LAKSHYA JEE

LAKSHYA KO HAR HAAL ME PAANA HAI

Electric Potential
&
Capacitance

-Er. Rohit Gupta

Today's GOALS!

PYQ

In free space, a particle A of charge $1\,\mu C$ is held fixed at a point P. Another particle B of the same charge and mass $4\,\mu g$ is kept at a distance of 1 mm from P. If B is released, then its velocity at a distance of 9 mm from P is

$$\left[\text{Take, } \frac{1}{4\pi\varepsilon_0} = 9\times 10^9\ \text{N-m}^2\text{C}^{-2}\right]$$

(Main 2019, 10 April II)

(a) 1.5×10^2 m/s
(b) 3.0×10^4 m/s
(c) 1.0 m/s
(d) 2.0×10^3 m/s

$1\ \text{kg} = 10^3\ \text{g}$

$1\ \text{g} = 10^6\ \mu\text{g}$

$1\mu C$ $1\mu C$

$A \cdot \longleftarrow 1\text{mm} \longrightarrow \cdot B$

P

fixed

$m = 4\mu g$

$$U_i + KE_i = U_f + KE_f$$

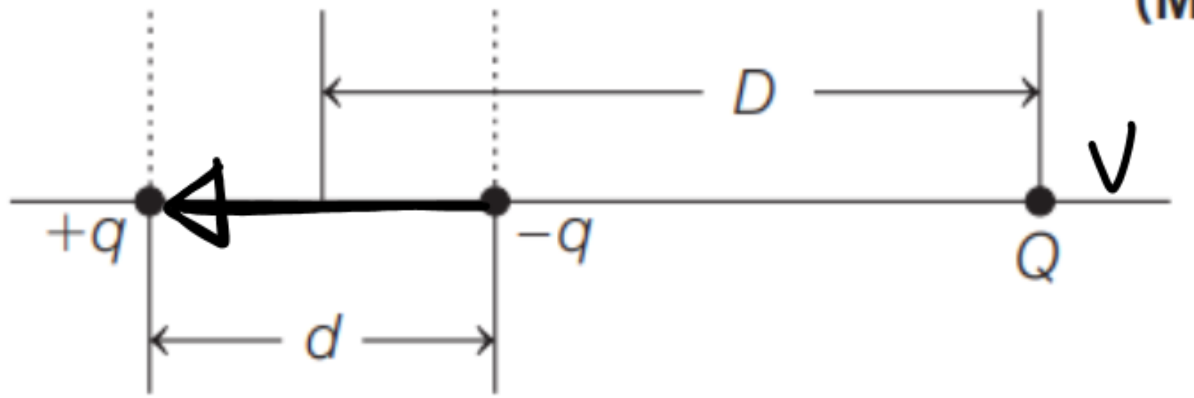
$$\frac{(9\times10^9)(1\times10^{-6})^2}{(10^{-3})} + 0 = \frac{9\times10^9\times(1\times10^{-6})^2}{(9\times10^{-3})} + \frac{1}{2}\times(4\times10^{-6}\times10^{-3})v^2$$

$$9 = 1 + 2\times10^{-9}v^2 \Rightarrow v^2 = 4\times10^9$$

$$\sqrt{40}\times10^4$$

A system of three charges are placed as shown in the figure

(Main 2019, 9 April I)



If $D >> d$, the potential energy of the system is best given by

(a) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d}+\frac{2qQd}{D^2}\right]$

(b) $\frac{1}{4\pi\varepsilon_0}\left[+\frac{q^2}{d}+\frac{qQd}{D^2}\right]$

(c) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d}-\frac{qQd}{2D^2}\right]$

(d) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d}-\frac{qQd}{D^2}\right]$

$-\frac{Kqq}{d}$

$V = -\frac{KP}{r^2}$ $\qquad$ $V = \frac{KP}{r^2}$

$\boxed{U = Q\,V_{dipole}}$

$= \left(-\frac{KP}{D^2}\right)Q$

$= -\frac{KqdQ}{D^2}$

$U_{total} = -\frac{Kq^2}{d} - \frac{KqdQ}{D^2}$

$= K\left[-\frac{q^2}{d} - \frac{qQd}{D^2}\right]$

The electric field in a region is given by $\mathbf{E} = (Ax + B)\hat{\mathbf{i}}$, where E is in NC^{-1} and x is in metres. The values of constants are $A = 20$ SI unit and $B = 10$ SI unit. If the potential at $x = 1$ is V_1 and that at $x = -5$ is V_2, then $V_1 - V_2$ is **(Main 2019, 8 April II)**

(a) -48 V (b) -520 V

(c) 180 V (d) 320 V

$$E = (Ax + B)\hat{i}$$

$$A = 20$$

$$B = 10$$

$x = 1$ V_1

$x = -5$ V_2

$$V \underset{\int}{\overset{d}{\rightleftarrows}} E$$

$$\int dV = -\int \vec{E}\cdot d\vec{x}$$

$$\int_{V_1}^{V_2} dV = -\int_{1}^{-5} (Ax + B)\,dx$$

$$V_2 - V_1 = -\left[\frac{Ax^2}{2} + Bx\right]_1^{-5}$$

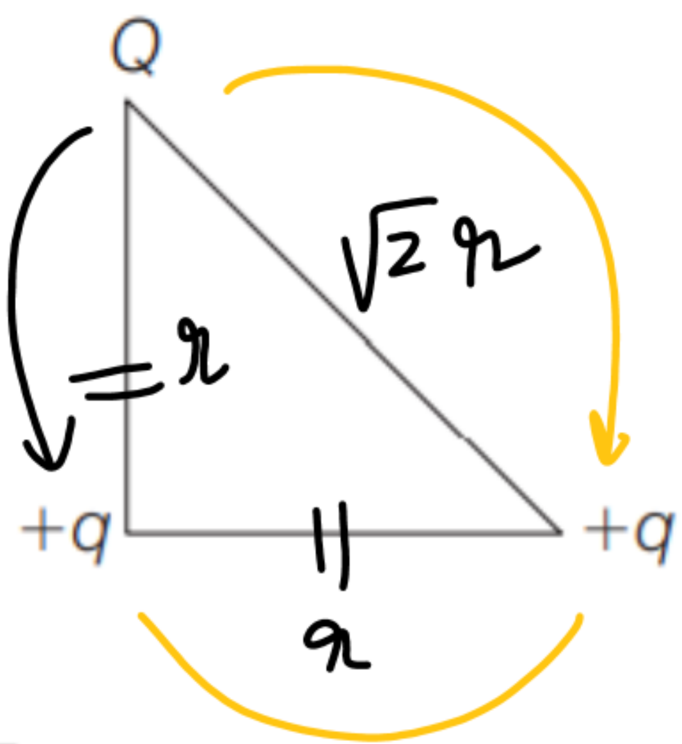
$$= -\left[\frac{A}{2}\left[(-5^2) - (1)^2\right] + B(-5-1)\right]$$

$$V_1 - V_2 = 10(24) + 10(-6)$$

$$= 240 - 60 = 180.$$

Three charges Q, $+q$ and $+q$ are placed at the vertices of a right angle isosceles triangle as shown below. The net electrostatic energy of the configuration is zero, if the value of Q is

