GATE CS Topic wise Questions

Theory of Computation

YEAR 2001

Question. 1

Consider the following two statements :

$S1 : \{0^{2n} | n \geq 1\}$ is a regular language

$S2 : \{0^m 1^n 0^{m+n} | m \geq 1 \text{ and } n \geq 1\}$ is a regular language

Which of the following statements is incorrect ?

(A) Only $S1$ is correct

(B) Only $S2$ is correct

(C) Both $S1$ and $S2$ are correct

(D) None of $S1$ and $S2$ is correct.

SOLUTION

S_1 can be represented using a DFA so it is regular S_1 is correct.
S_2 can't be represented by DFA but it requires PDA to accept. So is S_2 is CFG not regular. S_2 is false.
Hence (A) is correct option.

Question. 2

Which of the following statements true ?

(A) If a language is context free it can be always be accepted by a deterministic push-down automaton.

(B) The union of two context free language is context free.

(C) The intersection of two context free language is context free

(D) The complement of a context free language is context free

SOLUTION

(A) It is not necessary at all.

(B) $\{a^n b^n\} \cup \{a^n b^n c^n\} = \{a^n b^n c^n\}$ always true so correct.

(C) $\{a^n b^n\} \cap \{a^n\} = \{a^n\}$ not CFG so false.

(D) Not necessary.

Hence (B) is correct option

Question. 3

Given an arbitary non-deterministic finite automaton (NFA) with N states, the maximum number of states in an equivalent minimized DFA is at least.

(A) N^2

(B) 2^N

(C) $2N$

(D) $N!$

SOLUTION

In DFA the no. of states are always more than NFA, so if NFA has N states DFA will have $2N$ states.
Hence (C) is correct option.

Question. 4

Consider a DFA over $\Sigma = \{a, b\}$ accepting all strings which have number of $a's$ divisible by 6 and number of $b's$ divisible by 8. What is the minimum number of states that the DFA will have ?

(A) 8

(B) 14

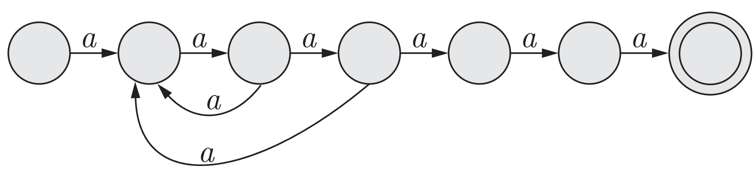
(C) 15

(D) 48

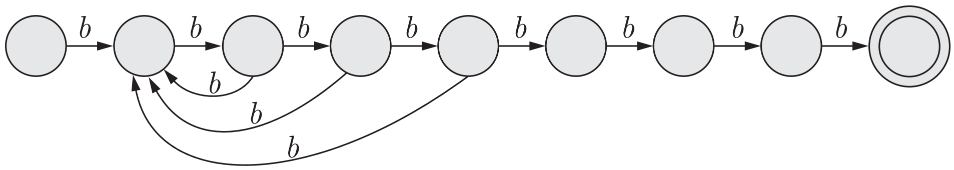
SOLUTION

The valid strings will be where no. of $a's$ 6, 12, 18, 24

No. of $b's = 8, 16, 24$



No. of states = 7 for a



No. of states = 9

Total $\quad 9+7-1=15$

1 subtracted due to 2 final states.
Hence (C) is correct option.

Question. 5

Consider the following languages :

$L1 \ = \{ww \mid w \in \{a,b\}^*\}$

$L2 \ = \{ww^R \mid w \in \{a,b\}^* \ w^R \text{ is the reverse of w}\}$

$L3 \ = \{0^{2i} \mid i \text{ is an integer}\}$

$L4 \ = \{0^{i^2} \mid i \text{ is an integer}\}$

Which of the languages are regular ?

(A) Only $L1$ and $L2$ (B) Only $L2$, $L3$ and $L4$

(C) Only $L3$ and $L4$ (D) Only $L3$

SOLUTION

L_1 would be accepted by PDA so can't be regular.
L_2 similarly can't be accepted by DFA so not regular.
L_3 & L_4 both require only finite no of zeros.
So both regular.
Hence (C) is correct option.

Question. 6

Consider the following problem x.

Given a Turing machine M over the input alphabet Σ, any state q of M.

And a word $w \in \Sigma^*$ does the computation of M on w visit the state

q ?

Which of the following statements about x is correct ?

(A) x is decidable

(B) x is undecidable but partially decidable

(C) x is undecidable and not even partially decidable

(D) x is not a decision problem

SOLUTION

Since it is possible to create a turing machine for the problem, 20 this problem is decidable.
Hence (A) is correct option.

YEAR 2002

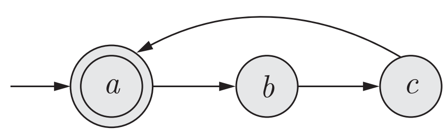
Question. 7

The smallest finite automaton which accepts the language $\{x |$ length of x is divisible by $3\}$ has

(A) 2 states (B) 3 states

(C) 4 states (D) 5 states

SOLUTION



Start & end are same (A) so the minimum no. of states required are 3.
Option (B) is correct
If string traversal doesn't stop at (A) then string length is not divisible by 3.

Question. 8

Which of the following is true ?

(A) The complement of a recursive language is recursive.

(B) The complement of a recursively enumerable language is recursively enumerable.

(C) The complement of a recursive language is either recursive or recursively enumerable.

(D) The complement of a context-free language is context-free.

SOLUTION

A recursive language has complement & its complement is also recursive.

Whereas complement of others is not recursive.

Hence (A) is correct option.

Question. 9

The C language is :

(A) A context free language

(B) A context sensitive language

(C) A regular language

(D) Parsable fully only by a Turing machine

SOLUTION

C language is context free language entirely based upon the productions.

Hence (A) is correct option.

Question. 10

The language accepted by a Pushdown Automaton in which the stack is limited to 10 items is best described as

(A) Context free (B) Regular

(C) Deterministic Context free (D) Recursive

SOLUTION

Pushdown Automaton uses stock as data structure & languages accepted by PDA is regular.

Hence (B) is correct option.

YEAR 2003

Question. 11

Ram and Shyam have been asked to show that a certain problem Π is NP-complete. Ram shows a polynomial time reduction from the 3-SAT problem to Π , and Shyam shows a polynomial time reduction from Π to 3-SAT. Which of the following can be inferred from these reduction?

(A) Π is NP-hard but not NP-complete

(b) Π is in NP, but is not NP-complete

(C) Π is NP-complete

(D) Π is neither Np-hard, nor in NP

SOLUTION

A problem is said to be NP- complete, if it is both NP & NP hard.
3-SAT problem is NP complete so a reduction of 3-SAT problem to Π & Π to 30-SAT.
So this infers that Π is NP complete, since it is reducible to a NP complete problem.
Hence (C) is correct option

Question. 12

Nobody knows yet if $P = NP$. Consider the language L defined as follows

$$L = \begin{cases} (0+1)^* & \text{if P = NP} \\ \phi & \text{othervise} \end{cases}$$

Which of the following statements is true?

