Currency Risk and Pricing Kernel Volatility

Federico Gavazzoni,[*] Batchimeg Sambalaibat,[†] and Chris Telmer[‡]

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Abstract

A basic tenet of lognormal asset pricing models is that a risky currency is associated with *low* pricing kernel volatility. Empirical evidence indicates that a risky currency is associated with a relatively *high* interest rate. Taken together, these two statements associate high-interest-rate currencies with low pricing kernel volatility. We document evidence suggesting that the opposite is true, thus contradicting a fundamental empirical restriction of lognormal models. Our identification strategy revolves around using interest rate *volatility* differentials to make inferences about pricing kernel volatility differentials. In most lognormal models the two are monotonic functions of one another. A risky currency, therefore, is one with relatively low pricing kernel volatility *and* relatively low interest rate volatility. In the data, however, we see the opposite. High interest rates are associated with *high* interest rate volatility. This indicates that lognormal models of currency risk are inadequate and that future work should emphasize distributions in which higher moments play an important role. Our results apply to a fairly broad class of models, including Gaussian affine term structure models and many recent consumption-based models.

[*]INSEAD; federico.gavazzoni@insead.edu

[†]Tepper School, Carnegie Mellon University; bsambala@andrew.cmu.edu

[‡]Tepper School, Carnegie Mellon University; chris.telmer@cmu.edu

1 Introduction

Currency risk is tricky. Unlike many financial securities, volatility and risk can be very different things. Consider the following anecdote. Producers of apples and bananas face supply shocks. The supply shocks become manifest in the relative price of fruit, the cost of one banana in units of apples. The producers, therefore, face price risk ... a positive supply shock to apples decreases the relative price of apples. Now consider volatility. Suppose that the apple shocks are volatile relative to the banana shocks. Apple producers notice two things: (i) the relative price of *bananas* isn't doing them any favors; it goes down whenever they get a bad shock, (ii) the supply shocks that they face are *dominating* the variability in the relative price. These apple producers therefore view the price of bananas as being *risky*. If you offer them a derivative security that has its value tied to the price of bananas, they will demand a risk premium in order to buy it. Banana producers, of course, face a reciprocal sort of situation. They notice that the relative price of *bananas* goes in their favor when they get a bad shock. A banana derivative security represents a hedge for them and they will pay an insurance premium in order to buy it. However, because the supply shocks that they face have relatively small volatility, this insurance premium will be small relative to the risk premium demanded by the apple producers. The equilibrium price will *tend to* — remember, this is an anecdote — feature a positive risk premium for bananas. Herein lies the tricky business. The *high* volatility commodity, apples, enjoys relatively *low* relative price risk.

If you replace apples and bananas with the pricing kernel for dollars and pounds, and the relative price of bananas with the price of one pound in units of U.S. dollars, you now understand what currency risk is in *any* lognormal model. The *currency* with the relatively volatile pricing kernel will pay a relatively low risk premium (in fact, negative). Many recent statistical models of the term structure of interest rates fall into this category. If you assume that financial markets are complete you can go further and replace "currency" with "country." In *any* complete-markets, lognormal equilibrium model, the *country* with the relatively volatile nominal marginal rate of substitution will have a currency that pays a negative risk premium. Many recent consumption-based asset pricing models fall into this category. High volatility in marginal utility growth coincides with low currency risk.

Algebraically, currency risk in lognormal models takes the form

$$E_t\, s_{t+1} - f_t \;\; = \;\; \frac{1}{2}\left(Var_t \log m_{t+1} - Var_t \log m^*_{t+1} \right) \;\; , \tag{1}$$

where s_t and f_t are the log spot and forward exchange rates (price of pounds in units of dollars), and m_t and m^*_t are respectively, the dollar and pound pricing kernels (derivation provided below). The left-hand-side is the (continuously compounded) expected excess return on pounds. We call it the *currency risk premium on the pound*. The right-hand-side is the pricing kernel conditional volatility differential. Eq. (1) says that the pound will pay a positive risk premium if its pricing kernel has relatively low volatility. Empirically, the LHS seems to be increasing in the pound less dollar interest rate differential. This is the "carry trade evidence." The question we ask is whether or not lognormal models are consistent with the carry trade evidence. This amounts to evaluating the restriction that *high* interest rate currencies have *low* pricing kernel volatility.

The pricing kernel, of course, is not observable. So neither is the volatility difference on the RHS of Eq. (1). Many previous papers have attempted to relate the LHS to the volatility of the currency depreciation rate, $s_{t+1} - s_t$. As we'll see below, this amounts to relating the risk premium to the volatility of the difference,

$$Var_t\big(s_{t+1} - s_t\big) \quad = \quad Var_t\big(\log m_{t+1} - \log m^*_{t+1}\big) \ \ ,$$

not the difference in the volatility from Eq. (1). So, strictly speaking, it's the wrong measure of volatility. Other papers have have implicitly evaluated Eq. (1) by formulating a specific model of m_t and m^*_t and asking whether or not the model can account for the carry trade evidence. If it can then, if the model is in the lognormal class that we consider (many have been), Eq. (1) holds by construction.

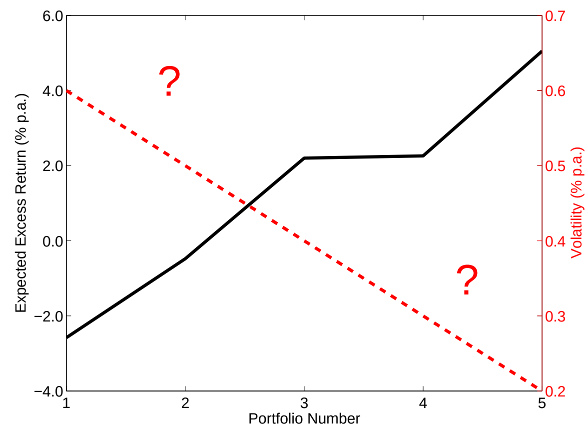
Our approach is to try to say something that is not so strongly tied to one particular model. We do so by associating the RHS with the interest rate *volatility* differential in the following way. We outline conditions under which

$$Var_t(m_{t+1}) > Var_t(m^*_{t+1}) \Leftrightarrow Var_t(i_{t+1}) > Var_t(i^*_{t+1}) \qquad (2)$$

If this condition is true, then we can say that currency risk is associated with a *negative* interest-rate volatility differential. A risky currency (in a lognormal model) is one with relatively *low* interest rate volatility. This is the restriction that we actually evaluate.

Figure 1 illustrates our question graphically. It is an adaptation of Table 1 from Lustig, Roussanov, and Verdelhan (2011) (LRV). We construct five portfolios of currencies by constructing a monthly sort by the level of interest rates. Figure 1 reports the annualized average excess return, vis-a-vis the U.S. dollar, on each of the currency portfolios. We see what LRV, Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011) and many others have documented; the 'carry-trade' of funding a long-position in a high-interest-rate currency by borrowing in a low-interest-rate currency seems to pay a positive excess expected return. Our question, then, is this. If Figure 1 also plots, for each portfolio, the average interest rate volatility of all the currencies in the portfolio, then, according to Eq. (2), the line should be decreasing. Is it?

Figure 1
Currency Risk and Return



The solid black line (left axis) reports the sample mean of the excess return on 5 interest-rate sorted currency portfolios, similar to that reported in Lustig, Roussanov, and Verdelhan (2011). The red dashed line (right axis) is, qualitatively, what lognormal pricing kernel models predict that pricing kernel and interest rates volatility should look like.

We find evidence suggesting that it is not. High expected return seems to be associated with high pricing kernel volatility. This implies one of two things. Either there is something wrong with method of inferring what the pricing kernel volatility differential in Eq. (1) looks like, or there's something wrong with conditional lognormality. We argue that the latter is most likely. We conclude that models of currency risk should incorporate departures from lognormality. Our results are supportive of recent work by Brunnermeier, Nagel, and Pedersen (2008) and others that emphasize conditional skewness and leptokurtosis.

It is important to understand that, while Eqns. (1) and (2) hold for any lognormal model, the particular pricing kernels for which they do must be carefully defined. Section 2 therefore begins with some background and a precise statement of which models our results restrict, and which they do not. Section 3 develops the overall affine Gaussian structure and articulates our main results using two theorems. Section 4 describes our data, Section 5 our results, and Section **??** concludes.

2 Notation and Basic Approach

We begin with a simple derivation of Eq. (1). The pricing kernel for claims denominated in U.S. dollars (USD) is m_{t+1}, so that $E_t\, m_{t+1} R_{t+1} = 1$ for all (gross) USD-denominated asset returns, R_{t+1}, realized between dates t and $t+1$. The analogous pricing kernel for claims denominated in foreign currency (say, British pounds, GBP) is m^*_{t+1} and GBP-denominated returns are R^*_{t+1}. Both m_t and m^*_t exist by virtue of no-arbitrage, but are only unique if markets are complete. For any of these pricing kernels, one period, continuously-compounded USD and GBP interest rates, i_t and i^*_t, satisfy

$$i_t = -\log E_t\, m_{t+1} \tag{3}$$
$$i^*_t = -\log E_t\, m^*_{t+1} \quad . \tag{4}$$

The date-t nominal spot exchange rate, USD per GBP, is S_t. Since the USD pricing kernel must also price USD-denominated returns on GBP-denominated assets, no-arbitrage implies that, for any m_{t+1},

$$E_t \left(m_{t+1} \frac{S_{t+1}}{S_t} R^*_{t+1} \right) = 1 \quad . \tag{5}$$

Consider one particular USD pricing kernel, m_{t+1}, along with the observable process S_{t+1}/S_t. Define

$$m^*_{t+1} = m_{t+1} \frac{S_{t+1}}{S_t} \tag{6}$$

and note that, by construction, $E_t\, m^*_{t+1} R^*_{t+1} = 1$, so that this particular m^*_{t+1} is a legitimate pricing kernel GBP-denominated claims. If markets are incomplete, then there are other legitimate GBP pricing kernels. This is discussed in the next section. Express Eq. (6) in terms of natural logarithms:

$$s_{t+1} - s_t = \log m^*_{t+1} - \log m_{t+1} \quad , \tag{7}$$

where $s_t \equiv \log S_t$, so that the LHS is the (continuously-compounded) depreciation rate of the USD. The one-period forward exchange rate and its logarithm are F_t and f_t. Subtract Eq. (4) from Eq. (3) and invoke covered interest parity, $f_t - s_t = i_t - i^*_t$:

$$f_t - s_t = -\log E_t\, m_{t+1} - (-\log E_t\, m^*_{t+1}) \quad . \tag{8}$$

Take the conditional mean of Eq. (7), and subtract from it Eq. (8):

$$\begin{aligned} E_t\, s_{t+1} - f_t &= \big(\log E_t\, m_{t+1} - E_t \log m_{t+1}\big) - \big(\log E_t\, m^*_{t+1} - E_t \log m^*_{t+1}\big) \\ &= \frac{1}{2}\big(Var_t \log m_{t+1} - Var_t \log m^*_{t+1} \big) \quad , \end{aligned} \tag{9}$$

where the last equation holds if we assume that m_t and m_t^* are jointly, conditionally lognormal.[1] It is the same as Eq. (1) from the introduction. The LHS is the *risk premium on GBP*: the (continuously-compounded) expected excess return on borrowing USD and investing the proceeds in GBP. The RHS says that, in this class of models, a necessary condition for GBP to be risky is that its pricing kernel exhibit relatively *low* volatility.

