Lecture IV

Harvard Econ 2416
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OUTLINE

1. FISCAL POLICY (BLANCHARD AND PEROTTI, QJE 2002)
2. MONETARY FAVAR (BERNANKE, BOIVIN, AND ELIASZ, QJE 2005)
3. EXTERNAL INSTRUMENTS
4. BAYESIAN IDENTIFICATION (BAUMEISTER AND HAMILTON, AER 2019)

Outline

1. Fiscal policy (Blanchard and Perotti, QJE 2002)
2. Monetary FAVAR (Bernanke, Boivin, and Eliasz, QJE 2005)
3. External instruments
4. Bayesian identification (Baumeister and Hamilton, AER 2019)

OVERVIEW

- Blanchard's second-most highly cited article.
- Revived interest in study of fiscal policy.
- Approach still widely used.
- I follow BP's notation with minor changes.

Specification

- $Y_t = \begin{pmatrix} T_t & G_t & X_t \end{pmatrix}' =$
  $\log \begin{pmatrix} \text{Taxes net of transfers}_t & \text{Government purchases}_t & \text{Output}_t \end{pmatrix}'$.
- Reduced form: $Y_t = A(L,q)Y_{t-1} + u_t$, or

$$\begin{pmatrix} T_t \\ G_t \\ X_t \end{pmatrix} = A(L,q) \begin{pmatrix} T_{t-1} \\ G_{t-1} \\ X_{t-1} \end{pmatrix} + \begin{pmatrix} u_{T,t} \\ u_{G,t} \\ u_{X,t} \end{pmatrix}.$$

- Structural shocks $e_t = \begin{pmatrix} e_{T,t} & e_{G,t} & e_{X,t} \end{pmatrix}'$:

$$\begin{aligned} u_{T,t} &= a_1 u_{X,t} + a_2 e_{G,t} + e_{T,t} \\ u_{G,t} &= b_1 u_{X,t} + b_2 e_{T,t} + e_{G,t} \\ u_{X,t} &= c_1 u_{T,t} + c_2 u_{G,t} + e_{X,t}. \end{aligned}$$

  - 3 equations in 3 unknowns define structural shocks implicitly.
  - Could rewrite in form $u_t = Re_t$.
  - BP write this way because they argue a_1 and b_1 have data counterparts without requiring further assumptions.

Identification

$$u_{T,t} = a_1 u_{X,t} + a_2 e_{G,t} + e_{T,t}$$
$$u_{G,t} = b_1 u_{X,t} + b_2 e_{T,t} + e_{G,t}$$
$$u_{X,t} = c_1 u_{T,t} + c_2 u_{G,t} + e_{X,t}$$

- a_1, b_1 are elasticities of surprises to taxes and government spending w.r.t output.
- Combine automatic stabilizers and discretionary policy response.
- Identification assumption: discretionary policy response is zero within the quarter ($b_1 = 0$).
- Is this plausible?
- Compare to timing assumption in traditional monetary policy VAR.

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$$u_{T,t} = a_1 u_{X,t} + a_2 e_{G,t} + e_{T,t}$$
$$u_{G,t} = b_1 u_{X,t} + b_2 e_{T,t} + e_{G,t}$$
$$u_{X,t} = c_1 u_{T,t} + c_2 u_{G,t} + e_{X,t}$$

- $b_1 = 0$: no automatic response of government purchases to output.
- $a_1 = \sum_i \frac{\exp(T_i)}{\exp(T)} \eta_{T_i,B_i} \eta_{B_i,X}$: weighted avg. elasticity of taxes to output.
  - Separately estimate elasticity of tax base (i.e. profits, or income) B_i w.r.t output, and elasticity of taxes w.r.t base.
- Construct cyclically adjusted reduced form residuals:

$$u'_{T,t} = u_{T,t} - a_1 u_{X,t}$$
$$u'_{G,t} = u_{G,t} - b_1 u_{X,t}.$$

- $u'_{T,t}, u'_{G,t}$ valid instruments for $u_{T,t}, u_{G,t}$ in

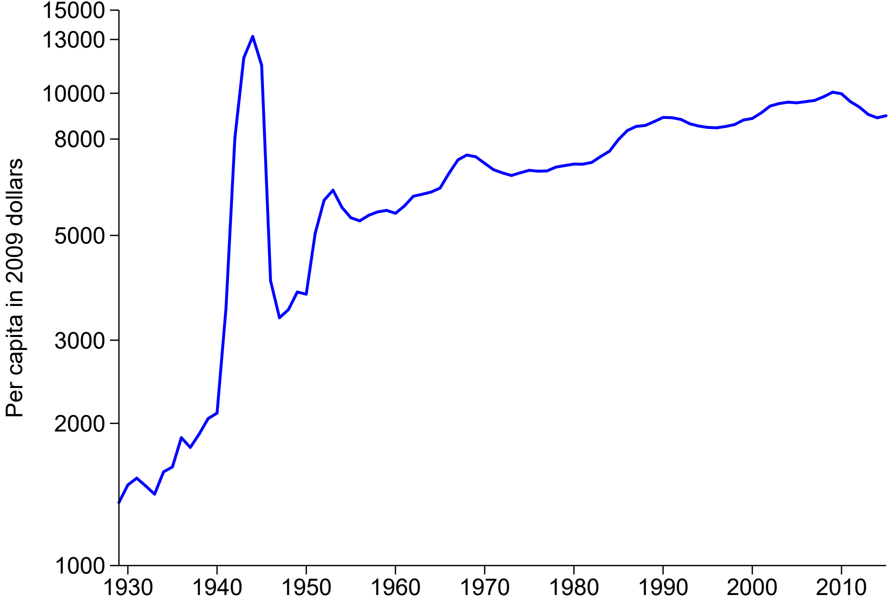
$$u_{X,t} = c_1 u_{T,t} + c_2 u_{G,t} + e_{X,t}.$$

Identification review

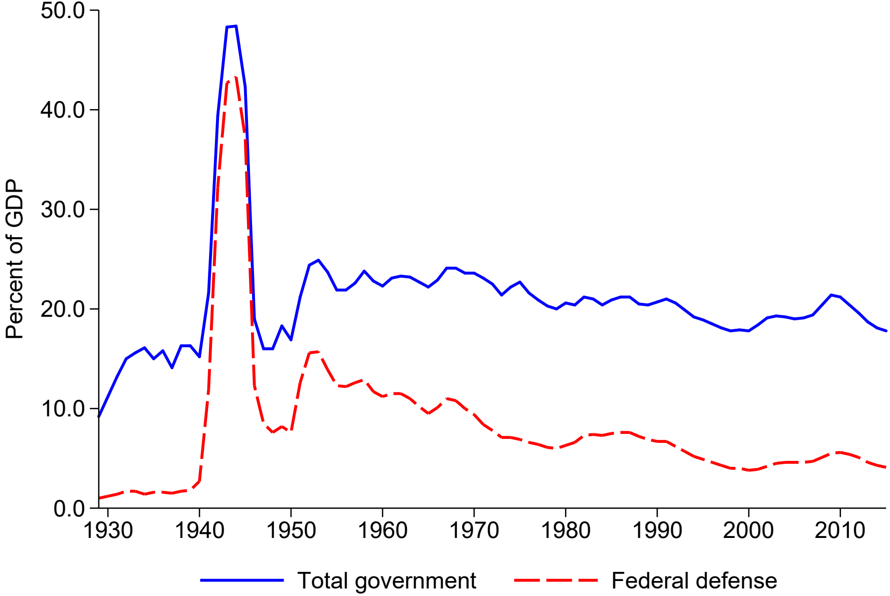
$$u_{T,t} = a_1 u_{X,t} + a_2 e_{G,t} + e_{T,t}$$
$$u_{G,t} = b_1 u_{X,t} + b_2 e_{T,t} + e_{G,t}$$
$$u_{X,t} = c_1 u_{T,t} + c_2 u_{G,t} + e_{X,t}$$

- a_1, b_1 constructed using auxiliary information.
- c_1, c_2 constructed from IV regression.
- Agnostic on a_2, b_2. Sufficient to set one or the other to zero, and paper reports both specifications.
- BP identify shocks e separately from estimation of IRFs. Could have used local projection.