(A) L is recursive

(B) L is recursively enumerable but not recu

(C) L is not recursively enumerable

(D) Whether L is recursive or not will be known after we find out if $P = NP$

SOLUTION

A language L is said to be recursive if there exists any rule to determine whether an element belong to language or not, if language

can be accepted by turning machine.
So there exist the rules so L is recursive.
Hence (A) is correct option.

Question. 13

The regular expression $0*(10)*$ denotes the same set as

(A) $(1*0)*1*$

(B) $0+(0+10)*$

(C) $(0+1)*10(0+1)*$

(D) None of the above

SOLUTION

Question. 14

If the strings of a language L can be effectively enumerated in lexicographic (i.e. alphabetic) order, which of the following statements is true?

(A) L is necessarily finite

(B) L is regular but not necessarily finite

(C) L is context free but not necessarily regular

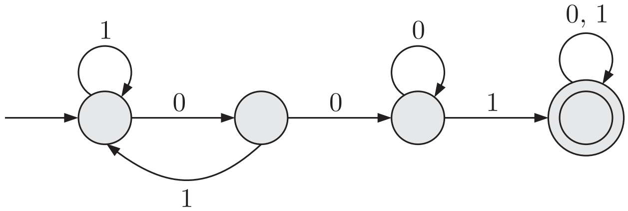
(D) L is recursive but not necessarily context free

SOLUTION

Since L can be effectively enumerated so L has to be regular, but is doesn't mean that the decisions are finite.
Hence (B) is correct option.

Question. 15

Consider the following deterministic finite state automaton M.



Let S denote the set of seven bit binary strings in which the first, the fourth, and the last bits are 1. The number of strings in S that are accepted by M is

(A) 1 (B) 5

(C) 7 (D) 8

SOLUTION

The strings accepted by the given automata are of type.

Option 1 2 3 4 5 6 7

1 — — 1 — — 1

These four blank spaces can have 0 or 1, so total $2^4 = 6$ strings are possible, but the given automata does not accept all of those.

1. 1 1 1 1 0 0 1
2. 1 1 0 1 0 0 1
3. 1 0 1 1 0 0 1
4. 1 0 0 1 0 0 1
5. 1 0 0 1 0 0 1
6. 1 0 0 1 1 0 1
7. 1 0 0 1 1 1 1

Hence (C) is correct option.

Question. 8

Let $G = (\{S\}, \{a, b\} R, S$ be a context free grammar where the rule set R is

$$S \rightarrow a\ S\ b \mid S\ S \mid \varepsilon$$

Which of the following statements is true?

(A) G is not ambiguous

(B) There exist $x, y, \in L(G)$ such that $xy \notin L(G)$

(C) There is a deterministic pushdown automaton that accepts $L(G)$

(D) We can find a deterministic finite state automaton that accepts $L(G)$

SOLUTION

(A) Incorrect since the production has same non terminal in both sides, so definitely ambiguous.

(B) Since $S \rightarrow SS$ this leads to conjunction of every possible string to make a valid string in $L(G)$.

(C) Context free languages are accepted by push down automata so true.

(D) The language is not regular so DFA is not possible.

Hence (C) is correct option.

Question. 9

Consider two languages L_1 and L_2 each on the alphabet $\sum$. Let $f: \sum \to \sum$ be a polynomial time computable bijection such that $(\forall x[x \in L_1 \;\; iff \;\; f(x) \in L_2]$. Further, let f^{-1} be also polynomial time commutable.

Which of the following CANNOT be true?

(A) $L_1 \in P$ and L_2 finite

(B) $L_1 \in NP$ and $L_2 \in P$

(C) L_1 is undecidable and L_2 is decidable

(D) L_1 is recursively enumerable and L_2 is recursive

SOLUTION

$$f : \Sigma \to \Sigma$$

So $\qquad f^{-1} : \Sigma \to \Sigma$

Bijection $\qquad (\forall \cdot X) : X \in L_1 \; iff \; f(x) \in L_2$

So L_1 is undecidable & L_2 is decidable L_1 depends on L_2 & L_2 dependent upon f^{-1}.

Hence (C) is correct option.

Question. 10

A single tape Turing Machine M has two states q^0 and q^1, of which q^0 is the starting state. The tape alphabet of M is {0,1,B} and its input alphabet is {0,1}. The symbol B is the blank symbol used to indicate end of an input string. The transition function of M is described in the following table

	0	1	B
q^0	$q^{1,1,R}$	$Q^{1,1,R}$	Halt
q^1	$q^{1,1,R}$	$q^{0,1,L}$	$qH0, B, L$

The table is interpreted as illustrated below.

The entry $(q^{1,1,R})$ in row q^0 and column 1 signifies that if M is in state q^0 and reads 1 on the current tape square, then it writes 1 on the same tape square, moves its tape head one position to the right and transitions to state q^1.

Which of the following statements is true about M?

(A) M does not halt on any string in $(0+1)^+$

(B) M dies not halt on any string in $(00+1)^*$

(C) M halts on all string ending in a 0

(D) M halts on all string ending in a 1

SOLUTION

This turning machine starts at 90 if it doesn't get any input symbol but B then it halts.
So if $(00+1)^*$ is chosen then the M/C can halt. Option (B) is wrong.
Option (C) & (D) are possible but not necessary.
Option (A) $(0+1)^*$, 1 or more occurrence of 0 or 1.
So 0, 1, 00, 01, 10, 11........are valid strings & the machine doesn't halt for them.
Hence (A) is correct option.

Question. 11

Define languages L_0 and L_1 as follows

$L_0 = \{< M, w, 0 >| \, M \text{ halts on } w\}$

$L_0 = \{< M, w, 1 >| \, M \text{ does not halts on } w\}$

Here$< M, w, i >$ is a triplet, whose first component. M is an encoding of a Turing Machine, second component,w, is a string, and third component, t, is a bit.

Let $L = L_0 \cup L_1$. Which of the following is true?

(A) L is recursively enumerable, but $\overline{L}$ is not

(B) $\overline{L}$ is recursively enumerable, but L is not

(C) Both L and $\overline{L}$ are recursive

(D) Neither L nor $\overline{L}$ is recursively enumerable

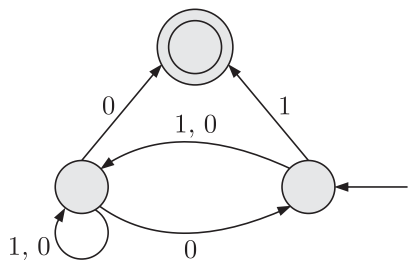
SOLUTION

$$L = L_0 VL, =< M, w, \{0,1\}$$
$$\overline{L} = (L_0 \cup L_1)^c$$
$= \overline{L_0} \cap \overline{L_1} = \phi$ so a regular language, so it is recursively enumerable.

So $\because \quad L = L_0 \cup L$, it is not. RE
Hence (B) is correct option.

Question. 12

Consider the NFAM shown below.

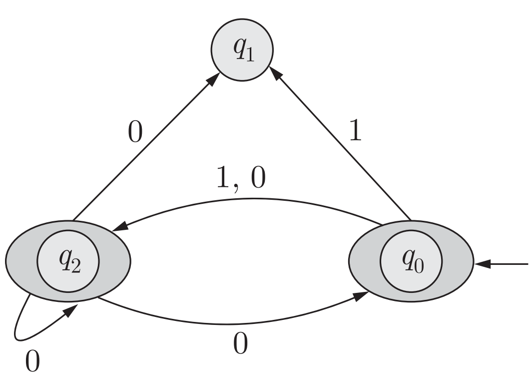


Let the language accepted by M be L. Let L_1 be the language accepted by the $NFAM_1$, obtained by changing the accepting state of M to a non-accepting state and by changing the non-accepting state of M to accepting states. Which of the following statements is true?

