Today's Goal

Potential energy

Q $\vec{E} = y\hat{i} + x\hat{j}$

find ΔV between $\underset{A}{(2,1)}$ & $\underset{B}{(3,-2)}$.

$$\boxed{d(xy) = x\,dy + y\,dx}$$

$$\Delta V = -\int E_x\,dx - \int E_y\,dy$$

$$= -\int y\,dx - \int x\,dy$$

$$= -\int (y\,dx + x\,dy)$$

$$\Delta V = -\int d(xy)$$

$$\Delta V \Big|_{V_A}^{V_B} = -\,xy \Big|_{(2,1)}^{(3,-2)}$$

$$V_B - V_A = -\left[3\times(-2) - 2\times 1\right]$$

$$= 6 + 2 = 8$$

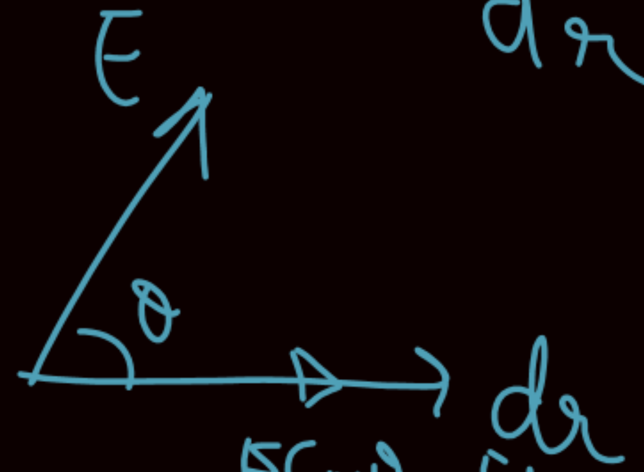
$$\vec{E} \xrightarrow{\int} \Delta V$$
$$\vec{E} \xleftarrow[d]{} \Delta V$$

Relation between field and potential

$$\int dV = -\int \vec{E} \cdot \vec{dr}$$

$$dV = -E\, dr \cos\theta$$

$$E\cos\theta = -\frac{dV}{dr}$$



$$E\cos\theta = E_{dr}$$

$$E_{dr} = -\frac{dV}{dr}$$

$$\boxed{E_x} = -\frac{\partial V}{\partial x} \Rightarrow \text{Partial differential}$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

$$V = f(x, y, z)$$

y & z are treated const

$$\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$$

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

$$= -\boxed{\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)} V$$

$\boxed{\vec{\nabla} \Rightarrow \text{delta operator.}}$

$$\vec{E} = -\vec{\nabla} V$$

$\boxed{\text{Electric field is negative of gradient of potential.}}$

Q $V = x^2 + xy + y^2 z$.

find $\vec{E}$ at $(1, 2, -1)$

$$\vec{E} = -\boxed{\frac{\partial V}{\partial x}}\hat{i} - \boxed{\frac{\partial V}{\partial y}}\hat{j} - \boxed{\frac{\partial V}{\partial z}}\hat{k}$$

$$\frac{\partial}{\partial x}\left(\boxed{x^2} + \boxed{xy} + y^2 z\right) = 2x + y + 0$$

$$\frac{\partial}{\partial y}\left(x^2 + xy + y^2 z\right) = 0 + x + 2yz$$

$$\frac{\partial}{\partial z}\left(x^2 + xy + y^2 z\right) = 0 + 0 + y^2$$

$$\boxed{\vec{E} = -(2x+y)\hat{i} - (x+2yz)\hat{j} - y^2\hat{k}}$$

$$\vec{E} = -(2\times 1 + 2)\hat{i} - (1 + 2\times 2\times(-1))\hat{j} - 2^2\hat{k}$$

$$= -4\hat{i} + 3\hat{j} - 4\hat{k}.$$

Q

$V = xy^2 + y(\ln z)$

find $\vec{E}$ at $(1, 2, 3)$

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}(xy^2) + \frac{\partial}{\partial x} y(\ln z)$$

$$= \underline{y^2} + 0$$

$$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}(xy^2) + \frac{\partial}{\partial y} y(\ln z)$$

$$\frac{\partial V}{\partial y} = \underline{x2y + \ln z}$$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z} xy^2 + \frac{\partial}{\partial z}(y \ln z)$$

$$= 0 + \frac{y}{z}$$

$$\vec{E} = -y^2\hat{i} - (2xy + \ln z)\hat{j} - \frac{y}{z}\hat{k}$$

$$= -4\hat{i} - (4 + \ln 3)\hat{j} - \frac{2}{3}\hat{k}$$

Field $(E) = \dfrac{F}{q_0}$

$$\Delta V = -\frac{W_{EF}}{q_0}$$

$$\Delta V = \frac{W_{ext\,agent}}{q_0} \Rightarrow (\text{slow})$$

$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

Uniform.

