

Department of Computer Science, Virtual University of Pakistan

CS702: Advanced Algorithms Analysis and Design

Assignment 1 Fall 2009 Maximum Points: 100

Due Date: Monday, 16th November 2009 Time: 6:00 PM

Instructions

The purpose of this assignment is to give you hands on practice. It is expected that students will solve the assignments themselves. Following rules will apply during the evaluation of assignment.

- ⌚ Cheating from any source will result in zero marks in the assignment.
- ⌚ Any student found cheating in any of the two assignments submitted

will be awarded "F" grade in the course.

- ⌚ No assignment after due date will be accepted
- ⌚ Full credit will be given only to correct solutions, attempt will also be appreciated.

Problem # 1 (((5+5+5) + (5+5) = 25 Points))

a)
Let N be a set of natural numbers. The symbols, < (less than), ≤ (less than or equal) and = (equal) are relations over N. Prove or disprove the following.

1. < is reflexive, symmetric and transitive
2. ≤ is reflexive, symmetric and tran

3. = is reflexive, symmetric and transitive

b)
Prove that $\neg C \Rightarrow \neg (B \wedge (B \Rightarrow C))$ by

1. Logical equivalences
2. Truth table

Problem # 2 (((5+5+5) + (5+5) = 25 Points))a)

1. Use Mathematical Induction to prove that sum of the first m odd positive integers is m^2 .
2. Use Mathematical Induction to prove that sum of first m even positive integers is $m(m+1)$.
3. Deduce the result of part 2 from part 1 and vice versa.

b)
Prove $\neg C \Rightarrow \neg (B \wedge (B \Rightarrow C))$ by

1. Rules of inference
2. Contradiction

Problem # 3 (((5+5+5) + 10 = 25 Points))a)

Start with some special kind of pair of rabbits, one male and one female, born on January 1. Assume all months are of equal length and that rabbits begin to produce four months after their own birth. After reaching age of four months, each pair produces two mixed pairs, two males and two females, and then other two mixed pairs each month, and no rabbit dies.

1. Develop a model using tree diagram
2. Describe a recursive mathematical model
3. Compute the number of pairs of rabbits after one year?

b) Suppose sequence, b_0, b_1, b_2, \dots , satisfies recurrence relation $b_n = 6b_{n-1} + 2b_{n-2}$: condition initial with $b_0 = 2, b_1 = 9$

Then find explicit formula for b_0, b_1, b_2, \dots , using characteristic equation of the above recursion.

Problem # 4 ((5+5) +5+10 = 25 Points))

a) Solve the recurrence relation given below using substitution method. otherwise)

$$T_n = 4T_{n-1} + 5T_{n-2} \quad (4 \text{ if } 1) \quad (1 \text{ if } 2)$$

B- Consider the recurrence given below. Find its general solution.

$$t_n - 4t_{n-1} = (n + 3)5^n$$

b) Show by mathematical induction that any amount in cents ≥ 20 cents can be obtained using 5 cents and 6 cents coins only.

c) Consider the recurrence

$$t_n = n \text{ if } n = 0, 1, 2$$

$$t_n = 6.t_{n-1} - 11.t_{n-2} + 6.t_{n-3} \text{ otherwise}$$

Find the general solution of the recurrence above.