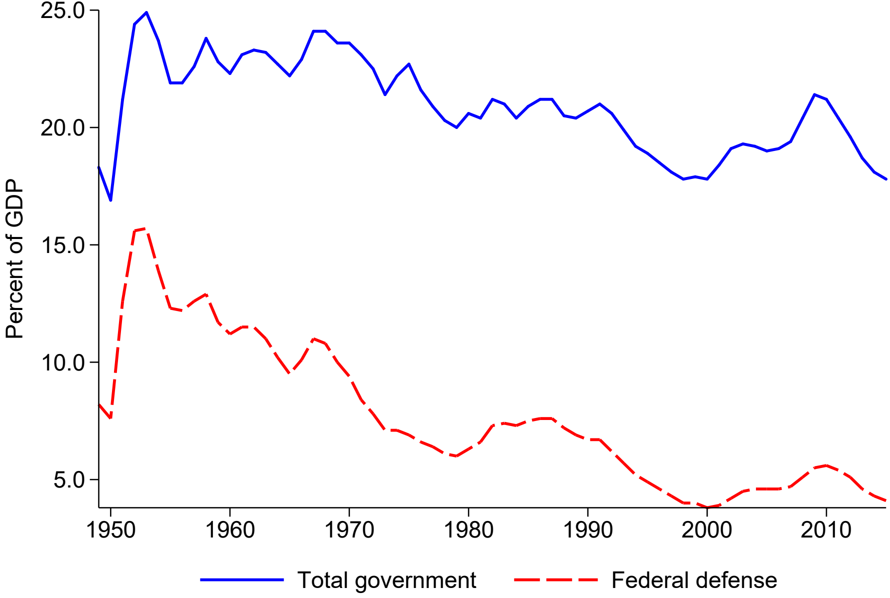
Government consumption and investment



Government consumption and investment



Government consumption and investment



Large Changes in Net Taxes and Spending

$\text{sd}(\Delta \log G) = 0.019$		$\text{sd}(\Delta \log T) = 0.049$	
$\Delta \log G > 3$ sd		$\Delta \log T > 3$ sd	
1951:1	0.103	1950:2	0.266
1951:2	0.112	1950:3	0.171
1951:3	0.108	1975:2	−0.335
		1975:3	0.240
$2 \text{ sd} < \Delta \log G < 3$ sd		$2 \text{ sd} < \Delta \log T < 3$ sd	
1948:2	0.039	1947:3	−0.117
1948:4	0.043	1947:4	0.107
1949:1	0.049	1951:1	0.097
1949:2	0.043		
1950:4	0.054		
1951:4	0.051		
1952:2	0.041		
1967:1	0.041		

TABLE II

ESTIMATED CONTEMPORANEOUS COEFFICIENTS

	c_1	c_2	b_2	a_2
		DT		
coeff.	−0.868	0.956	−0.047	−0.187
t-stat.	−3.271	2.392	−1.142	−1.142
p-value	0.001	0.018	0.255	0.255
		ST		
coeff.	−0.876	0.985	−0.057	−0.238
t-stat.	−3.255	2.378	−1.410	−1.410
p-value	0.001	0.019	0.161	0.161

DT: Deterministic Trend; ST: Stochastic Trend.
Sample: 1960:1–1997:4.
c_1: effect of t on x within quarter;
c_2: effect of g on x within quarter;
a_2: effect of g on t within quarter (assuming $b_2 = 0$, i.e., when spending is ordered first);
b_2: effect of t on g within quarter (assuming $a_2 = 0$, i.e., when net taxes are ordered first).
All effects are expressed as dollar for dollar.

TABLE III
Responses to Tax Shocks

	1 qrt	4 qrts	8 qrts	12 qrts	20 qrts	peak
			DT			
GDP	−0.69*	−0.74*	−0.72*	−0.42*	−0.22	−0.78* (5)
TAX	0.74*	0.13	−0.21*	−0.20*	−0.11	
GCN	−0.05*	−0.12*	−0.24*	−0.26*	−0.16*	
			ST			
GDP	−0.70*	−1.07*	−1.32*	−1.30*	−1.29*	−1.33* (7)
TAX	0.74*	0.31*	0.17	0.16	0.16	
GCN	−0.06*	−0.10*	−0.17*	−0.20*	−0.20*	

DT: Deterministic Trend; ST: Stochastic Trend. An asterisk indicates that 0 is outside the region between the two one-standard error bands. In parentheses besides the peak response is the quarter in which it occurs. All reduced-form equations include lags 0 to 4 of the 1975:2 dummy. Sample: 1960:1–1997:4.

TABLE IV
Responses to Spending Shocks

	1 qrt	4 qrts	8 qrts	12 qrts	20 qrts	peak
			DT			
GDP	0.84*	0.45	0.54	1.13*	0.97*	1.29* (15)
GCN	1.00*	1.14*	0.95*	0.70*	0.42*	
TAX	0.13	0.14	0.17	0.43*	0.52*	
			ST			
GDP	0.90*	0.55	0.65	0.66	0.66	0.90* (1)
GCN	1.00*	1.30*	1.56*	1.61*	1.62*	
TAX	0.10	0.18	0.33	0.36	0.37	