Note that a great deal of empirical work — from older papers such as Domowitz and Hakkio (1985) to newer papers such as Chernov, Graveline, and Zviadadze (2012) — has focused on the conditional variance of the exchange rate, $Var_t(s_{t+1} - s_t)$. Similarly, Brunnermeier, Nagel, and Pedersen (2008) focus on $Skew_t(s_{t+1} - s_t)$. While these moments are certainly interesting in-and-of-themselves, the combination of Eqns. (7) and (9) indicate that they emphasize moments of the difference, whereas currency risk is more directly about the difference of the moments. Sorting this out (in a new way) is the focal point of our paper.

2.1 Incomplete Markets

Eq. (9) holds by no-arbitrage for *given* processes m_t and S_{t+1}/S_t, and for the *particular* no-arbitrage pricing kernel, m_t^*, defined by Eq. (6). If markets are complete then these things are all unique and any two will tell you the third. If not, then there is a multiplicity that one must consider.

Our approach is to fix models for m_t and m_t^* and then use Eq. (6) to compute the depreciation rate, $S_{t+1}/S_t = m_{t+1}^*/m_{t+1}$. The multiplicity can be represented by a random variable η_{t+1} such that $\exp(\eta_{t+1}) m_{t+1}^*/m_{t+1}$ is also a legitimate depreciation rate. Complete markets imposes that $\eta_{t+1} = 0$. No arbitrage imposes that

$$\begin{aligned} E_t\big(m_{t+1}R_{t+1}\big) &= E_t\big(m_{t+1}e^{-\eta_{t+1}}R_{t+1}\big) = 1 \\ E_t\big(m_{t+1}^*R_{t+1}^*\big) &= E_t\big(m_{t+1}^*e^{\eta_{t+1}}R_{t+1}^*\big) = 1 \end{aligned}$$

for all asset returns, R_{t+1} and R_{t+1}^*, denominated in USD or GBP, respectively. These conditions restrict the admissible η_t processes in important ways but still leave open a potentially large set of admissible exchange rate processes that are consistent with no-arbitrage and the fixed processes m_t and m_t^*. However, if we restrict attention to affine term structure models, then all elements of this set are observationally equivalent and we can set $\eta_t = 0$ without loss of generality (Backus, Foresi, and Telmer (2001), Proposition 1 and subsequent discussion on page 289). In this sense, our results apply to *any* Gaussian affine term structure

[1] Backus, Foresi, and Telmer (2001), page 286, Eq. (12), shows the more general expression, involving higher-order moments. This will play a key role later, in Section #.

model of currency risk (*e.g.*, Backus, Foresi, and Telmer (2001), Bakshi and Chen (1997), Bansal (1997), Brenna and Xia (2006), Frachot (1996), Lustig, Roussanov, and Verdelhan (2011), and Saá-Requejo (1994)).

What exactly does this mean? The presumption of a two-currency affine term structure model is that all predictable and unpredictable movements in log bond prices and the depreciation rate are spanned by the model's state variables and innovations, respectively. Hence, the residual η_t obeys the same affine structure as m_t and m_t^*. Hence, we can redefine m_t^*, say, to encapsulate the residual, use the result in combination with the original m_t and Eq. (6) to define S_{t+1}/S_t, and the mapping between identifiable parameters and moments of the data will be identical to the case of $\eta_t = 0$. Basically, if we restrict ourselves to Gaussian affine models in which parameter values are identified only via *moments* of bond prices and exchange rates, then, conditional on the specifications for m_t and m_t^*, the distinction between complete and incomplete markets is moot. From a macroeconomic perspective, this might seem like throwing out the baby with the bathwater.[2] From the perspective of the large literature on statistical term structure models, perhaps not.

2.2 Changing Units Versus Changing Countries

When can we associate Eq. (1) with *countries*, not just *currencies*? Only when markets are complete. Only then does Eq. (**??**) describe an equilibrium condition in which marginal rates of substitution of domestic and foreign representative agents are equated, pointwise, via the change of units process, S_{t+1}/S_t. This is, of course, a restrictive assumption. But it is employed by a large majority of recent consumption-based models that employ either the Campbell and Cochrane (1999) or the Bansal and Yaron (2004) machinery to a multi-country setting. Recent examples include Alvarez, Atkeson, and Kehoe (2009), Bansal and Shaliastovich (2013), Colacito and Croce (2011), Stathopoulos (2012) and Verdelhan (2010). These are models in which Eq. (6) describes a unique relationship between marginal rates of substitution and exchange rates, and in which the pricing kernels are jointly lognormal (thanks in some cases to linearizations such as that from Hansen, Heaton, and Li (2008)). In most cases these models *work* in the sense that calibrated versions of them are consistent with the standard set of carry-trade facts. This means that, necessarily, Eq. (1) applies and that high risk is associated with low pricing kernel volatility. We will show that, for this class of models, data on interest rate volatility differentials calls this relationship into question and thus poses a challenge to these models.

[2]The bathwater is the difficult job of identifying η_t using, say, consumption data. The baby is what we might learn by doing so.

2.3 A Very Rough Approximation

Eq. (9) tells us that the currency risk premium is the difference between two unobservable variables. Many papers, of course, have written down an explicit (lognormal) model of m_t and m_t^* under which this difference is a function of observables. A successful model has been one with an interest rate differential, $i_t - i_t^*$, that is *negatively* correlated with the volatility difference, $Var_t \log m_{t+1} - Var_t \log m_{t+1}^*$. Our goal, instead, is to derive an empirical measure of this variance difference that applies more generally than to just one, particular (lognormal) model. We begin with a very informal derivation, intended to get at the main idea. In the next section we provide a tight, formalized analysis showing that what we do here applies much more generally.

With lognormality, Eq. (3) implies that the period-ahead domestic interest rate is

$$i_{t+1} \quad = \quad -E_{t+1} \log m_{t+2} - \frac{1}{2} Var_{t+1} \log m_{t+2} \quad . \tag{10}$$

Suppose that variation in conditional mean is relatively small. This is not as unreasonable as it might sound, at least for our question. Backus, Foresi, and Telmer (2001) show that Fama's (1984) necessary conditions for resolving the forward premium anomaly translate into conditions stating that 'variance in pricing kernel variance must be larger than variance in pricing kernel means.' Papers by **?** and **?** show that something similar is implied by the Campbell-Shiller, Fama-Bliss term structure regressions. Thus ignoring the first term in Eq. (10) we have

$$Var_t \, i_{t+1} \quad \approx \quad \frac{1}{4} Var_t \, Var_{t+1} \log m_{t+2} \quad .$$

Suppose that the conditional variance of the conditional variance isn't too different from the the conditional variance. For some processes, this is sensible. For example, for the canonical square-root process, $x_{t+1} = \mu + \varphi x_t + \sigma x_t^{1/2} \varepsilon_{t+1}$, we have that $Var_t \, Var_{t+1} x_{t+2} = \sigma^2 \, Var_t x_{t+1} = \sigma^4 x_t$. For other processes — *e.g.*, an autoregression with stochastic volatility, but where the volatility is homoskedastic — it makes no sense. Ignoring the latter,

$$Var_t \, i_{t+1} \quad \approx \quad \frac{1}{4} Var_t \log m_{t+1} \quad . \tag{11}$$

This is the main idea. Perhaps we can learn something about pricing kernel volatility by simply estimating interest rate volatility?

Suppose that this is valid. Substitute it into Eq. (9):

$$E_t \, s_{t+1} - f_t \quad \approx \quad 2\big(Var_t \, i_{t+1} - Var_t \, i_{t+1}^* \big) \quad .$$

In most lognormal models the LHS is a linear function of the interest rate differential:

$$a + b\big(i_t - i_t^*\big) \;\;\approx\;\; 2\big(\,Var_t\, i_{t+1} - Var_t\, i_{t+1}^*\big) \;\;. \tag{12}$$

There is overwhelming empirical evidence indicating that $b < 0$. This is the 'Uncovered Interest Rate Parity (UIP)' regression evidence first found by Bilson (1981), Fama (1984) and Tryon (1979). It is the basis for the foreign currency 'carry trade.' Assume that $a = 0$.[3] Eq. (12) implies that, according to lognormal models, (i) the interest rate differential, $i_t - i_t^*$ and the volatility differential, $Var_t\, i_{t+1} - Var_t\, i_{t+1}^*$, should have the opposite sign, and (ii) they should be negatively correlated.

Figure 3 shows results that are representative of our main findings. It shows that, for a classic 'carry trade' pair of currencies — USD and the Australian dollar (AUD) — restrictions (i) and (ii) seem to be strongly at odds with the data. Much more often than not, the LHS and RHS of Eq. (12) have the *same* sign. Moreover, they are positively correlated at 0.52.

Interest rate differentials and interest rate volatility differentials appear to be positively related. The high-interest rate currency — the carry-trade recipient currency that pays a positive risk premium — also appears to be the high interest-rate-volatility currency. This isn't the case for every currency pair and time period but, as we exhaustively demonstrate in Section 5, it is much more the rule than the exception. Our approximation, Eq. (11), suggests that we can restate this. The high-interest rate currency appears to be the high pricing-kernel-volatility currency. This is a stark contradiction of any lognormal model of currency risk.

We now tighten up the approximation from Section 2.3 and show precisely under what conditions high interest rate volatility is necessarily associated with high pricing kernel volatility.

3 Affine Models

The Duffie and Kan (1996) class of lognormal, affine pricing kernel models can be specified as follows. Uncertainty in the domestic country is described by the k-dimensional vector of state variables z that follows a square-root model:

$$z_{t+1} = (I - \Phi)\theta + \Phi z_t + \Sigma(z_t)^{1/2}\epsilon_{t+1} \quad ,$$

[3] This is implied by the UIP evidence if (i) $b = -1$, (ii) UIP holds unconditionally, and (iii) the unconditional mean of the interest rate differential is zero, $E(i_t - i_t^*) = 0$. For many currency pairs all of these conditions are empirically plausible (Engel (2011) provides some up-to-date evidence and his survey paper, Engel (1996), is a standard reference for a more exhaustive survey). For those for which they are not, our story is basically unchanged once we subtract out an innocuous mean.

where $\{\epsilon\} \sim$ NID(0,1), $\Sigma(z_t)$ is a $k \times k$ diagonal matrix with a typical element given by $\sigma_i(z_t) = \alpha_i + \beta_i^\top z_t$, where β_i has nonnegative elements, and Φ is a $k \times k$ stable matrix with positive diagonal elements. The process for z requires that the volatility functions, $\sigma_i(z)$, be positive, which places additional restrictions on the parameters. The pricing kernel is

$$-\log m_{t+1} = \delta + \gamma^\top z_t + \lambda^\top \Sigma(z_t)^{1/2}\epsilon_{t+1} \quad , \tag{13}$$

where the $k \times 1$ vector γ is referred to as the "factor loadings" for the pricing kernel, and the $k \times 1$ vector λ is referred to as the "price of risk" vector.