(A) $L_1 = \{0,1\}^* - L$
(B) $L_1 = \{0,1\}^*$
(C) $L_1 \subseteq L$
(D) $L_1 = L$

SOLUTION

L is accepted by M(NFA) but NFA M_1 has



So this accept L_1.
L_1 will accept not only L but also substrings of L.
So $\quad L_1 \subseteq L$
Hence (A) is correct option.

YEAR 2004

Question. 13

The problems 3-SAT and 2-SAT are

(A) both in P

(B) both NP-complete

(C) NP-complete and in P respectively

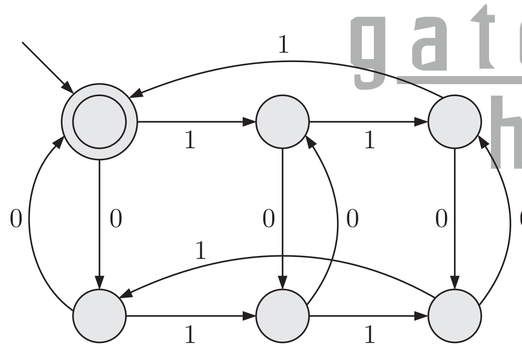
(D) undecidable and NP-complete respectively

SOLUTION

3 SAT problem is both NP & NP hard so it is NP complete, but 2 SAT problem is solvable in Polynomial time so it is in class P.
Hence (C) is correct option.

Question. 14

The following finite state machine accepts all those binary strings in which the number of 1's and 0's are respectively



(A) divisible by 3 and 2

(B) odd and even

(C) even and odd

(D) divisible by 2 and 3

SOLUTION

Due to the 3 one's in the upper edges & 3 one's in lower edges to reach to final state the no of 1's is always divisible by 3 & 0's are always in pair in forward & back edge so, no of zero's is divisible by 2.
Hence (A) is correct option.

Question. 15

The language $\{a^m b^{m+n} \mid m, n \leq 1\}$ is

(A) regular

(B) context-free but not regular

(C) context sensitive but not context free

(D) type-0 but not context sensitive

SOLUTION

Language $\{a^n b^n c^{m+n} / m, n \geq 1\}$ is a context free language since it can be represented by pushdown automata, but it is not regular since ΔFA can't count the no. of a's & b's and then check the sum for occurrence of c.
Hence (B) is correct option.

Question. 16

Consider the flowing grammar C

$$S \to bS \mid aA \mid b$$
$$A \to bA \mid aB$$
$$B \to bB \mid aS \mid a$$

Let $N_a(W)$ and $N_b(W)$ denote the number of a's and b's in a string W respectively. The language $L(G) \subseteq \{a, b\}^+$ generated by G is

(A) $\{W \mid N_a(W) > 3N_b(W)\}$

(B) $\{W \mid N_b(W) > 3N_a(W)\}$

(C) $\{W \mid N_a(W) = 3k, k \in \{0,1,2,...\}\}$

(D) $\{W \mid N_b(W) = 3k, k \in \{0,1,2,...\}\}$

SOLUTION

$$S \to bS \mid aA \mid b$$
$$A \to bA \mid aB$$
$$B \to bB \mid aS \mid a$$

Let $N_a(w)$ & $N_b(w)$ denote of a's & b's in strings.
Some valid strings are

1. $S \to bS \to bbS \to bbb$ (any no. of b)
2. $S \to bA \to abA \to abbA \to abbaB \to abbaa$
3. $abbaB \to abbaaS \to abbaab$

From (2) option (D) is false also from (1), (2) & (3) (a), (b) & (d) are false.
So only (C) satisfy.

Hence (C) is correct option.

Question. 17

L_1 is a recursively enumerable language over Σ. An algorithm A effectively enumerates its words as $w_1, w_2, w_3, \ldots$. Define another language L_2 over $\Sigma \cup \{\#\}$ as $\{w_i \# w_j : w_i, w_j \in L_1, i < j\}$. Here # is a new symbol. Consider the following assertion.

S_1: L_1 is recursive implies L_2 is recursive

S_2: L_2 is recursive implies L_1 is recursive

Which of the following statements is true?

(A) Both S_1 and S_2 are true

(B) S_1 is true but S_2 is not necessarily true

(C) S_2 is true but S_1 ins necessarily true

(D) Neither is necessarily true

SOLUTION

Problem can be solved using membership algorithm. If L_1 is recursive & $w_i \in L_1$ & $w_j \in L_1$ then we can check. Whether $i < j$, so here $w_i \# w_j \in L_2$ but if $w_i \notin L_1$ also $w_j \notin L_2$ or $i \geq j$ then $w_i \# w_j \notin L_2$
So L_2 is also recursive & S_1 is true.
L_2 is recursive said by S_2 but membership algorithm can't be applied here since L_1 doesn't has '#' symbol.
So S_2 is not necessarily true.
Hence (B) is correct option.

YEAR 2005

Question. 18

Consider three decision problem P_1, P_2 and P_3. It is known that P_1 is decidable and P_2 is undecidable. Which one of the following is TRUE?

(A) P_3 is decidable if P_1 is reducible to P_3

(B) P_3 is undecidable if P_3 is reducible to P_2

(C) $PL3$ is undecidable if P_2 is reducible to P_3

(D) P_3 is decidable if P_3 is reducible to P_2's complement

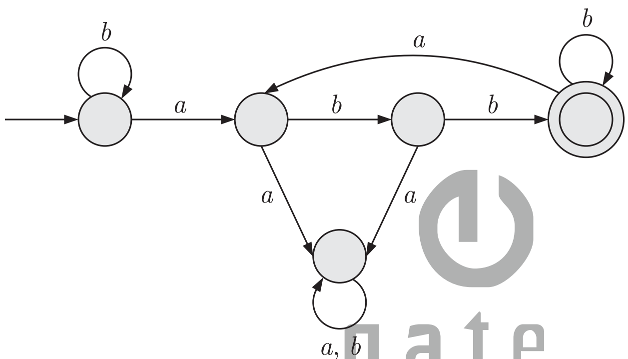
SOLUTION

$P_1 \rightarrow$ decidable
$P_2 \rightarrow$ undecidable

If P_1 or P_2 is reducible to P_3 then P_3 also has same properties as P_1 & P_2.
So if P_2 is reducible to P_3 then P_3 is also undecidable.
Hence (C) is correct option.

Question. 19

Consider the machine M



The language recognized by M is

(A) $\{W \in \{a, b\}^* /$ every a in w is followed by exactly two $b's\}$

(B) $\{W \in \{a, b\}^* /$ every a in w is followed by at least two $b's\}$

(C) $\{W \in \{a, b\}^* /$ w contains the substring 'abb'

(D) $\{W \in \{a, b\}^* /$ w does not contain 'aa' as a substring$\}$

SOLUTION

From the given FSM, it is clear that a not necessity followed by only $2b$ due to self loop at final state. But at least $2b's$ are there.
abb substring not always, Similarly aa not always.
Hence (B) is correct option.

