Segregation of Multiple Heterogeneous Units in a Robotic Swarm

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***Abstract*— Several natural systems adopt self-sorting mechanisms based on segregative behaviors. Among these, cell segregation is of particular interest since it plays an important role in the formation of tissues, organs, and living organisms. The Differential Adhesion Hypothesis states that cells naturally segregate because of differences in affinity, which lead similar cells to strongly adhere to each other. By exploring this principle, we propose a controller that can segregate a heterogeneous swarm of robots according to the characteristics of each agent, such that similar robots form homogeneous teams and dissimilar robots are segregated. We apply LaSalle's Invariance Principle to show convergence and perform simulated experiments in order to demonstrate the robustness and effectiveness of the proposed controller. Results show that our approach allows a swarm of multiple heterogeneous robots to segregate in a coherent and smooth fashion, without any inter-agent collisions.**

I. INTRODUCTION

Swarm robotics studies multi-agent systems which consist of a large number of relatively simple robots. These agents can solve complex problems by relying on system-level properties such as robustness, flexibility, and scalability [1]. Most researchers in swarm robotics usually focus on homogeneous systems, in which all robots have the same characteristics. However, several applications of multi-robot and swarm systems require the use of heterogeneous teams of agents in order to fulfill a given mission, as sometimes it is not possible to integrate all of the required sensing and actuation capabilities for the task in a single robot. These heterogeneous systems are specially useful on cooperative assignments such as search and rescue, surveillance, perimeter protection, and transportation of large objects.

In some cases, heterogeneous agents must be able to organize themselves in a specific manner to carry out their assigned tasks. For instance, robots that gather distinct types of materials may need to form teams which can maximize the gathering of a particular resource. One strategy would be to sort agents according to their specialization, such that gatherers of similar materials stay in the same team. Afterwards, these groups can be deployed to different regions where a specific resource is abundant. We can say that such system shows a segregative behavior since the sorting process leads dissimilar agents into distinct teams.

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Segregation is a particular sorting mechanism that is common in nature, being widely used by many individuals such as cells and animals to shape these populations into tissues as well as societies, respectively. This behavior has been extensively studied by biologists, but few robotics researchers have tried to simulate it on large swarm systems. Therefore, we propose in this paper a controller that can segregate a heterogeneous robotic swarm according to the characteristics of each robot, such that similar robots form a cohesive team, and dissimilar ones are separated from each other. We base our approach on the differential potential concept [2], an analogy for multi-agent systems of the biological mechanisms by which cells segregate, extending it to multiple groups. Furthermore, we employ LaSalle's Invariance Principle to demonstrate convergence and present several simulated experiments in 2D and 3D spaces, which validate the proposed controller.

This paper is organized as follows: Section II discusses related work on swarm control and segregation. In Section III, we present our controller which is able to achieve segregation in swarm systems. Experimental results in simulated scenarios are detailed in Section IV, and Section V closes the paper with our conclusions and directions for future work.

II. RELATED WORK

Reynolds [3] was one of the first researchers to realistically simulate the movement of a swarm of agents, more specifically a flock of birds, known as *boids*. His algorithm relies on three simple steering rules that an agent applies based on the information of its surrounding neighbors: *separation*, which avoids collisions; *alignment*, which steers the agents towards their average heading; and *cohesion*, which moves the agents towards their average position. Such interactions can be modeled as a special case of the social potential field method [4], an extension of the classical artificial potential field technique [5] that specifically deals with multi-agent systems. These works have been widely employed as foundations to several methodologies on the control of robotic swarms, such as behavior-based [6], leader-follower [7], [8], hierarchical abstractions [9], [10], hydrodynamic-based [11], and others [12], [13].

Segregation is a natural phenomenon that appears in several biological systems. For instance, ants sort their brood in annular patterns in which distinct broods tend to be placed at particular annuli [14]. Another example is cellular segregation, which is of central importance in embryogenesis, as the formation of many tissues require an initial subdivision of cells into regions, each with specific characteristics that will allow particular cell types to be generated [15]. Steinberg's

Differential Adhesion Hypothesis [16] states that differences in cell adhesion generate mechanical forces which drive cellular segregation. In other words, a cell population experience stronger cohesive forces from similar cells than from dissimilar ones, and this imbalance is responsible for the segregative behavior [15].

