Asymptotes $\iff$ Chapter 2.6

1 Asymptotes

Diagram

Goal: Come to terms with the infinite, ∞...

1.1 Intro and Definition

There are two kinds of asymptotes:

1. Vertical (either come from dividing by zero or taking $\ln(0)$. We've already seen these.
2. Horizontal Asymptotes. Look like

Diagram

Definition: A function f has a horizontal asymptote at L if the values of f get arbitrarily close to L as x gets very big or very small.

WRABD: What we actually mean by x gets very big or very small is that $x \to \infty$ or $x \to -\infty$. So, we actually are taking a limit to compute a horizontal asymptote

Definition: If $\lim_{x\to\infty} f(x) = L$ then we call L a horizontal asymptote. Similarly, if $\lim_{x\to-\infty} f(x) = L_2$ then we call L_2 a horizontal asymptote.

Fact: A function can have **at most 2** horizontal asymptotes (however a function can have infinitely many vertical asymptotes... can you think of such a function?).

1.2 Toolbox

Here are the barebones limits you need to know

1. $\lim_{x\to\pm\infty} \dfrac{1}{x^r} = 0$, for any constant $r > 0$
2. $\lim_{x\to-\infty} e^x = 0$
3. $\lim_{x\to\infty} \arctan x = \pi/2$, and $\lim_{x\to-\infty} \arctan x = -\pi/2$

Diagram

1.3 Some Examples

Example: Functions don't always have horizontal asymptotes. Consider the functions x^2 or $\ln(x)$.

Diagram

Example: "The Divide Out Method"... Consider $\lim_{x\to\infty} \dfrac{2x^2+2x+1}{5x^2+2}$.

Computation

In general given a rational functions $f(x) = \frac{p(x)}{q(x)}$ we divide out by $\frac{1}{x^n}$ with

$n =$ relative degree of $q(x) =$ (highest degree exponent in $q(x)$)·(fractional power of denominator).

Example: $q(x) = \sqrt[5]{4x^15 + 6x + 2}$ has relative degree $15 \cdot \frac{1}{3} = 3$.

Example: Squeeze Theorem. We can use the squeeze theorem to show that $\lim_{x\to\infty} \frac{\sin x}{x} = 0$.

Computation

Example: "You cannot treat ∞ like a number." Remember that ∞ is not a number, it is instead a symbol that represents the idea of unboundedness. So, we cannot treat ∞ and $-\infty$ like numbers. For example,

$$\lim_{x\to\infty} \sqrt{x^2+2x} - \sqrt{x^2+3x} \neq \infty - \infty = 0 \qquad \text{(Because can't use limit law in this case)}$$

Instead we need to manipulate (*get $-1/2$)

Computation