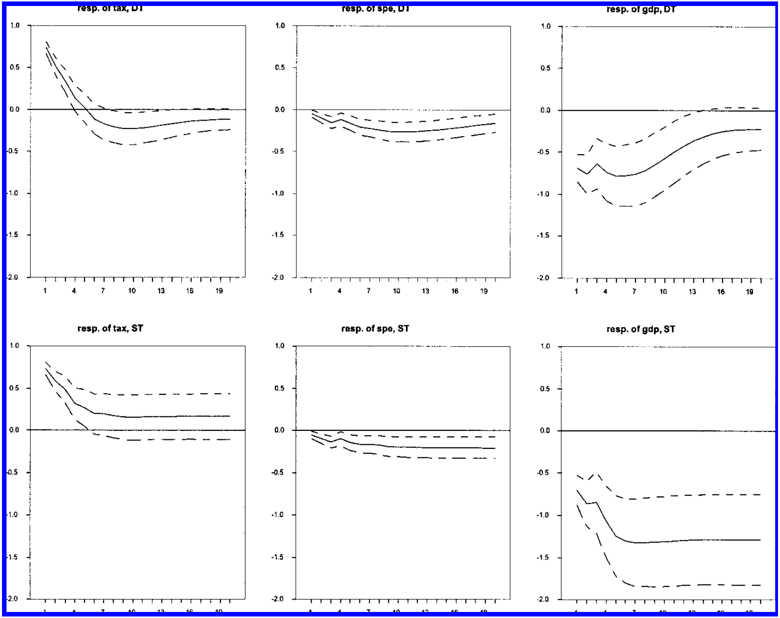


FIGURE III
Response to a Tax Shock

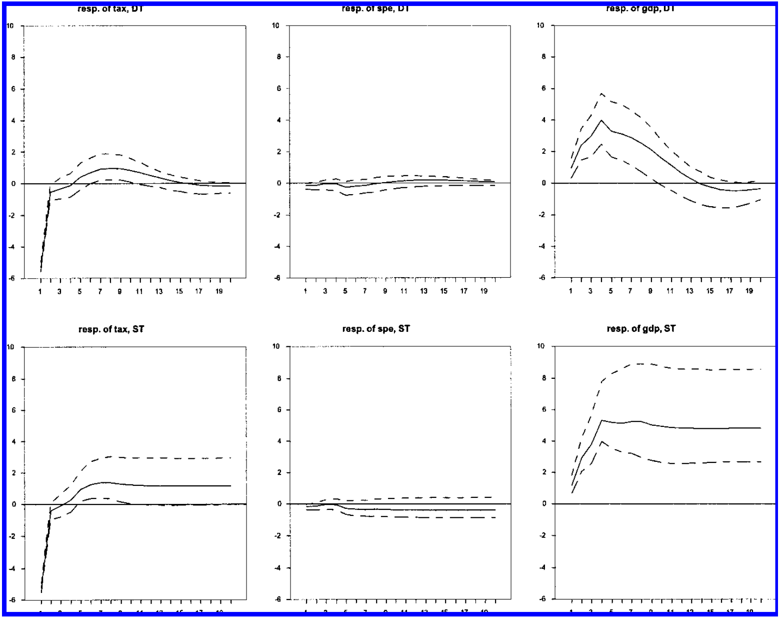


FIGURE IV
Response to a Shock to the 1975:2 Dummy

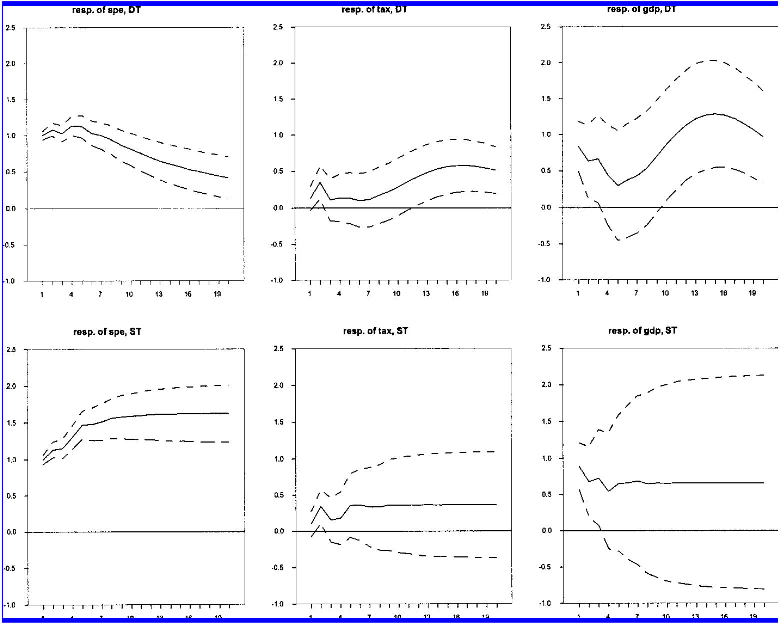


FIGURE V
Response to a Spending Shock

Competing theories

- Government spending shock:
  - Neoclassical theory: $C \downarrow, L^S \uparrow \Rightarrow w \downarrow$.
  - Old Keyneisan theory: $C \uparrow, L^D \uparrow \Rightarrow w \uparrow$.
  - New Keynesian theory: it depends.
- Tax shock:
  - Neoclassical theory: it depends.
  - Old Keyneisan theory: $C \downarrow, L^D \downarrow \Rightarrow w \downarrow$.
  - New Keynesian theory: it depends.

Responses of GDP Components

	1 qrt	4 qrts	8 qrts	12 qrts	20 qrts	peak
			DT, TAX			
GDP	−0.69*	−0.74*	−0.72*	−0.42*	−0.22	−0.78* (5)
GCN	−0.05*	−0.12*	−0.24*	−0.26*	−0.16*	−0.05* (1)
CON	−0.18*	−0.35*	−0.32*	−0.23*	−0.20*	−0.35* (5)
INV	−0.36*	−0.00	−0.00	0.18*	0.16*	−0.36* (1)
EXP	−0.02	0.01	−0.01	0.02	0.05	−0.08 (3)
IMP	−0.01	0.02	−0.14*	−0.06	0.04	−0.14* (7)
SUM	−0.60	−0.48	−0.43	−0.23	−0.18	−0.60 (1)
			ST, TAX			
GDP	−0.70*	−1.07*	−1.32*	−1.30*	−1.29*	−1.33* (7)
GCN	−0.06*	0.04*	−0.01*	−0.00*	−0.00*	0.04* (4)
CON	−0.15	−0.40*	−0.44*	−0.43*	−0.43*	−0.44* (7)
INV	−0.35*	−0.22	−0.30	−0.27	−0.27	−0.35* (1)
EXP	−0.00	−0.01	−0.06	−0.07	−0.07	−0.10 (3)
IMP	−0.01	−0.02	−0.12	−0.12	−0.11	−0.13 (3)
SUM	−0.55	−0.57	−0.68	−0.66	−0.66	−0.73 (6)
			DT, SPE			
GDP	0.84*	0.45	0.54	1.13*	0.97*	1.29* (15)
GCN	1.00*	1.14*	0.95*	0.70*	0.42*	1.14* (4)
CON	0.50*	0.63*	0.91*	1.21*	0.90*	1.26* (14)
INV	−0.03	−0.75*	−0.69*	−0.41*	−0.35*	−1.00* (5)
EXP	0.20*	−0.47*	−0.76*	−0.70*	−0.06	−0.80* (9)
IMP	0.64*	−0.19*	−0.46*	−0.42*	−0.16*	−0.49* (9)
SUM	1.03	0.74	0.86	1.22	1.07	1.39 (15)
			ST, SPE			
GDP	0.90*	0.55	0.65	0.66	0.66	0.90* (1)
GCN	1.00*	1.30*	1.56*	1.61*	1.61*	1.00 (1)
CON	0.33*	0.34	0.42	0.43	0.44	0.46* (2)
INV	0.02	−0.74*	−0.97*	−0.96*	−0.95*	−0.98* (9)
EXP	0.17*	−0.16	−0.30	−0.37*	−0.37	−0.37* (13)
IMP	0.56*	0.03	−0.06	−0.05	−0.04	−0.08 (9)
SUM	0.95	0.72	0.77	0.76	0.78	0.95 (1)

Sample: 1960:1–1997:4.

Kueng (WP) evidence of anticipated tax policy

- Uses Treasury-Muni yield spread to infer break-even expected top marginal tax rate.

- Evidence of expectations moving ahead of policy changes, for example during 1992 and 2000 elections.

