - \frac{\nu\sigma}{\nu + \sigma} \hat{r}_{s,t} + \frac{\nu}{\nu + \sigma} [\hat{\xi}_{j,s,t} + (\sigma - 1) \hat{a}_{j,s,t}],$$

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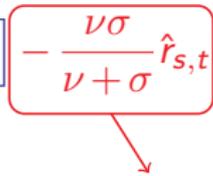
or:

$$\hat{l}_{j,s,t} = \beta_0$$

$\beta_0 = \alpha_1 \hat{C}_t - \alpha_2 [\hat{w}_t - \hat{P}_t]$ depends on aggregate output and the real wage;

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or:

$$\hat{l}_{j,s,t} = \beta_0 + \beta_1 m_{s,t}$$

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PARTIAL EQUILIBRIUM AND GENERAL EQUILIBRIUM

$$\begin{aligned} \text{Shortfall}^{PE} &= \int_0^1 \hat{l}_{j,1,t} dj - \int_0^1 \int_0^1 \hat{l}_{j,s,t} dj ds \\ &= \int_0^1 \beta_1 (m_{1,t} - m_{s,t}) ds. \end{aligned}$$

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Net GE effect determined by the sign of β_0 .

GENERAL EQUILIBRIUM EFFECTS

$$\beta_0 = \hat{C}_t - \frac{\nu\sigma}{\nu + \sigma} \left[(\hat{p}_{1,t} - \hat{P}_t) + (\hat{w}_t - \hat{p}_{1,t}) \right].$$

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- $\hat{w}_t - \hat{p}_{1,t}$: cost of labor at unconstrained firms $\downarrow \Rightarrow$ labor demand at unconstrained firms \uparrow .
- Elasticity of employment to reallocation rising in substitutability of the goods produced (σ) but falling in frictions to labor mobility ($\frac{1}{\nu}$).

SUFFICIENT STATISTIC: REAL RIGIDITY

- Solve the model:

$$\beta_0 = \kappa (\sigma - \Upsilon^{-1}) \tilde{r}_t.$$

- ▶ $\kappa = \frac{\nu}{(1-\gamma)\nu + \sigma(1+\gamma\nu)} \subseteq [0, 1]$.
 - ▶ $\tilde{r}_t = \int_0^1 \hat{r}_{s,t} ds$.
 - ▶ Υ is real rigidity as in Ball and Romer (1990): elasticity of optimal change in relative price to change in aggregate demand.
- Intuition: large real rigidity (Υ small) means large change in aggregate output required to generate equilibrium relative price gradient.

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- Direct summation
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3 CONCLUSION

RESTRICT ELASTICITY IN AGGREGATE SYSTEM

- Maybe cross-sectional elasticity identifies one aggregate elasticity directly.
- Example: "missing" wage deflation during Great Recession.
- Beraja, Hurst, Ospina (ECMA 2019): steep relationship between wage growth and unemployment in the cross-section of states.
- Cross-sectional relationship parameterizes Phillips curve allowing for identification of aggregate shocks.

MOTIVATING EVIDENCE

	Employment Rate	Nominal Wage	Real Wage
Cross-State			
Cross-State Wage Elasticity With Respect to Employment, 2007-2010		0.62 (0.10)	0.52 (0.15)
Aggregate			
Actual Aggregate Growth, 2007-2010	-7.7 percent	3.8 percent	-0.9 percent
Expected Aggregate Growth, 2007-2010 (Based on 2000-2007 Trend)	-0.9 percent	5.5 percent	-2.1 percent
Aggregate Deviation from Expected Growth, 2007-2010	-6.8 percent	-1.7 percent	1.2 percent
Aggregate Wage Elasticity With Respect to Employment, 2007-2010		0.25	-0.17

- Lots of people looked at aggregate wages during Great Recession and assumed wage rigidity.
- BHO say no, wages are flexible across states. So there must be an aggregate labor supply shock.

