Control Engineering

Control system engineering is the branch of engineering which deals with the principles of control theory to design a system which gives desired behavior in a controlled manner. Hence, this is interdisciplinary. **Control system** engineers analyze, design, and optimize complex systems which consist of highly integrated coordination of mechanical, electrical, chemical, metallurgical, electronic or pneumatic elements. Thus **control engineering** deals with diverse range of dynamic systems which include human and technological interfacing.

Control system engineering focuses on analysis and design of systems to improve the speed of response, accuracy and stability of system. The two methods of control system include classical methods and modern methods. The mathematical model of system is set up as first step followed by analysis, designing and testing. Necessary conditions for the stability are checked and finally optimization follows.

In classical method, mathematical modeling is usually done in time domain, frequency domain or complex s domain. Step response of a system is mathematically modeled in time domain differential analysis to find its settling time, % overshoot etc. Laplace transforms are most commonly used in frequency domain to find the open loop gain, phase margin, band width etc of system. Concept of transfer function, sampling of data, poles and zeros, system delays all comes under the **classical control engineering** stream. **Modern control engineering** deals with Multiple Input Multiple Output (MIMO) systems, State space approach, Eigen values and vectors etc. Instead of transforming complex ordinary differential equations, modern approach converts higher order equations to first order differential equations and solved by vector method. Automatic control systems are most commonly used as it does not involve manual control. The controlled variable is measured and compared with a specified value to obtain the desired result. As a result of automated systems for control purposes, the cost of energy or power as well as the cost of process will be reduced increasing its quality and productivity.

Types of Control Systems

Linear and Non Linear Control System

Before I introduce you the theory of control system it is very essential to know the various **types of control systems**. Now there are various types of systems, we are going to discuss only those types of systems that will help us to understand the theory of control system and detail description of these types of system are given below:

- **Linear Control Systems**

In order to understand the **linear control system**, we should know the principle of superposition. The principle of superposition theorem includes two the important properties and they are explained below: Homogeneity : A system is said to be homogeneous, if we multiply input with some constant 'A' then output will also be multiplied by the same value of constant (i.e. A). Additivity: Suppose we have a system 'S' and we are giving the input to this system as 'a_1' for the first time and we are getting output as 'b_1' corresponding to input 'a_1'. On second time we are giving input 'a_2' and correspond to this we are getting output as 'b_2'. Now suppose this time we giving input as summation of the previous inputs (i.e. $a_1 + a_2$) and corresponding to this input suppose we are getting output as ($b_1 + b_2$) then we can say that system 'S' is following the property of additivity. Now we are able to define the **linear control systems** as those **types of control systems** which follow the principle of homogeneity and additivity.

- **Examples of Linear Control System**

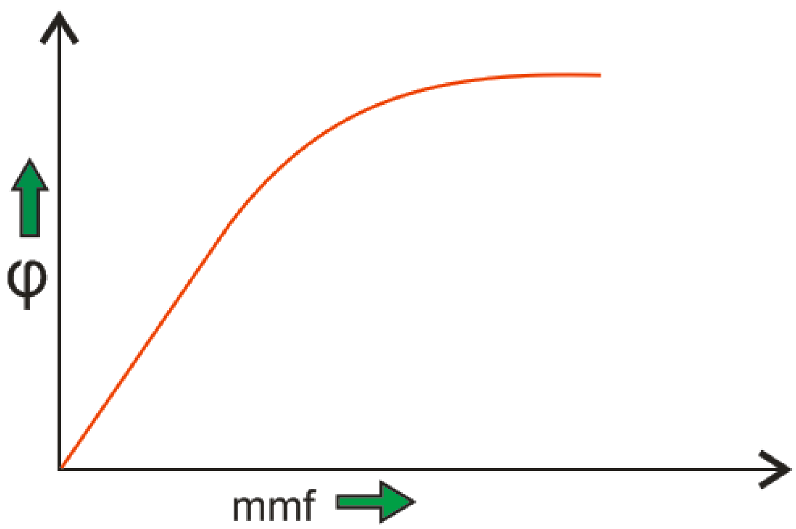
Consider a purely resistive network with a constant DC source. This circuit follows the principle of homogeneity and additivity. All the undesired effects are neglected and assuming ideal behavior of each element in the network, we say that we will get linear voltage and current characteristic. This is the example of **linear control system**.

- **Non-linear Systems**

We can simply define **non linear control system** as all those system which do not follow the principle of homogeneity. In practical life all the systems are non-linear system.

- **Examples of Non-linear System**

A well known example of non-linear system is magnetization curve or no load curve of a DC machine. We will discuss briefly no load curve of DC machines here: No load curve gives us the relationship between the air gap flux and the field winding mmf. It is very clear from the curve given below that in the beginning there is a linear relationship between winding mmf and the air gap flux but after this, saturation has come which shows the non linear behavior of the curve or characteristics of the **non linear control system**.



- **Analog or Continuous System**

In these **types of control system** we have continuous signal as the input to the system. These signals are the continuous function of time. We may have various sources of continuous input signal like sinusoidal type signal input source, square type of signal input source, signal may be in the form of continuous triangle etc.

- **Digital or Discrete System**

In these types of control system we have discrete signal (or signal may be in the form of pulse) as the input to the system. These signals have the discrete interval of time. We can convert various sources of continuous input signal like sinusoidal type signal input source, square type of signal input source etc into discrete form using the switch.

Now there are various advantages of discrete or digital system over the analog system and these advantages are written below:

1. Digital systems can handle non linear control systems more effectively than the analog type of systems.
2. Power requirement in case of discrete or digital system is less as compared to analog systems.
3. Digital system has higher rate of accuracy and can perform various complex computations easily as compared to analog systems.
4. Reliability of digital system is more as compared to analog system. They also have small and compact size.
5. Digital system works on the logical operations which increases their accuracy many times.
6. Losses in case of discrete systems are less as compared to analog systems in general.

- **Single Input Single Output Systems**

These are also known as SISO type of system. In this the system has single input for single output. Various example of this kind of system may include temperature control, position control system etc.

- **Multiple Input Multiple Output Systems**

These are also known as MIMO type of system. In this the system has multiple outputs for multiple inputs. Various example of this kind of system may include PLC type system etc.

- **Lumped Parameter System**

In these types of control systems the various active (resistor) and passive parameters (like inductor and capacitor) are assumed to be concentrated at a point and that's why these are called lumped parameter type of system. Analysis of such type of system is very easy which includes differential equations.

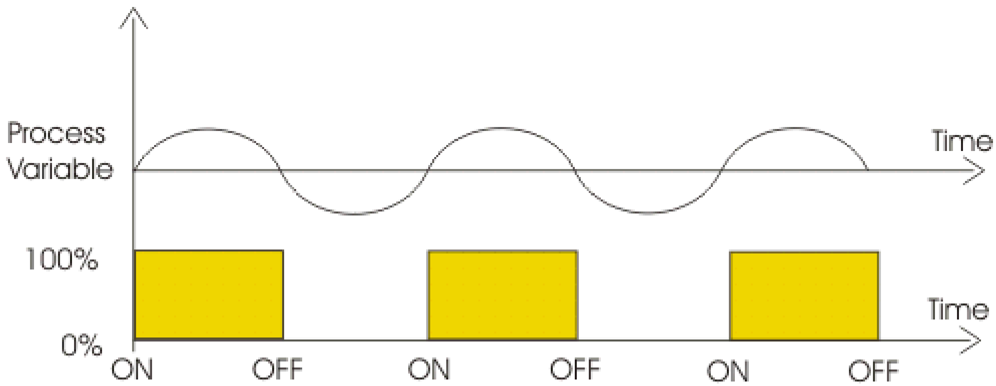
- **Distributed Parameter System**

In these **types of control systems** the various active (resistor) and passive parameters (like inductor and capacitor) are assumed to be distributed uniformly along the length and that's why these are called distributed parameter type of system. Analysis of such type of system is slightly difficult which includes partial differential equations.

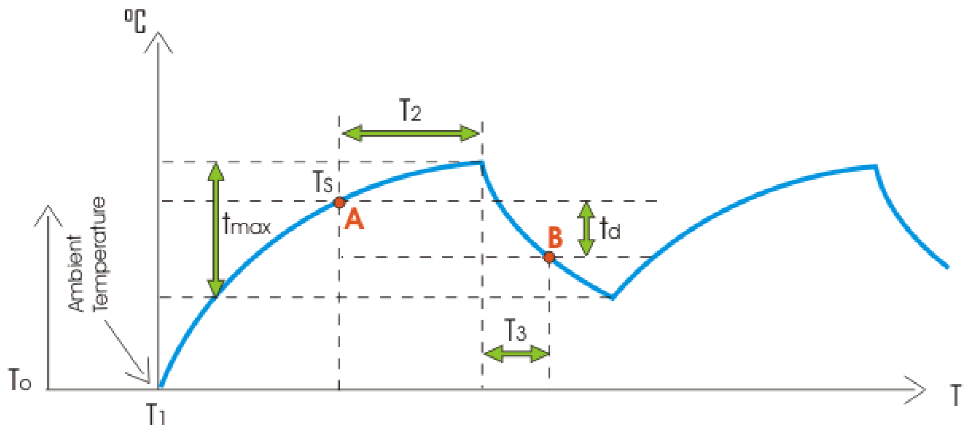
On Off Control Theory | Controller

Sometimes, the control element has only two position either it is fully closed or fully open. This control element does not operate at any intermediate position, i.e. partly open or partly closed position. The control system made for controlling such elements, is known as **on off control theory**. In this control system, when process variable changes and crosses certain preset level, the output valve of the system is suddenly fully opened and gives 100 % output.Generally in on off control system, the output causes change in process variable. Hence due to effect of output, the process variable again starts changing but in reverse direction. During this change, when process variable crosses certain predetermined level, the output valve of the system is immediately closed and output is suddenly reduced to 0 %. As there is no output, the process variable again starts changing in its normal direction. When it crosses the preset level, the output valve of the system is again fully open to give 100 % output. This cycle of closing and opening of output valve continues till the said on-off control system is in operation. A very common example of **on-off control theory** is fan controlling scheme of transformer cooling system. When transformer runs with such a load, the temperature of the electrical power transformer rises beyond the preset value at which the cooling fans start rotating with their full capacity. As the cooling fans run, the forced air (output of the cooling system) decreases the temperature of the transformer. When the temperature (process variable) comes down below a preset value, the control switch of fans trip and fans stop supplying forced air to the transformer. After that, as there is no cooling effect of fans, the temperature of the transformer again starts rising due to load.

