THE 3-CARD WHAMO!

Description

The mathemagician selects three volunteers who select three cards. Three stacks are made and each volunteer is allowed to cut and hide their card in the deck. The mathemagician proceeds to display the cards face up, face down instructing the volunteers to tell her to stop the second they see their card. The process continues until only three cards remain and WHAMO!!! - the remaining cards are those selected by the volunteers.

Instructions

- Allow the deck to be shuffled and inspected.
- Let volunteer (up to three volunteers) select three cards, $\#1, \#2, \#3$.
- Tell them to remember there cards carefully. If you have only one volunteer it is better to let them choose three memorable cards (i.e Q,K,A of hearts).
- Instruct the volunteers to hold their cards while you make three stacks.
- From your left to your right count out facedown stacks of 10 cards, 15 cards, 15 cards and 9 cards.

10	15	15	9

- Tell the first volunteer to place their card (#1) on face down on the stack with 10 cards, and then cut from the adjacent 15 card stack, placing the cut cards on top of their card.

$$\left|\begin{array}{c|c|c|c} \text{x} & & & \\ \#1 & & & \\ 10 & \text{15-x} & 15 & 9 \end{array}\right|$$

- Now let the second volunteer put their card (#2) on top of the second pile, and cut the third stack, placing the cut cards on top of their card.

$$\left|\begin{array}{c|c|c|c} \text{x} & \text{y} & & \\ \#1 & \#2 & & \\ 10 & \text{15-x} & \text{15-y} & 9 \end{array}\right|$$

- Instruct the third volunteer to place their card (#3) on the third stack. Let them put the stack of 9 cards on top of their card.
- You should end with three stacks:

$$\left|\begin{array}{c|c|c} \text{x} & \text{y} & 9 \\ \#1 & \#2 & \#3 \\ 10 & \text{15-x} & \text{15-y} \end{array}\right|$$

- From right to left consolidate the three stacks by placing the rightmost stack on top of the stack to its left.
- You now have one stack of cards.

$$\begin{bmatrix} 9 \\ \#3 \\ 15-y \\ y \\ \#2 \\ 15-x \\ x \\ \#1 \\ 10 \end{bmatrix}$$

- Take the top four cards and place them on the bottom of the deck. The arrangement is now:

$$\begin{bmatrix} 9-4 \\ \#3 \\ 15-y \\ y \\ \#2 \\ 15-x \\ x \\ \#1 \\ 10+4 \end{bmatrix} = \begin{bmatrix} 5 \\ \#3 \\ 15 \\ \#2 \\ 15 \\ \#1 \\ 14 \end{bmatrix}$$

- Tell the volunteer(s) you will begin alternate flipping the cards, first face-up then face down. Instruct them to tell you to stop the second they see their card.
- Proceed to flip the cards, FACE-UP, FACE-DOWN, FACE-UP, FACE-DOWN, $\cdots$. When you run out of cards, with the remaining face down cards start again by revealing FACE-UP, FACE-DOWN, $\cdots$.
- Continue until 3 cards remain. WHAMO!!!

Math-Magic

Notice that no matter where the volunteers cut the deck, the cards will always end in the same place. #1 will be in the 6^{th} place, #2 will be in the 22^{nd} place and #3 will be in the 38^{th}. *The mathematical question would be 'what's so special about 6, 22 and 38?'.*

~~1~~ 2 ~~3~~ 4 ~~5~~ $\boxed{\mathbf{6}}$ ~~7~~ 8 ~~9~~ 10 ~~11~~ 12 ~~13~~ 14 ~~15~~ 16 ~~17~~ 18 ~~19~~ 20 ~~21~~ $\boxed{\mathbf{22}}$ ~~23~~ 24 ~~25~~ 26 ~~27~~ 28 ~~29~~ 30 ~~31~~ 32 ~~33~~ 34 ~~35~~ 36 ~~37~~ $\boxed{\mathbf{38}}$ ~~39~~ 40 ~~41~~ 42 ~~43~~ 44 ~~45~~ 46 ~~47~~ 48 ~~49~~ 50 ~~51~~ 52

$\xrightarrow{\text{eliminate odds}}$

~~52~~ 50 ~~48~~ 46 ~~44~~ 42 ~~40~~ $\boxed{\mathbf{38}}$ ~~36~~ 34 ~~32~~ 30 ~~28~~ 26 ~~24~~ $\boxed{\mathbf{22}}$ ~~20~~ 18 ~~16~~ 14 ~~12~~ 10 ~~8~~ $\boxed{\mathbf{6}}$ ~~4~~ 2

$\xrightarrow{\text{eliminate divisible by 4}}$

~~2~~ $\boxed{\mathbf{6}}$ ~~10~~ 14 ~~18~~ $\boxed{\mathbf{22}}$ ~~26~~ 30 ~~34~~ $\boxed{\mathbf{38}}$ ~~42~~ 46 ~~50~~

$\xrightarrow{\text{eliminate } (2+8\text{k})^{th}}$

~~46~~ $\boxed{\mathbf{38}}$ ~~30~~ $\boxed{\mathbf{22}}$ ~~14~~ $\boxed{\mathbf{6}}$

$\xrightarrow{\text{eliminate } (14+16\text{k})^{th}}$

$\boxed{\mathbf{38}}$ $\boxed{\mathbf{22}}$ $\boxed{\mathbf{6}}$ $\rightarrow$ WHAMO!

