Discrete interacting particle systems

Math 690-40: Tuesday, Thursday 11:45-1:00 PM in Gross Hall 318

Instructor: Matthew Junge (www.mathjunge.com)

The following excerpt is from the introduction to [Ald13]:

> What is the most broad-ranging currently active field of applied probability? Measuring breadth by the number of different academic disciplines where it appears, it seems hard to beat a field appearing in Physics, Computer Science and Electrical Engineering, Economics and Finance, Psychology and Sociology, Population Genetics and Epidemiology and Ecology. Curiously, the field has no good name, though readers of Bernoulli will most likely know it, under the continuing influence of Liggett's 1985 book, as Interacting Particle Systems.

Discrete interacting particle systems are toy models of real world phenomena. They are often intuitive to describe, and powerful in their applications. Moreover, there is no dearth of easy-to-state, but difficult-to-solve problems. Consider this lingering open question for the frog model:

> *A sick individual is at the root of a full binary tree with height n. One healthy individual resides at each other vertex. Sick individuals perform a random walk. Upon contact, they infect healthy individuals, who then start their own random walk. The disease spreads in this manner indefinitely. Is the average time for everyone to be infected polynomial in n (i.e. bounded by n^k for some k)?.*

Our goal is to present the major theorems, and open questions for some subset of the following processes:

- the **frog model** [HJJ16b, HJJ16a, AMP02],
- the **bullet problem** (link to preprint),
- **coalescing random walk** [BFGG+16],
- **stochastic social dynamics** [Ald13],
- **choice interval splitting processes** [Jun15, MP16],
- the **spin and loop O(n) models**. (link to course notes)

We won't assume much background, but will call upon tools like couplings, duals, recursive distributional equations, and martingale techniques. Familiarity with graduate level real analysis, and understanding of probability distributions at the level of an advanced undergrad ought to suffice. *Some optional exercises will be assigned. Students will be expected to do a 30 minute presentation (solo or in pairs depending on enrollment levels), and also encouraged to attempt a few open problems.*

References

[Ald13] David Aldous. Interacting particle systems as stochastic social dynamics. *Bernoulli*, 19(4):1122–1149, 09 2013.

[AMP02] O. S. M. Alves, F. P. Machado, and S. Yu. Popov. The shape theorem for the frog model. *Ann. Appl. Probab.*, 12(2):533–546, 2002.

[BFGG+16] Itai Benjamini, Eric Foxall, Ori Gurel-Gurevich, Matthew Junge, and Harry Kesten. Site recurrence for coalescing random walk. *Electron. Commun. Probab.*, 21:12 pp., 2016.

[HJJ16a] Christopher Hoffman, Tobias Johnson, and Matthew Junge. From transience to recurrence with Poisson tree frogs. *Ann. Appl. Probab.*, 26(3):1620–1635, 2016.

[HJJ16b] Christopher Hoffman, Tobias Johnson, and Matthew Junge. Recurrence and transience for the frog model on trees. to appear in the *Annals of Probability*, available at arXiv:1404.6238, 2016.

[Jun15] Matthew Junge. Choices, intervals and equidistribution. *Electron. J. Probab.*, 20:18 pp., 2015.

[MP16] Pascal Maillard and Elliot Paquette. Choices and intervals. *Israel Journal of Mathematics*, 212(1):337–384, 2016.