Robotics researchers have mostly focused on segregation as a mechanism by which robots sort a collection of objects (e.g., see [17]), but some authors have specifically dealt with segregation of heterogeneous agents. For example, Groß [18] discussed a motor schema that allowed mobile robots to self-organize into annular structures. A distributed controller considers robots as having distinct virtual sizes, and local interactions make "larger" robots move outwards. This procedure was inspired by the *Brazil Nut Effect*, a granular convection phenomenon by which a mixture of granular material subjected to vibrations will lead its largest particles to the surface. This work was later extended to consider real e-puck robots [19]. In spite of the interesting results, the controller requires all robots to share a common target in order to simulate the gravitational forces responsible for the granular convection. This implies that a centralized broadcast or a consensus algorithm must be executed previously. Based on Steinberg's work [16], Kumar et al. [2] proposed the *differential potential concept*, which asserted that agents should experience different magnitudes of potential while interacting with agents of distinct types in order to achieve segregation. Stability analysis as well as convergence proofs were presented. Nevertheless, their approach is limited to only two types of robots, and the use of multiple types easily leads the system to local minima where segregation does not occur, a result which we have seen in many experiments performed with their controller. Finally, in a previous work [20], we maintained segregation among multiple robot groups during navigation by using velocity obstacles [21], but our method depends on teams being already segregated at the initial time step.

In the present work, we are interested in developing proper mechanisms that ensure segregation in the case of multiple robot types. We tackle this problem by adapting and extending Kumar's work [2] to deal with this scenario. Besides, we also introduce a new metric in order to define segregation in a more convenient way, which can be easily verified. Apart from 2D simulations, we explore our controller's behavior in 3D spaces as well.

III. METHODOLOGY

We consider a set of fully actuated mobile agents whose dynamics are given by the double integrator

$$\dot{q}_i = v_i \text{ and } \dot{v}_i = u_i \qquad i \in \Upsilon = \{1, 2, \ldots, n\}, \tag{1}$$

in which $q_i \in \mathbb{R}^p$, $v_i \in \mathbb{R}^p$, and $u_i \in \mathbb{R}^p$ denote the position, velocity, and control input of robot i, respectively. This set of mobile agents consists of different types of robots, which we represent by the partition $\tau = \{\tau_1, \tau_2, \ldots, \tau_m\}$, where each $\tau_k \subset \Upsilon$ contains all agents of type k. We assume that $\forall j,k : j \neq k \rightarrow \tau_j \cap \tau_k = \emptyset$ and $\forall j,k : |\tau_j| = |\tau_k|$, i.e., each robot is uniquely assigned to a single type and the type partition is fully balanced, respectively.

Our objective is to synthesize a controller that can sort robots of different types into m distinct clusters in the workspace, such that each cluster contains agents of a single type only. We refer to the latter as the *segregation problem*, and a control system which solves this problem is said to display a *segregative behavior*.

A. Control Law

Given a population of n heterogeneous mobile robots with partition τ and dynamics specified by (1), we propose the following control law:

$$u_i = -\sum_{j \neq i} \nabla_{q_i} U_{ij}(\|q_i - q_j\|) - \sum_{j \neq i} (v_i - v_j), \tag{2}$$

in which $U_{ij}(\|q_i - q_j\|)$ is an artificial potential function that rules the interaction between agents i and j, $\|q_i - q_j\|$ is the Euclidean norm of the vector $q_i - q_j$, and ∇_{q_i} is the gradient with respect to the coordinates of agent i. The first term represents the resultant force that acts on robot i given its interactions with all other agents, whereas the second term serves as damping and causes robots to match their velocities. This kind of controller equation is a common approach for potential-based swarm systems [2], [7], [12].