Using Eq. (3), together with the dynamics of the pricing kernel from Eq. (13), the interest rate is

$$i_{t+1} = \left(\delta - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \alpha_j\right) + \left(\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \beta_j^\top\right) z_{t+1} \quad .$$

The conditional variance of the home pricing kernel is

$$Var_t(m_{t+1}) = \left(\lambda^\top \Sigma(z_t)^{1/2}\right) Var_t(\epsilon_{t+1}) \left(\lambda^\top \Sigma(z_t)^{1/2}\right)^\top = \lambda^\top \Sigma(z_t)\lambda = \sum_{j=1}^{k} \lambda_j^2 \sigma_j(z_t) \quad ,$$

and the conditional variance of the home interest rate is

$$\begin{aligned} Var_t(i_{t+1}) &= \left(\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \beta_j^\top\right) \Sigma(z_t) \left(\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \beta_j^\top\right)^\top \\ &= \sum_{n=1}^{k} \left(\gamma_n - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \beta_{n,j}\right)^2 \sigma_n(z_t) \quad . \end{aligned}$$

Similarly to the domestic country, the foreign country is described by a k-dimensional vector z^*. Foreign parameters are denoted with an asterisk. It is straightforward to derive the expressions for the foreign pricing kernel, m^* and the foreign interest rate, i^*.

3.1 Theorems

Under what conditions can we say that

$$Var_t(m_{t+1}) > Var_t(m_{t+1}^*) \Leftrightarrow Var_t(i_{t+1}) > Var_t(i_{t+1}^*) \quad ? \tag{14}$$

The validity — or not — of result (14) is in general parameter dependent. However, for two special cases which are ubiquitous in the term structure and the currency risk premium literature, it turns out that we can say a fair bit.

Theorem 1: *Symmetric Coefficients*

> Let Ω (Ω^*) denote the vector containing the home (foreign) parameters. Let $\Omega = \Omega^*$, $\lambda \neq 0$ and $\left(\gamma^\top - \frac{1}{2}\sum_{j=1}^{k}\lambda_j^2\beta_j^\top\right) \neq 0$. Then $(\Sigma(z_t) - \Sigma(z_t^*))$ is positive definite if and only if $Var_t(m_{t+1}) > Var_t(m_{t+1}^*)$, if and only if $Var_t(i_{t+1}) > Var_t(i_{t+1}^*)$.

The proof is in Appendix A. Theorem 1 says that, with symmetric coefficients, a relatively large conditional variance of the home state variables is associated with a relatively large conditional variance of the home pricing kernel and a relatively large conditional variance of the home interest rate. A given ranking in the conditional variance of the state variables in each country is associated with the same ranking in the conditional variance of the pricing kernels and the conditional variance of the interest rates.

As an example, consider the single-factor (per-country) case, $k = 1$. Theorem 1 simplifies to

$$Var_t(z_{t+1}) > Var_t(z_{t+1}^*) \Leftrightarrow Var_t(m_{t+1}) > Var_t(m_{t+1}^*) \Leftrightarrow Var_t(i_{t+1}) > Var_t(i_{t+1}^*) \quad ,$$

which can be read as "high conditional variance at home means high conditional variance in the home kernel and high conditional variance in the home interest rate."

Theorem 2: *Common Factors*

> Consider the case of common factors, $z_t = z_t^*$. Assume that $\gamma = \gamma^*$, $\beta = \beta^*$, and that there exists a strong enough 'precautionary savings motive' associated with both the domestic and foreign pricing kernels, so that
>
> $$\gamma^\top - \frac{1}{2}\sum_{j=1}^{k}\lambda_j^2\beta_j^\top < 0 \quad \text{and} \quad \gamma^\top - \frac{1}{2}\sum_{j=1}^{k}(\lambda_j^*)^2\beta_j^\top < 0 \quad .$$
>
> Suppose that the prices of risk λ and λ^* can be *ordered*, so that either $\lambda > \lambda^*$ or $\lambda < \lambda^*$. Then, $|\lambda| > |\lambda^*|$ if and only if $Var_t(m_{t+1}) > Var_t(m_{t+1}^*)$ if and only if $Var_t(i_{t+1}) > Var_t(i_{t+1}^*)$.

The proof is in Appendix A. Theorem 2 says that, with common factors, a sufficient condition for the conditional variance of the domestic pricing kernel to be larger than the conditional variance of the foreign pricing kernel is that the price of risk of each of the home state variables is at least as large (in absolute value) than the price of risk of each of the foreign state variables. When this is the case, a strong enough precautionary saving demand in each country delivers a larger conditional variance of the home interest rate, relative to the conditional variance of the foreign interest rate. For the theorem to hold, symmetry in the factor loadings and in the sensitivity of the conditional variances to the state variables is required.

The condition that $\gamma^\top - \frac{1}{2}\sum_{j=1}^{k}\lambda_j^2\beta_j^\top < 0$ (and its foreign counterpart) is usually referred to in the literature as a *strong enough precautionary savings motive.* Lustig, Roussanov, and Verdelhan (2011) is an example of Theorem 2 at work. They show that an affine model with a common factor, common coefficients across countries with the exception of the price of risk, and a precautionary saving motive that is strong enough in each country delivers interest rate and exchange rate dynamics that are consistent with the carry trade facts. Hence, the assumptions of Theorem 2 are not stringent at all. They *must* be satisfied for an affine model of the Duffie-Kan class to fit the data.

As an example, consider the single factor case, $k = 1$. Here, a stronger result is available. Under the conditions of Theorem 2

$$|\lambda| > |\lambda^*| \Leftrightarrow Var_t(m_{t+1}) > Var_t(m^*_{t+1}) \Leftrightarrow Var_t(i_{t+1}) > Var_t(i^*_{t+1}) \quad ,$$

where now λ and λ^* are scalars. In words, in a world with one single common factor, when all other coefficients are symmetric, having a relatively large home price of risk (in absolute value) means having a relatively high domestic kernel volatility and a relatively high interest rate volatility.

4 Data and Estimation

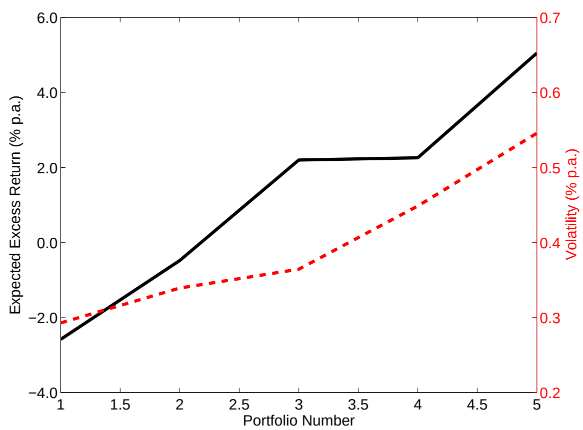
For our main analysis, we use data for 114 countries on 3-month treasury bill yields, forward and spot exchange rates from *Global Financial Data* (GFD) for the period 1950-2009. As a comparison, we also use eurocurrency interest rates for 27 countries from the *Financial Times/ICAP* (FT/ICAP), 1975-2009. Appendix B provides details. The number of countries is extensive, but, obviously, we do not have data for all countries and time periods, and the relevance of many of the countries for our question is questionable. We therefore report results for a number of different subsamples of time and country. Our main result is not sensitive to either.

We estimate a GARCH model for interest rate volatility. Details on the estimation procedure are given in Appendix B. We report results for both bilateral currency pairs and portfolios of currencies. The latter are formed in a manner identical to Lustig, Roussanov, and Verdelhan (2011), by sorting on interest rate levels at a monthly frequency.

5 Results

In the introduction we used Figure 1 to illustrate our main question: is pricing kernel volatility decreasing in the interest rate differential, as lognormal theory predicts it should? Figure 2 answers our question. We conduct the exact same exercise as Lustig, Roussanov, and Verdelhan (2011). We sort currencies by interest rate and, each month, form 5 portfolios. Figure 2 plots the time-averaged excess return on each portfolio, and the time-averaged interest rate volatility. Both are increasing as we move from low to high interest rate portfolios. The answer to the question is no.

Figure 2
High Interest Rates are Associated with High Interest Rate Volatility



The solid black line (left axis) is the same as that in Figure 1 from the introduction. It is the sample mean of the excess return on 5 interest-rate sorted currency portfolios, similar to that reported in Lustig, Roussanov, and Verdelhan (2011). The red dashed line (right axis) is, the time-averaged interest rate volatility on each of the portfolios. The class of lognormal models described in Section 3 restrict the red line to be decreasing if the black line is increasing. The data are, therefore, inconsistent with lognormal models in this dimension.

The remainder of our empirical work simply consists of further substantiating the point of Figure 2 by considering different samples and data sources. Figure 4 shows average volatility for the currency portfolios for a number of different subsamples, starting from 1950, 1975, 1987 and 1995. Figure 5 reports analogous results for bilateral currency pairs. Figure 6 compares results across the GFD and FT/ICAP data sources. In all cases, we see robust evidence of high interest rates differentials being associated with high interest rate volatility differentials.

6 Conclusions

We have shown that Gaussian models of currency risk face an empirical challenge. A high interest rate differential seems to be associated with a high interest-rate *volatility* differential. In a broad class of Gaussian models of the pricing kernel, this is inconsistent with (i) the empirical observation that high interest rates are associated with high excess expected returns, and (ii) the theoretical restriction that high excess expected returns accrue to the *low* volatility pricing kernel. This inconsistency derives from some conditions under which interest rate volatility and pricing kernel volatility are positively related. These conditions are not exhaustive. But they are satisfied by some models that are prominent in the literature. This suggests that we either enrich these models, or consider non-Gaussian alternatives.

We've couched our analysis in terms of trying to observe *directly* the difference in the moments of pricing kernels. This stands in contrast to much existing work, which tends to be based on moments of the difference in pricing kernels (*e.g.*, the variance of the exchange rate). We find our approach useful in that it emphasizes something fundamental about what currency risk *is*, while at the same time providing some links to other asset pricing models and results such as those from the literature on the term structure of interest rates. Nevertheless, we admit that ours does have the flavor of many previous papers that have gone searching for the unicorn by trying to identify the magical pricing kernel using very few assumptions and very little data (*i.e.*, asset return data only). An alternative interpretation, therefore, is as follows.