Question. 20

Let N_f and N_p denote the classes of languages accepted by non-deterministic finite automata and non-deterministic push-down automata, respectively. let D_f and D_P denote the classes of languages accepted by deterministic finite automata and deterministic pushdown automata, respectively. Which one of the following is TRUE?

(A) $D_f \subset N_f$ and $D_P \subset N_P$ (B) $D_f \subset N_f$ and $D_P = N_P$

(C) $D_f = N_f$ and $D_P = N_P$ (D) $D_f = N_f$ and $D_P \subset N_P$

SOLUTION

$N_f \rightarrow$ languages accepted by $NDFA$
$N_p \rightarrow$ accepted by $NDPDA$
$D_f \rightarrow$ deterministic FA
$D_p \rightarrow$ deterministic PDA

DFA & $NDFA$ both can accept all the regular languages, the difference is in no. of states.

So $D_f = N_f$

But $NPDA$ accepts only a subset of context free languages which are deterministic, whereas $NDPDA$ accept all the context free languages.

So $D_p \subset N_p$

Hence (D) is correct option.

Question. 21

Consider the languages

$L_1 + \{a^n b^n c^m \mid n, m > 0\}$ and $L_2 = \{a^n b^m c^m \mid n, m > 0\}$

(A) $L_1 \cap L_2$ is a context-free language

(B) $L_1 \cup L_2$ is a context-free language

(C) L_1 and L_2 are context-free language

(D) $L_1 \cap L_2$ is a context sensitive language

SOLUTION

$$L_1 = \{a^n b^n c^m \mid n, m > 0\}$$
$$L_2 = \{a^n b^m c^m \mid n, m > 0\}$$

Here $L_1 \cap L_2$ may not be CFL since CFL is not closed for intersection operation.

$L_1 \cap L_2$ would be context sensitive (A) is false.

Hence (A) is correct option.

Question. 22

Let L_1 be a recursive language, and let L_2 be a recursively enumerable but not a recursive language. Which one of the following is TRUE?

(A) $\overline{L_1}$ is recursive and $\overline{L_2}$ is recursively enumerable

(B) $\overline{L_1}$ is recursive and $\overline{L_2}$ is not recursively enumerable

(C) $\overline{L_1}$ and $\overline{L_2}$ are recursively enumerable

(D) $\overline{L_1}$ is recursively enumerable and $\overline{L_2}$ is recursive

SOLUTION

The rules here used will be.
All those languages which are recursive their complements are also recursive.
So option (A) & (B) can be correct.
Now languages which are recursively enumerable but not recursive, their complements can't be recursively enumerable.
So only option (B) is correct
Hence (B) is correct option

Question. 23

Consider the languages

$L_1 = \{WW^R \mid W \in \{0,1\}^*\}$

$L_2 = \{W\#W^R \mid W \in \{0,1\}^*\}$, where # is a special symbol

$L_3 = \{WW \mid W \in \{0,1\}^*\}$

Which one of the following is TRUE?

(A) L1 is a deterministic CFL

(B) L_2 is a deterministic CFL

(C) L_3 is a CFL, but not a deterministic CFL

(D) L_3 is a deterministic CFL

SOLUTION

In all the options there is linear relationship among strings so all $CFL's$, but L_1 & L_3 can be accepted by PDA, L_2 can be accepted by deterministic CFL due to presence of special symbol # which tells the middle of the string, so deterministic.
Hence (B) is correct option.

Question. 24

Consider the following two problems on undirected graphs

α: Given $G(V,E)$, does G have an independent set of size $|V|-4$?

β: Given $G(V,E)$, does G have an independent set of size 5?

Which one of the following is TRUE?

(A) α is in the P and β is NP-complete

(B) α is NP-complete and β is P

(C) Both α and β are NP-complete

(D) Both α and β are in P

SOLUTION

YEAR 2006

Question. 25

Let S be an NP-complete problem Q and R be two other problems not known to be in NP. Q is polynomial-time reducible to S and S is polynomial-time reducible to R. Which one of the following statements is true?

(A) R is NP-complete
(B) R is NP-hard
(C) Q is NP-complete
(D) Q is NP-hard

SOLUTION

S is NP complete and a NP complete problem is reducible to some unknown problem then that problem is also NP complete. So $S \to \Delta_p R$ the R is NP complete.
Hence (A) is correct option.

Question. 26

Let $L_1 = \{0^{n+m}1^n0^m \mid n, m \leq 0\}, L_2 = \{0^{n+m}1^{n+m}0^m \mid n, m \leq 0\}$, and $L_3 = \{0^{n+m}1^{n+m}0^{n+m} \mid n, m \leq 0\}$. Which of these languages are NOT context free?

(A) L_1 only
(B) L_3 only
(C) L_1 and L_2
(D) L_2 and L_3

SOLUTION

To accept CFL we require PDA & PDA accept CFG using stack
L_1 can be accepted using PDA firstly 0^{n+m} are pushed into stack then 1^n & 0^m times stack is poped. If stack empty then string accepted.
But for L_2 & L_3 0^m & 0^{n+m} are extra to accept by PDA.

Hence (D) is correct option.

Question. 27

If s is a string over (0+1)*, then let $n_0(s)$ denote the number of 0's in s and $n_1(s)$ the number of 1's in s. Which one of the following languages is not regular?

(A) $L = \{s \in (0+1)^* | \ n_0(s)$ is a 3-digit prime$\}$

(B) $L = \{s \in (0+1)^* |$ for every prefixes' of $s, | \ n_0(s') - n_1(s')| \leq 2\}$

(C) $L = \{s \in (0+1)^* || \ n_0(s) - n_1(s) \leq 4$

(D) $L = \{s \in (0+1)^* | \ n_0(s)$ mod $7 = n_1(s)$ mod$5 = 0\}$

SOLUTION

Option (A), (B) & (D) can be accepted by DFA, & there is no linear relationship between the no. of $0's$ & $1's$ in the string but in (C) $n_0(S) - n_1(S) \leq 4$ can't be accepted by DFA, we require a PDA.
So not regular.
Hence (C) is correct option.

Question. 28

For $s \in (0+1)^*$ let $d(s)$ denote the decimal value of $s(e.g. d(101) = 5)$

Let $L = \{s \in (0+1)^* | \ d(s)$ mod 5=2 and $d(s)$ mod $7 \neq 4\}$

Which one of the following statements is true?

(A) L is recursively enumerable, but not recursive

(B) L is recursive, but not context-free

(C) L is context_free, but not regular

(D) $L$is regular

SOLUTION

$S \ \in (0+1)^*$

$d(S)$ decimal value of S

$d(S)$ nod $5 = 2$

2, 12, 7, 17, 22, 27, 32

$d(S)$ mod $7 \neq 4$

$d(S) \neq 4, 11, 18, 25, 32$

So from the rules deterministic.

So it is regular can be accepted by DFA.

Hence (D) is correct option.

Question. 29

Let SHAM, be the problem of finding a Hamiltonian cycle in a graph $G + (V, E)$ with $[V]$ divisible by 3 and DHAM' be the problem of determining if a Hamltonian cycle exists in such graphs. Which one of the following is true?