The artificial potential field $U_{ij} : \mathbb{R} \rightarrow \mathbb{R}_{>0}$ is a function of the relative distance between a pair of agents that we express as

$$U_{ij}(\|q_{ij}\|) = \alpha \left(\frac{1}{2}(\|q_{ij}\| - d_{ij})^2 + \ln \|q_{ij}\| + \frac{d_{ij}}{\|q_{ij}\|} \right), \tag{3}$$

in which α is a scalar control gain, q_{ij} is a shortened form of writing $q_i - q_j$ (i.e., $q_{ij} = q_i - q_j$), and d_{ij} is a positive parameter that will be described later. The initial conditions and dynamics of the system exclude the situations where $\|q_{ij}\| = 0$, in which (3) is undefined. Furthermore, as we will show later, if robots do not collide at the initial configuration then there will be no collisions through all time steps.

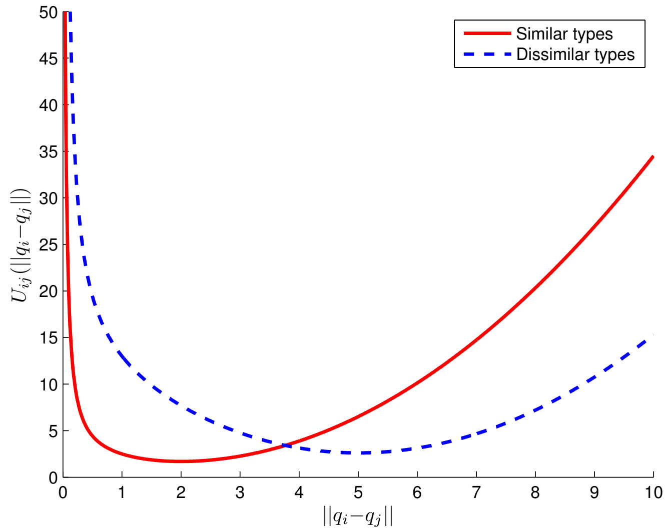
Although there are m distinct types of robots involved in the system, each agent classifies its neighbors as being either of its own type or of a different type. This means that agents see the system through a binary filter which reduces possible robot interactions to only two kinds thereof: among robots of the same type and among robots of distinct types. Formally, we say that an agent i has a local type partition

$${}^{i}\tau = \{\tau_k, \Upsilon \setminus \tau_k\} \quad i \in \tau_k, \tag{4}$$

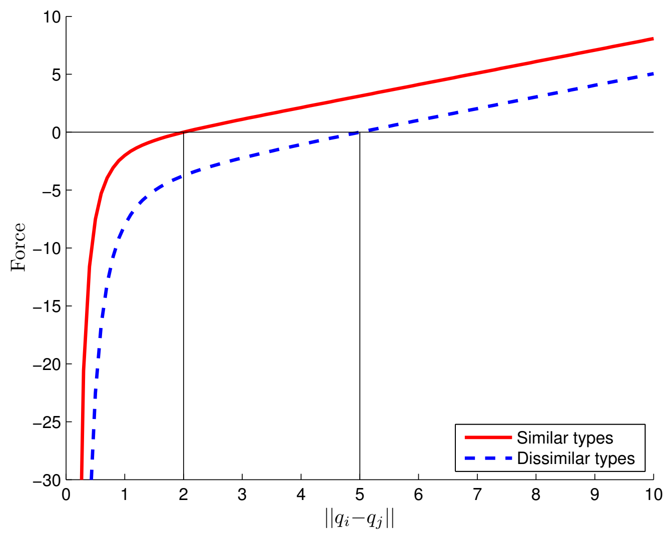
where $\tau_k \in \tau$, and $\Upsilon \setminus \tau_k$ represents the set difference.

In order to segregate robots, we apply the *differential potential* concept, i.e., pairs of dissimilar agents experience different magnitudes of potential than pairs of similar agents [2]. We can accomplish this by defining the parameter d_{ij} of (3) according to the local type partition ${}^{i}\tau$.

$$d_{ij}({}^{i}\tau) = \begin{cases} d_{AA}, & \text{if } i \in \tau_k \text{ and } j \in \tau_k \\ d_{AB}, & \text{if } i \in \tau_k \text{ and } j \notin \tau_k \end{cases} \tag{5}$$



(a) Inter-agent potential.