(Main 2019, 11 Jan I)



(a) $-2q$

(b) $\dfrac{-q}{1+\sqrt{2}}$

(c) $+q$

(d) $\dfrac{-\sqrt{2}q}{\sqrt{2}+1}$

$$\frac{\cancel{K}Q\cancel{q}}{\cancel{r}} + \frac{\cancel{K}Q\cancel{q}}{\sqrt{2}\,\cancel{r}} + \frac{\cancel{K}q^{\cancel{2}}}{\cancel{r}} = 0$$

$$Q + \frac{Q}{\sqrt{2}} + q = 0$$

$$Q\left[\frac{\sqrt{2}+1}{\sqrt{2}}\right] = -q$$

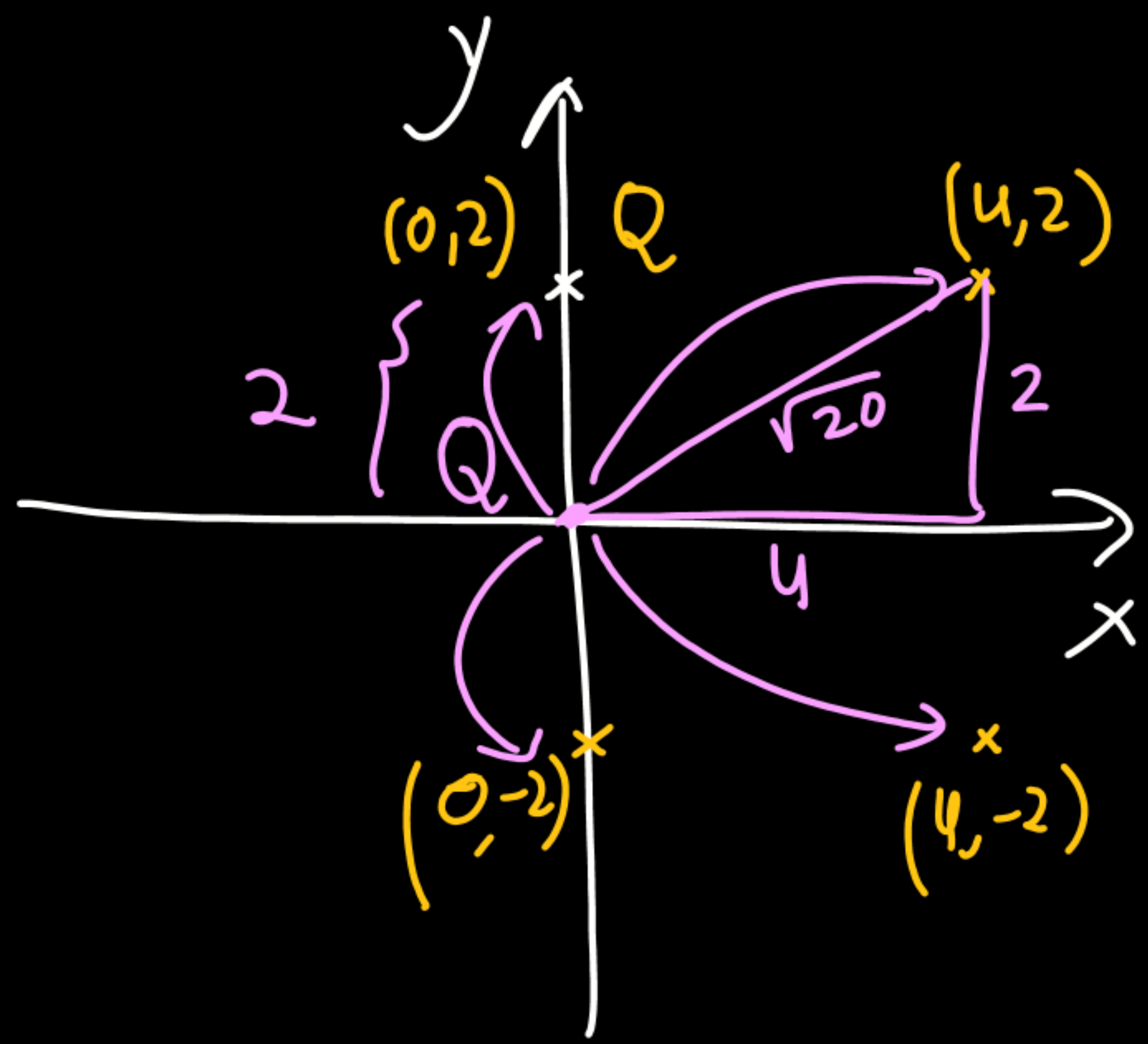
$$Q = -\frac{q\sqrt{2}}{\sqrt{2}+1}$$

Four equal point charges Q each are placed in the xy-plane at (0, 2), (4, 2), (4, –2) and (0, – 2). The work required to put a fifth charge Q at the origin of the coordinate system will be

(Main 2019, 10 Jan II)

(a) $\dfrac{Q^2}{4\pi\varepsilon_0}$ (b) $\dfrac{Q^2}{4\pi\varepsilon_0}\left(1+\dfrac{1}{\sqrt{3}}\right)$

(c) $\dfrac{Q^2}{2\sqrt{2}\pi\varepsilon_0}$ ~~(d)~~ $\dfrac{Q^2}{4\pi\varepsilon_0}\left(1+\dfrac{1}{\sqrt{5}}\right)$



$$\frac{KQ^2}{\cancel{2}} \times \cancel{2} + \frac{KQ^2}{\cancel{2}\sqrt{5}} \times \cancel{2}$$