(A) Both DHAM, and SHAM, are NP-hard

(B) SHAM, is NP-hard, but DHAM, is not

(C) DHAM, is NP-hard, but SHAM, is not

(D) Neither DHAM,nor SHAM, is NP-hard

SOLUTION

Question. 30

Consider the following statements about the context-free grammar,

$G = \{S \rightarrow SS, S \rightarrow ab, S \rightarrow ba, S \rightarrow \in\}$

1. G is ambiguous.
2. G produces all strings with equal number of a's and b's.
3. G can be accepted by a deterministic PDA.

Which combination below expresses all the true statements about G ?

(A) 1 only	(B) 1 and 3 only
(C) 2 and 3 only	(D) 1, 2 and 3

SOLUTION

Due to $S \rightarrow SS$ this Grammar is ambiguous right hand side has two Non terminals.

Also the strings like $aaabbb$ have equal no. of $a's$ & $b's$ but can't be produced by this grammar. So 2 is false.

Statement 3 is true since it is a CFG so accepted by PDA.

Hence (B) is correct option.

Question. 31

Let L_1 be regular language, L_2 be a deterministic context-free language and L_3 a recursively enumerable, but not recursive, language. Which one of the following statements is false?

(A) $L_1 \cap L_2$ is a deterministic CFL

(B) $L_3 \cap L_1$ is recursive

(C) $L_1 \cup L_2$ is context free

(D) $L_1 \cap L_2 \cap L_3$ is recursively enumerable

SOLUTION

L_1 is regular language
L_2 is CFL.
L_3 is recursively enumerable but not REC.

(A) $L_1 \cap L_2$ is CFL is true

(B) $L_3 \cap L_1$ is recursive, not necessary so false.

(C) & (D) are also true.

Hence (B) is correct option.

Question. 32

Consider the regular language $L = (111 + 111111)^*$. The minimum number of states in any DFA accepting this languages is

(A) 3 (B) 5

(C) 8 (D) 9

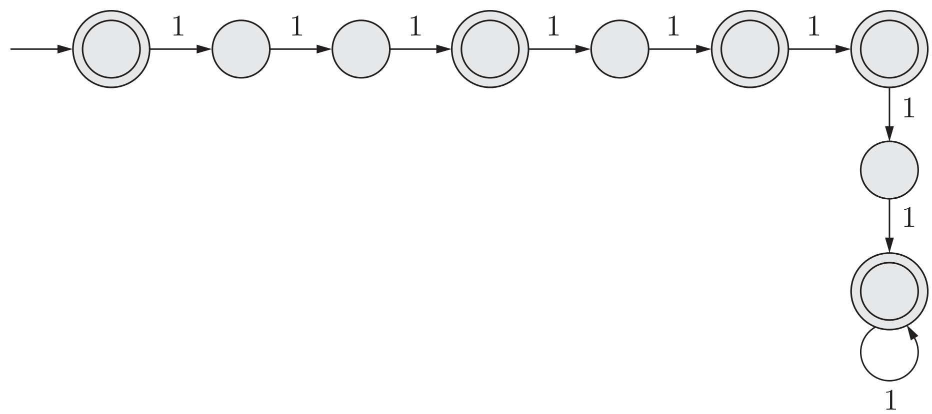
SOLUTION

The valid strings are

$\in$,111,11111,11111111,11111111111.........

No. of is can be 0, 3, 6, 9, 5, 8, 10, 15, 13, 11, 12

So the DFA accepts no. of 1's 8 to every digit i.e. 8, 9, 10, 11, 12.....



So no. of states required $= 9$

Hence (D) is correct option.

YEAR 2007

Question. 33

Which of the following problems is undecidable?

(A) Membership problem for $CFGs$

(B) Ambiguity problem for $CFGs$

(C) Finiteness problem for $FSAs$

(D) Equivalence problem for $FSAs$

SOLUTION

Finite state automata (FSA) has no undecidability CFL membership problem is also decidable.

So option (B) i.e Ambiguity of CFL cannot be decidable.

Hence (B) is correct option.

Question. 34

Which of the following is TRUE?

(A) Every subset of a regular set is regular

(B) Every finite subset of a non-regular set is regular

(C) The union of two non-regular sets is not regular

(D) Infinite union of finite sets is regular

SOLUTION

(A) Not necessary eg. $L_1 = \Sigma^*$ & $L_2 = \{a^n b^n, n \geq 0\}$ L_1 is regular but L_2 not so false.

(B) It is true all finite sets are regular.

(C) Can happen it $L_1 \cup L_1^c = \Sigma^*$

(D) also false.

Hence (B) is correct option.

Question. 35

A minimum state deterministic finite automation accepting the language $L = \{w \mid w \in (0,1)^*$, number of $0s$ &$1s$ in w are divisible by 3 and 5, respectively$\}$ has

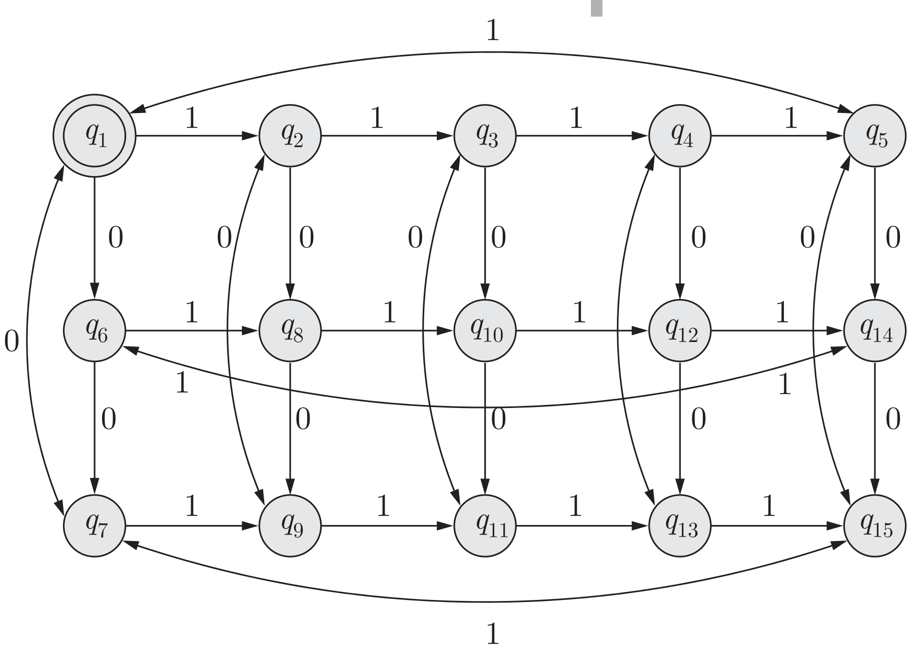
(A) 15 states (B) 11 states

(C) 10 states (D) 9 states

SOLUTION

To accept continuous string of 5 $1's$ & 3 $0's$ we require at least 7 states.

But $(0,1)^*$ is there so any combination of 0 & 1 can be there in the string. So a grid of states would be there.



Total states = 15

Hence (A) is correct option.

Question. 36

The language $L = \{0^T 21^i \mid i \leq 0\}$ over the alphabet $\{0,1,2)$ is

(A) not recursive

(B) is recursive and is a deterministic CFL

(C) us a regular language

(D) is not a deterministic CFI but a CFL

SOLUTION

$L = \{0^i 21^i | i \geq 0\}$, this language can't be accepted by DFA to regular, but it is recursive & can be accepted by PDA to CFL.

