1. Compute the following integrals.

(a) $\int x \arctan(x)\, dx$. First do integration by parts with $u = \arctan x$, $dv = xdx$. This gives

$$= \frac{1}{2}x^2 \arctan x - \frac{1}{2}\underbrace{\int \frac{x^2}{1+x^2}dx.}_{(i)}$$

Next we use the trig sub $x = \tan\theta$ to rewrite (i) as

$$\begin{aligned}(i) &= \int \frac{\tan^2\theta}{1+\tan^2\theta}\sec^2\theta d\theta \\ &= \int \frac{\tan^2\theta}{\sec^2\theta}\sec^2\theta d\theta \\ &= \int \tan^2\theta d\theta \\ &= \int \sec^2\theta - 1 d\theta \\ &= \tan\theta - \theta \\ &= x - \arctan x.\end{aligned}$$

So the answer is $\boxed{\frac{1}{2}x^2\arctan x - \frac{1}{2}x + \arctan x + C.}$

(b) $\int \frac{dx}{\sqrt{4x^2+8x-12}}$.

We need to complete the square. First let's pull out the 4 and consider $x^2+2x-3 = (x+1)^2 - 4$. This lets us rewrite as

$$\int \frac{dx}{\sqrt{4[(x+1)^2-4]}} = \frac{1}{4}\int \frac{dx}{\sqrt{(x+1)^2-4}}.$$

Make the trig-sub with $x+1 = 2\sec\theta$ to obtain

$$\begin{aligned}&= \frac{1}{4}\int \frac{\sec\theta\tan\theta d\theta}{\sqrt{4\sec^2 - 4}} \\ &= \frac{1}{8}\int \frac{\sec\theta\tan\theta d\theta}{\sqrt{\sec^2 - 1}} \\ &= \frac{1}{8}\int \frac{\sec\theta\tan\theta d\theta}{\tan\theta} \\ &= \int \sec\theta d\theta \\ &= \ln|\sec\theta + \tan\theta| + C \\ &= \ln\left|\frac{x+1}{2} + \frac{\sqrt{(x+1)^2-4}}{4}\right| + C.\end{aligned}$$

(c) $\displaystyle\int_1^2 \frac{dx}{x^2(x+1)}$. Use partial fractions to write as

$$\int_1^2 \frac{1}{x^2} - \frac{1}{x} + \frac{1}{x+1}dx = -\frac{1}{x} - \ln x - \ln(x+1)\Big|_1^2 = \frac{1}{2} - 2\ln 2 - \ln 3.$$

2. Compute $\displaystyle\int \sec^3 x dx$. *Hint:* $\sec^3 x = \frac{\cos x}{\cos^4 x}$, *make a substitution then use partial fractions.*

First we use the hint and the identity $\cos^2 x = 1 - \sin^2 x$

$$\int \sec^3 x dx = \int \frac{\cos x}{\cos^4 x}dx = \int \frac{\cos x}{(1-\sin^2 x)^2}dx.$$

Then we let $u = \sin x$ and rewrite as

$$= \int \frac{1}{(1-u^2)^2}du = \int \frac{1}{(1-u)^2(1+u)^2}.$$

If we whip out some partial fractions this becomes

$$= \frac{1}{4}\int \frac{1}{1-u} + \frac{1}{(1-u)^2} + \frac{1}{1+u} + \frac{1}{1-u}du.$$

And all of these can be integrated term by term.

3. You are helping Reggie get out of a 50 foot well by lifting him with a rope which weighs .2 lb/ft. Since Reggie is wet, he is dripping water and steadily becomes .05 pounds lighter with every foot you lift him. If wet Reggie initially weighs 200 pounds, find the total work done to get him out of the well. *Write your answer on the back of this page.*

$$\int_0^{50} (50-y)(.2) + (200 - .05y)\, dy$$