1. (a) $\frac{d}{dx}\left[e^{\sin(\sqrt{e^x})}\right]$

$e^{\sin\sqrt{e^x}}\cos(\sqrt{e^x})\frac{1}{2\sqrt{e^x}}e^x.$

(b) $\frac{d}{dx}\left[\sin(\cos(\tan(x^{3/2}+x)))\right]$

$\cos(\cos(\tan(x^{3/2}+x)))(-\sin(\tan(x^{3/2}+x))\sec^2(x^{3/2}+x)(\frac{3}{2}x^{1/2}+1)$

(c) $\frac{d}{dx}\left[xe^{\left(xe^{(xe^x)}\right)}\right]$

$e^{\left(xe^{(xe^x)}\right)}+x\left[\left(e^{(xe^x)}+x[e^x+xe^x]\right]\right)$

2. $x(t)=\cos(2t)$ and $y(t)=\sin(3t)$. Find the equations of the tangent lines at the $y$ intercepts.

We know the derivative is
$$\frac{dy}{dx}(t)=\frac{y'(t)}{x'(t)}=-\frac{3\cos(3t)}{2\sin(2t)}$$
The $y$-intercepts occur when $x(t)=0=\cos(2t)$. This is is when $t_1=\frac{\pi}{4}$ and $t_2=\frac{3\pi}{4}$. The lines are
$$y-y(t_1)=\frac{dy}{dx}(t_1)(x-0)\iff y=-\frac{3\cos(3\cdot\frac{\pi}{4})}{2\sin(2\cdot\frac{\pi}{4})}x+\sin(3\cdot\frac{\pi}{4})$$
$$y-y(t_2)=\frac{dy}{dx}(t_2)(x-0)\iff y=-\frac{3\cos(3\cdot\frac{3\pi}{4})}{2\sin(2\cdot\frac{3\pi}{4})}x+\sin(3\cdot\frac{3\pi}{4})$$

3. Let $f$ and $g$ be positive differentiable functions and $a$ and $b$ positive real numbers.

(a) Write a formula in terms of $f$ and $g$ for $\frac{d}{dx}\sqrt{f(g(x))}$.

$$\frac{f'(g(x))g'(x)}{2\sqrt{f(g(x))}}.$$

(b) Suppose that $f(a)=b$ and $\frac{d}{dx}\sqrt{f(ax)}=\sqrt{b}$. Write a formula for $f'(a)$.

$$\sqrt{b} = \frac{d}{dx}\sqrt{f(ax)} = \frac{af'(ax)}{2\sqrt{f(ax)}}.$$

When $x = 1$ we have

$$\sqrt{b} = \frac{af'(a)}{2\sqrt{f(a)}} = \frac{af'(a)}{2\sqrt{b}}.$$

Solving for $f'(a)$ gives

$$f'(a) = \frac{2b}{a}.$$

*Hint: Use part (a) and let $x = 1$.*

(c) When $x = a$ write the equation (involving $a$ and $b$) for the tangent line to $f$.

$$y - b = \frac{2b}{a}(x - a).$$

(d) Write $f(x)$ as a formula involving $a$ and $b$. (*Difficult.*)

$f(x) = \frac{b^2}{a^2}x^2$

4. A curve $\Gamma$ is defined by $x^2y + y^2x + 2x + y = \pi$. Find $\frac{dy}{dx}$.

5. Let $C$ be the curve defined by the relation

$$\sin(x^2 + y^2) = e^{2y}$$

(a) Find $\frac{dy}{dx}$ for $C$.

(b) Find the equation of the tangent line to $C$ at the point $(x, y) = (\sqrt{\pi}, 0)$.

(c) Use the tangent line at $(\sqrt{\pi}, 0)$ to estimate the number $a$ so that $(a, \frac{\pi}{10})$ lies on $C$.

6. A marble rolls on a flat table along the curve

$$y \ln(xy) = e^{2x-1}$$

At the point $(1, e)$ the $y$-coordinate of the marble is changing at $\pi$ cm/sec. At what rate is the $x$-coordinate changing? *Hint: differentiate with respect to $t$.*