THE 5-CARD CODE

Description

The mathemagician's assistant is dealt 5 random cards. After being shown 4 cards by her assistant, the mathemagician announces the final card!

Instructions

This trick takes a little bit of memorization and some on the fly calculations. You and your assistant need to work as a team.

- Deal 5 cards.
- Since there are only 4 suits, you will have at least two cards of the same suit. Choose two cards of the same suit (if there are multiple it doesn't matter which two you choose).
- Letting aces=1, jacks =11, queens = 12, kings =13 and number cards equal their number. Decide which of your two selected cards can be added six or less to in order to arrive at the other card. For example, if you have a $3\clubsuit$ and a $J\clubsuit$ you can count up five from the jack to get to 3

  $$J \xrightarrow{1} Q \xrightarrow{2} K \xrightarrow{3} A \xrightarrow{4} 2 \xrightarrow{5} 3$$

  But you would have to count up eight from 3 to get to 11=jack. So, you would display the jack.
- Now your partner knows the suit (since your secret card has the same suit as the first card you show). You need to use the remaining three cards to communicate how many to count up (it will be the number between 1 and 6 you have determined).
- Notice that there are six different ways to order the numbers 1,2,3.

  $$\underbrace{123}_{1} \quad \underbrace{132}_{2} \quad \underbrace{213}_{3} \quad \underbrace{231}_{4} \quad \underbrace{312}_{5} \quad \underbrace{321}_{6}$$

- So, we assign the numbers 1,2, and 3 to the three remaining cards by letting

  1 = low card, 2 = middle card, 3 = high card

  In the event that you have two cards of the same number, i.e. a seven of spades and a seven of diamonds, we break ties by ranking the suits alphabetically

  $$\clubsuit < \diamondsuit < \heartsuit < \spadesuit$$

  So, if your remaining 3 cards were $4\diamondsuit, 4\spadesuit, K\heartsuit$ and you wanted to communicate 'count 5 cards' you would display

  $$K\heartsuit, 4\diamondsuit, 4\spadesuit = 312 = 5$$

  WARNING: Be aware that what you read as left-to-right your partner will see a mirror image of and read right-to-left.

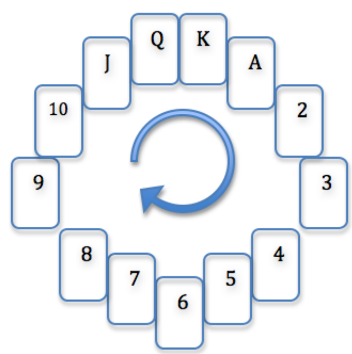
Math-Magic

Pigeon-Hole

The first mathematical idea being used is what is know as the *pigeon-hole principle*. The pigeon-hole principle is named after the idea that if you have more pigeons than pigeon holes then at least two birds will have to share a hole. In this trick we have 5 cards and only 4 suits, so you must be dealt at least two cards of the same suit.

Clock-Arithmetic

The next idea used is *modular-arithmetic*. Since there are 13 different types of cards (ignoring suit) it must be the case that any two cards are at most +6 from each other. This is very believable when you choose any two cards on the figure below and note they are separated by at most 6. Mathematically we are performing addition modulo 13.



Combinatorics

A very simple combinatorial idea used is that there are exactly $3! = 3 \cdot 2 \cdot 1 = 6$ distinct ways to arrange three objects.

Can you think of a different way to encode the numbers 1-6 with just three cards, that instead uses whether a card is shown face-up or face-down? (Hint: Binary)

Mind-Control Sequence

Description

The mathemagician begins flipping cards face-up on a table. Any number of volunteers are instructed to pick a starting card from the first few cards flipped over. From there they are choose the card the corresponds to the value of their initial card and continue in this way as you flip through the deck. After going through the deck the mathemagician reveals the final card viewed by the volunteers!