$\Delta V = -Ed$ → distance along field

− ↓ in dir. of field pot. dec.

$$\Delta V = -\int E_x\,dx - \int E_y\,dy - \int E_z\,dz$$

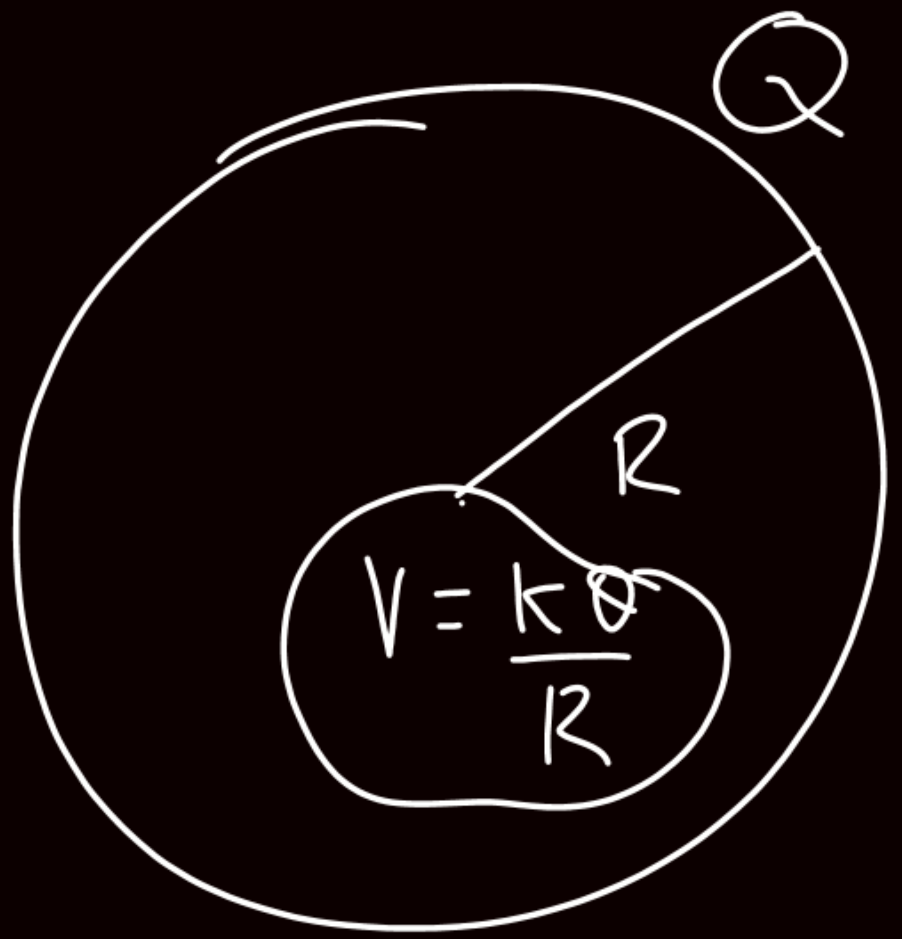
$$\boxed{\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}}$$