Kueng (WP)

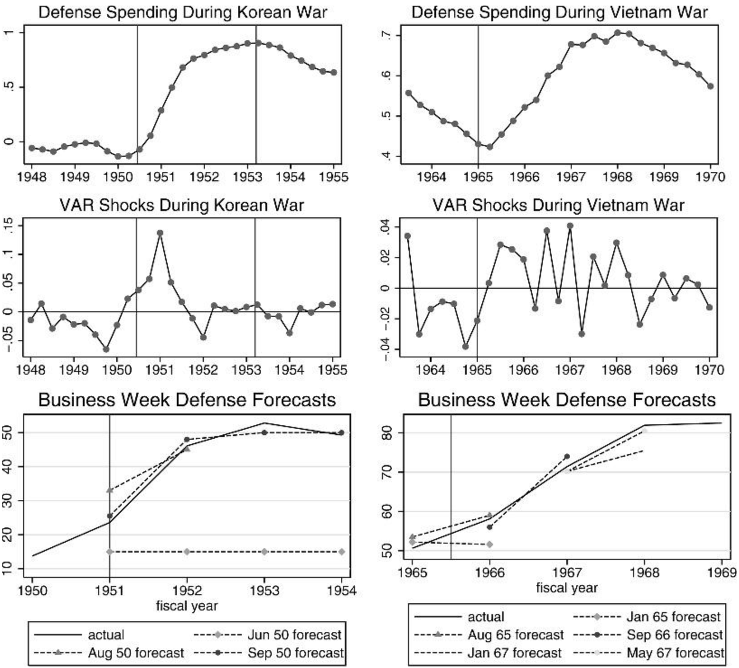
Table 2: Break-even tax rate responses to changes in election probabilities.

	Maturity (m)							
Break-Even Tax Rate $\theta_{t,m}$ (BETR) :	1-Year	2-Year	3-Year	5-Year	7-Year	10-Year	20-Year	30-Year
Price of Bush Contract in 2000 [in cents]	0.018	-0.018***	-0.031***	-0.033***	-0.028***	-0.024**	-0.006	0.003
	(0.013)	(0.007)	(0.007)	(0.007)	(0.010)	(0.011)	(0.009)	(0.013)
Price of Clinton Contract in 1992 [in cents]	0.140***	0.096**	0.140***	0.091***	0.097***	0.103***	0.042***	0.047**
	(0.048)	(0.047)	(0.040)	(0.026)	(0.022)	(0.021)	(0.015)	(0.018)

Notes: This table shows the results from regressing daily election probabilities on break-even tax rates for the presidential election of 2000 and 1992, respectively. The tax reform enacted in 1993 (OBRA 1993) increased the statutory top income rate by 8.6% from 31% to 39.6% retroactively to January 1, 1993. The tax reform enacted in 2001 (EGTRRA 2001) reduced the statutory top income rate by 4.6% from 39.6% to 35% over 5 years and the reform in 2003 (JGTRRA 2003) accelerated the phase-in period to three years. The contracts yield 100 cents if the candidate wins and zero otherwise. Therefore, an increase of the price by 1 cent corresponds to a 1% increase in the perceived probability of the candidate winning the presidential election. The full regression results are provided in the online appendix. Newey-West HAC robust standard errors in parentheses: ***, **, * mark significance at the 1, 5, and 10 percent level, respectively.

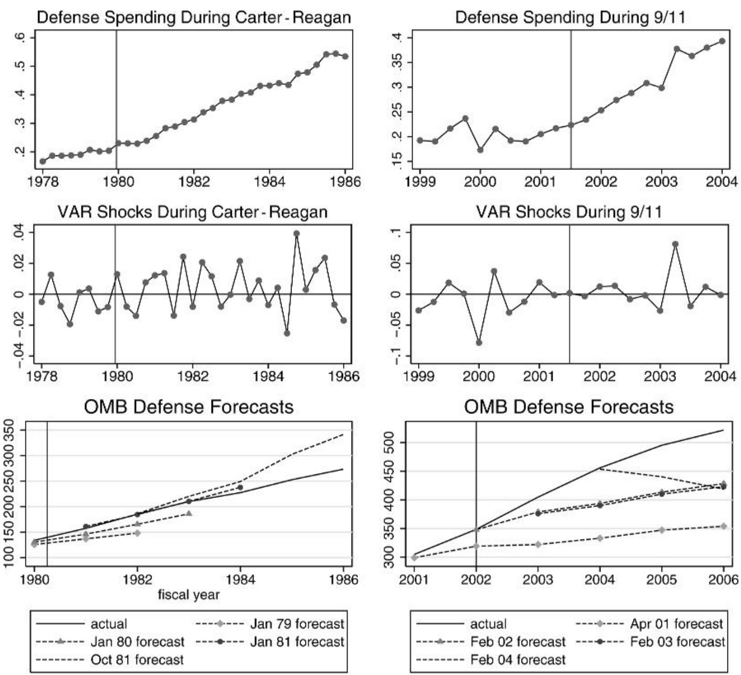
Ramey (QJE 2011) evidence of anticipated spending

- Ramey-Shapiro war dates: 1950Q3, 1965Q1, 1980Q1, 2001Q3, and VAR with indicator for war date ordered first.
- Business Week to construct series of defense spending news.
- Evidence defense spending news Granger cause Blanchard-Perotti shocks and affects VAR responses.
- Solutions:
  - Blanchard and Perotti: tighten identification requirement by increasing policy lag.
  - Ramey: Include news shocks in VAR ordered first: effectively restricts G shock to be unanticipated.
  - Auerbach and Gorodnichenko: replace G in VAR with unexpected G.



Defense Spending During Korean War
Defense Spending During Vietnam War
VAR Shocks During Korean War
VAR Shocks During Vietnam War
Business Week Defense Forecasts
fiscal year
actual
Jun 50 forecast
Aug 50 forecast
Sep 50 forecast
Business Week Defense Forecasts
fiscal year
actual
Jan 65 forecast
Aug 65 forecast
Sep 66 forecast
Jan 67 forecast
May 67 forecast

FIGURE V



Defense Spending During Carter - Reagan
Defense Spending During 9/11
VAR Shocks During Carter - Reagan
VAR Shocks During 9/11
OMB Defense Forecasts
fiscal year
actual
Jan 79 forecast
Jan 80 forecast
Jan 81 forecast
Oct 81 forecast
OMB Defense Forecasts
actual
Apr 01 forecast
Feb 02 forecast
Feb 03 forecast
Feb 04 forecast

FIGURE VI

Other details and dos and don'ts

- Logs versus shares of GDP versus shares of trend GDP.
  - Common problem: how to normalize variables?
  - Log GDP on log spending: compute multiplier using average G/Y in sample. Sensitive if G/Y changes a lot.
- Point-in-time versus maximal versus cumulative multipliers.
  - Full information: trace IRF of both government spending and output to government spending shock.
  - Summary: present value of output divided by present value of government spending.
- Sensitivity to estimation period (perhaps not surprising).

Outline

What is a shock?

- Something useful to identify monetary policy? Or more?

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- Suppose Cholesky with fed funds rate ordered last, you knew Fed sets rate at 11:59pm on last day of each month, and you have end of month fed funds rate series. Identified?

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- Fed funds rate less fed funds futures rate in small window around announcement?

What is a shock?

- Something useful to identify monetary policy? Or more?
- Suppose Cholesky with fed funds rate ordered last, you knew Fed sets rate at 11:59pm on last day of each month, and you have end of month fed funds rate series. Identified?
- Fed funds rate less fed funds futures rate in small window around announcement?
- Fed randomizes months ahead of time the interest rate at date t?

Critique of traditional VARs

- Basic problem is omitted variable bias.
- Limited information content relative to that used by economic agents.
- Degrees of freedom and curse of dimensionality: each added variable requires p additional terms in each equation.
- Example: price puzzle and central bank's response to supply shocks unobserved to the econometrician.

FAVAR OVERVIEW

- PCA summarizes information from a large number of data series into a small number of factors.

- FAVAR uses the factors to control for economic conditions.

