HOMEWORK 7 **Chapter 7:** 5, 11, 15, 21

5.

Ans.

(a)	A	B	C
A	0	1/2	1/2
B	3/4	0	1/4
C	3/4	1/4	0

(b) At time 2, A has probability 3/4, while B and C have probability 1/8 each. The probability of B at time 3 is then $(3/4)(1/2)+(1/8)(0)+(1/8)(1/4) = 13/32$.

11.

Ans. (a) 0.55, (b) 0.575, (c) 0.6

15.

Ans. 38%, 25%, $8/33 = 24\%$

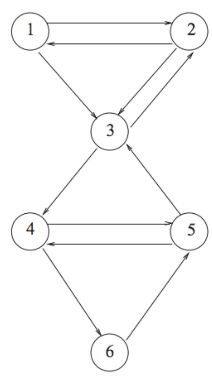
21.

Ans. The transition probability is

	H	M
H	.4	.6
M	.7	.3

By our formula the long run probability of H is $.7/(.7 + .6) = 7/13$.

1. Matt's website consists of six pages linked to one another as depicted below:



Arrows indicate which pages have a link pointing to others.

(a) Suppose a person clicks links randomly. Write the transition matrix, p, for this Markov chain.

Solution:

$$p = \begin{pmatrix} 0.0 & 0.5 & 0.5 & 0.0 & 0.0 & 0.0 \\ 0.5 & 0.0 & 0.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.5 & 0.0 & 0.5 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.5 & 0.5 \\ 0.0 & 0.0 & 0.5 & 0.5 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 \end{pmatrix}$$

(b) Here is p^{10}:

$$p^{10} = \begin{pmatrix} 0.076 & 0.151 & 0.225 & 0.22 & 0.22 & 0.108 \\ 0.075 & 0.152 & 0.225 & 0.22 & 0.22 & 0.108 \\ 0.076 & 0.148 & 0.223 & 0.225 & 0.217 & 0.111 \\ 0.073 & 0.146 & 0.217 & 0.22 & 0.228 & 0.116 \\ 0.073 & 0.149 & 0.225 & 0.228 & 0.22 & 0.108 \\ 0.076 & 0.141 & 0.223 & 0.217 & 0.232 & 0.111 \end{pmatrix}.$$

Suppose the clicker starts at page 3. What is the probability the random clicker is at page 5 after 10 clicks?

Solution:
$p^{10}(3,5) = .217$.

(c) The vector

$$\pi = \left[\frac{2}{27}, \frac{4}{27}, \frac{6}{27}, \frac{6}{27}, \frac{6}{27}, \frac{3}{27}\right]$$

satisfies $\pi p = \pi$. (This was found using a computer.) Which site is least likely to be occupied after 1000 clicks?

Solution: Site 1.

(d) Which sites are most likely to be occupied?

Solution: Sites 3,4,5

2. Gary is gambling and starts with **\$2**. He is flipping a fair coin and receives \$1 if he flips heads and loses \$1 if he flips tails. He stops playing when he has no money. Let X_n be the amount of flips for which he has n dollars.

(a) (4 points) Explain why X_2 is a geometric random variable. Find its parameter, p.

Solution: After each flip at \$2 Gary will either go broke or return to having \$2. If he returns the probability of going broke is the same. We have p_2 is the probability he flips tails twice in a row. This is $\frac{1}{4}$.

(b) (2 points) For $n \geq 2$, what is $P(X_n > 0)$? (Hint: Gambler's ruin.)

Solution: $\frac{2}{n}$.

(c) (3 points) Suppose Gary has reached n dollars. What is the probability, q, he goes broke before visiting n again?

Solution: He has to flip tails, then not return to n. Again this is gambler's ruin but this time the picture is reversed. So we have $\frac{1}{2}\frac{1}{n}$. The 1/2 is for flipping the first tails.

(d) (3 points) You have just shown that when Gary reaches $n \geq 2$ he visits it a Y = geometric(q) number of times. Use (b) and (c) to compute EX_n.

Solution:

$$EX_n = E[X_n \mid X_n > 0]P(X_n > 0) = E[X_n \mid X_n > 0]\frac{2}{n} = 2\frac{EY}{n} = 4.$$

(e) What is EX_n if instead Gary starts with $k \leq n$ dollars?

Solution: Now we have $P(X_n > 0) = \frac{k}{n}$, so we multiple by a factor of k. Thus, $EX_n = 2k$.