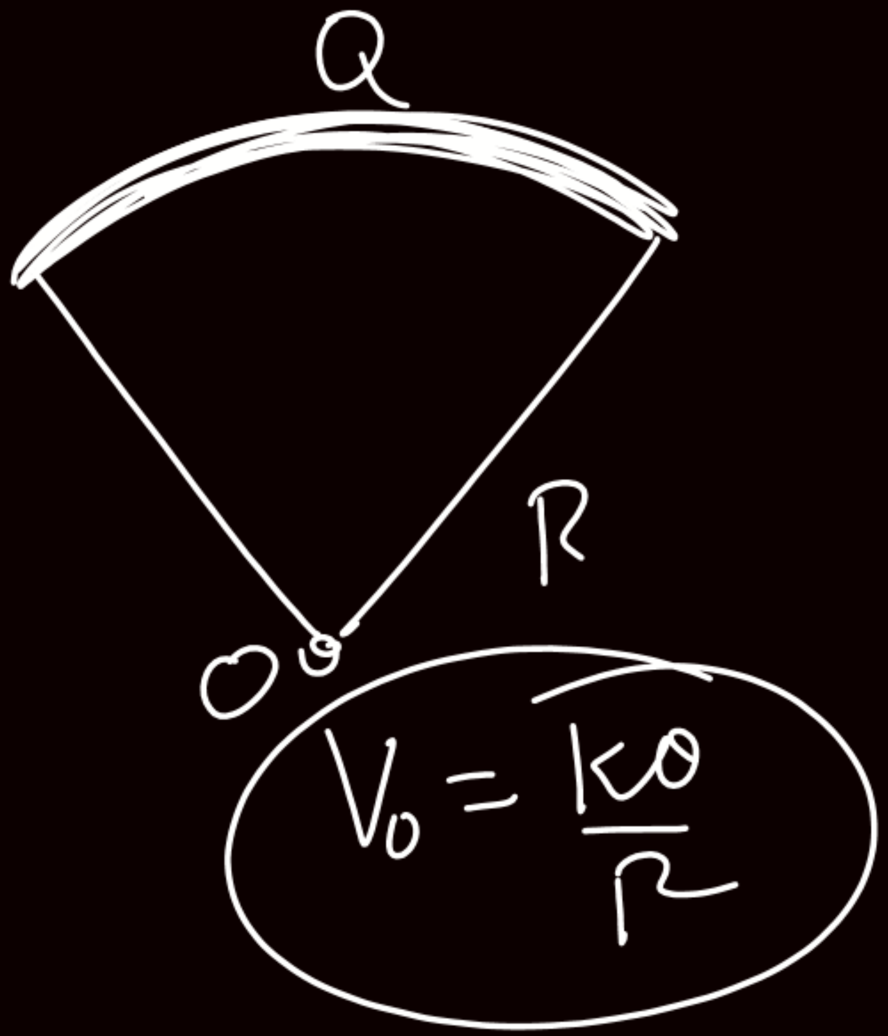
$= -\vec{\nabla} V \Rightarrow$ −ve of gradient of potential.

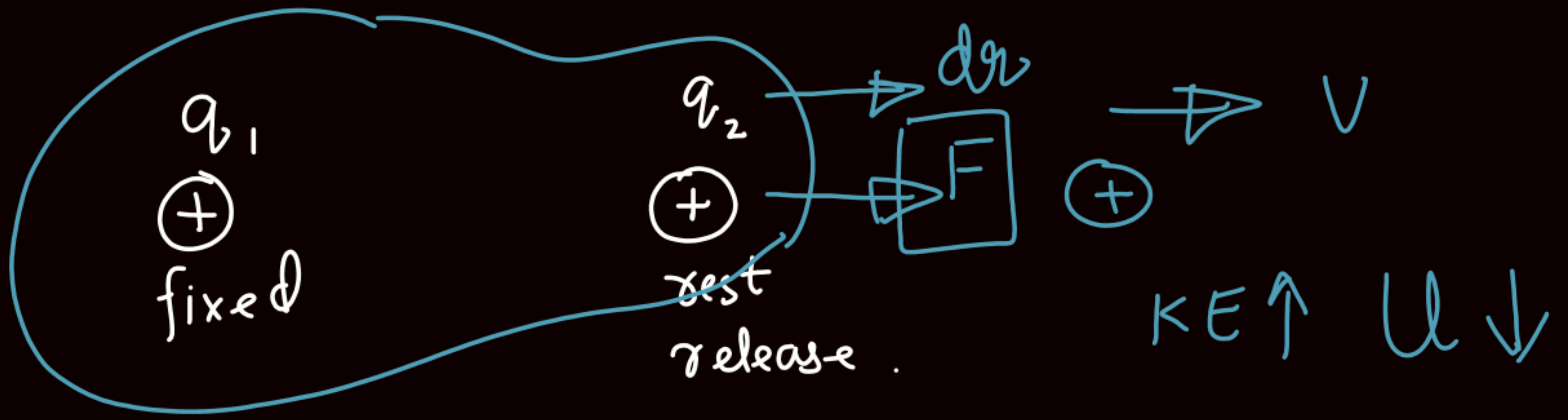
Electrostatic potential energy

Change in electrostatic potential energy of a system of charges is the -ve of work done by electric field in changing the configuration of the system.