Instructions

- Select as many volunteers as you like. One to four volunteers works best.
- First explain that each card has a value. Numbered cards have their number value and all other cards (aces and facecards) are value 1.
- Tell the volunteers you will begin continuously placing cards face up on the table and that you want them to at some time in the beginning choose a card.
- After they choose the initial card, say it has value **x**, they will choose the card that comes **x** after the first chosen card. Whatever card they land on next, say it has value **y**, they will count **y** cards, and so on...
- For example if the first cards you flipped over were

  $$3 \quad 5 \quad A \quad J \quad 3 \quad 4 \quad J \quad K \quad 2 \quad Q \quad 8$$

  If you initially select the A, then the cards you subsequently select would be $\{J, 3, K, 2, 8\}$

  $$3 \quad 5 \quad \underline{A} \quad \underline{J} \quad \underline{3} \quad 4 \quad J \quad \underline{K} \quad \underline{2} \quad Q \quad \underline{8}$$

  But if you started with the 3 then the cards you select would be $\{3, J, 3, K, 2, 8, \ldots\}$.
- Now, begin flipping through the cards. The mathemagician needs to choose one of the first cards and also begin counting. It doesn't matter which card you choose, but make sure you choose a card early on.
- Flip through the entire deck, keeping track of your sequence. Once you are close to finishing the deck begin paying attention to the exact cards you are landing on.
- When you have flipped through the entire deck, pick up whatever card you were last to select and ask whether this was the last card your volunteers chose. It (most likely) will be!

Math-Magic

This trick is probabilistic. It has the most difficult mathematics of the three tricks.

Sequences

A shuffled deck can be thought of as a *finite sequence* with 52 terms. Recall that a sequence is an ordered set of numbers.

Once a volunteer selects their initial card they are committing to a *subsequence* of the 52-term sequence. A subsequence is a subset of a sequence that still respects the initial ordering. For example, $\{2, 4, 6, 8\}$ is a subsequence of $\{1, 2, 3, 4, 5, 6, 7, 8\}$ but $\{8, 6, 4, 2\}$ is not a subsequence since it has a different ordering.

In this trick our 52-term sequence consists of the digits 1-10. Let's look at an example sequence with 26-terms

$$1, 4, 2, 3, 7, 6, 9, 1, 5, 1, 5, 3, 2, 7, 8, 3, 4, 4, 1, 6, 1, 7, 9, 10, 1, 2$$

Now, lets look at the subsequences a volunteer would commit to if they chose one of the first five numbers.

$$\begin{aligned} \mathbf{1} &\rightarrow \{1, 4, 6, 3, 8, 9\} \\ \mathbf{4} &\rightarrow \{4, 6, 3, 8, 9\} \\ \mathbf{2} &\rightarrow \{2, 7, 3, 8, 9\} \\ \mathbf{3} &\rightarrow \{3, 9, 3, 6, 2\} \\ \mathbf{7} &\rightarrow \{7, 3, 8, 9\} \end{aligned}$$

Notice that when choosing either 1,4,2, or 7 the subsequences almost immediately align. Meaning at a certain point they contain the same ordering of numbers, in this case it is the numbers 3,8,9. Since the previous number determines which you choose next, this means that each of these sequences will be the same from here on out.

The trick works because it is very high probability that the subsequences will eventually be the same, regardless of starting card. The calculation is too difficult to present here, but try and convince yourself (or better yet compute) why this is very likely.

Magic Square

Description

The mathemagician writes a number down and hands it to an audience member. They then make a 5×5 table with the numbers $1, 2, \ldots, 25$ in order:

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 \end{bmatrix}$$

The audience calls out numbers and the mathemagician circles them, then crosses out all remaining numbers in the row and column of the called number until 5 numbers remain. For example, this might happen

$$\begin{bmatrix} & \boxed{2} & & & \\ & & & \boxed{9} & \\ \boxed{11} & & & & \\ & & & & \boxed{20} \\ & & \boxed{23} & & \end{bmatrix}$$

The sum of those numbers is $2+9+11+20+23 = 65$ and so is the number the mathemagician wrote down initially!

Instructions

The description has the instructions.

Math-Magic

This works because the selection procedure forces one number from each row and column to be chosen. Say we select the number in the $r_1, r_2, \ldots, r_5$ columns from each row. These are all different and sum to $1+2+3+4+5 = 15$. We can represent the number chosen in each row as

$$\begin{aligned} 0 &+ r_1 \\ 5 &+ r_2 \\ 10 &+ r_3 \\ 15 &+ r_4 \\ 20 &+ r_5 \end{aligned}$$

If we sum these we get

$$0 + 5 + 10 + 15 + 20 + r_1 + r_2 + r_3 + r_4 + r_5$$

and no matter how we choose $r_1, \ldots, r_5$ this number is always $50 + 15 = 65$.

Note that this works in general using the formula $\sum_{i=1}^{n} i = n(n+1)/2$. If we make an $n \times n$ square then the numbers we choose have the form

$$\sum_{i=0}^{n-1} (ni + r_{i+1})$$

with the r_{i+1} are all of the numbers from $1, \ldots, n$. This simplifies to

$$n \sum_{i=0}^{n-1} i + \sum_{i=1}^{n} r_i$$

which is equal to

$$n \cdot n(n-1) + n(n+1)/2 = \frac{n}{2}(n^2+1).$$

So if the mathemagician wants to do this with a larger table then the number the mathemagician should write down is given by the following table

n	$(n/2)(n^2+1)$
2	5
3	15
4	34
5	65
6	111
7	175
8	260
9	369
10	505