$$\Rightarrow KQ^2\left(1+\frac{1}{\sqrt{5}}\right)$$

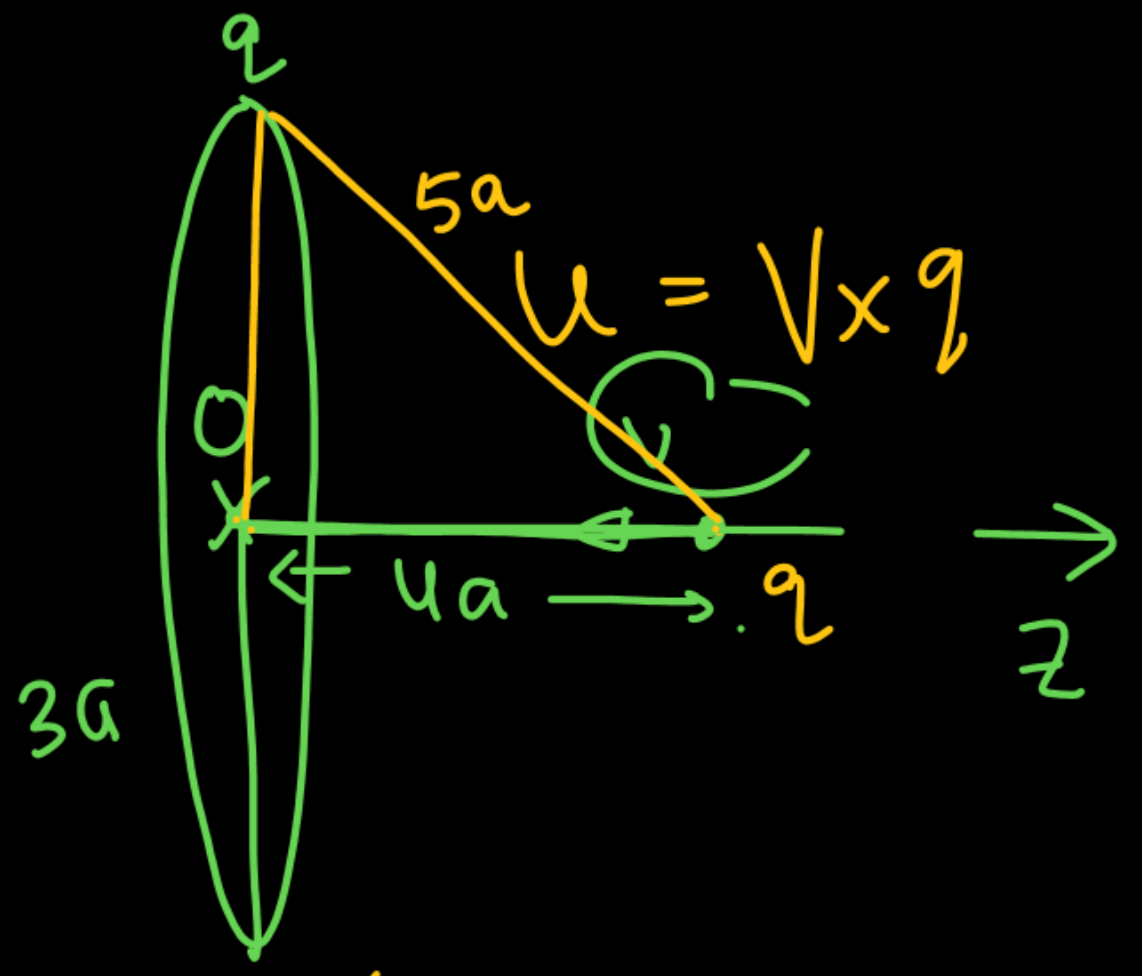
A uniformly charged ring of radius $3a$ and total charge q is placed in xy-plane centred at origin. A point charge q is moving towards the ring along the Z-axis and has speed v at $z = 4a$. The minimum value of v such that it crosses the origin is **(Main 2019, 10 April I)**

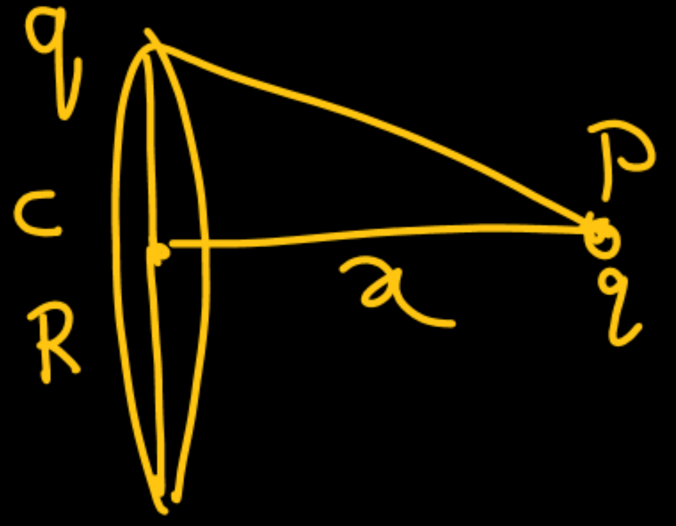
(a) $\sqrt{\frac{2}{m}}\left(\frac{1}{5}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

(b) $\sqrt{\frac{2}{m}}\left(\frac{4}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

(c) $\sqrt{\frac{2}{m}}\left(\frac{1}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

(d) $\sqrt{\frac{2}{m}}\left(\frac{2}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$





$V_c = \frac{Kq}{R}$

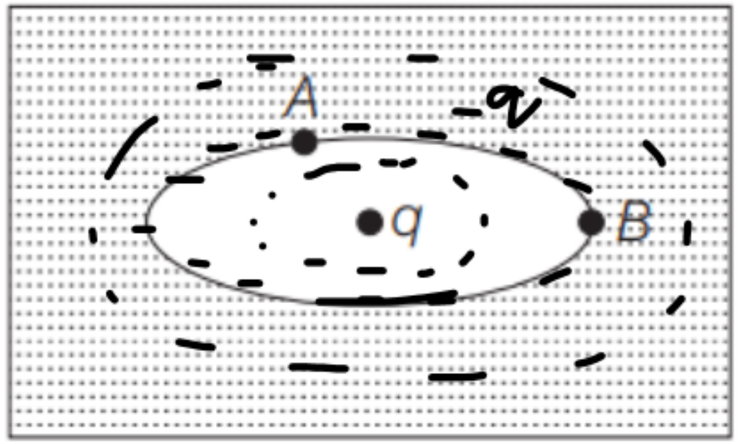
$V_p = \frac{kq}{\sqrt{R^2+x^2}}$

$\frac{1}{2}mv^2 + \left(\frac{Kq}{5a}\right)q = 0 + q\left(\frac{Kq}{3a}\right)$

$\frac{1}{2}mv^2 = \frac{Kq^2}{a}\left(\frac{1}{3}-\frac{1}{5}\right)$

$v^2 = \sqrt{\frac{kq^2 4}{15am}}$

An elliptical cavity is carved within a perfect conductor. A positive charge q is placed at the centre of the cavity. The points A and B are on the cavity surface as shown in the figure. Then **(1999, 3M)**