$$\Delta U = -W_{EF} = W_{ext} \text{ [slowly]}$$


Q
R
V = kQ/R


Q
R
O
V₀ = kQ/R



$$U_f < U_i$$

$$W_{EF} = +ve = \Delta KE \uparrow$$

$$= -\Delta U \qquad \Delta U = U_f - U_i < 0$$

$$\boxed{\Delta U = -W_{EF}}$$

Potential Energy of two charge particles

$$\boxed{\int \frac{1}{r^2}\,dr = -\frac{1}{r} + c}$$

$$\frac{r^{-2+1}}{-2+1} = \frac{r^{-1}}{-1} = -\frac{1}{r}$$

$$F = \frac{K q_1 q_2}{r^2}$$

$$dU = -dW = -\frac{K q_1 q_2}{r^2}\,dr$$

$$\int_{U(r_1)}^{U(r_2)} dU = -\int_{r_1}^{r_2} \frac{K q_1 q_2}{r^2}\,dr$$

$$\int_{U(r_1)}^{U(r_2)} dU = \cancel{-}K q_1 q_2 \left[\cancel{-}\frac{1}{r}\right]_{r_1}^{r_2}$$

$$U(r_2) - U(r_1) = K q_1 q_2 \left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

if $r_1 \to \infty$ | $r_2 \to r$

$U(\infty) \to 0$

$$U(r) - U(\infty) = K q_1 q_2 \left(\frac{1}{r} - \frac{1}{\infty}\right)$$

$$\boxed{U(r) = \frac{K q_1 q_2}{r}}$$

$q_1 \quad r \quad q_2$

$$u = K\frac{q_1 q_2}{r}$$ withsign

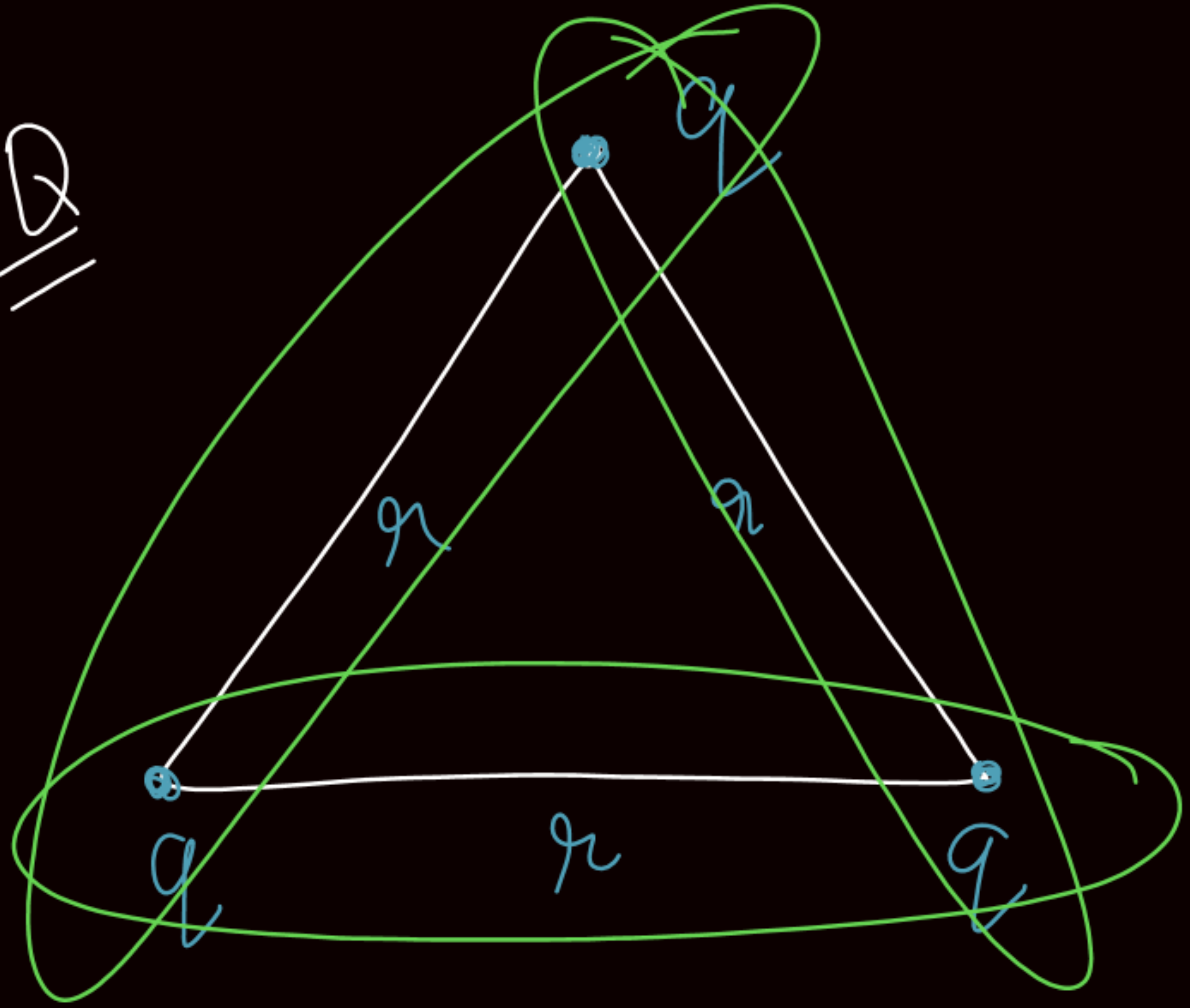
like charges have +ve pot. energy.