(b) Inter-agent force.

Fig. 1. Plot of the artificial potential field $U_{ij}(\|q_j - q_i\|)$ and its underlying forces given $d_{AA} = 2$ and $d_{AB} = 5$.

Equation (5) states that interactions among similar and dissimilar types of robots are ruled by d_{AA} and d_{AB}, respectively. Thus, the system exhibits a segregative behavior when we choose values for these parameters such that

$$0 < d_{AA} < d_{AB}. \tag{6}$$

We show in Figure 1(a) a plot of the artificial potential function $U_{ij}(\|q_{ij}\|)$, whose minimum is located at $\|q_{ij}\| = d_{ij}$. Furthermore, we depict the interaction forces among a pair of robots in Figure 1(b), in which constraint (6) holds true. The latter plot actually represents the scalar part of the gradient

$$\nabla U_{ij}(\|q_{ij}\|) = \alpha \left(\|q_{ij}\| - d_{ij} + \frac{1}{\|q_{ij}\|} - \frac{d_{ij}}{\|q_{ij}\|^2} \right) \frac{q_{ij}}{\|q_{ij}\|}, \tag{7}$$

in which we ignore the normalized vector term. It is easy to see that, at any given distance, forces among agents of similar types are greater than those among different types. Therefore, our controller respects the differential potential concept [2], and effectively implements an approximation of Steinberg's differential adhesion model [16].

B. Controller Analysis

In this section, we analyze the convergence of the multi-agent system when using the proposed control law. We start with the definition of the Lyapunov function

$$V(\mathbf{q},\mathbf{v}) = U(\mathbf{q}) + \frac{1}{2}\mathbf{v}^\mathsf{T}\mathbf{v}, \tag{8}$$

where $\mathbf{q} \in \mathbb{R}^{np}$ and $\mathbf{v} \in \mathbb{R}^{np}$ are stacked vectors whose components are the configurations and velocities of all robots, respectively, and $U(\mathbf{q}) : \mathbb{R}^{np} \to \mathbb{R}_{>0}$ is the collective artificial potential function, which we write as

$$U(\mathbf{q}) = \frac{1}{2} \sum_{\tau_k \in \tau} \sum_{i \in \tau_k} \sum_{j \in \tau_k, j \neq i} U_{ij}(\|q_i - q_j\|) + \frac{1}{2} \sum_{\tau_k \in \tau} \sum_{i \in \tau_k} \sum_{j \in \Upsilon \setminus \tau_k} U_{ij}(\|q_i - q_j\|). \tag{9}$$

Thus, we can model the collective dynamics of the system:

$$\dot{\mathbf{q}} = \mathbf{v} \tag{10}$$

$$\dot{\mathbf{v}} = -\nabla U(\mathbf{q}) - \hat{L}(\mathbf{q})\mathbf{v}, \tag{11}$$

in which $\hat{L}(\mathbf{q}) = L(\mathbf{q}) \otimes I_p$ is the Kronecker product of the system's graph Laplacian $L(\mathbf{q})$ and the $p \times p$ identity matrix I_p (for a complete description, see [12]). These definitions let us introduce the proposition below.

Proposition 1: Assuming that the underlying adjacency graph of the system is complete at all times, for any initial condition that belongs to the level set $\Omega_C = \{(\mathbf{q},\mathbf{v}) \mid V(\mathbf{q},\mathbf{v}) \leq C\}$, with $C > 0$, a heterogeneous system with type partition τ on n mobile agents, whose dynamics and control laws are respectively given by (1) and (2), asymptotically converges to the largest invariant set in $\Omega_I = \{(\mathbf{q},\mathbf{v}) \in \Omega_C \mid \dot{V}(\mathbf{q}) = 0\}$, without any inter-agent collisions. At the largest invariant set in Ω_I, the velocity of each agent is bounded, all velocities match, and the system's collective potential reaches a local minimum.