Hence (B) is correct option.

Question. 37

Which of the following languages is regular?

(A) $\{WW^R \mid W \in \{0,1\}^+\}$

(B) $\{WW^R X \mid X, W \in \{0,1\}^+\}$

(C) $\{WXW^R X \mid X, W \in \{0,1\}^+\}$

(D) $\{XWW^R X \mid X, W \in \{0,1\}^+\}$

SOLUTION

Option (C) is a regular language since it starts & ends with same symbol $w \times w^R$.

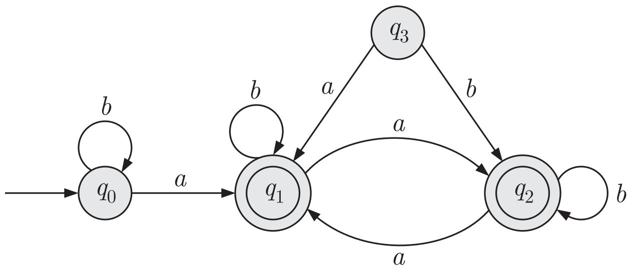
The regular expression $(1(0+1)+1)+(0(0+1)+0)$ satisfy this regular language.

Hence (C) is correct option.

Data for Q. 46 & 47 are given below

Solve the problems and choose the correct answers.

Consider the following Finite State Automation



Question. 38

The language accepted by this automaton is given by the regular expression

(A) $b^* ab^* ab^* ab^*$ (B) $(a+b)^*$

(C) $b^* a(a+b)^*$ (D) $b^* ab^* ab^*$

SOLUTION

From the given FSA to reach to find state $b^* a$ is necessary in REX remaining part is option.

REX $b^* a(a+b)^*$ followed by FSA.

Hence (C) is correct option.

Question. 39

The minimum state automaton equivalent to the above FSA has the following number of states

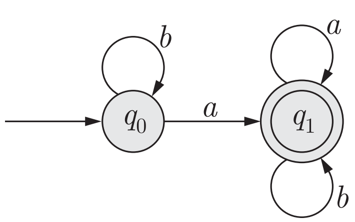
(A) 1 (B) 2

(C) 3 (D) 4

SOLUTION

State q_3 has no incoming edge, so automata will never reach in that state. So q_3 can be removed.

Also q_1 & q_2 works same so can be merged.



So only 2 states.
Hence (B) is correct option.

YEAR 2008

Question. 40

Which of the following in true for the language $\{a^P \mid P$ is a prime$\}$?

(A) It is not accepted by a Turning Machine

(B) It is regular but not context-free

(C) It is context-free but not regular

(D) It is neither regular nor context-free, but accepted by a Turing machine

SOLUTION

$\{a^p \mid P$ is a prime no.$\}$
This prime no. is extra constraint so this language is neither LFG nor RG but it can be accepted by turing machine.
Hence (D) is correct option.

Question. 41

Which of the following are decidable?

1. Whether the intersection of two regular languages is infinite
2. Whether a given context-free language is regular
3. Whether two push-down automata accept the same language
4. Whether a given grammar is context-free

(A) 1 and 2 (B) 1 and 4

(C) 2 and 3 (D) 2 and 4

SOLUTION

We can't determine whether a given CFL is regular or not, also similarity of PDA's on basis of language acceptance is not possible but intersection of two REL & whether grammar is CFG is decidable.
Hence (B) is correct option.

Question. 42

If L and $\overline{L}$ are recursively enumerable then L is

(A) regular (B) context-free

(C) context-sensitive (D) recursive

SOLUTION

L & $\overline{L}$ are recursively enumerable i.e. they can accept any element through some finite algorithm.
All algorithms are finite step procedures so these both has to be recursive.
Hence (D) is correct option.

Question. 43

Which of the following statements is false?

(A) Every NFA can be converted to an equivalent DFA

(B) Every non-deterministic Turing machine can be converted to an equivalent deterministic Turing machine

(C) Every regular language is also a context-free language

(D) Every subset of a recursively enumerable set is recursive

SOLUTION

(A) true since $NFA \rightarrow DFA$ conversion possible.

(B) $N.D$ turing M/C so true.

(C) every rex is a CFL but reverse is not true.

(D) false, since these may be proper subset of each other so not necessary.

Hence (D) is correct option.

Question. 44

Given below are two finite state automata(→indicates the start and F indicates a final state)

Y:

	a	b
→	1	2
2F	2	1

Z :

	a	b
→	2	2
2F	1	1

(A)

	a	b
-P	S	R
Q	R	S
R(F)	Q	P
S	Q	P

(B)

	a	b
-P	S	Q
Q	R	S
R(F)	Q	P
S	Q	P

(C)

	a	b
-P	Q	S
Q	R	S
R(F)	Q	P
S	Q	P

(D)

	a	b
-P	S	Q
Q	S	R
R(F)	Q	P
S	Q	P

SOLUTION

Z & Y each has 2 states so ZXY will have 2 states {(1, 1), (1, 2), (2, 1), (2, 2)}

Z	a	b
$\rightarrow 1$	2	2
$2(F)$	1	1

Y	a	b
$\rightarrow 1$	1	2
$2(F)$	2	1

ZXY	a	b
$\rightarrow$(1, 1)	(2, 1)	(2, 2)
(1, 2)	(2, 2)	(2, 1)
(2, 1)	(1, 1)	(1, 2)
(F)(2, 2)	(1, 2)	(1, 1)

ZXY	a	b
$\rightarrow P$	S	R
Q	R	S
$(F)R$	P	Q
S	Q	P

Let the states be P, Q, R & S.
So this transition table would be.

Fig.

Hence (A) is correct option

Question. 45

Which of the following statements are true ?

1. Every left-recursive grammar can be converted to a right-recursive grammar and vice-versa
2. All ε-productions can be removed from any context-free grammar by suitable transformations
3. The language generated by a context-free grammar all of whose production are of the form $X \rightarrow w$ or $X \rightarrow wY$ (where, w is a staring of terminals and Y is a non-terminal), is always regular
4. The derivation trees of strings generated by a context-free grammar in Chomsky Normal Form are always binary trees.

(A) 1, 2, 3 and 4

(B) 2, 3 and 4 only

(C) 1, 3 and 4 only

(D) 1, 2 and 4 only

SOLUTION

Yes, every left recursive grammar can be converted into right recursive grammar but all $\in$ period can't be removed only CFL that has λ -free $CFL's$ can be removed.
So this statement is false.
3 & 4 are also true.
Hence (C) is correct option.