Unlike charges have −ve pot. energy.

$q_1 \quad r \quad -q_2$

$$u = -K\frac{q_1 q_2}{r}$$

Q

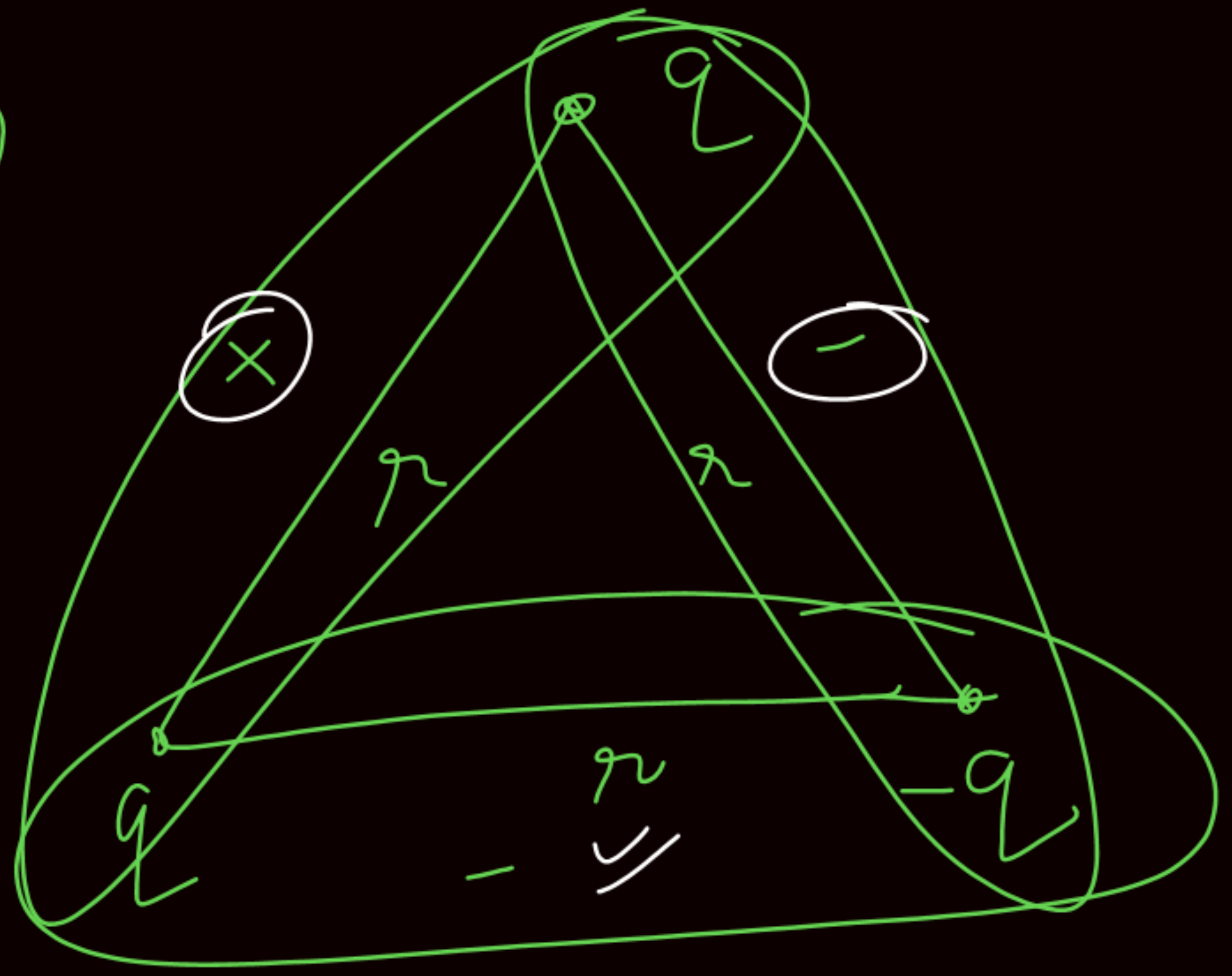


Pair

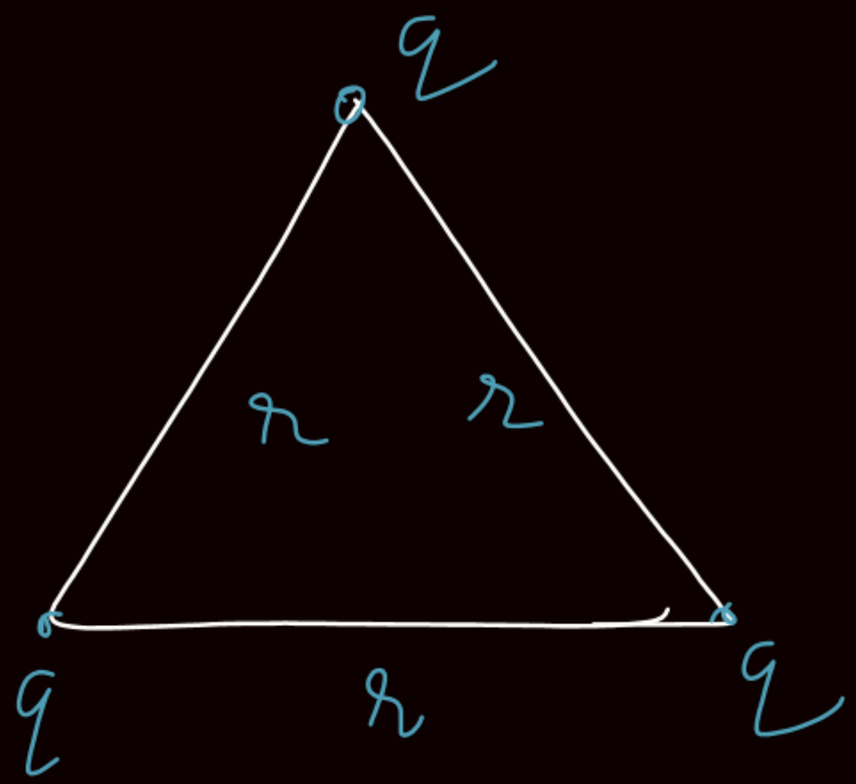
$$U = 3\left(\frac{Kqq}{r}\right)$$

$$= \frac{3Kq^2}{r}$$

Q



$$U = -\frac{Kq^2}{r}$$



Find the work done by Ext agent to reduce the side length to $r/2$.

