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Electrostatic Potential & Capacitance

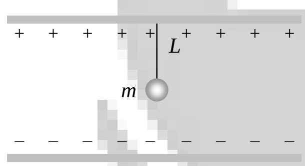
DPP-08

1. An infinite number of electric charges each equal to 5 *nano-coulomb* (magnitude) are placed along *X*-axis at $x = 1\,cm$, $x = 2\,cm$, $x = 4\,cm$ $x = 8\,cm$ ………. and so on. In the setup if the consecutive charges have opposite sign, then the electric field in *Newton/Coulomb* at $x = 0$ is $\left(\frac{1}{4\pi\varepsilon_0} = 9\times10^9\,N - m^2/c^2\right)$

(a) 12×10^4 (b) 24×10^4

(c) 36×10^4 (d) 48×10^4

2. A small sphere carrying a charge '*q*' is hanging in between two parallel plates by a string of length *L*. Time period of pendulum is T_0. When parallel plates are charged, the time period changes to T. The ratio T/T_0 is equal to



(a) $\left(\frac{g+\frac{qE}{m}}{g}\right)^{1/2}$ (b) $\left(\frac{g}{g+\frac{qE}{m}}\right)^{3/2}$

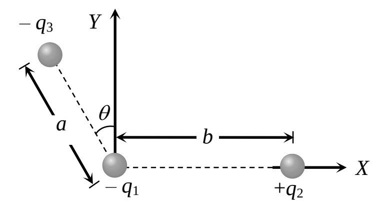
(c) $\left(\frac{g}{g+\frac{qE}{m}}\right)^{1/2}$ (d) None of these

3. A point charge of 40 stat coulomb is placed 2 *cm* in front of an earthed metallic plane plate of large size. Then the force of attraction on the point charge is

(a) 100 *dynes* (b) 160 *dynes*

(c) 1600 *dynes* (d) 400 *dynes*

4. Three charges $-q_1$, $+q_2$ and $-q_3$ are placed as shown in the figure. The *x*-component of the force on $-q_1$ is proportional to



(a) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\sin\theta$

(b) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\cos\theta$

(c) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta$

(d) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\cos\theta$

5. A solid conducting sphere having a charge *Q* is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be *V*. If the shell is now given a charge of –3*Q*, the new potential difference between the same two surfaces is

(a) *V* (b) 2*V*

(c) 4*V* (d) – 2*V*

6. Two point charges $+q$ and $-q$ are held fixed at $(-d, 0)$ and $(d, 0)$ respectively of a (X, Y) coordinate system. Then

(a) E at all points on the $Y-$axis is along $\hat{i}$

(b) The electric field $\vec{E}$ at all points on the $X-$axis has the same direction

(c) Dipole moment is $2qd$ directed along $\hat{i}$

(d) Work has to be done in bringing a test charge from infinity to the origin

7. A piece of cloud having area $25\times10^6\,m^2$ and electric potential of 10^5 volts. If the height of cloud is $0.75\,km$, then energy of electric field between earth and cloud will be

(a) $250\,J$ (b) $750\,J$

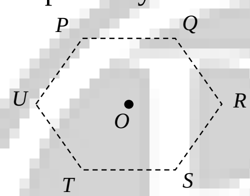
(c) $1225J$ (d) $1475J$

8. Two point charges $(+Q)$ and $(-2Q)$ are fixed on the X-axis at positions a and $2a$ from origin respectively. At what positions on the axis, the resultant electric field is zero

(a) Only $x=\sqrt{2}a$ (b) Only $x=-\sqrt{2}a$

(c) Both $x=\pm\sqrt{2}a$ (d) $x=\frac{3a}{2}$ only

9. Six charges, three positive and three negative of equal magnitude are to be placed at the vertices of a regular hexagon such that the electric field at O is double the electric field when only one positive charge of same magnitude is placed at R. Which of the following arrangements of charges is possible for P, Q, R, S, T and U respectively



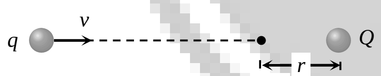
(a) $+,-,+,-,-,+$

(b) $+,-,+,-,+,-$

(c) $+,+,-,+,-,-$

(d) $-,+,+,-,+,-$

10. A charged particle q is shot towards another charged particle Q which is fixed, with a speed v. It approaches Q upto a closest distance r and then returns. If q were given a speed $2v$, the closest distances of approach would be



(a) r (b) $2r$

(c) $r/2$ (d) $r/4$

11. Four charges equal to $-Q$ are placed at the four corners of a square and a charge q is at its centre. If the system is in equilibrium the value of q is

(a) $-\frac{Q}{4}(1+2\sqrt{2})$ (b) $\frac{Q}{4}(1+2\sqrt{2})$

(c) $-\frac{Q}{2}(1+2\sqrt{2})$ (d) $\frac{Q}{2}(1+2\sqrt{2})$

12. A parallel plate air capacitor has a capacitance of $100\,\mu\mu F$. The plates are at a distance d apart. If a slab of thickness $t(t\le d)$ and dielectric constant 5 is introduced between the parallel plates, then the capacitance will be

