Choose and answer two questions from each of the sections and completely solve the two extra problems to increase your grade on MT2 to a 2.7. So you should turn in **nine** total answers.

Section 4.6 29, 40, 48

Section 6.7 (other book) 38, 41, 50

Section 6.5 26, 39, 58

1. Let $S_n = 2 + \sum_{i=1}^{n} X_i$ with $X_i \in \{-1, 1\}$ and $P(X_i = -1) = \frac{1}{2} = P(X_i = 1)$.
   (a) What is ES_n?
   (b) Let X be a random variable and A be an event. The *conditional expectation* formula is
   $$EX = E[X \mid A]P(A) + E[X \mid A^c]P(A^c). \tag{1}$$
   Let N be the smallest n such that $S_n \in \{1, k\}$. So N is the random time when it first occurs that $S_n = 1$ or $S_n = k$. Let $p = P(S_N = 1)$. Use (1) to write a formula for ES_N in terms of p.
   (c) Using the fact (called the optional stopping theorem) that $ES_N = ES_n$. Solve for p.

2. (a) The length of human pregnancies is approximately normal with mean 266 days and standard deviation 16 days. What is the probability that a pregnancy lasts less than ('<') 240 days (about 8 months)? Assume days are discrete (i.e. a 244.2345 day pregnancy is not possible). Write your answer in terms of $\Phi(x) = P(Y \leq x)$ with $Y = N(0, 1)$. You do not need to look anything up on the table, or do any arithmetic.
   (b) Write an algebraic expression that you could solve for the value c for which there is a 99% probability that a pregnancy lasts longer than c days. You do not need to solve it.

3. A marathon runner has observed that her mile splits for the first 16 miles of a marathon are i.i.d. with mean 7 minutes and standard deviation .5 minutes. Use normal approximation to estimate the probability she runs the first 16 miles in less than ('<') 108 minutes. Actually lookup Φ in the table. Note that $7 * 16 = 112$ and $.5^2 = .25$.