Again when during rising, the temperature crosses the preset value, the fans again start rotating to cool down the transformer. Theoretically, we assume that there is no lag in the control equipment. That means, there is no time day for on and off operation of control equipment. With this assumption if we draw series of operations of an ideal on off control system, we will get the graph given below.



But in practical on off control, there is always a non zero time delay for closing and opening action of **controller** elements. This time delay is known as dead time. Because of this time delay the actual response curve differs from the above shown ideal response curve. Let us try to draw actual response curve of an on off control system.



Say at time T_O the temperature of the transformer starts rising. The measuring instrument of the temperature does not response instantly, as it requires some time delay for heating up and expansion of mercury in temperature sensor bulb say from instant T_1 the pointer of the temperature indicator starts rising. This rising is exponential in nature. Let us at point A, the **controller** system starts actuating for switching on cooling fans and finally after period of T_2 the fans starts delivering force air with its full capacity. Then the temperature of the transformer starts decreasing in exponential manner.

At point B, the controller system starts actuating for switching off the cooling fans and finally after a period of T_3 the fans stop delivering force air. Then the temperature of the transformer again starts rising in same exponential manner. N.B.: Here during this operation we have assumed that, loading condition of the electrical power transformer, ambient temperature and all other conditions of surrounding are fixed and constant.

Closed Loop Open Loop Control System

When a number of elements are combined together to form a system to produce desired output then the system is referred as control system. As this system controls the output, it is so referred. Each element connected to the system has its own effect on the output.

- **Definition of Control System**

A **control system** is a system of devices or set of devices, that manages, commands, directs or regulates the behavior of other device(s) or system(s) to achieve desire results. In other words the **definition of control system** can be rewritten as A control system is a system, which controls other system. As the human civilization is being modernized day by day the demand of automation is increasing accordingly. Automation highly requires control of devices. In recent years, **control systems** plays main role in the development and advancement of modern technology and civilization. Practically every aspects of our day-to-day life is affected less or more by some control system. A bathroom toilet tank, a refrigerator, an air conditioner, a geezer, an automatic iron, an automobile all are control system. These systems are also used in industrial process for more output. We find control system in quality control of products, weapons system, transportation systems, power system, space technology, robotics and many more. The **principles of control theory** is applicable to engineering and non engineering field both.

- **Feature of Control System**

The main feature of control system is, there should be a clear mathematical relation between input and output of the system. When the relation between input and output of the system can be represented by a linear proportionality, the system is called linear control system. Again when the relation between input and output cannot be represented by single linear proportionality, rather the input and output are related by some non-linear relation, the system is referred as non-linear control system.

- **Requirement of Good Control System**

Accuracy : Accuracy is the measurement tolerance of the instrument and defines the limits of the errors made when the instrument is used in normal operating conditions. Accuracy can be improved by using feedback elements. To increase accuracy of any control system error detector should be present in control system.

Sensitivity : The parameters of control system are always changing with change in surrounding conditions, internal disturbance or any other parameters. This change can be expressed in terms of sensitivity. Any control system should be insensitive to such parameters but sensitive to input signals only.

Noise : An undesired input signal is known as noise. A good control system should be able to reduce the noise effect for better performance.

Stability : It is an important characteristic of control system. For the bounded input signal, the output must be bounded and if input is zero then output must be zero then such a control system is said to be stable system.

Bandwidth : An operating frequency range decides the bandwidth of control system. Bandwidth should be large as possible for frequency response of good control system.

Speed : It is the time taken by control system to achieve its stable output. A good control system possesses high speed. The transient period for such system is very small.

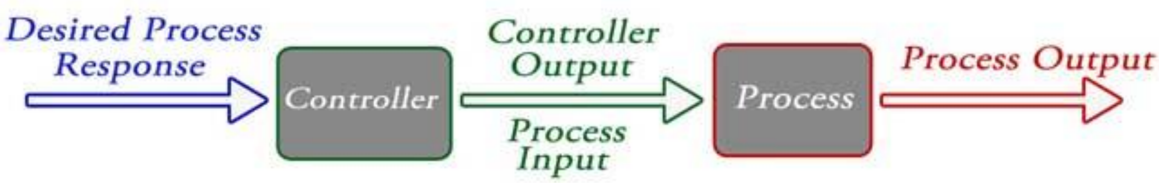
Oscillation : A small numbers of oscillation or constant oscillation of output tend to system to be stable.

Types of Control Systems

There are various **types of control system** but all of them are created to control outputs. The system used for controlling the position, velocity, acceleration, temperature, pressure, voltage and current etc. are examples of control systems. Let us take an example of simple temperature controller of the room, to clear the concept. Suppose there is a simple heating element, which is heated up as long as the electric power supply is switched on. As long as the power supply switch of the heater is on the temperature of the room rises and after achieving the desired temperature of the room, the power supply is switched off. Again due to ambient temperature, the room temperature falls and then manually the heater element is switched on to achieve the desired room temperature again. In this way one can manually control the room temperature at desired level. This is an example of **manual control system**. This system can further be improved by using timer switching arrangement of the power supply where the supply to the heating element is switched on and off in a predetermined interval to achieve desired temperature level of the room. There is another improved way of controlling the temperature of the room. Here one sensor measures the difference between actual temperature and desired temperature. If there is any difference between them, the heating element functions to reduce the difference and when the difference becomes lower than a predetermined level, the heating elements stop functioning. Both forms of the system are **automatic control system**. In former one the input of the system is entirely independent of the output of the system. Temperature of the room (output) increases as long as the power supply switch is kept on. That means heating element produces heat as long as the power supply is kept on and final room temperature does not have any control to the input power supply of the system. This system is referred as **open loop control system**. But in the later case, the heating elements of the system function, depending upon the difference between, actual temperature and desired temperature. This difference is called error of the system. This error signal is fed back to the system to control the input. As the input to output path and the error feedback path create a closed loop, this type of control system is referred as **closed loop control system**. Hence, there are two main **types of control system**. They are as follow **Open loop control system Closed loop control system**

Open Loop Control System

A control system in which the control action is totally independent of output of the system then it is called **open loop control system**. Manual control system is also an open loop control system. Fig - 1 shows the block diagram of open loop control system in which process output is totally independent of controller action.



- **Practical Examples of Open Loop Control System**

1. Electric Hand Drier – Hot air (output) comes out as long as you keep your hand under the machine, irrespective of how much your hand is dried.
2. Automatic Washing Machine – This machine runs according to the pre-set time irrespective of washing is completed or not.
3. Bread Toaster - This machine runs as per adjusted time irrespective of toasting is completed or not.
4. Automatic Tea/Coffee Maker – These machines also function for pre adjusted time only.
5. Timer Based Clothes Drier – This machine dries wet clothes for pre – adjusted time, it does not matter how much the clothes are dried.
6. Light Switch – lamps glow whenever light switch is on irrespective of light is required or not.
7. Volume on Stereo System – Volume is adjusted manually irrespective of output volume level.

- **Advantages of Open Loop Control System**

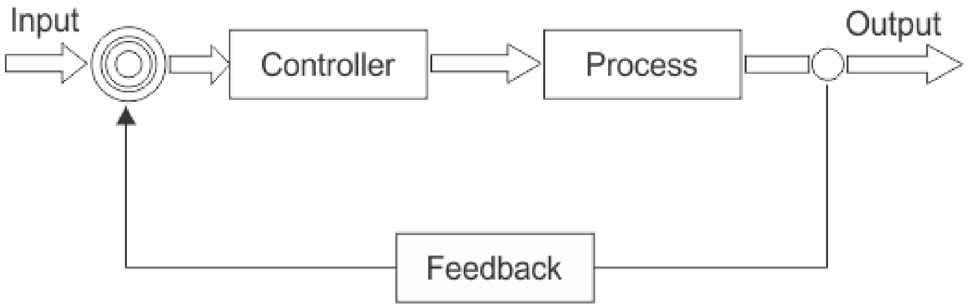
1. Simple in construction and design.
2. Economical.
3. Easy to maintain.
4. Generally stable.
5. Convenient to use as output is difficult to measure.

- **Disadvantages of Open Loop Control System**

1. They are inaccurate.
2. They are unreliable.
3. Any change in output cannot be corrected automatically.

Closed Loop Control System

Control system in which the output has an effect on the input quantity in such a manner that the input quantity will adjust itself based on the output generated is called **closed loop control system**. Open loop control system can be converted in to closed loop control system by providing a feedback. This feedback automatically makes the suitable changes in the output due to external disturbance. In this way closed loop control system is called automatic control system. Figure below shows the block diagram of closed loop control system in which feedback is taken from output and fed in to input.