(a) $50\mu\mu F$ (b) $100\mu\mu F$

(c) $200\mu\mu F$ (d) $500\mu\mu F$

13. A dielectric slab of thickness d is inserted in a parallel plate capacitor whose negative plate is at $x=0$ and positive plate is at $x=3d$. The slab is equidistant from the plates. The capacitor is given some charge. As one goes from 0 to $3d$

(a) The magnitude of the electric field remains the same

(b) The direction of the electric field remains the same

(c) The electric potential increases continuously

(d) The electric potential increases at first, then decreases and again increases

ANSWER KEY

1. (c)
2. (c)
3. (a)
4. (c)
5. (a)
6. (a)
7. (d)
8. (b)
9. (d)
10. (d)
11. (b)
12. (c)
13. (b, c)

HINTS AND SOLUTIONS

1. **(c)**

$$E=\frac{1}{4\pi\varepsilon_0}.\left[\frac{5\times10^{-9}}{(1\times10^{-2})^2}-\frac{5\times10^{-9}}{(2\times10^{-2})^2}+\frac{5\times10^{-9}}{(4\times10^{-2})^2}-\frac{(5\times10^{-9})}{(8\times10^{-2})^2}+.....\right]$$

$$\Rightarrow E=\frac{9\times10^{9}\times5\times10^{-9}}{10^{-4}}\left[1-\frac{1}{(2)^2}+\frac{1}{(4)^2}-\frac{1}{(8)^2}+...\right]$$

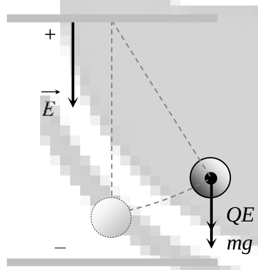
$$\Rightarrow E=45\times10^{4}\left[1+\frac{1}{(4)^2}+\frac{1}{(16)^2}+...\right]$$

$$-45\times10^{4}\left[\frac{1}{(2)^2}+\frac{1}{(8)^2}+\frac{1}{(32)^2}+...\right]$$

$$\Rightarrow E=45\times10^{4}\left[\frac{1}{1-\frac{1}{16}}\right]-\frac{45\times10^{4}}{(2)^{2}}\left[1+\frac{1}{4^2}+\frac{1}{(16)^2}+..\right]$$

E = 48 × 10^4 – 12 × 10^4 = 36 × 10^4 N/C

2. **(c)**



Net downward force $mg' = mg + QE$

$\Rightarrow$ Effect acceleration $g' = \left(g+\frac{QE}{m}\right)$

Hence time period

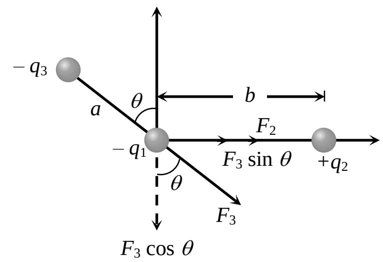
$$T=2\pi\sqrt{\frac{l}{g'}}=2\pi\sqrt{\frac{l}{\left(g+\frac{QE}{m}\right)}}$$

3. **(a)**

By the concept of electrical image, it is considered that an equal but opposite charge present on the other side of the plate at equal distance. Hence force

$$F=\frac{40\times40}{4^2}=100\,dynes$$

4. **(c)**



F_2 = Force applied by q_2 on $-q_1$

F_3 = Force applied by $(-q_3)$ on $-q_1$

x-component of Net force on $-q_1$ is

$$F_x = F_2 + F_3\sin\theta = k\frac{q_1q_2}{b^2}+k.\frac{q_1q_3}{a^2}\sin\theta$$

$$\Rightarrow F_x = k\left[\frac{q_1q_2}{b^2}+\frac{q_1q_3}{a^2}\sin\theta\right]$$

$$\Rightarrow F_x = k\cdot q_1\left[\frac{q_2}{b^2}+\frac{q_3}{a^2}\sin\theta\right]\Rightarrow$$

$$F_x \propto \left(\frac{q_2}{b^2}+\frac{q_3}{a^2}\sin\theta\right)$$

5. **(a)**

In case of a charged conducting sphere

$$V_{\text{inside}} = V_{\text{centre}} = V_{\text{surface}} = \frac{1}{4\pi\varepsilon_o}.\frac{q}{R},$$

$$V_{\text{outside}} = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$$

If a and b are the radii of sphere and spherical shell respectively, then potential at their surface will be

$$V_{\text{sphere}} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{a} \text{ and } V_{\text{shell}} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{b}$$

$$\therefore V = V_{\text{sphere}} - V_{\text{shell}} = \frac{1}{4\pi\varepsilon_0}.\left[\frac{Q}{a}-\frac{Q}{b}\right]$$

Now when the shell is given charge (–3Q), then the potential will be

$$V'_{sphere} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{a} + \frac{(-3Q)}{b}\right],$$

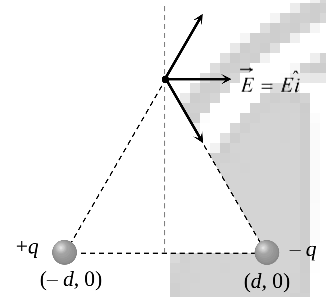
$$V'_{shell} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{b} + \frac{(-3Q)}{b}\right]$$

$$\therefore V'_{sphere} - V'_{shell} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{a} - \frac{Q}{b}\right] = V$$

6. **(a)**

From figure, it is clear that $\vec{E}$ at all points on the y-axis is along $\hat{i}$. Here $\vec{E}$ of all points on x-axis cannot have the same direction.