$$W_{ext\,agent} = \Delta U$$

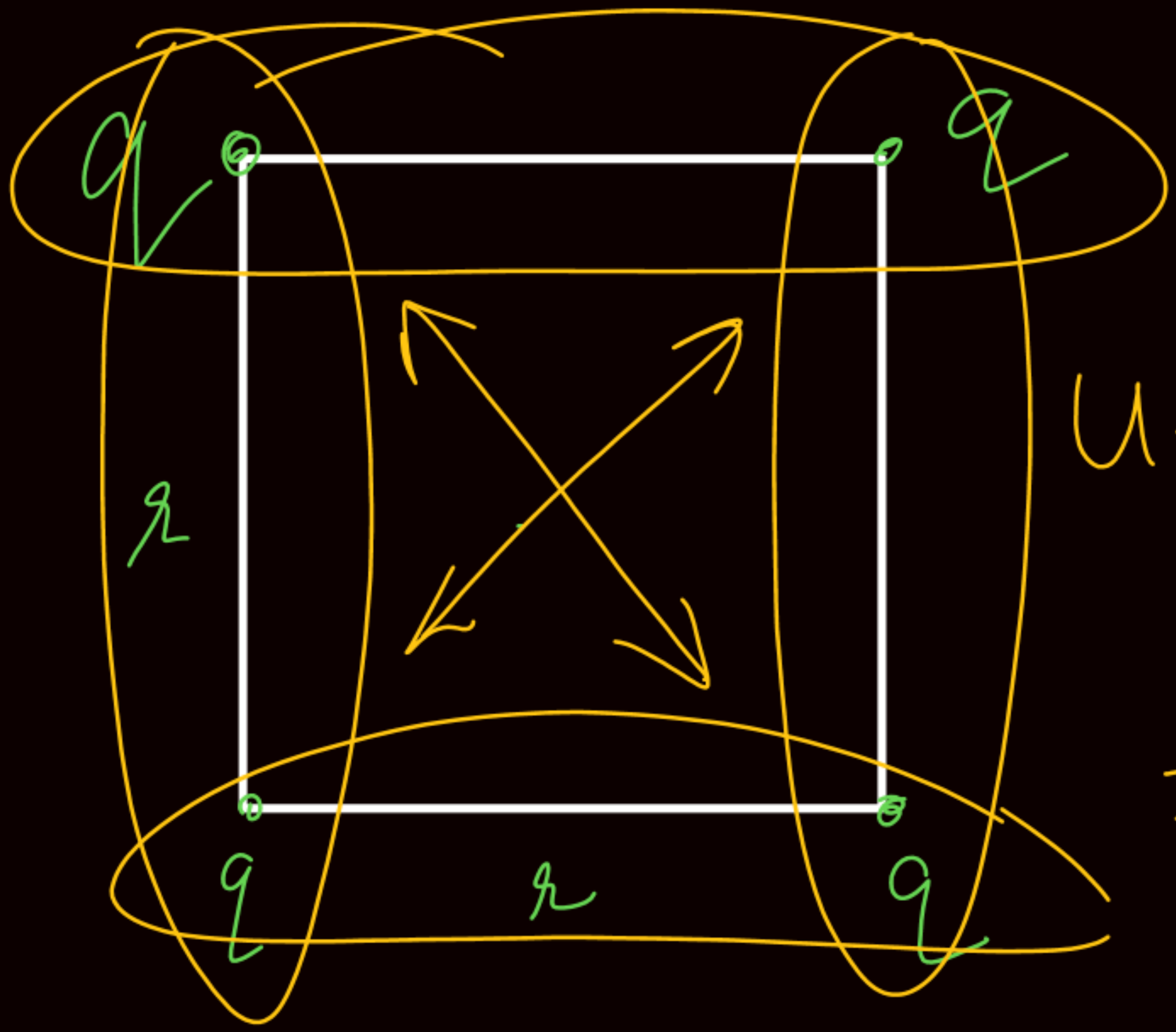
$$= U_f - U_i$$

$$= (6-3)\frac{Kq^2}{r}$$

$$= \frac{3Kq^2}{r}.$$