Ever since Bilson (1981), Fama (1984) and Tryon (1979) discovered the existence of excess expected returns in currency markets, many models have been developed to account for this behavior. The first models focused almost exclusively on the UIP regression coefficient. For subsequent models the bar has been raised higher. Other moments of the joint distribution of exchange rates, interest rates, consumption and so on have been emphasized. Sharpe ratios on traded currency *portfolios* have been emphasized. Our results can be viewed as simply suggesting one more moment: the interest-rate volatility differential. There is clear evidence on this, and it places very binding restrictions on state-of-the-art models. Moreover, it seems (to us) to be a particularly important moment. There is a clear link to theory and this link might be pointing us to non-Gaussian behavior, something that has been strongly emphasized in the recent literature on crashes, disasters and the like. We like this moment.

Finally, we close with a broad, interpretive point. Currency risk is a particular type of "change of units risk." In Gaussian models, an inescapable characteristic of this is that high risk is associated with low volatility. If the units that I care about are subject to *relatively* volatile shocks, then financial securities with payoffs denominated in these units will pay a *negative* risk premium (relative to the other units in question). Our goal has been to ask if this characteristic fits the facts. To find out, one must first take a stand on what are "the units that I care about." The obvious answer (to an economist) is "real marginal utility units," where the word "real" makes clear that this is "marginal utility per unit of *goods*." We have circumvented this, focusing instead on *nominal* marginal utility units: "utils per dollar." We've done this because most financial securities are denominated in dollars, not in goods. We must therefore be silent on whether or not Gaussian models of *real* exchange rates fit the facts. But we can say something about nominal models, which, one-way-or-another, have been most prevalent in the literature. We find evidence suggesting that nominal exchange rate risk is more than just a Gaussian phenomenon. We also note that, while our data have not addressed them, Gaussian models of real marginal utility have become more and more prevalent in the literature ever since Hansen, Heaton, and Li (2008) developed their linearization of the recursive class of preferences. Much of the long-run risk literature initiated by Bansal and Yaron (2004) features models that are conditionally Gaussian. A prominent example featuring exchange rates is Bansal and Shaliastovich (2013). Our approach is easily applied in this setting, and will feature relative consumption volatility in addition to interest rate volatility. This is work-in-progress.

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A Appendix: Proofs of Theorems

Proof of Theorem 1

Note that, since $(\Sigma(z_t) - \Sigma(z_t^*))$ is diagonal, positive definiteness requires $\sigma_j(z_t) \geq \sigma_j(z_t^*)$, for every j, with at least one strict inequality (i.e. $\exists i$ such that $\sigma_i(z_t) > \sigma_i(z_t^*)$). Now, when $\Omega = \Omega^*$, we have

$$Var_t(m_{t+1}) - Var_t(m_{t+1}^*) = \lambda^\top \left(\Sigma(z_t) - \Sigma(z_t^*)\right) \lambda \quad .$$

Therefore, by the definition of positive definiteness, we have

$$(\Sigma(z_t) - \Sigma(z_t^*)) \text{ is positive definite } \Leftrightarrow Var_t(m_{t+1}) > Var_t(m_{t+1}^*) \quad ,$$

whenever $\lambda \neq 0$. Similarly,

$$Var_t(i_{t+1}) - Var_t(i_{t+1}^*) = \left(\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \beta_j^\top\right) (\Sigma(z_t) - \Sigma(z_t^*)) \left(\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \beta_j^\top\right)^\top \quad .$$

Again, positive definiteness gives

$$(\Sigma(z_t) - \Sigma(z_t^*)) \text{ is positive definite } \Leftrightarrow Var_t(i_{t+1}) > Var_t(i_{t+1}^*) \quad ,$$

whenever $\left(\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \beta_j^\top\right) \neq 0$. Last, let $a \neq 0$ be any $k \times 1$ vector and note that the matrix $(\Sigma(z_t) - \Sigma(z_t^*))$ is diagonal. Therefore,

$$a^\top \left(\Sigma(z_t) - \Sigma(z_t^*)\right) a = \sum_{j=1}^{k} a_j^2 \left(\sigma(z_t) - \sigma(z_t^*)\right) > 0 \Leftrightarrow \sigma_j(z_t) \geq \sigma_j(z_t^*), \text{ for every } j \quad ,$$

with at least one strict inequality associated with a non zero element of a. $\square$

Proof of Theorem 2

$$Var_t(m_{t+1}) - Var_t(m_{t+1}^*) = (\lambda^2 - (\lambda^*)^2)^\top \; diag\{\Sigma(z_t)\} \quad ,$$

which, under the assumption that λ and λ^* can be ordered, is positive if and only if $\lambda^2 > (\lambda^*)^2$, that is $|\lambda| > |\lambda^*|$. Moreover,

$$Var_t(i_{t+1}) - Var_t(i_{t+1}^*) = \left(\left(\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \beta_j^\top\right)^2 - \left(\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} (\lambda_j^*)^2 \beta_j^\top\right)^2\right)^\top diag\{\Sigma(z_t)\} \quad ,$$

which, under the conditions that $\gamma = \gamma^*$, $\beta = \beta^*$, $\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \beta_j^\top < 0$, and $\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} (\lambda_j^*)^2 \beta_j^\top < 0$ is positive if only if $\lambda^2 > (\lambda^*)^2$, that is $|\lambda| > |\lambda^*|$. $\square$

B Appendix: Data

B.1 Interest Rates

Our main analysis is done with 3-month treasury bill yields for 114 countries from *Global Financial Data* (GFD) for the period 1950-2009 as it is the most comprehensive interest rate data available. As a robustness check, we also use eurocurrency interest rates from FT/ICAP through Datastream. FT/ICAP data consists of data for 27 mostly developed countries where for many of them data starts in 1975.

Here we document the availability of various interest rates data from Datastream and why we thought FT/ICAP was the best interbank interest rate data available. In Datastream, the two sources for eurocurrency interest rates are FT/ICAP and ICAP while the British Bankers Association (BBA) gives the London interbank rates (LIBOR).[4] Each have different set of countries and data years.

In 2006, Financial Times (FT) stopped providing euro currency rates to Datastream for the FT/ICAP series (for which Datastream mnemonic all starts with "EC") series. So Datastream continued these same series by splicing them with Intercapital (or ICAP, but formerly Garban Information Services (GS)) data so that starting in 2006, FT/ICAP and ICAP series are identical. For the countries for which GS is not available (e.g. Hong Kong, Singapore, South Africa), Tullett Prebon (TP) data was used instead. The TP data itself starts in 2006. The source column in Datastream's *Navigator* is not very informative. If the name of the series has FT/ICAP/TR, the source column will say "Thomson Reuters" but the source is really FT spliced with ICAP. If the name series has FT/TP, the source column appropriately says Tullett Prebon. The ones that have "dead" in the name have FT as the source and are dead series because neither ICAP nor TP has data on these countries. In Table 1, the countries of these dead series start with Greece and end with Thailand. For all the ICAP series (which start with "GS...."), the source column says "Thomson Reuters" but it is actually all ICAP.

A Datastream documentation file regarding this discontinuation of FT series gives alternative series that can be used instead. This list includes above mentioned ICAP, Tullett Prebon, BBA LIBORs, as well as "locally supplied" rates which are interbank rates from mainly national sources. These rates from national sources can be found under "National Interest Rates." The column "Alt IB" in Table 1 demonstrates the starting dates of the specific series that were listed in this document although there can be more than one interbank type rate under "National Interest Rates".

[4] There is also Tullett Prebon through Datastream that provides eurocurrency rates, but their data starts in 2006 only.

Datastream has various interest rate data under "National Sources." In the future, our treasury bill yields data from GFD could be potential supplemented with the combination of Eurocurrency rates (mainly, FT/ICAP) and interbank rates from national sources. A Datastream documentation about risk free rates discusses this:

> A risk free interest rate is the internal rate of return that can be obtained by investing in a financial instrument without (or very limited) credit risk. Normally this will relate to a short term investment in a financial instrument backed by the government. These money market securities bear no credit risk and have a limited re-investment risk, when the investment is rolled over for another short term period. In general we recommend the (annualised) yield on 3-month treasury bills, as the best instrument to use for any analysis involving risk free rates. However, for currencies where no liquid treasury bill market exists (or this market is subject to institutional distortions), interbank rates such as LIBOR or EURIBOR rates can be used. These do, however, bear a minimal credit risk inherent to the banks active in the market. Currencies with liquid repo-markets where the general collateral is a risk free long-term government bond, offer another alternative for an interest rate wich comes closest to 'risk-free', but are not available for as many currencies as interbank rates. A good example of this latter altenative is the Japanese 'Gensaki' market. The series we recommend for the main markets are detailed on the following table.

Column titled "Risk Free" in Table 1 demonstrates starting dates for the series specified in the document. Not shown in the table are Datastream's recommendations for risk free rates on Russia, China, Korea, Pakistan, Taiwan, Argentina, Brazil, Chile, Mexico and Venezuela also. Overall, this recommended list is not comprehensive (even few of the series above is suspended) and one would have to manually go through the various interest rates available for each country under "National Interest Rates" and pick the most relevant rate (most likely some interbank rate is the best) in order to supplement GFD data.

Lastly, for the seven (for BBA, it is six) euro legacy countries, since the introduction of the Euro in 1999, GS, EC, and BBA interest rates are identical across these countries. Obviously, we will not have the same problem with GFD treasury yields data.