Proof: We aim to demonstrate that $\dot{V}(\mathbf{q},\mathbf{v}) \leq 0$ in order to apply LaSalle's Invariance Principle to show convergence. To achieve this, we can differentiate $V(\mathbf{q},\mathbf{v})$ with respect to time and then substitute (10) and (11) as follows:

$$\begin{aligned} \dot{V}(\mathbf{q},\mathbf{v}) &= \dot{\mathbf{q}}^\mathsf{T}\nabla U(\mathbf{q}) + \mathbf{v}^\mathsf{T}\dot{\mathbf{v}} \\ &= \mathbf{v}^\mathsf{T}\nabla U(\mathbf{q}) + \mathbf{v}^\mathsf{T}(-\nabla U(\mathbf{q}) - \hat{L}(\mathbf{q})\mathbf{v}) \\ &= -\mathbf{v}^\mathsf{T}\hat{L}(\mathbf{q})\mathbf{v} = -\frac{1}{2}\sum_i \sum_j \|v_j - v_i\|^2 \leq 0. \end{aligned} \tag{12}$$

The last step holds because the system's adjacency graph is complete [12]. From LaSalle's Invariance Principle, all initial conditions that lie on Ω_C will lead the system to the largest invariant set in Ω_I, where $\dot{V}(\mathbf{q},\mathbf{v}) = 0$. Therefore, (12) implies that all velocities match, i.e., $\forall i,j : v_i = v_j$. By (8) and (10), we have $\mathbf{v}^\mathsf{T}\mathbf{v} \leq 2C$ because $V(\mathbf{q},\mathbf{v}) \leq C$, which leads to $\|\mathbf{v}\| \leq \sqrt{2C}$. Consequently, all velocities are bounded by $\sqrt{2C}$ as well. Matching velocities imply that inter-agent distances remain constant, hence $\forall i, j : \dot{q}_{ij} = \mathbf{0}$, and thus

$$\dot{U}(\mathbf{q}) = \frac{1}{2} \sum_{\tau_k \in \tau} \sum_{i \in \tau_k} \sum_{j \in \tau_k, j \neq i} \dot{q}_{ij}^{\mathsf{T}} \nabla_{q_{ij}} U_{ij}(\|q_{ij}\|) + \frac{1}{2} \sum_{\tau_k \in \tau} \sum_{i \in \tau_k} \sum_{j \in \Upsilon \setminus \tau_k} \dot{q}_{ij}^{\mathsf{T}} \nabla_{q_{ij}} U_{ij}(\|q_{ij}\|) = 0, \quad (13)$$

which implies that $U(\mathbf{q})$ is constant at the steady state. Moreover, as $\hat{L}(\mathbf{q})\mathbf{v} = \mathbf{0}$ because of matching velocities, we can reduce (11) to

$$\dot{\mathbf{v}} = -\nabla U(\mathbf{q}). \quad (14)$$

Therefore, $\nabla U(\mathbf{q})$ must be the zero vector, as otherwise the collective potential would reach a lower value instead of being constant. This implies that the system has reached a local minimum and velocities must not change.

Finally, assume that robots i and j collide, i.e., $\|q_{ij}\| = 0$. We can see from (3) and (9) that this would take $U(\mathbf{q}) \to \infty$, but this contradicts the fact that $V(\mathbf{q}, \mathbf{v}) \leq C$. Hence, no agent collides with each other. ∎

C. Metric

In order to measure segregation among clusters quantitatively, we propose a metric that is based on the pairwise intersection area of their convex hulls:

$$M(\mathbf{q}, \tau) = \sum_{\tau_k \in \tau} \sum_{\tau_l \in \tau, l \neq k} A\left(CH(\bigcup_{i \in \tau_k} q_i) \bigcap CH(\bigcup_{j \in \tau_l} q_j) \right), \quad (15)$$

in which $A(Q)$ and $CH(Q)$ denote the area and the convex hull of set Q, respectively. We have chosen this metric because the convex hull can be used as a simple and well-defined shape representation of a cluster. This means that segregation occurs when there is no overlap among clusters. In other words, we say that the system is fully segregated when $M(\mathbf{q}, \tau)$ approaches zero.