$$U_i = \frac{3Kq^2}{r}$$

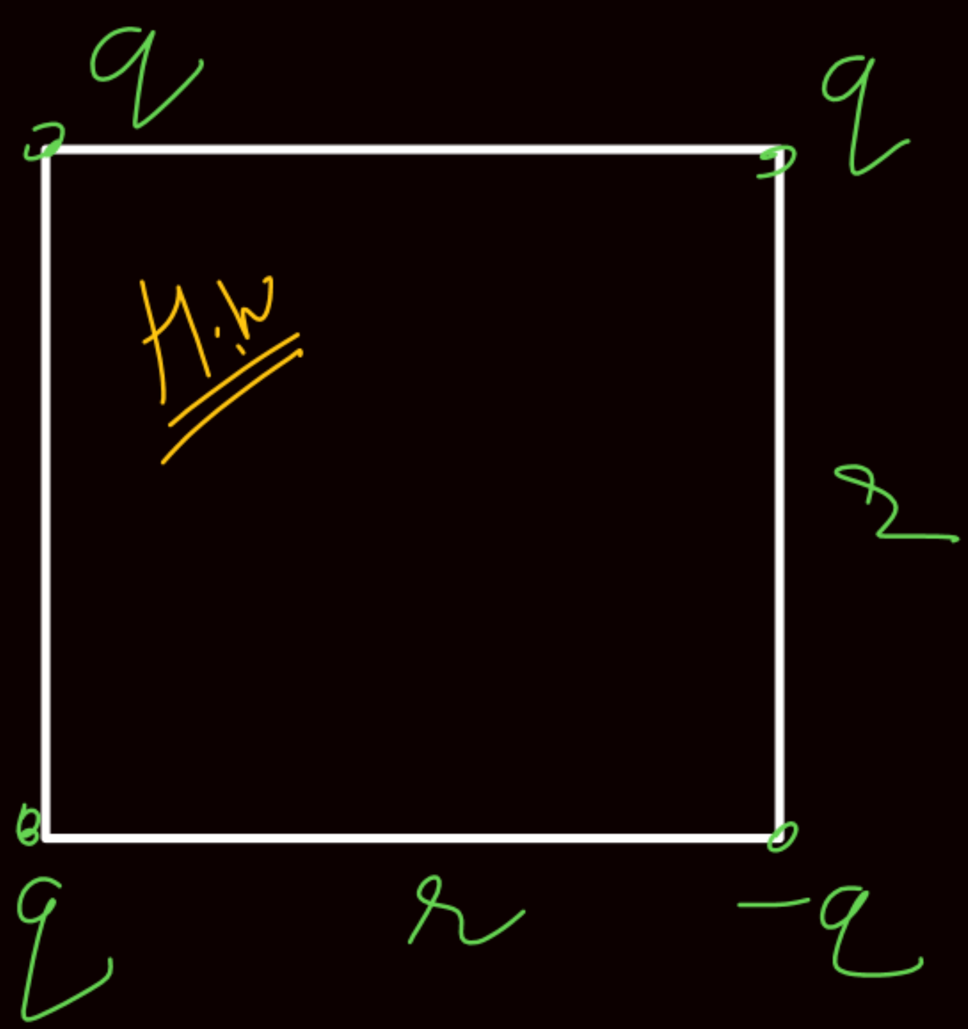
$$U_f = \frac{3Kq^2}{(r/2)} = \frac{6Kq^2}{r}$$



