1. (a) $\dfrac{d}{dx}\left[\pi x^e e^x \sqrt{x}\right]$

> Rewrite as $\pi x^{e+\frac{1}{2}} e^x$ then apply product rule to get
> $$\pi(e+\frac{1}{2})x^{e-\frac{1}{2}}e^x + \pi e^x x^{e+\frac{1}{2}}.$$

(b) $\dfrac{d}{dx}\left[\dfrac{\left(\frac{e^x}{x^{300}+x}\right)}{\sin x \cos x}\right]$ *Hint:* $\frac{d}{dx}\cos x = -\sin x$.

> $$\frac{\frac{e^x(x^{300}+x)-(300x^{299}+1)e^x}{(x^{300}+x)^2}\sin x\cos x - (\cos x\cos x + \sin x(-\sin x))\frac{e^x}{x^{300}+x}}{(\sin x\cos x)^2}$$

(c) Find all tangent lines to $g(x) = x - \sqrt{x}$ which go through the origin.

> The derivative is $f'(x) = 2x + 2$. So the tangent line at $(a, f(a))$ satisfies $y - f(a) = (2a+2)(x-a)$ this can be written as
> $$y - (a^2 + 2a + 1) = (2a+2)(x-a)$$
> We want to go through the point $(x, y) = (0, 0)$. Plugging this in gives a must satisfy
> $$0 - (a^2 + 2a + 1) = (2a+2)(0-a),$$
> simplifying gives the relation
> $$-a^2 - 2a - 1 = -2a^2 - 2a$$
> and so
> $$a^2 - 1 = 0$$
> which has solutions
> $$a = \pm 1$$
> This means our two tangent lines are
> $$y - 4 = 4(x-1)$$
> and
> $$y = 0$$

2. Matt is grinding a rail on his skateboard with position $p(t) = \frac{1}{3}t^3 + \frac{1}{2}t^2 - 4t + 10$.

We can find $p'(t) = t^2 + t - 4$. Using the quadratic formula this has roots $\frac{-1\pm\sqrt{17}}{2}$. We then check values near these roots and see that when $t?\frac{-1+\sqrt{17}}{2}$ it holds that $p'(t) > 0$. If $0 < t < \frac{-1+\sqrt{17}}{2}$ then $p'(t) < 0$. We can use this information to solve (a), (b) and (c).

(a) Find all times when Matt is grinding right (positive direction).

$(\frac{1}{2}(1+\sqrt{17}), \infty)$.

(b) Find all times when Matt is grinding left (negative direction).

$(0, \frac{1}{2}(-1+\sqrt{17})$.

(c) Use this to figure out Matt's leftmost position for $t \geq 0$.

Plug in $p(\frac{1}{2}(-1+\sqrt{17})) \approx -1.87$.

3. (a) Let $g(x) = x|x|$. Is $g(x)$ differentiable at $x = 0$? If so find $g'(0)$. If not explain why.

Yes. $g'(0) = \lim_{h\to 0} \frac{g(0+h)-g(0)}{h} = \lim_{h\to 0} \frac{h|h|}{h} = \lim_{h\to 0} |h| = 0$.

(b) Let $h(x) = \begin{cases} a^2x^3 + \frac{1}{bx}, & x > 1 \\ a(x-1)e^x, & x \leq 1 \end{cases}$. Find a and b so h is differentiable at $x = 1$.

We need two things. First, $\lim_{x\to 1^+} h(x) = \lim_{x\to 1^-} h(x)$. This is equivalent to

$$a^2 + \frac{1}{b} = 0.$$

Solving we get $b = -\frac{1}{a^2}$. For differentiability we need $\lim_{x\to 1^+} h(x) = \lim_{x\to 1^-} h(x)$. This is equivalent to

$$3a^2 - \frac{1}{b} = ae.$$

Making the substition $b = -\frac{1}{a^2}$ we then have

$$-3a^2 + a^2 = ae.$$

Dividing by a we then have

$$-2a = e$$

and so $a = -e/2$. This implies $b = -\frac{4}{e^2}$.

(c) Below is a graph of a function $g(x)$. Draw $g'(x)$.