Notation (follows BBE)

- F_t is vector containing period t realizations of K factors.
- X_t is vector containing period t realizations of N time series driven by the factors.
- Y_t is vector containing period t realizations of M time series observable to the econometrician and which enter directly into the economic system.
- Joint dynamics:

$$\begin{pmatrix} F_t \\ Y_t \end{pmatrix} = \Phi(L) \begin{pmatrix} F_{t-1} \\ Y_{t-1} \end{pmatrix} + v_t$$

$$X_t = \Lambda^f F_t + \Lambda^y Y_t + e_t.$$

Example

$$
\begin{aligned}
\text{Phillip's curve:} \quad & \pi_t = \delta \pi_{t-1} + \kappa \left(y_{t-1} - y^n_{t-1} \right) + s_t \\
\text{IS curve:} \quad & y_t = \phi y_{t-1} - \psi (R_{t-1} - \pi_{t-1}) + d_t \\
\text{Productivity process:} \quad & y^n_t = \rho y^n_{t-1} + \eta_t \\
\text{Cost push process:} \quad & s_t = \alpha s_{t-1} + v_t \\
\text{Taylor rule:} \quad & R_t = \beta \pi_t + \gamma (y_t - y^n_t) + \varepsilon_t \\
\text{Observed variables:} \quad & X_t = \Lambda \left(y^n_t \quad s_t \quad \pi_t \quad y_t \quad R_t \right)' + e_t.
\end{aligned}
$$

- ε_t is a monetary policy shock. Recovering it is the goal of the exercise.
- Economic model has 5 factors: $y^n_t, s_t, \pi_t, y_t, R_t$.
- If factors all observed, recursive VAR identifies monetary shocks because y^n_t, s_t, π_t, y_t are functions of R_{t-1} but not R_t.

BBE IMPLEMENTATION

- Assumption: space spanned by latent economic factors is equivalent to space spanned by factors from PCA of large number of macroeconomic time series, including R_t.
- Complication I: R_t potentially correlated with factors.
  - BBE solution:
    1. For each estimated factor $\tilde{F}_{k,t}$, regress

       $$\tilde{F}_{k,t} = \sum_{j=1}^{J} b_j \hat{H}_{j,t} + b_R R_t + u_t,$$

       where $\{\hat{H}_{j,t}\}$ are the factors from performing PCA on a subset of J slow moving variables assumed not to respond contemporaneously to R_t.
    2. Construct

       $$\hat{F}_{k,t} = \tilde{F}_{k,t} - \hat{b}_R R_t.$$

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- Recursive identification: $Y_t = \left(\hat{F}_{1,t} \quad \dots \quad \hat{F}_{k,t} \quad R_t\right)'$.

- Complication II: $\hat{F}_{k,t}$ are generated regressors.

  - Standard errors via bootstrap.

- Sample: 1959-2001, monthly frequency, 120 macroeconomic time series.

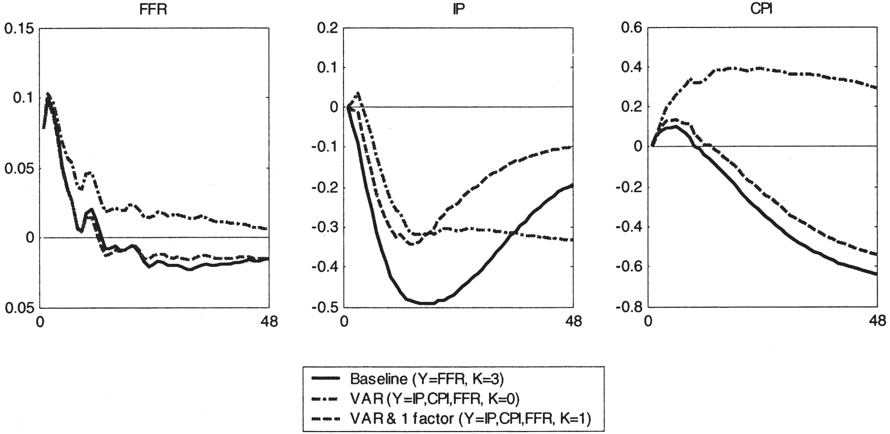


FIGURE I

Estimated Impulse Responses to an Identified Policy Shock for Alternative FAVAR Specifications, Based on the Two-Step Principal Component's Approach

Informal performance assessment

- Adding the estimated factor resolves the price puzzle.
- Adding the estimated factor generates U shaped response of IP rather than permanent effect.
- VAR with only R_t observed and 3 factors and VAR with R_t, IP_t, π_t observed and 1 factor behave similarly.
- Generate IRF at horizon h for any variable $x_t \in X_t$ by taking inner product of horizon h impulse responses of the factors $\hat{F}_{k,t}$ and the loadings of x_t on the factors.
  - By construction, IRFs will be similar for variables that have similar loadings.
  - Alternative is local projection.

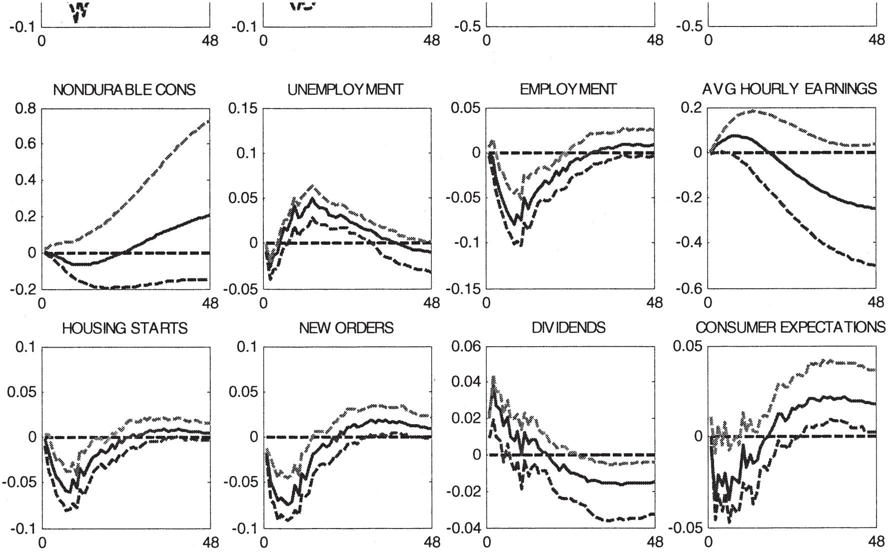


FIGURE II

Impulse Responses Generated from FAVAR with Three Factors and FFR Estimated by Principal Components with Two-Step Bootstrap

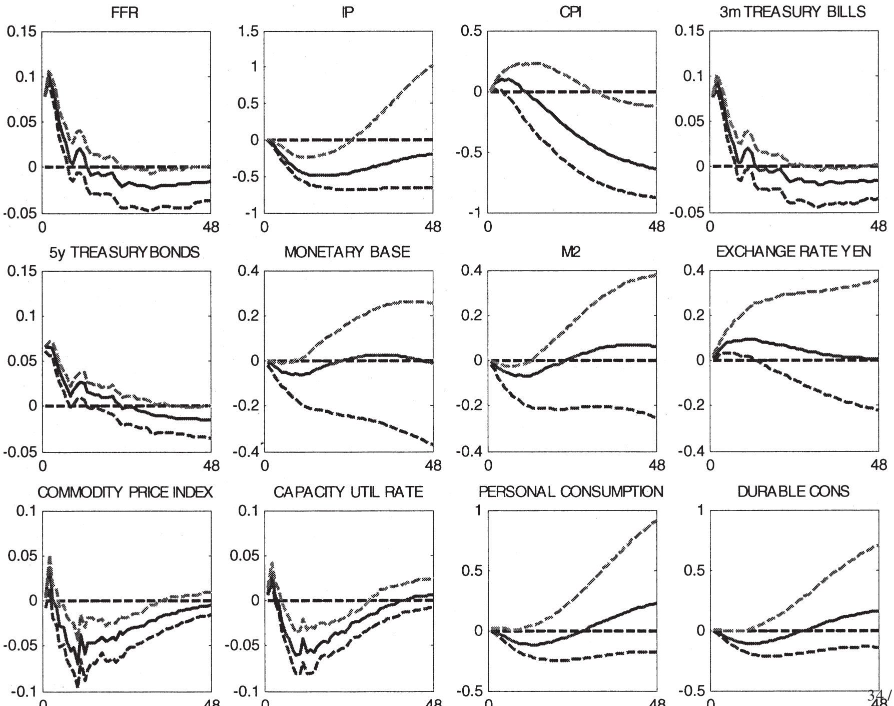

FFR
IP
CPI
3m TREASURY BILLS
5y TREASURY BONDS
MONETARY BASE
M2
EXCHANGE RATE YEN
COMMODITY PRICE INDEX
CAPACITY UTIL RATE
PERSONAL CONSUMPTION
DURABLE CONS