Here electric potential at origin is zero so no work is done in bringing a test charge from infinity to origin.



Here dipole moment is in – x direction (– q to +q).

Hence only option (a) is correct.

7. **(d)**

$$\text{Energy} = \frac{1}{2}\varepsilon_0 E^2 \times (A \times d) = \frac{1}{2}\varepsilon_0 \left(\frac{V^2}{d^2}\right) Ad$$

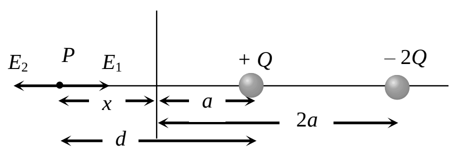
$$= \frac{1}{2} \times \frac{8.85 \times 10^{-12} \times (10^5)^2 \times 25 \times 10^6}{0.75 \times 10^3} = 1475 J$$

8. **(b)**

Suppose electric field is zero at a point P lies at a distance d from the charge + Q.

At P $\quad \frac{kQ}{d^2} = \frac{k(2Q)}{(a+d)^2}$

$$\Rightarrow \frac{1}{d^2} = \frac{2}{(a+d)^2} \Rightarrow d = \frac{a}{(\sqrt{2}-1)}$$



Since $d > a$ i.e. point P must lies on negative x-axis as shown at a distance x from origin

hence $\quad x = d - a \quad = \frac{a}{(\sqrt{2}-1)} - a = \sqrt{2}\,a.$

Actually P lies on negative x-axis so $x = -\sqrt{2}\,a$

9. **(d)**

If the charges are arranged according to the option (d), the electric fields due to P and S and due to Q and T add to zero, while due to U and R will be added up.

10. **(d)**

Charge q will momentarily come to rest at a distance r from charge Q when all it's kinetic energy converted to potential energy

i.e. $\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qQ}{r}$

Therefore the distance of closest approach is given by

$$r = \frac{qQ}{4\pi\varepsilon_0} \cdot \frac{2}{mv^2} \Rightarrow r \propto \frac{1}{v^2}$$

Hence if v is doubled, r becomes one fourth.

11. **(b)**

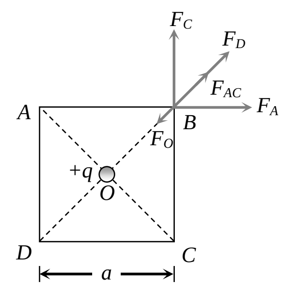
If all charges are in equilibrium, system is also in equilibrium.

Charge at centre : charge q is in equilibrium because no net force acting on it corner charge :

If we consider the charge at corner B. This charge will experience following forces

$$F_A = k\frac{Q^2}{a^2},\ F_C = \frac{kQ^2}{a^2},$$

$$F_D = \frac{kQ^2}{(a\sqrt{2})^2} \text{ and } F_O = \frac{KQq}{(a\sqrt{2})^2}$$



Force at B away from the centre = $F_{AC}+F_D$

$$=\sqrt{F_A^2+F_C^2}+F_D=\sqrt{2}\frac{kQ^2}{a^2}+\frac{kQ^2}{2a^2}$$

$$=\frac{kQ^2}{a^2}\left(\sqrt{2}+\frac{1}{2}\right)$$

Force at B towards the centre $=F_O=\frac{2kQq}{a^2}$

For equilibrium of charge at B, $F_{AC}+F_D=F_O$

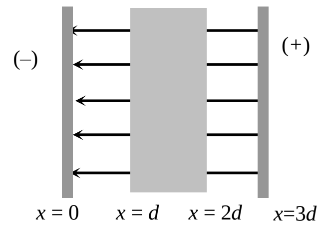
$$\Rightarrow \frac{KQ^2}{a^2}\left(\sqrt{2}+\frac{1}{2}\right)=\frac{2KQq}{a^2} \Rightarrow$$

$$q=\frac{Q}{4}\left(1+2\sqrt{2}\right)$$

12. **(c)**

Capacitance will increase but not 5 times (because dielectric is not filled completely). Hence new capacitance may be 200 $\mu\mu F$.

13. **(b, c)**



Even after introduction of dielectric slab, direction of electric field will be perpendicular to the plates and directed from positive plate to negative plate.

Further, magnitude of electric field in air $=\frac{\sigma}{\varepsilon_0}$

Magnitude of electric field in dielectric $=\frac{\sigma}{K\varepsilon_0}$

Similarly electric lines always flows from higher to lower potential, therefore, electric potential increases continuously as we move from x = 0 to x = 3d.