Market	FT/ICAP	ICAP	BBA	Alt IB	"Risk Free"	GFD		
	Start	Start	Start	Start	Start	Start	End	nmiss
Australia	1997	1988	1986	1987	1977	1950	2009	3
Canada	1975	1995	1990	1992	1981	1950	2009	1
Denmark	1985	1995	2003	1989	1993	1976	2007	29
Hong Kong	1997			1986		1991	2009	1
Japan	1978	1995	1986	1996		1960	2009	1
New Zealand	1997	1988	2003	1987	1989	1978	2009	3
Norway	1997	1995		1986	1986	1984	2009	1
Singapore	1988			1987	1989	1987	2009	1
South Africa	1997			2000		1950	2009	1
Sweden	1997	1995	2006	1993	1994	1955	2009	1
Switzerland	1975	1995	1986	1974	1950	1980	2009	3
United Kingdom	1975	1995	1986	1975	1972	1950	2009	1
United States	1975	1995	1986	1971	1955	1950	2009	1
Other Western European	1999	1999	1998	1999	1999	1984	2009	2
Belgium	1978	1995				1950	2009	2
France	1975	1995	1989			1950	2009	61
Germany	1975	1995	1986			1953	2009	2
Italy	1978	1995	1990			1950	2009	2
Netherlands	1975	1995	1991			1950	2009	2
Portugal	1992	1995	1994			1985	2009	2
Spain	1992	1996	1990			1982	2009	2
Greece	1999					1980	2009	2
India	1999			1999	1994	1993	2009	1
Indonesia	1997			2001	1985	2000	2003	72
Malaysia	1997			1994	1998	1961	2009	1
Philippines	1999					1976	2009	3
Thailand	1997			2005	1995	1977	2009	88
Total	27	18	16	18	15		27	

Table 1: This table compares various interest rates data available from Datastream (the first five columns) with GFD data. “nmiss” stands for the number of missing months

Market	Start	End	nmiss	Market	Start	End	nmiss
ALBANIA	1994	2009	4	KOREA, REPUBLIC OF	1987	2009	2
ALGERIA	1998	2009	5	KUWAIT	1979	2005	59
ANGOLA	2000	2009	3	KYRGYZSTAN	1994	2009	3
ARGENTINA	2002	2009	16	LATVIA	1994	2008	16
ARMENIA	1995	2009	3	LEBANON	1977	2009	4
AUSTRIA	1960	1990	228	LITHUANIA	1994	2009	4
AZERBAIJAN	1997	2009	9	MACEDONIA	1997	2009	3
BAHAMAS	1971	2009	3	MADAGASCAR	2000	2009	3
BAHRAIN	1987	2009	3	MALTA	1987	2009	3
BANGLADESH	1984	2009	29	MAURITIUS	1996	2009	3
BARBADOS	1966	2009	5	MEXICO	1978	2009	6
BELIZE	1978	2009	4	MOLDOVA, REPUBLIC OF	1995	2009	3
BOLIVIA	1994	2009	6	MONGOLIA	2006	2008	20
BOTSWANA	1996	2009	3	MONTENEGRO	2004	2009	3
BRAZIL	1965	2009	62	MOROCCO	2008	2009	4
BULGARIA	1992	2008	23	MOZAMBIQUE	2000	2009	4
BURUNDI	2001	2009	29	NAMIBIA	1991	2009	8
CAPE VERDE	1998	2009	3	NEPAL	1981	2008	17
CENTRAL AFRICAN REPUBLIC	1996	2004	139	NICARAGUA	2003	2007	24
CHILE	1997	2009	2	NIGERIA	1970	2009	229
CHINA	2002	2009	1	PAKISTAN	1991	2009	1
COLOMBIA	1998	2009	2	POLAND	1991	2009	1
COSTA RICA	1996	2009	3	ROMANIA	1994	2005	51
CROATIA	2000	2009	3	RUSSIAN FEDERATION	1994	2009	18
CYPRUS	1975	2008	21	RWANDA	1999	2009	5
CZECH REPUBLIC	1993	2009	5	SAUDI ARABIA	1991	2008	12
EGYPT	1991	2009	1	SERBIA	1997	2009	34
EL SALVADOR	2001	2005	51	SIERRA LEONE	1965	2008	12
ETHIOPIA	1985	2008	12	SLOVAK REPUBLIC	1993	2007	24
FIJI	1975	2009	4	SLOVENIA	1998	2009	5
GEORGIA	2001	2005	54	SRI LANKA	1981	2009	1
GHANA	1978	2009	9	SWAZILAND	1981	2006	45
GUYANA	1972	2009	6	TAIWAN	1974	2009	3
HAITI	1996	2009	2	TANZANIA, UNITED REPUBLIC OF	1993	2009	5
HONDURAS	1998	2000	108	TRINIDAD AND TOBAGO	1964	2009	7
HUNGARY	1988	2009	3	TUNISIA	1990	2009	2
ICELAND	1987	2009	2	TURKEY	1985	2009	3
IRAQ	2004	2009	3	UGANDA	1980	2009	3
IRELAND	1969	2009	2	URUGUAY	1992	2009	49
ISRAEL	1992	2009	2	VENEZUELA	1996	2003	72
JAMAICA	1953	2009	3	VIET NAM	1997	2009	9
JORDAN	2000	2009	36	ZAMBIA	1978	2009	3
KAZAKHSTAN	1994	2009	2	ZIMBABWE	1962	2009	11
KENYA	1972	2009	8				

Table 2: The remaining 87 GFD interest rates (mainly treasury yields). “nmiss” stands for number of missing months.

B.2 Exchange Rate Data

Our main exchange rate data is from GFD for 172 countries for the period 1950-2009. This dataset is most comprehensive compared to three different data sources of forward and spot exchange rates available through Datastream: Barclays Bank PLC (BBI), Tenfore, and WM/Reuters (WMR). For each of these sources, full sample of countries and data years vary and are a lot more limited compared to GFD as shown in the table belows.

	WM/R	Tenfore	Barclays		WM/R	Tenfore	Barclays
Argentine Peso	2004			Latvian Lat	2004		
Australian Dollar	1996	1990	1984	Lithuanian Lita	2004		
Austrian Schilling	1996			Malaysian Ringgit	1996		
Belgian Franc	1996			Maltese Lira	2004		
Brazilian Real	2004			Mexican Peso	1996		
Bulgarian Lev	2004			Moroccan Dirham	2004		
Canadian Dollar	1996	1990	1984	New Zealand Dollar	1996	1990	1984
Chilean Peso	2004			Norwegian Krone	1996	1990	1984
Chinese Yuan Renminbi	2002			Omani Rial	2004		
Colombian Peso	2004			Pakistani Rupee	2004		
Croatian Kuna	2004			Peruvian Nuevo Sol	2004		
Cyprian Pound	2004			Philippine Peso	1996	2006	
Czech Koruna	1996	1996		Polish Zloty	2002	1996	
Danish Krone	1996	1990	1984	Portuguese Escudo	1996		
Dutch Guilder	1996			Qatari Riyal	2004		
Egyptian Pound	2004			Romanian Leu	2004	2008	
Estonian Kroon	2004			Russian Federation Rouble	2004		
Euro	1998	1990	1999	Saudi Arabian Riyal	1996	1990	
Finnish Markka	1996			Singaporean Dollar	1996	1990	1984
French Franc	1996			Slovak Koruna	2002		
German Mark	1996			Slovenian Tolar	2004		
Greek Drachma	1996			South African Rand	1996	1990	1983
Hong Kong Dollar	1996	1990	1983	Spanish Peseta	1996		
Hungarian Forint	1997			Swedish Krona	1996	1990	1984
Icelandic Krona	2004	2006		Swiss Franc	1996	1990	1983
Indian Rupee	1997			Taiwanese Dollar	1996		
Indonesian Rupiah	1996			Thai Baht	1996	1995	
Irish Punt or Pound	1996			Tunisian Dinar	2004		
Israeli Sheqel	2004	2006		Turkish Lira	1996	2006	
Italian Lira	1996			Ukrainian Hryvnia	2004	2008	
Japanese Yen	1996	1990	1983	United Arab Emirates Dirham	1996	1995	
Jordanian Dinar	2004			United Kingdom Pound	1996	1990	1983
Kazakh Tenge	2004			Venezuelan Bolivar	2004		
Kenyan Shilling	2004						
Korean Won	2002			Total	69	25	13
Kuwaiti Dinar	1996	1990					

Table 3: Exchange rate data sources from Datastream

B.3 GARCH Model

Interest rates, r_t, are AR(p) process:

$$r_t = c + \beta_1 r_{t-1} + ...\beta_p r_{t-p} + u_t$$

where u_t is $GARCH(r, m)$:

$$u_t \sim N(0, \sigma_t^2) \tag{15}$$

$$\sigma_t^2 = \kappa + \sum_{i=1}^{r} \delta_i \sigma_{t-i}^2 + \sum_{i=1}^{m} \alpha_i u_{t-i}^2 \tag{16}$$

For all countries, we first fit AR(5) to the interest rates, and fit the residuals to GARCH(1,1).[5]

[5] In the future, we will update our procedure to estimate the parameters of AR and GARCH processes at the same time.

	all	1975	1987	1995		all	1975	1987	1995
ALB	1	0	0	1	KOR	1	0	1	1
DZA	1	0	0	0	KWT	1	0	1	1
AGO	1	0	0	0	KGZ	1	0	0	1
ARG	1	0	0	0	LVA	1	0	0	1
ARM	1	0	0	0	LBN	1	0	1	1
AUS	1	1	1	1	LTU	1	0	0	1
AUT	1	1	1	0	MKD	1	0	0	0
AZE	1	0	0	0	MDG	1	0	0	0
BHS	1	1	1	1	MYS	1	1	1	1
BHR	1	0	0	1	MLT	1	0	0	1
BGD	1	0	1	1	MUS	1	0	0	0
BRB	1	1	1	1	MEX	1	0	1	1
BEL	1	1	1	1	MDA	1	0	0	0
BLZ	1	0	1	1	MNG	1	0	0	0
BOL	1	0	0	1	MNE	1	0	0	0
BWA	1	0	0	0	MAR	1	0	0	0
BRA	1	1	1	1	MOZ	1	0	0	0
BGR	1	0	0	1	NAM	1	0	0	1
BDI	1	0	0	0	NPL	1	0	1	1
CAN	1	1	1	1	NLD	1	1	1	1
CPV	1	0	0	0	NZL	1	0	1	1
CAF	1	0	0	0	NIC	1	0	0	0
CHL	1	0	0	0	NGA	1	0	0	1
CHN	1	0	0	0	NOR	1	0	1	1
COL	1	0	0	0	PAK	1	0	0	1
CRI	1	0	0	0	PHL	1	0	1	1
HRV	1	0	0	0	POL	1	0	0	1
CYP	1	1	1	1	PRT	1	0	1	1
CZE	1	0	0	1	ROU	1	0	0	1
DNK	1	0	1	1	RUS	1	0	0	1
EGY	1	0	0	1	RWA	1	0	0	0
SLV	1	0	0	0	SAU	1	0	0	1
ETH	1	0	1	1	SRB	1	0	0	0
EUR	1	0	1	1	SLE	1	1	1	1
FJI	1	1	1	1	SGP	1	0	0	1
FRA	1	1	1	1	SVK	1	0	0	1
GEO	1	0	0	0	SVN	1	0	0	0
DEU	1	1	1	1	ZAF	1	1	1	1
GHA	1	0	1	1	ESP	1	0	1	1
GRC	1	0	1	1	LKA	1	0	1	1
GUY	1	1	1	1	SWZ	1	0	1	1
HTI	1	0	0	0	SWE	1	1	1	1
HND	1	0	0	0	CHE	1	0	1	1
HKG	1	0	0	1	TWN	1	1	1	1
HUN	1	0	0	1	TZA	1	0	0	1
ISL	1	0	0	1	THA	1	0	1	0
IND	1	0	0	1	TTO	1	1	1	1
IDN	1	0	0	0	TUN	1	0	0	1
IRQ	1	0	0	0	TUR	1	0	1	1
IRL	1	1	1	1	UGA	1	0	1	1
ISR	1	0	0	1	GBR	1	1	1	1
ITA	1	1	1	1	USA	1	1	1	1
JAM	1	1	1	1	URY	1	0	0	1
JPN	1	1	1	1	VEN	1	0	0	0
JOR	1	0	0	0	VNM	1	0	0	0
KAZ	1	0	0	1	ZMB	1	0	1	1
KEN	1	1	1	1	ZWE	1	1	1	1