- **Practical Examples of Closed Loop Control System**

1. Automatic Electric Iron – Heating elements are controlled by output temperature of the iron.
2. Servo Voltage Stabilizer – Voltage controller operates depending upon output voltage of the system.
3. Water Level Controller– Input water is controlled by water level of the reservoir.
4. Missile Launched & Auto Tracked by Radar – The direction of missile is controlled by comparing the target and position of the missile.
5. An Air Conditioner – An air conditioner functions depending upon the temperature of the room.
6. Cooling System in Car – It operates depending upon the temperature which it controls.

- **Advantages of Closed Loop Control System**

1. Closed loop control systems are more accurate even in the presence of non-linearity.
2. Highly accurate as any error arising is corrected due to presence of feedback signal.
3. Bandwidth range is large.
4. Facilitates automation.
5. The sensitivity of system may be made small to make system more stable.
6. This system is less affected by noise.

- **Disadvantages of Closed Loop Control System**

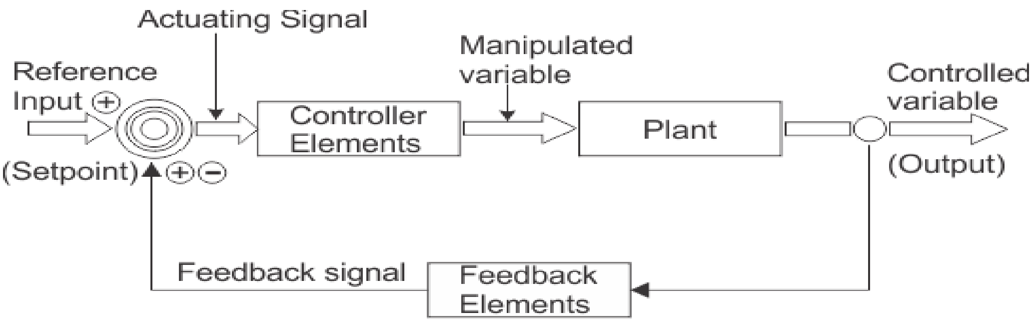
1. They are costlier.
2. They are complicated to design.
3. Required more maintenance.
4. Feedback leads to oscillatory response.
5. Overall gain is reduced due to presence of feedback.
6. Stability is the major problem and more care is needed to design a stable closed loop system.

- **Comparison of Closed Loop And Open Loop Control System**

Sr. No.	Open loop control system	Closed loop control system
1	The feedback element is absent.	The feedback element is always present.
2	An error detector is not present.	An error detector is always present.
3	It is stable one.	It may become unstable.
4	Easy to construct.	Complicated construction.
5	It is an economical.	It is costly.
6	Having small bandwidth.	Having large bandwidth.
7	It is inaccurate.	It is accurate.
8	Less maintenance.	More maintenance.
9	It is unreliable.	It is reliable.
10	Examples: Hand drier, tea maker	Examples: Servo voltage stabilizer, perspiration

- **Feedback Loop of Control System**

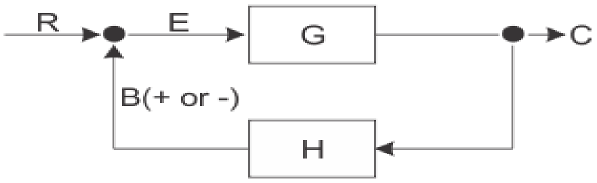
A feedback is a common and powerful tool when designing a control system. Feedback loop is the tool which take the system output into consideration and enables the system to adjust its performance to meet a desired result of system. In any control system, output is affected due to change in environmental condition or any kind of disturbance. So one signal is taken from output and is fed back to the input. This signal is compared with reference input and then error signal is generated. This error signal is applied to controller and output is corrected. Such a system is called feedback system. Figure below shows the block diagram of feedback system.



When feedback signal is positive then system called positive feedback system. For positive feedback system, the error signal is the addition of reference input signal and feedback signal. When feedback signal is negative then system is called negative feedback system. For negative feedback system, the error signal is given by difference of reference input signal and feedback signal.

- **Effect of Feedback**

Refer figure beside, which represents feedback system where R = Input signal E = Error signal G = forward path gain H = Feedback C = Output signal B = Feedback signal



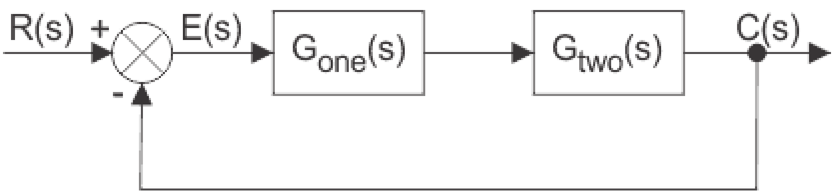
Block Diagram

1. Error between system input and system output is reduced.
2. System gain is reduced by a factor 1/(1±GH).
3. Improvement in sensitivity.
4. Stability may be affected.
5. Improve the speed of response.

Block Diagrams of Control System

The block diagram is to represent a control system in diagram form. In other words practical representation of a control system is its block diagram. It is not always convenient to derive the entire transfer function of a complex control system in a single function. It is easier and better to derive transfer function of control element connected to the system, separately. The transfer function of each element is then represented by a block and they are then connected together with the path of signal flow. For simplifying a complex control system, block diagrams are used. Each element of the control system is represented with a block and the block is the symbolic representation of transfer function of that element. A complete control system can be represented with a required number of interconnected such blocks. In the figure below, there are two elements with transfer function $G_{one}(s)$ and $G_{two}(s)$. Where $G_{one}(s)$ is the transfer function of first element and $G_{two}(s)$ is the transfer function of second element of the system.

In addition to that, the diagram also shows there is a feedback path through which output signal C(s) is fed back and compared with the input R(s) and the difference between input and output E(s) = R(s) – C(s) is acting as actuating signal or error signal.



In each block of diagram, the output and input are related together by transfer function. Where, transfer function

$$G(s) = \frac{C(s)}{R(s)}$$

where, C(s) is the output and R(s) is the input of that particular block.

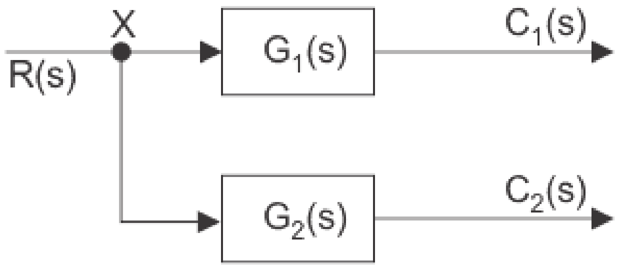
A complex control system consists of several blocks. Each of them has its own transfer function. But overall transfer function of the system is the ratio of transfer function of final output to transfer function of initial input of the system. This overall transfer function of the system can be obtained by simplifying the control system by combining this individual blocks, one by one. Technique of combining of these blocks is referred as **block diagram reduction technique**. For successful implementation of this technique, some rules for block diagram reduction to be followed. Let us discuss these rules, one by one for reduction of block diagram of control system.

If the transfer function of input of control system is R(s) and corresponding output is C(s), and the overall transfer function of the control system is G(s), then the control system can be represented as

$$Where,\ G(s) = \frac{C(s)}{R(s)}$$

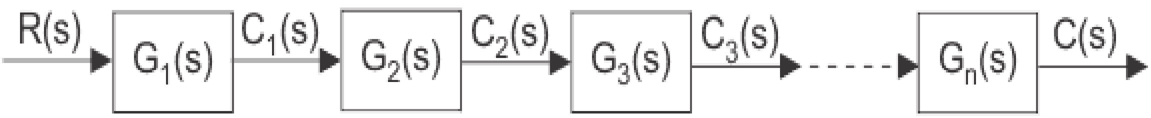
- **Take off Point of Block Diagram**

when we need to apply one or same input to more than one blocks, we use **take off point**. A point is where the input gets more than one paths to propagate. This to be noted that the input does not get divided at a point, hence input propagates through all the paths connected to that point without affecting its value. Hence, by take off point same input signals can be applied to more than one systems or blocks. Representation of a common input signal to more than one blocks of control system is done by a common point as shown in the figure below with point X.



- **Cascade Blocks**

When several systems or control blocks are connected in cascaded manner, the transfer function of the entire system will be the product of transfer function of all individual blocks. Here it also to be remembered that the output of any block will not be affected by the presence of other blocks in the cascaded system.



Now, from the diagram it is seen that,

$$G_1(s) = \frac{C_1(s)}{R(s)},\ G_2(s) = \frac{C_2(s)}{C_1(s)},\ G_3(s) = \frac{C_3(s)}{C_2(s)},$$

$$\cdots\cdots\cdots G_n(s) = \frac{C(s)}{C_{n-1}(s)}$$

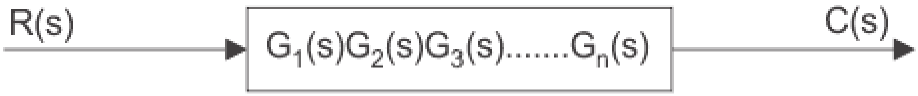
$$\therefore G(s)\ can\ be\ rewritten\ as,$$

$$G(s) = \frac{C(s)}{R(s)} = \frac{C_1(s)}{R(s)} \cdot \frac{C_2(s)}{C_1(s)} \cdot \frac{C_3(s)}{C_2(s)} \cdots \frac{C(s)}{C_{n-1}(s)}$$

$$= G_1(s)G_2(s)G_3(s)\cdots\cdots G_n(s)$$

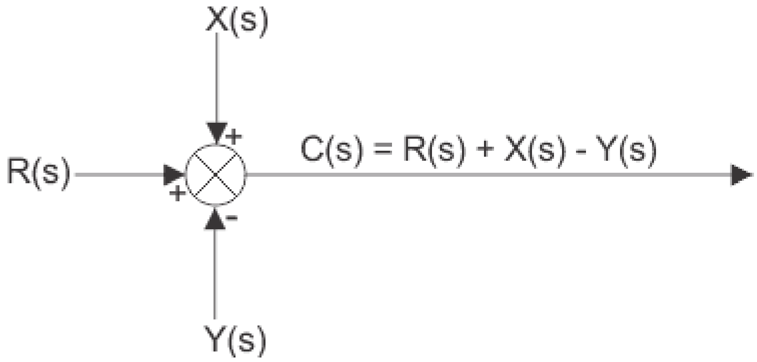
$$\therefore G(s) = G_1(s)G_2(s)G_3(s)\cdots\cdots G_n(s)$$

Where, G(s) is the overall transfer function of cascaded control system.



