Choose and answer two questions from each of the sections and completely solve the two extra problems to increase your grade on MT1 to a 2.7. So you should turn in **eight** total answers.

Section 1.5 24, 30, 36, 51

Section 2.5 9, 12, 34, 45

Section 3.5 8, 12, 32

1. A bacteria colony is multiplying. It starts with a single bacterium. Each bacterium takes Ber(p) minutes with $p = 1/30^2$ to split into three (yes, usually it takes 0 minutes). The children then take an independent Ber(p) minutes to split in three and so on.

   A biologist is interested in studying all of the bacteria in G_n, the nth generation of this colony (i.e. the bacteria formed after n splits). So there are 3^n of them, each with an ancestry of n splits back to the original.

   (a) Let $T_1, \ldots, T_{3^n}$ be the time it took for each bacterium in G_n to be born starting from the first bacterium. Explain why each T_i is a $\text{Bin}(n, p)$ random variable.

   (b) Are the T_i independent? Briefly explain.

   (c) Let $E_{i,n,k}$ for $i = 1, 2, \ldots, 3^n$ be the event that the ith bacteria from G_n took k minutes to be born. What is $P(E_{i,n,k})$?

   (d) Let $E_{n,k}$ be the event that there exists a bacterium from G_n that is k minutes old. Use a union bound to upper bound $P(E_{n,k})$.

   (e) Use the bound $C_{n,k} \leq 2^n$ and $1 - p \leq 1$ to give an upper bound on $P(E_{n,k})$ that only involves p^k.

   (f) Let E_n be the event that there exists a bacterium from G_n that is at least $n/2$ minutes old. Use the previous part and the bound $k \geq n/2$ and $n/2 \leq n$ to show that $P(E_n) \leq n(6\sqrt{p})^n$.

   (g) Let E be the event that there exists an n such that a bacterium from G_n is at least $n/2$ minutes old. Write E in terms of the E_n and apply a union bound to upper bound $P(E)$. Simplify by using the formula $\sum_{n=1}^{\infty} na^n = \frac{a}{(1-a)^2}$.

2. There are one hundred 20, 30, and 40 year olds in a study on car accidents. So, three hundred people in total. The probability, p_x, that an x-year old gets in an accident the next year are: $p_{20} = 5/1000$, $p_{30} = 3/1000$ and $p_{40} = 2/1000$.

   (a) Write the mean of a Poisson random variable that predicts the total number of car accidents for these 300 people over the next year. Simplify as much as you can.

   (b) Use Poisson approximation to estimate the probability there is at least one accident in the next year.

   (c) A car accident is fatal with probability 10^{-2}. Use Poisson thinning and Poisson approximation to estimate the probability that no one in the study is in a fatal car accident.