Question. 46

Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I

A. Checking that identifiers are declared before their use

B. Number of formal parameters in the declaration to a function agress with the number of actual parameters in a use of that function

C. Arithmetic expressions with matched pairs of parentheses

D. Palindromes

List-II

1. $L = \{a^n b^m c^n d^m \mid n \le 1, m \le 1\}$
2. $X \to XbX \mid XcX \mid dXf \mid g$
3. $L = \{wcw \mid w \in (a \mid b)^*\}$
4. $X \to bXb \mid cXc \mid \varepsilon$

Codes:

	A	B	C	D
(A)	1	3	2	4
(B)	3	1	4	2
(C)	3	1	2	4
(D)	1	3	4	2

SOLUTION

A. To check whether identifiers are declared before their use, are shown by

$$L = \{wcw \mid w \in (a/b)^*\}$$

B. No. of formal parameters matching will be done by

$\{a^n b^m c^n d^m \mid n \ge 1, m \ge 1\}$

C. Arithmetic matching of parentheses. done by

$\{X \to XbX \mid XcX \mid dXf \mid g\}$

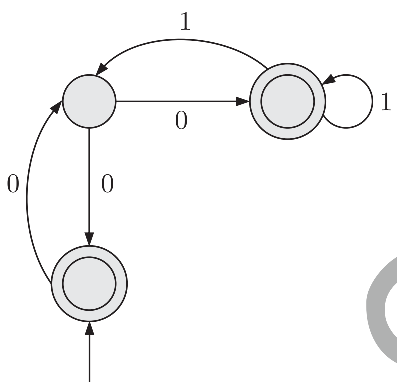
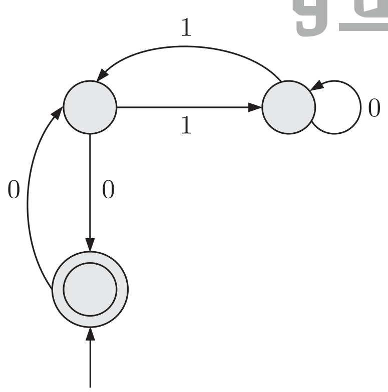
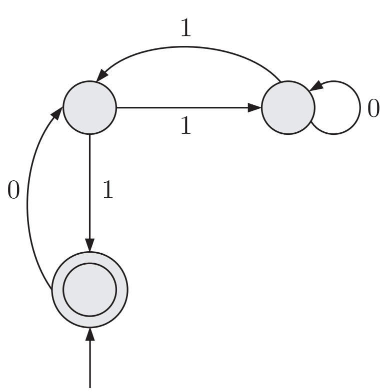
D. $X \to bXb \mid cXc \mid \in$, shows set of all even length palindromes.

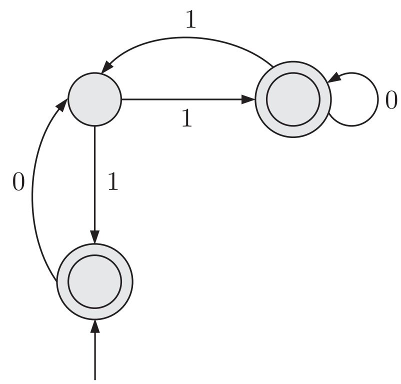
So $A \to 3, B \to 1, C \to 2, D \to 4$

Hence (C) is correct option.

Question. 47

Match **List I** with **List II** and select the correct answer using the codes given below the lists:

List I	List II
a. 	1. $\varepsilon+0(01^*1+00)^*01^*$
b. 	2. $\varepsilon+0(10^*1+00)^*0$
c. 	3. $\varepsilon+0(10^*1+10)^*1$

d. 

4. ε+0(10*1+10)*10*

Code:

	a	b	c	d
(A)	2	1	3	4
(B)	1	3	3	4
(C)	1	2	3	4
(D)	3	2	1	4

SOLUTION

A. →this DFA is regulated by regular expression $\in + 0(01^*100)^*01^*$, since this REX require two 00 in sub-expression & accepted by DFA.

B. The string 010001100010 is accepted by DFA, & string follows $e + 0(10^*1 + 00)^*0$

So $A \to 1$, $B \to 2$, $C \to S$, $D \to 4$.

Hence (C) is correct option.

Question. 48

Which of the following are regular sets?

1. $\{a^n b^{2m} \mid n \leq 0, m \leq 0\}$
2. $\{a^n b^m \mid n = 2m\}$
3. $\{a^n b^m \mid n \neq m\}$
4. $\{xcy \mid x, y \in \{a, b\}^*\}$

(A) 1 and 4 only
(B) 1 and 3 only
(C) 1 only
(D) 4 only

SOLUTION

I. $\{a^n b^{2m}, n \geq 0, m \geq 0\}$ is a regular language, since we can represent it by regular expression $(a)^*(bb)^*$ both $n \,\&\, m$ do not have any linear relationship.

II. & III. Not regular since there exists linear relationship between $m \,\&\, n$, neither they can be represented by DFA or REX.

IV. Also regular language, $(a+b^*)\,c(a+b)^*$ REX is possible & no linear relationship between $m \,\&\, n$ that require PDA.

Hence (A) is correct option.

YEAR 2009

Question. 49

$S \rightarrow a\,S\,a | b\,S\,b | a | b$

The language generated by the above grammar over the alphabet $\{a, b\}$ is the set of

(A) all palindromes

(B) all odd length palindromes

(C) strings that begin and end with the same symbol

(D) all even length palindromes

SOLUTION

Given grammar $S \rightarrow aSa | bSb | a | b$.
The strings generated through this grammar is definitely palindromes, but not all it can only generate palindromes of odd length only so (A) & (D) are false, (B) is correct.
Also it can generate palindromes which start and end with same symbol, but not all strings eg. $aabababba$.
Hence (B) is correct option.

Question. 50

Which one of the following languages over the alphabet {0, 1} is described by the regular expression :

$(0+1)^* 0 (0+1)^* 0 (0+1)^*$?

(A) The set of all strings containing the substring 00

(B) The set of all strings containing at most two 0's

(C) The set of all strings containing at least two 0's

(D) The set of all strings that being and end with either 0 or 1

SOLUTION

Given regular expression
$(0+1)^*0(0+1)^*0(0+1)^*$ due to two 0 in between, every string would contain at least two 0's.
Hence (C) is correct option.

Question. 51

Which one of the following is FALSE ?

(A) There is a unique minimal DFA for every regular language

(B) Every NFA can be converted to an equivalent PDA

(C) Complement of every context-free language is recursive

(D) Every nondeterministic PDAcan be converted to an equivalent deterministic PDA

SOLUTION

(A) true, since minimal DFA for every regular language is possible.

(B) true , NFA can be converted into an equivalent PDA.

(C) $CG'S$ are not recursive but their complements are.

(D) false, since non deterministic PDA represents, non deterministic CFG, since $NDCFG$ and CFG are proper subsets so conversion required.

Hence (D) is correct option.

Question. 52

Match all items in Group I with correct options from those given in Group 2

	Group 1		Group 2
P.	Regular expression	1.	Syntax analysis
Q.	Pushdown automata	2.	Code generation
R.	Data flow analysis	3.	Lexical analysis
S.	Register allocation	4.	Code Optimization

(A) P-4, Q-1, R-2, S-3
(B) P-3, Q-1, R-4, S-2
(C) P-3, Q-4, R-1, S-2
(D) P-2, Q-1, R-4, S-3

SOLUTION

Regular expressions are meant for lexical analysis to define tokens.
Pushdown Automata is used to accept context free language which are used for syntax analysis.
Data flow analysis is a technique for code optimization.
Register allocation is used for code generation.
So $P-3, Q-1, R-4, S-2$.
Hence (B) is correct option

Question. 53

Given the following state table of an FSM with two states A and B , one input and one output :

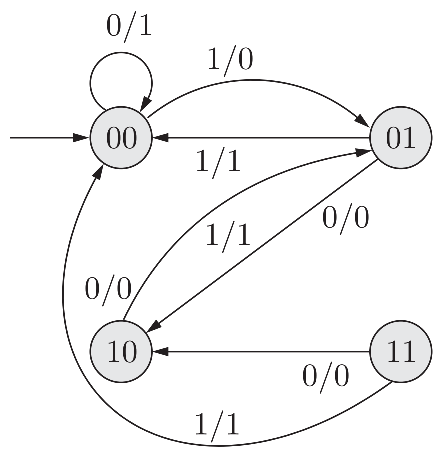
Present State A	Present State B	Input	Next State A	Next State B	Output
0	0	0	0	0	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	1	0	0
0	0	1	0	1	0
0	1	1	0	0	1
1	0	1	0	1	1
1	1	1	0	0	1

If the initial state is $A=0, B=0$, what is the minimum length of an input string which will take the machine to the state $A=0, B=1$ with Output$=1$?

