1. Compute the derivative of the following functions:

   (a) $F(x) = \int_{2x} x^2 e^{t^2} dt.$

   $$e^{x^4} 2x - 2e^{x^2}$$

   (b) $G(x) = \int_{g(x)}^{h(x)} f(t)dt.$

   $$f(h(x))h'(x) - f(g(x))g'(x)$$

   (c)

   (d) (Sp14 Final) Let $A(t)$ denote the area under the curve $y = \sqrt{1 - x^3}$ and above the axis, between the vertical lines $x = t$ and $x = 2t$. Find the value of t for which $A(t)$ is a maximum on the interval $0 \leq t \leq 1/2$.

   $$A'(t) = 2\sqrt{1 - (2t)^3} - \sqrt{1 - t^3}$$

   Solve $A'(t) = 0$:

   $$\begin{aligned} A'(t) &= 0 \\ 2\sqrt{1 - (2t)^3} &= \sqrt{1 - t^3} \\ 4(1 - 8t^3) &= 1 - t^3 \\ 3 &= 31t^3 \\ (3/31)^{1/3} &= t. \end{aligned}$$

2. (Wi03 MT1) Compute the total area bounded by the curves $f(x) = x^2$ and $g(x) = x^3 - 6x^2 + 10x$.

   Set equal to get $x = 0, 2, 5$. Figure out which is on top and compute

   $$\int_0^2 g(x) - f(x)dx + \int_2^5 f(x) - g(x) = 253/12.$$

3. Find positive numbers a and b so that both of the following hold

   $$\frac{1}{2} \int_{\sqrt{b}}^{a} x \, dx = 1$$
   $$\int_0^{a^3} \frac{1}{x^{2/3}} \, dx = \int_0^{\ln 7} e^x \, dx.$$

   If we evaluate the first integral we get

   $$\frac{1}{2} \int_{\sqrt{b}}^{a} x dx = \frac{1}{2} \left(\frac{1}{2} x^2 \Big|_{\sqrt{b}}^{a} \right) = \frac{1}{4} \left(a^2 - b \right).$$

   And we know that this is equal to 1. So we have

   $$\frac{1}{4} \left(a^2 - b \right) = 1.$$

If we solve the second integral we obtain

$$3a^3 = 6.$$

So we must have $a = 2$ and using this to solve for b gives $b = 8$.