SVAR

- Driving forces $z_t, \gamma_t, \epsilon_t$:
 - ▶ z_t : Productivity/markup shock.
 - ▶ γ_t : Time preference/interest rate deviation shock.
 - ▶ ϵ_t : Labor supply shifter shock.
- Each variable $x \in \{z_t, \gamma_t, \epsilon_t\}$ follows AR(1) in region k and aggregate:

$$x_{k,t} = \rho_x x_{k,t-1} + \sigma_x^u u_t^x + \sigma_x^v v_{k,t}^x,$$

$$x_t = \rho_x x_{t-1} + \sigma_x^u u_t^x,$$

- Observed: price inflation π_t , wage inflation π_t^w , employment n_t .
- Large class of models with AR(1) shocks have SVAR representation:

$$B(L) \begin{pmatrix} \pi_t \\ \pi_t^w \\ n_t \end{pmatrix} = R \begin{pmatrix} u_t^\epsilon \\ u_t^z \\ u_t^\gamma \end{pmatrix} = R u_t.$$

- Estimate $B(L)$ and recover reduced form VCV matrix $\Omega = R \Sigma R'$.
- Three restrictions needed for identification.

SVAR RESTRICTIONS

Frictionless wage: $W_t^* = P_t \times MRS_t = -P_t u_N / u_C.$

Nominal wage rigidity: $W_t = (W_t^*)^\lambda W_{t-1}^{1-\lambda}.$

GHH preferences: $u(C, N) = g(C - v(N))$

$$\Rightarrow MRS_t = -u_N / u_C = v'(N) = e^{\epsilon_t} N_t^{1/\phi}.$$

Wage equation: $W_t = \left(P_t e^{\epsilon_t} N_t^{1/\phi} \right)^\lambda W_{t-1}^{1-\lambda}.$

Log wage inflation: $\pi_t^w = \lambda \left(\pi_t + \Delta \epsilon_t + \phi^{-1} \Delta n_t \right) + (1 - \lambda) \pi_{t-1}^w.$

Forecast error: $E_2' R u_t = \lambda E_1' R u_t + \lambda \sigma_\epsilon^u u_t^\epsilon + \lambda \phi^{-1} E_3' R u_t$
 $\Rightarrow 0 = \begin{pmatrix} \lambda & -1 & \lambda \phi^{-1} \end{pmatrix} R u_t + \lambda \sigma_\epsilon^u u_t^\epsilon.$

- Two restrictions on R given λ, ϕ and $0 = (\lambda - r_{21} + \lambda \phi^{-1} r_{31} + \lambda \sigma_\epsilon^u) u_t^\epsilon + (\lambda r_{12} - 1 + \lambda \phi^{-1} r_{32}) u_t^z + (\lambda r_{13} - r_{23} + \lambda \phi^{-1}) u_t^\gamma.$
- Last restriction comes from forecast revision.

WHERE DO λ AND ϕ COME FROM?

- Cross-sectional data! (Or anywhere else...)

$$\begin{aligned}\text{State wage: } \pi_{k,t}^w &= \lambda \left(\pi_{k,t} + \sigma_\epsilon^u u_t^\epsilon + \sigma_\epsilon^v v_{k,t}^\epsilon + (\rho_\epsilon - 1) \epsilon_{t-1} + \phi^{-1} \Delta n_{k,t} \right) \\ &+ (1 - \lambda) \pi_{k,t-1}^w \\ &= \lambda \pi_{k,t} + \lambda \phi^{-1} \Delta n_{k,t} + (1 - \lambda) \pi_{k,t-1}^w \\ &+ \lambda (\sigma_\epsilon^u u_t^\epsilon + (1 - \rho_\epsilon) \epsilon_{t-1}) + \lambda \sigma_\epsilon^v v_{k,t}^\epsilon.\end{aligned}$$

- $\pi_{k,t}^w, \pi_{k,t}, \Delta n_{k,t}$ observed. $\lambda (\sigma_\epsilon^u u_t^\epsilon + (1 - \rho_\epsilon) \epsilon_{t-1})$ absorbed by time fixed effect.
- Strategy 1: argue $v_{k,t}^\epsilon$ “small” during Great Recession.
- Strategy 2: instrument for $\Delta n_{k,t}, \pi_{k,t}$ using contemporaneous and lagged house price growth.