IV. EXPERIMENTS

We executed a sequence of simulations in order to study the performance and feasibility of our proposed approach. We first present their results according to metric $M(\mathbf{q}, \tau)$ and then display some snapshots of these experiments. Finally, we close the section with a discussion on the behavior of the system as well as on particular details of our method.

A. Simulations

We have performed an extensive series of simulations in order to analyze our controller under metric $M(\mathbf{q}, \tau)$. Each simulation consisted of 150 robots and a varying number of agent types. At the initial state, all velocities were set to zero, and robots were positioned according to a two-dimensional uniform distribution, which is independent of a robot's type. Additionally, we have set $d_{AA} = 2$ and $d_{AB} = 5$ for all experiments. We present in Figure 2 the mean and standard deviation of $M(\mathbf{q}, \tau)$ among 100 experiments given these initial conditions. In all cases, both the mean and standard deviation approach zero as the number of iterations increase. Moreover, systems with less types tend to achieve segregation faster than those with more types. This is expected, as given a fixed number of robots, the use of a large number of types would result in few robots per cluster, which in turn would lower the magnitude of the resultant force towards it.

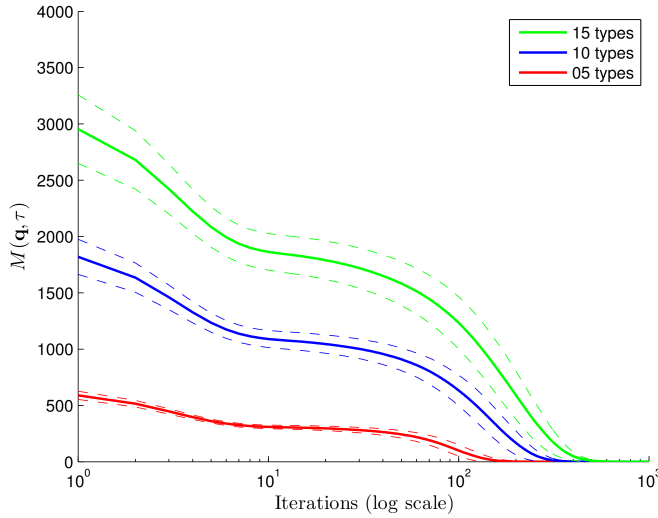
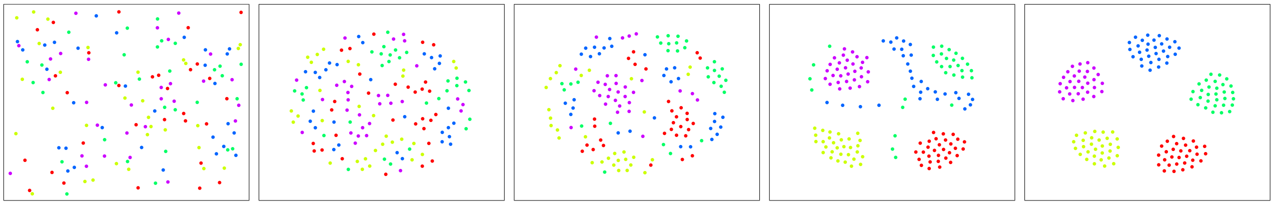


Fig. 2. Mean intersection area of convex hulls for 100 experiments with a varying number of robot types. Dashed lines represent one standard deviation from the mean.