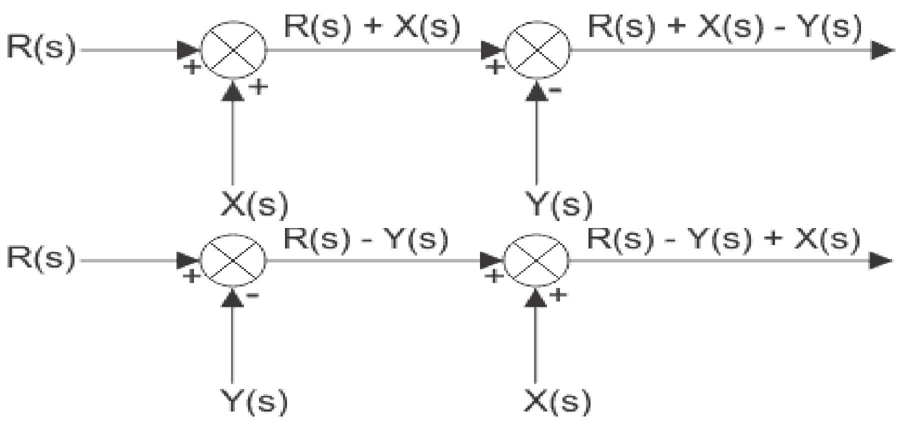
- **Summing Point of Block Diagram**

Instead of applying single input signal to different blocks as in the previous case, there may be such situation where different input signals are applied to same block. Here, resultant input signal is the summation of all input signals applied. Summation of input signals is represented by a point called summing point which is shown in the figure below by crossed circle. Here R(s), X(s) and Y(s) are the input signals. It is necessary to indicate the fine specifying the input signal entering a summing point in the block diagram of control system.



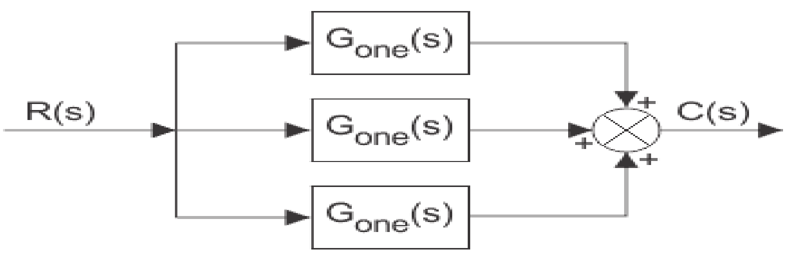
- **Consecutive Summing Point**

A summing point with more than two inputs can be divided into two or more consecutive summing points, where alteration of the position of consecutive summing points does not effect the output of the signal. In other words - if there are more than one summing points directly inter associated, then they can be easily interchanged from their position without affecting the final output of the summing system.



- **Parallel Blocks**

When same input signal is applied different blocks and the output from each of them are added in a summing point for taking final output of the system then over all transfer function of the system will be the algebraic sum of transfer function of all individual blocks.



If C_{one}, C_{two} and C_{three} are the outputs of the blocks with transfer function G_{one}, G_{two} and G_{three}, then

$$G_{one}(s) = \frac{C_{one}(s)}{R(s)}, \; G_{two}(s) = \frac{C_{two}(s)}{R(s)} \; and \; G_{three}(s) = \frac{C_{three}(s)}{R(s)}$$

$$From \; summing \; point$$

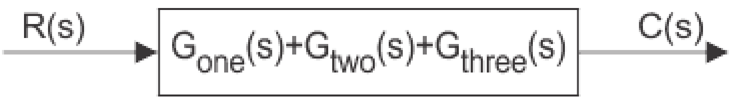
$$C(s) = C_{one}(s)+C_{two}(s)+C_{three}(s) = R(s)G_{one}(s)+R(s)G_{two}(s)+R(s)G_{three}(s)$$

$$\Rightarrow C(s) = R(s)\left[G_{one}(s) + G_{two}(s) + G_{three}(s)\right]$$

$$\Rightarrow \frac{C(s)}{R(s)} = G_{one}(s) + G_{two}(s) + G_{three}(s)$$

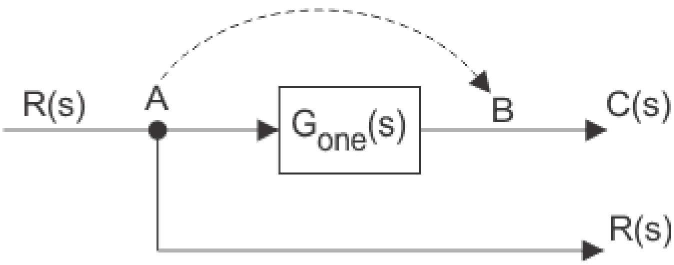
$$This \; is \; over \; all \; transfer \; function \; the \; system \; G(s),$$

$$\therefore G(s) = \frac{C(s)}{R(s)} = G_{one}(s) + G_{two}(s) + G_{three}(s)$$

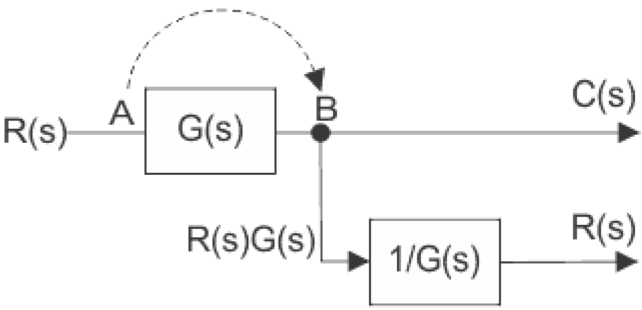


- **Shifting of Take off Point**

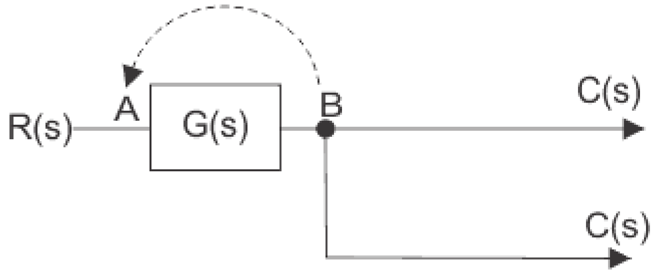
If same signal is applied to more than one system, then the signal is represented in the system by a point called take off point. Principle of **shifting of take off point** is that, it may be shifted either side of a block but final output of the branches connected to the take off point must be un-changed. The take off point can be shifted either sides of the block.



In the figure above the take off point is shifted from position A to B. The signal R(s) at take off point A will become G(s)R(s) at point B. Hence another block of inverse of transfer function G(s) is to be put on that path to get R(s) again.

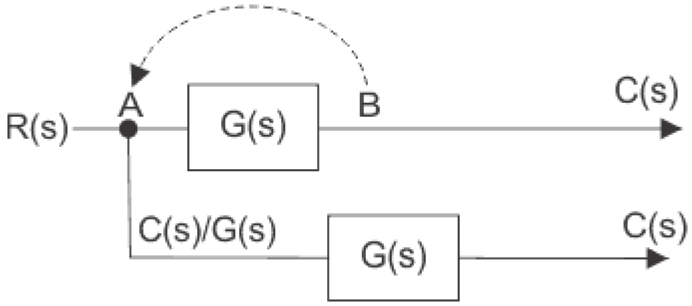


Now let us examine the situation when take off point is shifted before the block which was previously after the block.



Here the output is C(s) and input is R(s) and hence

$$G(s) = \frac{C(s)}{R(s)} \Rightarrow R(s) = \frac{C(s)}{G(s)}$$



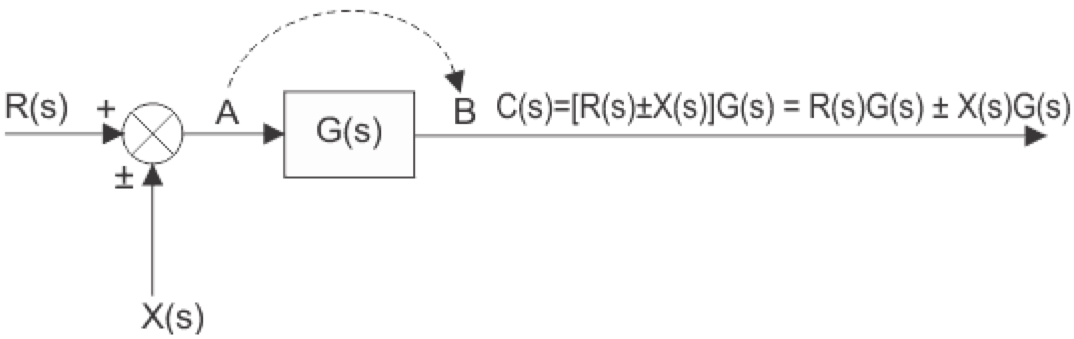
Here, we have to put one block of transfer function G(s) on the path so that output again comes as C(s).