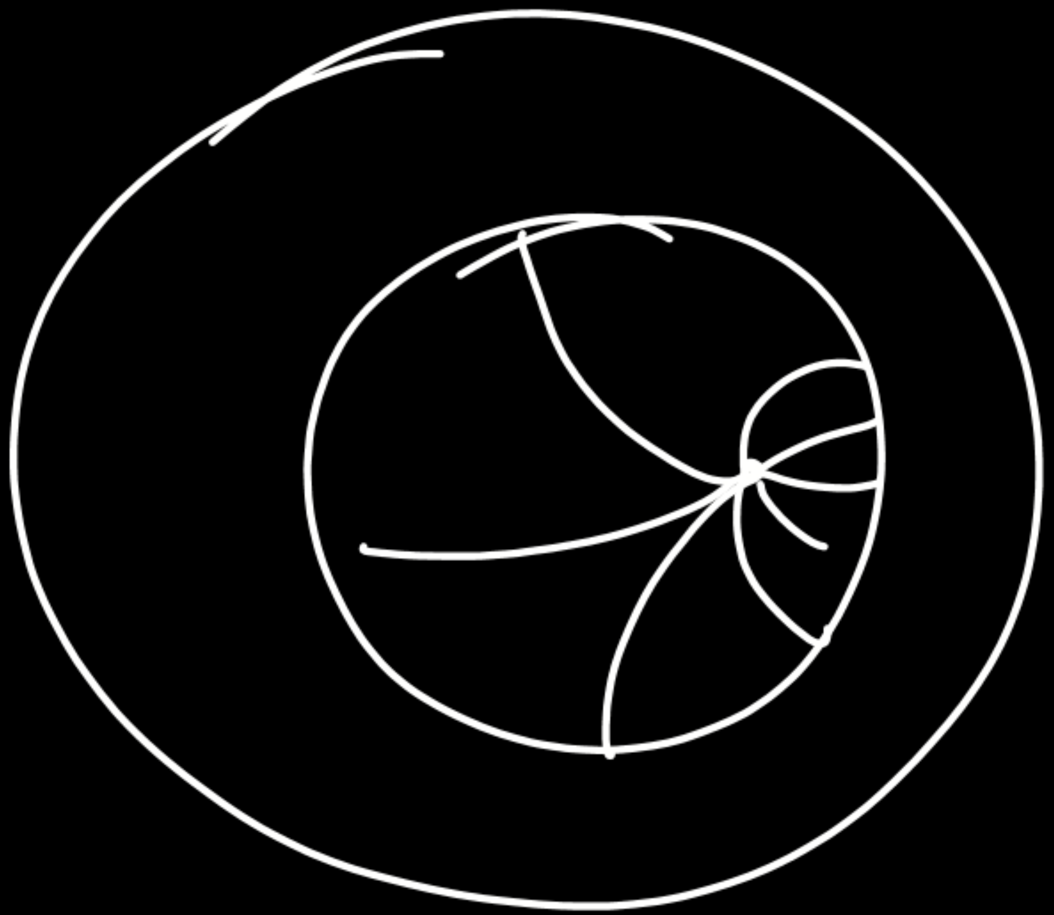


(a) electric field near A in the cavity = electric field near B in the cavity.

(b) charge density at A = charge density at B

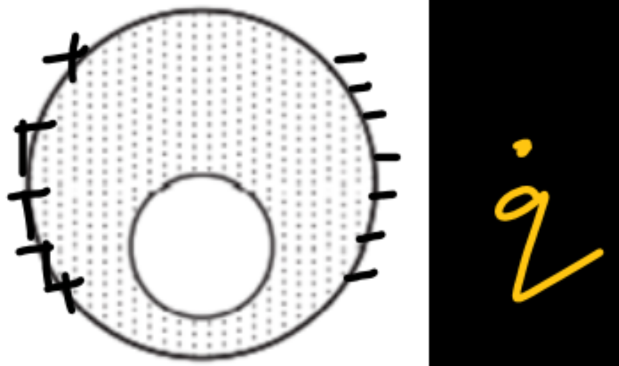
(c) potential at A = potential at B

(d) total electric field flux through the surface of the cavity is q/ε_0.



Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then

(2007, 3M)



(a) negative and distributed uniformly over the surface of the sphere

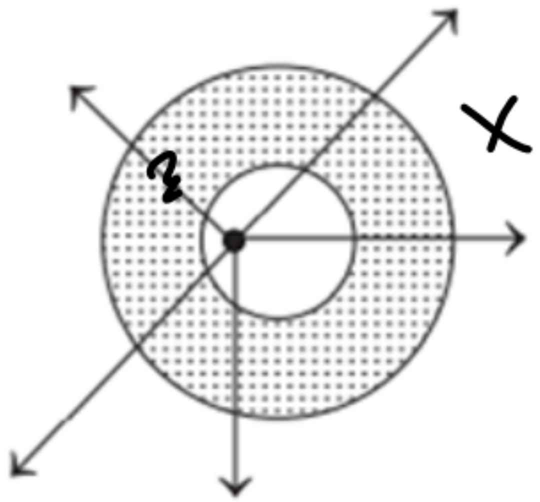
(b) negative and appears only at the point on the sphere closest to the point charge

(c) negative and distributed non-uniformly over the entire surface of the sphere

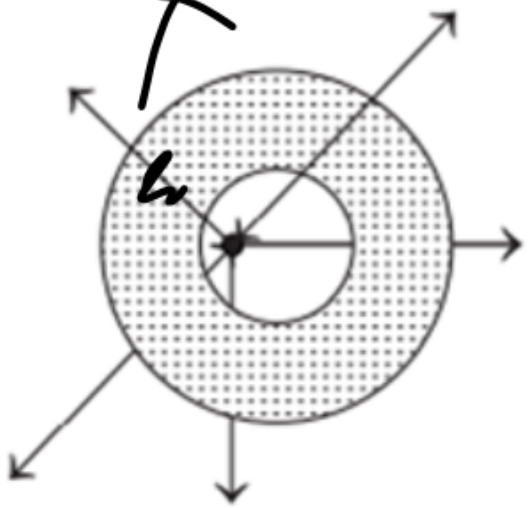
(d) zero

A metallic shell has a point charge q kept inside its cavity. Which one of the following diagrams correctly represents the electric lines of forces ? **(2003, 1M)**

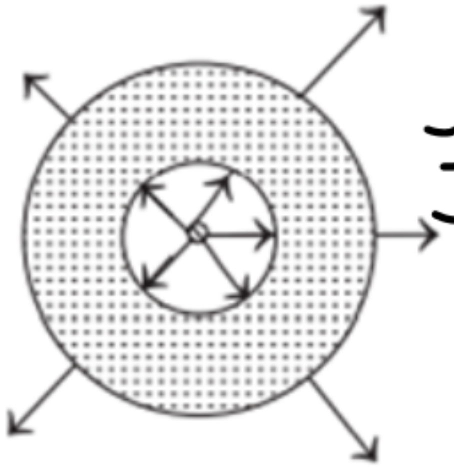
(a)