Table 4: The countries that get included in the various data period subsamples. One means it is included, zero otherwise. For example, countries with ones in the column for 1975 means these were the countries that had non-missing interest rate data in Jan 1975

	rate				vol				
	min	max	mean	std	min	max	mean	std	nobs
ALB	5.05	38.23	12.32	8.32	0.19	4.34	0.70	0.68	183
DZA	0.09	10.13	3.72	3.75	0.42	1.70	0.57	0.26	135
AGO	2.80	134.00	55.13	47.86	1.04	25.02	6.10	5.76	110
ARG	1.10	59.77	10.02	8.81	-	-	-	-	22
ARM	3.24	80.42	22.85	20.02	0.42	17.20	3.57	3.85	170
AUS	0.75	19.40	6.30	4.19	0.08	2.72	0.34	0.41	717
AUT	3.61	10.38	6.63	1.66	0.10	0.81	0.14	0.06	373
AZE	3.92	22.10	11.76	4.37	0.97	3.03	1.54	0.46	144
BHS	0.06	9.90	4.44	2.56	0.28	1.73	0.55	0.26	463
BHR	0.69	9.98	4.73	2.18	0.14	0.52	0.24	0.08	269
BGD	1.86	11.50	8.03	1.80	0.10	1.07	0.19	0.17	205
BRB	0.24	16.02	5.88	2.42	0.17	1.75	0.37	0.25	513
BEL	0.34	14.03	6.18	2.83	0.12	1.67	0.25	0.16	718
BLZ	3.22	14.46	6.38	3.06	0.03	2.27	0.18	0.32	370
BOL	0.75	26.60	11.69	5.69	0.85	3.87	1.18	0.45	187
BWA	8.16	14.31	12.39	0.98	0.19	0.79	0.28	0.11	165
BRA	8.65	933.60	68.71	115.38	1.42	185.95	10.15	21.50	295
BGR	2.12	1232.75	48.10	131.98	6.08	1006.04	24.15	82.10	194
BDI	6.41	19.84	9.85	3.05	0.11	1.17	0.47	0.30	66
CAN	0.20	20.90	5.69	3.73	0.11	1.77	0.38	0.28	719
CPV	2.00	11.08	5.69	2.29	0.14	1.51	0.43	0.32	142
CAF	2.08	3.73	2.63	0.57	-	-	-	-	12
CHL	0.46	19.17	6.73	4.49	0.24	4.55	0.86	0.77	149
CHN	1.21	4.50	2.63	0.91	0.31	1.28	0.39	0.16	96
COL	4.35	52.64	13.75	9.33	0.26	6.38	1.33	1.55	108
CRI	3.33	24.50	15.44	4.90	1.10	5.50	1.37	0.56	163
HRV	1.90	7.60	4.27	1.62	0.31	1.03	0.50	0.17	107
CYP	2.46	6.23	5.30	0.84	0.02	1.04	0.07	0.11	400
CZE	1.66	15.54	5.75	3.71	0.16	1.52	0.31	0.22	193
DNK	2.00	20.70	9.94	5.90	0.40	4.62	0.60	0.48	380
EGY	5.26	19.40	10.53	3.26	0.03	3.79	0.44	0.53	228
SLV	2.82	6.99	3.77	0.95	-	-	-	-	56
ETH	0.04	12.00	3.83	3.71	0.02	6.74	0.35	0.71	284
EUR	0.34	11.75	5.81	3.03	0.07	1.10	0.27	0.19	311
FJI	0.07	18.65	4.06	2.74	0.05	5.47	0.67	0.81	417
FRA	0.37	18.92	6.31	3.71	0.08	1.77	0.36	0.26	600
GEO	9.95	58.44	31.79	14.05	-	-	-	-	55
DEU	0.34	12.05	4.39	2.02	0.15	1.55	0.32	0.17	683
GHA	9.38	46.75	22.22	10.27	0.53	7.69	1.05	0.93	376
GRC	0.72	25.50	11.44	6.13	0.18	6.55	0.44	0.60	359
GUY	2.84	33.75	10.67	6.91	0.04	12.99	0.44	0.95	420
HTI	4.00	27.83	16.42	7.36	1.53	6.26	2.07	0.90	157
HND	13.97	18.00	14.71	1.31	-	-	-	-	26
HKG	-0.08	12.24	3.53	2.27	0.27	3.14	0.56	0.45	223
HUN	5.55	35.30	16.81	8.95	0.52	2.90	1.01	0.47	251
ISL	4.44	34.30	11.46	6.53	0.27	5.01	1.01	0.93	270
IND	3.39	14.00	8.19	2.53	0.57	1.71	0.81	0.25	204
IDN	3.50	14.50	8.97	2.42	-	-	-	-	48
IRQ	1.20	22.00	11.63	6.32	2.16	3.91	2.56	0.46	70
IRL	0.31	39.94	8.07	4.60	0.32	15.05	0.88	1.27	480
ISR	0.29	17.96	8.73	4.54	0.23	1.58	0.58	0.28	215
ITA	0.31	22.08	7.48	5.19	0.05	2.44	0.43	0.42	718
JAM	1.75	51.98	12.81	10.16	0.23	6.48	0.97	1.11	682
JPN	0.00	8.27	3.78	2.50	0.03	0.64	0.14	0.13	600
JOR	2.05	6.88	4.93	1.71	0.23	0.80	0.35	0.12	71
KAZ	2.09	318.78	28.70	60.71	0.17	29.70	2.48	4.87	188
KEN	0.11	70.64	11.98	9.39	0.43	8.72	1.39	1.36	405

Table 5: Descriptive statistics of GFD interest rates and the computed volatility. The volatility is based on the entire interest rate data available for that country. The last column (nobs) shows the number of consecutive months with non-missing data.

	rate				volatility				
	min	max	mean	std	min	max	mean	std	nobs
KOR	2.48	19.20	9.78	4.78	0.21	1.56	0.61	0.33	275
KWT	0.60	8.87	6.12	1.78	0.03	1.19	0.18	0.19	311
KGZ	3.47	216.50	28.94	36.62	1.79	33.82	6.19	5.79	190
LVA	2.30	33.98	8.11	8.06	0.50	4.67	0.94	0.74	173
LBN	2.54	34.18	12.34	5.72	0.12	7.86	0.66	0.95	382
LTU	1.96	37.00	8.81	8.46	0.21	1.09	0.42	0.25	115
MKD	4.66	18.00	8.63	2.72	0.33	2.47	0.67	0.43	143
MDG	3.92	24.04	12.49	5.01	0.53	3.64	0.85	0.57	111
MYS	1.82	9.98	4.39	1.35	0.04	1.48	0.18	0.18	588
MLT	1.46	5.49	4.26	0.81	0.04	0.88	0.10	0.11	264
MUS	3.68	12.91	8.50	2.32	0.26	1.71	0.51	0.24	155
MEX	4.56	153.91	29.03	25.98	0.46	13.42	2.39	2.35	251
MDA	1.21	74.30	19.73	13.39	1.99	6.98	2.94	1.25	176
MNG	5.56	7.91	6.93	0.77	-	-	-	-	27
MNE	0.45	10.80	2.65	3.59	0.17	2.65	0.65	0.58	64
MAR	3.24	3.70	3.46	0.16	-	-	-	-	21
MOZ	7.15	31.65	16.70	6.53	1.53	8.99	1.93	0.93	117
NAM	6.66	21.68	11.60	3.22	0.35	1.79	0.54	0.23	217
NPL	0.62	12.88	5.48	2.62	0.03	1.76	0.50	0.40	332
NLD	0.23	13.80	4.36	2.63	0.10	1.61	0.34	0.25	718
NZL	2.62	27.20	9.51	4.74	0.11	3.92	0.63	0.67	380
NIC	1.02	6.50	3.91	1.83	-	-	-	-	55
NGA	2.00	27.50	12.95	5.42	0.00	8.69	0.79	0.94	220
NOR	0.10	15.75	7.09	3.85	0.30	9.93	0.70	0.99	312
PAK	1.21	17.42	10.05	3.80	0.21	1.79	0.59	0.30	226
PHL	2.92	43.39	12.50	6.57	0.26	5.91	1.05	0.96	399
POL	3.90	49.02	16.57	12.10	0.16	7.64	1.06	1.31	224
PRT	0.31	22.19	7.54	5.07	0.15	1.16	0.38	0.22	292
ROU	7.82	179.94	48.65	32.35	5.11	39.99	9.25	6.77	140
RUS	0.26	355.80	33.46	59.80	1.53	3.00	1.70	0.21	114
RWA	5.24	12.85	9.57	1.80	0.24	2.21	0.57	0.35	128
SAU	1.12	7.16	4.41	1.65	0.16	0.75	0.29	0.12	207
SRB	4.20	99.25	17.52	12.85	0.78	5.92	1.57	0.97	108
SLE	3.80	95.20	16.56	14.98	0.34	23.59	1.46	2.26	519
SGP	0.20	4.90	1.90	1.17	0.16	1.30	0.46	0.22	265
SVK	1.95	26.00	9.36	5.90	0.15	5.81	1.17	1.12	180
SVN	0.55	12.70	6.37	3.30	0.08	1.47	0.47	0.27	96
ZAF	1.00	22.15	8.06	5.11	0.08	1.91	0.34	0.31	719
ESP	0.31	15.27	7.39	4.40	0.15	1.45	0.36	0.24	329
LKA	6.54	21.30	13.08	3.20	0.40	3.07	0.94	0.49	345
SWZ	4.93	19.50	10.83	3.27	0.53	1.90	0.77	0.26	293
SWE	0.13	18.00	6.46	3.78	0.25	3.70	0.53	0.39	660
CHE	0.00	9.30	3.30	2.43	0.09	1.05	0.36	0.22	358
TWN	0.17	14.99	5.46	3.18	0.08	2.52	0.27	0.32	428
TZA	2.60	62.30	13.25	11.01	0.69	11.80	2.36	1.90	189
THA	1.02	19.32	6.19	4.12	0.10	3.28	0.40	0.59	156
TTO	2.30	12.11	5.92	2.52	0.04	1.44	0.18	0.22	535
TUN	4.02	11.62	7.38	2.17	0.45	0.93	0.46	0.05	239
TUR	7.92	159.44	56.71	32.63	1.07	25.30	7.21	5.85	170
UGA	2.97	43.50	16.74	11.90	0.86	10.88	1.73	1.34	358
GBR	0.42	16.27	6.77	3.58	0.18	1.29	0.48	0.28	719
USA	0.01	15.52	4.77	2.87	0.13	1.97	0.37	0.29	719
URY	2.26	146.47	26.98	25.14	1.12	9.09	2.96	2.20	87
VEN	8.89	57.05	21.98	10.40	2.58	13.77	3.60	1.70	86
VNM	3.34	15.60	7.06	2.67	0.21	4.49	0.54	0.62	147
ZMB	4.38	181.78	26.51	26.56	1.14	36.66	3.10	4.38	382
ZWE	3.05	525.00	34.97	78.92	1.60	259.00	5.55	18.79	566

Table 6: Descriptive statistics of GFD interest rates and the computed volatility. The volatility is based on the entire interest rate data available for that country. The last column (nobs) shows the number of consecutive months with non-missing data.