$$G(s) \cdot \frac{C(s)}{G(s)} = C(s)$$

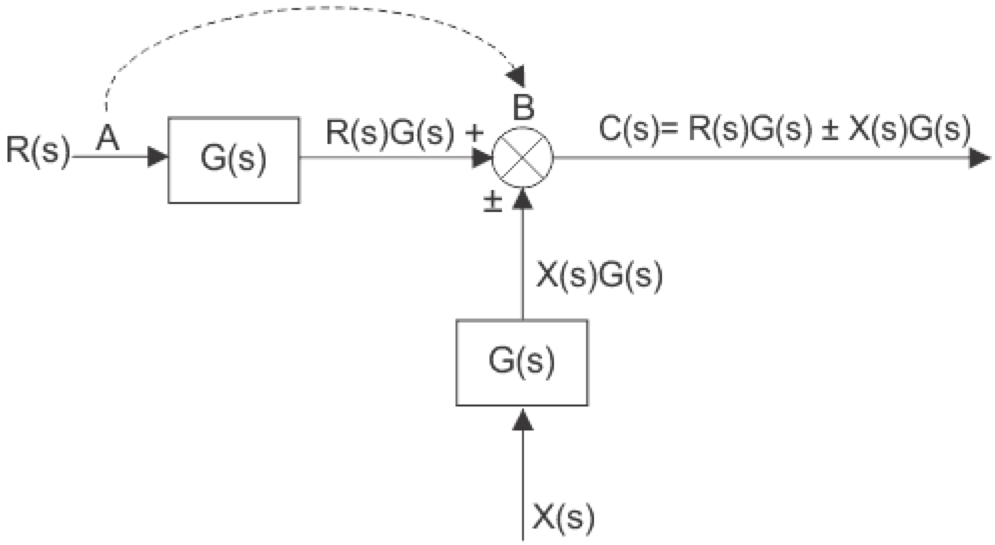
- **Shifting of Summing Point**

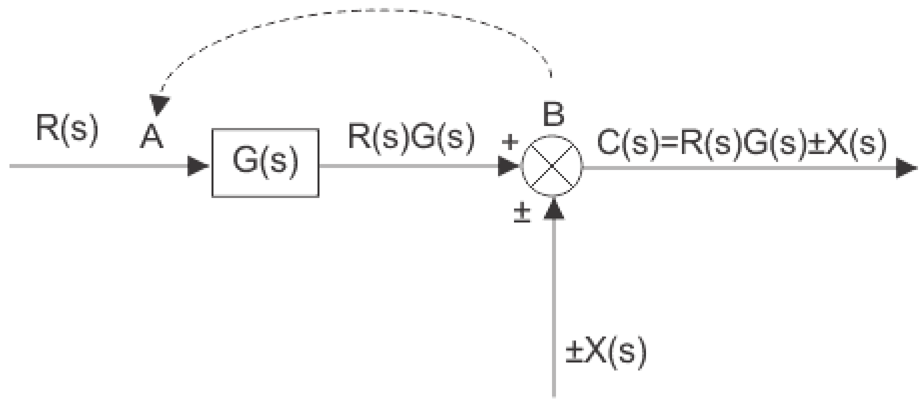
Let us examine the shifting of summing point from a position before a block to a position after a block. There are two input signals R(s) and ±X(s) entering in a summing point at position A. The output of the summing point is R(s) ± X(s). The resultant signal is the input of a control system block of transfer function G(s) and the final output of the system is

$$C(s) = [(R(s) \pm X(s)] \, G(s) = R(s)G(s) \pm X(s)G(s)$$



Hence, a summing point can be redrawn with input signals R(s)G(s) and ± X(s)G(s)

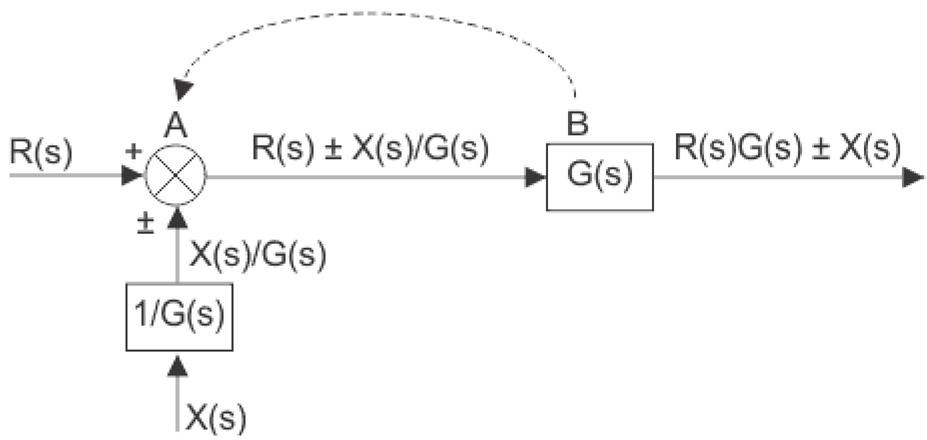




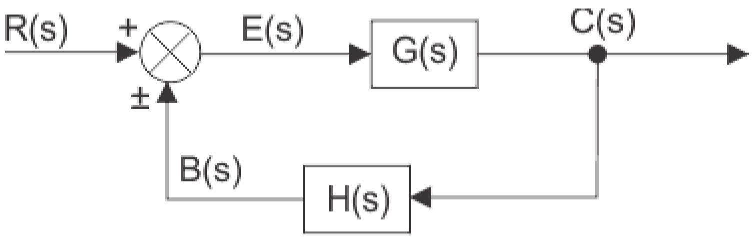
In the above block diagrams of control system output can be rewritten as

$$C(s) = R(s)G(s) \pm X(s) = G(s)\left[R(s) \pm \frac{X(s)}{G(s)}\right]$$

The above equation can be represented by a block of transfer function G(s) and input R(s)±X(s)/G(s) again R(s)±X(s)/G(s) can be represented with a summing point of input signal R(s) and ±X(s)/G(s) and finally it can be drawn as below.



Block Diagram of Closed Loop Control System



In a closed loop control system, a fraction of output is fed-back and added to input of the system. If H (s) is the transfer function of feedback path, then the transfer function of feedback signal will be B(s) = C(s)H(s). At summing point, the input signal R(s) will be added to B(s) and produces actual input signal or error signal of the system and it is denoted by E(s).

$$E(s) = R(s) \pm B(s) = R(s) \pm C(s)H(s)$$

$$Now,\ G(s) = \frac{C(s)}{E(s)} = \frac{C(s)}{R(s) \pm C(s)H(s)} [\because E(s) = R(s) \pm C(s)H(s)]$$

$$Now,\ overall\ transfer\ function\ of\ the\ system\ is$$

$$G'(s) = \frac{C(s)}{R(s)} = \frac{G(s)E(s)}{R(s)} [\because\ from\ above\ equation\ C(s) = G(s)E(s)]$$

$$\Rightarrow G'(s) = \frac{G(s)\,[R(s) \pm C(s)H(s)]}{R(s)} [\because\ from\ above\ equation\ E(s) = R(s) \pm C(s)H(s)]$$

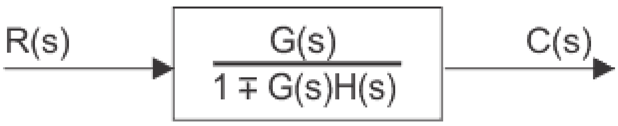
$$\Rightarrow G'(s) = G(s)\left[1 \pm \frac{C(s)H(s)}{R(s)}\right]$$

$$\Rightarrow G'(s) = G(s)\,[1 \pm G'(s)H(s)] \left[\because G'(s) = \frac{C(s)}{R(s)}\right]$$

$$\Rightarrow G'(s) = G(s) \pm G(s)G'(s)H(s)$$

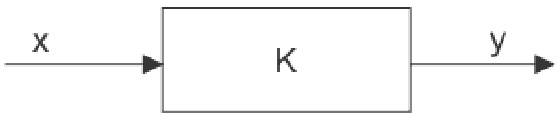
$$\Rightarrow G'(s)\,[1 \mp G(s)H(s)] = G(s)$$

$$\Rightarrow G'(s) = \frac{G(s)}{1 \mp G(s)H(s)}$$



Signal Flow Graph of Control System

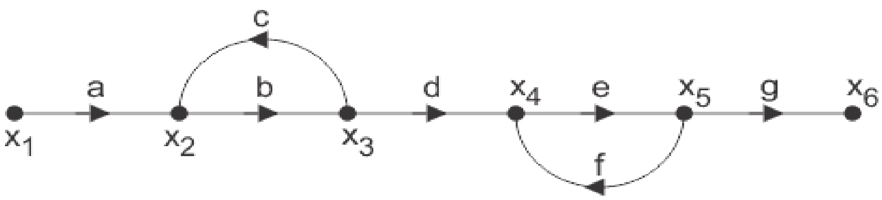
Signal flow graph of control system is further simplification of block diagram of control system. Here, the blocks of transfer function, summing symbols and take off points are eliminated by branches and nodes.The transfer function is referred as transmittance in signal flow graph. Let us take an example of equation y = Kx. This equation can be represented with block diagram as below



The same equation can be represented by signal flow graph, where x is input variable node, y is output variable node and a is the transmittance of the branch connecting directly these two nodes.

Rules for Drawing Signal Flow Graph

1. The signal always travels along the branch towards the direction of indicated arrow in the branch.
2. The output signal of the branch is the product of transmittance and input signal of that branch.
3. Input signal at a node is summation of all the signals entering at that node.
4. Signals propagate through all the branches, leaving a node.