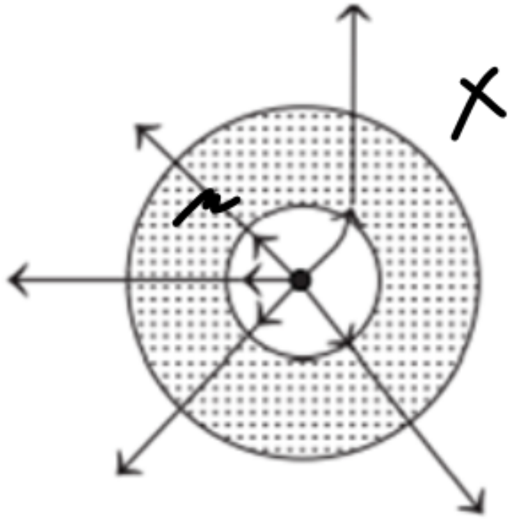
(b)



(c)

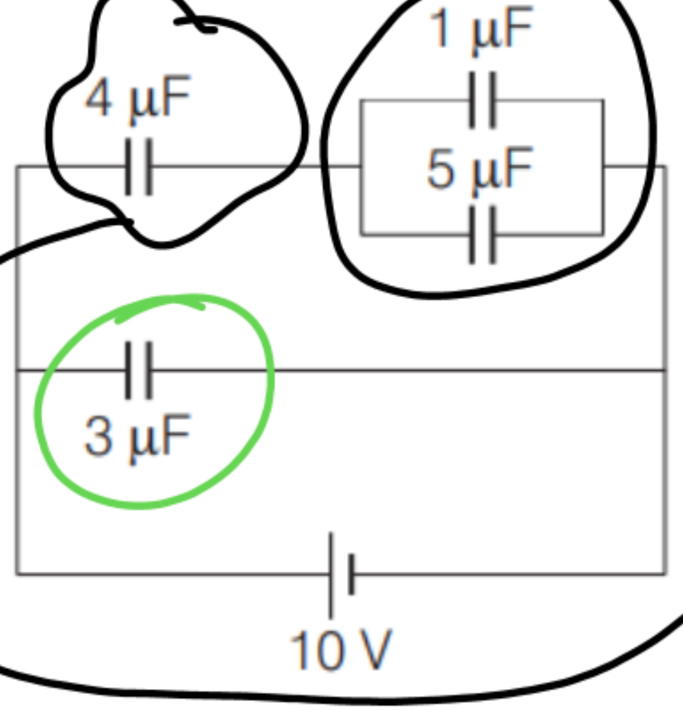


(d)



In the given circuit, the charge on 4 μF capacitor will be

(Main 2019, 12 April II)

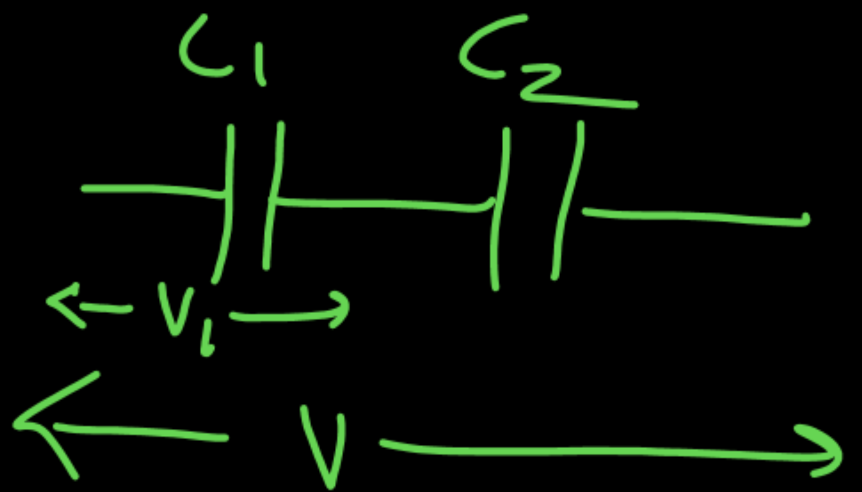


6

$$V_4 = \frac{10 \times 6}{6+4} = 6\text{ V}$$

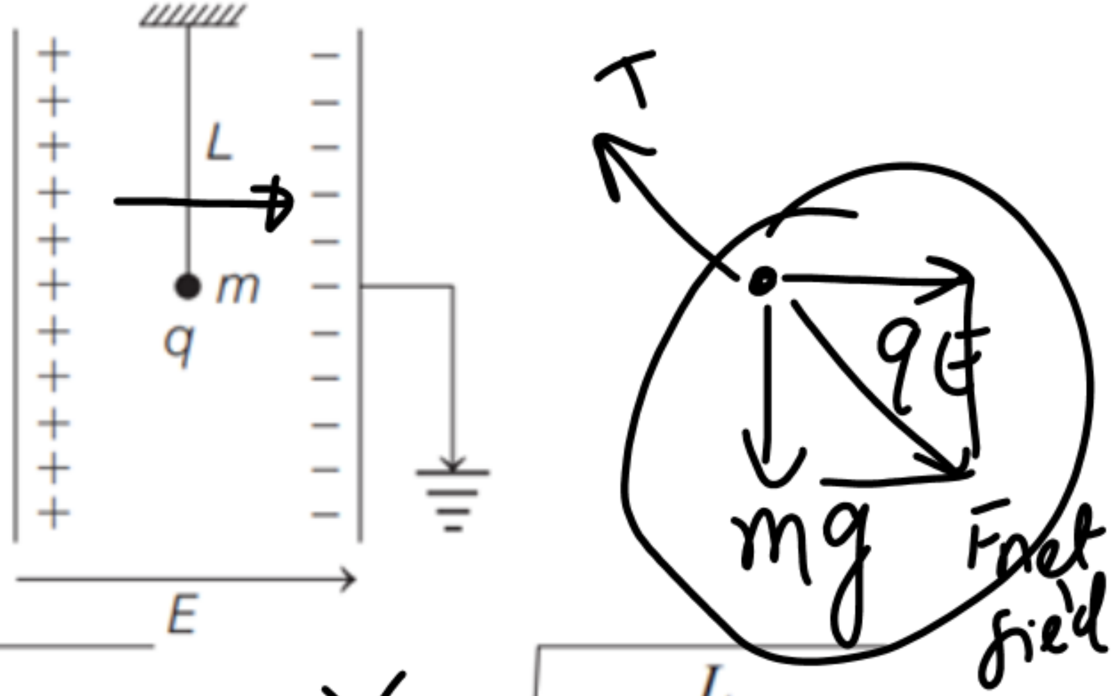
$$Q = CV = 4 \times 6 = 24\,\mu$$

(a) 5.4 μC (b) 9.6 μC (c) 13.4 μC (d) 24 μC



$$\boxed{V_1 = \frac{C_2 V}{C_1 + C_2}}$$

A simple pendulum of length L is placed between the plates of a parallel plate capacitor having electric field E, as shown in figure. Its bob has mass m and charge q. The time period of the pendulum is given by **(Main 2019, 10 April II)**