TABLE I

CONTRIBUTION OF THE POLICY SHOCK TO VARIANCE OF THE COMMON COMPONENT

Variables	Variance decomposition	R^2
Federal funds rate	0.454	*1.000
Industrial production	0.054	0.707
Consumer price index	0.038	0.870
3-month treasury bill	0.433	0.975
5-year bond	0.403	0.925
Monetary base	0.005	0.104
M2	0.005	0.052
Exchange rate (Yen/$)	0.007	0.025
Commodity price index	0.049	0.652
Capacity utilization	0.100	0.753
Personal consumption	0.006	0.108
Durable consumption	0.005	0.062
Nondurable cons.	0.002	0.062
Unemployment	0.103	0.817
Employment	0.066	0.707
Aver. hourly earnings	0.007	0.072
Housing starts	0.032	0.387
New orders	0.081	0.624
S&P dividend yield	0.062	0.549
Consumer expectations	0.036	0.700

The column titled Variance decomposition reports the fraction of the variance of the forecast error, at the

Discussion

- How much progress does FAVAR method make relative to previous methods?

- What threats to identification does FAVAR method not address?

Outline

Overview

- Often have measure that is a "proxy shock".
- External instruments properly scales proxy shocks.
- Conceptually clarifies controversies such as whether two candidate series must be correlated with each other.
- Raises issue of heterogeneous treatment effects and LATE.
- Some subtleties to introduce into VAR.

External instruments (my notation)

$$
\begin{aligned}
Y_t &= B_1 L Y_t + \ldots B_p L^p Y_t + e_t, \\
e_t &= R v_t.
\end{aligned}
$$

- Partition the structural shocks:

$$v_t = \left(\underbrace{v_{1,t}'}_{1xd} \quad \underbrace{v_{2,t}'}_{1x(k-d)} \right)'.$$

- Partition R:

$$R = \left(\underbrace{R_1}_{kxd} \quad \underbrace{R_2}_{kx(k-d)} \right) = \begin{pmatrix} \underbrace{R_{11}}_{dxd} & \underbrace{R_{12}}_{dx(k-d)} \\ \underbrace{R_{21}}_{(k-d)xd} & \underbrace{R_{22}}_{(k-d)x(k-d)} \end{pmatrix}.$$

- Let m_t be a $dx1$ instrument vector and Φ a non-singular dxd matrix such that

$$
\begin{aligned}
E\left[m_t v_{1,t}'\right] &= \Phi \\
E\left[m_t v_{2,t}'\right] &= 0.
\end{aligned}
$$

Algebra

$$
\begin{aligned}
\begin{pmatrix} E\left[m_t e'_{1,t}\right] & E\left[m_t e'_{2,t}\right] \end{pmatrix} &= E\left[m_t e'_t\right] \\
&= E\left[m_t v'_t R'\right] \\
&= E\left[m_t \begin{pmatrix} v'_{1,t} & v'_{2,t} \end{pmatrix}\right] \begin{pmatrix} R'_1 \\ R'_2 \end{pmatrix} \\
&= \Phi R'_1 \\
&= \Phi \begin{pmatrix} R'_{11} & R'_{21} \end{pmatrix}.
\end{aligned}
$$

Equating the left terms:

$$
E\left[m_t e'_{1,t}\right]^{-1} = \left(R'_{11}\right)^{-1} \Phi^{-1}.
$$

Equating the right terms and premultiplying by the left terms:

$$
\begin{aligned}
E\left[m_t e'_{1,t}\right]^{-1} E\left[m_t e'_{2,t}\right] &= \left(R'_{11}\right)^{-1} R'_{21} \\
\Longrightarrow \left(E\left[m_t e'_{1,t}\right]^{-1} E\left[m_t e'_{2,t}\right]\right)' R_{11} &= R_{21}.
\end{aligned}
$$

ORDER CONDITION

$$\left(E\left[m_t e_{1,t}'\right]^{-1} E\left[m_t e_{2,t}'\right]\right)' R_{11} = R_{21}.$$

- $\left(E\left[m_t e_{1,t}'\right]^{-1} E\left[m_t e_{2,t}'\right]\right)$ is a function of reduced form residuals e_t and instruments m_t and can be estimated.
- R_{21} is $(k-d)xd \Rightarrow (k-d)xd$ imposable restrictions on the matrix R.
- If $d = 1$, external instruments sufficient for partial identification. Otherwise, further structure on R_{11} required, such as short or long run restrictions.

Interpretation

$$\left(E\left[m_t e_{1,t}'\right]^{-1} E\left[m_t e_{2,t}'\right]\right)' R_{11} = R_{21}.$$

- $\left[\frac{1}{T} m' e_1\right]^{-1}\left[\frac{1}{T} m' e_2\right] \to E\left[m_t e_{1,t}'\right]^{-1} E\left[m_t e_{2,t}'\right]$ is the *plim* of a 2SLS regression of $e_{2,t}$ on $e_{1,t}$ using m_t as instruments.
- R_{21} determines how structural innovations in $v_{1,t}$ affect reduced form surprises in $e_{2,t}$:

$$e_t = \begin{pmatrix} e_{1,t} \\ e_{2,t} \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} v_{1,t} \\ v_{2,t} \end{pmatrix} = v_t.$$

- This relationship is exactly the IV regression of $e_{2,t}$ on $e_{1,t}$.
- Multiplying by R_{11} allows for cross effects among the variables in $e_{1,t}$.

Practicalities

- Restrictions may be used alone or with other restrictions.
- Implementation:
  1. Estimate reduced form VAR.
  2. Estimate $\hat{E}\left[m_t e'_{1,t}\right]$ and $\hat{E}\left[m_t e'_{2,t}\right]$ by regressing reduced form residuals on instruments.
  3. Impose restrictions on R matrix.

Alternative: LP-IV

- Can also use external instruments in LP framework.
- For simplicity, assume $d = 1$. Estimate for variable n:

$$\text{Second stage:} \qquad Y_{n,t+h} = c_{0,n,h}\hat{Y}_{1,t} + \sum_{\ell=1}^{L} C_{\ell,n,h} Y_t + error,$$

$$\text{First stage:} \qquad \hat{Y}_{1,t} = \pi_0 m_t + \sum_{\ell=1}^{L} \Pi_\ell Y_t.$$

- Note: first stage does not depend on second stage dependent variable or horizon.
- Recovers impulse response of Y_n *relative to* Y_1 to time t shock to Y_1.
- Relaxes invertibility requirement.
- Requires lead/lag exogeneity of external instrument (Stock and Watson, 2018).
- Note: does first stage need to control for lags of Y?