$x_2 = ax_1 + cx_3$
$x_3 = bx_2$
$x_4 = dx_3 + fx_5$
$x_5 = ex_4$
$x_6 = gx_5$

$$\Rightarrow x_6 = gx_5 = gex_4 = ge\left(dx_3 + fx_5\right) = gedx_3 + gefx_5 = gedbx_2 + efx_6$$

$$= gedb\left(ax_1 + cx_3\right) + efx_6 = abdegx_1 + bcdegx_3 + efx_6$$

$$\Rightarrow \left(1 - ef\right) x_6 = abdegx_1 + bceg\left(x_4 - fx_5\right) = abdegx_1 + bcegx_4 - bcegfx_5$$

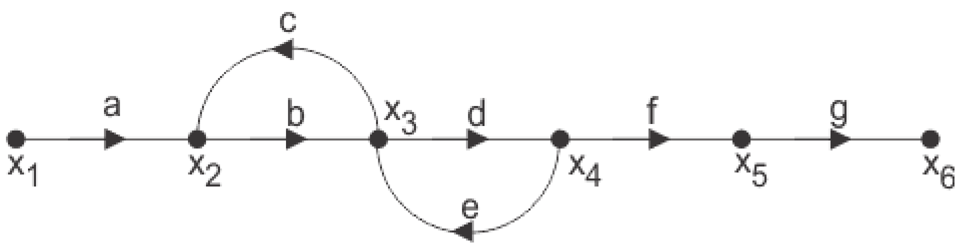
$$= abdegx_1 + bcgx_5 - bcefx_6 = abdegx_1 + bcx_6 - bcefx_6$$

$$\Rightarrow \left(1 - ef - bc + bcef\right) x_6 = abdegx_1$$

$$\Rightarrow \frac{x_6}{x_1} = \frac{abdeg}{1 - \left(bc + ef\right) + bcef}$$

- **Simple Process of Calculating Expression of Transfer Function for Signal Flow Graph**

1. First, the input signal to be calculated at each node of the graph. The input signal to a node is summation of product of transmittance and the other end node variable of each of the branches arrowed towards the former node.
2. Now by calculating input signal at all nodes will get numbers of equations which relating node variables and transmittance. More precisely, there will be one unique equation for each of the input variable node.
3. By solving these equations we get, ultimate input and output of the entire signal flow graph of control system.
4. Lastly by dividing inspiration of ultimate output to the expression of initial input we calculate the expiration of transfer function of that signal flow graph.



$$x_2 = ax_1 + cx_3$$
$$x_3 = bx_2 + ex_4$$
$$x_4 = dx_3$$
$$x_5 = fx_4$$
$$x_6 = gx_5$$
$$\Rightarrow x_6 = gx_5 = fgx_4 = dfgx_3 = dfg\left(bx_2 + ex_4\right) = bdfgx_2 + dfgex_4$$

$$= bdfg\left(ax_1 + cx_3\right) + dfgex_4 = abdfgx_1 + bcdfgx_3 + dfgex_4$$

$$\Rightarrow x_6 = abdfgx_1 + bcfgx_4 + dgex_5 = abdfgx_1 + bcfgx_4 + dgex_5$$

$$= abdfgx_1 + bcgx_5 + dex_6 = abdfgx_1 + bcx_6 + dex_6$$

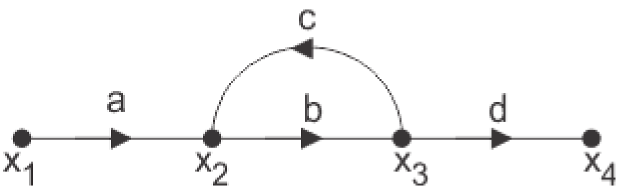
$$\Rightarrow x_6 - \left(bcx_6 + dex_6\right) = abdfgx_1$$

$$\Rightarrow \frac{x_6}{x_1} = \frac{abdfg}{1 - (bc + de)}$$

$$x_2 = ax_1$$
$$x_3 = bx_2$$
$$x_4 = cx_3$$
$$x_5 = dx_4$$
$$x_6 = ex_5$$

$$\Rightarrow x_6 = ex_5 = edx_4 = edcx_3 = edcbx_2 = edcbax_1$$

$$\frac{x_6}{x_1} = abcde$$

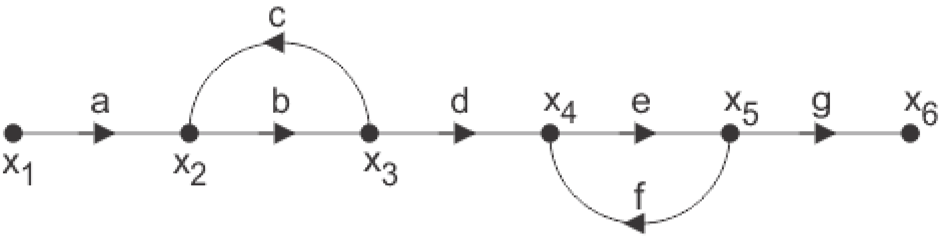


$$x_2 = ax_1 + cx_3$$
$$x_3 = bx_2$$
$$x_4 = dx_3$$

$$\Rightarrow x_4 = dx_3 = dbx_2 = db\left(ax_1 + cx_3\right) = abdx_1 + cbdx_3 = abdx_1 + cbx_4$$

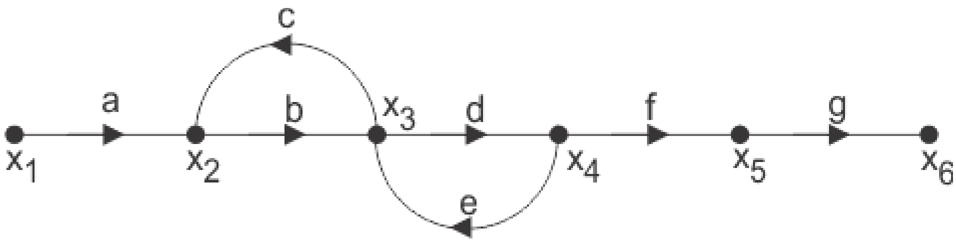
$$\Rightarrow \frac{x_4}{x_1} = \frac{abd}{1 - cb}$$

If P is the forward path transmittance between extreme input and output of a signal flow graph. L_1, L_2..................... loop transmittance of first, second,........loop of the graph. Then for first signal flow graph of control system, the overall transmittance between extreme input and output is

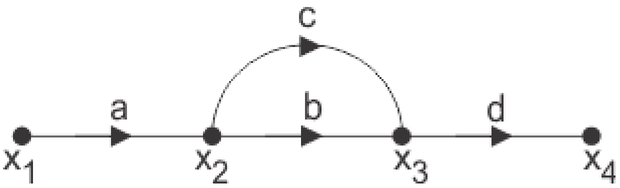


$$T = \frac{x_6}{x_1} = \frac{abdeg}{1 - (bc + ef) + bcef} = \frac{P}{1 - (L_1 + L_2) + L_1 L_2}$$

Then for second signal flow graph of control system, the overall transmittance between extreme input and output is



$$T = \frac{x_6}{x_1} = \frac{abdeg}{1-(bc+ef)} = \frac{P}{1-(L_1+L_2)}$$



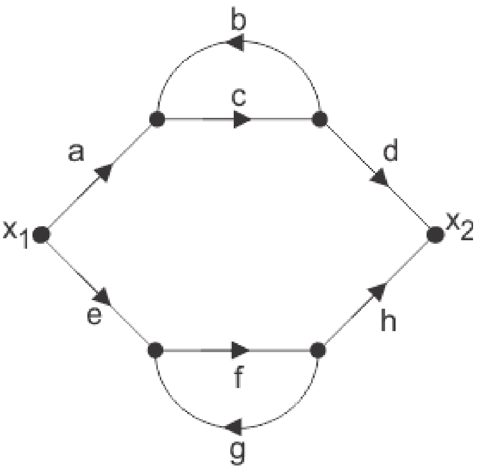
$x_2 = ax_1$
$x_3 = bx_2 + cx_2$
$x_4 = dx_3$

$$\Rightarrow x_4 = d(bx_2 + cx_2) = bdx_2 + cdx_2 = bdax_1 + cdax_1$$

$$\Rightarrow \frac{x_4}{x_1} = bda + cda = abd + acd$$

$$\therefore T = \frac{x_4}{x_1} = abd + acd = P_1 + P_2$$

$Where\ P_1\ \&\ P_2 forward\ path\ transmittance\ of\ two\ parallel\ path\ respectively.$



Here in the figure above, there are two parallel forward paths. Hence, overall transmittance of that signal flow graph of control system will be simple arithmetic sum of forward transmittance of these two parallel paths.