(a) $2\pi\sqrt{\dfrac{L}{\sqrt{g^2+\left(\dfrac{qE}{m}\right)^2}}}$

(b) $2\pi\sqrt{\dfrac{L}{\sqrt{g^2-\dfrac{q^2E^2}{m^2}}}}$

(c) $2\pi\sqrt{\dfrac{L}{\left(g+\dfrac{qE}{m}\right)}}$

(d) $2\pi\sqrt{\dfrac{L}{\left(g-\dfrac{qE}{m}\right)}}$

$T = 2\pi\sqrt{\dfrac{l}{g_{eff}}} = 2\pi\sqrt{\dfrac{l}{\sqrt{g^2+\left(\dfrac{qE}{m}\right)^2}}}$

$\vec{g}_{eff} = \dfrac{\text{Fnet field}}{\text{mass}}$

$= \dfrac{\sqrt{(mg)^2+(qE)^2}}{m}$

$g_{eff} = \sqrt{g^2+\left(\dfrac{qE}{m}\right)^2}$

The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is **(Main 2019, 9 April II)**

(a) $\frac{(n+1)V}{(K+n)}$ ✓

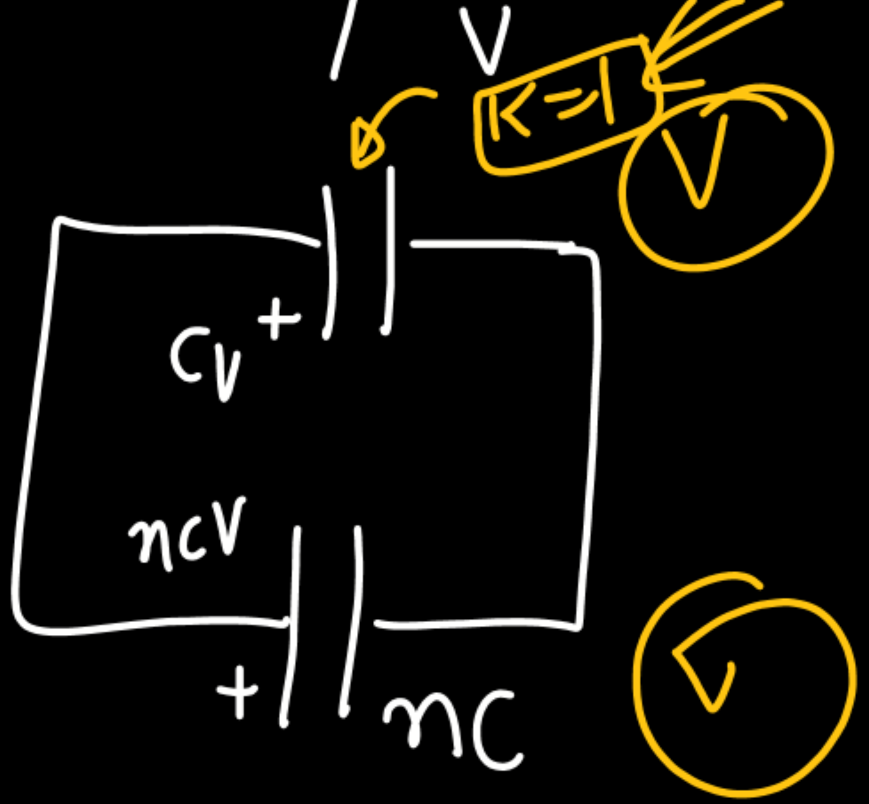
(b) $\frac{nV}{K+n}$

(c) V

(d) $\frac{V}{K+n}$

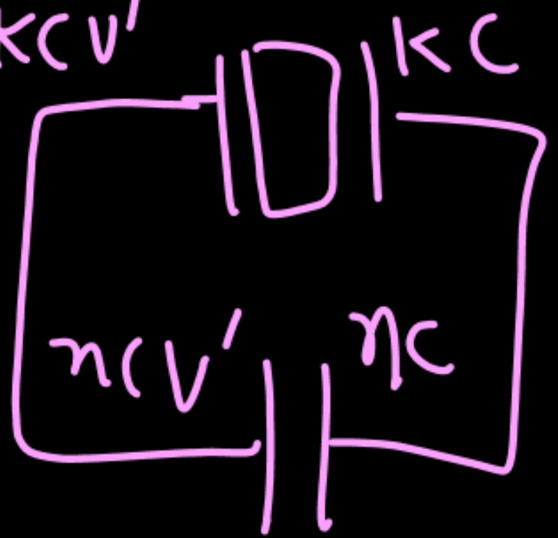
$K = 1$

$V' = V$



$K\cancel{C}V' + n\cancel{C}V' = n\cancel{C}V + \cancel{C}V$

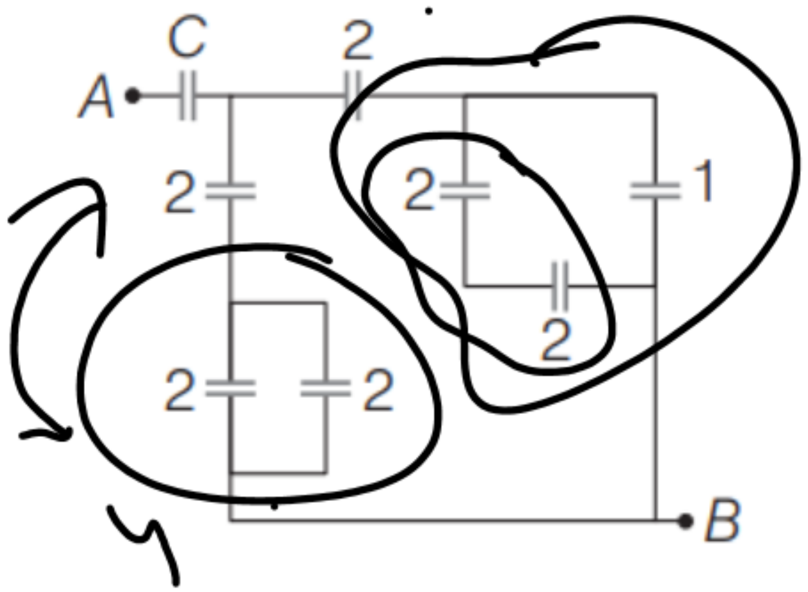
V'



$V' = \frac{(n+1)V}{K+n}$

In the circuit shown, find C if the effective capacitance of the whole circuit is to be $0.5\,\mu F$. All values in the circuit are in μF.