Alternative: order m_t first in VAR

- Assume m is scalar for simplicity.
- By FWL and local projection, ordering m_t first is equiavlent to LP on shock to m (see VAR/LP lecture).
- The impulse response of $Y_{1,t}$ to m_t is the first stage.
- The impulse response of $Y_{n,t}, n \neq 1$ to m_t is the reduced form.
- The ratio of the reduced form to the first stage is the IV IRF.
- This implementation makes clear it is *relative* IRF.

Gertler, Karadi (AEJM, 2015)

- Monetary policy VAR with high frequency interest rate surprises as external instruments.

- HFI: change in interest rates in narrow window around FOMC announcement due to monetary policy surprise news, not other shocks.

- Previous literature (e.g. Kuttner, 2001) used HFI to assess impact of monetary policy on credit spreads, real interest rates.

GK implementation

1. Monthly VAR over 1979:7-2012:6.
2. Partial identification: only interested in response to monetary policy shock.
3. Monetary policy indicator: one-year government bond rate.
4. Exernal instruments: changes in federal fund futures in current month, three month ahead, and 6,9,12 month ahead Eurodollar futures in 30 minute window around FOMC announcement.
5. "Stripped-down" VAR: policy indicator, log industrial production, log CPI. GZ excess bond premium.

CONVENTIONAL HFI ANALYSIS

TABLE 1—YIELD EFFECTS OF MONETARY POLICY SHOCKS (*Daily*, 1991–2012)

Indicator and instruments	2 year (1)	5 year (2)	10 year (3)	30 year (4)	5×5 forw (5)	Baa$^+$ (6)	Mortg.$^+$ (7)
FF, FF1	0.367***	0.233**	0.0980	0.00637	−0.0369	0.139	0.170
	(3.467)	(2.241)	(1.053)	(0.103)	(−0.388)	(1.475)	(1.445)
1 YR, FF1	0.739***	0.469***	0.197	0.0128	−0.0744	0.280	0.343
	(8.493)	(3.094)	(1.173)	(0.103)	(−0.379)	(1.544)	(1.416)
1 YR, FF4	0.880***	0.683***	0.375***	0.145*	0.0668	0.333**	0.427**
	(15.81)	(8.201)	(4.410)	(1.694)	(0.614)	(2.176)	(2.239)
2 YR, FF4		0.778***	0.432***	0.169*	0.0848	0.355**	0.483**
		(11.80)	(5.306)	(1.839)	(0.702)	(1.986)	(2.141)
2 YR, GSS		0.878***	0.575***	0.234***	0.271***	0.231*	0.350**
		(18.70)	(11.84)	(4.139)	(3.601)	(1.844)	(2.049)

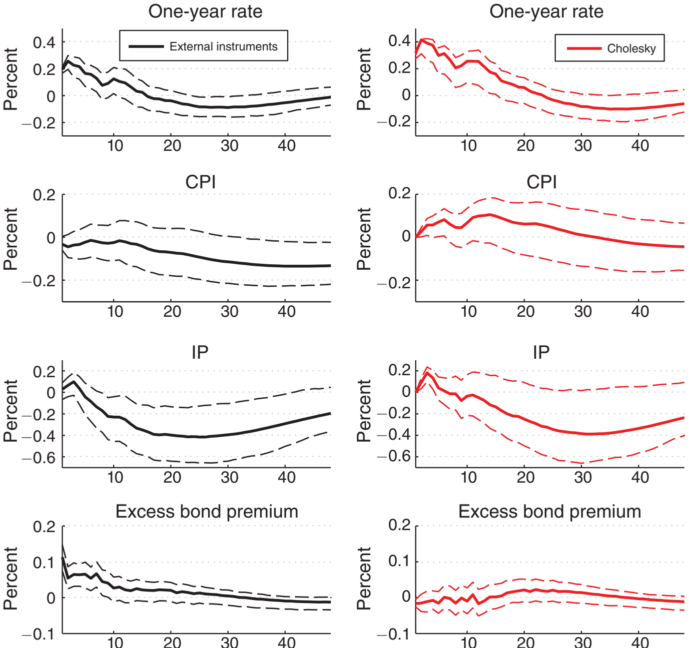
Note: Robust z-statistics in parentheses; QE dates and crisis period are excluded, 188 observations.
$^+$Two-week cumulative changes

External instruments first stage

Table 3—Effects of High-Frequency Instruments on the First Stage Residuals of the Four Variable VAR (*Monthly*, 1991–2012)

Variables	1 year (1)	1 year (2)	1 year (3)	1 year (4)	1 year (5)
FF1	0.890***				0.394
	(4.044)				(1.129)
FF4		1.151***		1.266***	1.243***
		(4.184)		(4.224)	(3.608)
ED2					1.440
					(1.244)
ED3					−4.443***
					(−2.635)
ED4			0.624**	−0.167	2.674**
			(2.039)	(−0.476)	(2.493)
Observations	258	258	258	258	258
R^2	0.066	0.078	0.025	0.079	0.110
F-statistic	16.36	17.50	4.159	11.00	8.347

VAR RESULTS


One-year rate
External instruments
Cholesky
CPI
IP
Excess bond premium
Percent
0.4
0.2
0
−0.2
−0.4
−0.6
0.1
−0.1
10
20
30
40

Blanchard-Perotti redux

$$u_{T,t} = a_1 u_{X,t} + a_2 e_{G,t} + e_{T,t}$$
$$u_{G,t} = b_1 u_{X,t} + b_2 e_{T,t} + e_{G,t}$$
$$u_{X,t} = c_1 u_{T,t} + c_2 u_{G,t} + e_{X,t}$$

- $b_1 = 0$: no automatic response of government purchases to output.
- $a_1 = \sum_i \frac{\exp(T_i)}{\exp(T)} \eta_{T_i,B_i} \eta_{B_i,X}$: weighted avg. elasticity of taxes to output.
- Construct cyclically adjusted reduced form residuals:

$$u'_{T,t} = u_{T,t} - a_1 u_{X,t}$$
$$u'_{G,t} = u_{G,t} - b_1 u_{X,t}.$$

- $u'_{T,t}, u'_{G,t}$ valid instruments for $u_{T,t}, u_{G,t}$ in

$$u_{X,t} = c_1 u_{T,t} + c_2 u_{G,t} + e_{X,t}.$$

- This is external instruments! (almost)

Outline

1. Fiscal policy (Blanchard and Perotti, QJE 2002)
2. Monetary FAVAR (Bernanke, Boivin, and Eliasz, QJE 2005)
3. External instruments
4. Bayesian identification (Baumeister and Hamilton, AER 2019)

Overview

- Traditional approaches to identification can be viewed as dogmatic priors on some parameters combined with complete agnosticism on others.
    - Cholesky example: some elements of contemporaneous response zero, others unrestricted.
    - Sign restriction example: some responses inadmissable, others unrestricted.
- Generalizes naturally to any prior beliefs.

Mechanics (my notation)

- VAR: $B(L)Y_t = e_t, e_t = Rv_t, Var(v_t) = \Sigma, R = A_0^{-1}$.
- Separate parameters into three blocks: $R, \Sigma, B(L)$.
- Specify prior over parameters $p(R, \Sigma, B(L))$.
- Computationally convenient to use distributions which conjugate together nicely – see paper for details.
- Compute posterior $p(R, \Sigma, B(L)|Y_1, \ldots, Y_T)$.
- Setup accommodates priors over A_0, structural IRFs, etc.