	1	2	3	4	5	Total
Spot Change: Δs	-1.028	-0.184	-1.845	-3.797	-2.935	-1.955
	(13.03)	(13.83)	(14.32)	(13.85)	(46.44)	(24.15)
$i^* - i$	-1.594	0.0685	1.724	3.823	9.282	2.660
	(2.004)	(1.882)	(2.231)	(3.326)	(8.661)	(5.820)
Excess Return: rx	-2.713	-0.143	-0.207	-0.0346	6.667	0.716
	(13.70)	(14.03)	(14.33)	(13.68)	(46.56)	(24.48)
Volatility: $\sigma(i^*)$	0.301	0.308	0.426	0.548	1.079	0.533
	(0.134)	(0.146)	(0.212)	(0.337)	(0.979)	(0.560)

Table 7: The whole sample of countries. This table shows for each portfolio 1 through 5 the average 3-month change in log spot exchange rates (i.e. the appreciation of the foreign currency) Δs, the average interest rate relative to USD $i^* - i$, the average log 3 month excess return rx, and the average volatility of interest rates $\sigma(i^*)$. All moments are annualized and in percentage points. In brackets are standard deviations. The average excess return and volatility of interest rates are plotted in the left panels of figure 4.

	1	2	3	4	5	Total
Spot Change: Δs	-0.805	0.627	-0.776	-5.063	-9.395	-3.082
	(11.85)	(14.42)	(19.64)	(19.66)	(21.48)	(18.15)
$i^* - i$	-2.292	-0.570	1.082	3.438	11.60	2.653
	(2.416)	(2.198)	(2.024)	(2.446)	(7.739)	(6.304)
Excess Return: rx	-3.241	0.0177	0.244	-1.677	2.506	-0.430
	(12.47)	(14.80)	(19.86)	(19.62)	(21.24)	(18.00)
Volatility: $\sigma(i^*)$	0.303	0.305	0.415	0.511	1.372	0.581
	(0.158)	(0.142)	(0.260)	(0.383)	(0.927)	(0.621)

Table 8: The sample of countries that had interest rate data on January 1975. This table shows for each portfolio 1 through 5 the average 3-month change in log spot exchange rates (i.e. the appreciation of the foreign currency) Δs, the average interest rate relative to USD $i^* - i$, the average log 3 month excess return rx, and the average volatility of interest rates $\sigma(i^*)$. All moments are annualized and in percentage points. In brackets are standard deviations. The average excess return and volatility of interest rates are plotted in the left panels of figure 4.

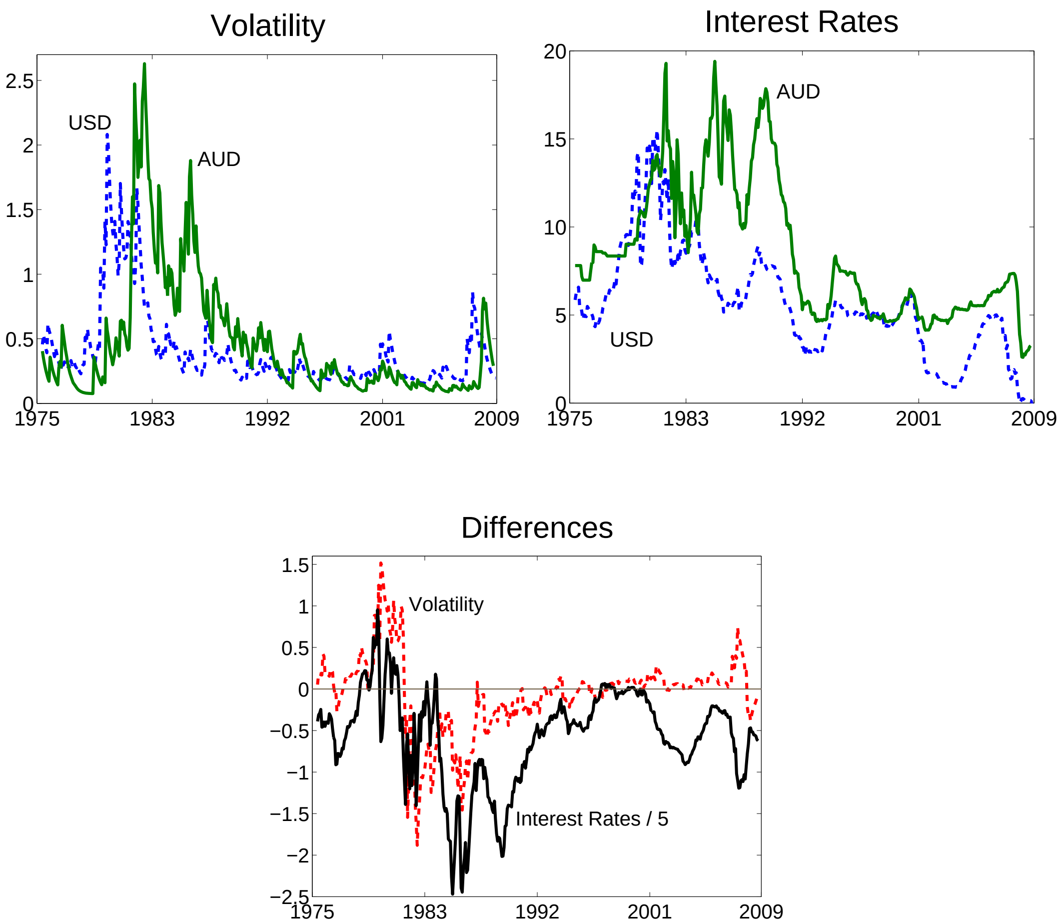
	1	2	3	4	5	Total
Spot Change: Δs	-0.443	1.060	0.0293	-4.578	-6.897	-2.166
	(11.75)	(17.22)	(16.03)	(13.65)	(54.49)	(27.87)
$i^* - i$	-0.964	0.725	2.394	5.551	17.18	4.977
	(1.609)	(1.764)	(2.165)	(2.630)	(7.755)	(7.571)
Excess Return: rx	-1.555	1.753	2.385	0.926	10.66	2.834
	(12.11)	(17.26)	(15.92)	(13.35)	(53.12)	(27.47)
Volatility: $\sigma(i^*)$	0.321	0.308	0.401	0.684	1.912	0.725
	(0.139)	(0.131)	(0.225)	(0.397)	(1.050)	(0.800)

Table 9: The sample of countries that had interest rate data on January 1987. This table shows for each portfolio 1 through 5 the average 3-month change in log spot exchange rates (i.e. the appreciation of the foreign currency) Δs, the average interest rate relative to USD $i^* - i$, the average log 3 month excess return rx, and the average volatility of interest rates $\sigma(i^*)$. All moments are annualized and in percentage points. In brackets are standard deviations. The average excess return and volatility of interest rates are plotted in the left panels of figure 4.

	1	2	3	4	5	Total
Spot Change: Δs	-0.861	1.839	-0.740	-4.033	-0.234	-0.806
	(11.84)	(14.77)	(13.07)	(13.27)	(41.09)	(21.91)
$i^* - i$	-1.144	0.408	2.171	5.311	13.56	4.062
	(1.172)	(1.040)	(1.229)	(1.928)	(4.855)	(5.781)
Excess Return: rx	-2.152	2.220	1.373	1.218	13.80	3.292
	(12.14)	(14.80)	(12.92)	(12.82)	(40.06)	(22.10)
Volatility: $\sigma(i^*)$	0.278	0.288	0.433	0.645	1.371	0.603
	(0.0753)	(0.0918)	(0.119)	(0.187)	(0.419)	(0.461)

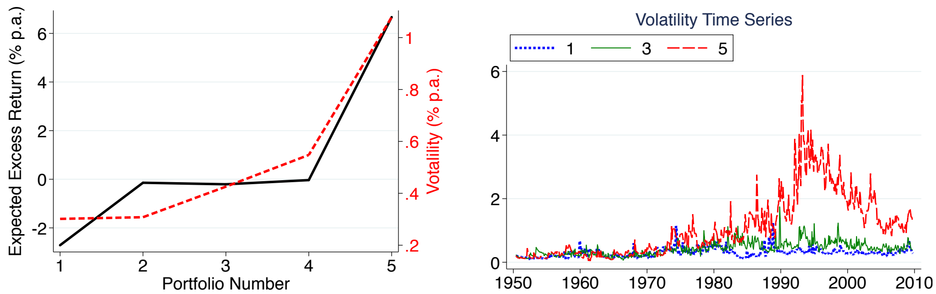
Table 10: The sample countries that had interest rate data on January 1995. This table shows for each portfolio 1 through 5 the average 3-month change in log spot exchange rates (i.e. the appreciation of the foreign currency) Δs, the average interest rate relative to USD $i^* - i$, the average log 3 month excess return rx, and the average volatility of interest rates $\sigma(i^*)$. All moments are annualized and in percentage points. In brackets are standard deviations. The average excess return and volatility of interest rates are plotted in the left panels of figure 4.

Figure 3
U.S. and Australian Interest Rates: Volatility and Spread

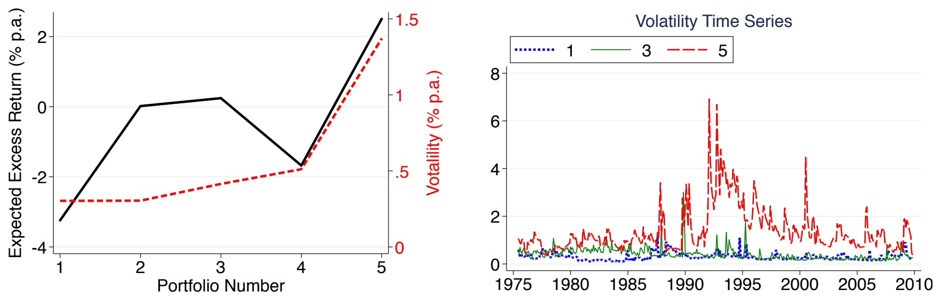


GARCH estimates of U.S. and Australian interest rate volatility appear in the upper left panel. Interest rates appear in the upper-right panel. U.S. data are the blue dashed-lines and Australian data are the green solid lines. The lower panel plots the differences, U.S. minus Australia from the two top panels. The red dashed line is the volatility difference and the the black solid line is the interest rate differential, divided by 5. Lognormal models predict that, in the lower panel, the lines appear on opposite sides of zero, and are negatively correlated. By-and-large, the opposite seems to be true. The correlation is 0.52. Data source: 3-month interbank deposits from Global Financial Data (www.globalfinancialdata.com).