(Main 2019, 12 Jan II)

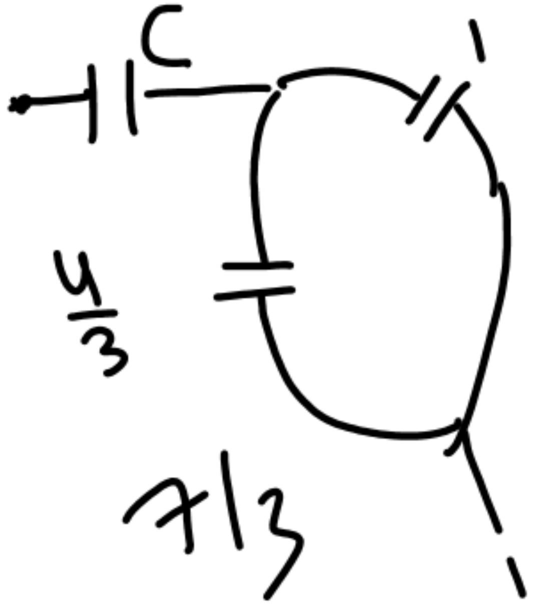


(a) $\frac{6}{5}\,\mu F$

(b) $4\,\mu F$

(c) $\frac{7}{10}\,\mu F$

(d) $\frac{7}{11}\,\mu F$



$$\frac{1}{C} + \frac{3}{7} = \frac{1}{0.5} = 2$$

$$\frac{1}{C} = 2 - \frac{3}{7}$$

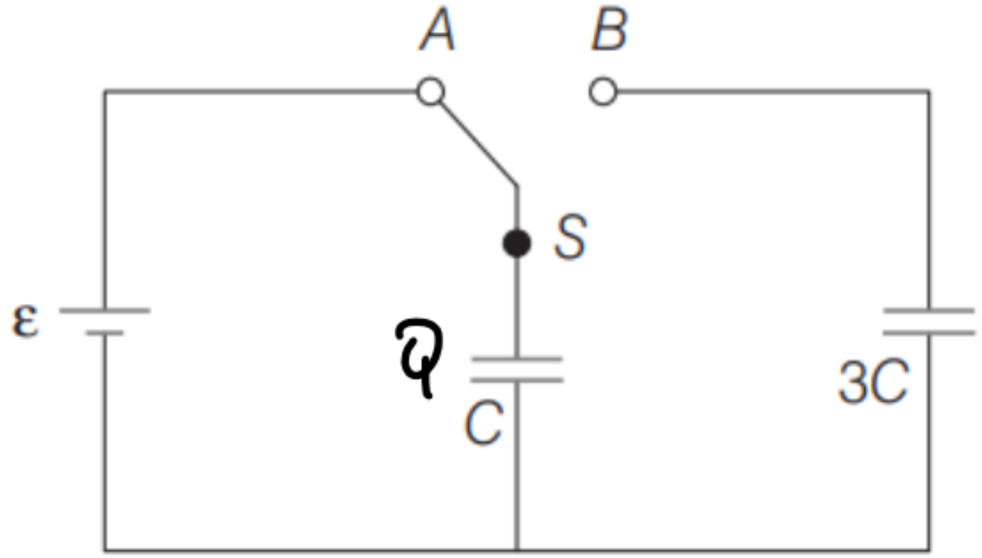
$$= \frac{14-3}{7}$$

$$= \frac{11}{7}$$

$$C = \frac{7}{11}$$

In the figure shown, after the switch 'S' is turned from position 'A' to position 'B', the energy dissipated in the circuit in terms of capacitance 'C' and total charge 'Q' is

(Main 2019, 12 Jan I)



$V = \frac{Q}{4C}$

$C_{eq} = 4C$

$U = \frac{1}{2} C_{eq} V^2$

(a) $\frac{3}{4} \cdot \frac{Q^2}{C}$ (b) $\frac{5}{8} \cdot \frac{Q^2}{C}$ (c) $\frac{1}{8} \cdot \frac{Q^2}{C}$ (d) $\frac{3}{8} \cdot \frac{Q^2}{C}$ ✓

$U_i = \frac{1}{2} \frac{Q^2}{C}$

$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$

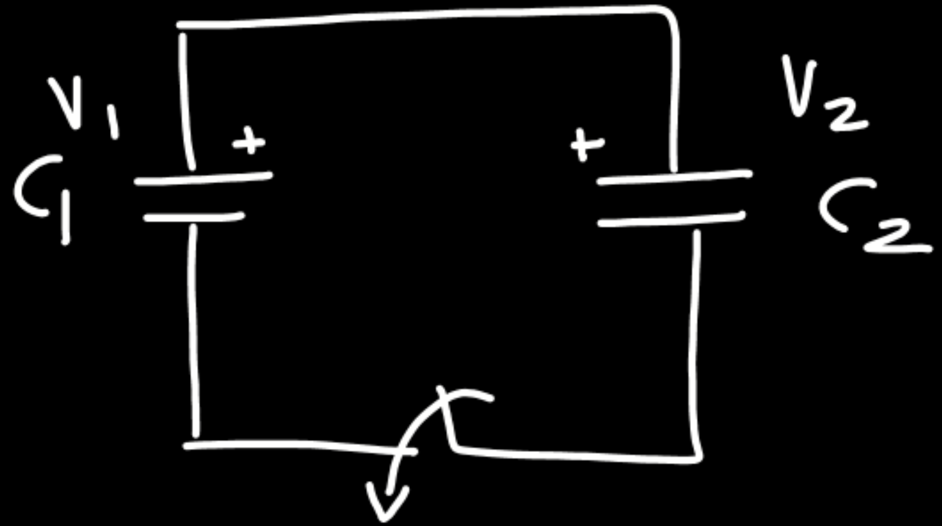
$V = \frac{\cancel{C} \frac{Q}{\cancel{C}} + 3C\, 0}{C + 3C}$

