INSTRUCTIONS:

- Write the following sentence on the top of your exam. *On my honor, I have neither received nor given any unauthorized assistance on this examination.*

- You are encouraged to consult the textbook and your class notes. You are also encouraged to reach out to Prof. Allender (via e-mail). That is likely to be much more productive than trying to search for help on-line.

- Submit your completed exam by 1:40 PM on Thursday, November 12 (in the same way that you submit your homework assignments).

- Show your work and provide some explanations; you get little or no credit for an unexplained answer.

- If a question asks for a numerical answer, it is fine to express the answer using well-known functions (such as factorials or binomial coefficients or multinomial coefficients); no extra credit will be given for expressing your answer as a fraction or as a percentage, etc.

- The value of each question appears in parentheses. There are 80 points in total.

1. (1 point) Write "On my honor, I have neither received nor given any unauthorized assistance on this examination."

2. (13 points) The New Jersey Widget factory boasts that 99% of its widgets are free of defects. You take a random sample of 1000 widgets and find that 30 are defective. What is the probability that the New Jersey Widget factory's claim is correct?

3. (5 points) Consider the experiment of picking a binary string of length 10 at random, and let X be the random variable that gives the number of 0's in the string plus twice the number of ones. What is the expected value of X?

4. (5 points) What is the variance of X in the preceding problem?

5. (5 points) Use Markov's Inequality to give an upper bound on $\Pr[X \geq 18]$.

6. (5 points) Use Tchebycheff's Inequality to give an upper bound on $\Pr[X \geq 18]$.

7. (5 points) Use the Chernoff Bound to give an upper bound on $\Pr[X \geq 18]$. (Note that the Chernoff bound can only be applied to a sum of random variables with values between 0 and 1. Can you express $X - 10$ in this way? This insight may be useful on some of the preceding problems as well.)

8. Now consider the experiment of picking a binary string of length 100 at random, and let Y be the random variable that gives the number of 0's in the string plus twice the number of ones. What is the expected value of Y?

9. (5 points) What is the variance of Y in the preceding problem?

10. (5 points) Use Markov's Inequality to give an upper bound on $\Pr[Y \geq 180]$.

11. (5 points) Use Tchebycheff's Inequality to give an upper bound on $\Pr[Y \geq 180]$.

12. (5 points) Use the Chernoff Bound to give an upper bound on $\Pr[Y \geq 180]$.

13. (2 points) Provide answers to the following two True/False questions:

    - In situations when the Chernoff Bound and the Tchebycheff Inequality apply, the Chernoff Bound always gives a better estimate of the probability.

    - In situations when the Chernoff Bound and the Tchebycheff Inequality apply, the Tchebycheff Inequality always gives a better estimate of the probability.

14. (14 points) Use generating functions to find the value of a_n in this recursively-defined sequence:

    - $a_0 = 1$
    - $a_1 = 2$
    - for $n \geq 2, a_n = a_{n-1} + 6a_{n-2} - 3$