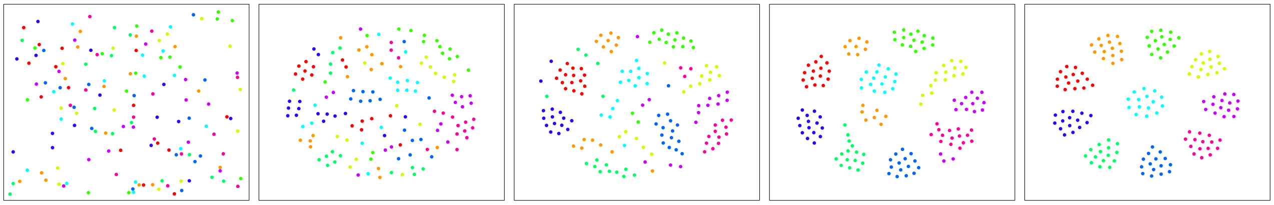
We display in Figure 3 a series of snapshots from particular instances of our experiments. Through a visual inspection, we can see that similar robots quickly form clusters whose size grows with time as other agents join them. Furthermore, interesting geometrical patterns are organized at the stable state. We have also observed two particular behaviors which might be difficult to notice in the figures[1]: large ensembles usually move to the outside of the main aggregate, the one that embodies all agents of the system, and adjacent dissimilar clusters form corridors which are used by agents of a third type to move at higher speeds. Both of these behaviors are compelling because they contribute to the opening of free spaces, whereby smaller clusters and lone robots can take advantage of the situation and form larger ensembles.

We have executed simulations in 3D space as well. Figure 4 contains two images from the initial and final configurations of experiments comprising 150 robots and a varying number of agent types. Initial conditions were chosen exactly as in the 2D simulations. The controller was able to achieve segregation, and robots have displayed the same overall behavior as of their 2D counterparts. Particularly, we have seen that it is easier for robots to form clusters in this scenario because of the additional degree of freedom, which allows agents to maneuver in new directions. Thus, in the 3D case, it is unusual to find lone robots wandering towards their cluster in later iterations, since larger clusters are aggregated more quickly.

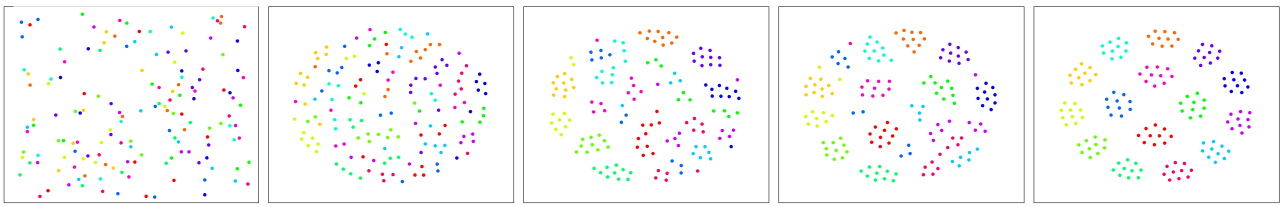
[1] An illustrative video of the simulations in Figures 3 and 4 can be found in the Multimedia Attachment of this paper.



(a) 5 heterogeneous types.



(b) 10 heterogeneous types.



(c) 15 heterogeneous types.

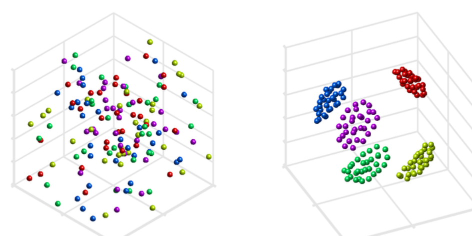
Fig. 3. Snapshots of simulated executions with 150 robots and a varying number of heterogeneous types. In each sequence, the initial configuration is depicted on the left, whereas the final configuration is displayed on the right. Each robot type is represented by a different color.

B. Discussion

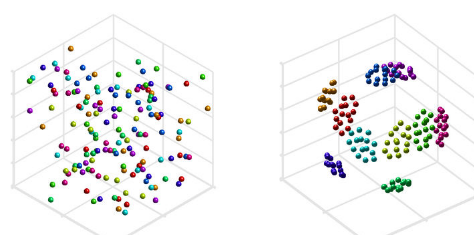
Equation (3) represents an important distinction between our controller and Kumar's prior work [2]. Regarding their potential function, we noticed that its gradient may vanish on the free space among adjacent dissimilar clusters. This is the reason why our initial experiments, which used their controller with multiple types, had often reached undesirable local minima. Thus, by adding the quadratic term in (3), we have actually increased the norm of the gradient and biased its direction, which allowed robots to keep moving towards their respective cluster.