$\boxed{V = \frac{Q}{4C}}$

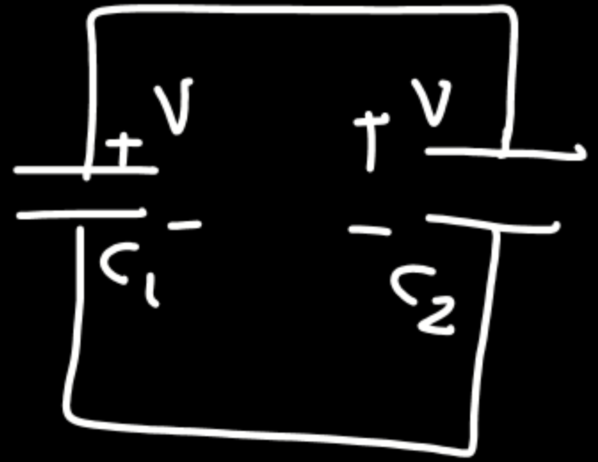
$U_f = \frac{1}{2}(4C)\left(\frac{Q}{4C}\right)^2$

$\frac{1}{8} \frac{Q^2}{C}$

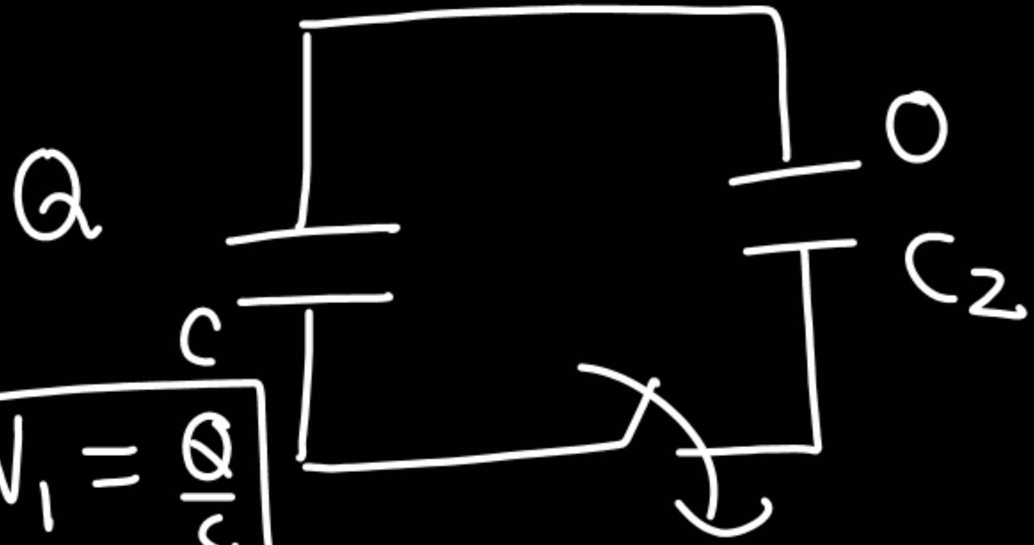
$\boxed{U_{loss} = U_i - U_f}$



$\Rightarrow$



$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

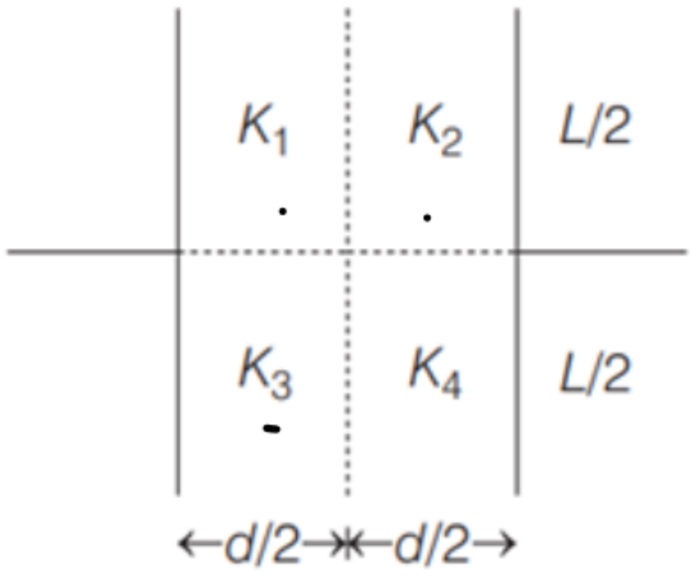


$$\boxed{V_1 = \frac{Q}{C}}$$

$$\Rightarrow V = \frac{\cancel{C}\,\frac{Q}{\cancel{C}} + C_2(0)}{C + 3C}$$

$$\boxed{V = \frac{Q}{4C}}$$

A parallel plate capacitor with square plates is filled with four dielectrics of dielectric constants K_1, K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be **(Main 2019, 9 Jan II)**



(a) $K = \dfrac{(K_1 + K_2)(K_3 + K_4)}{2(K_1 + K_2 + K_3 + K_4)}$

(b) $K = \dfrac{(K_1 + K_2)(K_3 + K_4)}{K_1 + K_2 + K_3 + K_4}$

$\frac{3 \times 8}{\cancel{6}\; 2}$

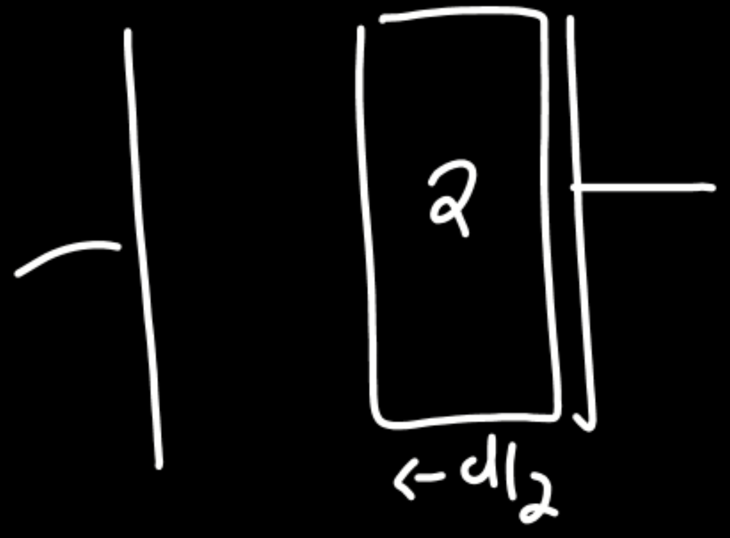
(c) $K = \dfrac{(K_1 + K_3)(K_2 + K_4)}{K_1 + K_2 + K_3 + K_4}$

(d) $K = \dfrac{(K_1 + K_4)(K_2 + K_3)}{2(K_1 + K_2 + K_3 + K_4)}$

Pehle series, fir parallel

K_1 & K_3 = 1

K_2 & K_4 = 2



$$\frac{\varepsilon_0 A}{\frac{d}{2\times 1} + \frac{d}{2\times 2}}$$

$$\frac{\varepsilon_0 A}{\frac{d}{2}\left(1+\frac{1}{2}\right)} = \frac{4}{3}\frac{\varepsilon_0 A}{d}$$

Thank You Lakshyians