1. Compute the following integrals

   (a) $\int \frac{x^3 - 4x + 7}{x^2 + 2x - 3} dx$

   Use polynomial division to rewrite as

   $$= \int x - 2 + \frac{3x + 13}{x^2 + 2x - 3} dx = \int x - 2 + \underbrace{\frac{2x + 13}{(x+3)(x-1)}}_{(i)} dx.$$

   We compute (i) using partial fractions

   $$= \int x - 2 + \frac{4}{x-1} - \frac{1}{x-3} dx$$
   $$= \frac{1}{2}x^2 - 2x + 4\ln(x-1) - \ln(x-3) + C.$$

   (b) $\int \frac{\ln x}{x} dx.$

   Oops. I thought this would be harder. It is actually too easy to be good practice. Let $u = \ln x$ then the integral becomes

   $$\int u du = \frac{1}{2}\ln(x)^2 + C.$$

2. A spring has natural length 30cm. It requires 2J of work to stretch it from 40 cm to 45 cm. How far beyond its natural length will a force of 64 N keep the spring stretched? *Remember Newtons are measured in* ***meters***.

   We know that for a spring with natural length L and spring constant K that

   $$\text{work to stretch from } a \text{ to } b = \int_{a-L}^{b-L} Kx dx.$$

   So, in this problem we know that

   $$2 = \int_{.10}^{.15} Kx dx = .003125K \implies K = 640.$$

   Now if we apply 64 N of force, we know from Hooke's law that

   $$\text{force to stretch } x \text{ meters beyond natural length} = Kx.$$

   Solving $64 = 640x$ gives $x = .1 = 10$ cm.

3. Is $\int_{-\infty}^{\infty} \frac{x}{\sqrt{2} + x^4} dx$ convergent or divergent? If it converges compute it's value.

In class I rushed it. Let's do this carefully. First we compute the indefinite integral by making the substitutions $u = x^2$ and $t = \frac{u}{\sqrt[4]{2}}$.

$$\begin{aligned}
\int \frac{x}{\sqrt{2}+x^4}dx &= \frac{1}{2}\int \frac{1}{\sqrt{2}+u^2}du \\
&= \frac{1}{2\sqrt{2}}\int \frac{1}{1+\frac{u^2}{\sqrt{2}}}du \\
&= \frac{1}{2\sqrt{2}}\int \frac{1}{1+\left(\frac{u}{\sqrt[4]{2}}\right)^2}du \\
&= \frac{\sqrt[4]{2}}{2\sqrt{2}}\int \frac{1}{1+t^2}dt \\
&= \frac{\sqrt[4]{2}}{2\sqrt{2}}\arctan t \\
&= \frac{\sqrt[4]{2}}{2\sqrt{2}}\arctan\left(\frac{x^2}{\sqrt[4]{2}}\right).
\end{aligned}$$

Now that that's out of the way let's take some limits.

$$\begin{aligned}
\int_{-\infty}^{\infty} \frac{x}{\sqrt{2}+x^4}dx &= \lim_{b\to\infty}\frac{\sqrt[4]{2}}{2\sqrt{2}}\arctan\left(\frac{b^2}{\sqrt[4]{2}}\right) - \lim_{a\to\infty}\frac{\sqrt[4]{2}}{2\sqrt{2}}\arctan\left(\frac{a^2}{\sqrt[4]{2}}\right) \\
&= \frac{\sqrt[4]{2}}{2\sqrt{2}}[\frac{\pi}{2}-\frac{\pi}{2}] \\
&= 0.
\end{aligned}$$