As the each of the parallel paths having one loop associated with it, the forward transmittances of these parallel paths are

$$T_1 = \frac{P_1}{1-L_1}\ and\ T_2 = \frac{P_2}{1-L_2}\ respectively.$$

Therefore overall transmittance of the signal flow graph is

$$T = T_1 + T_2 = \frac{P_1}{1-L_1} + \frac{P_2}{1-L_2}$$

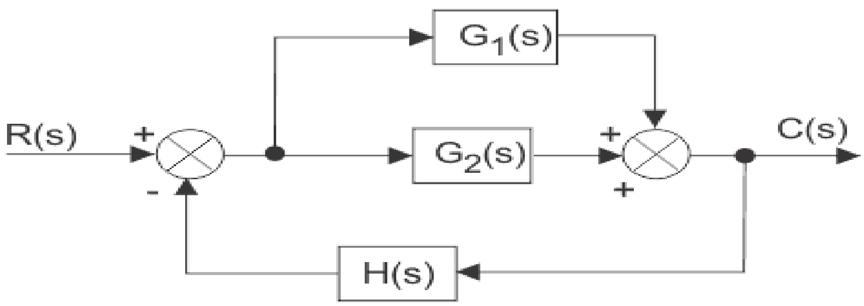
Mason's Gain Formula

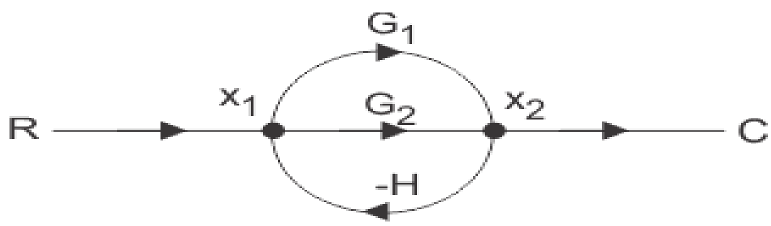
The overall transmittance or gain of signal flow graph of control system is given by Mason's Gain Formula and as per the formula the overall transmittance is

$$T = \sum_{k=1}^{k} \frac{P_k \Delta_k}{\Delta}$$

Where, P_k is the forward path transmittance of k_{th} in path from a specified input is known to an output node. In arresting P_k no node should be encountered more than once. Δ is the graph determinant which involves closed loop transmittance and mutual interactions between non-touching loops. Δ = 1 - (sum of all individual loop transmittances) + (sum of loop transmittance products of all possible pair of non-touching loops) - (sum of loop transmittance products of all possible triplets of non-touching loops) + (……) - (……) Δ_k is the factor associated with the concerned path and involves all closed loop in the graph which are isolated from the forward path under consideration. The path factor Δ_k for the k_{th} path is equal to the value of grab determinant of its signal flow graph which exist after erasing the K_{th} path from the graph.

By using this formula one can easily determine the overall transfer function of control system by converting a block diagram of control system (if given in that form) to its equivalent signal flow graph. Let us illustrate the below given block diagram





$$P_1 = G_1,\ P_2 = G_2,\ L_1 = -G_1H,\ L_2 = -G_2H$$

$$\Delta_1 = 1,\ \Delta_2 = 1\ \because\ both\ loops\ L_1\ \&\ L_2\ touch\ both\ forward\ paths\ G_2\ \&\ G_1\ respectively$$

$$\Delta = 1 - \{L_1 + L_2\} = 1 - \{(-G_1H) + (-G_2H)\} = 1 + G_1H + G_2H$$

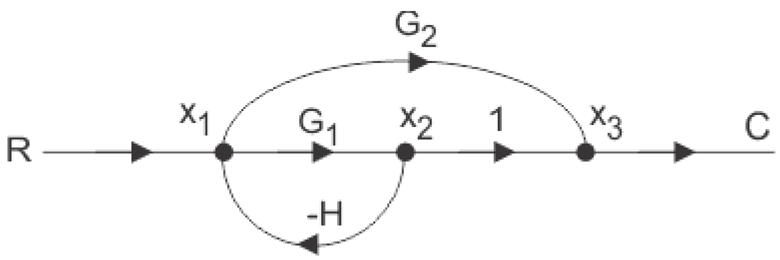
$$Now,\ T = \frac{C}{R} = \sum_{k=1}^{2} \frac{P_k \Delta_k}{\Delta} = \frac{P_1 \cdot \Delta_1 + P_2 \cdot \Delta_2}{\Delta} = \frac{G_1 + G_2}{1 + G_1H + G_2H}$$

$$Here,\ x_1 = R - Hx_2$$
$$x_2 = x_1G_1 + x_1G_2$$
$$C = x_2$$

$$\Rightarrow C = x_2 = x_1G_1 + x_1G_2 = x_1(G_1 + G_2) = (R - Hx_2)(G_1 + G_2)$$
$$= (R - HC)(G_1 + G_2)$$

$$\Rightarrow C(1 + G_1H + G_2H) = R(G_1 + G_2)$$

$$\Rightarrow T = \frac{C}{R} = \frac{G_1 + G_2}{1 + G_1H + G_2H}$$



$$P_1 = G_1,\ P_2 = G_2,\ L_1 = -G_1H$$

$$\Delta_1 = 1,\ \Delta_2 = 1\ \because loop\ L_1\ touches\ both\ forward\ paths\ G_1\ \&\ G_2$$

$$\Delta = 1 - \{L_1\} = 1 - \{(-G_1H)\} = 1 + G_1H$$

$$Now,\ T = \frac{C}{R} = \sum_{k=1}^{2}\frac{P_k\Delta_k}{\Delta} = \frac{P_1\cdot\Delta_1 + P_2\cdot\Delta_2}{\Delta} = \frac{G_1 + G_2}{1 + G_1H}$$

Basic Signal Operations

A signal, comprises of a set of information expressed as a function of any number of independent variables, that can be given as an input to a system, or derived as output from the system, to realize its true practical utility. The signal we derive out of a complex system might not always be in the form we want, ∴ being well acquainted with some **basic signal operations** may come really handy to enhance the understandability and applicability of signals.The mathematical transformation from one signal to another can be expressed as Y(t) = **T**X(t) ; where Y(t) represents the modified signal derived from the original signal X(t), having only one independent variable t. The **basic set of signal operations** can be broadly classified as below.

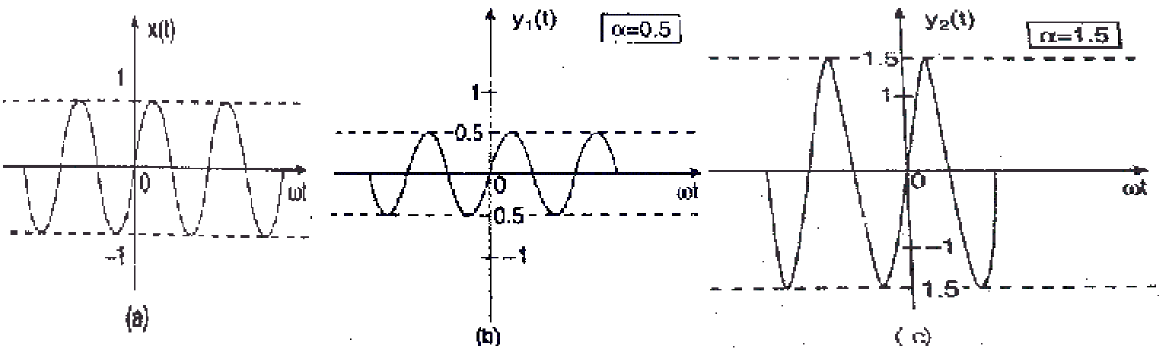
Basic Signal Operations Performed on Dependent Variables

In this transformation, only the quadrature axis values are modified i.e magnitude of the signal changes, with no effects on the horizontal axis values or periodicity of signals like.

1. Amplitude scaling of signals.
2. Addition of signals.
3. Multiplication of signals.
4. Differentiation of signals.
5. Integration of signals.

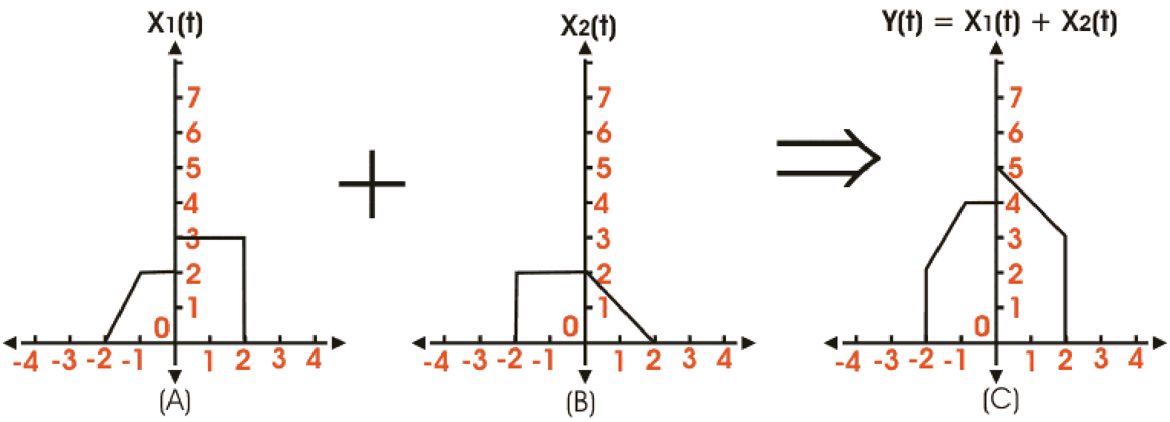
- **Amplitude Scaling of Signals**

Amplitude scaling is a very basic operation performed on signals to vary its strength. It can be mathematically represented as Y(t) = **α** X(t). Here, **α** is the scaling factor, where:- α<1 → signal is attenuated. α>1 → signal is amplified. This is illustrated in the diagram, where the signal is attenuated when α = 0.5 in fig (b) and amplified when α = 1.5 as in fig (c).