EXAMPLE: OIL MARKETS

- Trivariate monthly VAR in growth rate of world crude oil production q_t, real economic activity y_t, and real oil price p_t.
- Structural model:

$$
\begin{aligned}
\text{Oil supply curve:} \quad & q_t = \alpha_{qy} y_t + \alpha_{qp} p_t + {b_1}' x_{t-1} + v_{1,t}, \\
\text{Total economic activity:} \quad & y_t = \alpha_{yq} q_t + \alpha_{yp} p_t + {b_2}' x_{t-1} + v_{2,t}, \\
\text{Oil demand curve:} \quad & p_t = \alpha_{pq} q_t + \alpha_{py} y_t + {b_3}' x_{t-1} + v_{3,t}, \\
& x_{t-1} = \left(q_{t-1}, y_{t-1}, p_{t-1}, \ldots, q_{t-p}, y_{t-p}, p_{t-p}\right)'.
\end{aligned}
$$

- In VAR notation:

$$
\underbrace{\begin{pmatrix} 1 & -\alpha_{qy} & -\alpha_{qp} \\ -\alpha_{yq} & 1 & -\alpha_{yp} \\ -\alpha_{pq} & -\alpha_{py} & 1 \end{pmatrix}}_{A_0} \underbrace{\begin{pmatrix} q_t \\ y_t \\ p_t \end{pmatrix}}_{Y_t} = \underbrace{\begin{pmatrix} b_1' \\ b_2' \\ b_3' \end{pmatrix}}_{b} x_{t-1} + \underbrace{\begin{pmatrix} v_{1,t} \\ v_{2,t} \\ v_{3,t} \end{pmatrix}}_{v_t}.
$$

- Could rewrite as $B(L) Y_t = e_t = R v_t$.

Example: Cholesky (Kilian AER 2009)

- Ordering $(q_t, y_t, p_t) \Rightarrow \alpha_{qy} = \alpha_{qp} = \alpha_{yp} = 0$.
- Implement with flat prior over unrestricted elements of A_0 and all elements of Σ and $B(L)$.

Example: Cholesky (Kilian AER 2009)

- Ordering $(q_t, y_t, p_t) \Rightarrow \alpha_{qy} = \alpha_{qp} = \alpha_{yp} = 0$.

- Implement with flat prior over unrestricted elements of A_0 and all elements of Σ and $B(L)$.

- Result 1: Bayesian approach numerically equivalent to Cholesky.

- Result 2: Demand elasticity α_{pq}^{-1} either extremely flat or upward sloping. BH: "The key feature in the data that forces us to impute such unlikely values for the demand elasticity is the very low correlation between the reduced-form residuals for q_t and p_t. If we assume that innovations in q_t represent pure supply shifts, the lack of response of price would force us to conclude that the demand curve is extremely flat."

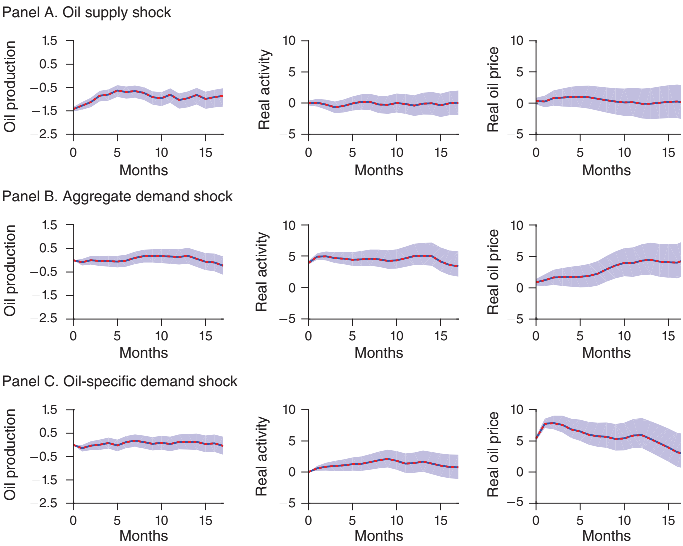
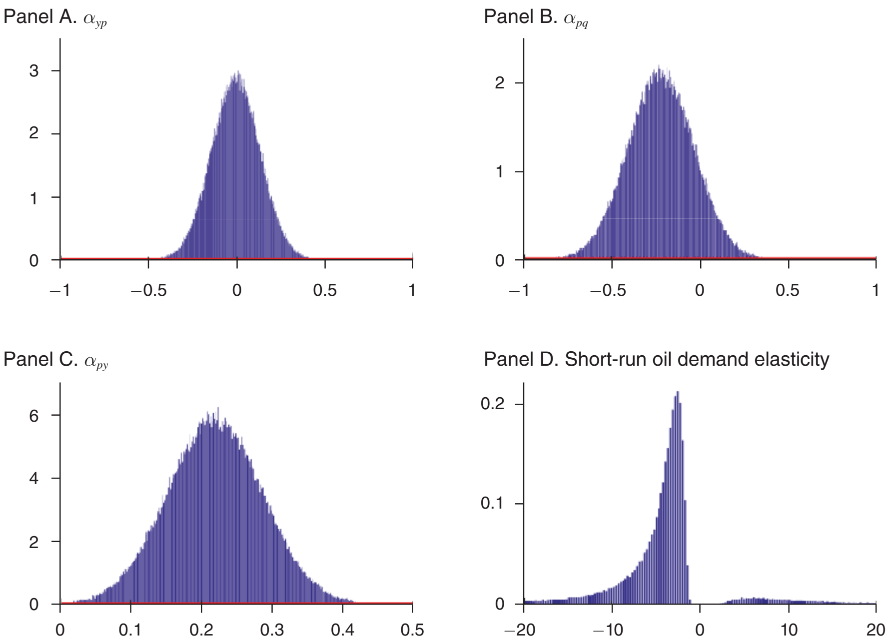


FIGURE 1. IMPULSE-RESPONSE FUNCTIONS FOR THREE-VARIABLE MODEL UNDER TRADITIONAL CHOLESKY IDENTIFICATION

Note: Red dotted lines: point estimates arrived at using Kilian's (2009) original methodology; blue solid lines median of Bayesian posterior distribution; shaded regions: 95 percent posterior credible set.


Panel A. α_{yp}
Panel B. α_{pq}
Panel C. α_{py}
Panel D. Short-run oil demand elasticity
0
1
2
3
−1
−0.5
0
0.5
1
0.1
0.2
0.3
0.4
0.5
4
6
−20
−10
10
20

BH implementation

- External information on short-run supply and demand elasticities discipline priors.
  - Like external instruments, incorporate auxiliary information for identification.

- Oil price changes and production largely unforecastable $\Rightarrow$ small coefficients in lag matrices.

- Down-weight earlier observations.

- General principle to use all information to construct priors.
  - Contrast with minimal assumptions in standard setup.

Miscellaneous research advice

1. Talk to each other.

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2. Stay organized.
    - Topic folders, subfolders within topic folder, programming files, etc.
    - Track your steps so at the end with "one click" you can go from raw data to published tables and figures (ideally).

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6. Be purposeful in topic selection, in specification, and in writing.
   - Don't do X just because ABC did X, unless point is contrast with ABC.

Miscellaneous presentation advice

1. Keep slides clean.
   - Ideally one line per bullet.
   - Text, figures, and tables legible from the back of the room.
   - Model yourself on other presentation slides, not teaching slides.
2. Adapt presentation to presentation slot:
   - Rule of thumb: two minutes per slide.
   - Explain everything or tell us what we can gloss over.
   - Preliminary lunch presentation different format and objective than job market seminar.
3. Practice: I have seen senior professors give a paper multiple times using *exactly* the same "script".