Desirable properties in swarm systems include scalability, flexibility, and robustness [1]. These are specially important on applications in which robots may be inserted or removed from the system dynamically. One of the main advantages of our approach is that a robot does not need to know either how many agents or how many types exist in the system. This is due to the second case of (5), in which we write $j \notin \tau_k$ instead of $j \in \Upsilon \setminus \tau_k$ as the former explicitly states that robot i needs to recognize only agents that are similar to itself. Consequently, robots can be added or subtracted from the system at any time.

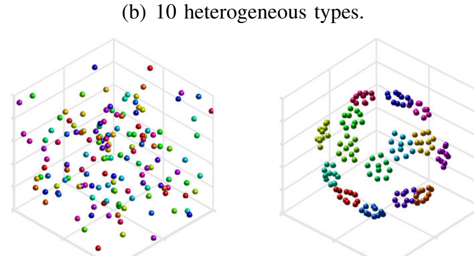
As can be seen in (2), our controller requires global perception capabilities, i.e., each robot must know the position and velocity of all agents. Thus, in spite of the robustness of our approach, this constraint may hinder its applicability on physical systems, as real sensors usually have constrained capabilities which restrict robots to gather local information only. This is also a limitation of the method in [2]. Regarding our metric, there is a possible drawback in real



(a) 5 heterogeneous types.



(b) 10 heterogeneous types.



(c) 15 heterogeneous types.

Fig. 4. Initial and final configurations of simulated experiments in 3D space with 150 robots and a varying number of heterogeneous types. We apply an orthographic projection and slightly rotate the stable state to better depict the separation among clusters.

robotic systems. In case of robot failures, the metric could give unsatisfactory results for an otherwise well-segregated swarm, since the location of a single agent can have a large impact on the metric.

Although we have constrained our approach to balanced type partitions, we executed some simulations with unbalanced partitions as well. In these experiments, robots occasionally achieved segregation depending on the relative balance of the partition, but several experiments reached local minima when the type partition had been severely unbalanced. In these local minima, robots were not segregated in the sense of our proposed metric. This usually happens when robots cannot reach their cluster as the attractive forces towards it are weaker than those repelling them away from other agents. On the other hand, these types of local minima in experiments with balanced type partitions are not common. For example, among the 100 experiments with 15 types presented in Figure 2, there was only one instance which did not segregate. However, given the same initial conditions, by choosing a larger value for d_{AB} the controller was able to achieve segregation. Thus, we think that this value should be chosen according to how many robots and types exist in the system, as larger numbers thereof may require wider corridors between dissimilar clusters so that the gradient of the potential function will not vanish.

Thus, we believe that, given a balanced type partition τ, and assuming that the underlying adjacency graph of the system is complete at all times, for any initial condition that belongs to the level set Ω_C, there exists a finite value r such that if $\frac{d_{AB}}{d_{AA}} > r$ then $M(\mathbf{q},\tau) \to 0$ as the number of iterations approaches infinity. Nevertheless, it is still necessary to gather more evidence in order to formally prove this claim.

V. CONCLUSION

In this work, we proposed a controller that can sort a system of multiple heterogeneous mobile robots into homogeneous clusters, such that agents of similar types are segregated from dissimilar ones. We based our approach on the differential potential concept, a conceptualization of the mechanisms by which biological systems achieve segregation. In this framework, agents experience distinct magnitudes of potential when they interact with agents of different types. We presented stability analyses and several experiments in 2D and 3D scenarios, which demonstrated the effectiveness of the proposed approach.

Despite the good results, we still see room for improvement. For instance, our assumptions that robots have global sensing capabilities may not hold in real scenarios, and severely unbalanced type partitions may lead them to stable states that are not segregated in the sense of our metric. Regarding the former, we think it would be interesting to explore control laws and possibly other potential functions that model local sensing into their methodologies, whereas the latter could be tackled by employing non-symmetrical potential functions. All in all, both problems can be regarded as a more general one, and we believe that, by further studying these, our controller can be improved to solve the segregation problem in a wider variety of instances.

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