1. Find the solution to the following differential equation

$$\frac{dy}{dx} = \frac{x\sin(x^2)}{y}, \qquad y(0) = -2.$$

Hint: $t^2 = c$ *implies* $t = \pm\sqrt{c}$. Cross multiply to obtain the integral equation

$$\int y\,dy = \int x\sin(x^2)dx.$$

Using the u-sub $u = x^2$ this gives

$$\frac{1}{2}y^2 = -\frac{1}{2}\cos(x^2) + C.$$

Solving for y we get

Text

$$y = \pm\sqrt{\cos(x^2) + C}.$$

Since $y(0)$ is negative we take the negative solution

$$y = -\sqrt{-\cos(x^2) + C}.$$

If we solve for C using $y(0) = -2$ we get $C = 5$ and so

$$y = -\sqrt{5 - \cos(x^2)}.$$

2. A tank initially contains 1000 L of pure water. Brine that contains 0.07 kg of salt per liter of water enters the tank at a rate of 5 L/min. In addition, brine that contains 0.04 kg of salt per liter of water enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at a rate of 15 L/min. Find the amount of salt (in kg) in the tank as a function of time t in minutes.

Let $y(t)$ represent the amount of kg of salt in the vat. So $c(t) = \frac{y(t)}{1000}$ is the concentration. We know that the concentration is increasing at the rate

$$\text{Rate In} = \frac{[5(.07) + 10(.04)]}{1000}.$$

And the concentration is diminishing at the rate

$$\text{Rate Out} = \frac{15 \cdot c(t)}{1000}.$$

We then have

$$\frac{dc}{dt} = \frac{[5(.07) + 10(.04)]}{1000} - \frac{15 \cdot c(t)}{1000}.$$

Solve the resulting differential equation to get

$$c(t) = \frac{.75}{1000}(1 - e^{-.015t}).$$

Since $y(t) = 1000c(t)$ we multiply by 1000 and obtain

$$y(t) = 50(1 - e^{-.015t}).$$