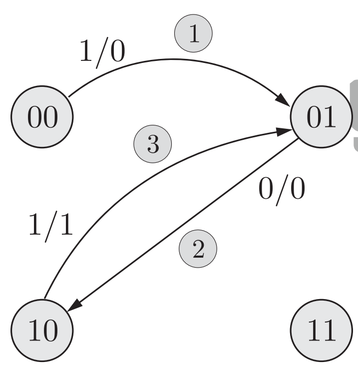
(A) 3
(B) 4

(C) 5 (D) 6

SOLUTION



The path to follow for given state.



So the length of input string is three.
Hence (A) is correct option.

Question. 54

Let $L = L_1 \cap L_2$ where L_1 and L_2 are language as defined below :

$L_1 = \{a^m b^m c a^n b^n | m, n \geq 0\}$

$L_2 = \{a^i b^j c^k | i, j, k \geq 0\}$

Then L is

(A) Not recursive

(B) Regular

(C) Context-free but not regular

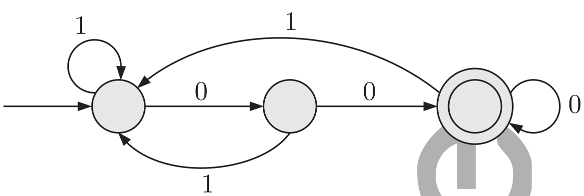
(D) Recursively enumerable nut not context-free

SOLUTION

$L = L_1 \cap L_2$, so L would contain $a^m b^m c$ where $m = i = j$ & $k = 1$ in L_2 $a^i b^j c^k$.
So the language $L = \{a^m b^m c$ is context free, but since it is recursive it is not regular, can't be represented using DFA.
Hence (C) is correct option.

Question. 55

The following DFA accept the set of all string over {0, 1} that



(A) Begin either with 0 or 1
(B) End with 0
(C) End with 00
(D) Contain the substring 00

SOLUTION

From the DFA it is clear that to reach to the end state two zero's would be there.
So all the strings that are accepted will end with 00.
Hence (C) is correct option.

YEAR 2010

Question. 56

Let $L1$ be a recursive language. Let $L2$ and $L3$ be language that are recursively enumerable but not recursive. What of the following statements is not necessarily true ?

(A) $L1 - L1$ is recursively enumerable
(B) $L1 - L3$ is recursively enumerable
(C) $L2 \cap L3$ is recursively enumerable
(D) $L2 \cap L3$ is recursively enumerable

SOLUTION

$$L_1 \rightarrow \text{recursive}$$
$$L_2, L_3 \rightarrow \text{recursively enumerable but not recursive.}$$

So L_1 can be recursive enumerable.

$$RE - RE = RE$$

So $L_1 - L_3$ is recursively enumerable.

Hence (B) is correct option.

Question. 57

Let $L = \{\omega \in (0+1)^* | \omega \text{ has even number of 1s}\}$, i.e., L is the set of all bit strings with even number of 1s. Which one of the regular expressions below represents L ?

(A) $(0^*10^*1)^*$

(B) $0^*(10^*10^*)^*$

(C) $0^*(10^*1)^*0^*$

(D) $0^*1(10^*1)^*10^*$

SOLUTION

We require strings to have even no. of $1's$, so to prove options false we need to find those strings which doesn't satisfy languages but have even no. of $1's$

Choice (A) $(0^*10^*1)^*$ is incorrect

1010101 string can't be derived

Choice (B) accepts 1010101

Choice (C) $0^*(10^*1)^*0^*$

Same 1010101 string is not accepted

Choice (D) $0^*1(10^*1)^*10^*$

010101010 can't be accepted.

Hence (B) is correct option.

Question. 58

Consider the language $L1 = \{0^i1^j | i \neq j\}$, $L2 = \{0^i1^j | i = j\}$, $L3 = \{0^i1^j | i = 2j+1\}$ $L4 = \{0^i1^j | i \neq 2j\}$.Which one of the following statements is true ?

(A) Only $L2$ is context free

(B) Only $L2$ and $L3$ are context free

(C) Only $L1$ and $L2$ are context free

(D) All are context free

SOLUTION

These sort of languages are accepted by PDA, so all should be context free languages. L_2 & L_3 are definitely CFL since accepted by stock of PDA.
And also L_1 & L_4 are linear comparisons of i & j so can also be represented using PDA.
So all are context free languages.
Hence (D) is correct option.

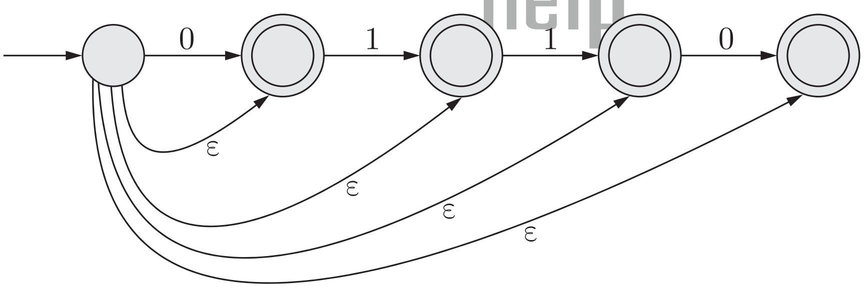
Question. 59

Let ω by any string of length n in$\{0,1\}^*$. Let L be the set of all substring so ω. What is the minimum number of states in a non-deterministic finite automation that accepts L ?

(A) $n-1$
(B) n
(C) $n+1$
(D) 2^{n+1}

SOLUTION

L is the set of all substrings of w where $w \in \{0,1\}^*$
Any string in L would have length 0 to n, with any no. of 1's and 0's
The $NDFA$



Here $n = 4$
So to accept all the substrings the no. of states required are

$$n + 1 = 4 + 1 = 5$$

Hence (C) is correct option.

****

GATE Multiple Choice Questions
For Computer Science

By NODIA and Company

Available in Two Volumes

FEATURES

- The book is categorized into units and the units are sub-divided into chapters.
- Chapter organization for each unit is very constructive and covers the complete syllabus
- Each chapter contains an average of 40 questions
- The questions are standardized to the level of GATE examination
- Solutions are well-explained, tricky and consume less time. Solutions are presented in such a way that it enhances you fundamentals and problem solving skills
- There are a variety of problems on each topic
- Engineering Mathematics is also included in the book