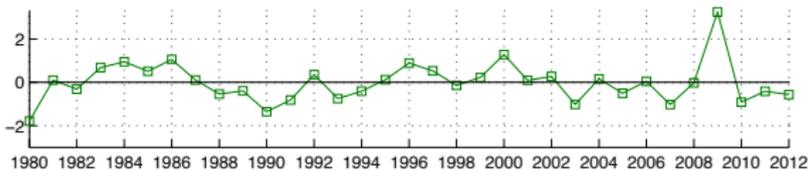
ESTIMATES OF λ, ϕ

	Specification						
	OLS			IV			
	2007-2011		(3)	2007-2009		2007-2011	2007-2009
(1)	(2)	(5)		(6)	(7)	(8)	
λ	0.69 (0.13)	0.69 (0.13)	0.75 (0.13)	0.73 (0.17)	0.73 (0.17)	0.77 (0.13)	0.79 (0.18)
$\lambda\phi$	0.31 (0.08)	0.32 (0.08)	0.31 (0.07)	0.39 (0.09)	0.39 (0.10)	0.76 (0.17)	0.99 (0.25)
Implied ϕ	2.2	2.2	2.4	1.9	1.9	1.0	0.8
Year Fixed Effects	Yes						
Industry Controls	No	Yes	Yes	No	Yes	Yes	Yes
Scaling Factor of Prices	1.4	1.4	1.0	1.4	1.4	1.4	1.4

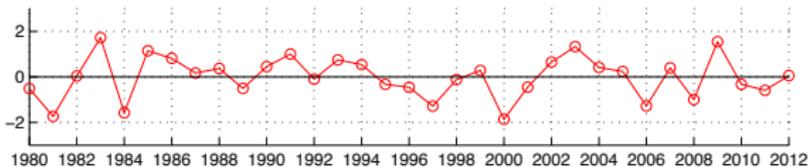
$$\pi_{k,t}^W = \lambda\pi_{k,t} + \lambda\phi^{-1}\Delta n_{k,t} + (1-\lambda)\pi_{k,t-1}^W + \lambda(\sigma_\epsilon^U u_t^\epsilon + (1-\rho_\epsilon)\epsilon_{t-1}) + \lambda\sigma_\epsilon^V v_{k,t}^\epsilon.$$

SHOCK TIME SERIES

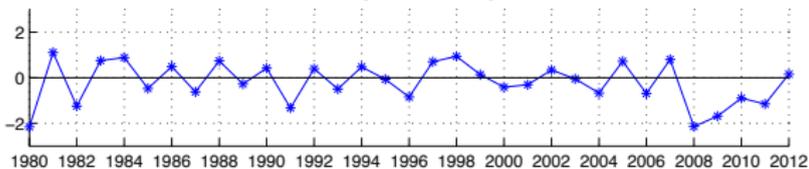
Leisure Shock



Discount rate Shock



Productivity / Markup Shock



WHAT IS A LEISURE SHOCK?

- Casey Mulligan: *The Redistribution Recession*.
- But little evidence from micro/cross-state data for this.
- Grigsby (JMP): mis-specification due to multiple types of workers and compositional effects on national wages.

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MATCH MOMENT IN STRUCTURAL MODEL

- Structural model can be identified from unconditional moments. Why use conditional moments?
- Loosely, conditional moments better if model's purpose is closely related to conditional moment.
- Long history in macro using structural VARs to identify DSGE models: Rotemberg and Woodford (NBERMA 1997); Christiano, Eichenbaum, Evans (JPE 2005).
- Similar to indirect inference in structural micro (and see critique by Chari, Kehoe, McGrattan).
- Recent applications: Nakamura, Steinsson (AER 2014); Catherine, Chaney, Huang, Sraer, Thesmar (2017).

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LOTS TO DO

- Aggregation problem shows up everywhere.
- Many approaches. Best approach depends on context.
- Closely related to spatial models, heterogeneous agent models, network models, etc.
- Active area of research.