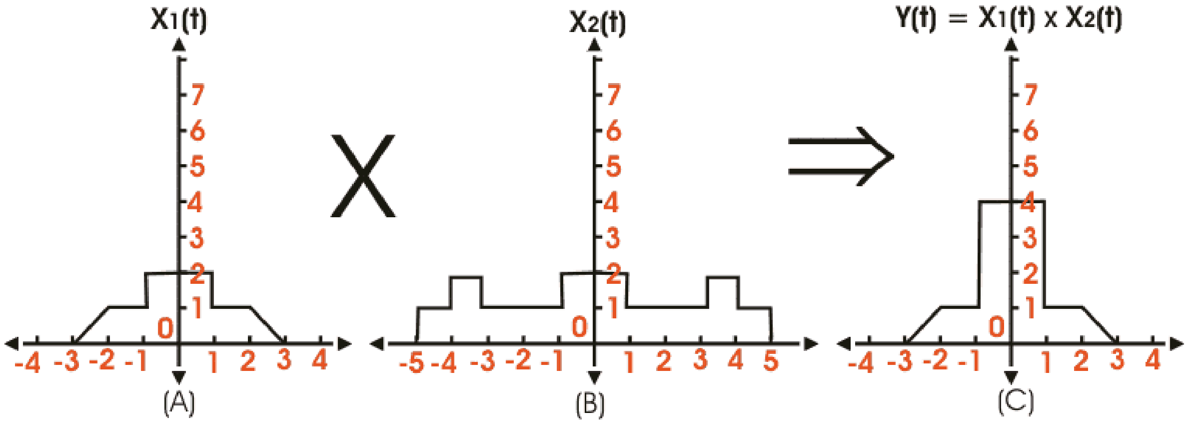
- **Addition of Signals**

This particular operation involves the addition of amplitude of two or more signals at each instance of time or any other independent variables which are common between the signals. Addition of signals is illustrated in the diagram below, where $X_1(t)$ and $X_2(t)$ are two time dependent signals, performing the additional operation on them we get, $Y(t) = X_1(t) + X_2(t)$

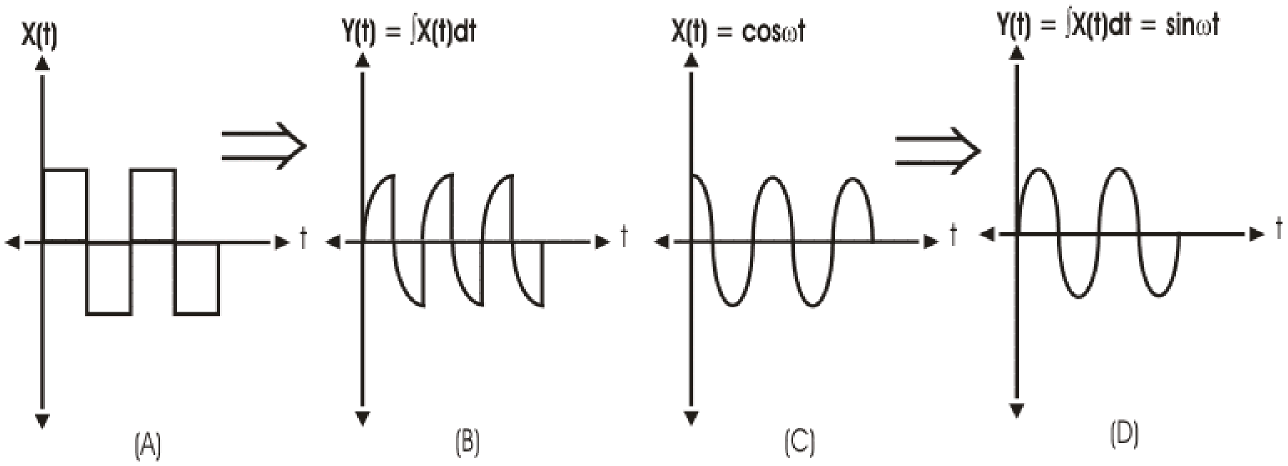


- **Multiplication of Signals**

Like addition multiplication of signals also falls under the category of basic signal operations. Here multiplication of amplitude of two or more signals at each instance of time or any other independent variables is done which are common between the signals. The resultant signal we get has values equal to the product of amplitude of the parent signals for each instance of time. Multiplication of signals is illustrated in the diagram below, where $X_1(t)$ and $X_2(t)$ are two time dependent signals, on whom after performing the multiplication operation we get, $Y(t) = X_1(t)\, X_2(t)$



Differentiation of Signals



For differentiation of signals, it must be noted that this operation is only applicable for only continuous signals, as a discrete function cannot be differentiated. The modified signal we get on differentiation has tangential values of the parent signal at all instance of time. Mathematically it can be expressed as:- Differentiation of a standard square and sine wave is shown in the figure below.

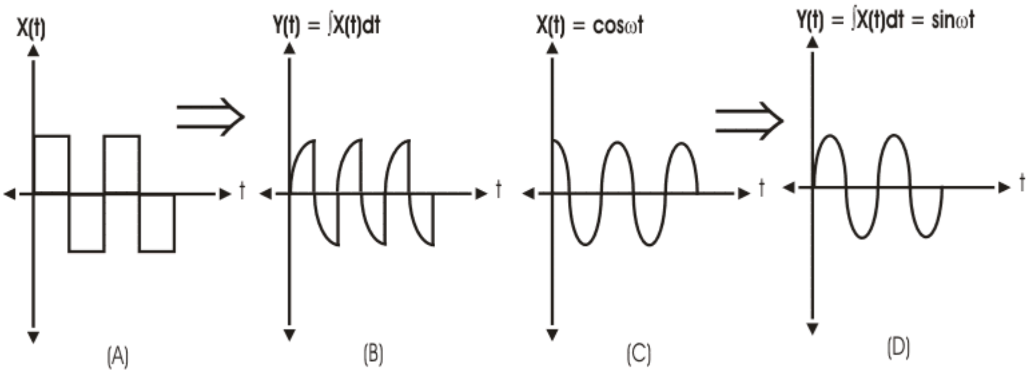
$$Y(t) = \frac{d}{dt}X(t)$$

- **Integration of Signals**

Like differentiation, integration of signals is also applicable to only continuous time signals. The limits of integration will be from − ∞ to present instance of time t. It is mathematically expressed as,

$$Y(t) = \int_{-\infty}^{t} X(t)dt$$

Integration of some continuous time signals is shown in the diagram below.



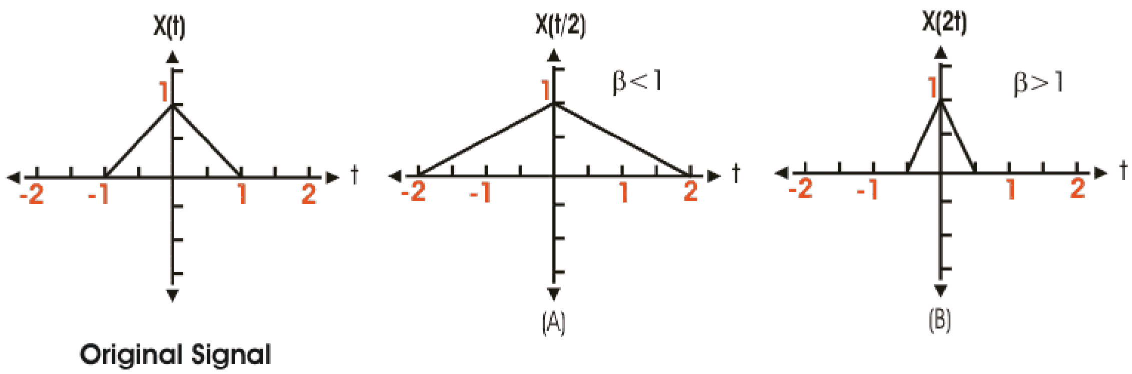
Basic Signal Operations Performed on Dependent Variables

This is exactly the opposite of the above mentioned case, here the periodicity of the signal is varied by modifying the horizontal axis values, while the amplitude or the strength remains constant. These are:-

1. Time scaling of signals
2. Reflection of signals
3. Time-shifting of signals.

- **Time Scaling of Signals**

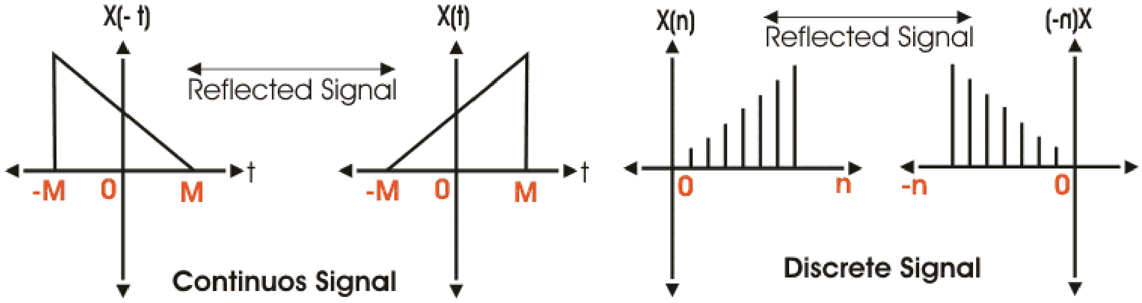
Time scaling of signals of signals involves the modification of a periodicity of the signal, keeping its amplitude constant. Its mathematically expressed as, Y(t) = βX(t) Where, X(t) is the original signal, and β is the scaling factor. If β > 1 implies, the signal is compressed. And β < 1 implies, the signal is expanded. This is illustrated diagrammatically for better understanding.



- **Reflection of Signals**

Reflection of signal is a very interesting operation applicable on both continuous and discrete signals. Here in this case the vertical axis acts as the mirror, and the transformed image obtained is exactly the mirror image of the parent signal. It can be defined as Y(t) = X(- t) Where, X(t) is the original signal. But if the reflected signal X(- t) = X(t); then its called an even signal. Where as when X(- t) = – X(t); then its known as an odd signal.

Its explained diagrammatically as,



- **Time Shifting of Signals**

Time shifting of signals is probably the most important one, and most widely used amongst all basic signal operations. Its generally used to fast-forward or delay a signal, as is necessary in most practical circumstances. Time shifting is mathematically expressed as, Y(t) = X(t - t_0). Where, X(t) is the original signal, and t_0 represents the shift in time. For a signal X(t) if the position shift $t_0 > 0$. Then the signal is said to be right shifted or delayed. In the same manner, if $t_0 < 0$, implies the signal is left shifted or delayed. This has been explained diagrammatically in the figure below. Where the original signal fig(a) is right shifted and also left shifted in figure (b) and (c) respectively.