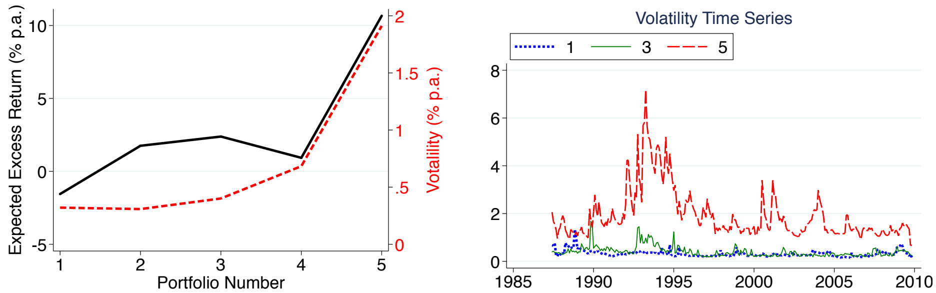
Figure 4
Average Volatility of Currency Portfolios



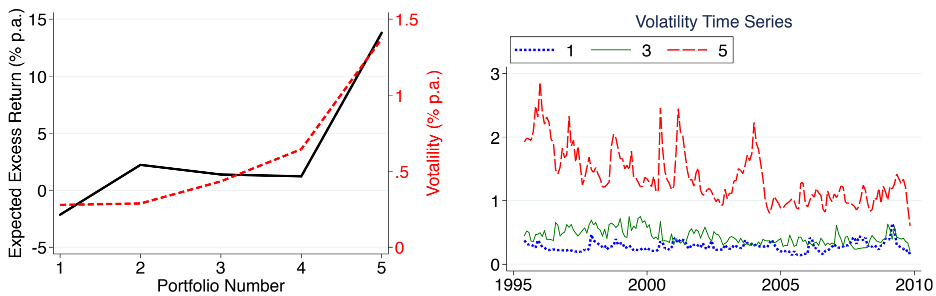
GFD interest rates, the whole sample.



GFD interest rates, only the countries that had non missing data on January 1975. See tables 4, 5, and 6 to see which countries are in this sample.



GFD interest rates, only the countries that had non missing data on January 1987 Jan. See tables 4, 5, and 6 to see which countries are in this sample.

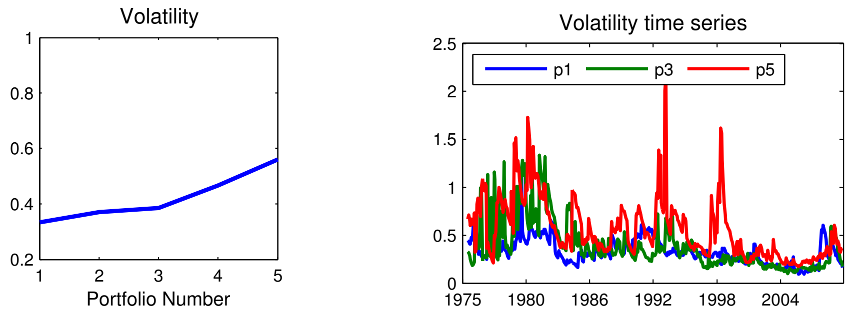


GFD interest rates, only the countries that had non missing data on January 1995 Jan. See tables 4, 5, and 6 to see which countries are in this sample.

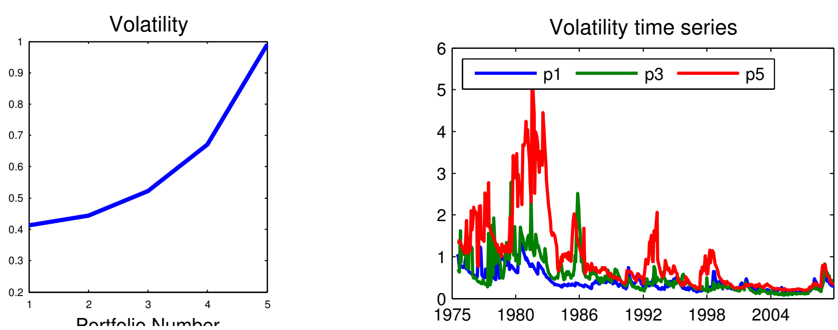
Figure 5
Bilateral Currency Pairs

GFD interest rates: currency pairs

Figure 6
Data Source Comparison: GFD Versus FT/ICAP

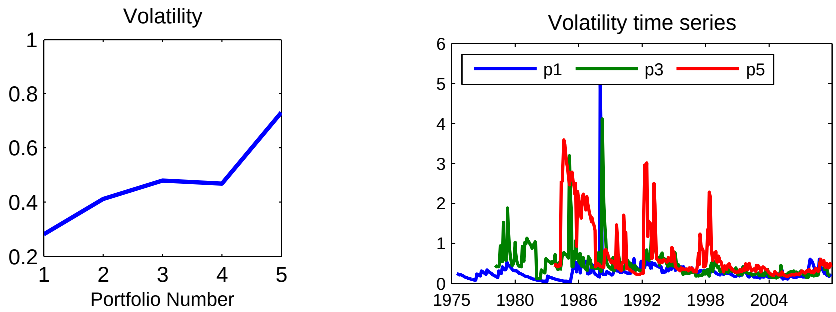


GFD interest rates, the same set of countries (and years) that is in the FT/ICAP interest rate data source in below figure.

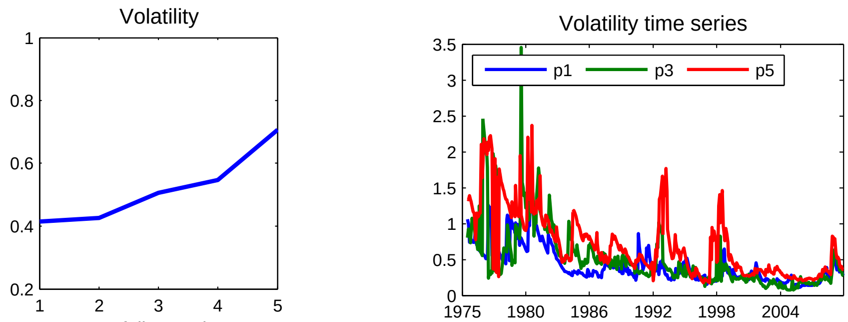


FT/ICAP interest rates, the whole sample.

The difference in volatilities between GFD and FT/ICAP interest rates (especially in early 1980's) is due to how different the GFD and FT/ICAP interest rates look even for the same country and the same date. Good examples were BEL, FRA, and ITA. So in figures 9 and 10, we replot above two figures without BEL, FRA, and ITA, after which the average volatility figure looks similar (in scale) between GFD and FT/ICAP.

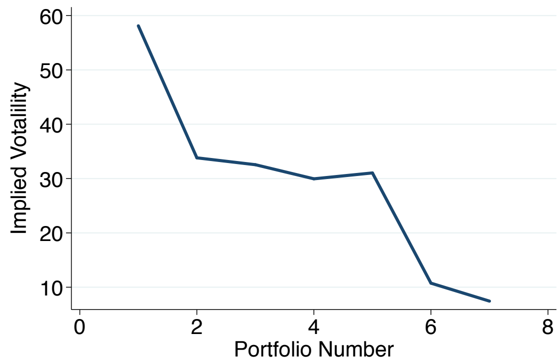


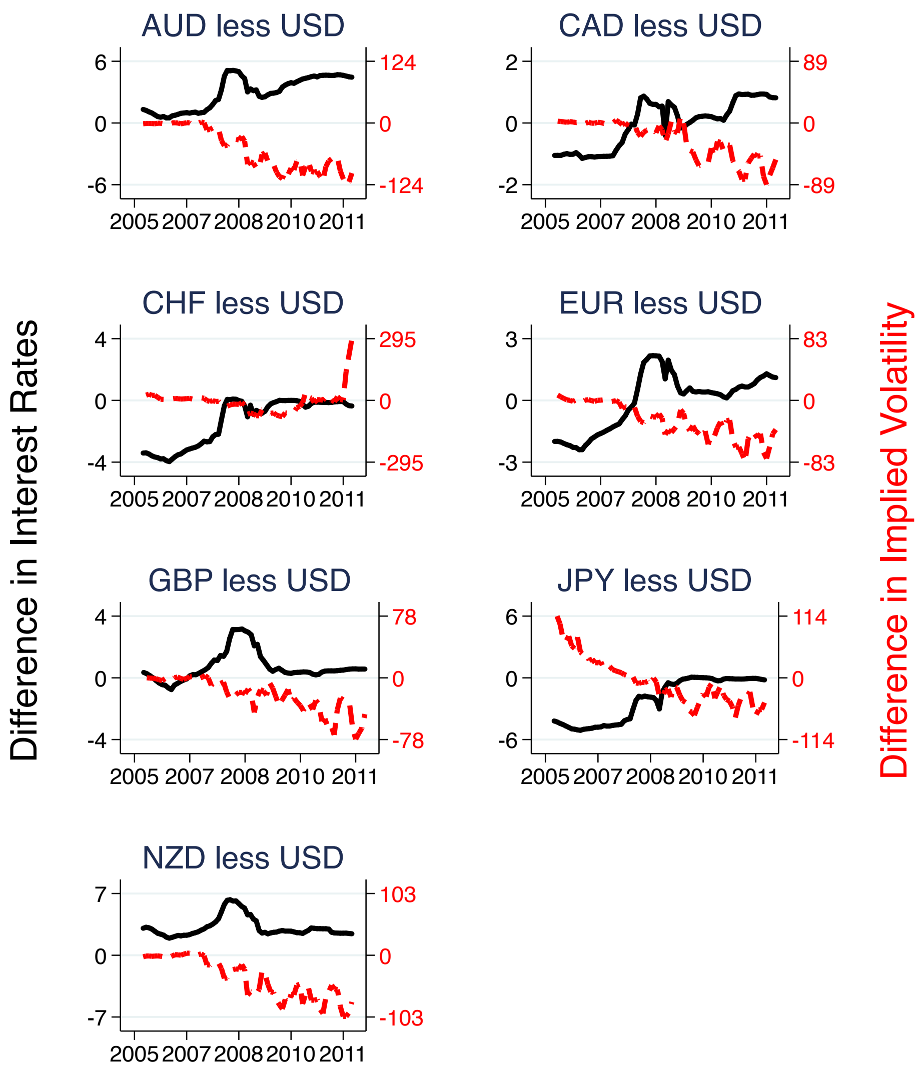
GFD interest rates, same as in figure 7 but without BEL, FRA, ITA



FT/ICAP interest rates, same as in figure 8 but without BEL, FRA, ITA

Figure 7
Implied Volatility of Options on Interest Rate Futures





AUD less USD
CAD less USD
CHF less USD
EUR less USD
GBP less USD
JPY less USD
NZD less USD
Difference in Interest Rates
Difference in Implied Volatility
2005 2007 2008 2010 2011