1. (a) Using $n = 4$ rectangles approximate the area between the curves $y = 2x$ and $y = x^2$ by averaging left and right sums then **also** by using midpoints.

The curves intersect at $x = 0$ and $x = 2$. Since $2x$ is above x^2 we consider the function $h(x) = 2x - x^2 = x(2 - x)$ on [0,2] with 4 rectangles.

$$\begin{aligned}\text{LEFT AREA} &= \frac{1}{2}(h(0) + h(1/2) + h(1) + h(3/2))\\ &= \frac{1}{2}(0 + (1/2)(3/2) + 1 + (3/2)(1/2))\\ &= \frac{5}{4}.\end{aligned}$$

$$\begin{aligned}\text{RIGHT AREA} &= \frac{1}{2}(h(1/2) + h(1) + h(3/2) + h(2))\\ &= \frac{1}{2}((1/2)(3/2) + 1 + (3/2)(1/2) + 0)\\ &= \frac{5}{4}.\end{aligned}$$

They are the same! Do you know why?

$$\begin{aligned}\text{MIDPOINT AREA} &= \frac{1}{2}(h(1/4) + h(3/4) + h(5/4) + h(7/4))\\ &= \frac{1}{2}((1/4)(7/4) + (3/4)(5/4) + (5/4)(3/4) + (7/4)(1/4))\\ &= \frac{11}{8}.\end{aligned}$$

(b) Let $f(t) = t^3$. Write a formula for a lefthand sums with 10 rectangles for the value of $\int_0^x f'(t)dt$.

We know that $f'(t) = 3t^2$, plug this into the formula below.

$$\frac{x}{10}(f'(x/10) + f'(2x/10) + \cdots f'(10x/10)) = 3\frac{x^2}{10^2}\frac{x}{10}(1^2 + 2^2 + \cdots + 10^2) \approx 1.15x^3.$$

(c) (DIFFICULT! Feel free to skip.) Suppose there are n rectangles instead of 10. Any guess what $\lim_{n\to\infty} \int_0^x f'(t)dt$ is equal to? The answer is $f(x)$. Fancier than you would ever have to do on a test, so please **don't stress over this** but we could write a righthand sum with n rectangles as

$$\begin{aligned}\frac{x}{n}\sum_{j=1}^{n} 3(jx/n)^2 &= \frac{3x^3}{n^3}\sum_{j=1}^{n} j^2\\ &= \frac{3x^3}{n^3}\frac{n}{3}(n+1)(n+\frac{1}{2})\\ &= \frac{x^3}{n^3}n(n+1)(n+\frac{1}{2})\end{aligned}$$

We just used the formula $\sum_{j=1}^{n} j^2 = \frac{n}{3}(n+1)(n+\frac{1}{2})$. So if we take the limit we only need to look at the n^3 terms and we end up with

$$\lim_{n\to\infty} \frac{x^3}{n^3} n(n+1)(n+\frac{1}{2}) = x^3 = f(x).$$

2. Differentiate the following functions:

   (a) $f(x) = \sin^2(\cos^2(x))$.

   $$2\sin(\cos^2(x))\cos(\cos^2(x))2\cos(x)(-\sin x)$$

   (b) $g(x) = x^2 e^{x^2}$.

   $$2xe^{x^2} + e^{x^2}(2x)x^2$$

   (c) $h(x) = e^{e^{e^x}}$.

   $$e^{e^{e^x}} e^{e^x} e^x$$

3. Find the antiderivative, $F(x)$, for $f(x) = \frac{1}{2}xe^{x^2} + \frac{1}{3}x^2\sin(x^3)$. *Hint: reverse chain rule.*

$$F(x) = \frac{1}{4}e^{x^2} + \frac{1}{9}\sin(x^3) + C$$