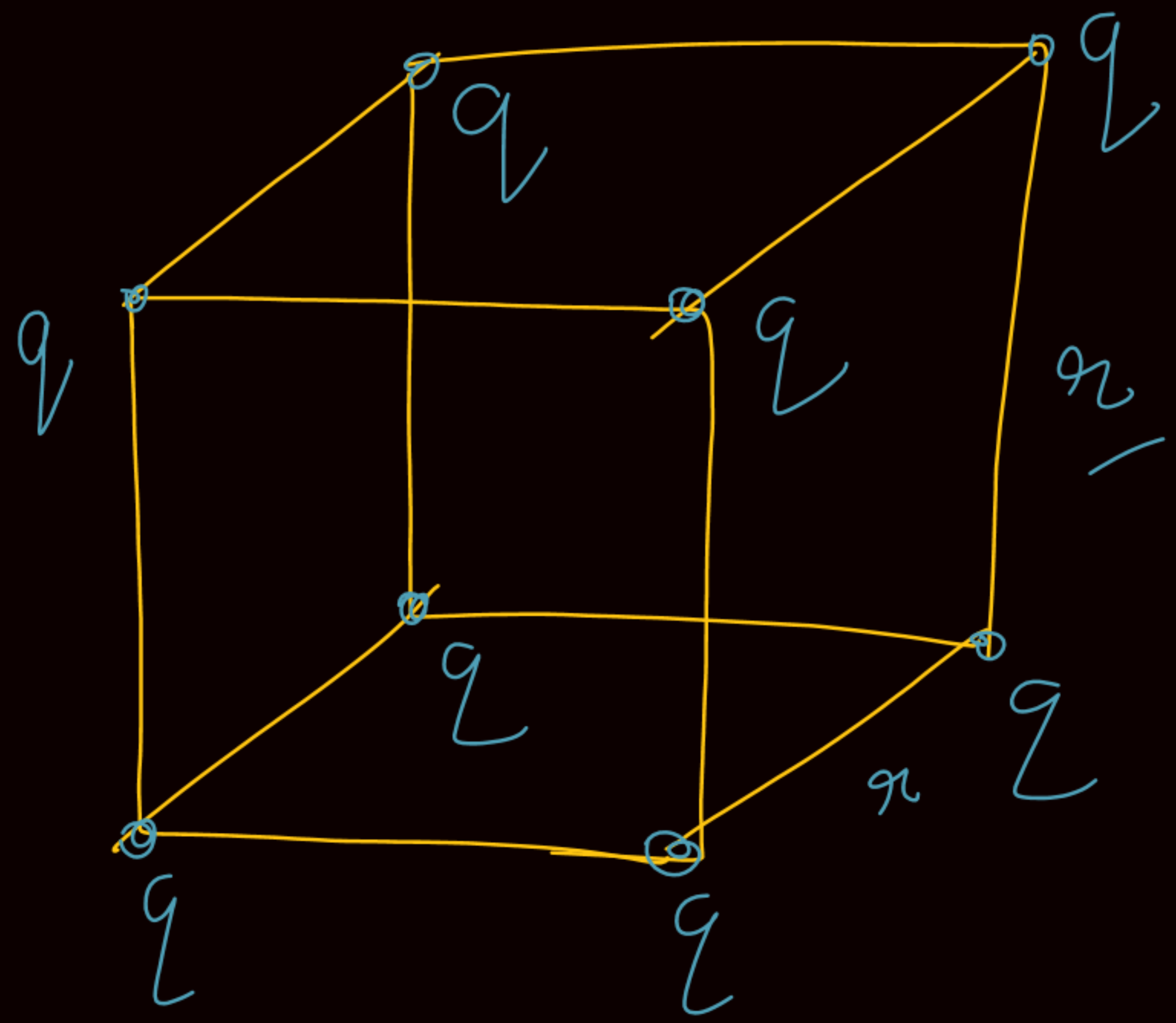
$$U_f = ?$$

$$U = 4\frac{kq^2}{r} + 2\frac{kq^2}{\sqrt{2}\,r}$$

$$= \left(4+\sqrt{2}\right)\frac{kq^2}{r}$$



$$U_f = ?$$



Cube

find U?